Hydrodynamic attractors in periodically driven systems

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Attractors and thermalization in nuclear collisions and cold quantum gases

Work in collaboration with Tilman Enss and Toshali Mitra
AM, Enss arXiv:2501.19240
see also Fujii, Enss, PRL (2024), arXiv:2404.12921

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

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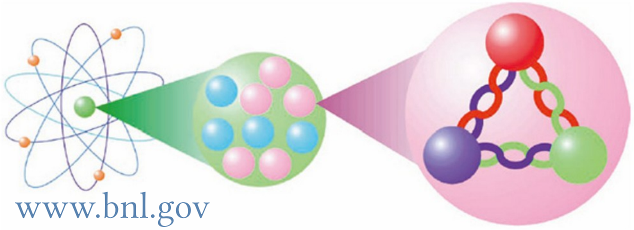
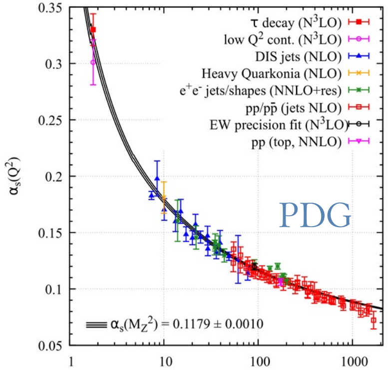
Strongly correlated quantum systems

Quantum Chromodynamics

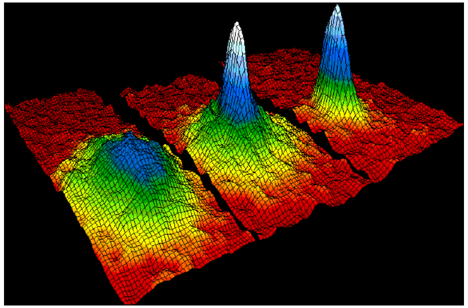


www.bnl.gov

QCD coupling α_s runs with energy scale

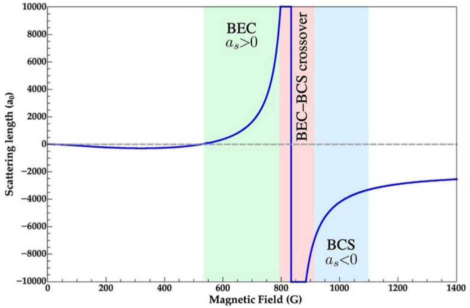


Ultracold quantum gases



NIST/JILA/CU-Boulder

Interaction strength tunable by external field



Hernández-Rajkov et al., 2008.05046

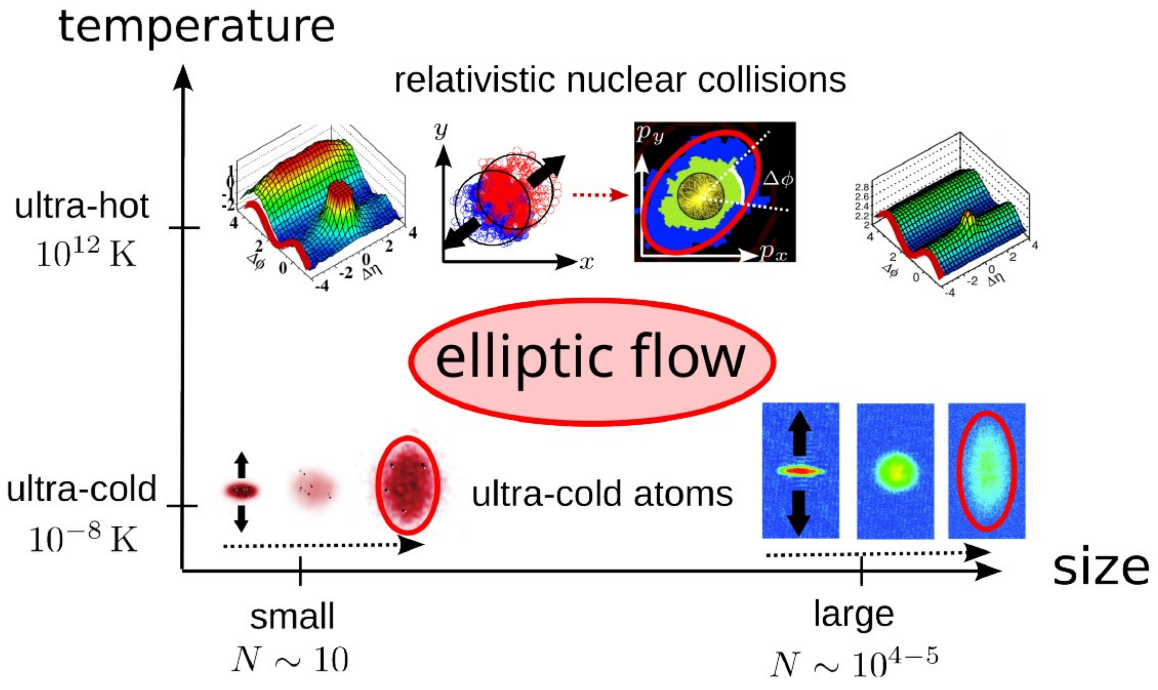
Many-body dynamics in mesoscopic systems

EMMI Rapid Reaction Task Force

Few is different: deciphering many-body dynamics in mesoscopic quantum gases

Juergen Berges[1], Sandra Brandstetter[2], Jasmine Brewer[3], Georg Bruun[4], Tilman Enss (organizer)[1], Stefan Floerchinger[5], Keisuke Fujii[1,6,7], Maciej Gałka[2], Giuliano Giacalone* (organizer)[8,1], Qingze Guan[9], Carl Heintze[2], Lars H. Heyen[10,1], Ilya Selyuzhenkov[11], Selim Jochim (organizer)[2], Jesper Levinsen[12], Philipp Lunt[2], Silvia Masciocchi (organizer)[2,8], Aleksas Mazeliauskas* (organizer)[1], Nir Navon[13,14], Alice Ohlson[15], Meera Parish[12], Stephanie M. Reimann[16], Francesco Scazza[17,18], Thomas Schäfer[19], Derek Teaney[20], Joseph Thywissen[21], Raju Venugopalan[22], Yangqian Yan[23], Matteo Zaccanti[24,25], and Torsten V. Zache[26,27]

arXiv:2509.05049



cold atoms

heavy ions

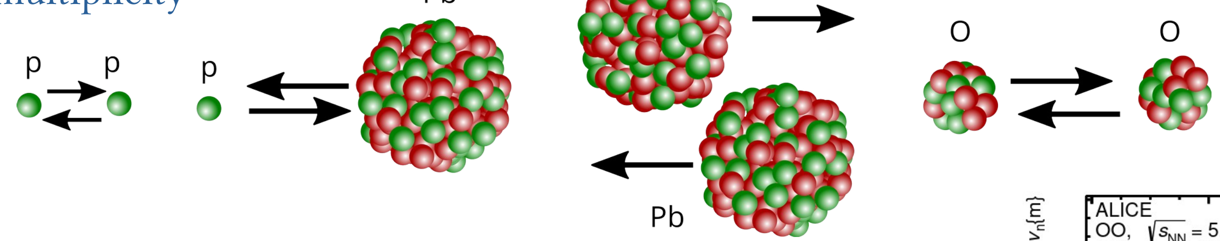
experiment theory

ABC project: collectivity with few ultracold fermionic atoms

Short OO and NeNe run July 1-7 at LHC

Small system toolbox

high-multiplicity asymmetric peripheral minimum-bias

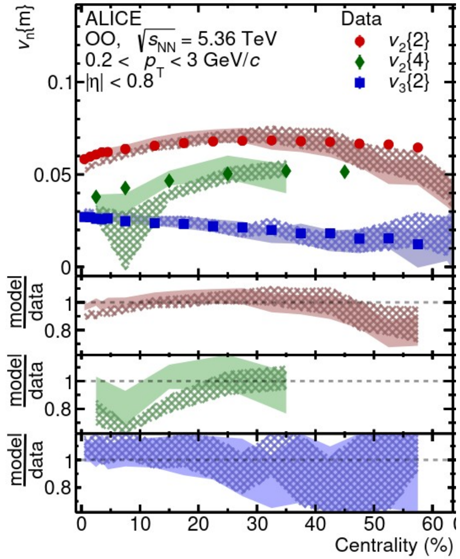


Light ions:
- "natural" small system
- good control of initial geometry

First results

→ great agreement with hydro predictions

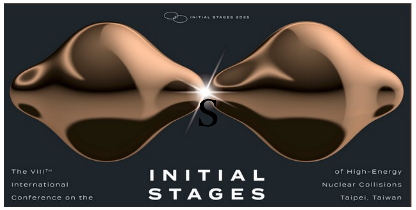
Light-ions: precision tests of emergent collectivity in QCD



