Kelvin waves and nonthermal fixed points

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Attractors and thermalisation in nuclear collisions and cold quantum gases, ECT workshop, Trento*

Based on V Noel, T Gasenzer, K Boguslavski, Phys. Rev. Research 7 (2025)

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STRUCTURES CLUSTER OF EXCELLENCE

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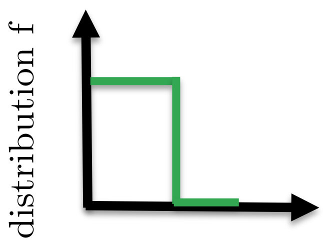
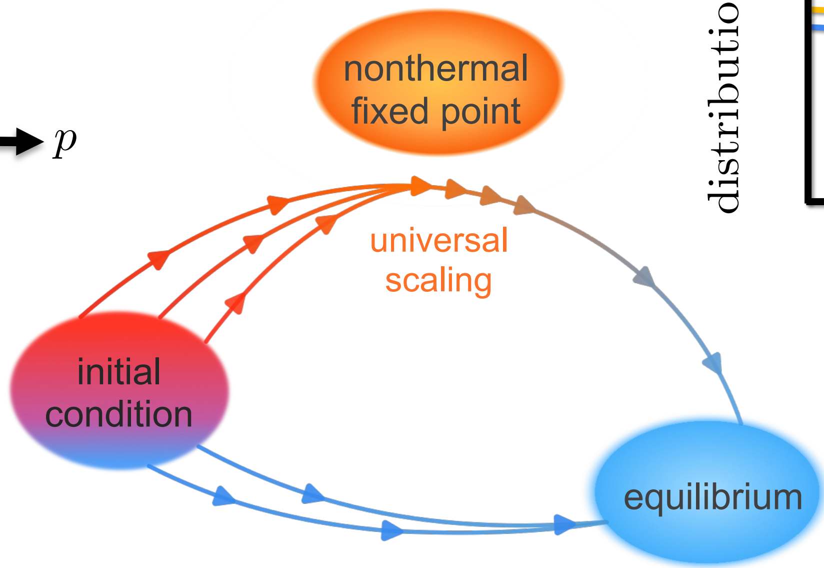
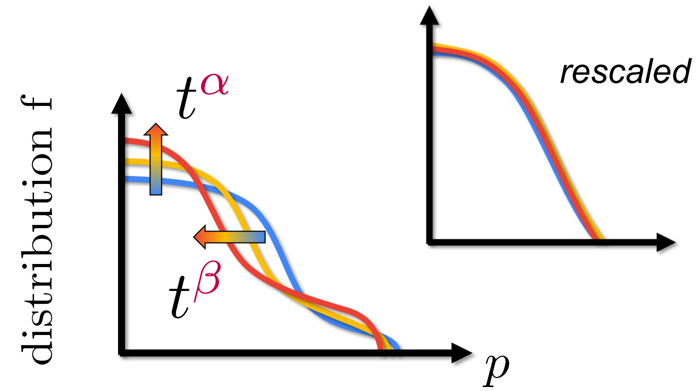
- **Far from equilibrium attractors**







scaling exponents

$$f(t,p) = t^\alpha f_S(t^\beta p)$$

scaling function

J Berges, A Rothkopf, J Schmidt, PRL 101 (2008),
J Berges, K Boguslavski, S Schlichting,
R Venugopalan, PRL 114 (2015)
Prüfer et al., Nature 563, 217 (2018),
Erne et al., Nature 563, 225 (2018)

What kind of (quasi)particles are in f(t,p)?

- **Relativistic *O(N)* symmetric scalars:**

$$S = \int d^d x \left[\frac{1}{2} \partial^\mu \varphi_a \partial_\mu \varphi_a - \frac{m^2}{2} \varphi_a \varphi_a - \frac{\lambda}{4! N} \left(\varphi_a \varphi_a\right)^2 \right]$$

$\rightarrow$ *also nonrelativistic scalars, spin-1 Bose gas, gauge fields…*

Nonthermal fixed points and quasiparticles

- **Effective kinetic description**
  → *elastic scattering of quasiparticles with ~p^2 dispersion*
  → *track evolution of f(t,p)*
  → *large-N resummations: also applicable at N=1,2...?*

A P Orioli, K Boguslavski, J Berges PRD 92 (2015), I Chantesana, A P Orioli, T Gasenzer, PRA 99 (2019)…

- **Universal scaling**
  → *Insensitivity of scaling to N: all O(N) behave the same?*

$$f(t,p) = t^{\alpha} f_S(t^{\beta} p) \qquad \alpha = d\beta, \ \beta \sim 0.5$$

(particle number conserved, underlying U(1) symmetry)

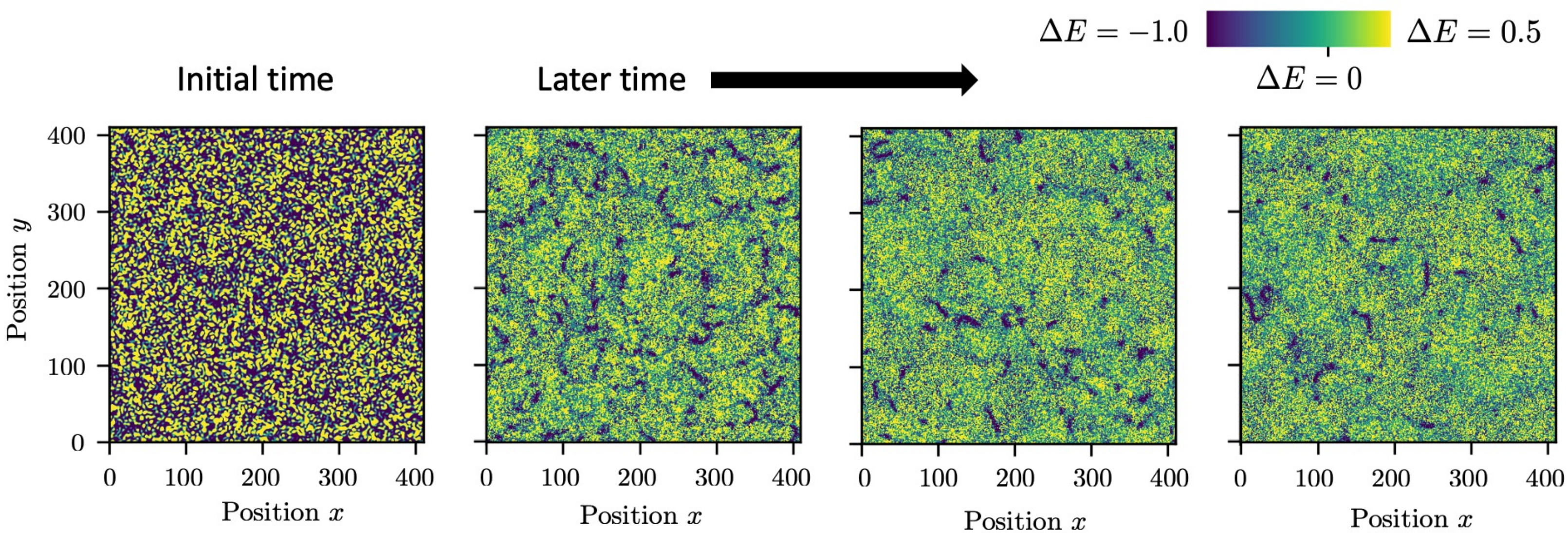
- **Low-energy effective theory of an *N*-comp. Bose gas (*U(N)* sym.)**
  → *collective low-energy excitations* *A N Mikheev, C-M Schmied, T Gasenzer, PRA 99 (2019)*
  → *U(N): 1 Bogoliubov, N-1 "quadratic modes" (Goldstones)*

- **What about topological defects?**
  → *NTFP has been connected to coarsening before*

B Nowak, J Schole, D Sexty, T Gasenzer, PRA 85 (2012), M Karl, T Gasenzer, New J. Phys. 19 (2015)…

Nonthermal fixed points and topological defects

- **Typical coarsening (e.g., energy densities)**
  → *vortex-antivortex annihilation* → *self-similar scaling*



V Noel, D Spitz, PRD 109 (2024)

- **Topological defects at "low *N*":**

→ *vortex defects in O(N), N=1-3, scaling in geometric observables (persistent homology)*

G Moore, PRD 93 (2016) *V Noel, D Spitz, PRD 109 (2024)*

→ *no coarsening-related exponent found above N>3*
→ *and defects are large structures in the deep infrared…*

how to "combine" coarsening-related self-similar scaling with quasiparticle picture?

