Transport at Large N

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Unexpected New Results for Old Problems

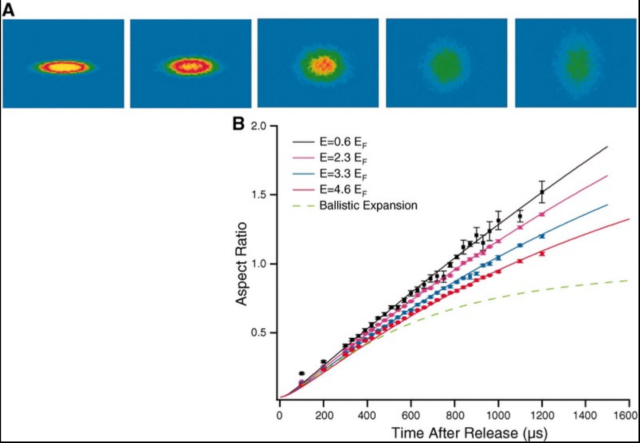
Apologies: my talk entirely misses the topic of this workshop!

Specific viscosity for ordinary fluids (in $\frac{\hbar}{k_B}$)

- Water, standard conditions: $\frac{\eta}{s} \sim 280$
- Water near critical point: $\frac{\eta}{s} \sim 2$
- Helium near critical point: $\frac{\eta}{s} \sim 0.8$

How low does $\frac{\eta}{s}$ get?

Ultracold Quantum Gases near Unitarity



[John Thomas' group, NC State]

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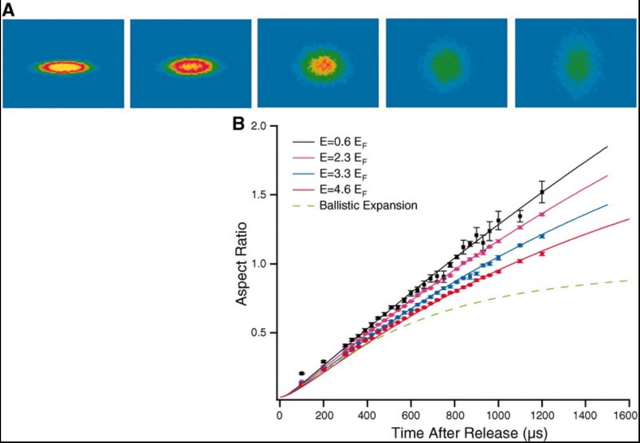
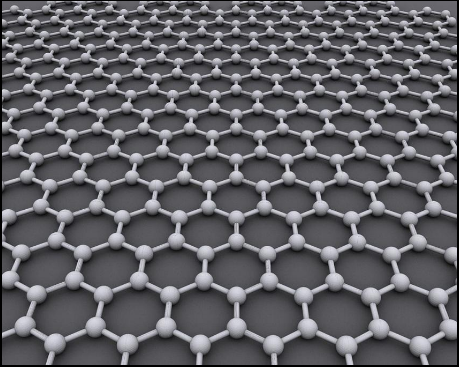


[John Thomas' group, NC State]

$$\frac{\eta}{s} \sim 0.5$$

Graphene



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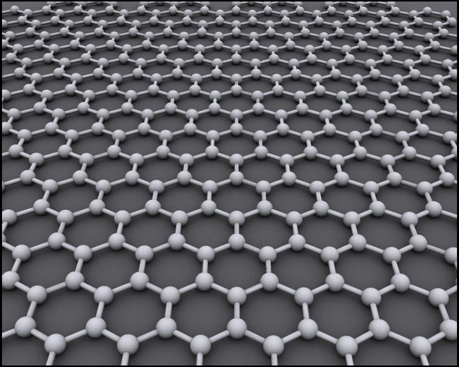
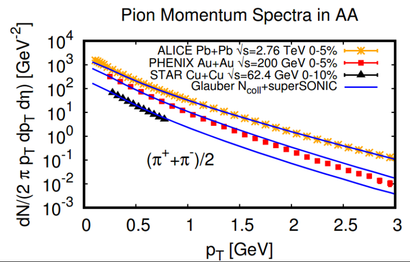
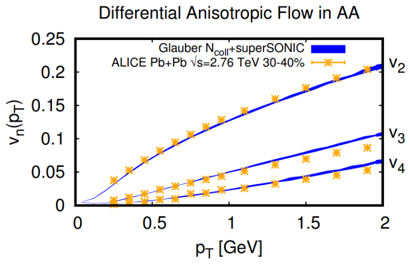


