Quasinormal modes of nonthermal fixed points

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Based on 2502.01622 and work in progress

With Michal P. Heller

Introduction

- NTFPs are attractors → Why?

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- Presence of decaying Quasinormal modes

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- Adiabatic hydrodynamization: decay of excited states (see Bruno’s talk)

Introduction

- NTFP is attractor → Why?

- Presence of decaying Quasinormal modes

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- Adiabatic hydrodynamization: decay of excited states (see Bruno's talk)

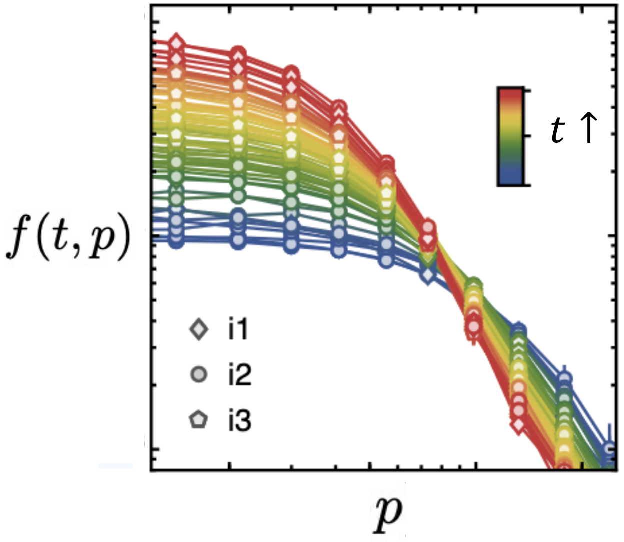
- Isotropic and Bjorken flow NTFPs

Self-similarity

- $f(t,p) = \left(\frac{t}{t_{ref}}\right)^{\alpha} f_s\left(\left(\frac{t}{t_{ref}}\right)^{\beta} p\right)$

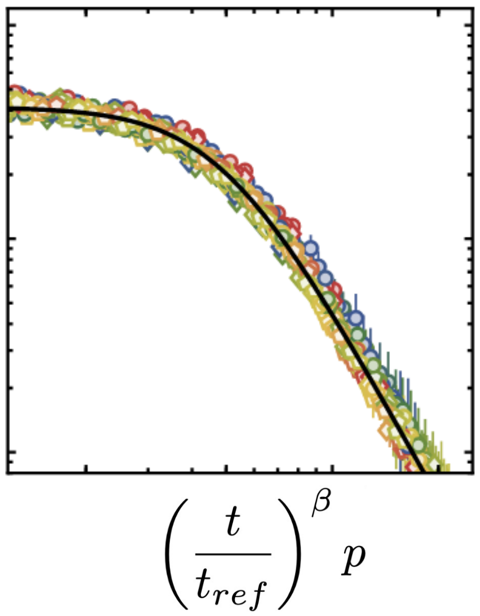
α, β and f_s universal

- Function of 2 variables → function of 1 variable



→ $\left(\frac{t}{t_{ref}}\right)^{-\alpha} f(t,p)$

"Static"



reference

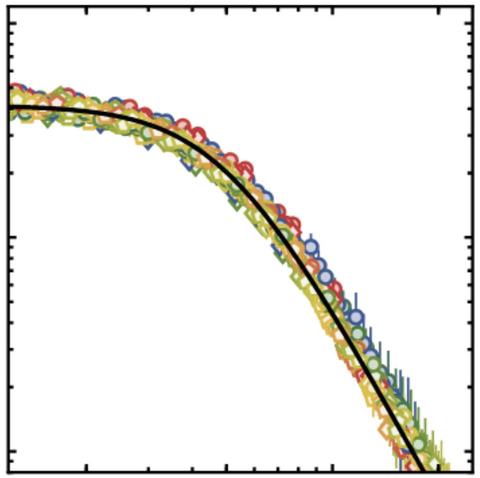
M. Gazo, A. Karailiev, T. Satoor, C. Eigen, M. Gałka, Z. Hadzibabic 2312.09248

Static frame

$$f(t,p) = \left(\frac{t}{t_{ref}}\right)^{\alpha} f_s\left(\left(\frac{t}{t_{ref}}\right)^{\beta} p\right)$$

Static frame

$$\left(\frac{t}{t_{ref}}\right)^{-\alpha} f(t,p)$$



$$\bar{p} \equiv \left(\frac{t}{t_{ref}}\right)^{\beta} p$$

- Effectively time independent
- Analogous to equilibrium distribution function or static BH
- Perturbation in static frame

$$\left(\frac{t}{t_{ref}}\right)^{-\alpha} f(t,p) = f_s(\bar{p}) + \delta f(t,\bar{p})$$

