Far-from equilibrium 1D bose gases from Kibble-Zurek to NTFPs

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Attractors and thermalization in nuclear collisions and cold quantum gases
24-09-2025 @ ECT*

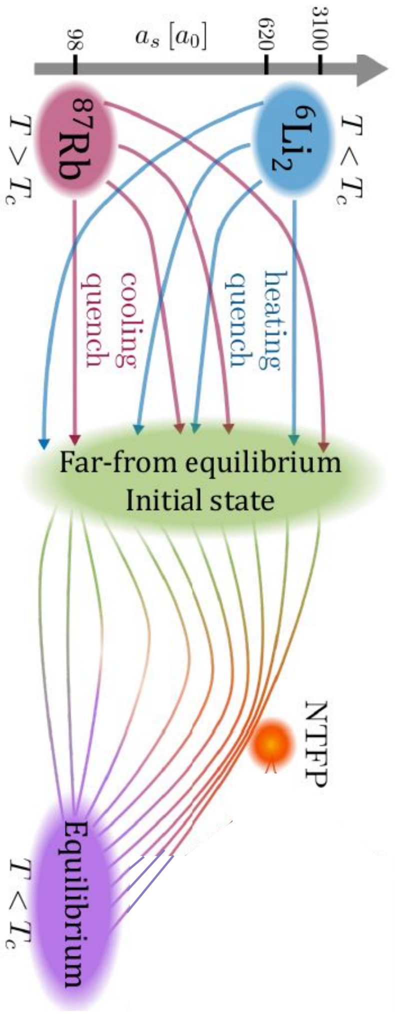
From theory to experiments

If you cannot measure it, it’s not physics

(paraphrased: William D. Phillips, FQMT’19)

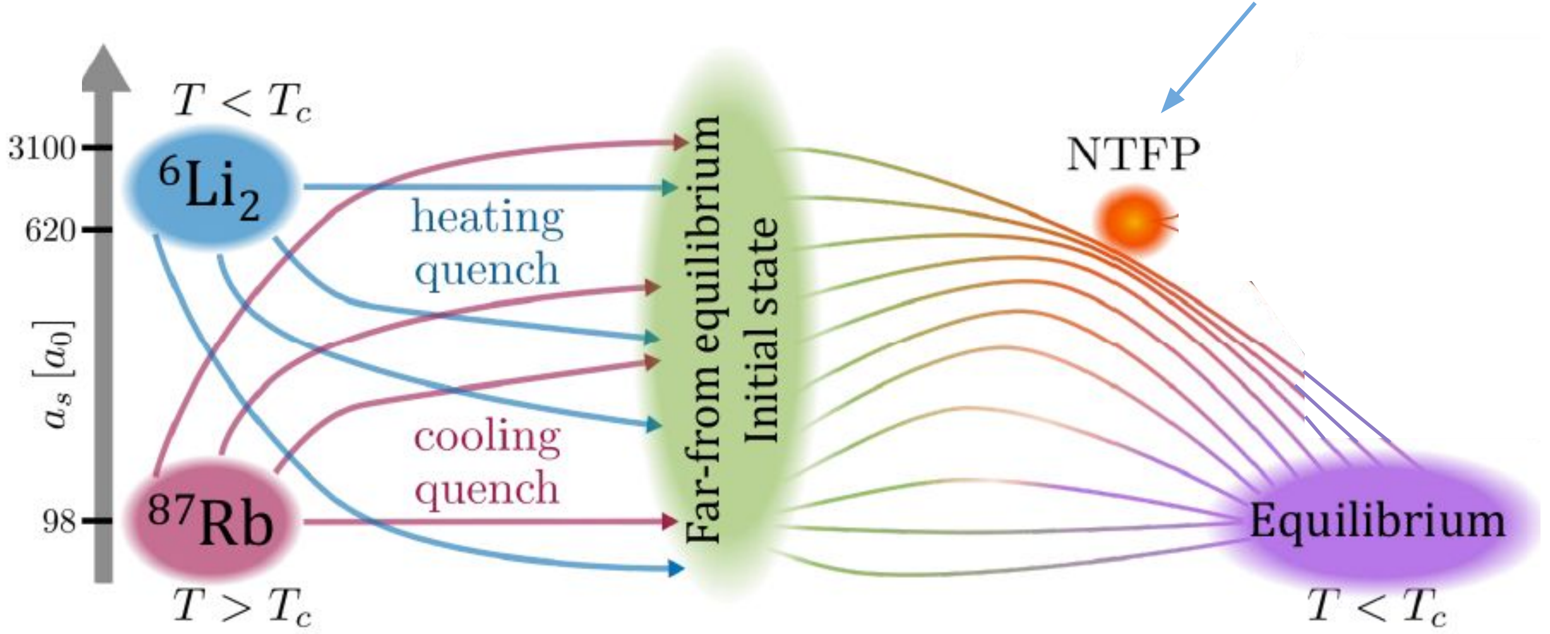
Outline

- ❑ *Cooling and heating quenches in ^{87}Rb and $^{6}Li_2$ bose gases*
- ❑ *Kibble-Zurek mechanism <-> short times and quench dynamics*
- ❑ *Non-thermal fixed points <-> intermediate times and relaxation*
- ❑ *Strongly interacting systems <-> universality of 1D attractor*
- ❑ *Outlook & open questions*



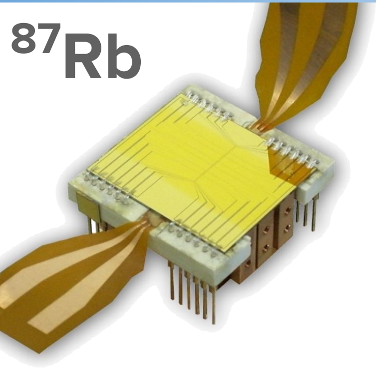
Outline

Universal attractor for
quasi-1D bosonic systems
far from equilibrium



Cold atom systems

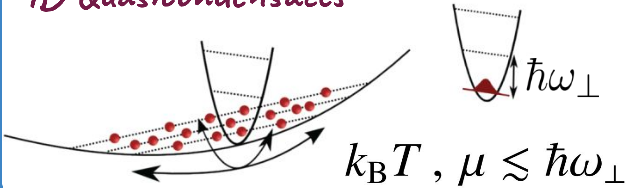
^{87}Rb



AtomChip Integrated Circuits for Ultracold Quantum Matter

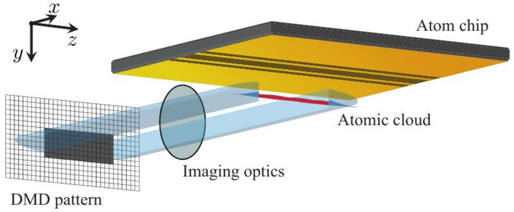
Combine the robustness of nano-fabrication and the quantum tools of atomic physics and quantum optics to build quantum experiments