$$\frac{\eta}{s} \sim 0.25$$

Quark-Gluon Plasma





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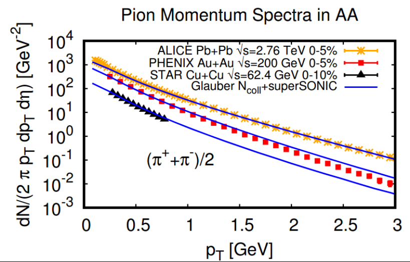
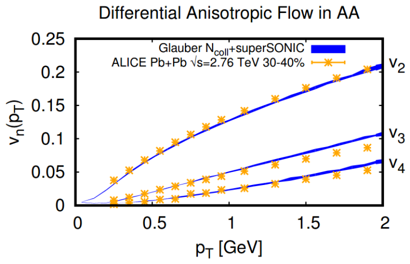




$$\frac{\eta}{s} \sim 0.12$$

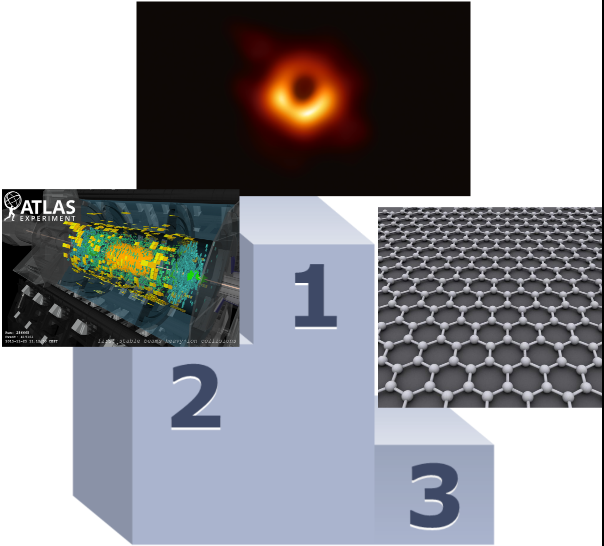
Black Holes

Black Holes

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$$

[Policastro, Son, Starinets]

Viscosity Olympic Games



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BHs are *conjectured to be* dual to a QFT at **infinite coupling**

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(At least at large N?)

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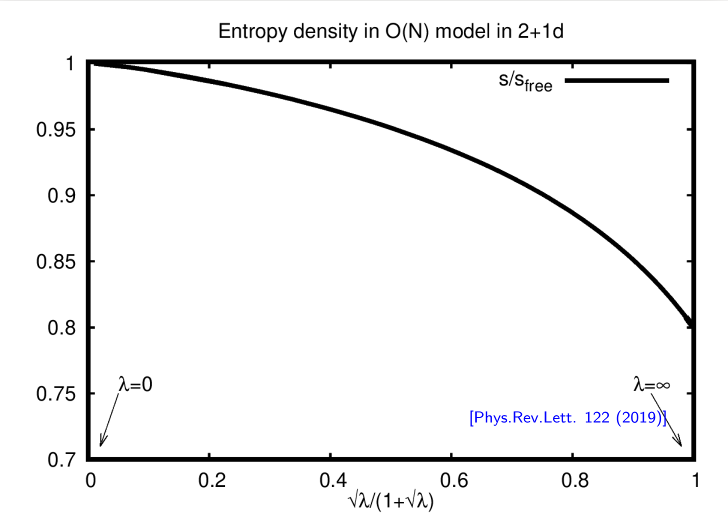
In 2+1 dimensions: "straightforward" results



Curious finding: 3d O(N) model at infinite coupling

$$\lim_{\lambda \to \infty} \frac{s}{s_{\text{free}}} = \frac{4}{5}$$

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compare to *conjectured result* for 4d $\mathcal{N} = 4$ SYM