Excitations ‘on top of’ defects

- **Vortex dynamics in two dimensions**

  → *vortex-antivortex annihilation → sound waves (phonons)*

  C. F. Barenghi, R. J. Donnelly, and W. Vinen, Quantized vortex dynamics and superfluid turbulence (2001)

- **Vortex lines in three dim.: added flexibility from extended structure**

  → *vortex reconnection, ring dynamics etc. → sound wave emission*

  → *Kelvin waves: helical excitations propagating along vortex lines*

  → *in a superfluid: helical perturbations of quantised vortex lines (“kelvon”)*

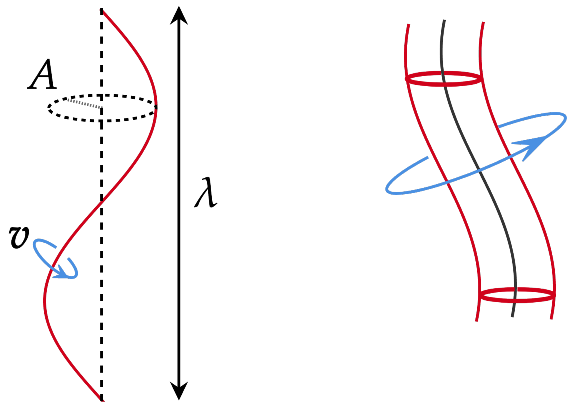
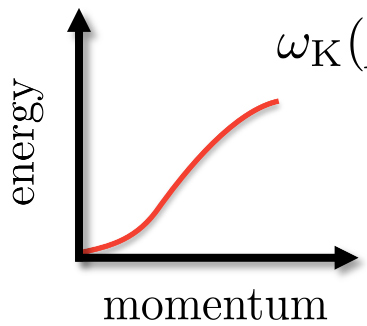


Figure from Wikipedia, Quantum Turbulence



$$\omega_K(p) \approx \frac{\kappa p^2}{4\pi} \ln\left(\frac{1}{p a_0}\right)$$

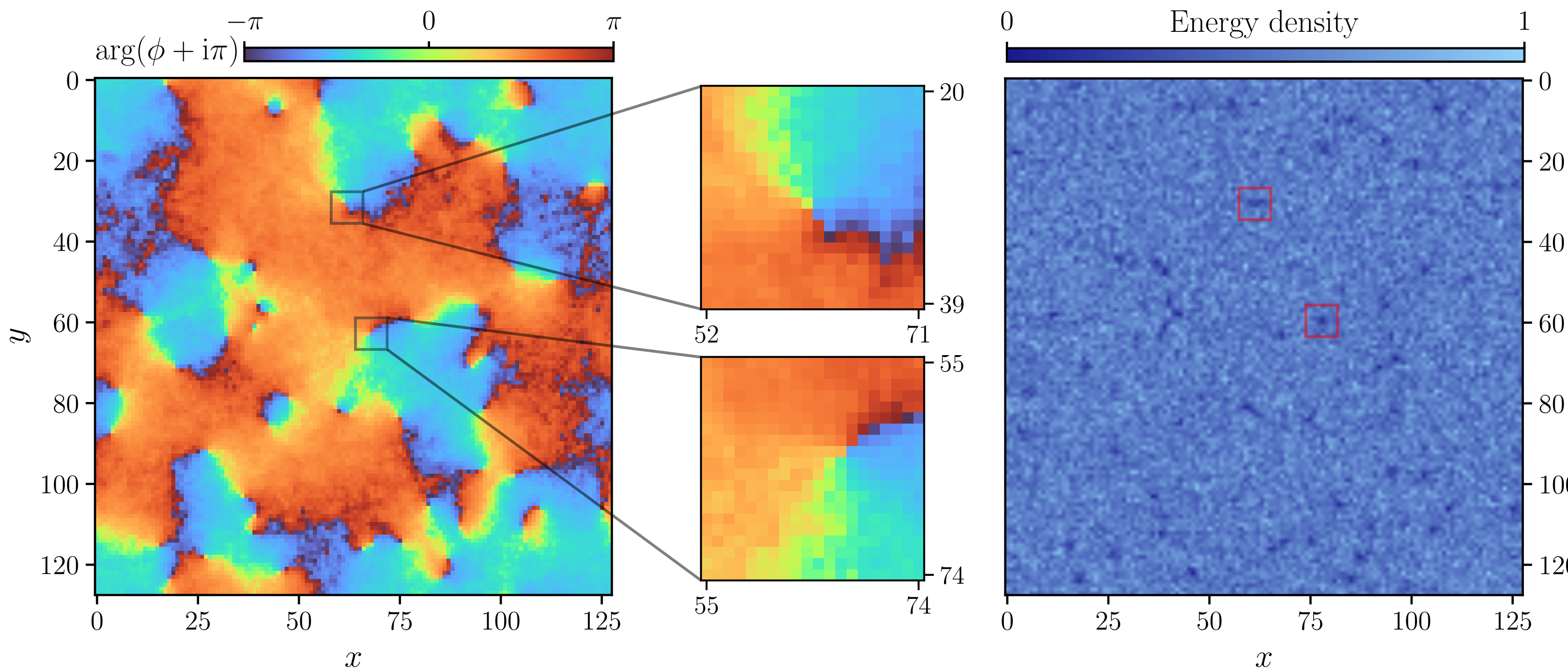
TP Simula, T Mizushima, K Machida, PRL (2008)
P Clarck di Leoni, PD Mininni, ME Brachet, PRA 92 (2015)

→ *“kelvons” also exist in 2D superfluids*

T Simula, PRA 97 (2017), T Simula, PRA 101 (2020)

Simulations on a lattice & finding vortices

- **Overoccupied quantum system → classical statistical field theory**
  → *evolve many different trajectories & take average for observables*
  → *for relativistic O(1), evolve ϕ and π (= $\dot{\phi}$) on a lattice*
  → *identify vortex defects*



V Noel, T Gasenzer, K Boguslavski, PRR 7 (2025)