ALICE, 2509.06428

2021 - 1st workshop

2024 - 2nd workshop



2025 – first results

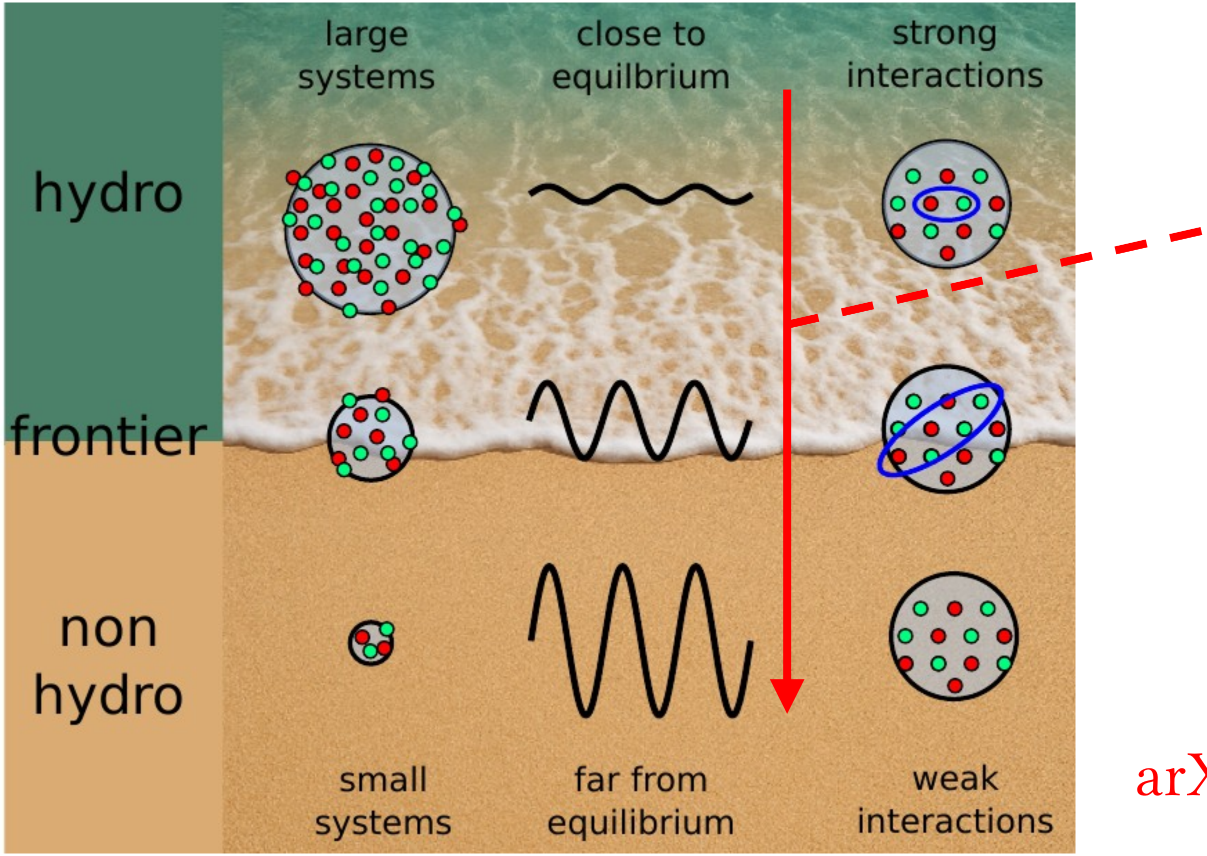
TBD

2026 - 3rd workshop

Aleksas Mazeliauskas, aleksas.eu

Frontiers of hydrodynamic (in)applicability

Quantifying emergence: when the whole is more than the sum of its parts



attractors

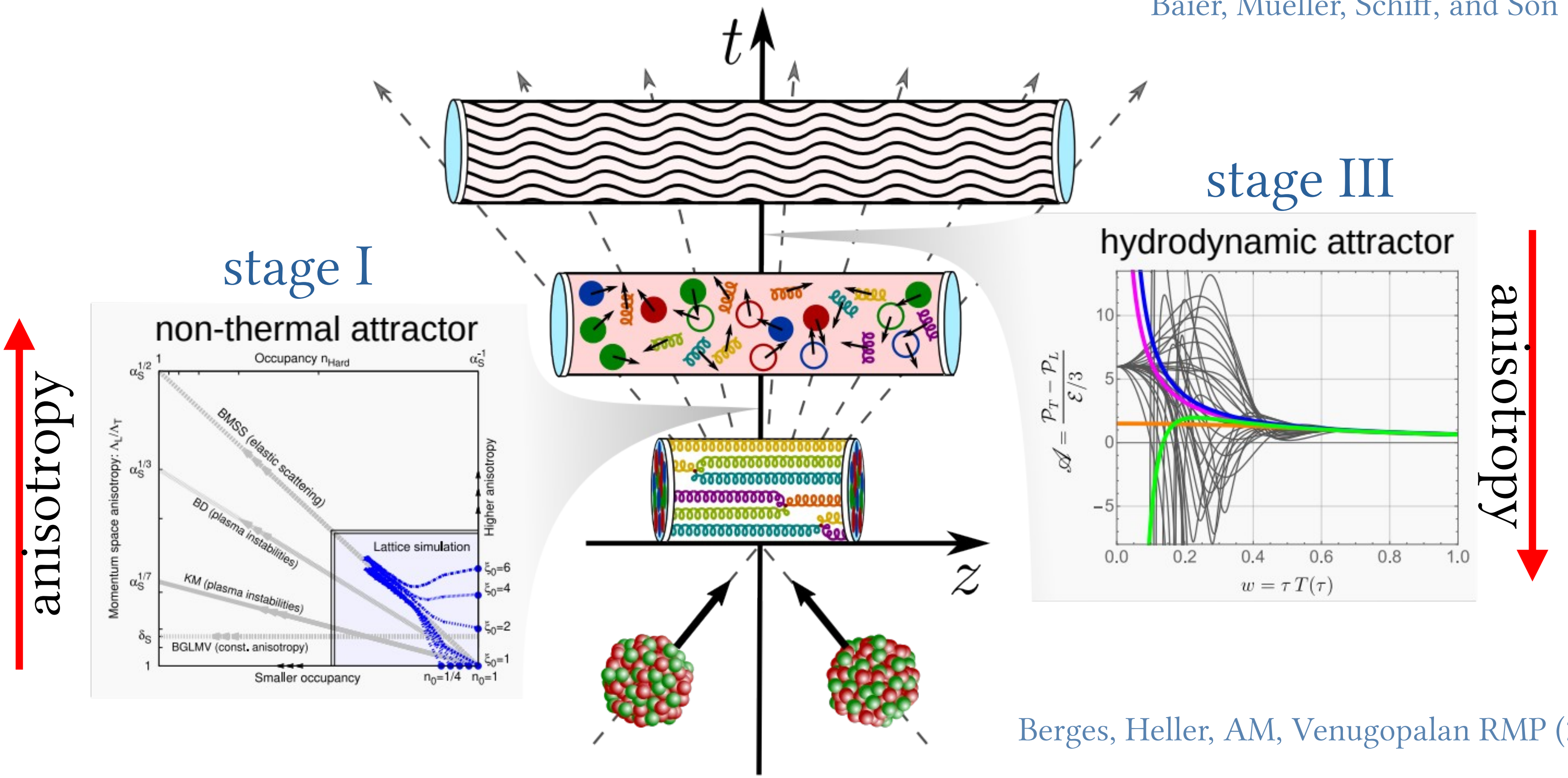
arXiv:2509.05049

Hydrodynamic attractors in heavy-ion collisions

Attractors in QCD thermalisation

Universality and simplification in bottom-up thermalization

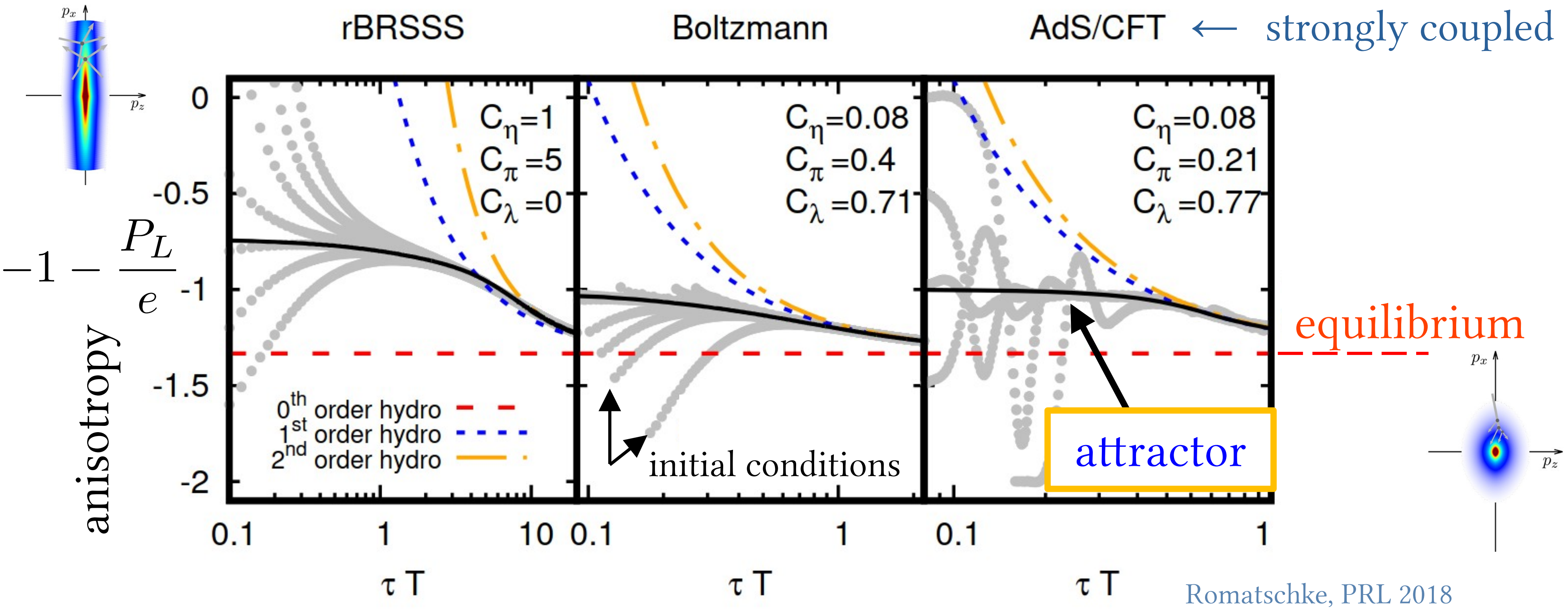
Baier, Mueller, Schiff, and Son (2001)



Berges, Heller, AM, Venugopalan RMP (2021)

Stage III: hydrodynamic attractor

Heller, Spaliński PRL (2015)



Romatschke, PRL 2018

Microscopic evolution rapidly collapses to hydrodynamic attractor function.

→ similar results with QCD kinetic theory Kurkela, AM, Paquet, Schlichting and Teaney, PRC, PRL (2018)

Hydrodynamics as gradient expansion

First viscous correction

$$\frac{T^{xx}_{\rm NS} - P_{\rm eq}}{P_{\rm eq}} = \frac{2}{3}\frac{\eta}{\tau P_{\rm eq}} \sim \frac{1}{\tilde{w}}$$

Higher-order $\tilde{w}^{-n}$ expansion diverges!

Heller, Spaliński PRL (2015)

$$\tilde{w} = \frac{\tau T}{4\pi\eta/s} = \frac{\text{relaxation rate}}{\text{expansion rate}}$$

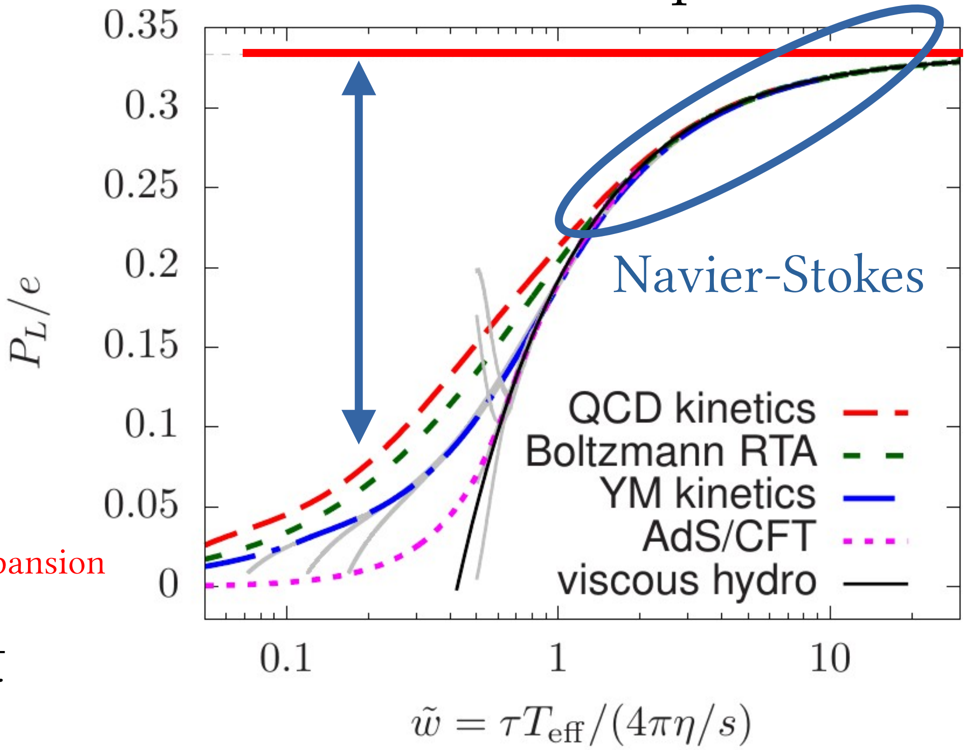
monotonic expansion

$$\partial_\mu u^\mu = \frac{1}{\tau}$$

What if expansion is non-monotonic?

$\longrightarrow$ periodically driven systems

$\tilde{w} \to \infty \Rightarrow$ equilibrium



Giacalone, AM, Schlichting, PRL (2019)

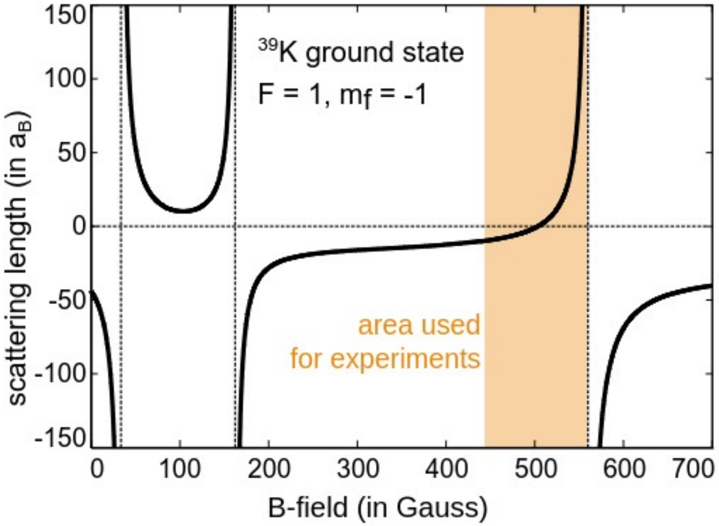
see however work by Toshali and Alexander on Gubser flow, 2408.04001, 2307.10384