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https://pt.wikipedia.org/wiki/Teoria_das_cordas

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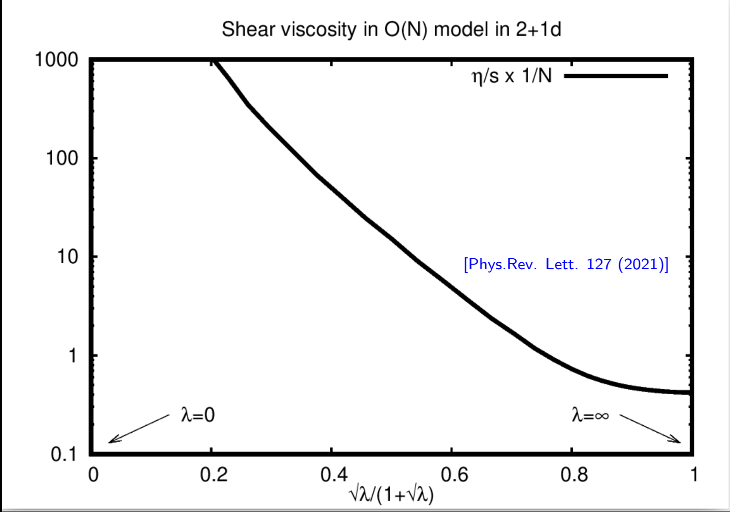
Dilema dos três quartos

Um dos avanços mais significativos na teoria das cordas sugere que tanto os buracos negros quanto a matéria são aproximadamente dois lados da mesma moeda. Essa dualidade permitiu aos pesquisadores mapear as propriedades da matéria (como pressão) à pressão dos buracos negros encontrados na relatividade geral de Einstein, o que abriria a teoria das cordas para uma exploração matemática ainda maior. No entanto, após mais de 20 anos da descoberta dessa dualidade, os teóricos das cordas têm tentado esclarecer esse obstáculo com equações progressivamente mais complicadas. Toda vez que eles comparam essa dualidade, todos obtêm o mesmo resultado: a energia livre (a habilidade de um sistema de fazer o trabalho) a partir de uma forte interação (ou acoplamento) dos dois é aproximadamente três quartos da força do acoplamento fraco.[5]

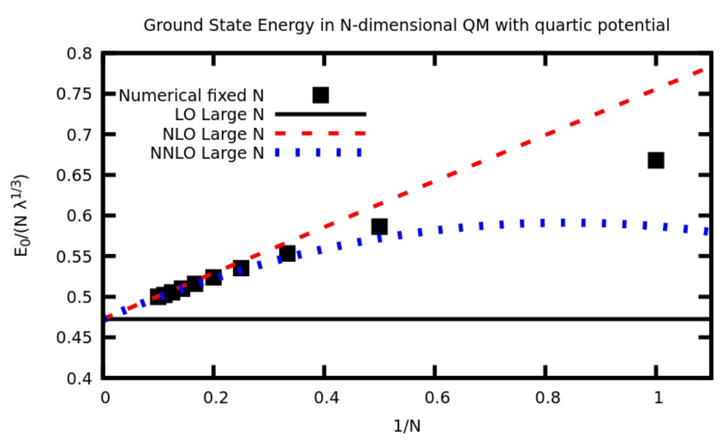
Paul Romatschke, em 2019, inventou um conjunto alternativo de ferramentas para aqueles que criaram o dilema de três quartos da teoria das cordas. Romatschke trabalhou em um mundo que só tem duas dimensões. Usando algumas das equações de pesquisas existentes sobre o assunto, bem como técnicas modernas de teoria de campo quântico, ele conseguiu provar que existe uma relação forçando a matéria (neste caso, a pressão) a interagir da interação zero à interação infinita. Romatschke descobriu que a pressão do acoplamento infinito é exatamente quatro quintos do que em acoplamentos nulos. Isso implica não apenas uma conexão mais forte nessa dimensão menor do que a encontrada anteriormente, mas também pode fornecer uma abordagem padrão para resolver esses tipos de quebra-cabeças.[6]

In 2+1 dimensions: “straightforward” results



$$\lim_{\lambda\to\infty} \frac{\eta}{s} = 0.42(1) \times N$$

Down to which N does Large N work?


