Quark production in bottom-up thermalization

ECT* Attractors and thermalization
in nuclear collisions and cold quantum gases

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Xiaojian Du UNIVERSITY OF JYVÄSKYLÄ

Introduction

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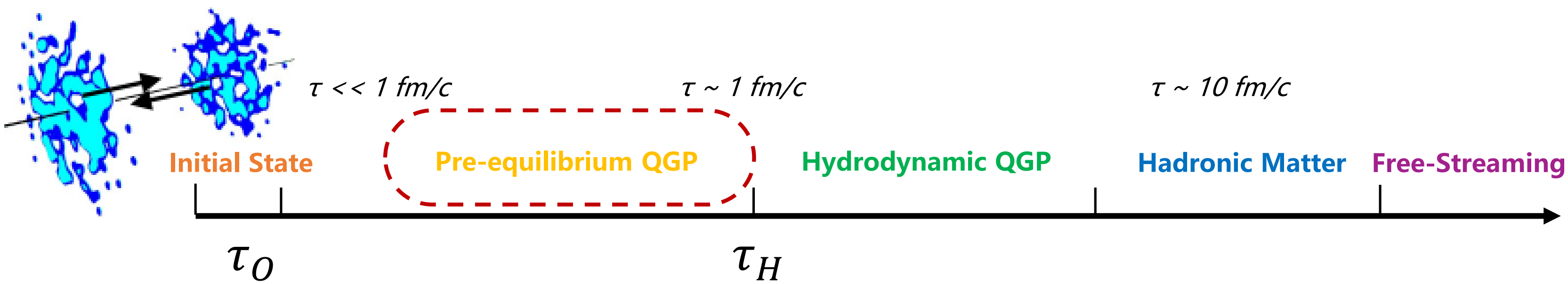
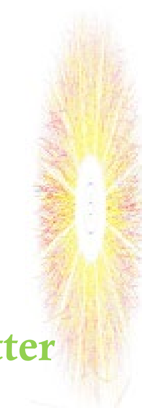
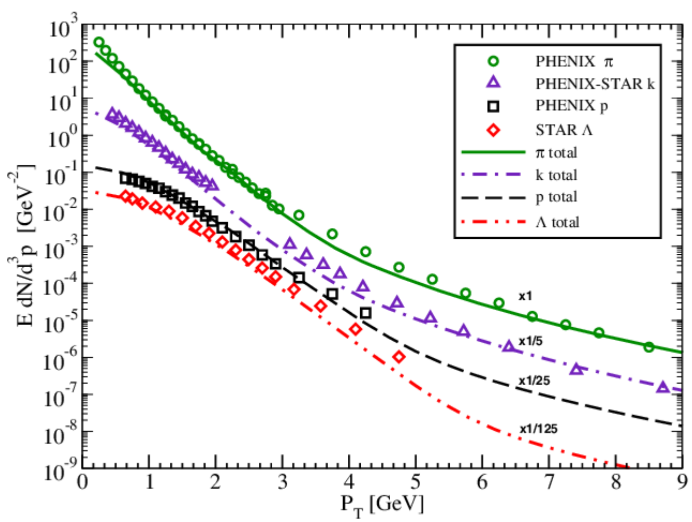
Intro: Heavy-ion collisions

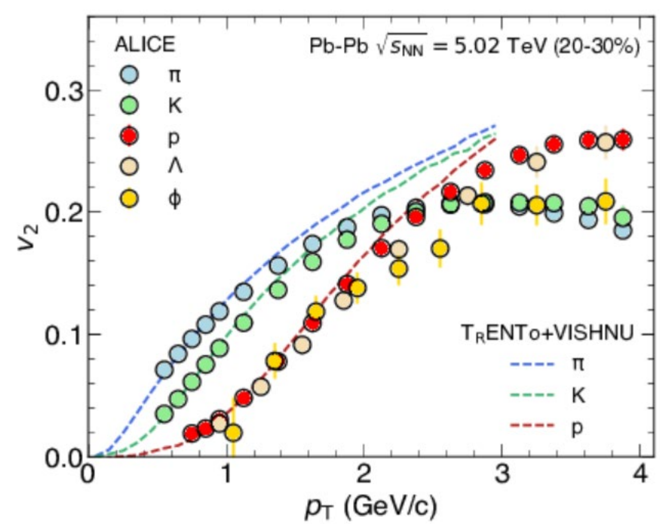
Thermalization of QCD plasma in heavy-ion collisions





Initially non-equilibrium QCD matter ⟹ Eventually in-equilibrium QCD matter





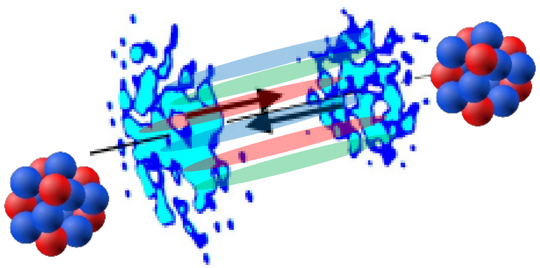
Hydrodynamic behavior (thermalization)

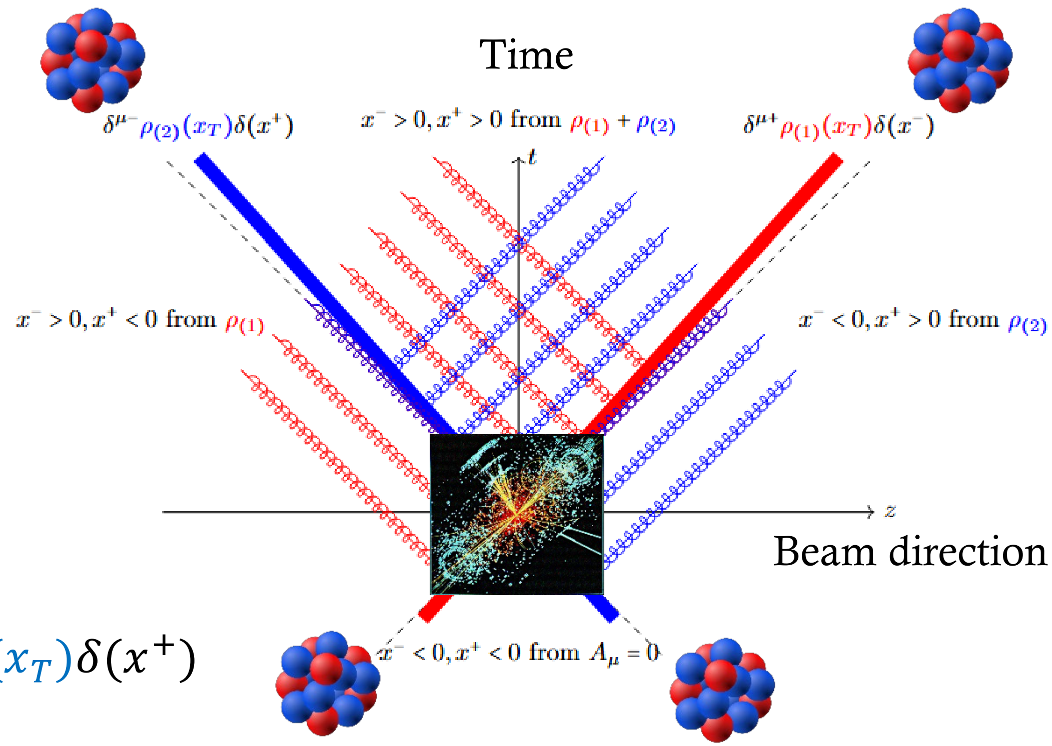
How do QCD matter thermalizes?