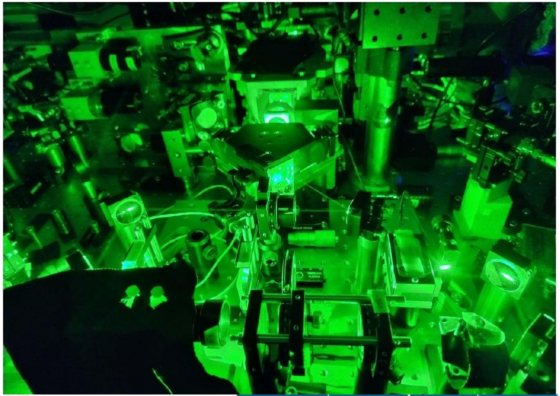
1D Quasicondensates



Tailored potential

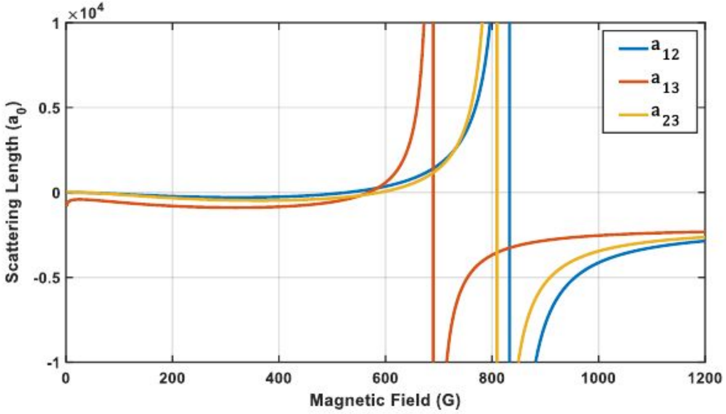
DMD potential painting





^{6}Li$_{(2)}$

Channel	B_0 (G)	Δ (G)	a_{bg} (a_0)
|1⟩|2⟩	834.1	-300	-1405
|1⟩|2⟩	543.25	0.1	60
|1⟩|3⟩	690.4	-122.3	-1727
|2⟩|3⟩	811.2	-222.3	-1490



Feshbach Resonance

➡ interaction control

C. Chin et al., Rev. Mod. Phys. 82, 1225 (2010)
G. Zürn et al., Phys. Rev. Lett. 110, 13530 (2013)

Long lived Feshbach molecules

➡ stable at strong interactions

Petrov, D. S., C. Salomon, G. V. Shlyapnikov
Phys. Rev. Lett. 93, 090404 (2004)
Phys. Rev. A 71, 012708 (2005)

Matter wave focusing

➡ Momentum measurement

I. Shvarchuck et al., PRL 89, 270404 (2002)
P. A. Murthy et al., Phys. Rev. A 90, 043611 (2014)

What can we measure in experiments?

Commonly: **Destructive measurements** ➡ The best we can **measure** is **every constituent** (and their internal states)

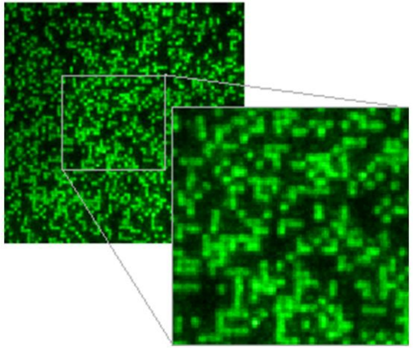
➡ Markus talk yesterday

Experiment → $\mathcal{U}$ → **'image'** → **Evaluation**

Quench, … & isolated evolution — *Unitary operation* — *Projective measurement of many body wavefunction*

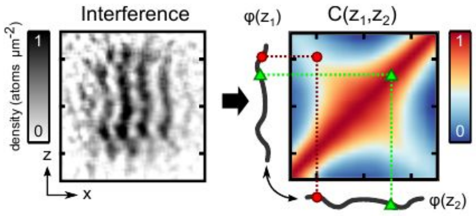
Extracted observables, full counting statistics, (higher-order) correlations, …

in situ → position



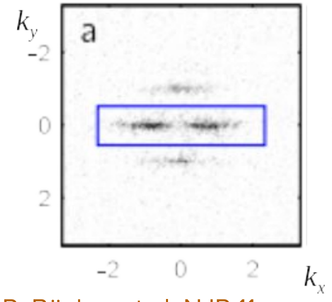
W.S. Bakr, et al.,Nature 462, 74 (2009).
J.F. Sherson, et al. Nature 467, 68 (2010).

interference → phase



Langen et al., Nat. Phys. 9, 460 (2013)
Gring et al., Science 337, 1318 (2012)
Langen et al., Science 348, 207 (2015)
Schweigler et.al, Nature 545, 323, (2017)
Rauer et al., Science 360, 307 (2018)
Zache et.al, PRX 10, 011020 (2020)
…

time of flight → momentum



R. Bücker et al. NJP 11, 103039 (2009)
R. Bücker et al. Nature Physics 7, 608 (2011)

$$n(k,t) \equiv \alpha\rho\left(\alpha^{-1} z_{\mathrm{tof}}\right)$$

$$= \int \mathrm{d}z\, f\left(z, k - \alpha^{-1}(z + v t_{\mathrm{tof}}), t\right)$$

Additional bulk velocity v

→ *condensate focusing*

Enables measuring momenta in IR for finite time of flight

New: **Partial/Continuous measurements:** M. Prüfer et al Phys. Rev. Lett. **133**, 250403 (2024), C. Gooding et al arXiv:2508.01080 (2025)

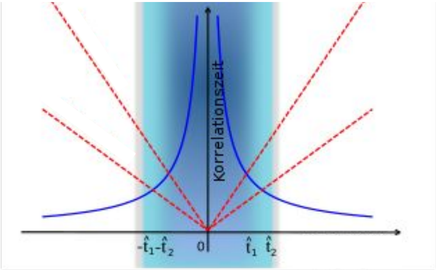
Cooling quenches and KZ mechanism

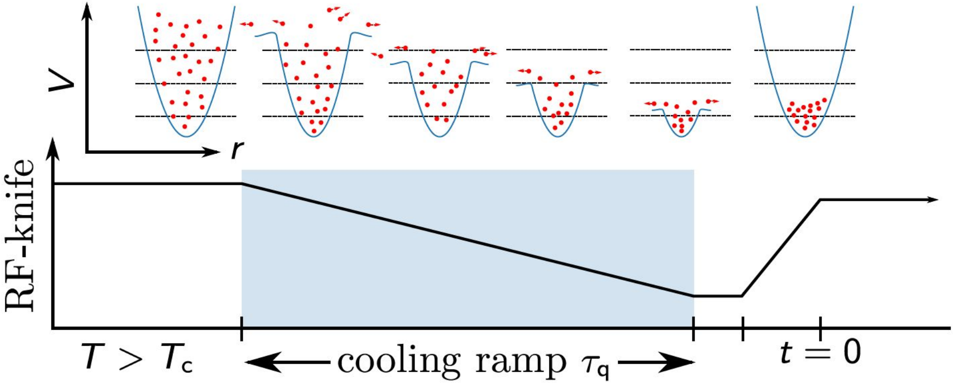
Critical slowing down near a 2nd-order phase transition

$$\tau = \tau_0/|\epsilon|^{\nu z}$$

$$\xi = \xi_0/|\epsilon|^{\nu}$$

$$\epsilon = \frac{T_C - T}{T_C}$$





→ system departs from equilibrium near the critical point (due to causality / sonic horizons)