Ground State Energy in N-dimensional QM with quartic potential
Numerical fixed N
LO Large N
NLO Large N
NNLO Large N
$E_0/(N\,\lambda^{1/3})$
1/N
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0
0.2
0.4
0.6
0.8
1

[2310.00048]

Is this approach limited to 3d?

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No,

Is this approach limited to 3d?

No, but....

Is this approach limited to 3d?
No, but....people say it doesn't work in 4d

I like people who do, not people who talk!
[Reinhold Messner]

Transport for O(N) model in 3+1 dimensions

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Same method as for 2+1d
only difference: coupling λ *gets renormalized*

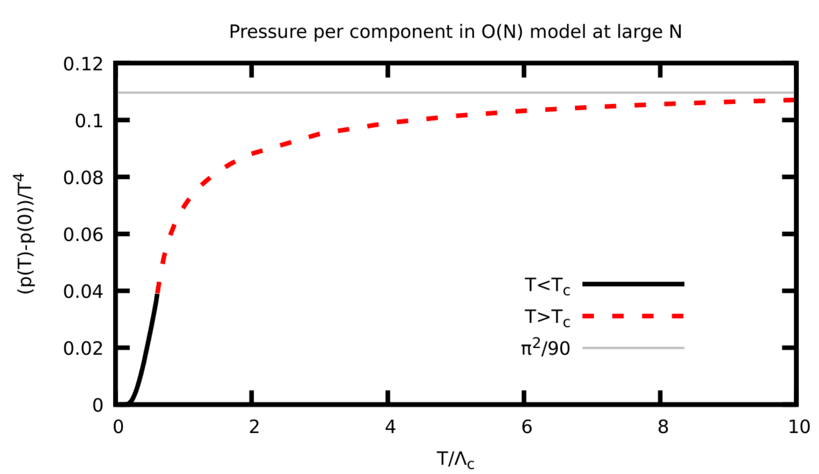
In 3+1 dimensions: "straightforward" results

$$p(m) = \frac{m^4}{64\pi^2} \ln \frac{\Lambda^2_{\overline{\mathrm{MS}}} e^{\frac{3}{2}}}{m^2} + \frac{m^2 T^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{K_2(n\beta m)}{n^2} \, .$$

where m is solution to saddle point equation

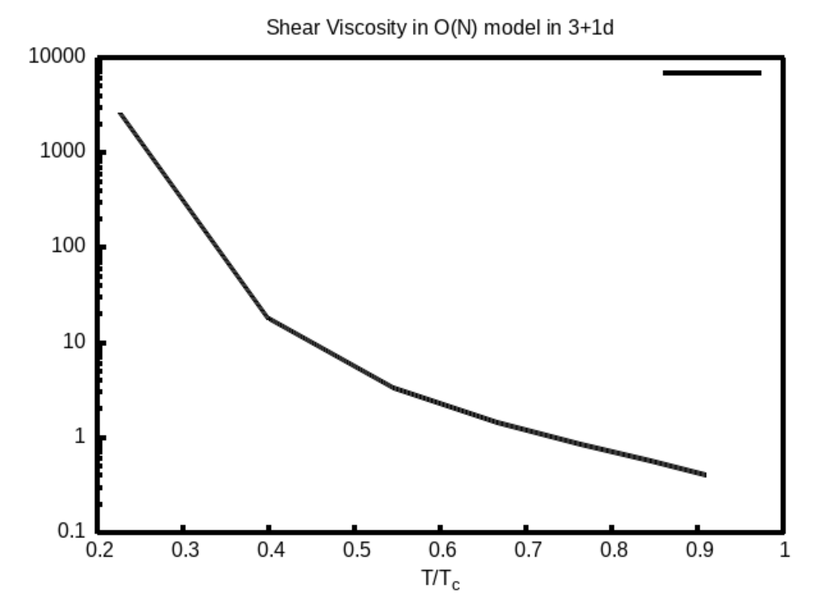
$$0 = \frac{dp(m)}{dm^2} = \frac{m^2}{32\pi^2} \ln \frac{\Lambda^2_{\overline{\mathrm{MS}}} e^1}{m^2} - \frac{mT}{4\pi^2} \sum_{n=1}^{\infty} \frac{K_1(n\beta m)}{n} \, .$$

