Limiting attractors: new universality in heavy-ion collisions and jet quenching

Based on
PLB 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]
and work in preparation with K. Boguslavski & L. Hörl

Florian Lindenbauer (MIT Center for Theoretical Physics – a Leinweber Institute)

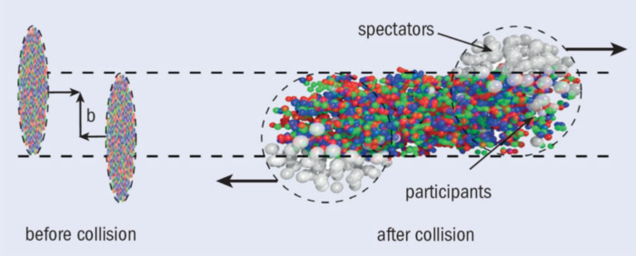
September 23, 2025, ECT* Trento workshop
Attractors and thermalization in nuclear collisions and cold quantum gases

Outline

Outline

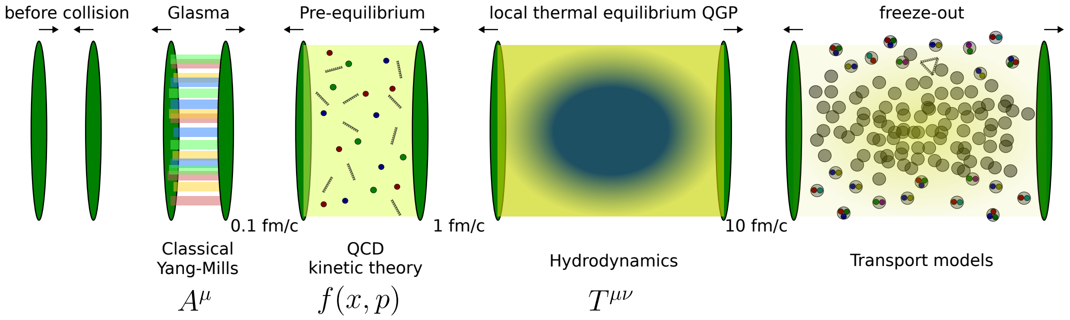
Heavy-ion collisions and the quark-gluon plasma

- Study properties of the strong interaction
- Collision of atomic nuclei at LHC or RHIC
- Creates high-temperature QCD matter = Quark-Gluon plasma (QGP)



[Alberica Toia 2013, CERN COURIER]

Time-evolution of the QGP in heavy-ion collisions



Interested in pre-equilibrium stages ("Initial stages")
$\rightarrow$ **QCD out of equilibrium**

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

Outline

Effective kinetic theory (EKT) description of the QGP

- **Microscopic description**
- Gluons with **distribution function** $f(t, \mathbf{p})$

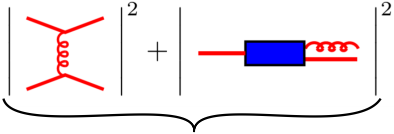
[1] [JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Effective kinetic theory (EKT) description of the QGP

- **Microscopic description**
- Gluons with **distribution function** $f(t, \mathbf{p})$

- Time evolution described by **Boltzmann equation** at leading-order[1]

$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}) f =$$



Collision term

- Azimuthal symmetry around beam axis $\hat{z}$,
  Bjorken expansion, homogeneous in transverse plane

[1] [JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Observables in EKT

- Fundamental quantity: Distribution function $f(\mathbf{p})$
- **Energy-Momentum tensor**:

$$T^{\mu\nu} = \nu_g \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p} f(\mathbf{p})$$

- Longitudinal pressure $P_L = T_{zz}$
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- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$
- Occupancy of the hard sector

$$\frac{\langle pf \rangle}{\langle p \rangle} = \frac{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})^2}{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})}$$

Bottom-up thermalization in heavy-ion collisions

- Initial condition[2], with $\lambda = g^2 N_C$

$$f(p_\perp, p_z) = \text{“squeezed”}\ \frac{1}{\lambda} \times \exp\left(-p^2\right)/p$$

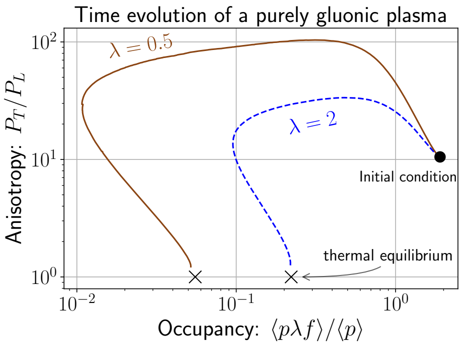

Figure from arXiv:2509.05904 [Altenburger, Boguslavski, FL]

→See talks by B. Scheihing and X. Du

[2] [Phys.Rev.Lett. 115 (2015) [Kurkela, Zhu]]

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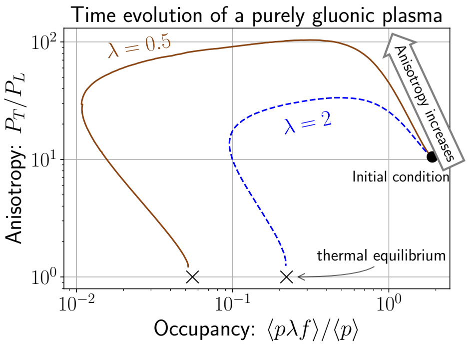

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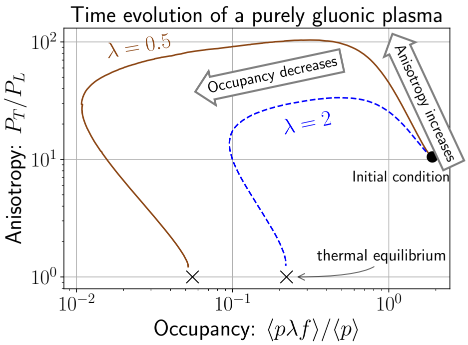

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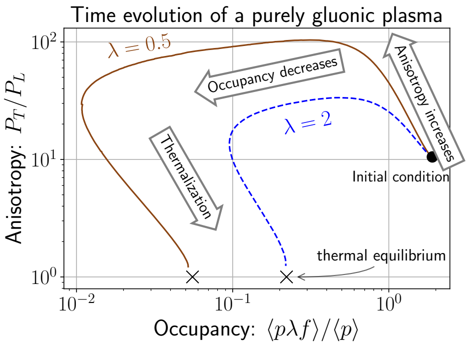
    $\tau_{\mathrm{BMSS}} = \left(\frac{\lambda}{12\pi}\right)^{-13/5} / Q_s$