Intro: Pre-equilibrium stage

Color glass condensate (CGC)

- Static source fields due to time dilation (Weizsacker-William fields)





- Current:

$$J^\mu = \delta^{\mu+}\rho_1(x_T)\delta(x^-) + \delta^{\mu-}\rho_2(x_T)\delta(x^+)$$

- Generates anisotropic chromo-electromagnetic fields
- McLerran-Venugopalan model (MV):

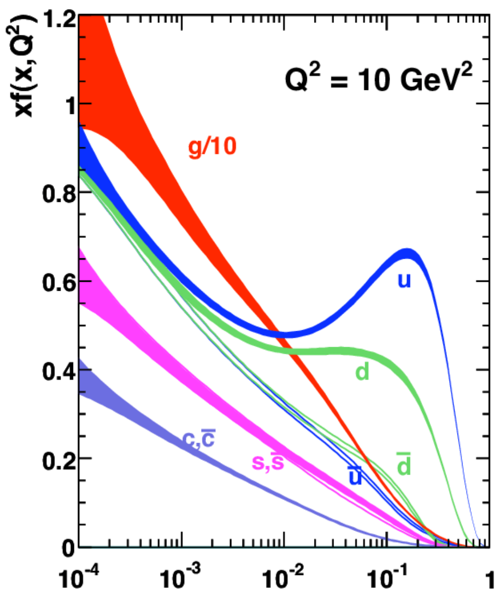
$$\langle \rho_i^a(x_T)\rho_j^b(y_T)\rangle = g^4\mu^2\delta_{ij}\delta^{ab}\delta^{(2)}(x_T - y_T)$$

Intro: Pre-equilibrium stage

Color glass condensate (CGC)

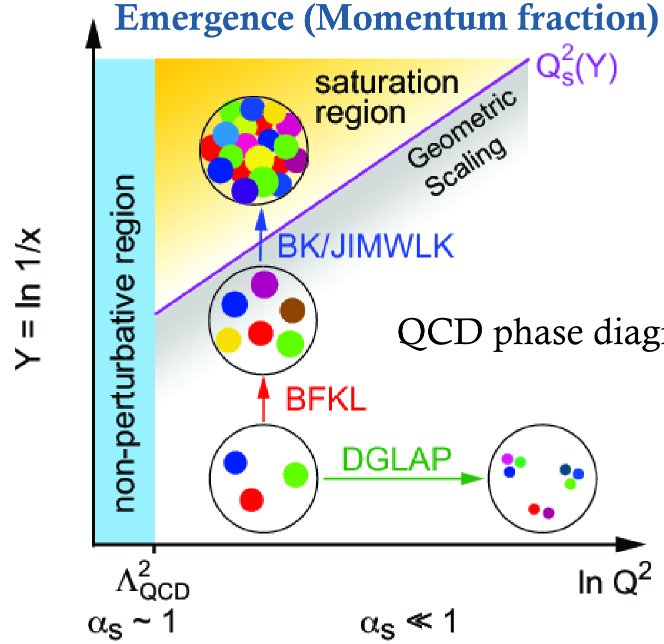
- Static source fields due to time dilation (Weizsacker-William fields)

Parton distribution



Momentum fraction

Emergence (Momentum fraction)



Reductionism (energy)

QCD phase diagram for CGC

- **Gluon saturated**, quantum correction small → Classical Yang-Mills fields

Classical Yang-Mills field evolution

- Real-time lattice gauge theory

Intro: Real-time lattice gauge theory

Real-time lattice gauge theory (LGT)

Hamiltonian's Canonical equation (spatial component)

- Gauge link (canonical variable)

$$\partial_t U_i(x) = \{U_i(x), H\} = igaE_i^a(x + \bar{\imath}/2)t^a U_i(x)$$

- Electric field (conjugate momentum)

$$\partial_t E_i^a(x) = \{E_i^a(x), H\} = -\frac{1}{ga^2}\sum_{i \neq j} \mathrm{Im\,tr}\left[t^a V_{ij}(x)\right]$$

Poisson bracket

Gauss's law (temporal component)

- Parallel transporter to maintain gauge invariance

$$[D_i, E_i^a(x)] = \frac{1}{a}\sum_i \left(E_i^a(x + \bar{\imath}/2) - U_i^+(x - \bar{\imath})E_i^a(x - \bar{\imath}/2)U_i(x - \bar{\imath})\right) = -J_0^a(x)$$

Left electric field $L_i^a(x)$ **Right electric field** $R_i^a(x - \bar{\imath})$ **Charge** $Q^a(x)$

- Gauss's Law in LGT

$$\sum_i \left(L_i^a(x) - R_i^a(x - \bar{\imath})\right) + Q^a(x) = 0$$

Intro: Real-time lattice gauge theory

Turbulence: classical attractor

Instability

- Magnetic fields will isotropize plasma (Weibel instability)

Attractor

- Memory loss … **to fixed point … but not thermal**

Self-similarity (non-thermal fixed point)

- Stationary solution … **but not thermal**

Thermalization: quantum attractor?