Hydrodynamic attractor in periodically driven systems

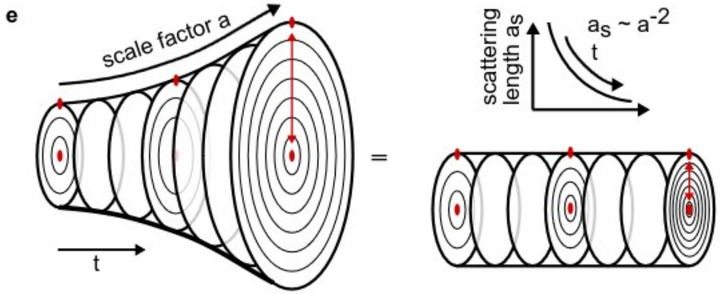
Ultracold atom experiments

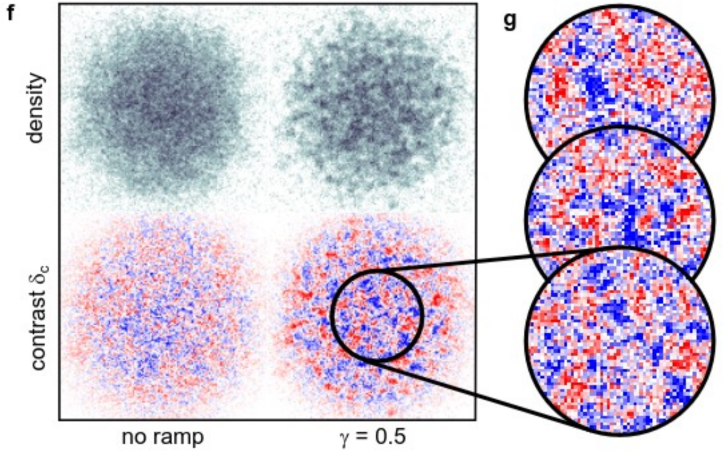
- Controllable interaction strength
- Time-resolved imaging
- Spatial or momentum imaging

$$g = \frac{4\pi\hbar^2}{m} a_s$$



Expanding space simulator in static BEC





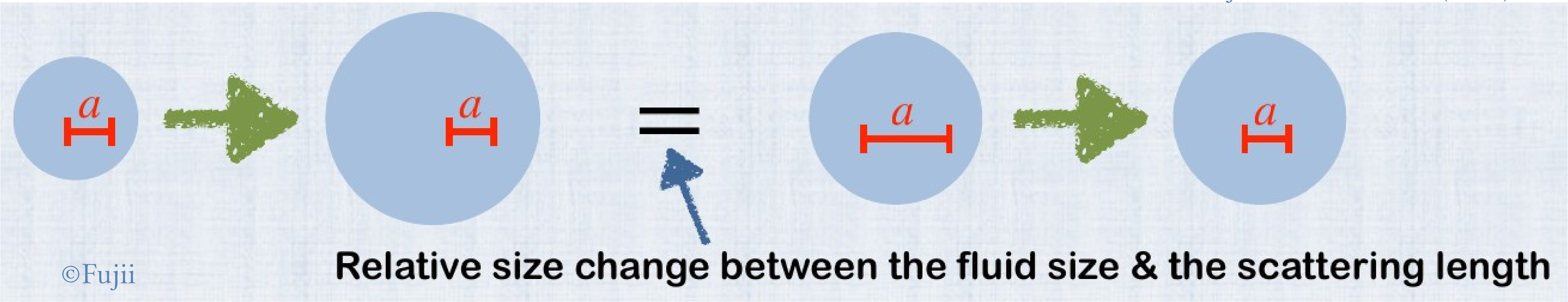
Viermann et al. Nature (2022)

Time dependent scattering length

See talk by Keisuke

Equivalence of isotropic expansion and changing scattering length

Fujii, Nashida, PRA (2018)



Bulk viscosity resists expansion

$$\Pi_{\rm NS}(t) = \frac{1}{3}T^i_{\ i} - P_{\rm eq} = -\zeta \vec{\nabla}\cdot\vec{v} \quad \Longleftrightarrow \quad \Pi_{\rm NS} = -\zeta 3 \frac{\partial_t a^{-1}(t)}{a^{-1}}$$

Can study isotropic expansion in static gas with time-dependent scattering length.

Müller-Israel-Stewart theory

Linear response relaxation of bulk pressure

See talk by Tilman

$$\tau_\zeta \dot{\Pi}(t) = -\Pi(t) + \Pi_{\rm NS}(t)$$

microscopic relaxation time

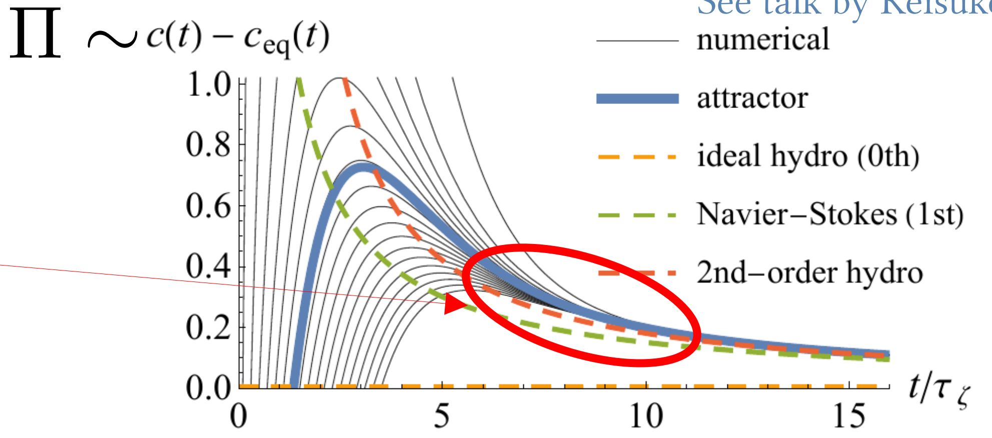
MIS solution: $$\Pi(t) = e^{-t/\tau_\zeta}\Pi(0) + \frac{1}{\tau_\zeta}\int_0^t dt'\, e^{(t'-t)/\tau_\zeta}\Pi_{\rm NS}(t')$$

Hydrodynamic attractor in cold atoms near unitarity $a^{-1}(t) \propto t^{-\alpha}$

See talk by Keisuke

$$\Pi_{\rm NS}(t) = -\underset{\to 0}{\zeta}\ \underset{\to\infty}{\vec{\nabla}\cdot\vec{v}} \to 0$$

Small window of
universal & non Navier-Stokes
hydrodynamic behaviour



Fujii & Enss, PRL (2024) 2404.12921

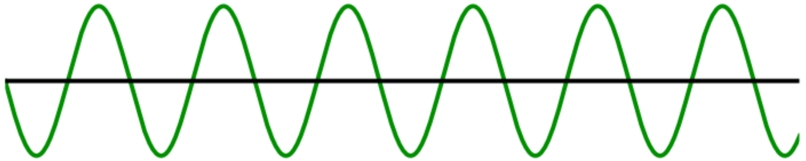
Periodic expansion and contraction

Consider non-monotonic expansion:

$$a^{-1}(t) = a_0^{-1}(1 + A\sin\omega t)$$

$$\partial_\mu u^\mu = 3A\cos\omega t$$

$$\tilde{w} \sim (\partial_\mu u^\mu)^{-1} \quad \leftarrow \text{diverges}$$



Scaled time not the right variable!

Instead consider deviation from equilibrium vs Navier-Stokes expectation

$$\frac{T^{xx} - P_{\text{eq}}}{P_{\text{eq}}} \quad \text{vs} \quad \frac{T^{xx}_{\text{NS}} - P_{\text{eq}}}{P_{\text{eq}}} = -\frac{\zeta}{P_{\text{eq}}\tau_\zeta} 3A\omega\tau_\zeta \cos\omega t$$

τ_ζ --- bulk relaxation time $\quad \dfrac{\zeta}{P_{\text{eq}}\tau_\zeta}$ --- bulk property

Periodically driven systems

Navier-Stokes expectation

$$\Pi_{\rm NS}(t) = -3A\omega\zeta\cos(\omega t)$$

MIS attractor

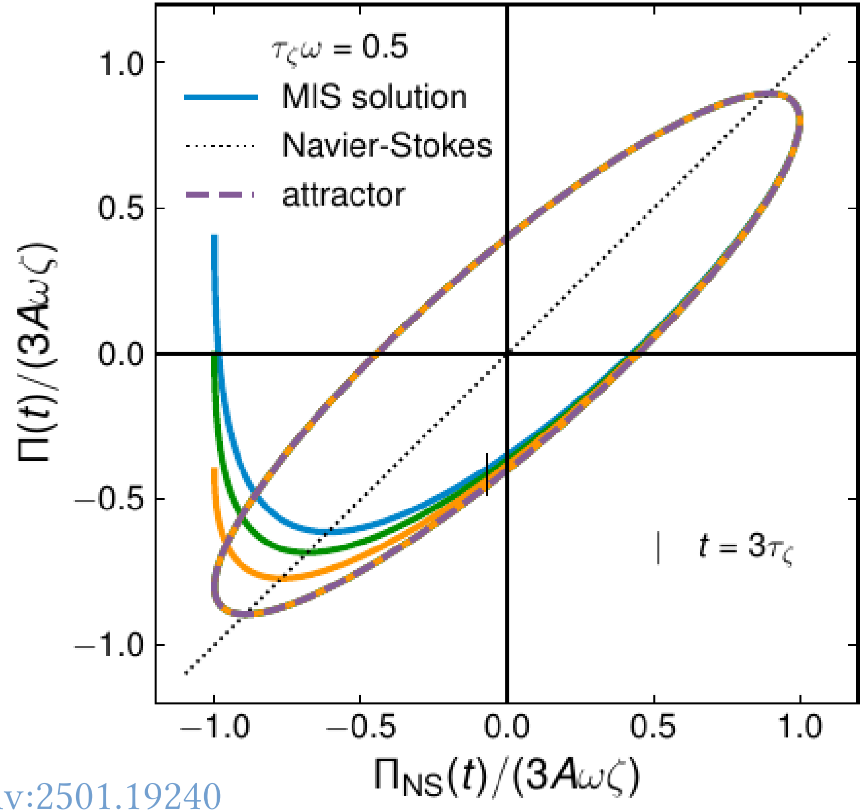
$$\Pi(t) = -3A\omega\zeta\frac{\cos(\omega t - \phi)}{\sqrt{(\omega\tau_\zeta)^2+1}}$$

Phase-shift: $\tan\phi = \omega\tau_\zeta$

Navier-Stokes is not the late-time limit of periodically driven system!

→ new type of cyclic attractor

$$\frac{\Pi}{P_{\rm eq}}\left(\frac{\zeta}{P_{\rm eq}\tau_\zeta}\right)^{-1}(3A\omega\tau_\zeta)^{-1} = \frac{\Pi}{3A\omega\zeta}$$

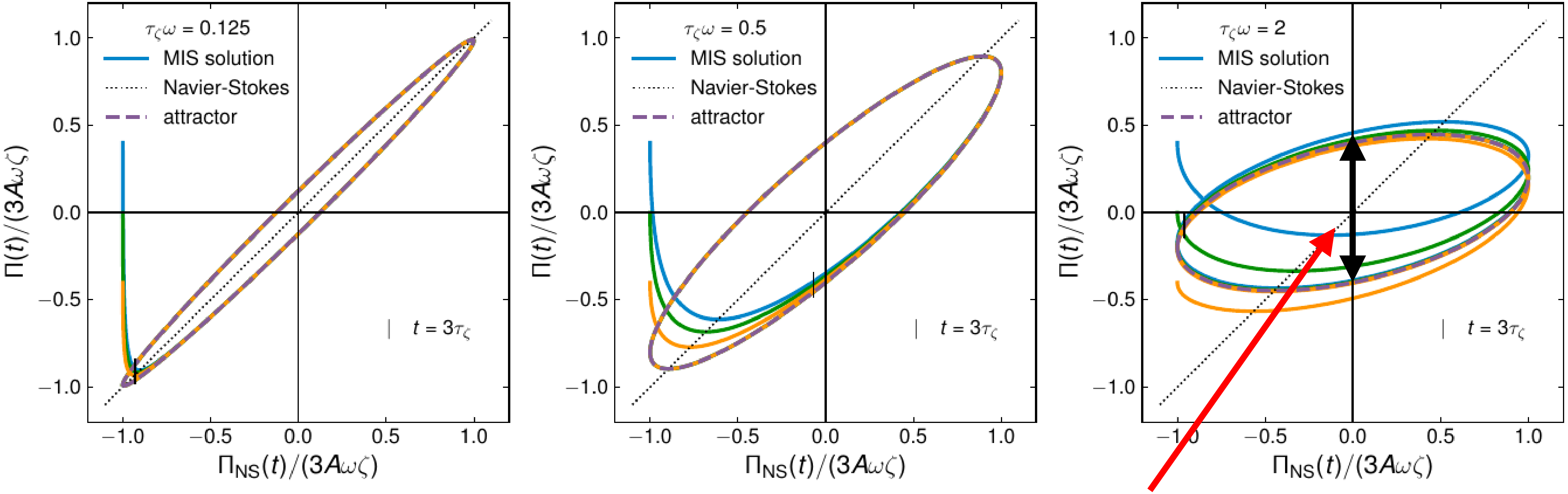


AM, Enss arXiv:2501.19240

Dependence on relaxation time

AM, Enss arXiv:2501.19240

For very slow drives or fast relaxation → closer to Navier-Stokes



Cyclic attractor can be observed for long time!