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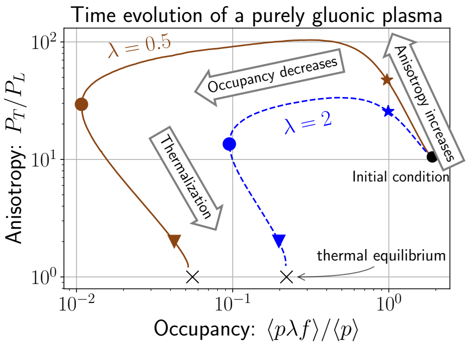
Markers represent **different stages**



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Outline

Hydrodynamic description of the QGP

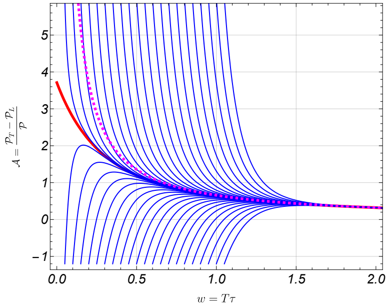
- **Macroscopic description**
- Expansion around local equilibrium

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{(0)} + \underbrace{T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \dots}_{\text{shear and bulk stress } \pi^{\mu\nu}, \Pi}$$

- Macroscopic properties (pressure, temperature, energy density, ...)
- Works **close to equilibrium** (small gradients)
- Approach to equilibrium governed by **transport coefficients** η, ζ, ...

Hydrodynamic attractors

- Attractor found by solving hydro equations
- Complexity of initial state quickly reduced, rapid loss of information



→See talk by A. Soloviev

[Eur.Phys.J.C 82 (2022) [Soloviev], Prog.Part.Nucl.Phys. 132 (2023) [Jankowski, Spaliński]]

[Phys.Rev.Lett. 115 (2015) [Heller, Spalinski]]

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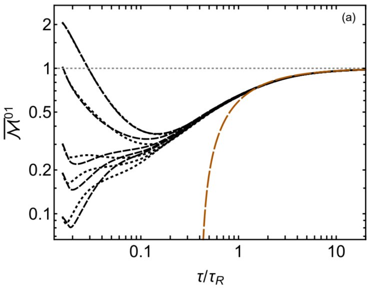

[Phys.Rev.Lett. 125 (2020) [Almaalol, Kurkela, Strickland]]

Hydrodynamic attractors

- Attractor found by solving hydro equations
- Complexity of initial state quickly reduced, rapid loss of information
- Similarly observed in kinetic theory
- Different couplings $\lambda \to$ same curve (when rescaled with η/s)

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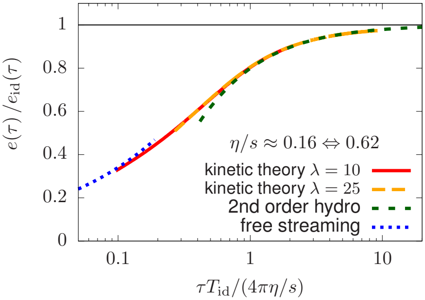

[Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, Phys.Rev.Lett. 122 (2019)]

Relaxation time τ_R

- Conformal (first order) hydro:

$$\frac{P_L}{P_T} = 1 - 8 \underbrace{\frac{\eta/s}{\tau T}}_{\sim \tau_R/\tau}$$

[[Romatschke, Romatschke] (2019)]

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- Conformal: $P_L + 2P_T = \epsilon \rightarrow$ similar relation for P_T/ϵ, P_L/ϵ, ...

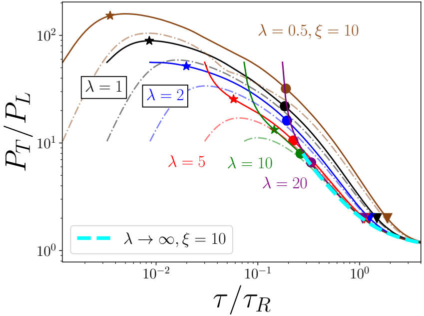
[[Romatschke, Romatschke] (2019)]

Outline

Pressure ratio

$$\tau_R = \frac{4\pi\eta/s}{T},\ \tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$$

- **Kinetic theory simulations** for **different couplings** $0.5 \leq \lambda \leq 20$ and **initial conditions**.



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- Curves **approach limiting attractors** after •

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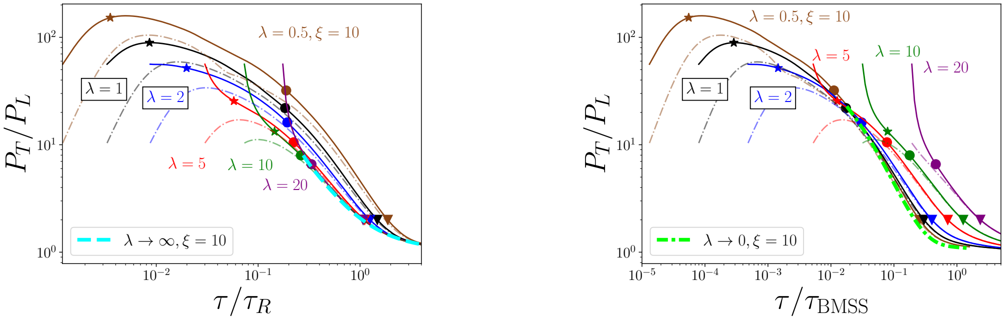

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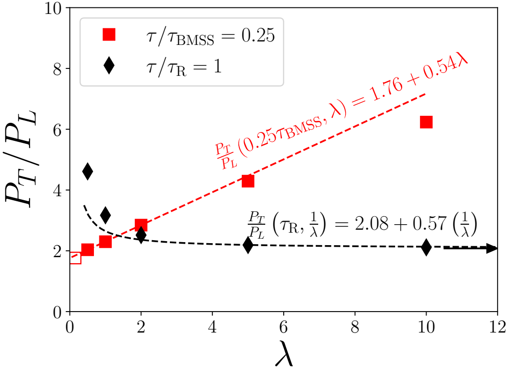
Extrapolation to limiting attractors

$$\tau_R = \frac{4\pi\eta/s}{T}$$
$$\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$$

- Obtain limiting attractors by extrapolating at fixed τ/τ_R or $\tau/\tau_{\mathrm{BMSS}}$
- **Bottom-up attractor**: Linear extrapolation to $\lambda \to 0$, i.e.,

$$\frac{P_T}{P_L}(\tau/\tau_{\mathrm{BMSS}}) = a(\tau/\tau_{\mathrm{BMSS}}) + \lambda b(\tau/\tau_{\mathrm{BMSS}})$$

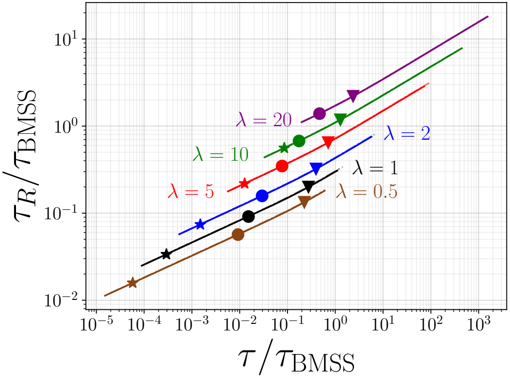
- **Hydro attractor**: Linear extrapolation to $1/\lambda \to 0$