→ *new broken symmetry phase is chosen locally*

→ *for transition at finite rate, 'causality' limits establishment of coherence over large scales*

→ *nucleation of defects between different regions*

Defect density predicted by universal (inhomogeneous) **Kibble-Zurek scaling:**

$$n_s \sim R_q^{\frac{1+2\nu}{1+\nu z}}$$

Theory: Kibble 1980, Zurek 1993, … e.g. Review: Int. J. Mod. Phys. A 29, 1430018 (2014) for 1d (solitons): e.g. Zurek PRL 102, 105702 (2009)

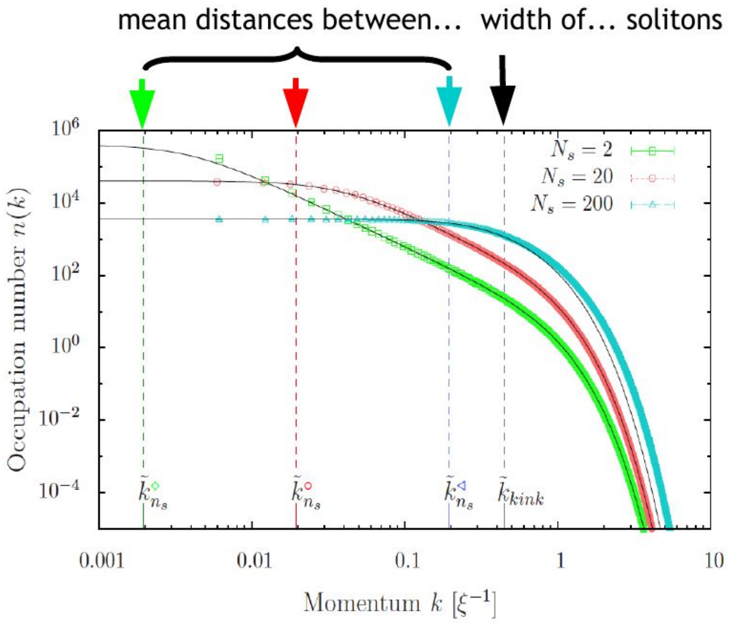
Experiments: Donner et al Science 315, 1556 (2007), Kessling et al Nature 568, 207 (2019), Navon et al Science 347, 167 (2015), Lamporesi et al Nat. Phys. 9, 656 (2013), ...

Solitonic defects

M. Schmidt, SE, et al., New J. Phys. 14, 075005 (2012)
SE, 10.11588/heidok.00025106 (2018)

Counting defects: ➔ very subjective (especially for solitons)
➔ requires additional 'waiting time'
➔ non-linear relaxation before measurement

Better: ➔ determine defects through ***correlation measures*** (correlation length, momentum distribution, …)
➔ enables measurement directly following the quench



→ *Random defect model for solitonic excitations*

$$n(k) = \int \mathrm{d}n_s\, p(n_s)\, [n_0(k,T) * n_s(k,n_s)]$$

$$n_s(k) = \left[\frac{k/k_{\xi_s}}{\sinh(k/k_{\xi_s})}\right]^2 n_{\xi_s \to 0}(k)$$

$$g_1(z_1,z_2) = n_{1D}\left|1 - \frac{2}{L}\,\mathcal{I}_L(z_1,z_2)\right|^{N_s}$$

$$\mathcal{I}_L = \frac{L}{2}\int_{(z_1,-1)}^{(z_2,+1)} \mathrm{d}z\,\mathrm{d}v\, P(z,v)\left[1 - \mathrm{e}^{\mathrm{i}\beta(v)}\right] = L\chi \int_{z_1}^{z_2} \mathrm{d}z\, P_1(z)$$

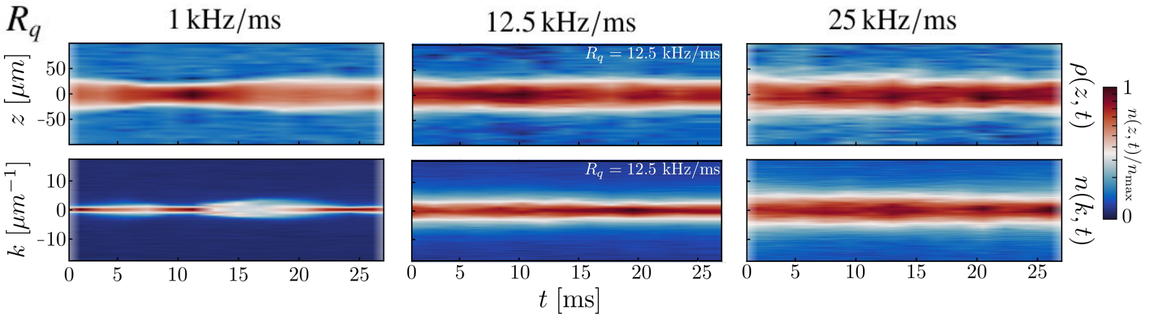
mean *density of defects*

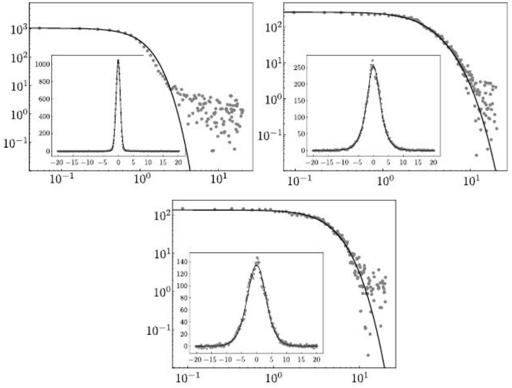
$$n_s(k) = \left[\frac{2n_{1D}k_{n_s}}{k_{n_s}^2 + (k-k_0)^2}\right]\left[\frac{k/k_{\xi_s}}{\sinh(k/k_{\xi_s})}\right]^2$$