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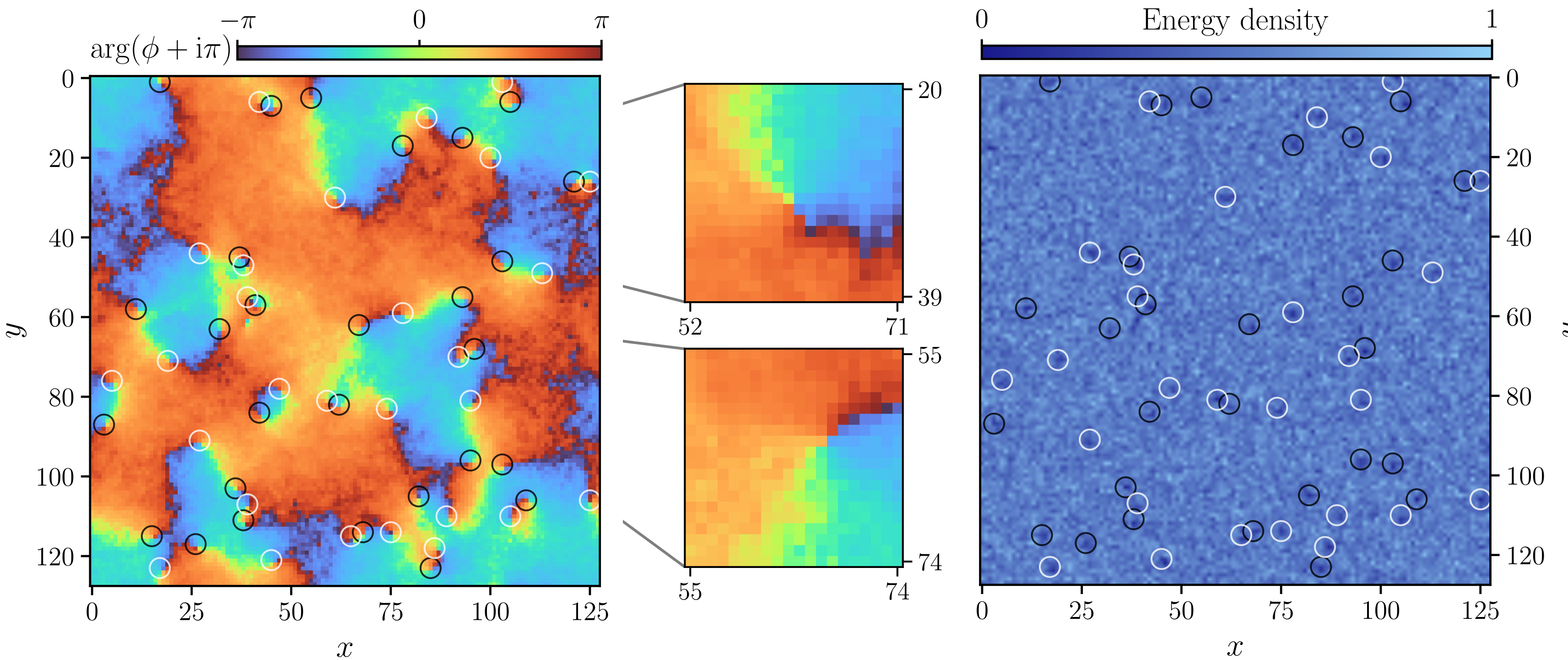

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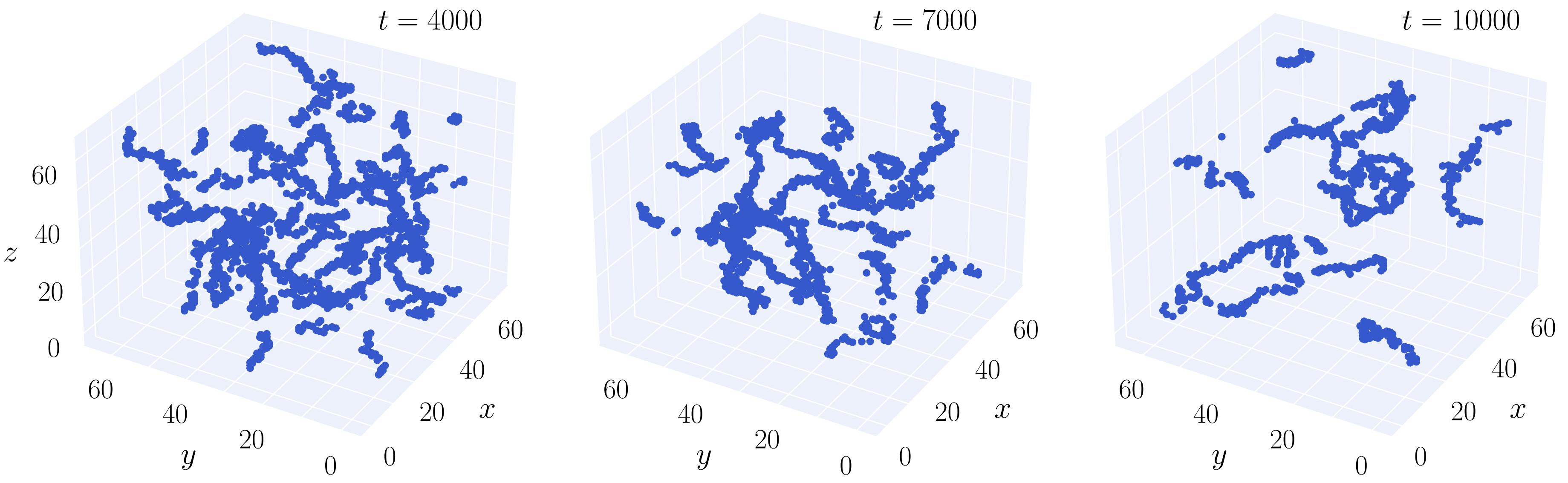
Energy densities in 3D



V Noel, T Gasenzer, K Boguslavski, PRR 7 (2025)

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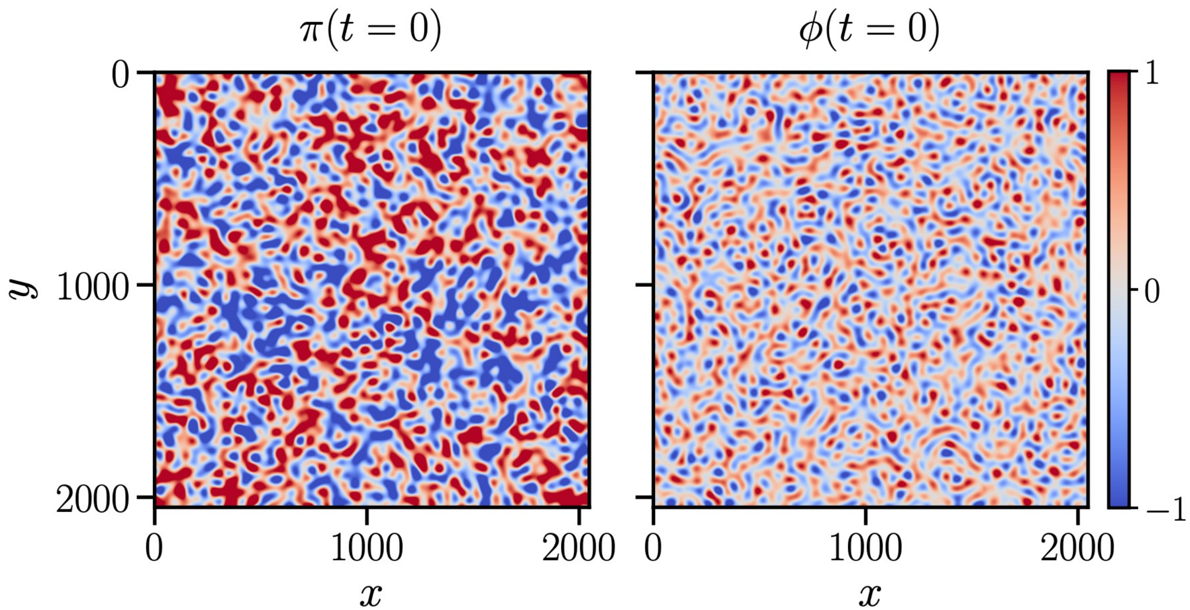
Initial field configurations in 2D