Isotropic Quasinormal modes

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$$t\partial_t \delta f(t,\bar{p}) = \hat{L}[f_s](\bar{p})\delta f(t,\bar{p})$$

- Time dependence factorizes

$$\delta f(t,\bar{p}) = \left(\frac{t}{t_{ref}}\right)^{i\Omega\beta} \delta f_\Omega(\bar{p})$$

- Eigenvalue equation

$$i\Omega\delta f_\Omega(\bar{p}) = \frac{1}{\beta}\hat{L}[f_s](\bar{p})\delta f_\Omega(\bar{p})$$

Ω and $\delta f_\Omega(\bar{p})$ define QNM of NTFP

Power laws!

Fokker-Planck kinetic theory

- Gluons
- Elastic scatterings
- Small angle approximation of QCD kinetic theory

$$\partial_t f = \frac{\lambda^2}{4\pi} \xcancel{\mathcal{L}} \left[\frac{I_a}{p^2} \partial_p \left(p^2 \partial_p f \right) + \frac{I_b}{p^2} \partial_p \left(p^2 f \left(f + \xcancel{1} \right) \right) \right]$$

$\beta = -1/7$ and $\alpha = 4\,\beta$

$$I_a = \int \frac{d^3p}{(2\pi)^3} f(f + \xcancel{1})$$

$$I_b = 2 \int \frac{d^3p}{(2\pi)^3} \frac{f}{p}$$

[1] M.P. Heller, A. Mazeliauskas, T. Preis 2307.07545
[2] J. Brewer, B. Scheihing-Hitschfeld, Y. Yin 2203.02427

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Role of conservation laws

$$\epsilon = \int \frac{d^d p}{(2\pi)^d} p^z f(t,p) = \int \frac{d^d \bar{p}}{(2\pi)^d} \bar{p}^z f_s(\bar{p}) + \delta\epsilon(t)$$

- Energy density of perturbation

$$\delta\epsilon(t) = \left(\frac{t}{t_{ref}}\right)^{i\Omega\beta} \underbrace{\int d^d\bar{p}\bar{p}^z \delta f_\Omega(\bar{p})}_{\equiv \delta\epsilon_\Omega}$$

→

Constant if:

1) $\Omega = 0$

Or

2) $\delta\epsilon_\Omega = 0$

Isotropic QNM spectrum

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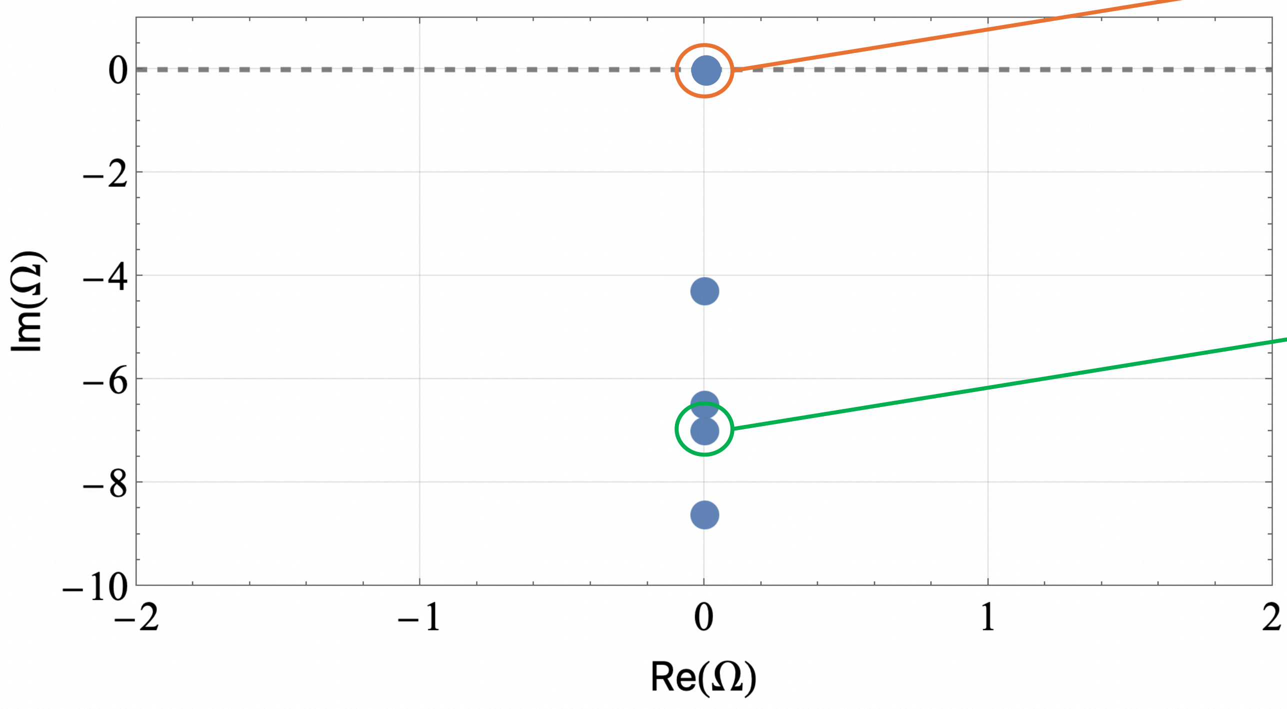