Timescale comparison

[Phys.Lett.B 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

- $\tau_R = \frac{4\pi\eta/s}{T(\tau)}$
  $\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$
- Difficult comparison, τ_R function of τ
- Also: Bottom-up[4]: $T_{\mathrm{max}} \sim \alpha_s^{2/5}$, $\eta/s \sim \alpha_s^{-2} \to \tau_R \sim \alpha_s^{-12/5}$
  But: **Strong coupling attractor** only valid for large λ, where $\eta/s \neq \alpha_s^{-2}$
- **Thermalization dominated by largest time scale**



[4] [Phys.Lett.B 502 (2001) [Baier, Mueller, Schiff, Son]]

Extrapolation revisited

[Phys.Lett.B 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

- Extrapolation to bottom-up attractor robust?
- Different times?

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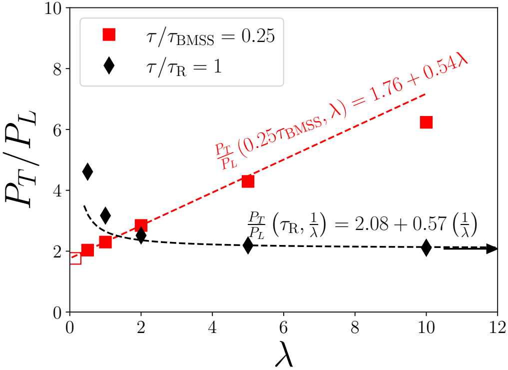

Extrapolation revisited

[See my PhD thesis]

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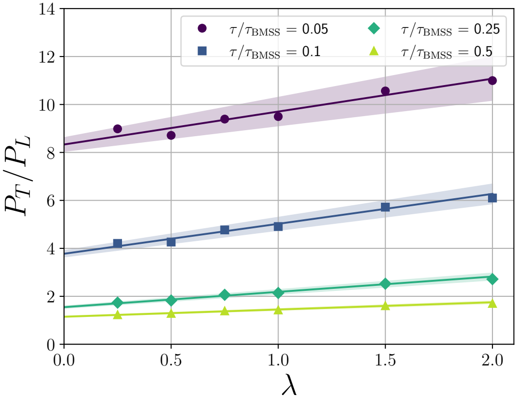
$$\frac{P_T}{P_L}(\tau/\tau_{\text{BMSS}}) = a(\tau/\tau_{\text{BMSS}}) + \lambda b(\tau/\tau_{\text{BMSS}})$$

- Additional simulations with smaller couplings



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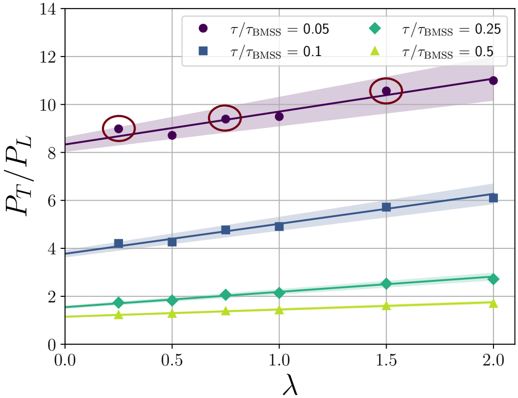
Note: Runs with $\lambda \in \{0.25, 0.75, 1.5\}$ with slightly different systematics



Are there **Limiting attractors in other quantities?**

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- Jet quenching parameter $\hat{q}$

- Heavy quark diffusion coefficient κ

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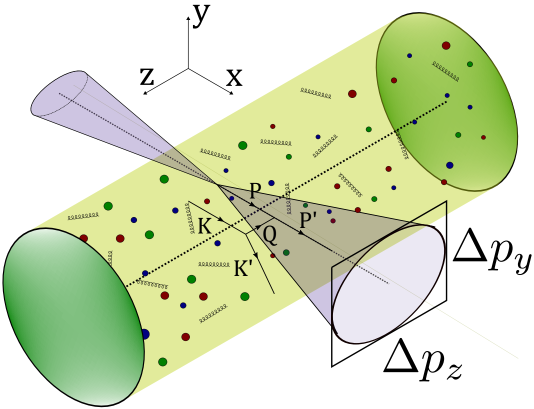
- Jet quenching parameter $\hat{q}$
- Heavy quark diffusion coefficient κ (not in this talk)

Outline

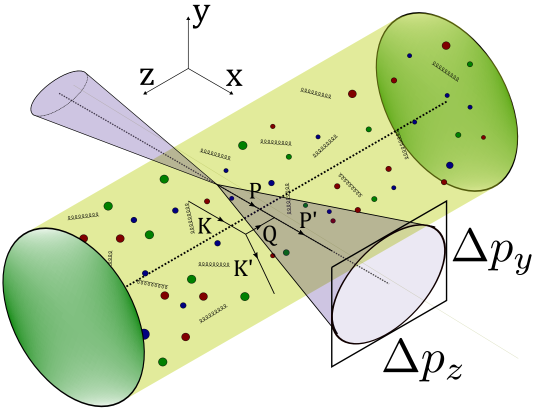
Jets in heavy-ion collisions

- Study **modifications of jets**



Jets in heavy-ion collisions

- Study **modifications of jets**
  - **Highly energetic partons** created in initial collision
  - Splits into many particles
    $\rightarrow$ then measured in the detectors
  - Imprints of **medium interactions**



Jets in heavy-ion collisions

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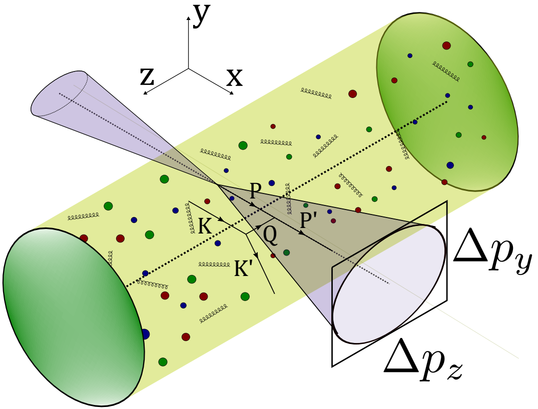
  - **Highly energetic partons** created in initial collision
  - Splits into many particles $\rightarrow$ then measured in the detectors
  - Imprints of **medium interactions**
- Momentum broadening quantified by $\hat{q} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}t}$
- Input to simple **jet energy loss** models



Obtaining $\hat{q}$

Phys.Rev.D 110 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]