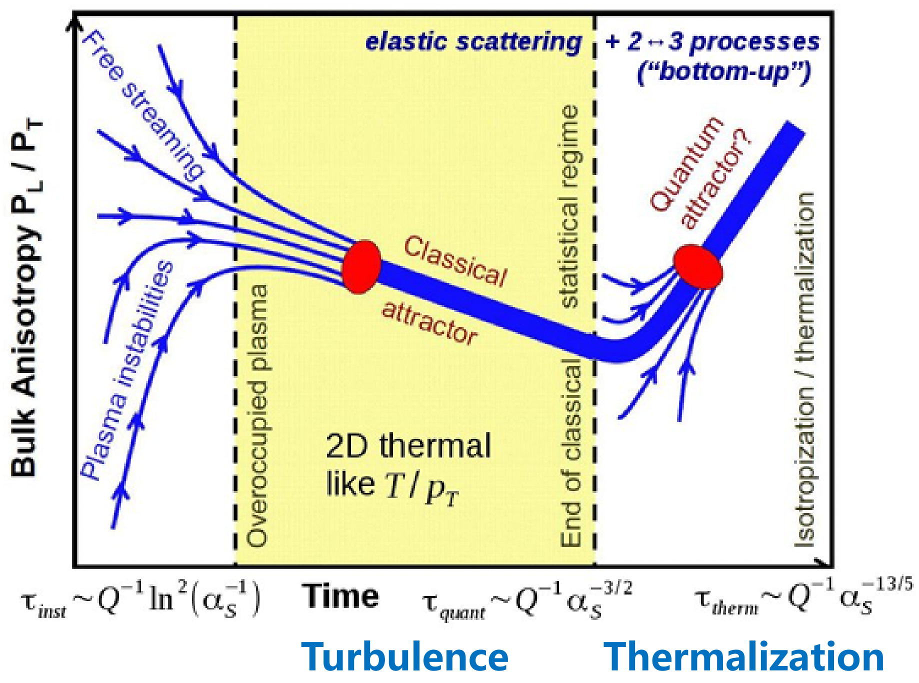
☒ Classical lattice gauge theory

- Fail at small density (quantum fluctuation)
- Difficult with quark (no large density)

☑ Quantum kinetic theory

- Correct **quantum statistics**

Berges, Boguslavski, Schlichting, Venugopalan (2013, 2014); …



Intro: Quantum kinetic theory

QCD effective kinetic theory (EKT)

Arnold, Moore, Yaffe (2003)

2-point correlations from the QCD $A_\mu, \psi_f \rightarrow f_g, f_q$

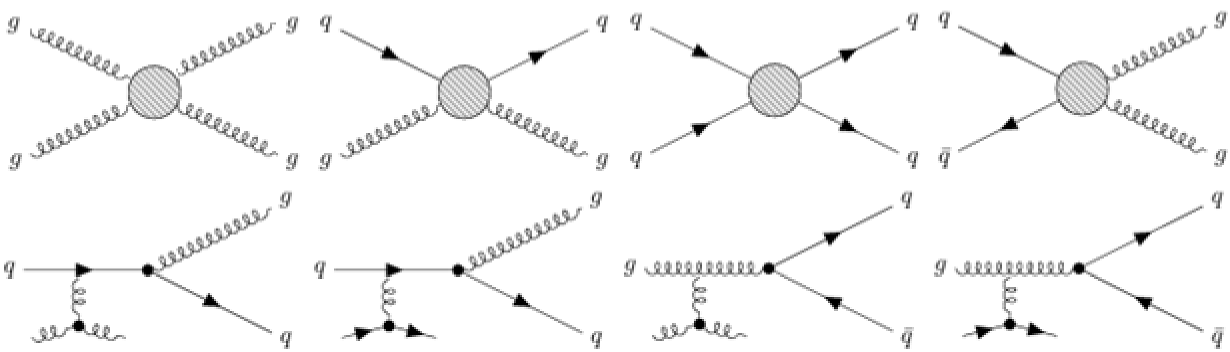
$$\mathcal{L}_{\text{QCD}} = \sum_{f}^{N_f} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m\right)\psi_f \; - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

Set of coupled Boltzmann equations for quarks and gluon distribution:

$$\left(\frac{\partial}{\partial \tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) f_a(\tau, p_T, p_\parallel) = C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_\parallel) + C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_\parallel)$$

$$a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$$

Including both elastic and inelastic scatterings in the QCD:



$C_a^{2\leftrightarrow 2}$

$C_a^{1\leftrightarrow 2}$

Semi-classical:

- Classical: distribution level
- Quantum: quantum statistics (collision integral), amplitudes (QCD diagrams), …

Intro: Quantum kinetic theory

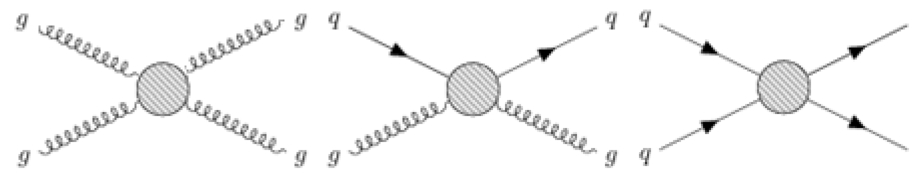
Small angle approximation (numerically less expensive)

Simplified 2-2 Kernel

$$C_a^{2\leftrightarrow 2} = D_a + S_a$$

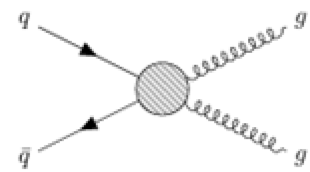
Diffusion

$$D_a = \frac{1}{4} C_a \hat{q}(t) \nabla_p \cdot \left[\nabla_p f_a + \frac{v}{T^*(t)} f_a (1 + \epsilon f_a) \right]$$



Conversion

$$S_q = \frac{2\pi \alpha_s^2 C_F^2 \mathcal{L}}{p} \mathcal{I}_c [f_g(1 - f_q) - f_q(1 + f_g)]$$



$$S_g = \frac{N_f}{C_F} S_q$$

Coefficients

$$\hat{q} = 8\pi \alpha_s^2 \mathcal{L} \int \frac{d^3p}{(2\pi)^3} [N_c f_g (1 + f_g) + N_f f_q (1 - f_q)]$$

$$T^* = \frac{C_A \hat{q}}{\alpha_s \mathcal{L} m_D^2} \qquad m_D^2 = 16\pi \alpha_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} [N_c f_g + N_f f_q]$$

$$\mathcal{L} = ln \left(\frac{\langle p_T^2 \rangle}{m_D^2} \right) \qquad \mathcal{I}_c = \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} [f_g + f_q]$$