Max width for $\omega = \tau_\zeta^{-1}$

Non-linear response with massive kinetic theory

Time-dependent space-time metric

$$ds^2 = dt^2 - b(t)^2(dx^2 + dy^2 + dz^2) \qquad \vec{\nabla}\cdot\vec{v} = 3\frac{\dot{b}}{b}$$

Co-moving fluid expands isotropically

Massive particle in relaxation-time approximation

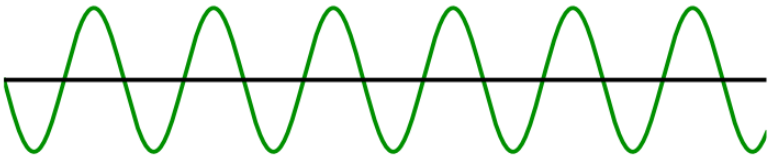
Florkowski et al. PRC (2014)

$$\left[\partial_t - 2\frac{\dot{b}}{b}p\frac{\partial}{\partial p}\right] f(t,p) = -\frac{f(t,p) - f_{\mathrm{eq}}}{\tau_R}$$

constant (τ_R)

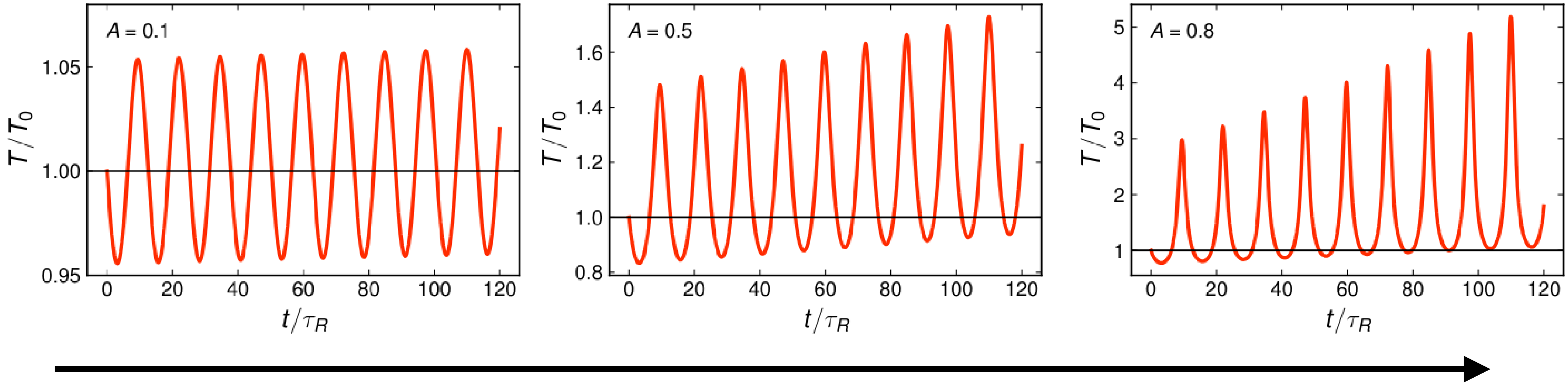
→ solve numerically for periodic expansion/contraction

$$b(t) = 1 + A\sin\omega t$$



Temperature evolution in periodic expansion

Viscous heating $\left\langle \dot{T} \right\rangle \propto A^2\omega^2\zeta$

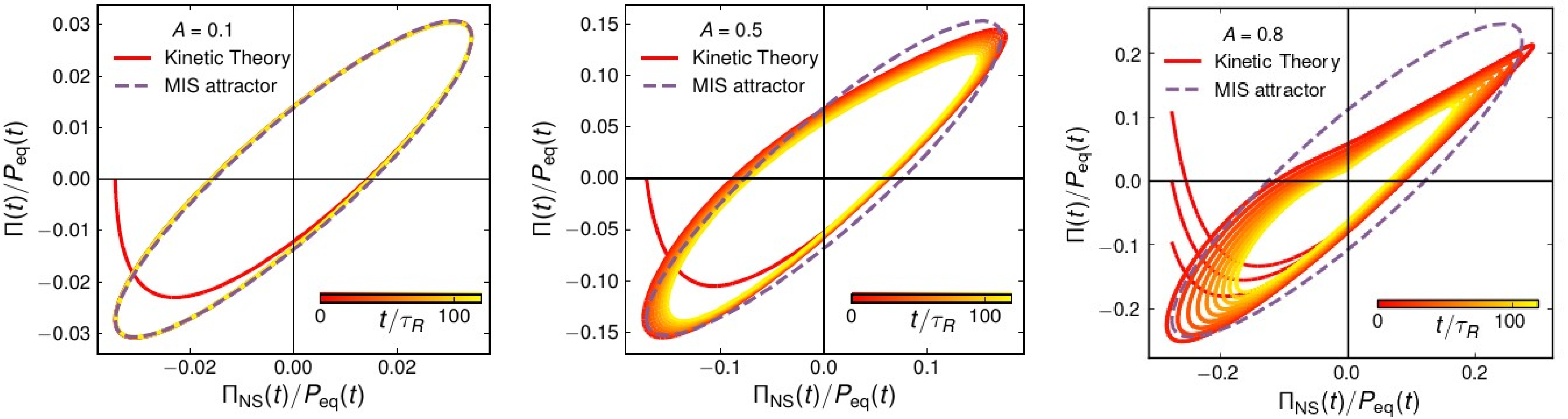


drive strength A

Non-linear hydro attractor

AM, Enss arXiv:2501.19240

For small amplitudes kinetic theory reproduce linear MIS attractor



For large amplitudes attractor evolves in time due to viscous heating.

Periodically driven conformal systems

(work in progress with Toshali Mitra and Tilman Enss)

Expansion and contraction in z-direction

Time-dependent space-time metric

$$ds^2 = dt^2 - dx^2 - dy^2 - b(t)^2 dz^2$$

$$\vec{\nabla} \cdot \vec{v} = \frac{\dot{b}}{b}$$

Co-moving fluid expands/contracts in z-direction

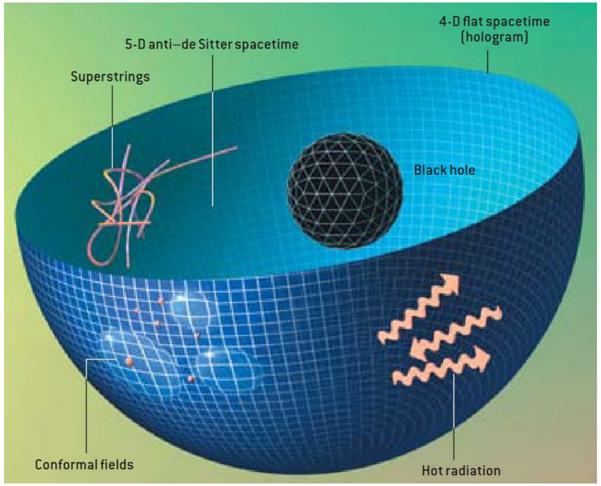
$\longrightarrow$ excite shear response in conformal systems

Can apply existing theory machinery:

- Holography
- Kinetic theory
- Generalized hydro, e.g. MIS

Gauge/gravity duality

Illustration by Alfred T. Kamajian

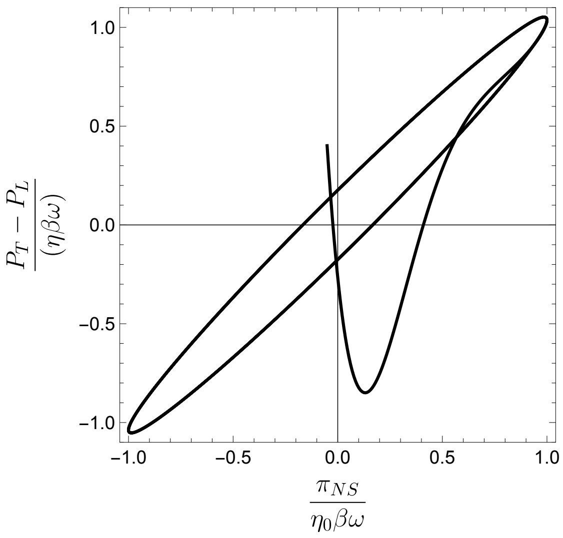


Bekenstein, Scientific American (2006)

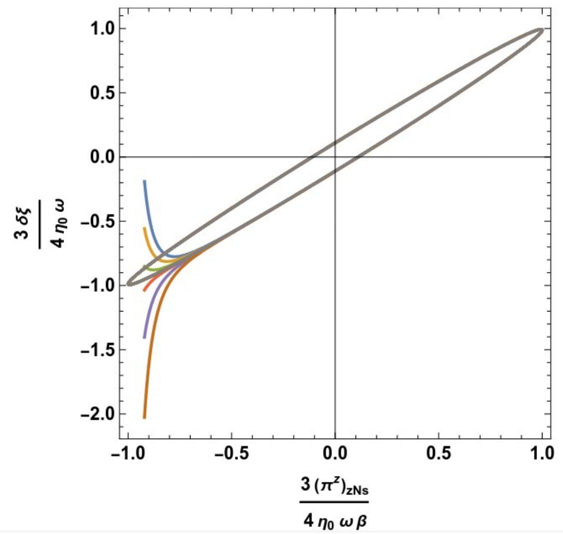
Cyclic attractor in different theories

Work in progress with Toshali Mitra and Tilman Enss

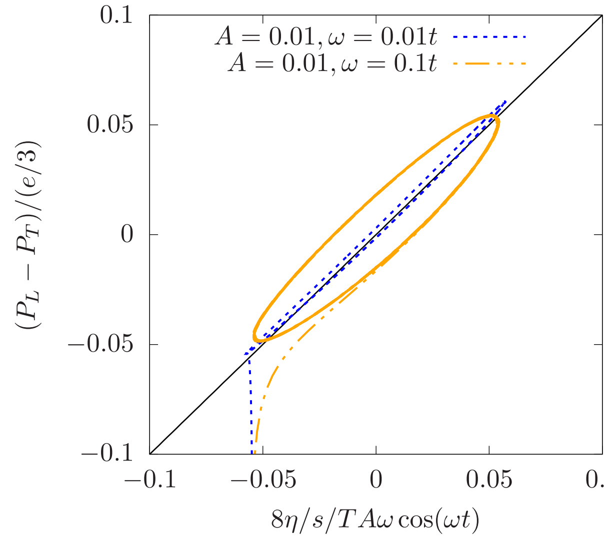
holography



MIS



QCD kinetic theory



Different theories exhibit the same cyclic attractor behaviour

Summary

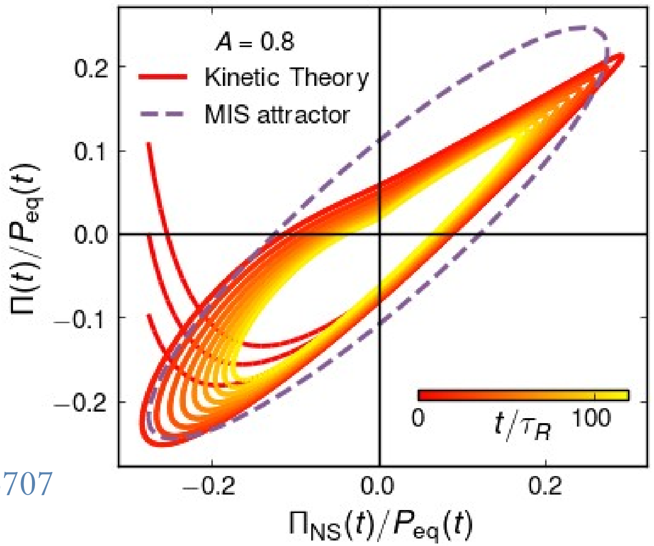
Conclusions

Hydro attractors ⟶ universal non-Navier-Stokes hydrodynamics

- Studied in many high-energy descriptions
- Rich mathematical structure

Cyclic hydrodynamic attractor AM, Enss arXiv:2501.19240

- New class of attractors
- Easier to simulate with existing systems



Thywissen lab arXiv:2506.13707

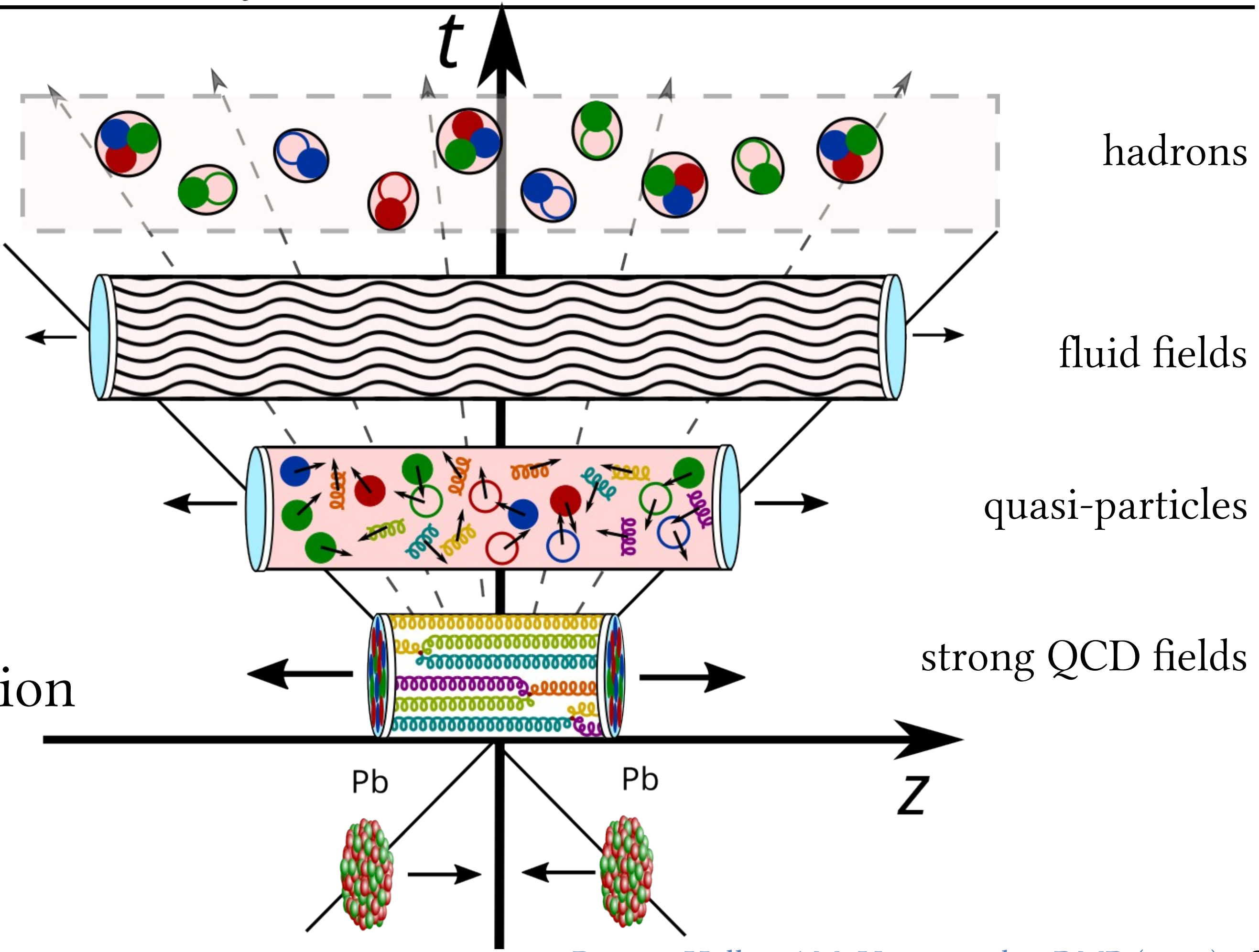
⟶ Should study attractors for general expansion