- Provided we know $f(\mathbf{k})$:

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- Provided we know $f(\mathbf{k})$:

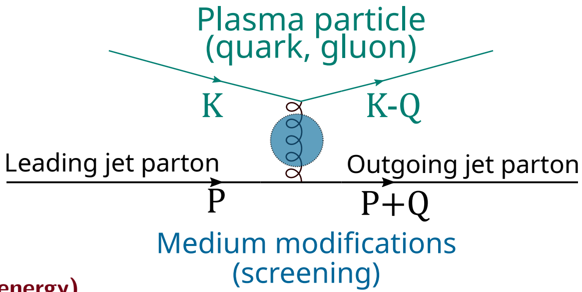
Outgoing plasma particle

$$\hat{q}^{ij} = \int_{\substack{q_\perp<\Lambda_\perp \\ p\to\infty}} \mathrm{d}\Gamma_{\mathrm{PS}}\, q^i q^j\, |\mathcal{M}|^2\, f(k)\,(1+f(k'))$$

Incoming plasma particles with momentum k

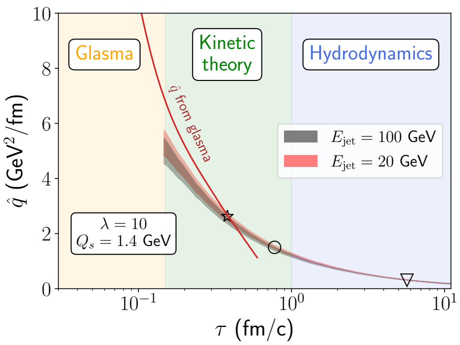
Matrix element **with medium corrections (self-energy)**

appropriate phase-space measure



Time evolution of jet quenching parameter

- Recently computed $\hat{q}(\tau)$
- **Bands:** Vary cutoff and initial conditions
- Supports **large values** from **Glasma**[5] and lower values in hydrodynamic stage

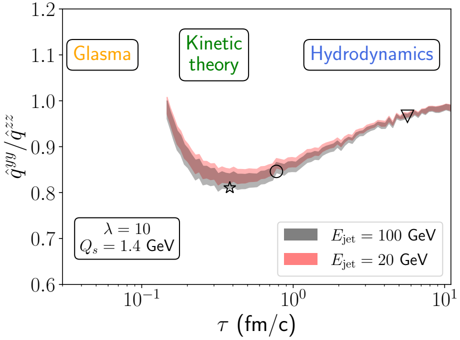


[Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

5 [Phys.Lett.B 810 (2020) [Ipp, Müller, Schuh]]

Time evolution of jet quenching parameter

- Recently computed $\hat{q}(\tau)$
- **Bands:** Vary cutoff and initial conditions
- Broadening **anisotropy** up to 15 %
- Mostly $\hat{q}^{zz} > \hat{q}^{yy}$ $\rightarrow$ **Enhanced broadening along beam axis**
- Possible impact on polarization[5], azimuthal and spin observables[6]

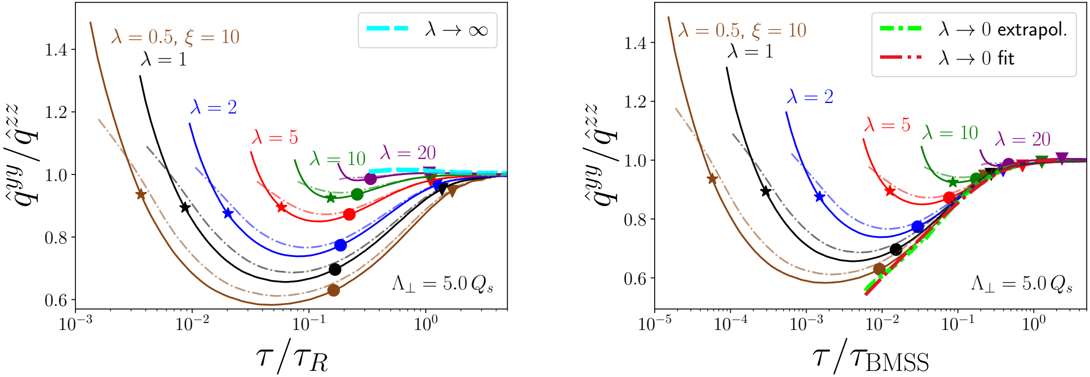


[Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

5 [JHEP 08 (2023) [Hauksson, Iancu]]
6 [JHEP 12 (2024) [Barata, Salgado, Silva]]

$\hat{q}$ and the limiting attractors

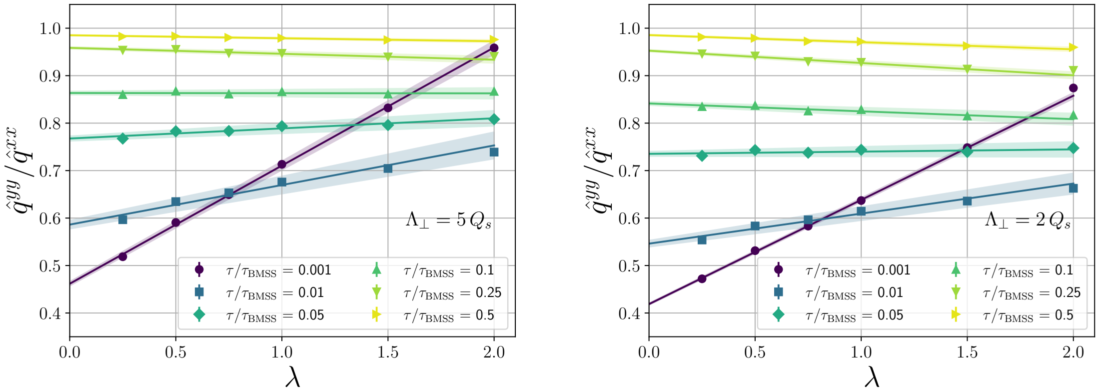
[Phys.Lett.B 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]



- $\hat{q}$ allows extrapolation to **both limiting attractors**
- **Weak-coupling attractor** approached even at **moderate couplings**

Check linear extrapolation

[See my PhD thesis]



- Excellent extrapolation at different times and cutoffs $\Lambda_\perp$.

But: $\hat{q}$ not directly measurable

$\rightarrow$ do these attractors survive in experimental observables?