Bottom-up thermalization

Bottom-up thermalization

Initial saturation time $\tau \sim 1/Q_s$, hard gluon $p \sim Q_s$, occupation $f \sim 1/\alpha_s$

I. Very early stage: $1 \leq \tau Q_s \leq \alpha_s^{-3/2}$ (weakly coupled $\alpha_s \ll 1$)

$$p_z \sim \frac{Q_s}{(\tau Q_s)^{1/3}} \qquad f_{\text{hard}} \sim \frac{1}{\alpha_s}\frac{1}{(\tau Q_s)^{2/3}} \qquad n_{\text{hard}} \sim \frac{1}{\alpha_s}\frac{Q_s^3}{\tau Q_s} \qquad n_{\text{soft}} \sim \frac{1}{\alpha_s}\frac{Q_s^3}{(\tau Q_s)^{4/3}}$$

Until $f_{\text{hard}} \sim 1$ (classical field theory becomes invalid)

II. Setting-up stage for thermalization:

$$\alpha_s^{-3/2} \leq \tau Q_s \leq \alpha_s^{-5/2}$$

$$n_{\text{hard}} \sim \frac{1}{\alpha_s}\frac{Q_s^3}{\tau Q_s} \qquad n_{\text{soft}} \sim \alpha_s^{1/4}\frac{Q_s^3}{(\tau Q_s)^{1/2}}$$

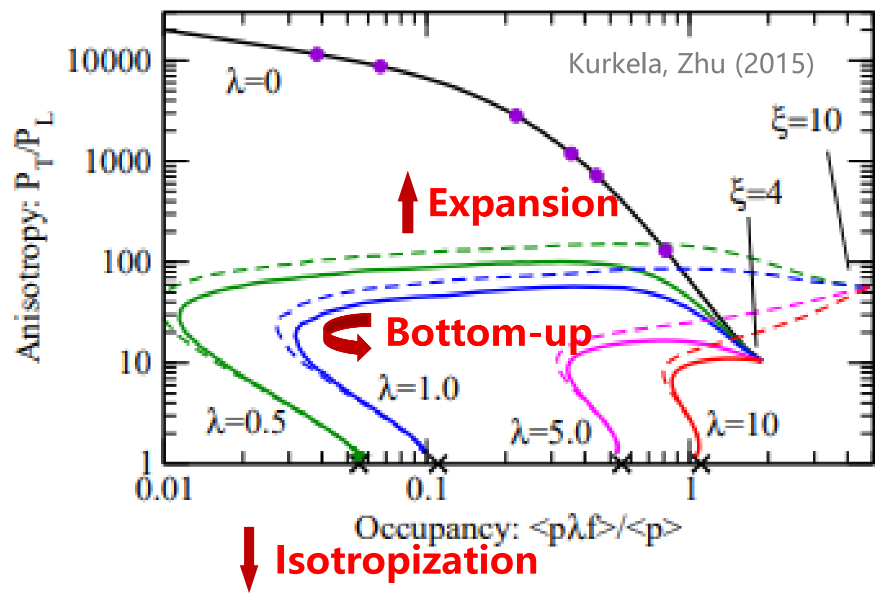
Until $n_{\text{hard}} \sim n_{\text{soft}} \sim \alpha_s^{3/2} Q_s^3$

III. Thermalization stage:

$$\alpha_s^{-5/2} \leq \tau Q_s \leq \alpha_s^{-13/5}$$

Until hard gluon fully quenched

Parameterization



Expansion + Collision → Attractor

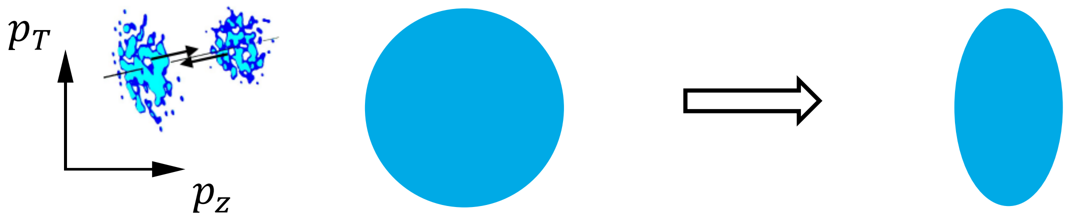
Theory: **Baier, Mueller, Schiff, Son (2000);** Arnold, Lenaghan, Moore (2004);

Simulation: Kurkela, Zhu (2015); Du, Schlichting (2020); Boguslavski, Kurkela, Lapp, Lindenbauer, Peuron (2023); Barrera Cabodevila, Du, Salgado, Wu (2025); ......

Bottom-up thermalization

I. Very early stage: diluting an over-occupied system $1 \leq \tau Q_s \leq \alpha_s^{-3/2}$ $(\alpha_s \ll 1)$

Initial time $\tau \sim 1/Q_s$, hard gluon (assume isotropic) $p_T \sim p_z \sim Q_s$, occupation $f_{h,g} \sim 1/\alpha_s$



Longitudinal expansion

$$n_{h,g} = \frac{1}{Q_s \tau} \int \frac{d^3p}{(2\pi)^3} f_{h,g} \sim \frac{p_T^2 p_z f_{h,g}}{Q_s \tau} \sim \frac{Q_s^3}{\alpha_s Q_s \tau}$$

Longitudinal momentum broadening

$$p_z^2 \sim \hat{q}\tau \sim \alpha_s^2 \int \frac{d^3p}{(2\pi)^3} f_{h,g}^2 \, \tau \sim \alpha_s^2 \frac{n_{h,g}^2}{p_T^2 p_z} \tau \quad \Longrightarrow \quad p_z \sim \frac{Q_s}{(Q_s\tau)^{1/3}}$$

Distribution

$$f_{\mathrm{h,g}} \sim \frac{n_{h,g}}{p_T^2 p_z} \sim \frac{1}{\alpha_s} \frac{1}{(Q_s\tau)^{2/3}}$$

Reach $f \sim 1$ when $\tau Q_s \sim \alpha_s^{-3/2}$ (classical LGT invalid)