Zero mode

$$t_{ref} \to t_{ref} + \delta t_{ref}$$

Prescaling mode[1,2]

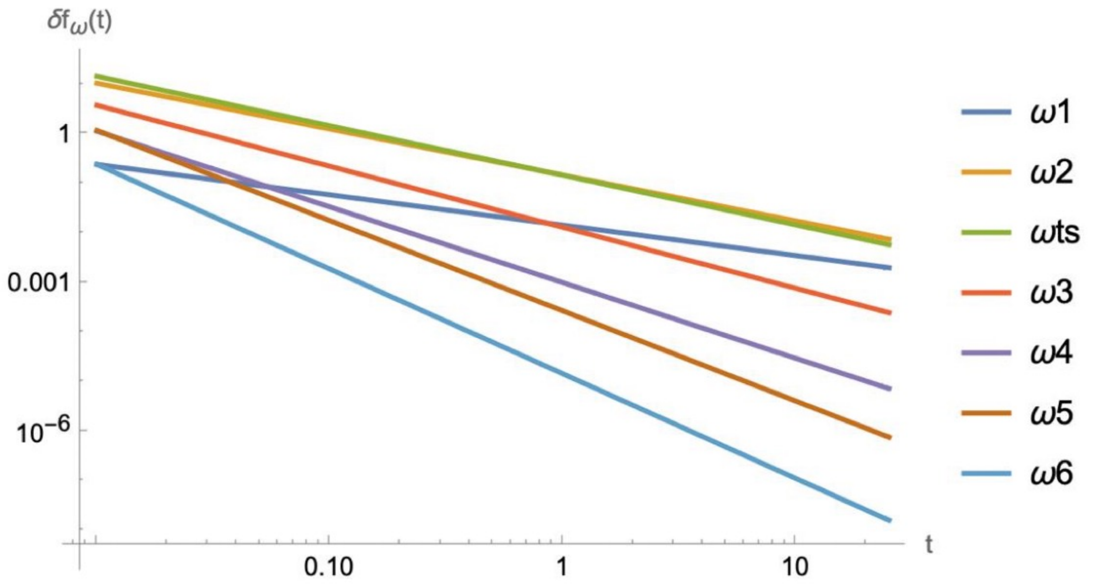
$$\left(\frac{t - t_*}{t_{ref}}\right)^{\beta}$$