(Spoiler: cannot give concrete answer yet, but consider jet energy loss calculations)

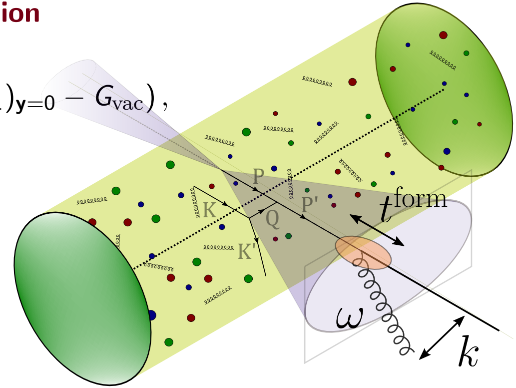
Outline

Obtaining the gluon emission spectrum

JETP Lett. 65 (1997) [Zakharov]
Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\frac{dI}{d\omega} \sim \mathrm{Re} \int_{t_0}^{\infty} dt_2 \int_{t_0}^{t_2} dt_1 \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} \left(G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\mathrm{vac}} \right),$$



- ω: emitted gluon energy
- $t^{\mathrm{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

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- Greens function G of 2D Schrödinger equation

$$\left(\partial_t - \frac{\partial_x^2}{2\omega} + \frac{i}{4}\hat{q}(t)x^2\right) = i\delta^2(\mathbf{x} - \mathbf{y})\delta(t_1 - t)$$

[7] [Phys.Rev.D 79 (2009) [Arnold]]

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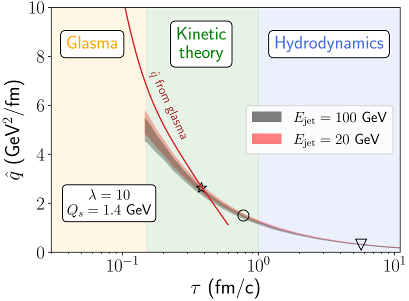
- Isotropic (in transverse plane): Trick: Find spectrum using 'simple formula'[7]

$$\ddot{c}(t) = i\frac{\hat{q}(t)}{2\omega}c(t), \qquad \rightarrow \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \ln|c(0)|$$

[7] [Phys.Rev.D 79 (2009) [Arnold]]

Simple formula for jet energy loss (isotropic $\hat{q}$)

Input: $\hat{q}(\tau)$, any time-dependence



'Simple formula'

Output: Spectrum $\frac{\mathrm{d}I}{\mathrm{d}\omega}$

Naive picture:
Mean energy of
single emitted gluon

mean energy loss: $E = \int \mathrm{d}\omega\, \omega \frac{\mathrm{d}I}{\mathrm{d}\omega}$

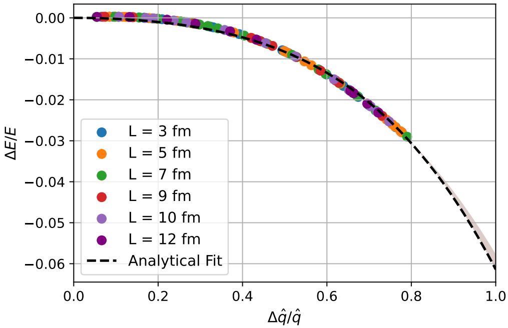
Generalizing 'simple formula' to anisotropic $\hat{q}$

- Previous trick[8] not applicable
- Need to perform time integrals numerically

[8] [Phys.Rev.D 79 (2009) [Arnold]]

Generalizing 'simple formula' to anisotropic $\hat{q}$

- Previous trick[8] not applicable
- Need to perform time integrals numerically
- Calculate in static brick
- Only **small effect** $< 2\%$ for realistic $\hat{q}$ $\rightarrow$ (may be **observable dependent!**)
- $\hat{q} = \frac{\hat{q}^{zz}+\hat{q}^{yy}}{2}$, $\Delta\hat{q} = \frac{\hat{q}^{zz}-\hat{q}^{yy}}{2}$
- $\Delta E = E_{\mathrm{aniso}} - E_{\mathrm{iso}}$



[8] [Phys.Rev.D 79 (2009) [Arnold]]

Plugging $\hat{q}(\tau)$ evolution

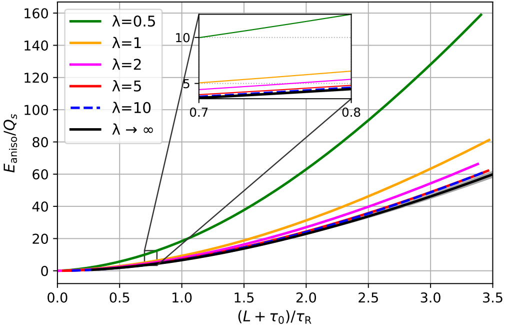
Other estimates using hydro attractor:
Phys.Rev.Lett. 122 (2019) [Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney],
Phys.Rev.Lett. 123 (2019) [Giacalone, Mazeliauskas, Schlichting],
JHEP 06 (2024) [Zhou, Brewer, Mazeliauskas], ...

- Now use $\hat{q}(t)$ from previous simulation
- Use 'corresponding brick'[9]
  $\hat{q}_x^{\mathrm{eff}}(L) = \frac{2}{L^2}\int_{\tau_0}^{L+\tau_0}(\tau-\tau_0)\,\hat{q}_x(\tau)\,\mathrm{d}\tau$
- Extrapolation to **limiting attractor**

$$E_{\mathrm{aniso}}/Q_s \approx 6.5 \times \tilde{w}^{1.8},$$

- Obtain estimate

$$E_{\mathrm{aniso}} \approx 74\,\mathrm{GeV} \times \left(\frac{Q_s}{2\,\mathrm{GeV}}\right)\left(\frac{\langle s\tau\rangle}{4.1\,\mathrm{GeV}^2}\right)^{3/5} \times \left(\frac{\nu}{40}\right)^{-3/5}\left(\frac{\tau}{5\,\mathrm{fm}}\right)^{6/5}\left(\frac{4\pi\eta/s}{2}\right)^{-9/5}$$

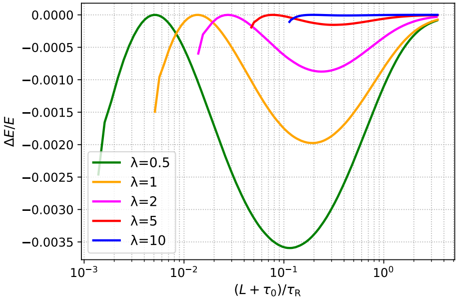


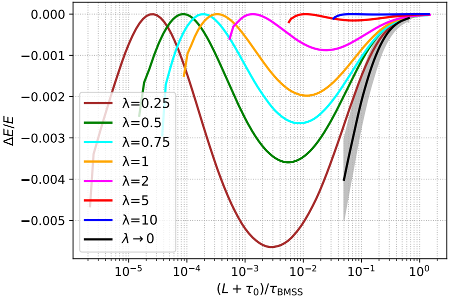
Preliminary

[9] [Phys.Rev.Lett. 89 (2002) [Salgado, Wiedemann]]

Relative change $\Delta E/E$

Preliminary





- **Bottom-up attractor** visible in relative change (despite averaging)
- **Both** attractors visible in **mean energy loss**

Outline

Summary and outlook

- Performed **kinetic theory simulations** of **early stages** in **heavy-ion collisions**
- **Limiting attractors** emerge for P_T/P_L, $\hat{q}$
  - Strong coupling: $\lambda \to \infty$ Hydrodynamic attractor
  - Weak coupling: $\lambda \to 0$ Weak-coupling bottom-up attractor
- For $\hat{q}^{yy}/\hat{q}^{zz}$ even at moderate coupling:
  universal curves in $\tau/\tau_{\mathrm{BMSS}}$
- Application to **jet energy loss**:
  - **Both attractors** visible/survive despite averaging procedure

Outlook:

- Which experimental observables are sensitive and what can be predicted?
- Detailed understanding of their emergence?
- Similar concept(s) in cold atoms?