Discovery of hydro attractor in cold atoms would further strengthen connections to high-energy nuclear physics

Backup

Standard model of heavy ion collisions

- Hadronisation
  $t > 10\,\text{fm}/c$
- Fluid expansion
  $t \sim 1 - 10\,\text{fm}/c$
- QGP equilibration
  $t \sim 1\,\text{fm}/c$
- Initial energy deposition
- Incoming nuclei



Berges, Heller, AM, Venugopalan RMP (2021)

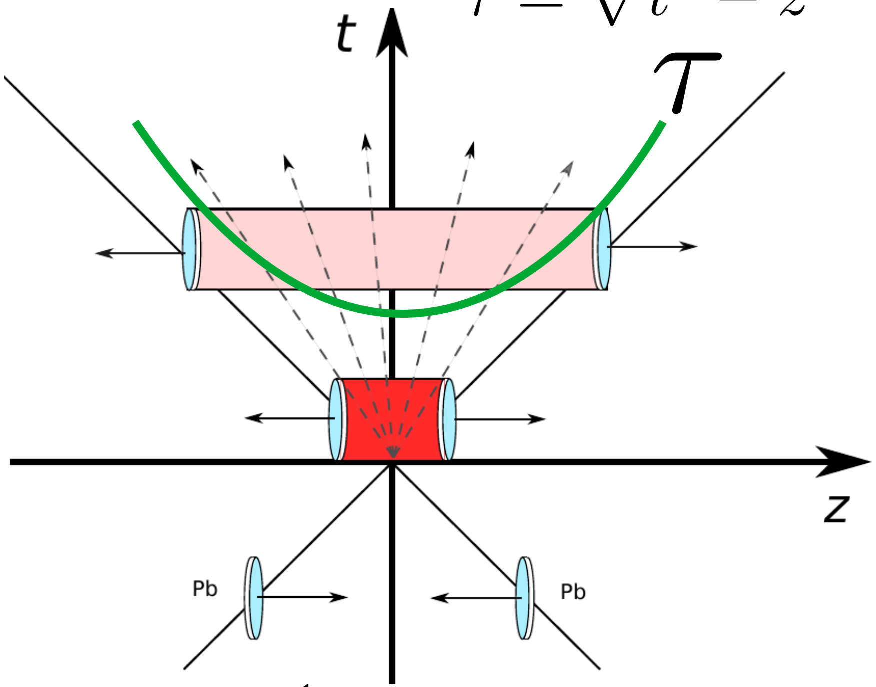
Longitudinal (Bjorken) expansion

1D fluid expansion: $v^z = \frac{z}{t}$

$$D_\mu T^{\mu\nu} = 0$$

proper-time

$$\tau = \sqrt{t^2 - z^2}$$



Energy density evolution:

$$\Rightarrow \partial_\tau e = -\frac{1}{\tau}(e + T^{zz})$$

Navier-Stokes sonstituent equation:

$$T^{zz} = P_{\text{eq}} - \frac{4}{3}\frac{\eta}{\tau} + \ldots$$

η/s - shear viscosity over entropy density

$$e \sim \tau^{-1}$$

Free-streaming $\eta/s = \infty$

$$e \sim \tau^{-\frac{4}{3}}$$

Ideal hydrodynamics $\eta/s = 0$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

in strongly interacting QI

Kovtun, Son, Starinets (2005)

QCD effective kinetic theory

Arnold, Moore, Yaffe JHEP (2003)

underlying quantum field theory

2-point functions

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - m\right)q - \frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu}_{a}$$

Boltzmann equation for quarks and gluons

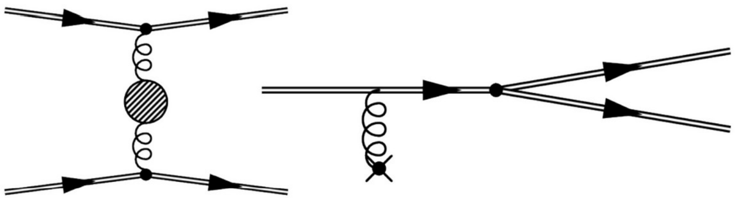
phase-space density

$$\partial_t f(t,\mathbf{x},\mathbf{p}) + \frac{\mathbf{p}}{|p|}\cdot\nabla_{\mathbf{x}} f(t,\mathbf{x},\mathbf{p}) = -\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]$$

multi-dimensional integrals

Leading order collision processes:

- Elastic scatterings
- Medium induced radiation



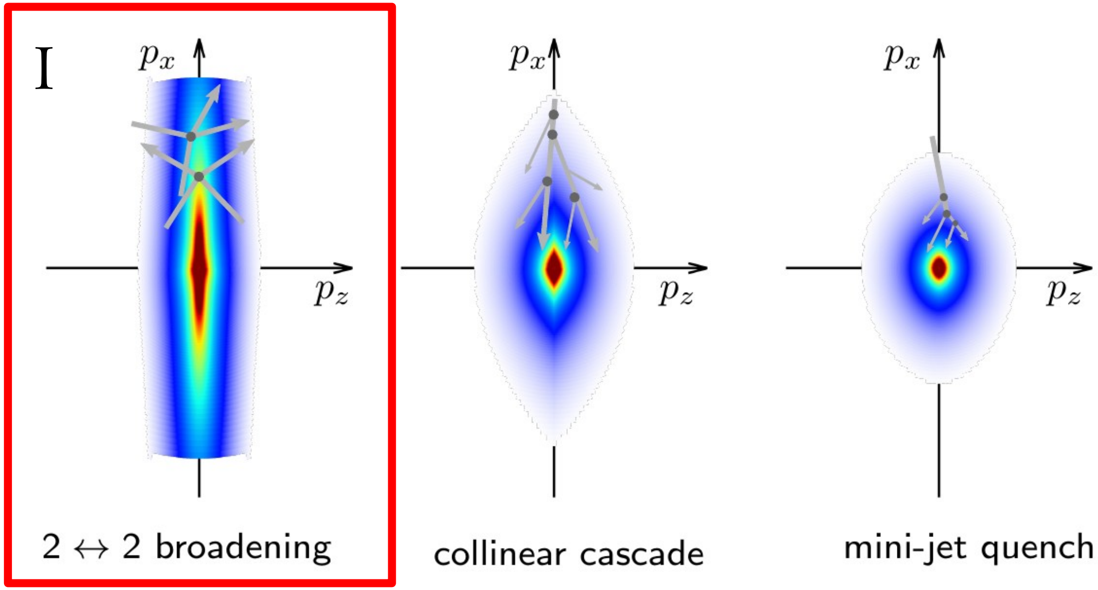
QCD kinetic theory ⟶ real-time dynamics of quark-gluon plasma.

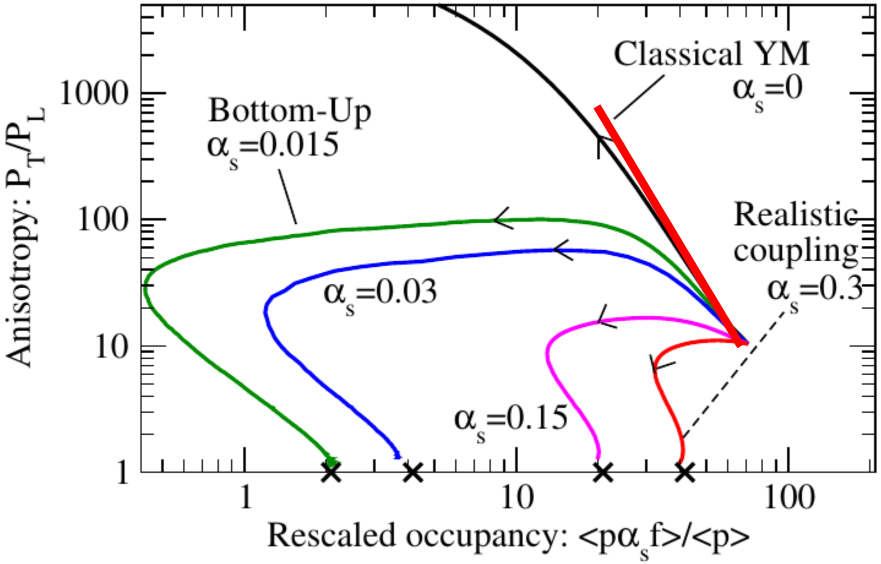
"Bottom-up" thermalization scenario

Baier, Mueller, Schiff, and Son (2001)

Boltzmann eq.: $\partial_\tau f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{longitudinal expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]}_{\text{in-medium QCD collisions}}$

gluon distribution





Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)

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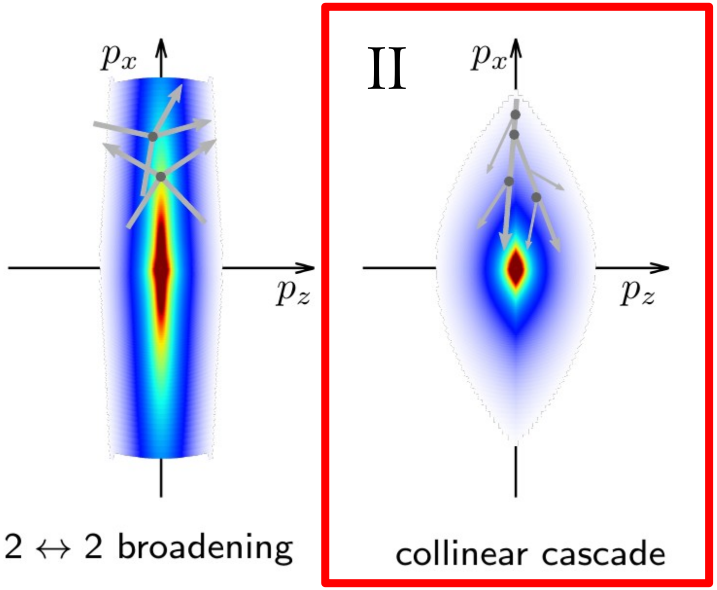