mean *width of defects*

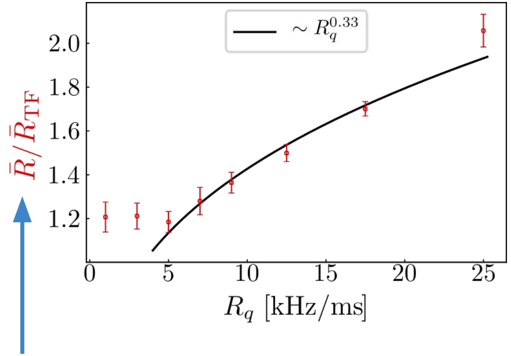
Soliton defect nucleation

Increasing quenchrate



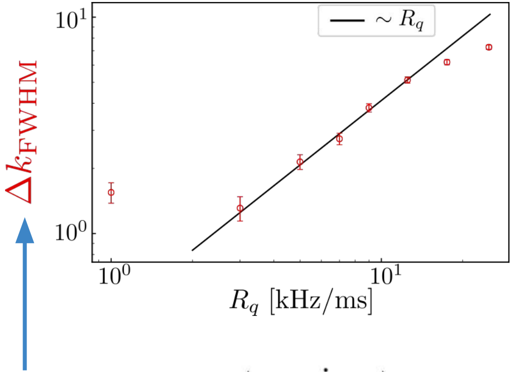


Strongly broadened density and momentum distribution



Via scaling Ansatz

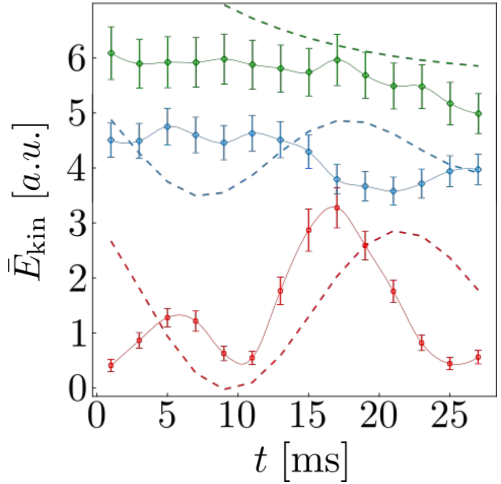
$$\rho(z,t) = b(t)^{-1}\rho_0(z/b(t))$$



measured at focus point

$$\left(1 + \frac{\dot{b}}{b} t_{tof}\right) = 0$$

Condensate 'stiffness'



'stiff'

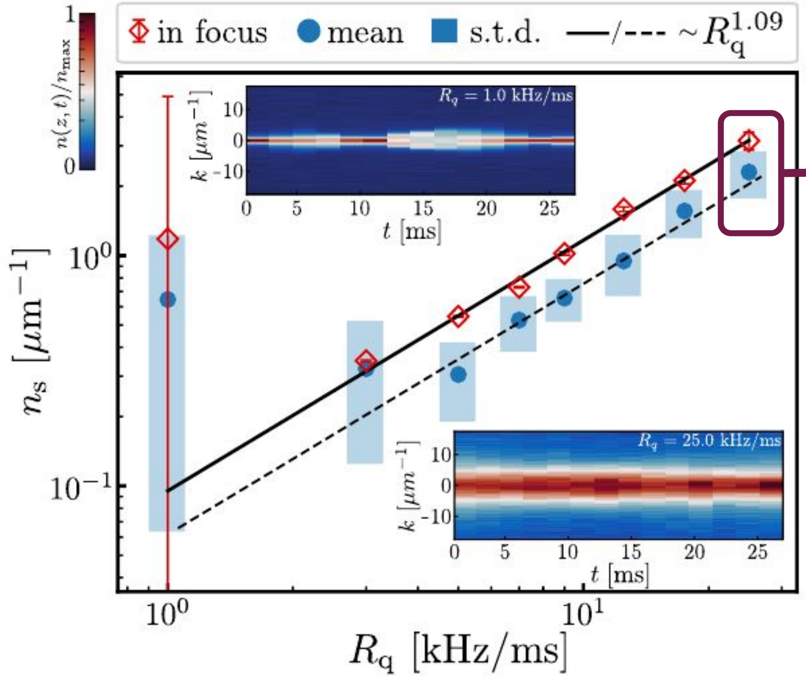
no-doubling

frequency doubling

Increasing quenchrate

Close to equilibrium see e.g.: Fang et al, PRL 113, 035301 (2014)

Reaching the far from equilibrium regime



Experiment

$\zeta = 1.09 \pm 0.04$

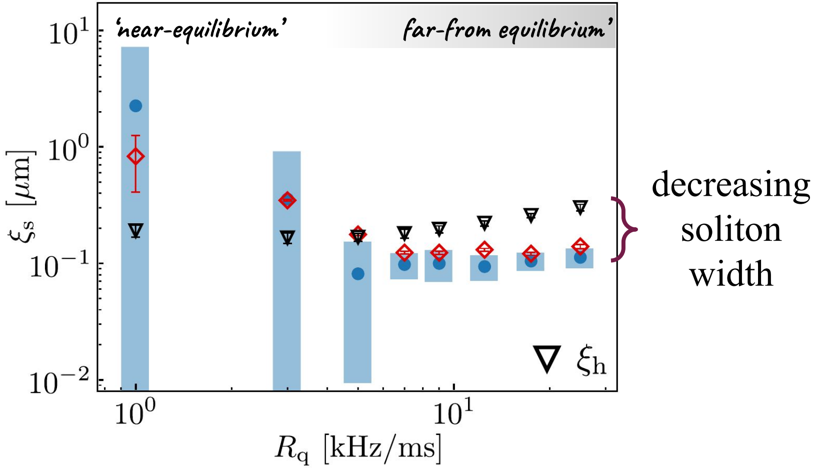
$\zeta = 1.20 \pm 0.07$

Theory

$\zeta = 1$ (MF)

$\zeta = 7/6$ (Model F)

- ➔ **KZ** scaling prediction valid for **almost instantaneous quenches** (~O(100) faster than typical experiments)
- ➔ Strong **overpopulation** of **high-momentum modes**
- ➔ **Very high density of defects** leads to **deformation of solitons**