Quenching parameter

$$\hat{q} \sim \alpha_s^2 \int \frac{d^3p}{(2\pi)^3} f_{h,g}^2 \sim \alpha_s^2 p_T^2 p_z f_{h,g}^2 \sim \frac{Q_s^3}{(Q_s\tau)^{5/3}}$$

Other parameters

$$m_D^2 \sim \alpha_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} f_{h,g} \sim \alpha_s \frac{n_{h,g}}{Q_s} \sim \frac{Q_s^2}{Q_s \tau}$$

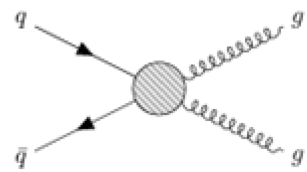
$$\mathcal{I}_c \sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} f_g \sim \frac{m_D^2}{\alpha_s} \qquad T^* \sim \frac{\hat{q}}{\alpha_s m_D^2} \sim \frac{Q_s}{\alpha_s (Q_s\tau)^{2/3}}$$

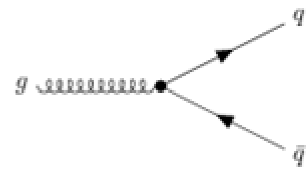
Bottom-up thermalization

I. Very early stage: diluting an over-occupied system $1 \leq \tau Q_s \leq \alpha_s^{-3/2}$ $(\alpha_s \ll 1)$

Initial time $\tau \sim 1/Q_s$, hard gluon (assume isotropic) $p_T \sim p_z \sim Q_s$, occupation $f_{h,g} \sim 1/\alpha_s$

Quark production



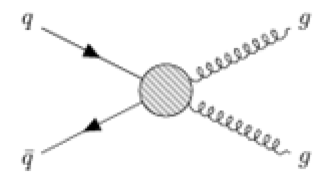


Relevant coefficients $n_{h,g} \sim \frac{Q_s^3}{\alpha_s Q_s \tau}$ $m_D^2 \sim \frac{Q_s^2}{Q_s \tau}$ $\mathcal{I}_c \sim \frac{m_D^2}{\alpha_s}$ $\hat{q} \sim \frac{Q_s^3}{(Q_s \tau)^{5/3}}$

Conversion

$$S_q \sim \frac{\alpha_s^2 \mathcal{I}_c f_{h,g}}{p} \sim \frac{1}{\tau} \frac{1}{(Q_s \tau)^{2/3}}$$

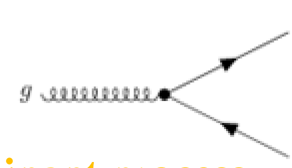
$$n_q \sim \int \frac{d^3p}{(2\pi)^3} S_q \tau \sim \alpha_s^2 \mathcal{I}_c \int \frac{d^3p}{(2\pi)^3} \frac{f_{h,g}}{p} \tau \sim m_D^4 \tau \sim \frac{Q_s^3}{Q_s \tau}$$



Splitting

$$C_q^{1\leftrightarrow 2} \sim \alpha_s \Gamma f_{h,g} \qquad \Gamma \sim \sqrt{\frac{\hat{q}}{p}} \qquad \text{LPM}$$

$$n_q \sim \int \frac{d^3p}{(2\pi)^3} C_q^{1\leftrightarrow 2} \tau \sim \alpha_s \sqrt{\frac{\hat{q}}{p}} n_{h,g} \tau \sim \frac{Q_s^3}{(Q_s \tau)^{5/6}}$$



Dominant process

Quark number small compare to gluon

Soft gluon (from radiation) contributes small portion

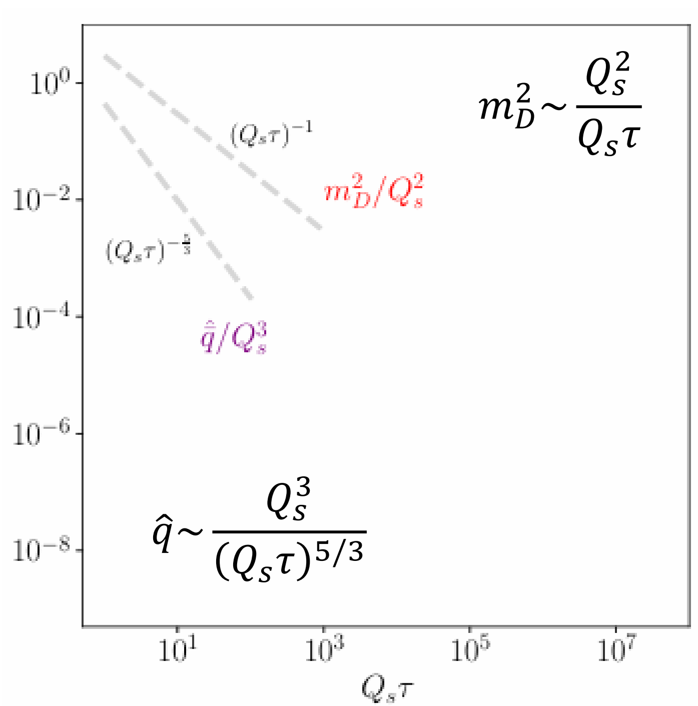
Bottom-up thermalization

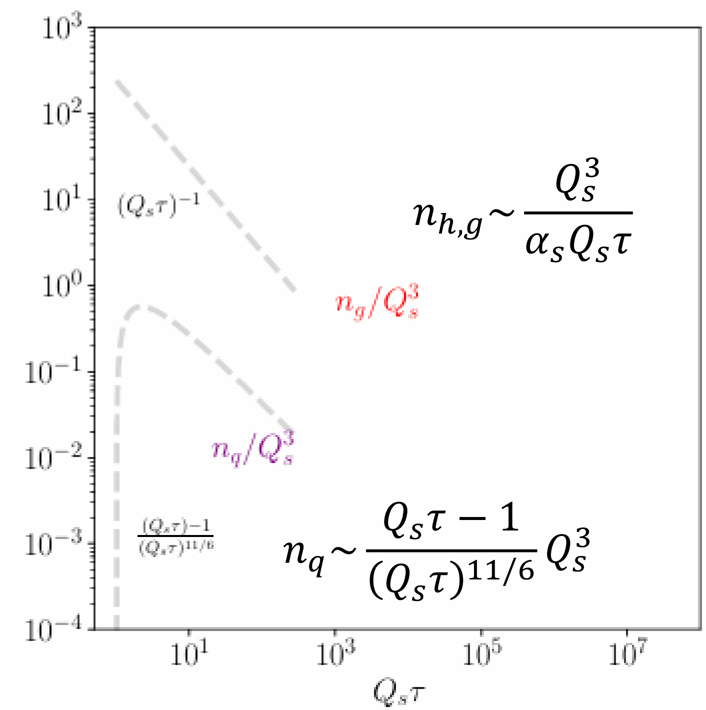
I. Very early stage: diluting an over-occupied system $1 \leq \tau Q_s \leq \alpha_s^{-3/2}$ $(\alpha_s \ll 1)$