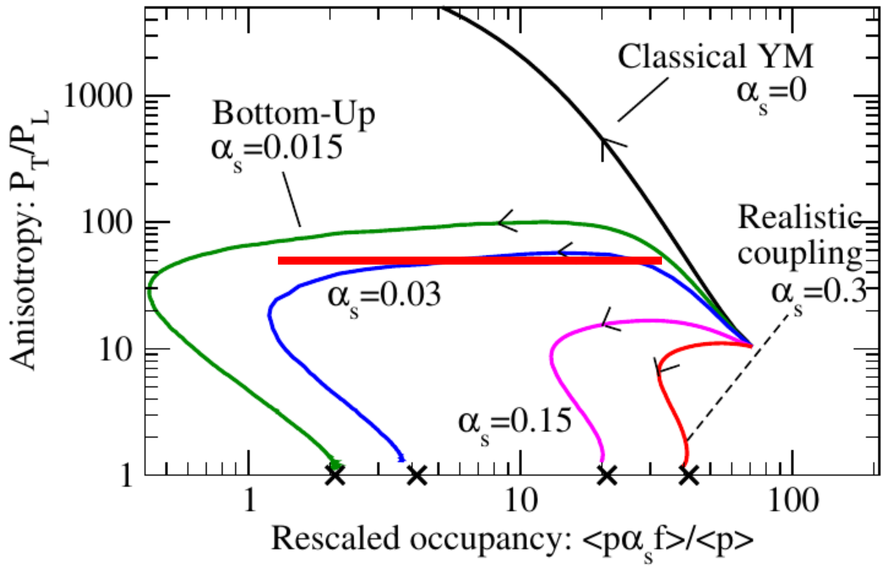

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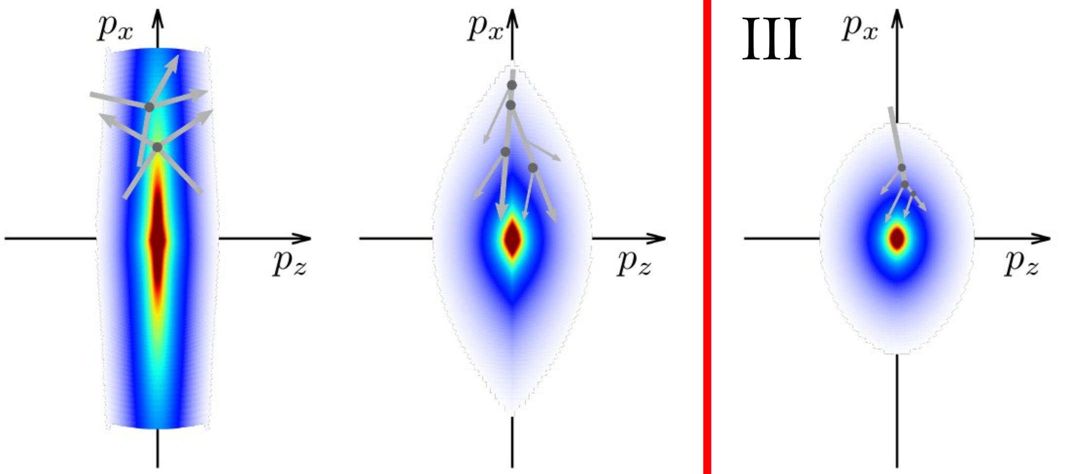
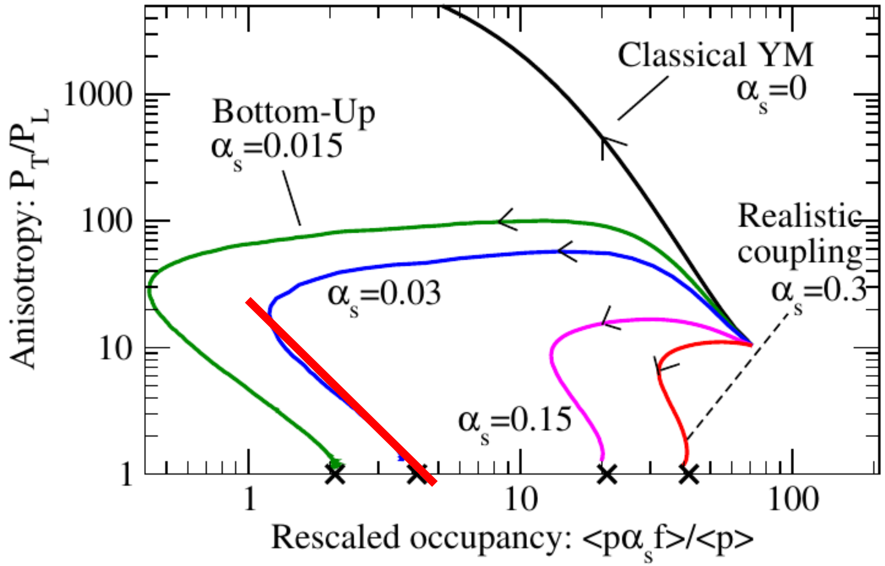
gluon distribution



$2 \leftrightarrow 2$ broadening — collinear cascade — mini-jet quench

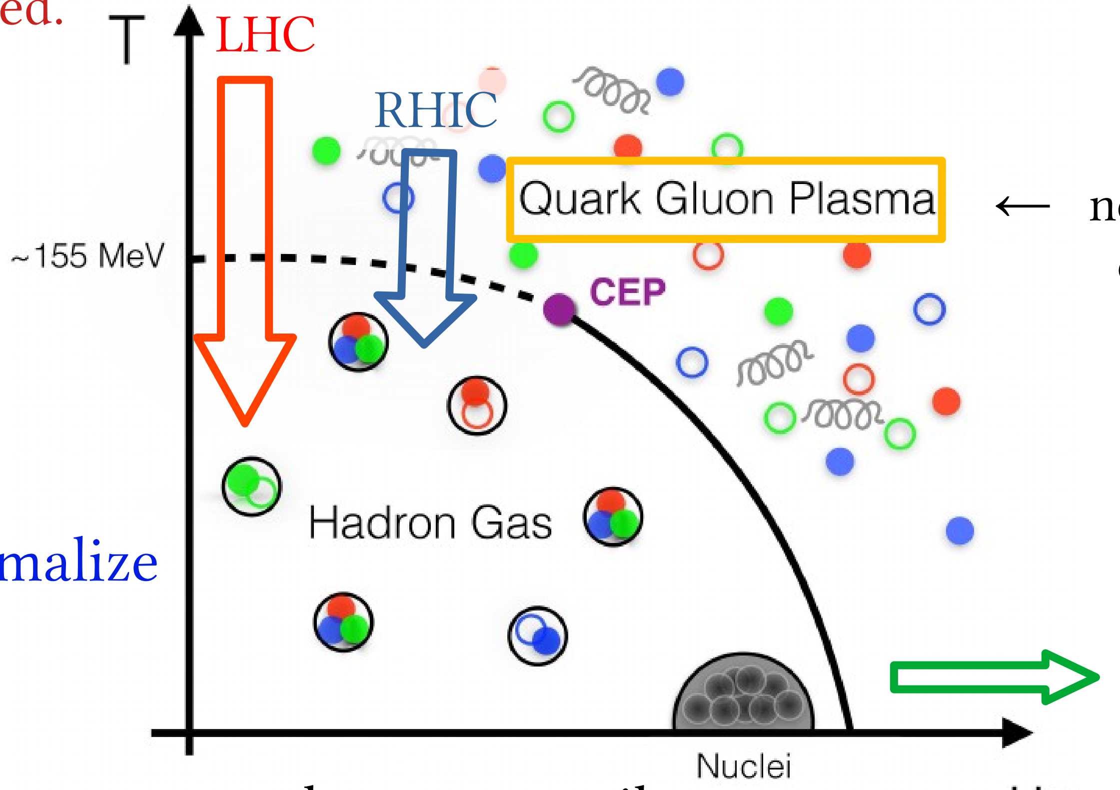


Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)

QCD phase diagram

Cabibbo, Parisi (1975): existence of a different phase of the vacuum in which quarks are not confined.

thermodynamic (static) picture →



← new phase of matter

neutron stars

baryon - antibaryon

(How) does QCD thermalize in nuclear collisions?

Ding, Karsch, Mukherjee (2016) QGP 5

Non-thermal attractor

Universality far from equilibrium

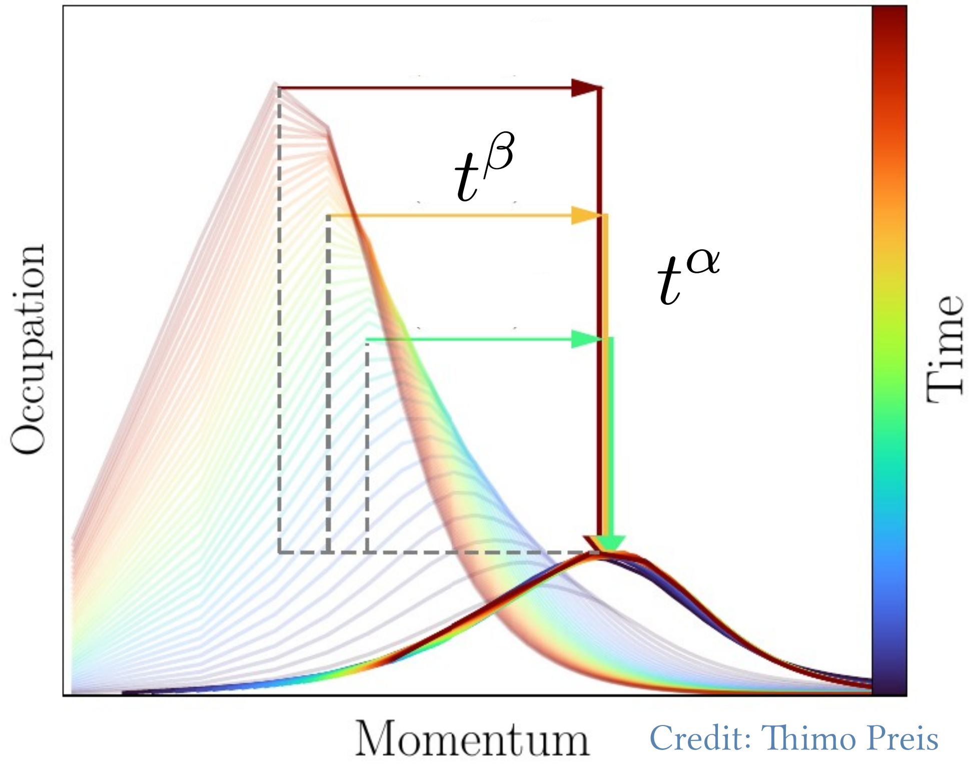
Self-similar evolution of distribution function

$$f(\tau, p_T, p_z) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

f_S - scaling function

α, β, γ - scaling exponents

scaling example



Credit: Thimo Preis

Non-thermal attractor:

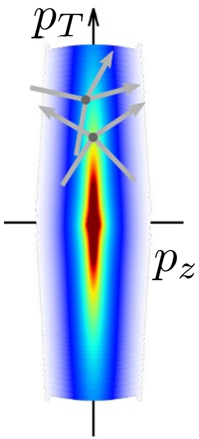
- Loss of initial information
- Simplification of dynamics

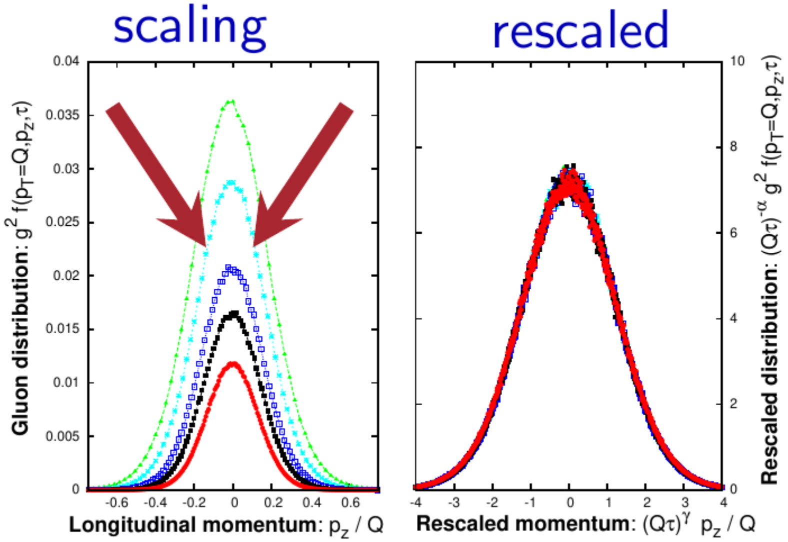
Stage I: non-thermal attractor

- Solve classical-statistical Yang-Mills equations with expansion
- Gluon distribution scales according to the bottom-up predictions

$$f(\tau, p_T, p_z) = \tau^{-2/3} f_S(p_T, \tau^{1/3} p_z)$$

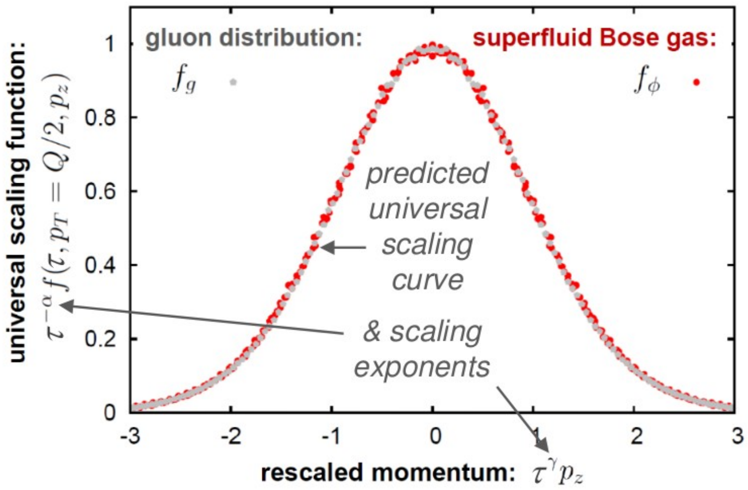
gluons





Berges, Boguslavski, Schlichting, Venugopalan (2014, 2015)