Thank you very much for your attention!

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Outline

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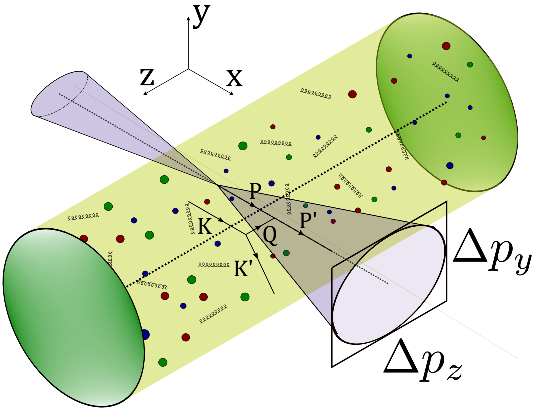
Jet energy loss through medium-induced radiation

- Very many works on energy loss of energetic parton
- **Difficulties**: Correctly including the LPM suppression



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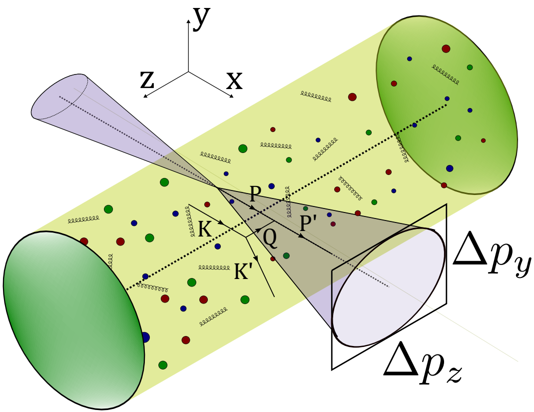
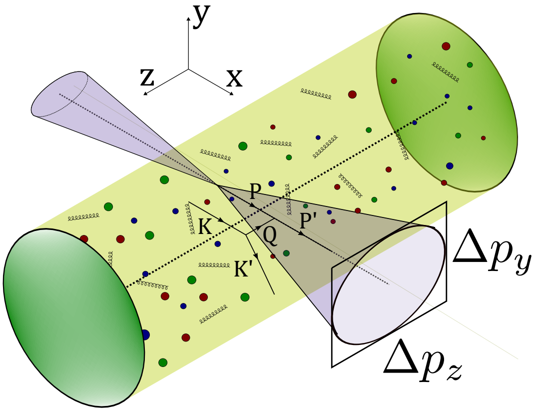


Jet energy loss through medium-induced radiation

- Very many works on energy loss of energetic parton
- **Difficulties**: Correctly including the LPM suppression
- **Harmonic approximation**: Depend on single medium parameter $\hat{q}$ **"Jet quenching parameter"**
- Quantifies **momentum broadening**

$$\hat{q} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}t} = \int \mathrm{d}^2 q_\perp \, q_\perp^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$



Making sense of the cutoff

- Cutoff $\Lambda_\perp$ restricts transverse momentum transfer $q_\perp < \Lambda_\perp$ (needed in eikonal limit $p \to \infty$)

$$\hat{q} \sim \int \mathrm{d}^2 q_\perp \, q_\perp^2 \underbrace{\frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}}_{1/q_\perp^4 \text{ for large } q_\perp} \sim \int \frac{\mathrm{d}q_\perp}{q_\perp}$$

Making sense of the cutoff

- Cutoff $\Lambda_\perp$ restricts transverse momentum transfer $q_\perp < \Lambda_\perp$ (needed in eikonal limit $p \to \infty$)
- Cutoff somehow grow with jet energy

[arXiv:2312.00447 [Boguslavski, Kurkela, Lappi, FL, Peuron]]

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  obtained from comparing leading log behavior for large p and $\Lambda_\perp$
- **LPM cutoff** $\Lambda_\perp^{\mathrm{LPM}}(E,T) = \zeta^{\mathrm{LPM}} g(ET^3)^{1/4}$
  Estimate for momentum broadening during LPM 'formation time': $Q_\perp^2 \sim \hat{q} t^{\mathrm{form}}$, $t^{\mathrm{form}} \sim \sqrt{E/\hat{q}}$, approximately $\hat{q} \sim g^4 T^3$

[arXiv:2312.00447 [Boguslavski, Kurkela, Lappi, FL, Peuron]]

Generalization of $\hat{q} \to \hat{q}^{ij}$ for anisotropic systems

- **Previously** (isotropic definition):

$$\hat{q} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}t} = \int \mathrm{d}^2 q_\perp \, q_\perp^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

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  with elastic scattering rate Γ^{el}

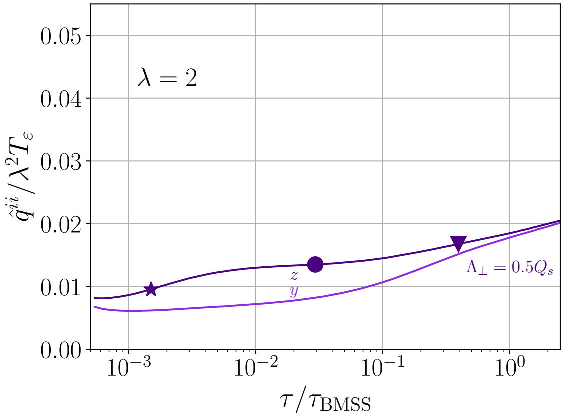
- **To take into account anisotropies:**
  Define matrix

$$\hat{q}^{ij} = \int \mathrm{d}^2 q_\perp \, q_\perp^i q_\perp^j \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

  Thus $\hat{q} = \hat{q}^{yy} + \hat{q}^{zz}$ (and $\hat{q}^{yz} = 0$)