Since quarks are initially absent

$$n_q \sim \int_{1/Q_s}^{\tau} d\tau' \frac{Q_s^3}{(Q_s\tau')^{5/6}} \frac{1}{\tau'} \sim \int_{1/Q_s}^{\tau} d\tau' \frac{Q_s^4}{(Q_s\tau')^{11/6}} \sim \frac{Q_s\tau - 1}{(Q_s\tau)^{11/6}} Q_s^3$$

Parametric estimations





Bottom-up thermalization

II. Setting up stage: cooling of soft sector $\alpha_s^{-3/2} \leq \tau Q_s \leq \alpha_s^{-5/2}$ $(\alpha_s \ll 1)$

$$f_{\mathrm{h,g}} \sim \frac{1}{\alpha_s} \frac{1}{(Q_s \tau)^{2/3}} \qquad n_{h,g} \sim \frac{Q_s^3}{\alpha_s Q_s \tau}$$

Occupation $f_{h,g} < 1$

Quenching parameter $$\hat{q} \sim \alpha_s^2 \int \frac{d^3p}{(2\pi)^3} f_{h,g} \sim \alpha_s^2 p_T^2 p_z f_{h,g} \sim \alpha_s^2 n_{h,g} \sim \frac{\alpha_s Q_s^3}{Q_s \tau}$$

Longitudinal momentum broadening $p_z^2 \sim \hat{q}\tau \Longrightarrow p_z \sim \alpha_s^{1/2} Q_s$

Soft gluon from radiation $$n_{s,g} \sim \int \frac{d^3p}{(2\pi)^3} C_g^{1\leftrightarrow 2} \tau \sim \alpha_s \sqrt{\frac{\hat{q}}{p_z}}\, n_{h,g} \tau \sim \frac{\alpha_s^{1/4} Q_s^3}{(Q_s \tau)^{1/2}}$$

Consequently the soft thermal bath contributes to quenching parameters remains smaller than hard sector **until $n_{s,g} \sim n_{h,g}$ when $\tau Q_s \sim \alpha_s^{-5/2}$**

Soft sector dominates Debye mass since $$m_{h,D}^2 \sim \alpha_s \frac{n_{h,g}}{Q_s} < m_{s,D}^2 \sim \alpha_s \frac{n_{s,g}}{p_z}$$

$$m_D^2 \sim \alpha_s \frac{n_{s,g}}{p_z} \sim \frac{\alpha_s^{3/4} Q_s^2}{(Q_s \tau)^{1/2}} \qquad T^* \sim \frac{\hat{q}}{\alpha_s m_D^2} \sim \frac{Q_s}{\alpha_s^{3/4} (Q_s \tau)^{1/2}}$$

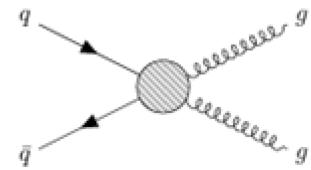
Bottom-up thermalization

II. Setting up stage: cooling of soft sector $\alpha_s^{-3/2} \le \tau Q_s \le \alpha_s^{-5/2}$ $(\alpha_s \ll 1)$

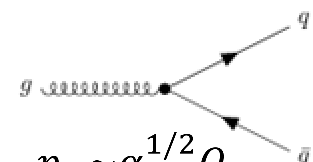
Quark production

Relevant coefficients $\quad n_{h,g} \sim \frac{Q_s^3}{\alpha_s Q_s \tau} \quad n_{s,g} \sim \frac{\alpha_s^{1/4} Q_s^3}{(Q_s\tau)^{1/2}} \quad m_D^2 \sim \frac{\alpha_s^{3/4} Q_s^2}{(Q_s\tau)^{1/2}} \quad \hat{q} \sim \frac{\alpha_s Q_s^3}{Q_s \tau}$

Conversion
(soft dominate) $\quad n_q \sim m_D^4 \tau \sim \alpha_s^{3/2} Q_s^3$



Hard splitting $\quad n_{h,q} \sim \alpha_s \sqrt{\frac{\hat{q}}{p}} n_{h,g} \tau \sim \frac{\alpha_s^{1/2} Q_s^3}{(Q_s\tau)^{1/2}}$



Soft splitting $\quad n_{s,q} \sim \alpha_s \sqrt{\frac{\hat{q}}{p_z}} n_{s,g} \tau \sim \alpha_s^{3/2} Q_s^3$

$p_z \sim \alpha_s^{1/2} Q_s$

(Comparable to conversion)

Competing density of hard n_q and soft $n_{s,q}$, two sub-stages

$n_q \sim \frac{\alpha_s^{1/2} Q_s^3}{(Q_s\tau)^{1/2}}$ When $\alpha_s^{-3/2} \le \tau Q_s \le \alpha_s^{-2}$

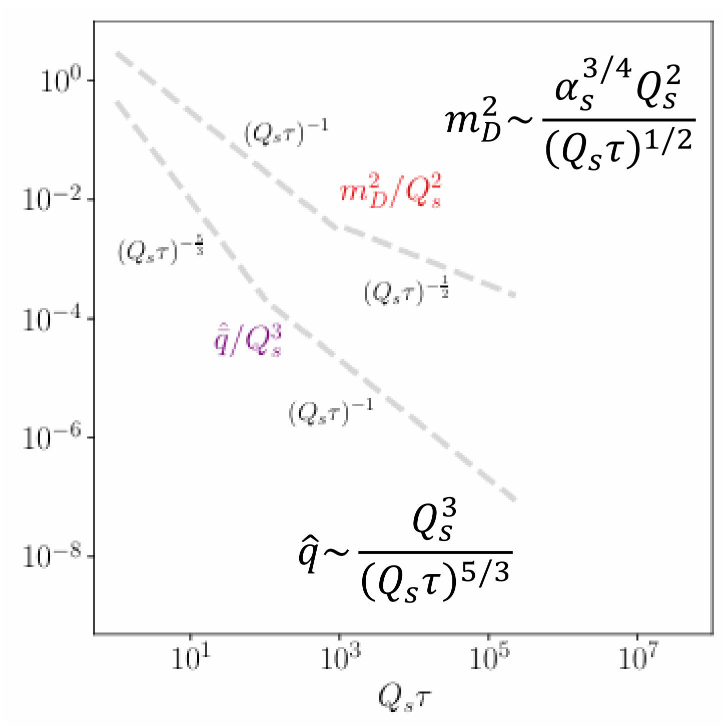
$n_q \sim \alpha_s^{3/2} Q_s^3$ When $\alpha_s^{-2} \le \tau Q_s \le \alpha_s^{-5/2}$