scalars



universality between theories

Generalization: time dependent exponents

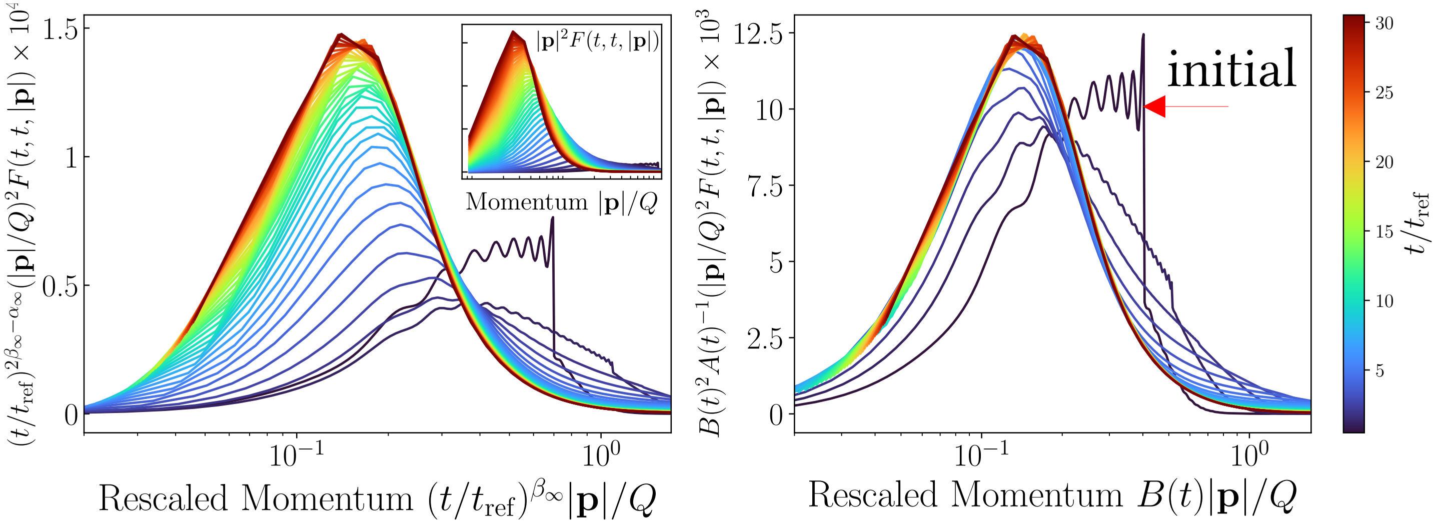
$$f(t,p) = t^{\alpha(t)} f_S(t^{\beta(t)} p)$$ $\rightarrow$ earlier collapse (pre-scaling)

AM, Berges PRL (2019)

Quantum (2PI) evolution of O(N) model

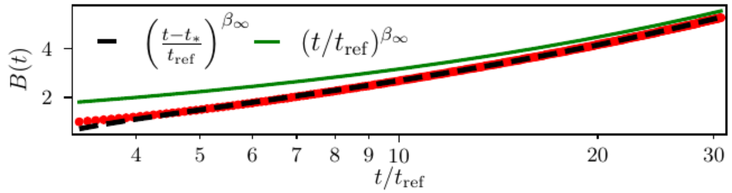
$\rightarrow$ similar results with QCD kinetic theory

constant exponents | pre-scaling | time



Analytic prediction: $B(t) \equiv t^{\beta(t)} \rightarrow (t - t_*)^\beta$

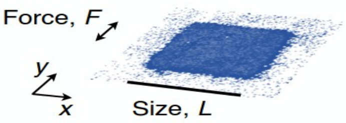
Heller, AM, Preis, PRL (2024)



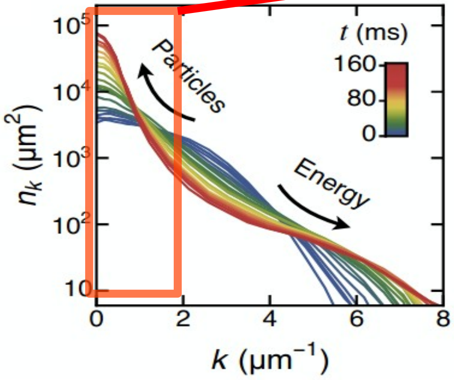
Dynamical scaling in ultracold atomic gases

- Highly controllable → tunable interactions and initial conditions.
- Observation of dynamical scaling in Bose gases   Prufer et al. , Erne et al., Nature (2018)
- Observation of pre-scaling dynamics in 2D Bose gas   Hadzibabic group, 2312.09248

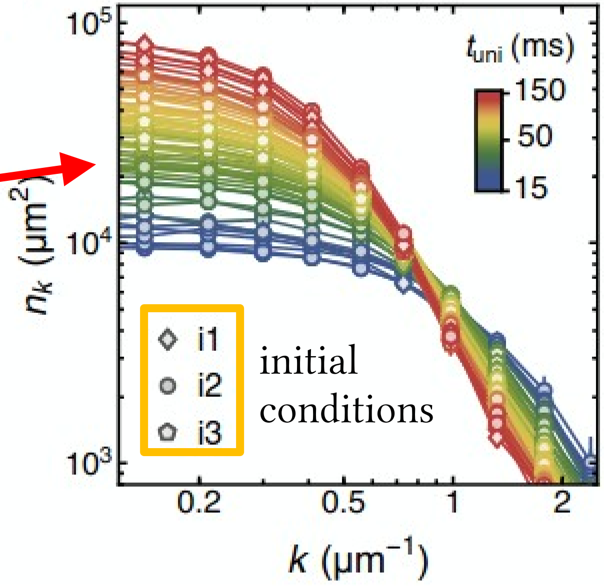
initial shaking



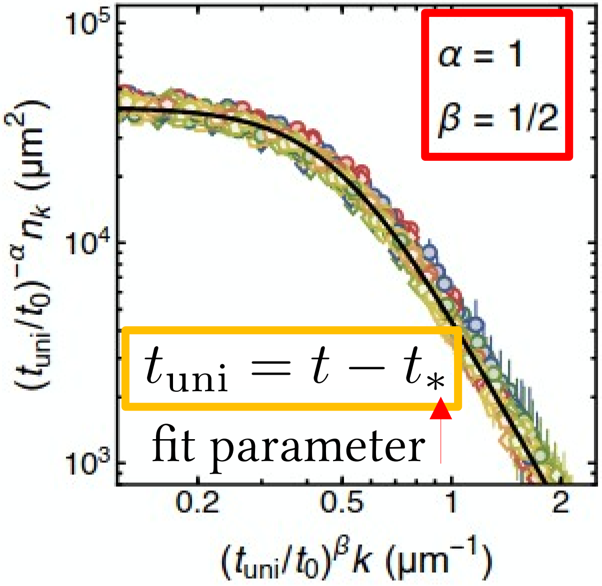
bidirectional cascade



distribution evolution

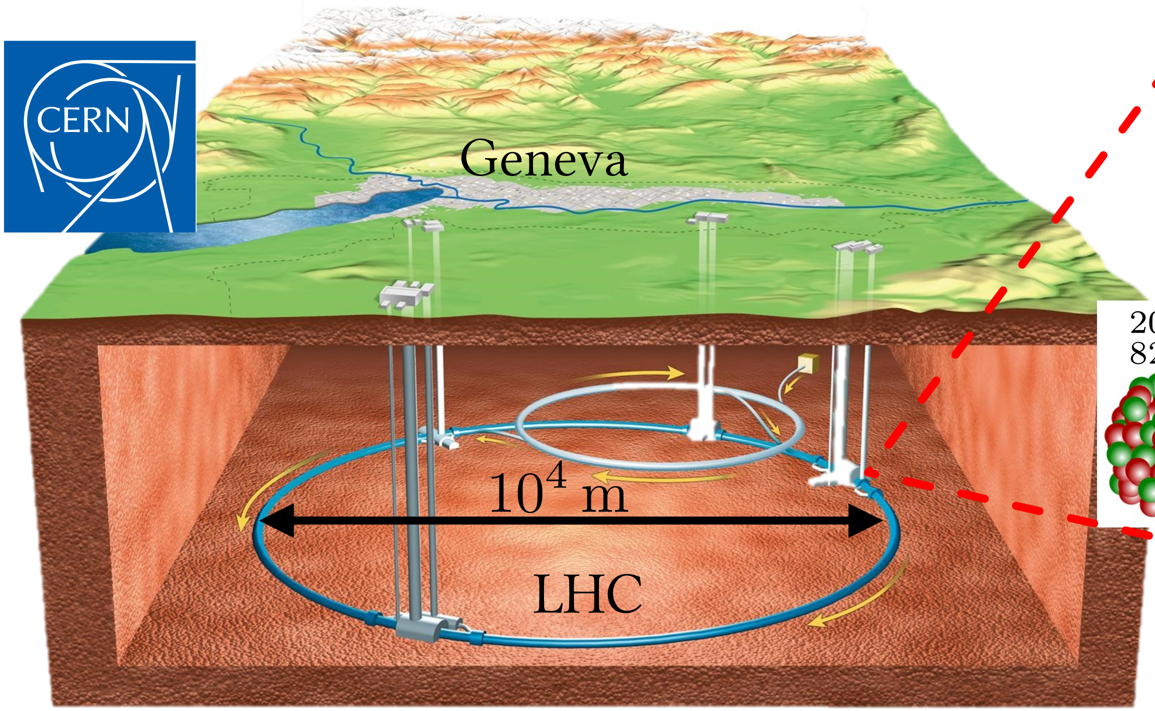


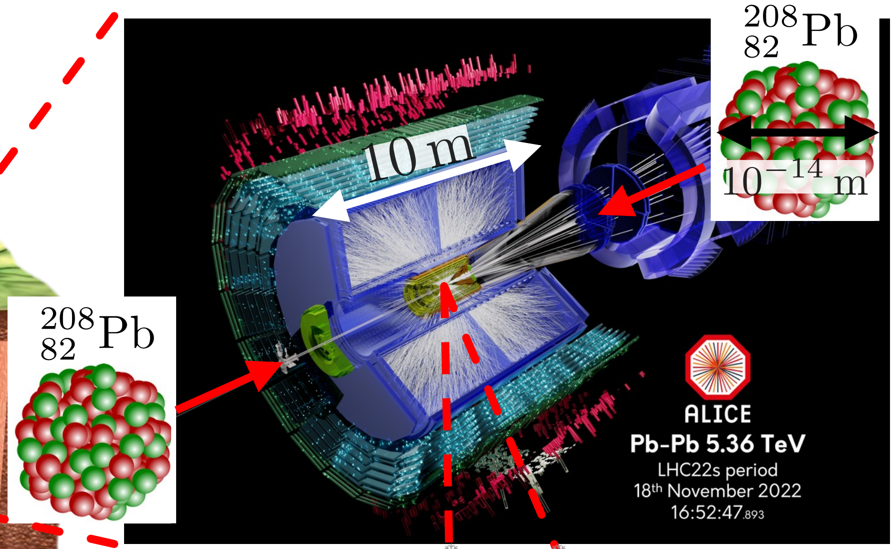
rescaled distributions

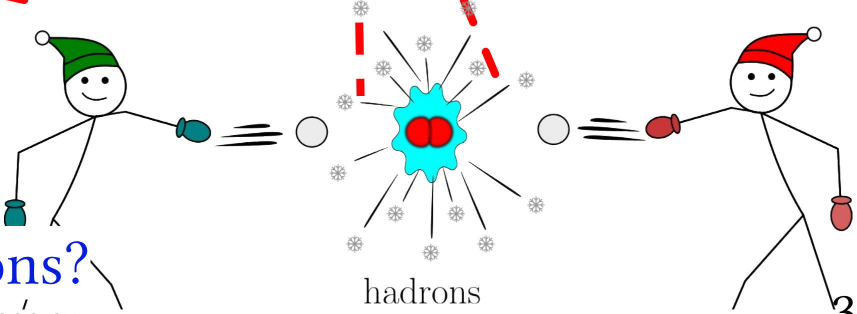


Ion collisions at the Large Hadron Collider

T.D. Lee, 1974: distribute high energy density over a relatively large volume.