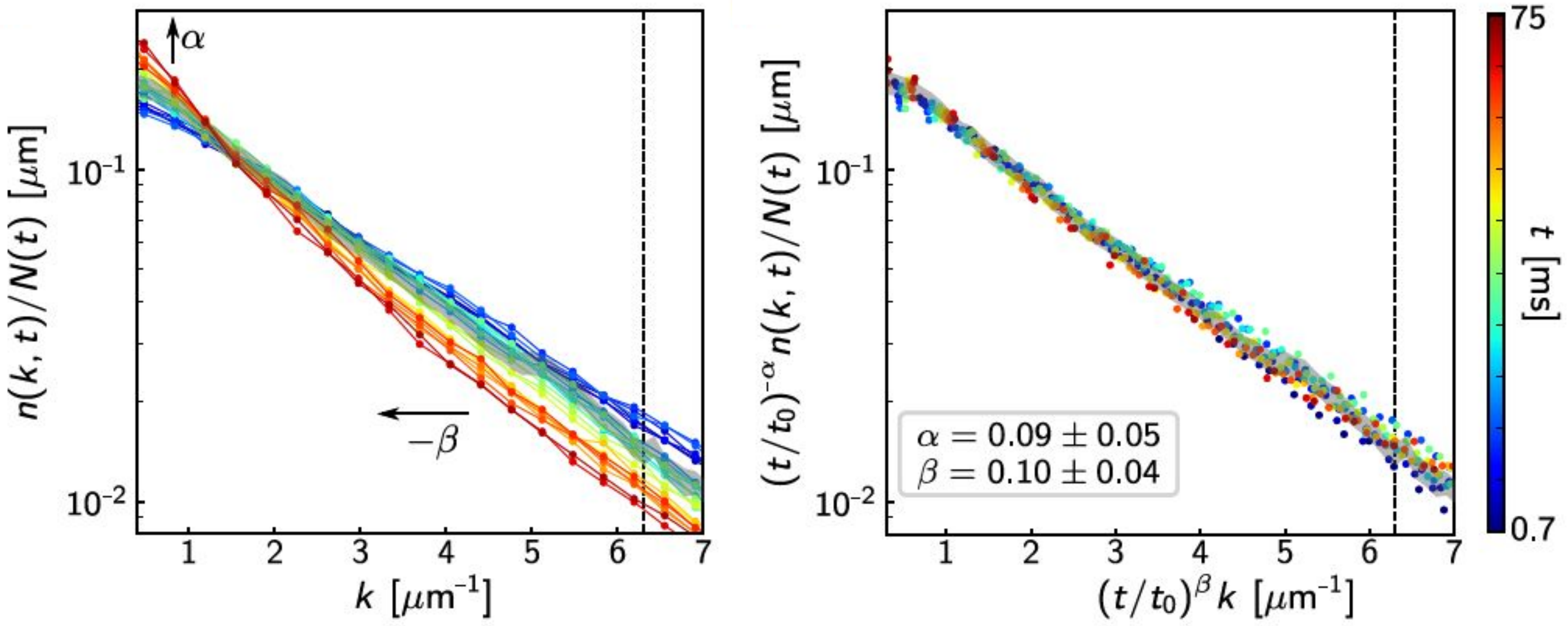
→ reaching far from equilibrium initial conditions

Scaling evolution

SE, R. Bücker, T. Gasenzer, J. Berges, J. Schmiedmayer Nature 563, p 225–229 (2018)

Following the scaling ansatz

$$n(k,t) = (t/t_0)^{\alpha} f([t/t_0]^{\beta} k)$$

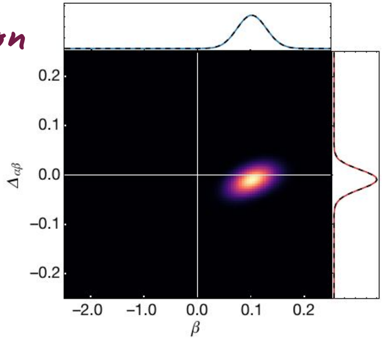


Scaling exponents

Determine via *maximum likelihood function*

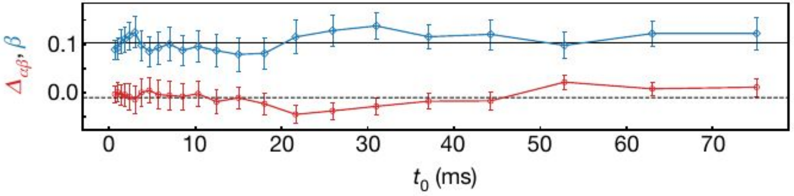
$$L(\Delta_{\alpha\beta}, \beta) = \exp\left[-\frac{1}{2}\chi^2(\Delta_{\alpha\beta}, \beta)\right]$$

$$\chi^2(\alpha, \beta) = \frac{1}{N_t^2}\sum_{t,t_0}^{N_t} \chi^2_{\alpha,\beta}(t, t_0)$$



$$\alpha \approx \beta = 0.1 \pm 0.03$$
$$\Delta_{\alpha\beta} = \alpha - \beta = -0.01 \pm 0.02$$

Independent of reference time

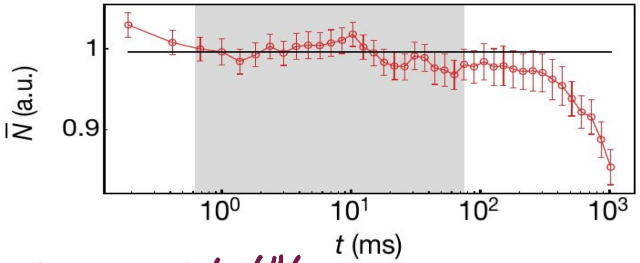


Scaling for *moments of the distribution function*

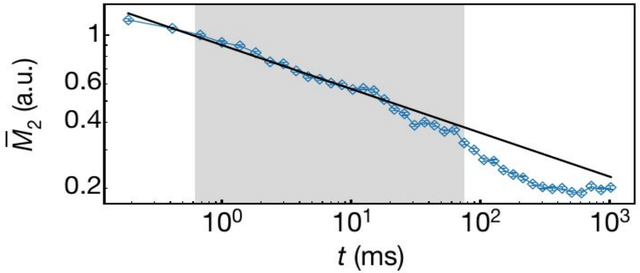
$$\bar{N} = \int_{|k| \le (t/t_0)^{-\beta} k_S} \frac{n(k,t)}{N(t)} \mathrm{d}k \qquad \propto (t/t_0)^{-\Delta_{\alpha\beta}}$$

$$\bar{M}_{n\ge 1} = \int_{|k| \le (t/t_0)^{-\beta} k_S} \frac{|k|^n\, n(k,t)}{N\bar{N}(t)} \mathrm{d}k \qquad \propto (t/t_0)^{-n\beta}$$