V Noel, T Gasenzer, K Boguslavski, PRR 7 (2025)

→ *box initial condition leads to small "domains" in real space => phase singularities*

→ *resulting topological defects are stable for O(N), N=1,2,3*

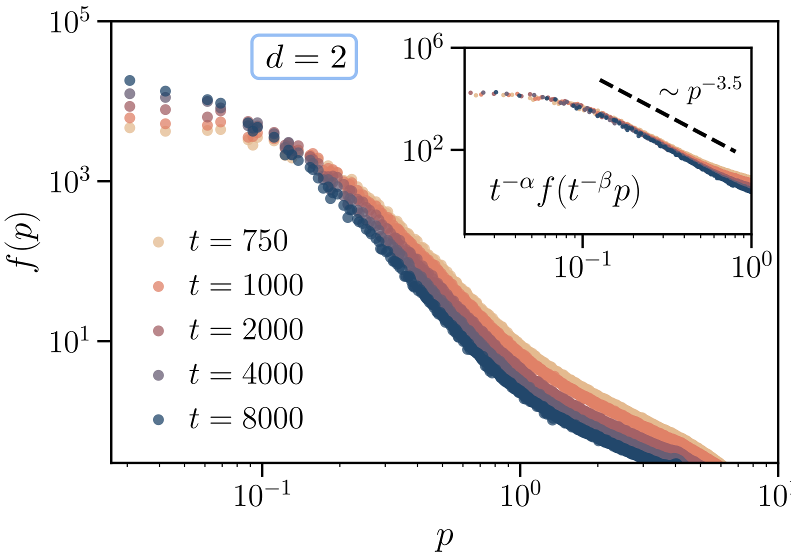
G Moore, PRD 93 (2016)

V Noel, D Spitz, PRD 109 (2024)

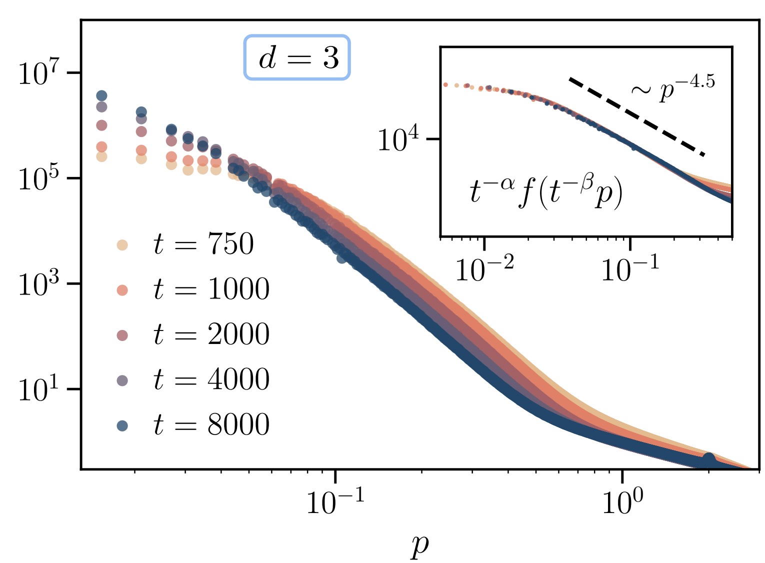
Scaling of the distribution function

- **Self-similar scaling**
  - → *3D: O(1) – O(N) all scale with β~0.5 and α~1.5*
  - → *2D: O(1) scales with β~0.25, O(N) scale with β~0.5*
  - → *β~0.25: alternative explanations based on vortex-antivortex annihilation*

$d = 2 \;\; O(1) \;\; \alpha = 0.5, \beta = 0.25$



$d = 3 \;\; O(1) \;\; \alpha = 1.0, \beta = 0.5$



2D: also J Deng, S Schlichting, R Venugopalan, Q Wang, PRA 97 (2018)

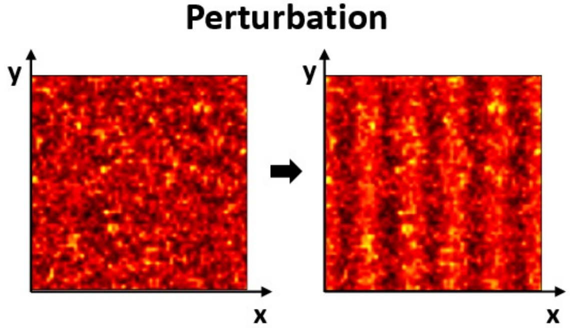
V Noel, T Gasenzer, K Boguslavski, PRR 7 (2025)

→ *vortex-antivortex annihilation is ~$t^{0.25}$ in 2D & vortex lines shrink at ~$t^{0.5}$ in 3D*

Methods & Observables

- **Unequal time correlation functions**
  - → *statistical function:* $F\left(t,t',\mathbf{x}-\mathbf{x}'\right) = \frac{1}{2N}\left\langle\left\{\hat{\phi}_a(t,\mathbf{x}),\hat{\phi}_a\left(t',\mathbf{x}'\right)\right\}\right\rangle_C$
  - → *occupancy of excitations*

    equal time: $f(t,p) = \sqrt{F(t,p)\ddot{F}(t,p)}$
  - → *spectral function:* $\rho\left(t,t',\mathbf{x}-\mathbf{x}'\right) = \frac{1}{N}\left\langle\left[\hat{\phi}_a(t,\mathbf{x}),\hat{\phi}_a\left(t',\mathbf{x}'\right)\right]\right\rangle$
  - → *excitation spectrum of system*
  - → *obtain via linear response* *A Piñeiro Orioli, J Berges, PRL 101 (2019)*



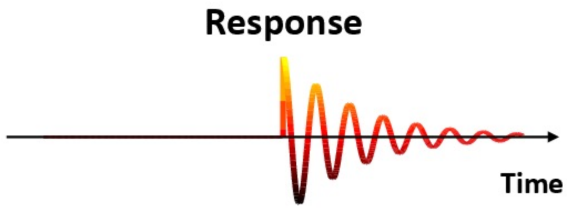


Fig.: K. Boguslavski, A. Kurkela, T. Lappi, J. Peuron, PRD 98 (2018)

  - → *FFT: frequency-momentum resolved spectrum*

- **Equilibrium connection:**
  - → *fluctuation dissipation relation* $F_{(\text{eq})}(\omega,p) = \left[f_{BE}(\omega)+1/2\right]\rho_{(\text{eq})}(\omega,p)$
  - → *even far from eq. "generalised FDR" has been observed*