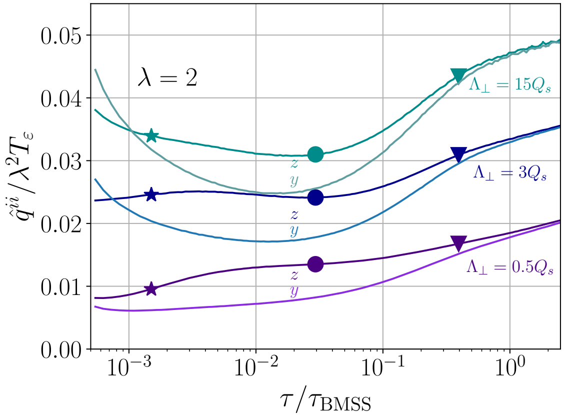
Cutoff dependence and comparison with equilibrium

- $\hat{q}$ for fixed coupling $\lambda = 2$



Cutoff dependence and comparison with equilibrium

- $\hat{q}$ for fixed coupling $\lambda = 2$ and varying cutoffs $\Lambda_\perp$
- **Ordering $\hat{q}^{yy} \lessgtr \hat{q}^{zz}$ depends on cutoff**



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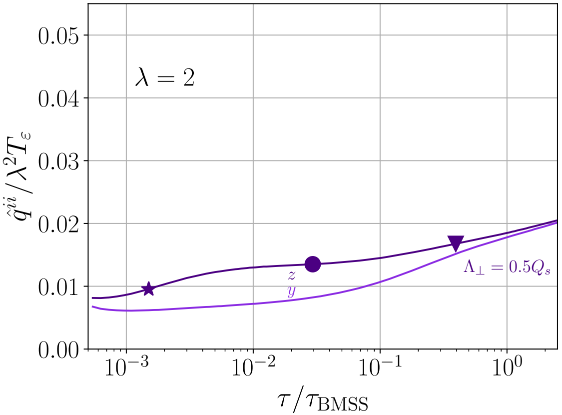

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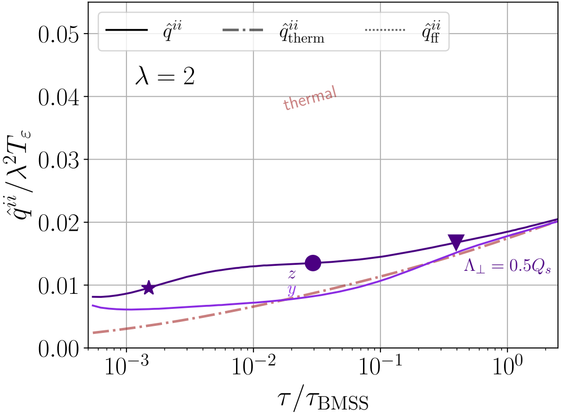
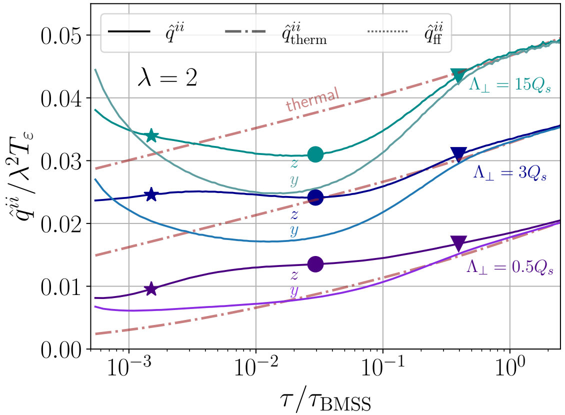
- Compare with **energy-density matched thermal equilibrium**



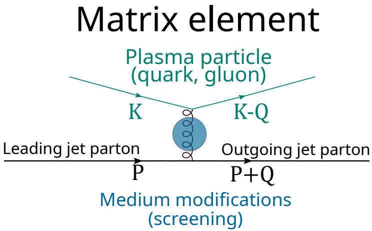
Cutoff dependence and comparison with equilibrium

- $\hat{q}$ for fixed coupling $\lambda = 2$ and varying cutoffs $\Lambda_\perp$
- **Ordering $\hat{q}^{yy} \lessgtr \hat{q}^{zz}$ depends on cutoff**
- Energy-matched **equilibrium over- or underestimates $\hat{q}$, depending on cutoff**



Screening in the matrix element of $\hat{q}$

- Scattering matrix element includes **in-medium propagator**
- Receives **self-energy corrections**
- Anisotropic hard thermal loop (HTL) self-energy $\rightarrow$ unstable modes[10]
- **Approximation: Use isotropic HTL matrix element**
  Similar approximation also in EKT implementations[11]



[10] [Phys.Rev.D 68 (2003) [Romatschke, Strickland]]

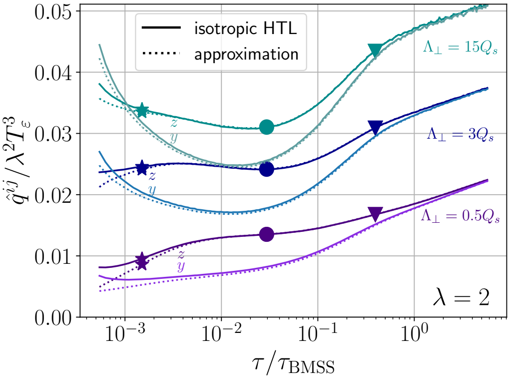
[11] [Phys.Rev.Lett. 115 (2015) [Kurkela, Zhu]; Phys.Rev.Lett. 122 (2019) [Kurkela, Mazeliauskas]; Phys.Rev.D 104 (2021) [Du, Schlichting]]

Screening approximation to the matrix element

- Compare with simple screening approximation

$$\frac{(s-u)^2}{t^2} \to \frac{(s-u)^2}{t^2}\frac{q^4}{(q^2+\xi_T^2 m_D^2)^2}$$

- Longitudinal[12] $\xi_L = e^{5/6}/\sqrt{8}$
- Transverse broadening: $\xi_T = e^{1/3}/2$
- **Good agreement**



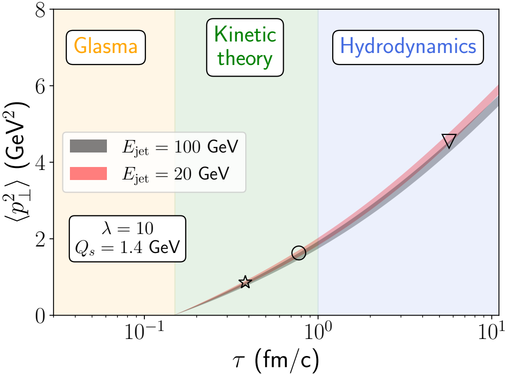
s, u, t: Mandelstam variables

[12] [Phys.Rev.D 89 (2014) [York, Kurkela, Lu, Moore]]

Outline

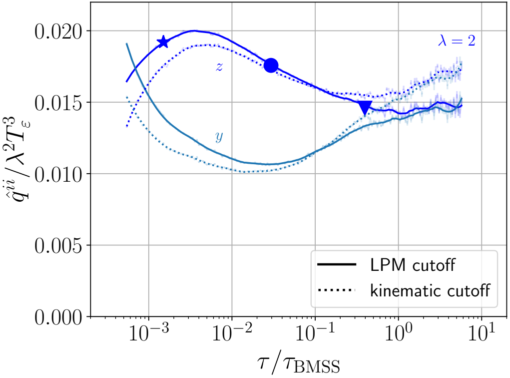
What about momentum broadening?