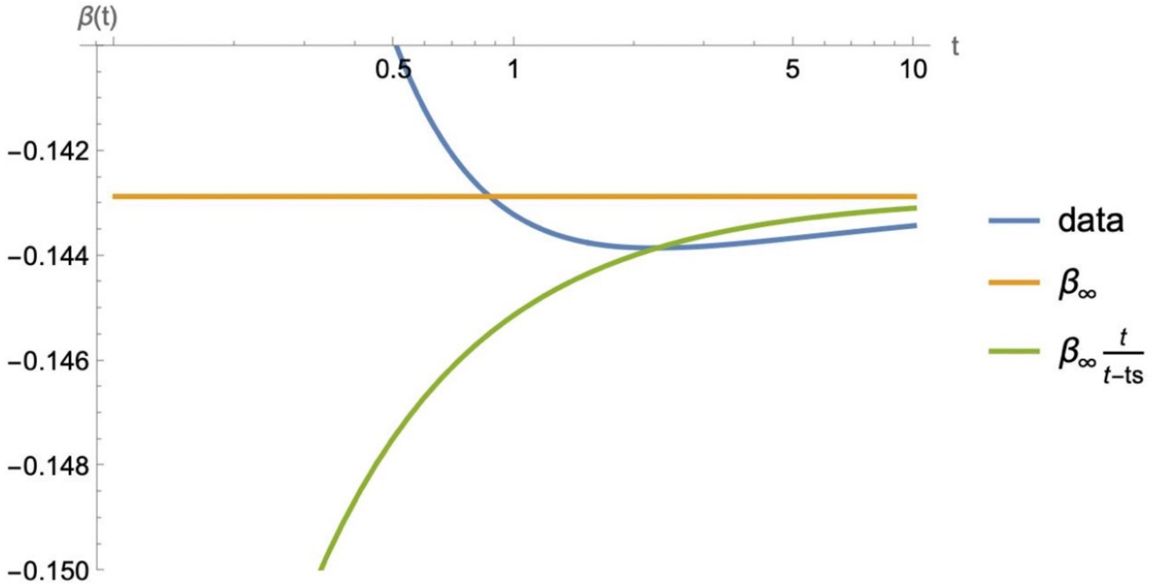
$$t_* \to t_* + \delta t_*$$

[1] M.P. Heller, A. Mazeliauskas, T. Preis 2307.07545

[2] M. Gazo, A. Karailiev, T. Satoor, C. Eigen, M. Gałka, Z. Hadzibabic 2312.09248

No clear prescaling





BMSS NTFP

- Longitudinal expansion

- First stage of bottom up thermalization picture[1]

$$(\alpha, \beta, \gamma) = (-2/3, 0, 1/3)$$

$$f(\tau, p_\perp, p_z) = \tau^\alpha f_s\left(\tau^\beta p_\perp, \tau^\gamma p_z\right)$$

[1] R. Baier, A.H. Mueller, D. Schiff, and D.T. Son (2001)

Kinetic theory set up

See Bruno’s talk

- Early times: longitudinal momentum derivatives dominate
- Simplification of collision kernel

$$\partial_\tau f - \frac{p_z}{\tau}\partial_{p_z} f = \lambda_0 \cancel{\mathcal{L}} I_a \partial_{p_z}^2 f$$

$$I_a = \int \frac{d^3p}{(2\pi)^3} f(f + \cancel{1})$$

Scaling function

$$f(\tau, p_\perp, p_z) = \tau^\alpha f_s\left(\tau^\beta p_\perp, \tau^\gamma p_z\right) \qquad (\alpha, \beta, \gamma) = (-2/3, 0, 1/3)$$

- Equation for scaling function[1]

$$\alpha f_s(\bar{p}_\perp, \bar{p}_z) + \beta \bar{p}_\perp \partial_{\bar{p}_\perp} f_s(\bar{p}_\perp, \bar{p}_z) + (\gamma - 1)\, \bar{p}_z \partial_{\bar{p}_z} f_s(\bar{p}_\perp, \bar{p}_z) = \tau_{ref} \lambda_0 I_a[f_s] \partial^2_{\bar{p}_z} f_s(\bar{p}_\perp, \bar{p}_z)$$

- Analytical solution[2]

$$f_s(\bar{p}_\perp, \bar{p}_z) = f_\perp(\bar{p}_\perp) e^{-\frac{\bar{p}_z^2}{2c1}}$$

[1] J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan 1303.5650

[2] J. Brewer, B. Scheihing-Hitschfeld, Y. Yin 2203.02427

QNM calculation

Work in progress

$$f(\tau, p_\perp, p_z) = \tau^\alpha f_s(\bar{p}_\perp, \bar{p}_z) + \tau^\alpha \delta f(\tau, \bar{p}_\perp, \bar{p}_z)$$

$$\alpha^{-1} \tau \partial_\tau \delta f = -\delta f - \bar{p}_z \partial_{\bar{p}_z} \delta f - \partial^2_{\bar{p}_z} \delta f - \frac{\delta I_a}{I_{as}} \partial^2_{p_z} f_s$$

$$\delta f(\tau, \bar{p}_\perp, \bar{p}_z) = \left(\frac{\tau}{\tau_{ref}}\right)^{\alpha\Omega} \delta f_\Omega(\bar{p}_\perp, \bar{p}_z)$$

Again power laws!