Emergent conserved quantity



Energy transport to UV

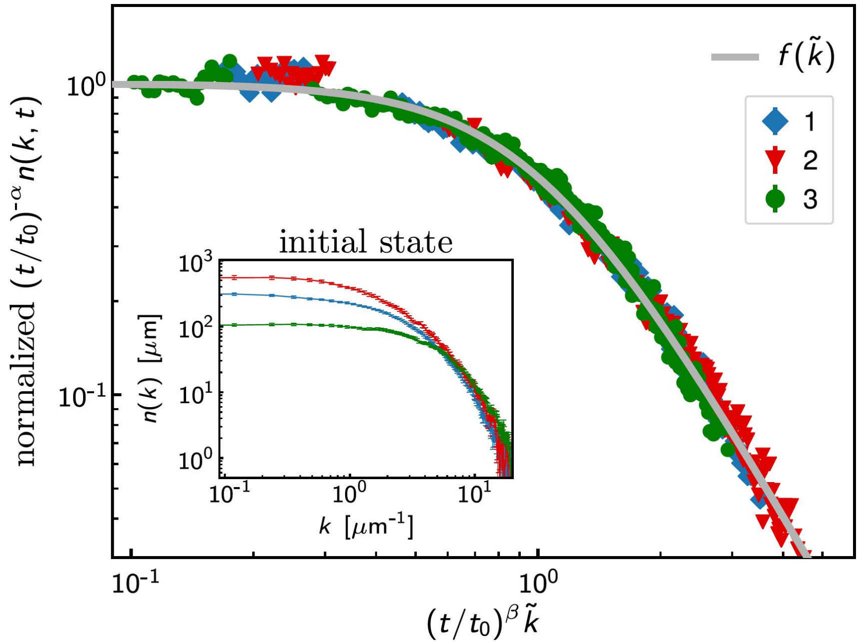


Bi-directional transport ➜ inverse particle transport towards IR
➜ direct energy transport towards UV

Universal scaling function

For varying initial conditions the system quickly approaches the same non-thermal fixed point

→ *efficient loss of information about initial state long before thermalization*



Universal scaling function

$$f_S \sim [1 + (\tilde{k}/k_0)^\zeta]^{-1} \qquad \zeta = 2.39 \pm 0.18$$

What determines the universal parameters?

$$\alpha \approx \beta = 0.1 \pm 0.03$$
$$\zeta = 2.39 \pm 0.18$$

➔ **Expected**

d=1:		$\beta = 0$
d≥2:	$\zeta = d + 1$	$\beta = 1/2$

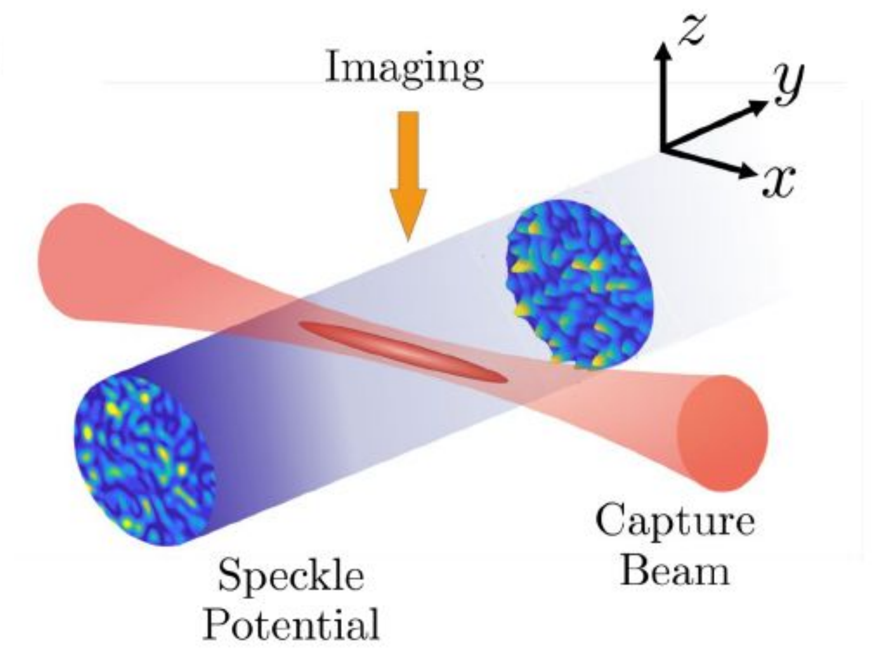
➔ **However:** transverse excited states break integrability

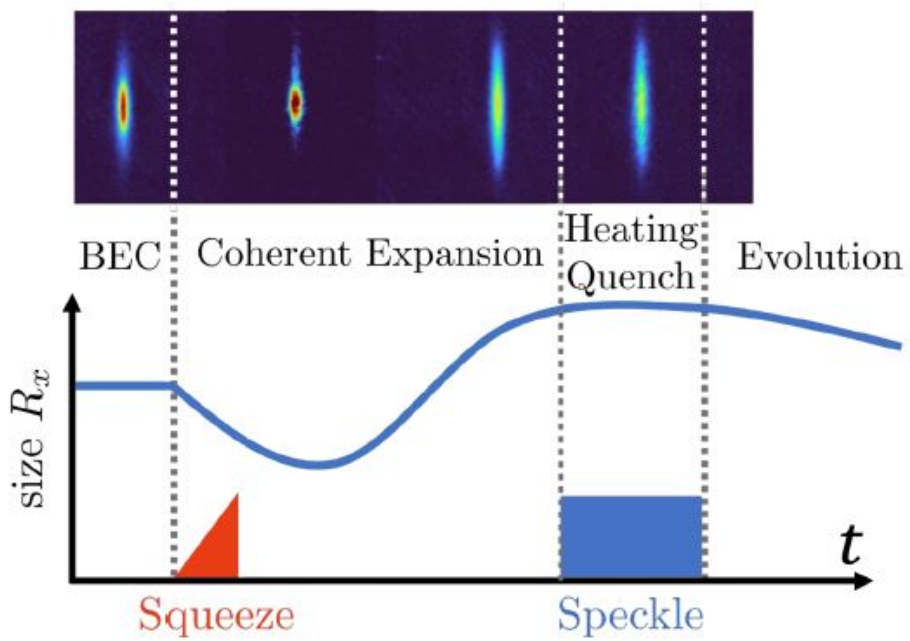
Is there a continuous connection (fractal dimensions) of fixed-points between 1D and 2D?