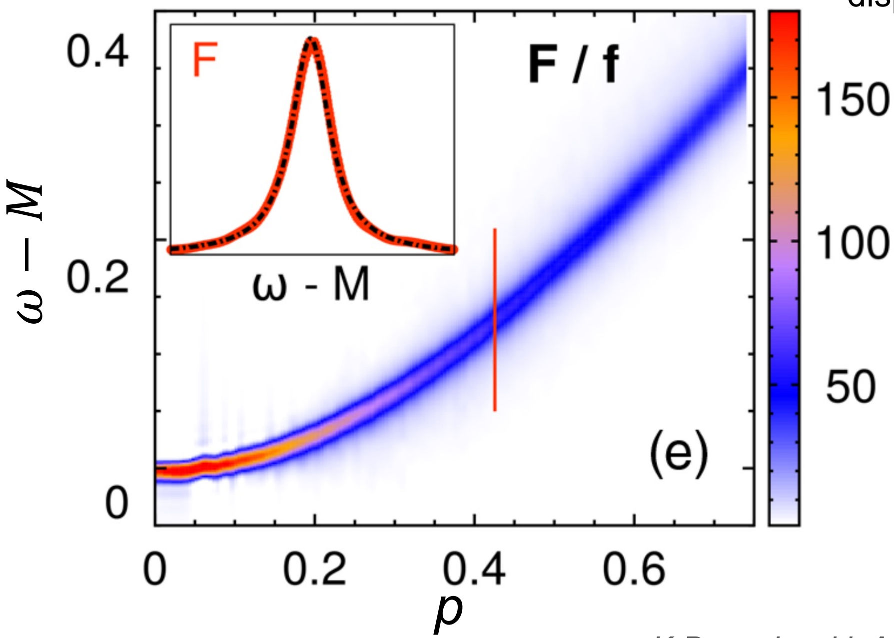
K Boguslavski, A Piñeiro Orioli, PRD 101 (2019)

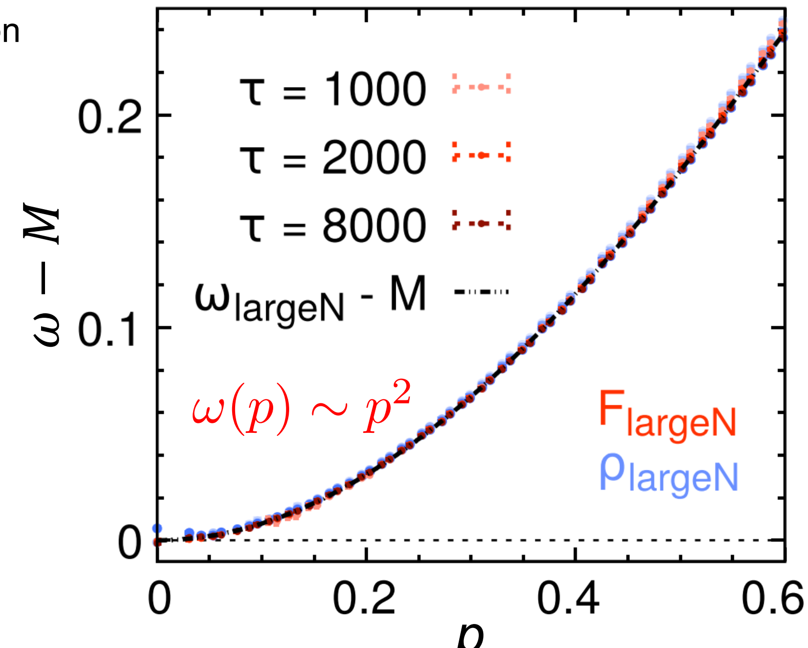
Unequal time correlators for large-N

- **Single, "large-*N*" (*N*=8) excitation**
  → *Lorentzian fit (for F)*

$$F_{\text{largeN}}(t,\omega,p) \simeq \frac{A_{\text{largeN}}(t,p)\gamma_{\text{largeN}}(t,p)}{(\omega-\omega_{\text{largeN}}(p))^2+\gamma_{\text{largeN}}(t,p)^2}$$

amplitude (A_{largeN}), decay width (γ_{largeN}), dispersion (ω_{largeN})





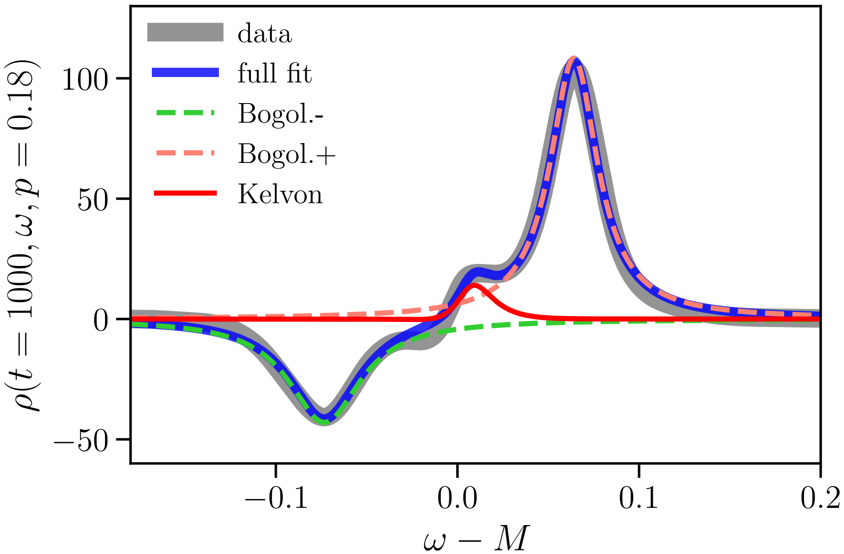
K Boguslavski, A P Orioli, PRD 101 (2020)

→ *similarly for the spectral function*

→ *as expected from large-N kinetic theory & low energy EFT*

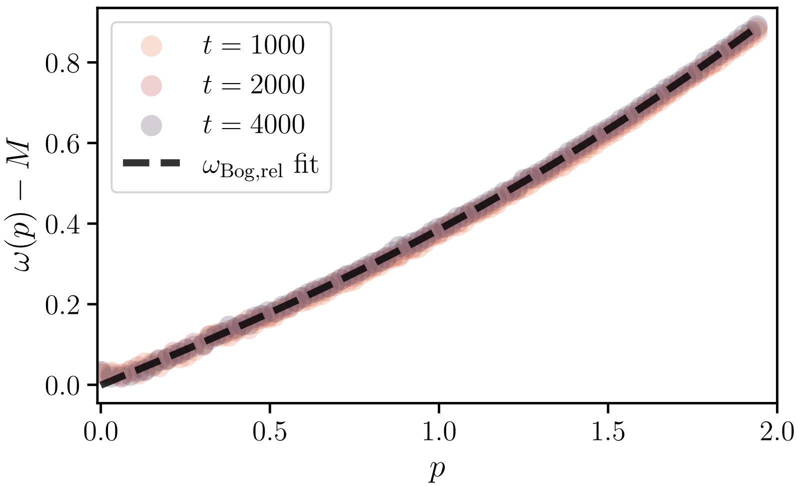
Different excitations from ρ, F for $O(1)$

- **Spectral function**

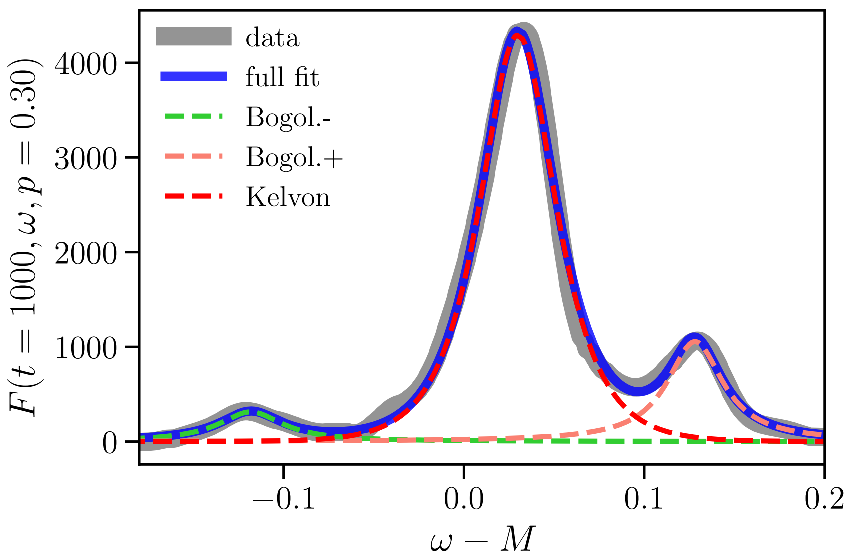


→ *peak at higher momenta: Bogoliubov dispersion, but it's not the dominant infrared excitation…*