Large N thermodynamics 4d: two phases

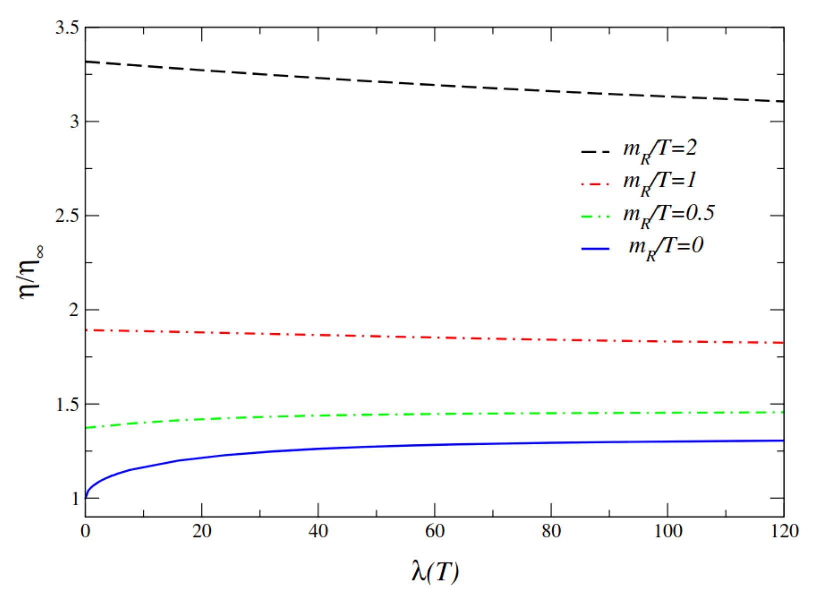


[2211.15683]

Large N transport 4d: shear viscosity



Large N transport 4d: matches "old" results



[Aarts & Resco, 2004]

Only difference between “old” and “new results”: express in terms of physical observables, such as temperature T

(Aside: $\lambda(T)$ is **not** a physical observable)

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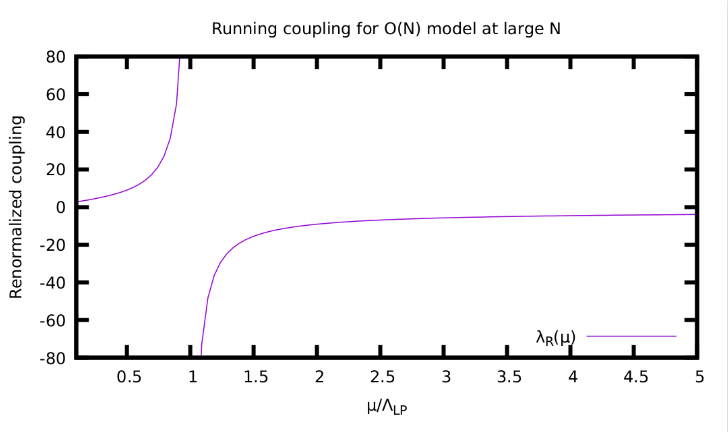
(Aside: $\lambda(T)$ is **not** a physical observable)

(Neither is α_s; **Caution! Heresy!**)

Once you recognize that λ is not a physical observable, then it can do whatever it wants:



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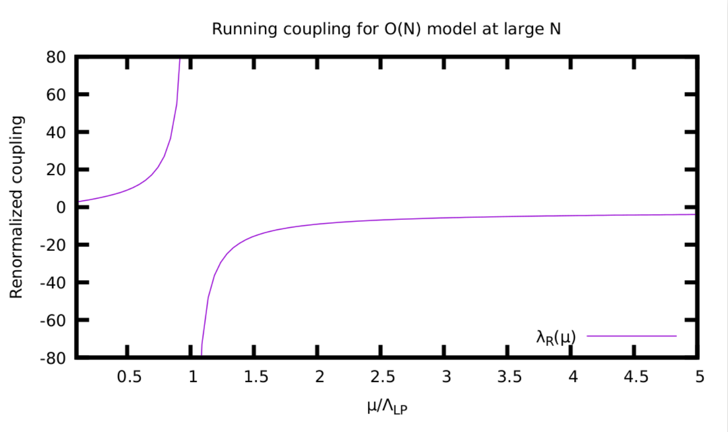