Gresista, Zache, Berges, Phys. Rev. A **105**, 013320 (2022)

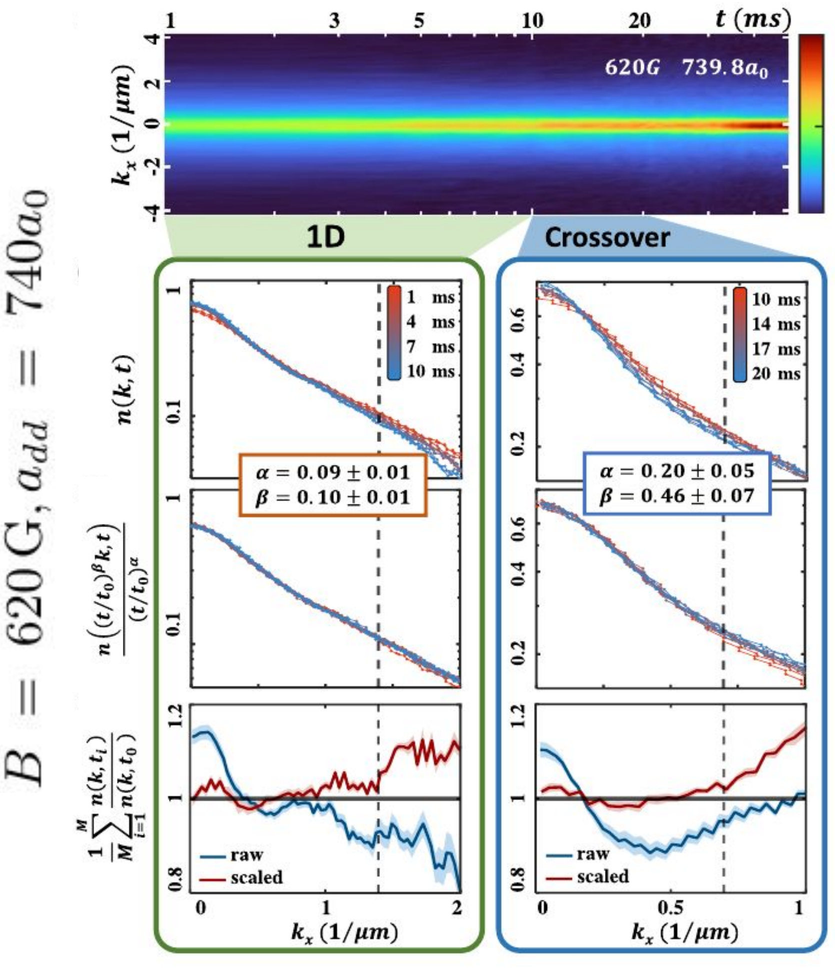
Heating quenches

Qi Liang, R. Wu, P. Paranjape, B. Schittenkopf, C. Li, J. Schmiedmayer, SE arXiv:2505.20213

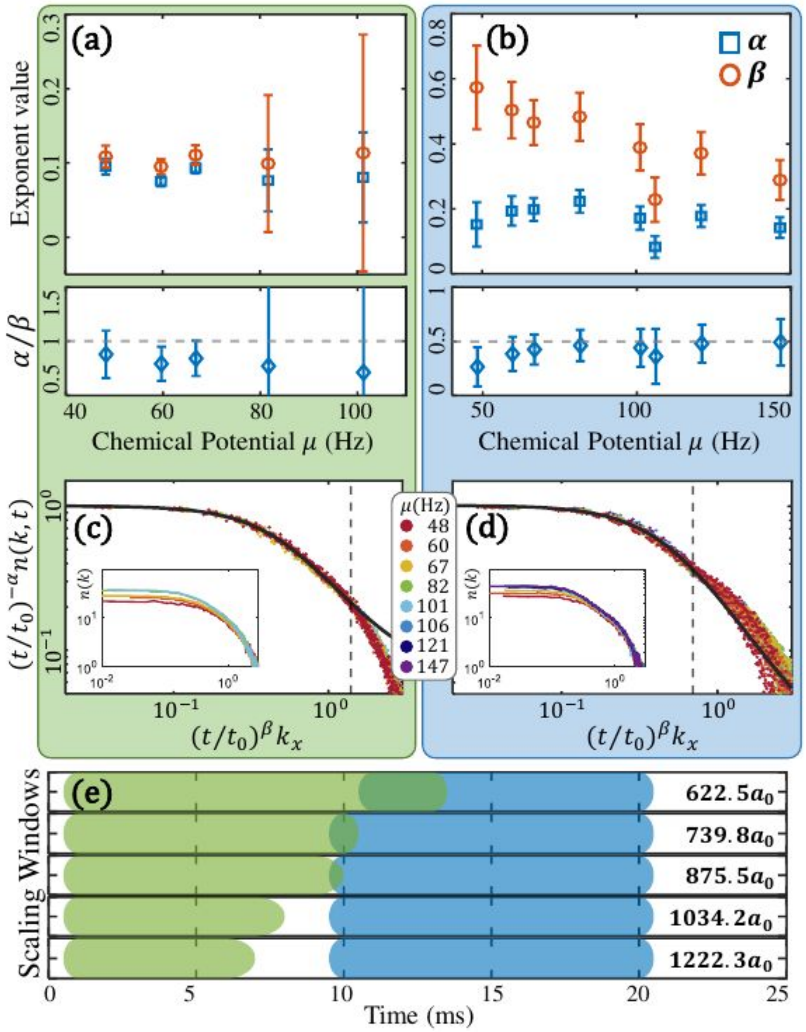




Scaling regimes

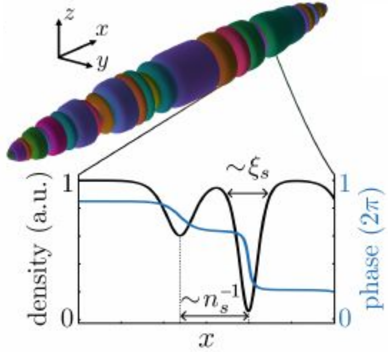


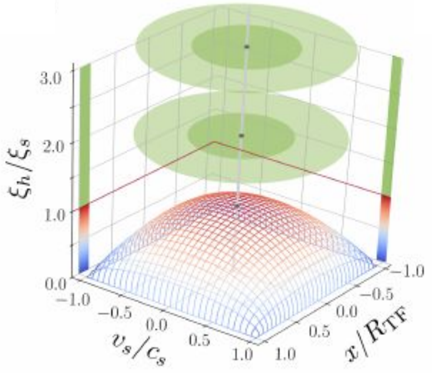
varying interactions



Generalized solitonic defect states

State immediately following the quench ⟶ well described by generalized solitonic defect (GSD) states





dimensionless units

Typical length-scale:

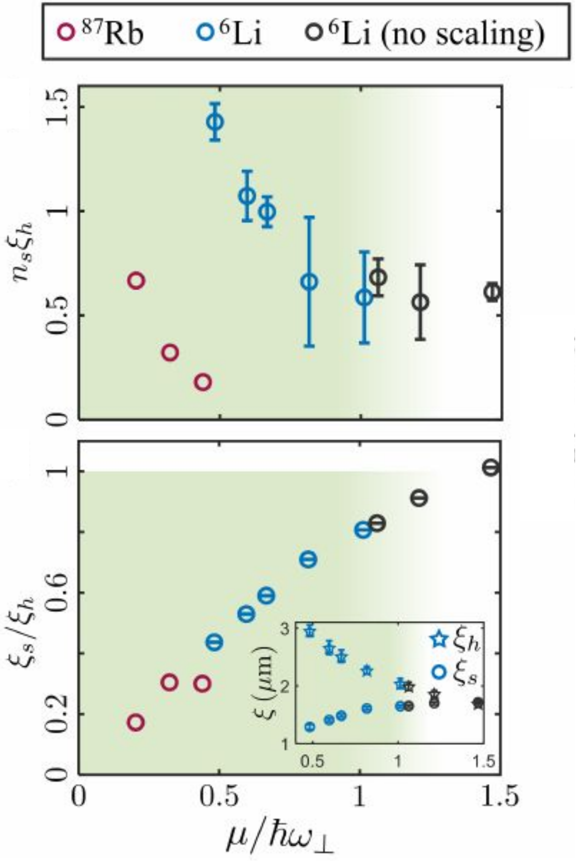
$$\xi_h = \hbar/\sqrt{2m\mu}$$

Interaction energy ⟷ 1d-ness

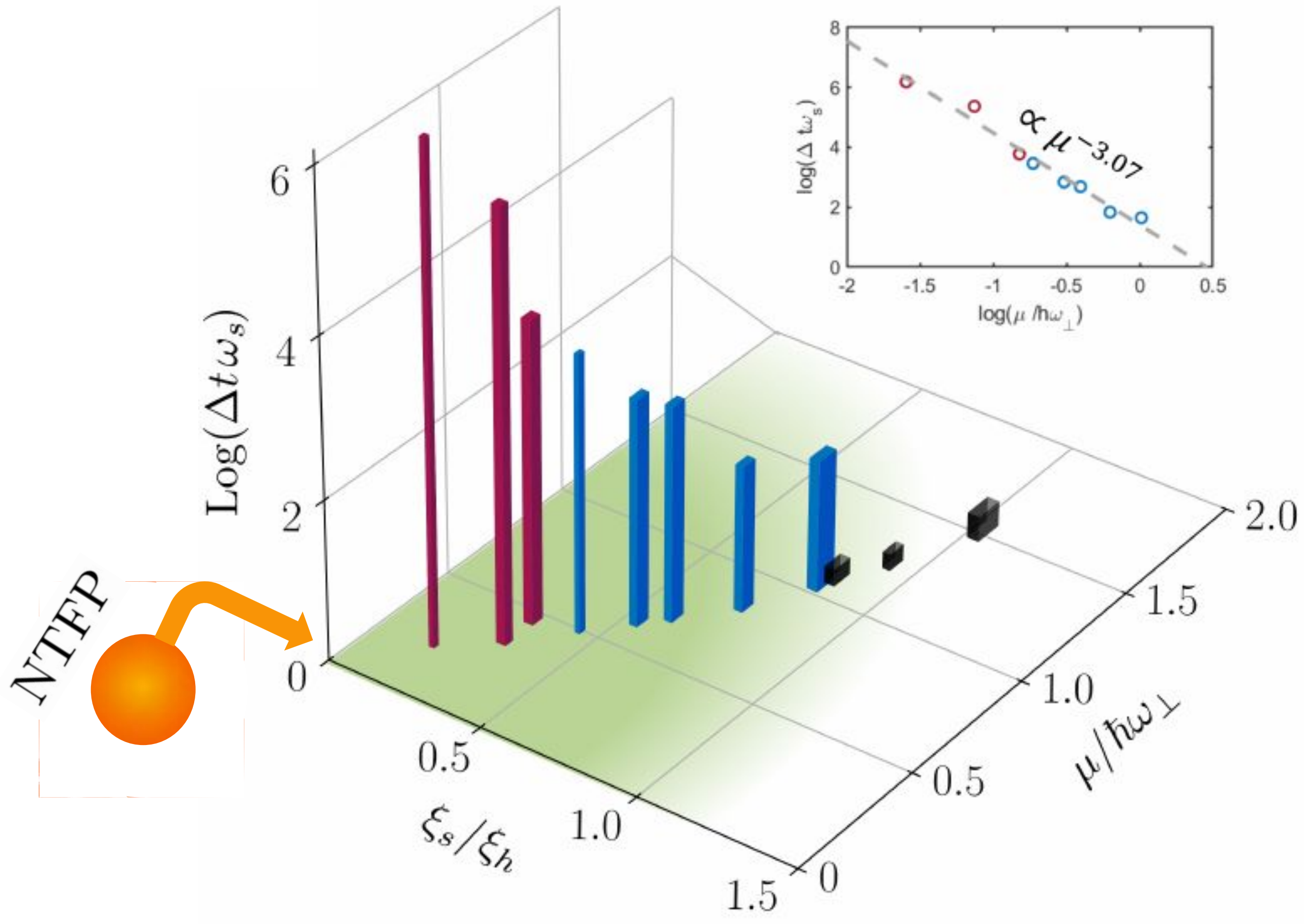
$$\mu/\hbar\omega_\perp$$

Typical time-scale in IR

$$\omega_s = 2\hbar n_s^2/m$$



Basin of attraction



From NTFPs to (generalized) Hydrodynamics

A lot of citations the last years! For an easy intro/review see: B. Doyon, et. al Phys. Rev. X **15**, 010501 (2025)

Generalized Hydrodynamics $\leftrightarrow$ Hydro for **integrable systems**

$$\partial_t q_n + \partial_x j_n = 0$$

Follows same logic as usual hydrodynamics but taking into account all conserved quantities
→ **local GGE instead of thermal state**

➡ Large/infinite number of extensive conserved quantities
➡ strongly restricts dynamics (Gibbs ensemble → Generalized Gibbs ensemble)
➡ Long-lived quasiparticles

Gradient expansion of currents leads to simplified equations at Euler scale

$$\partial_t \rho_\lambda + \partial_x \left(v_\lambda^{\mathrm{eff}}[\rho]\rho_\lambda \right) = 0$$

Generally: integrability is rare, but **near-integrability** has pretty large regime of applicability

$$\partial_t q_n + \partial_x j_n = \mathcal{I}_n\left[q\right]$$

Integrability breaking terms $\leftrightarrow$ generalized Boltzmann collision term $\mathcal{I}_n$

Can universality of the quasi-1D NTFP be phrased as universality of integrability breaking corrections?

Outlook & questions

Thank you for your attention!

- Kibble-Zurek scaling
  - ➜ Reach low quench-rate regime through flat-bottom potentials
  - ➜ Is there a general connection to NTFPs for ultra-fast quenches
- Universal scaling far from equilibrium and NTFPs
  - ➜ New perspective on attractor through integrability breaking corrections in generalized Hydro
  - ➜ Dimensional crossover, Pre-scaling and departure from NTFP, …
- Defect dominated NTFPs and connection to e.g. coarsening dynamics (Victorias talk)

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J. Schmiedmayer (TU Wien)

Qi Liang (TU Wien)

RuGway Wu (TU Wien)

& the whole AtomChip group!

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