QNM solution

Work in progress

$$\delta f(\tau, \bar{p}_\perp, \bar{p}_z) = \left(\frac{\tau}{\tau_{ref}}\right)^{-\frac{2}{3}\Omega} \delta f_\Omega(\bar{p}_\perp, \bar{p}_z)$$

$$\Omega_n = 0, 1, 4, 6, 8, \ldots \quad \Longrightarrow \text{Decay!}$$

$$\delta f_{\Omega_n} \propto \left(\mathrm{He}_{2n}(\bar{p}_z) + r_{2n}(-1 + \bar{p}_z^2)\right) e^{-\bar{p}_z^2/2}$$

$$r_{2n} = \frac{\int_{-\infty}^{\infty} d\bar{p}_z e^{\bar{p}_z^2} \mathrm{He}_{2n}(\bar{p}_z)}{(3/2 - n)\sqrt{\pi}}$$

Adiabatic Hydrodynamization

See Bruno's talk

- J. Brewer, B. Scheihing-Hitschfeld, Y. Yin 2203.02427 considered same NTFP in AH framework

$$f(t, p_\perp, p_z) = A(t) w(t, B(t) p_\perp, G(t) p_z)$$

$$\tau \partial_\tau w = -H[w] w$$

- AH frame

Adiabaticity:

$$\frac{\lambda_0 \mathcal{L} A(\tau)^2 G(\tau) I_a[w](\tau)}{1 + \gamma(\tau)} = 1$$

$$\gamma(\tau) = \tau \frac{\partial_\tau G}{G}$$

Ground state energy=0 :

$$\tau A(\tau) G(\tau)^{-1} = \text{Const}$$

AH results

Adiabatic hydrodynamization
$w(\tau,\xi) = \sum_{n=0} d_n A(\tau)^{\mathcal{E}'_n} \phi_n(\xi)$
$\mathcal{E}'_n = 2n$
$\phi_n(\xi) = \frac{1}{\sqrt{2\pi}(2n)!} \mathrm{He}_{2n}(\xi) e^{-\frac{\xi^2}{2}}$
$\xi = G(t) p_z$

$$\frac{\lambda_0 \mathcal{L} A(\tau)^2 G(\tau) I_a[w](\tau)}{1+\gamma(\tau)} = 1$$

$$\tau A(\tau) G(\tau)^{-1} = \mathrm{Const}$$

J. Brewer, B. Scheihing-Hitschfeld, Y. Yin 2203.02427

Comparison

Adiabatic hydrodynamization	Quasinormal mode picture
$w(\tau,\xi) = \sum_{n=0} d_n A(\tau)^{\mathcal{E}'_n} \phi_n(\xi)$	$\delta f(\tau,\bar{p}_z) = \sum_{n=0} c_n \tau^{\alpha\Omega_n} \delta f_{\Omega_n}(\bar{p}_z)$
$\mathcal{E}'_n = 2n$	$\Omega_n = 0, \boxed{1}, 4, 6, 8, \ldots$
$\phi_n(\xi) = \frac{1}{\sqrt{2\pi}(2n)!} \mathrm{He}_{2n}(\xi) e^{-\frac{\xi^2}{2}}$	$\delta f_{\Omega_n} \propto \left(\mathrm{He}_{2n}(\bar{p}_z) + r_{2n}(-1+\bar{p}_z^2)\right) e^{-\bar{p}_z^2/2}$
$\xi = G(t) p_z$	$\bar{p}_z = \left(\frac{\tau}{\tau_{ref}}\right)^{\gamma} p_z$

J. Brewer, B. Scheihing-Hitschfeld, Y. Yin 2203.02427

Work in progress

Work in progress

Can we understand discrepancy?

- AH frame : $\frac{\lambda_0 \mathcal{L} A(\tau)^2 G(\tau) I_a[w](\tau)}{1+\gamma(\tau)} = 1$ and $\tau A(\tau) G(\tau)^{-1} = \text{Const}$
- A and G depend on w through $I_a[w]$
- Consider $w(\tau,\xi) = \phi_0(\xi) + \epsilon A(\tau)^{2n} \phi_n(\xi)$
- To linear order in ϵ:

$$f_s(\bar{p}_z) + \epsilon \left(c1\tau^{-\alpha} \delta f_{\Omega_1}(\bar{p}_z) + c2\tau^{\Omega_n \alpha} \delta f_{\Omega_n}(\bar{p}_z) \right)$$

Conclusion

- Both QNM and AH predict decay → explains attractor
- QNM: power law decay for both NTFPs
- AH: approach determined by adiabatic frame → non-trivial
- Near NTFP the AH result reduces to sum of QNMs

Outlook

- QNMs for dual cascade
- Observe more QNMs in cold atom experiments
- Spatial dependence and hydro