- Per definition, $\hat{q} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}\tau}$
- Naïvely $\Delta p_\perp^2 = \int \mathrm{d}\tau\, \hat{q}(\tau)$ over lifetime of jet
- But: only true if no splittings occur.
- Think of $\hat{q}$ as medium parameter.



Results for $\hat{q}$

- Use cutoffs
  - $\Lambda_\perp^{\mathrm{LPM}}(E, T_\varepsilon) = \zeta^{\mathrm{LPM}} g(E T_\varepsilon^3)^{1/4}$
  - $\Lambda_\perp^{\mathrm{kin}}(E, T_\varepsilon) = \zeta^{\mathrm{kin}} g(E T_\varepsilon)^{1/2}$



[2303.12595 [Boguslavski, Kurkela, Lappi, FL, Peuron]]

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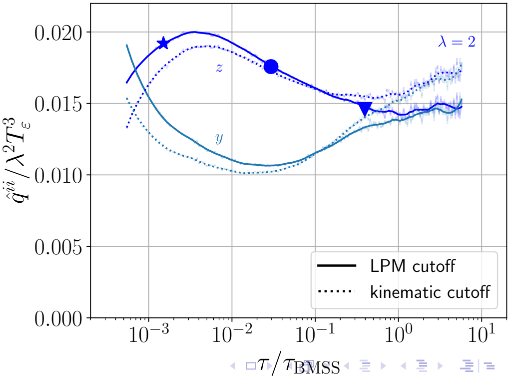

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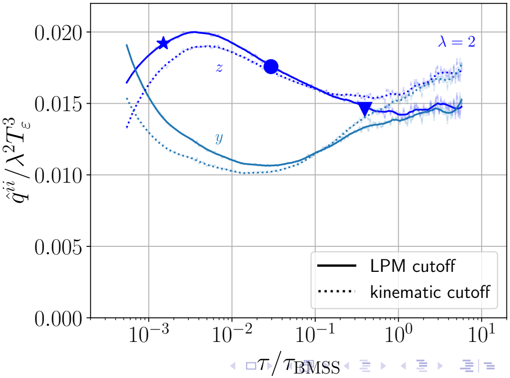
- Obtain $\hat{q}$ for multiple fixed $\Lambda_\perp$.
- Interpolate, using[14]

$$\hat{q}^{xx}(\Lambda_\perp \gg T_\varepsilon) \simeq a_x \ln \frac{\Lambda_\perp}{Q_s} + b_x$$



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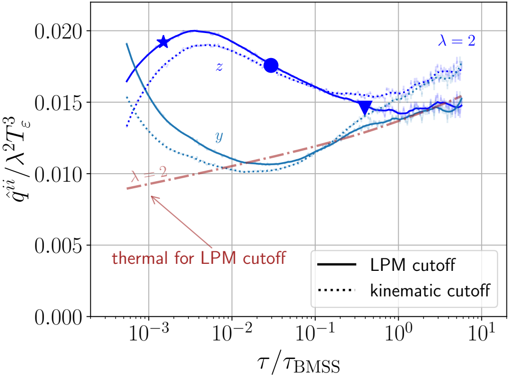
- Mostly $\hat{q}^{zz} > \hat{q}^{yy} \rightarrow$ **Momentum broadening along beam axis enhanced**
- Similar results for both cutoffs



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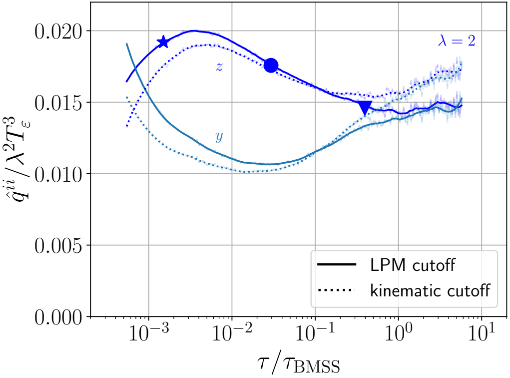

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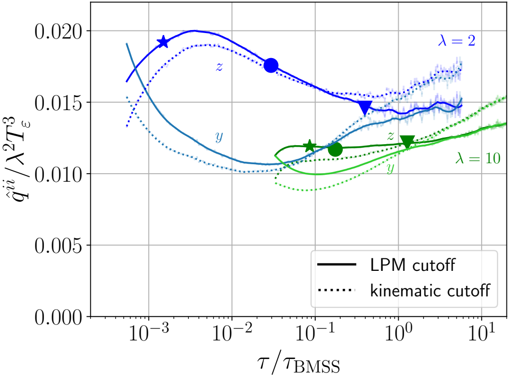

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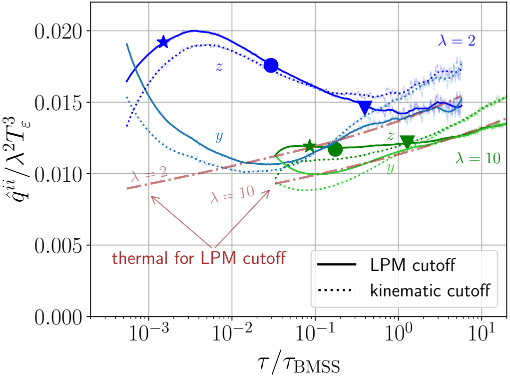

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Transport coefficients

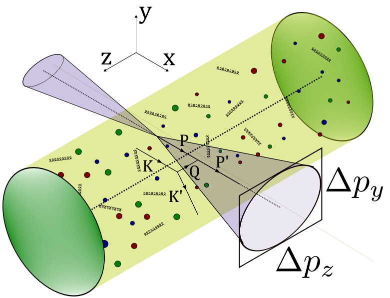
- **Jet quenching parameter:**
  Transverse momentum broadening of jets is quantified by $\hat{q} = \hat{q}^{yy} + \hat{q}^{zz}$,

$$\hat{q}^{ii} = \int \mathrm{d}\Gamma\, (q^i)^2\, |\mathcal{M}|^2\, f(k)(1 + f(k'))$$

- **Heavy-quark diffusion coefficient κ**

$$\kappa^i = \int \mathrm{d}\Gamma_\kappa\, (q^i)^2\, |\mathcal{M}_\kappa|^2\, f(k)(1 + f(k'))$$

  measures momentum transfer to (infinitely) heavy quark



Transport coefficients

Jets: $v \to c$, $m \to 0$
Heavy quark: $v \to 0$, $m \to \infty$

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