A P Orioli, J Berges, PRL 101 (2019)
K Boguslavski, A P Orioli, PRD 101 (2020)



- **Statistical function**



→ *dominant IR peak stronger in statistical function (based on which f(t,p) is defined!)*
→ *not understood by existing effective theories*

$$F_{\mathrm{K}}(t,\omega,p) \simeq \frac{\pi}{2}\frac{A_{\mathrm{T}}(t,p)}{\gamma_{\mathrm{T}}(p)}\,\mathrm{sech}\left[\frac{\pi}{2}\frac{\omega-\omega_{\mathrm{T}}(t,p)}{\gamma_{\mathrm{T}}(p)}\right]$$

amplitude (A_{T}), decay width (γ_{T}), dispersion (ω_{T})

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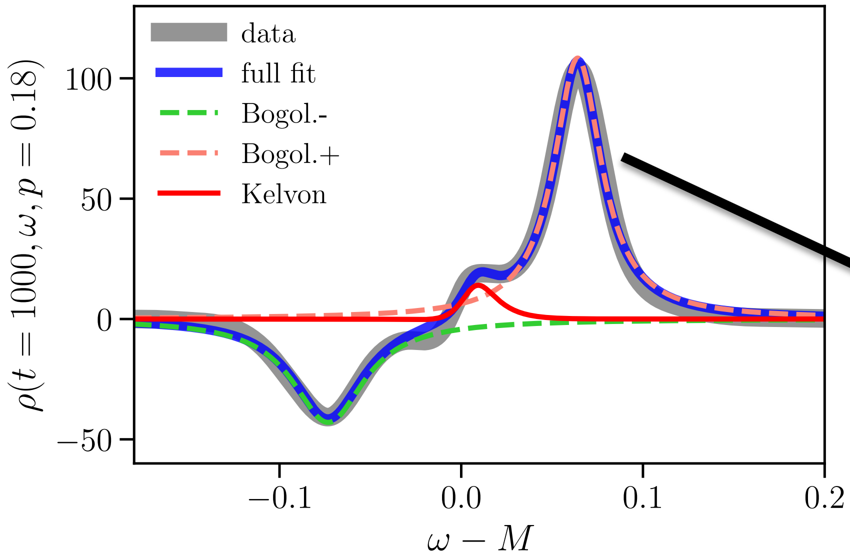

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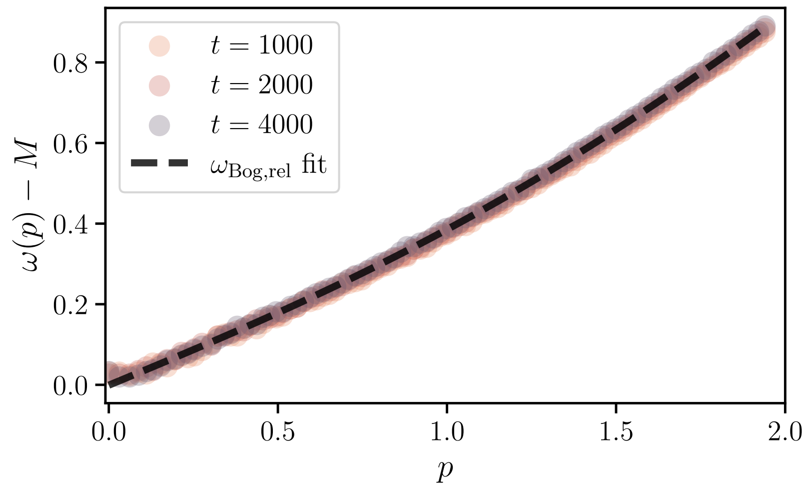

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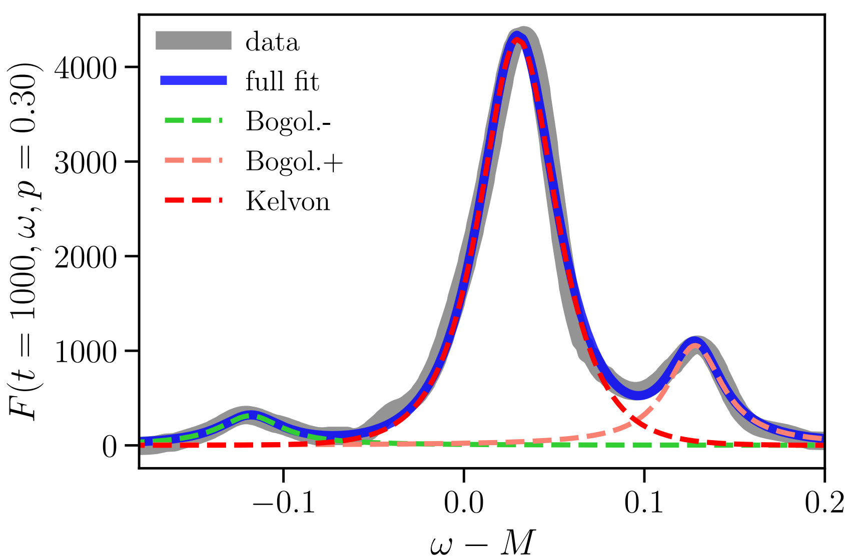

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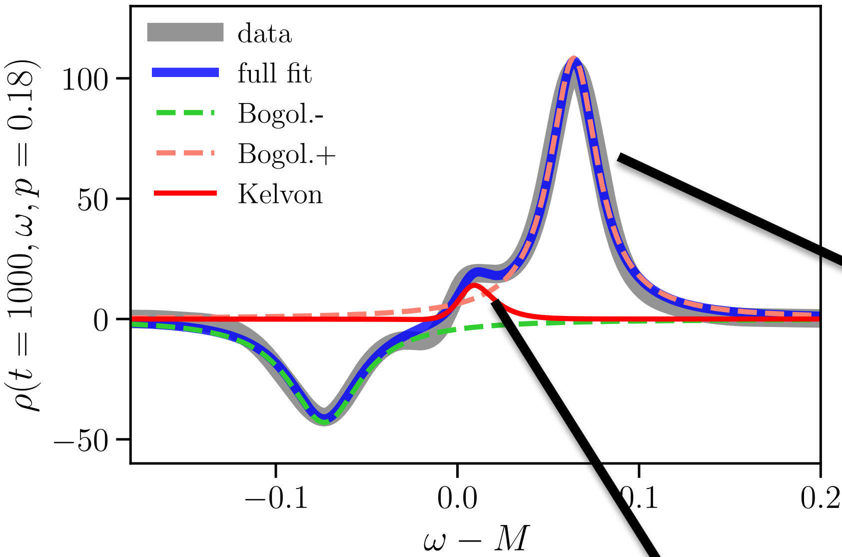