Large N Math: nothing bad happens to **physical** quantities, despite $\lambda < 0$

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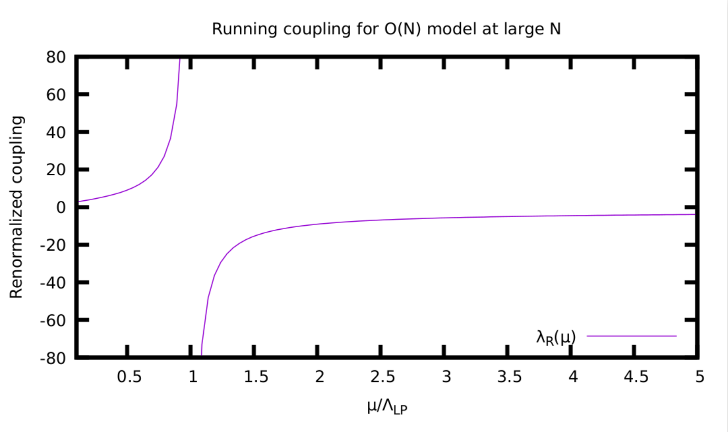
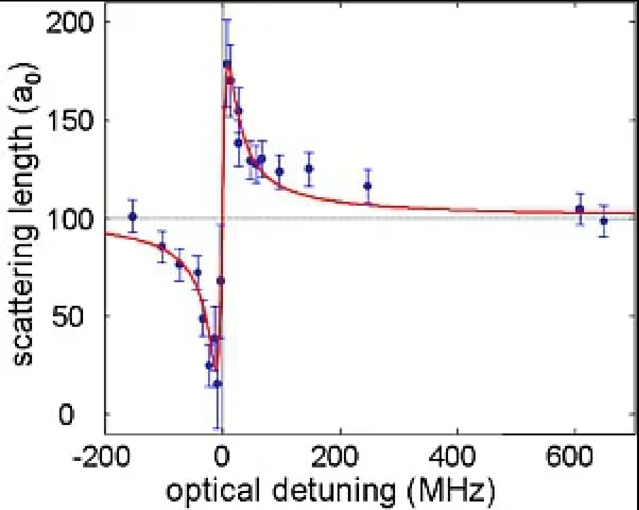


Large N Math: nothing bad happens to **physical** quantities, despite $\lambda < 0$
Analytic Continuation!



A bit like in cold quantum gases with Feshbach resonance where $\lambda \sim a_0$

But what about quantum triviality? Isn't there a math proof that scalar fields cannot give interacting QFTs in 4 dimensions?

What if ϕ^4 theory in 4 dimensions is non-trivial in the continuum?

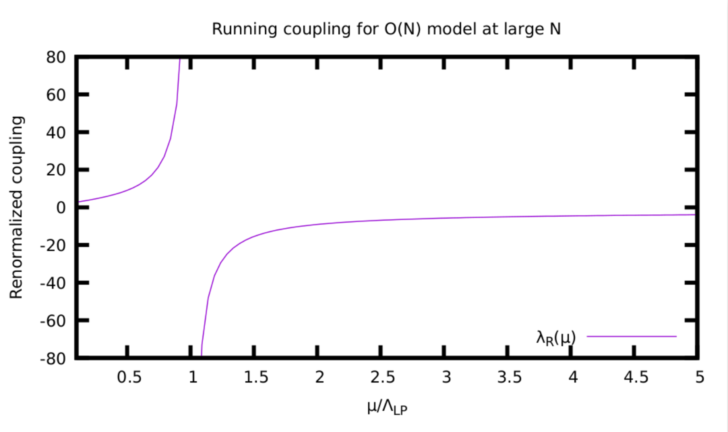
Paul Romatschke[1,2]