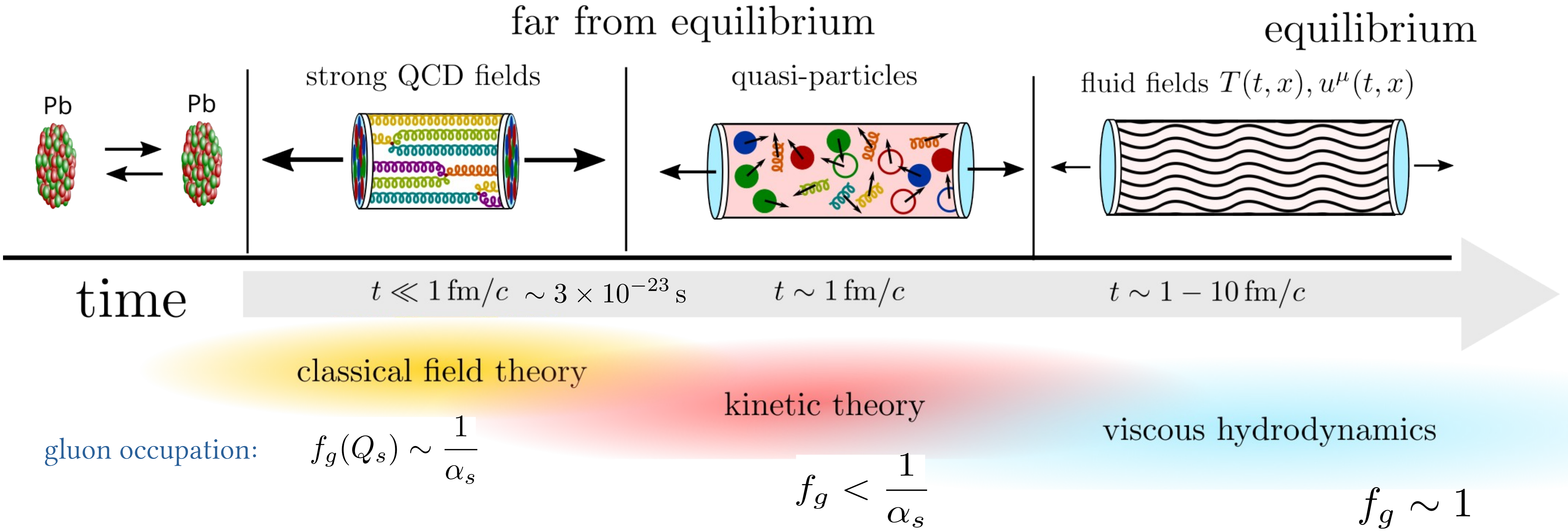


(How) does QCD thermalize in nuclear collisions?

QCD thermalization

Berges, Heller, AM, Venugopalan RMP (2021)

High-energy, high-density limit $\alpha_s(Q_s) \ll 1$

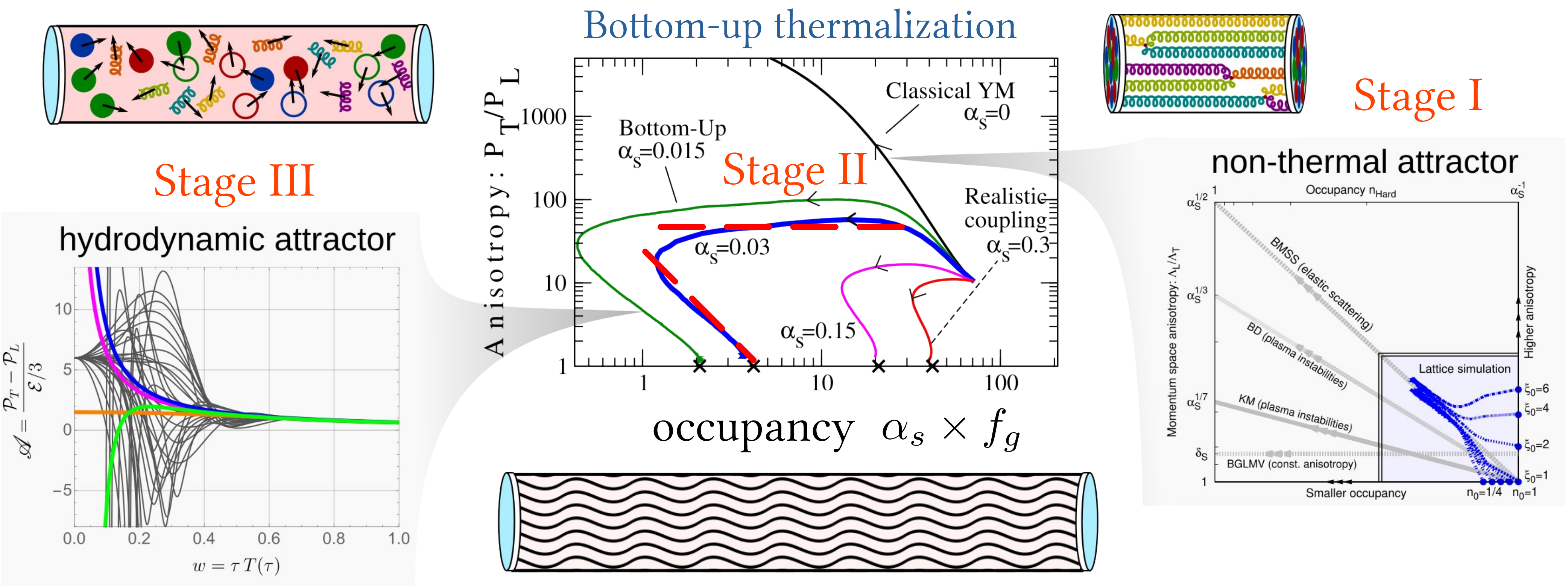


Consistent QCD description from initial conditions to equilibrium.

Attractors in Bottom-up thermalization

Baier, Mueller, Schiff, and Son (2001)

Simplification and universality in QCD thermalization → attractors



Applications

- Early applicability of Navier-Stokes eqs.

$$\tau_{\text{hydro}} \approx 1\,\text{fm}/c \propto (\eta/s)^{3/2}$$

- Entropy production during equilibration

$$s\tau \propto (\eta/s)^{1/3}$$

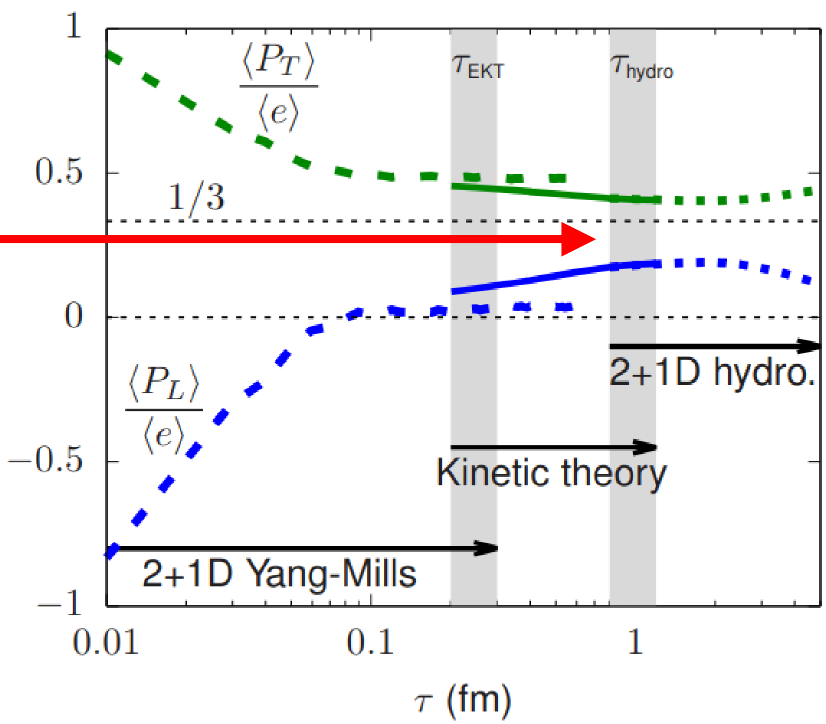
Giacalone, AM, Schlichting PRL (2019)

- Pre-equilibrium response functions
  $\longrightarrow$ KøMPøST code

Kurkela, AM, Paquet, Schlichting and Teaney, PRC, PRL (2018)

- Hydrodynamics beyond gradient expansion

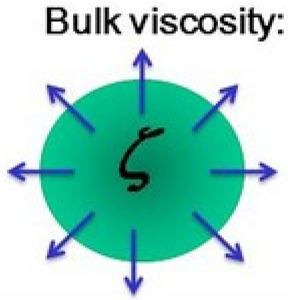
recent review: Jankowski, Spaliński, PPNP (2023), arXiv:2303.09414



Kurkela, AM, Paquet, Schlichting and Teaney PRL (2018)

Hydro attractor discovered theoretically in studies of QCD equilibration.
$\longrightarrow$ Can hydro attractors be observed experimentally?

Bulk pressure evolution



Bulk viscosity resists expansion

$$\Pi_{\rm NS} = \frac{1}{3}T^i_{\ i} - P_{\rm eq} = -\zeta \vec{\nabla}\cdot\vec{v} \qquad \vec{\nabla}\cdot\vec{v} \leftrightarrow 3\frac{\partial_t a^{-1}(t)}{a^{-1}}$$

Müller-Israel-Stewart theory ⟶ linear relaxation

$$\tau_\zeta \dot{\Pi}(t) = -\Pi(t) + \Pi_{\rm NS}(t)$$

- Monotonic drive ⟶ traditional hydro attractor.

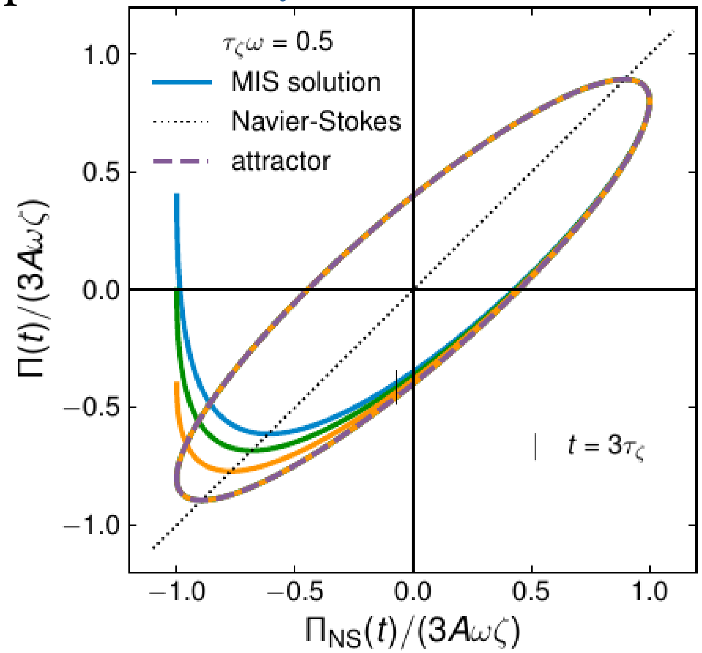
  $a(t)^{-1} \propto t^{-n}$ Fujii & Enss, PRL (2024)

- Periodic drive ⟶ new cyclic attractor.

  $a(t)^{-1} \propto (1 + A\cos\omega t)$ AM, Enss arXiv:2501.19240

⟶ ideally suited for experimental measurement

cyclic attractor

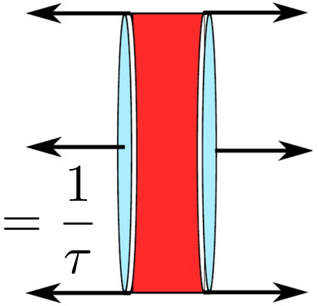


Navier-Stokes expectation

Relativistic viscous fluid dynamics

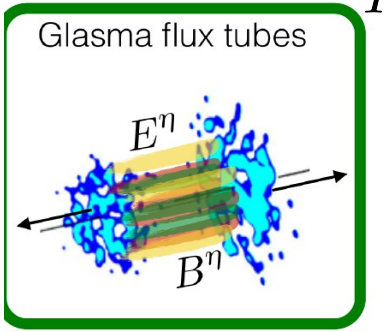
- Energy-momentum conservation: $\partial_\mu T^{\mu\nu} = 0$
- Equation of state $\longrightarrow$ lattice QCD $\quad P_{\rm eq}(e)$
- Constitutive equations: $\quad T^{zz} = P_{\rm eq} - \frac{4}{3}\frac{\eta}{\tau} + \dots$

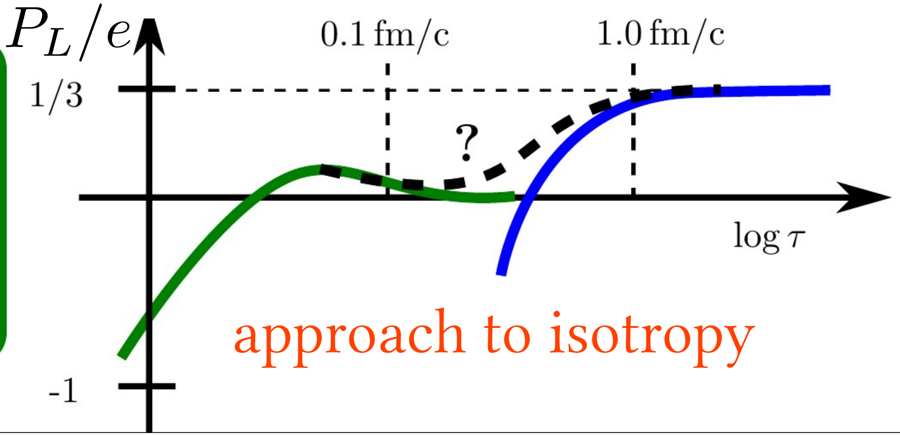
1st viscous correction in gradients



Conventional regime of applicability: near isotropic systems

$$\Rightarrow P_L \equiv T^{zz} \quad \approx \quad P_T \equiv (T^{xx} + T^{yy})/2$$





hydrodynamics