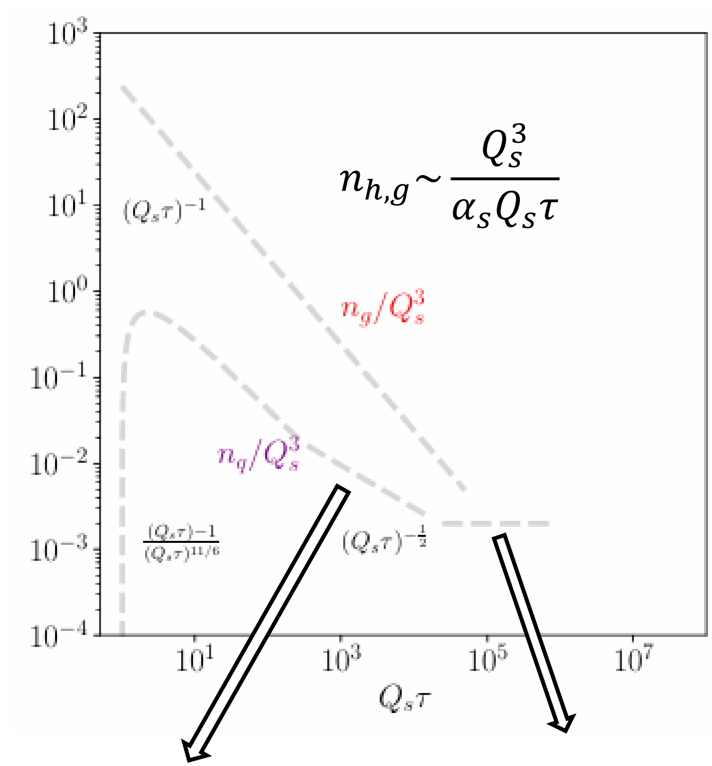
Reach $n_q \sim n_{s,g} \sim n_{h,g}$ when $\tau Q_s \sim \alpha_s^{-5/2}$

Bottom-up thermalization

II. Setting up stage: cooling of soft sector $\alpha_s^{-3/2} \le \tau Q_s \le \alpha_s^{-5/2}$ $(\alpha_s \ll 1)$

Parametric estimations





Sub-stages I: decreasing

$$n_q \sim \frac{\alpha_s^{1/2} Q_s^3}{(Q_s\tau)^{1/2}}$$

Sub-stages II: flat

$$n_q \sim \alpha_s^{3/2} Q_s^3$$

Bottom-up thermalization

III. Thermalization: heating up a QGP bath and mini-jet quenching

$\alpha_s^{-5/2} \leq \tau Q_s \leq \alpha_s^{-13/5}$ $(\alpha_s \ll 1)$

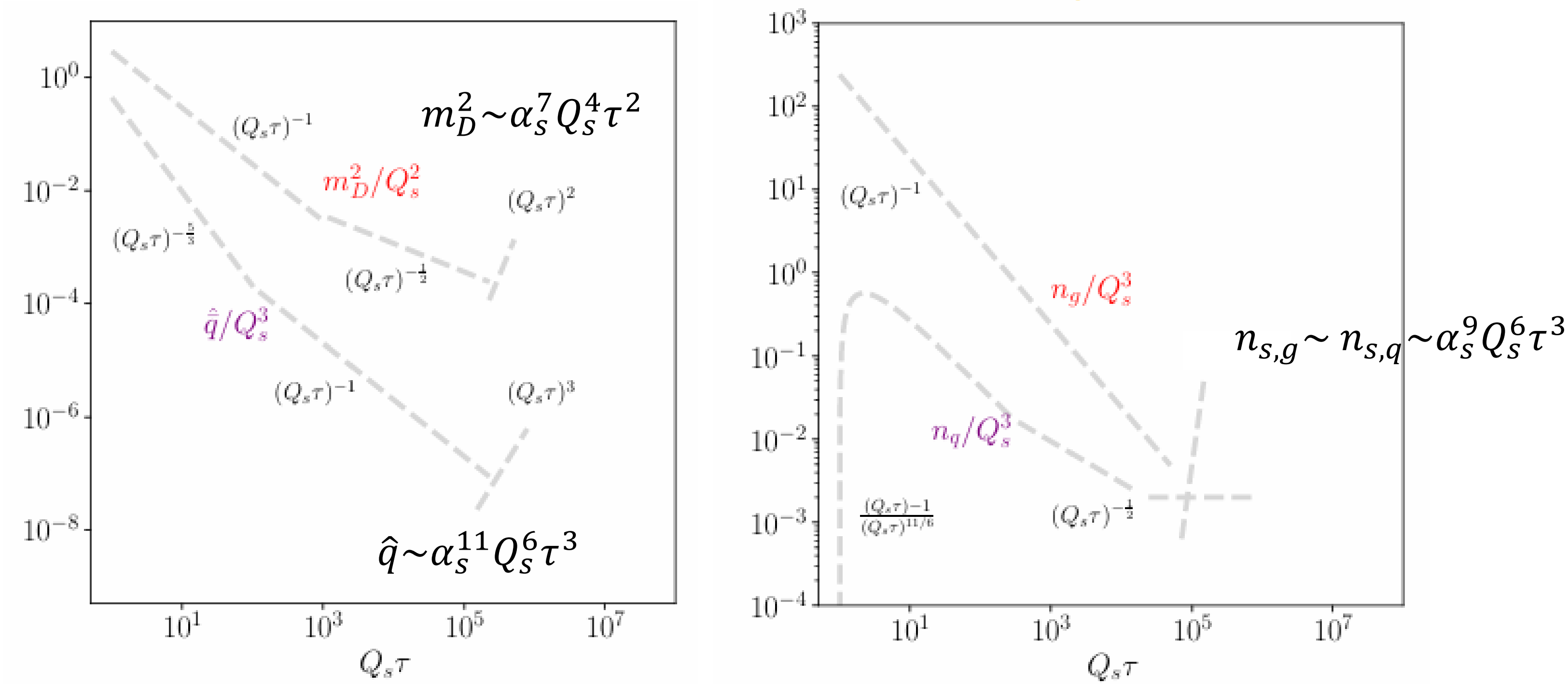
Energy loss/quenching of hard gluon/mini-jet