[1] *Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*
[2] *Center for Theory of Quantum Matter, University of Colorado, Boulder, Colorado 80309, USA*

Traditionally, scalar ϕ^4 theory in four dimensions is thought to be quantum trivial in the continuum. This tradition is apparently well grounded both in physics arguments and mathematical proofs. Digging into the proofs one finds that they do not actually cover all physically meaningful situations, in particular the case of multi-component fields and non-polynomial action. In this work, I study multi-component scalar field theories in four dimensions in the continuum and show that they do evade the apparently foregone conclusion of triviality. Instead, one finds a non-trivial inter-

There is a math proof for triviality, but it has loopholes!

Interaction becomes **weaker** at high energy



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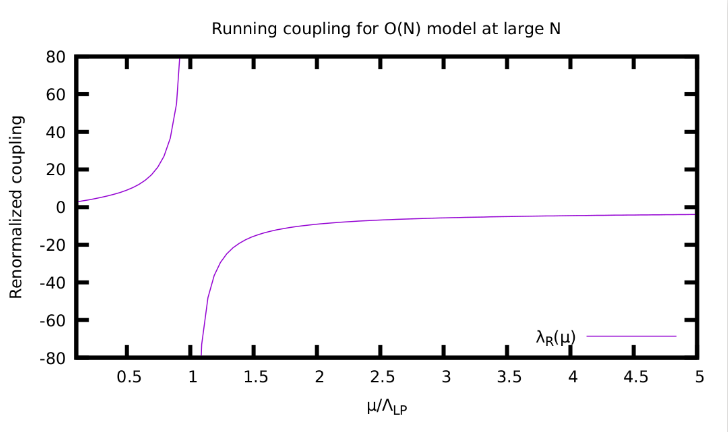

This is **asymptotic freedom!** [Symanzik, 1973]

Large N scalar field theory is fully solvable non-perturbative 4d QFT

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- Old (established) results, new interpretation!
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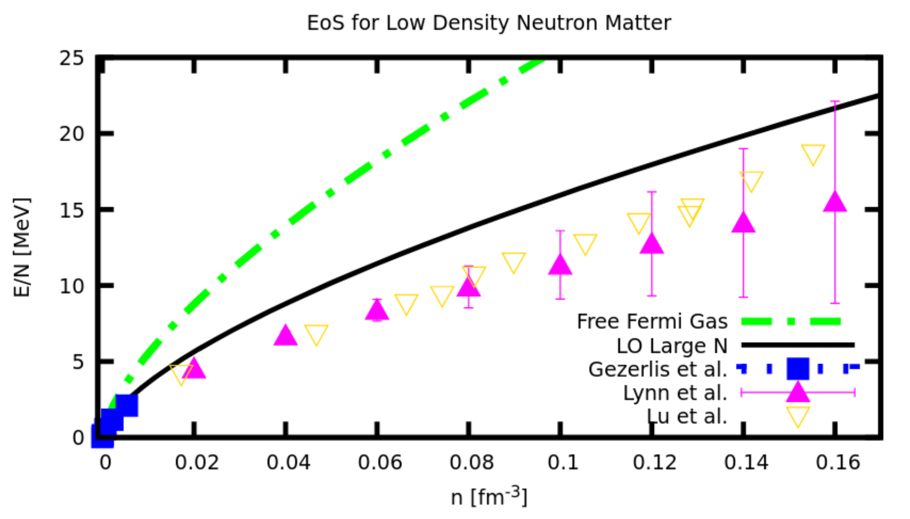
Cutting edge: real-time system evolution solvable at large N!

new non-thermal fixed points? [Chun-Wei Su, 2508.18821]

If you want to hear more on the 4d scalar field theory story, I invite you to look at my colloquium recording “Life on an Endless Hill” from Dec 2024:

Large N expansions for NR fermions



[2310.00048]

Summary

- I advocate for a radical departure from textbook QFT wisdom
- Extraordinary claims, evidence: predict new Standard Model particle (Higgs-Higgs bound state)
- Large N techniques more broadly applicable (e.g. cold quantum gases, large N_f QCD)
- Lecture notes for 3-hour mini-course on this topic: 2310.00048