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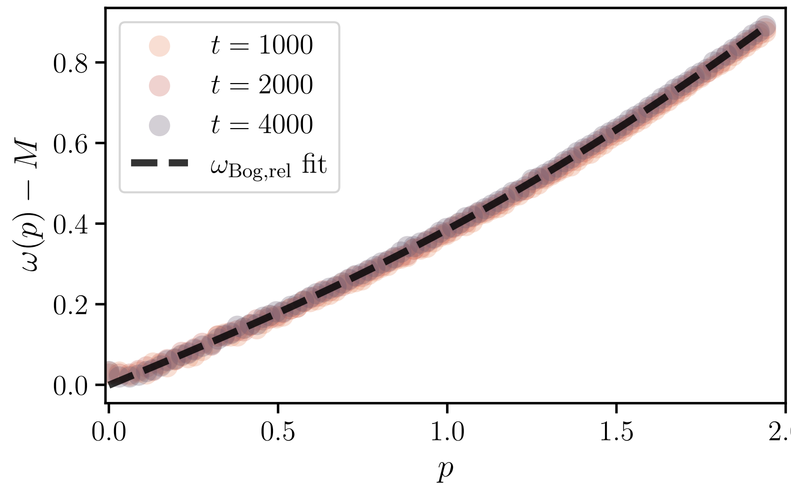

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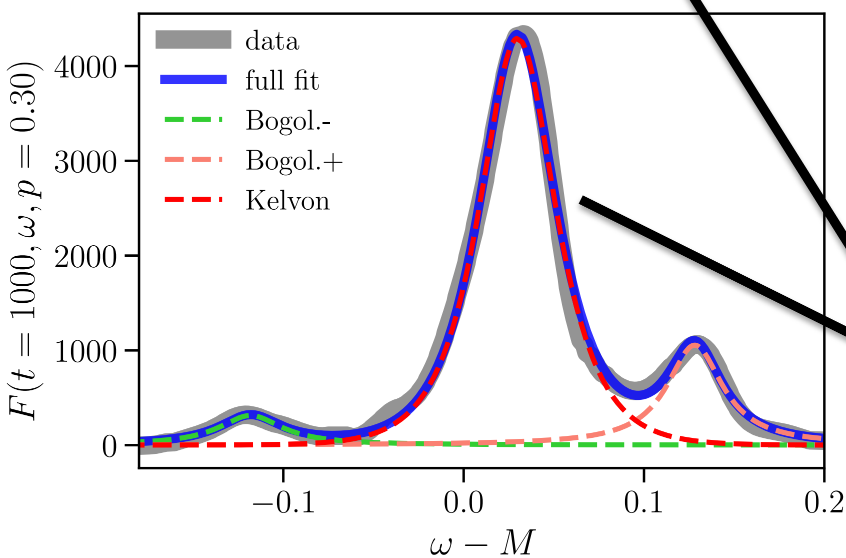

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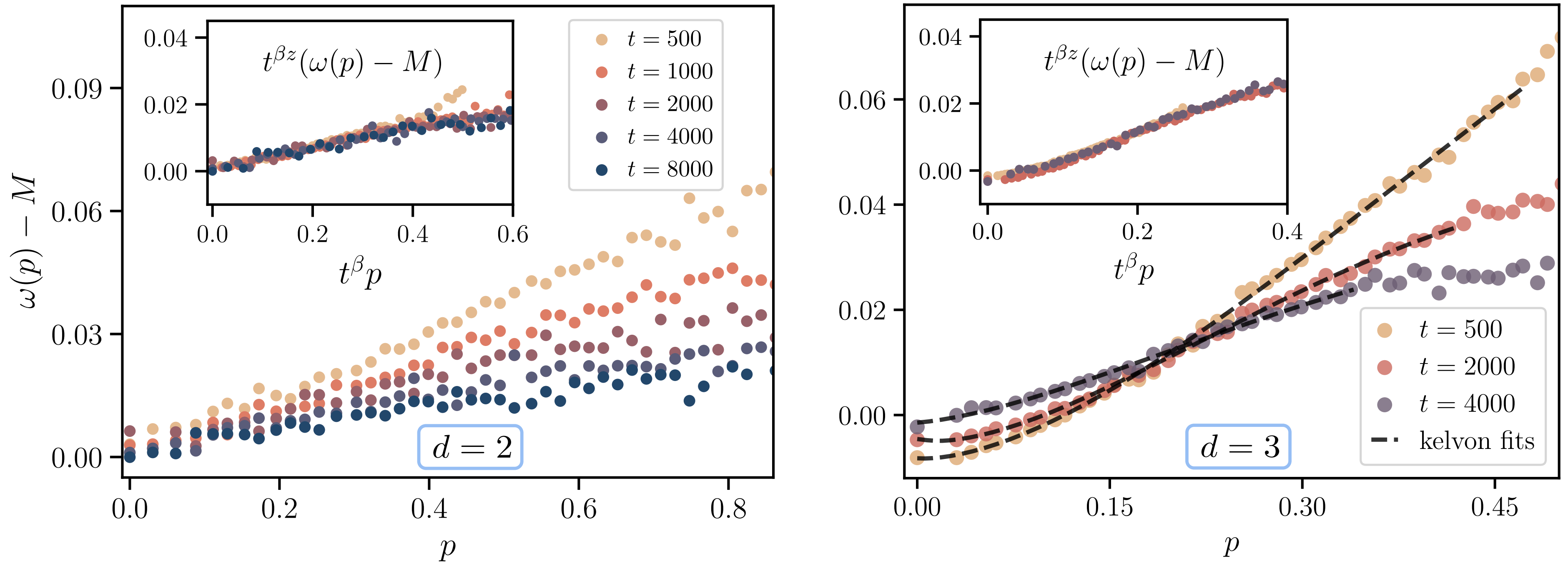
amplitude (A_{T}), decay width (γ_{T}), dispersion (ω_{T})

Dispersion relation of dominant (kelvon) peak

- **Time dependent, self-similar dispersion**
  → *Kelvin wave dispersion shows a very good fit in 3D*
  → *in 3D, $\omega(p)$ scales with β~0.5 and z~2, in 2D, it's β~0.25 and z~2*



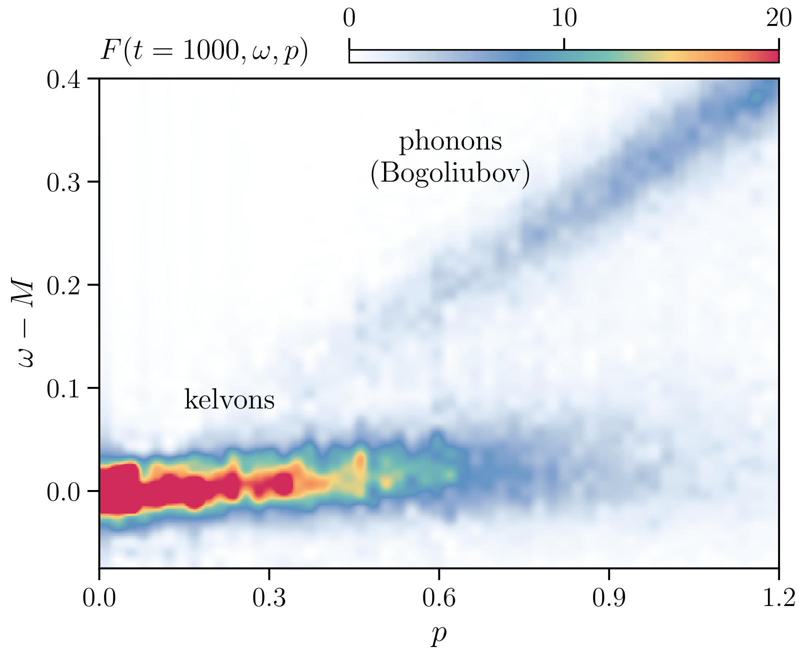
V Noel, T Gasenzer, K Boguslavski, PRR 7 (2025)