Each parton $p_{br} \sim \alpha_s^2 \hat{q} \tau^2$ Total $\mathcal{E}_{loss} \sim p_{br} n_{h,g} \sim (T^*)^4$ $n_{h,g} \sim \dfrac{Q_s^3}{\alpha_s Q_s \tau}$

Parameters $T^* \sim \alpha_s^3 Q_s^2 \tau$ $\Longrightarrow$ $m_D^2 \sim \alpha_s (T^*)^2 \sim \alpha_s^7 Q_s^4 \tau^2$ $\hat{q} \sim \alpha_s^2 (T^*)^3 \sim \alpha_s^{11} Q_s^6 \tau^3$

$\mathcal{E}_{loss} \sim (T^*)^4 \sim \alpha_s^{12} Q_s^8 \tau^4$ $n_{s,g} \sim n_{s,q} \sim (T^*)^3 \sim \alpha_s^9 Q_s^6 \tau^3$

Thermalized until hard partons all quenched $\mathcal{E}_{loss} \sim Q_s n_{h,g}$ when $\tau Q_s \sim \alpha_s^{-13/5}$



Bottom-up thermalization

IV. Post-thermalization: Bjorken expansion

$\alpha_s^{-13/5} \leq \tau Q_s \ (\alpha_s \ll 1)$

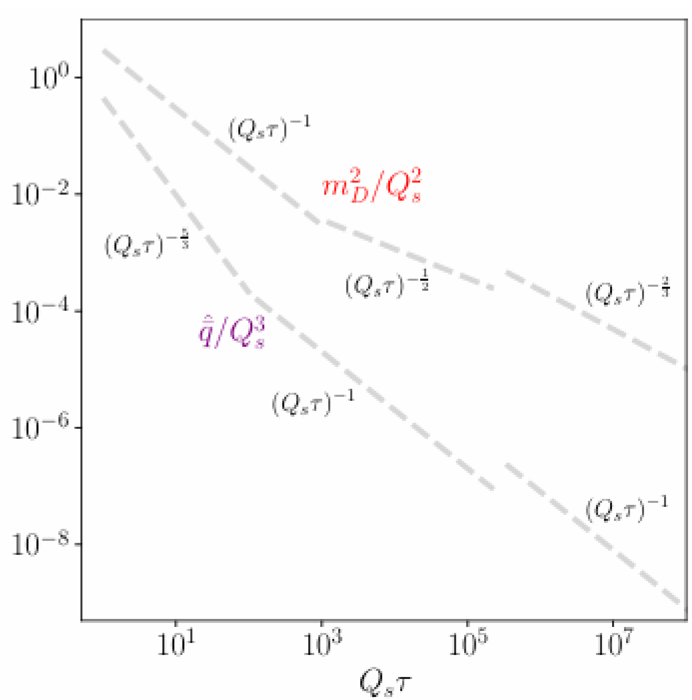
Expanding thermal system

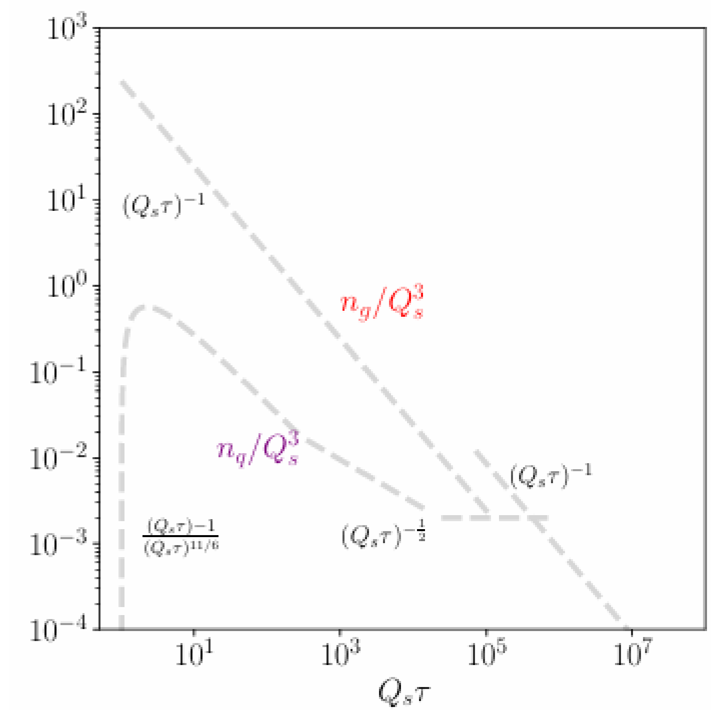
$n_g \sim n_q \sim \tau^{-1}$ $\quad T^* \sim \tau^{-1/3}$ $\quad \hat{q} \sim \tau^{-1}$ $\quad m_D^2 \sim \tau^{-2/3}$

Matching to values at time $\tau Q_s = \alpha_s^{-13/5}$

$n_g \sim n_q \sim \alpha_s^{-7/5} Q_s^3 (Q_s\tau)^{-1}$ $\quad T^* \sim \alpha_s^{-7/15} Q_s (Q_s\tau)^{-1/3}$

$\hat{q} \sim \alpha_s^{3/5} Q_s^3 (Q_s\tau)^{-1}$ $\quad m_D^2 \sim \alpha_s^{1/15} Q_s^2 (Q_s\tau)^{-2/3}$





Summary

Bottom-up thermalization

- Intro: Road to thermalization with quantum kinetic theory
- Scaling behaviors for quark production in bottom-up thermalization
- Numerical comparison with kinetic theory simulation

Thanks!

Numerical results