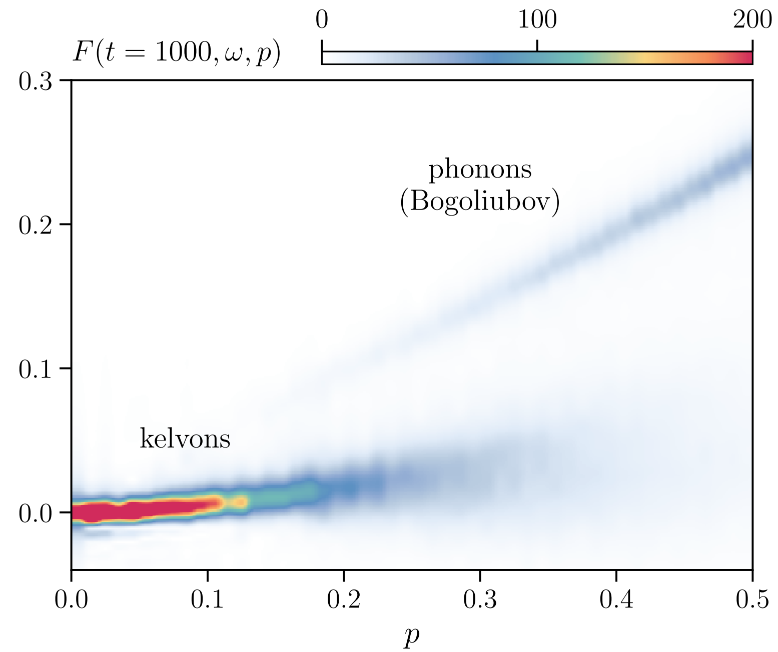
→ *β exponents agree across f(t,p) and coarsening*
→ *dispersions flatten due to underlying coarsening*

Excitations in the frequency-momentum heatmap

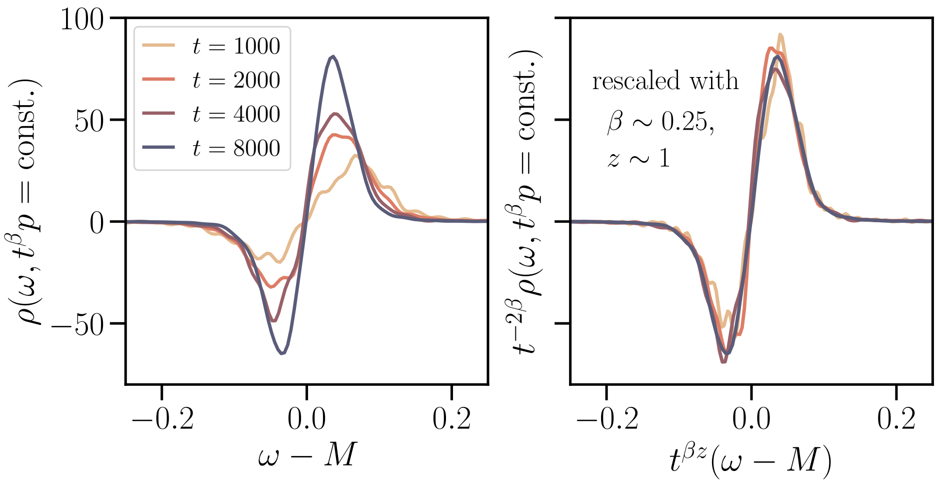
- **Qualitative similarity in F in d=2,3**

V Noel, T Gasenzer, K Boguslavski, PRR 7 (2025)





→ *"kelvon peak" dominant infrared excitation in f(t,p)*



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  → *exponent inherited from kelvons*
  → *new effective theory?*

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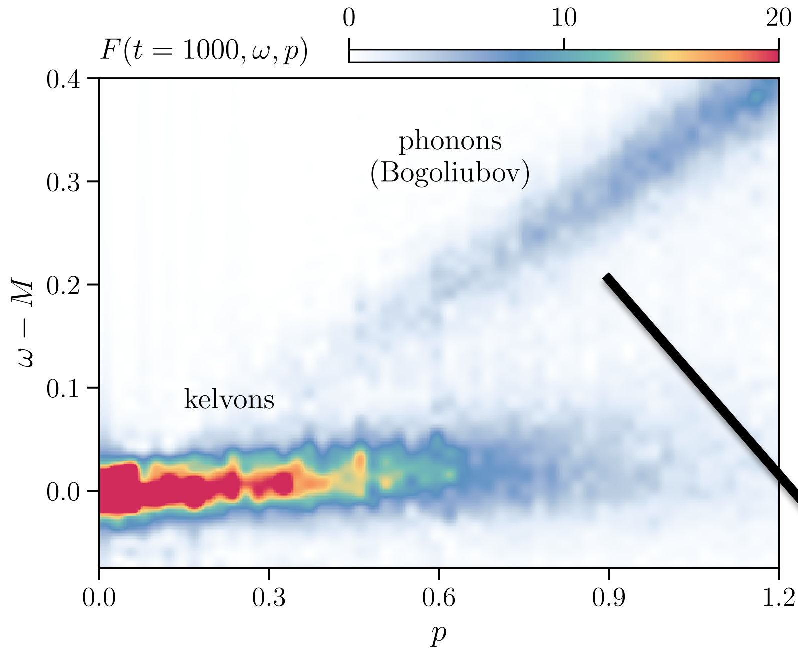

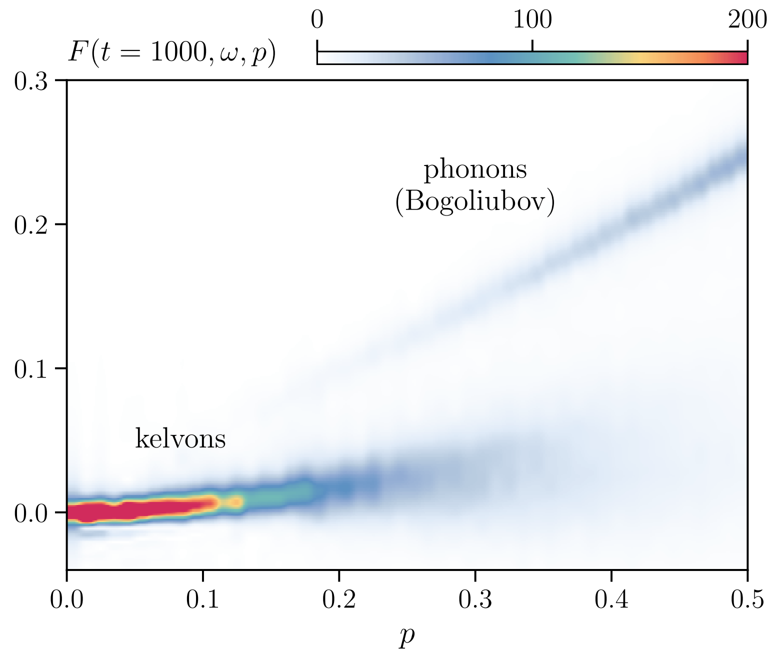

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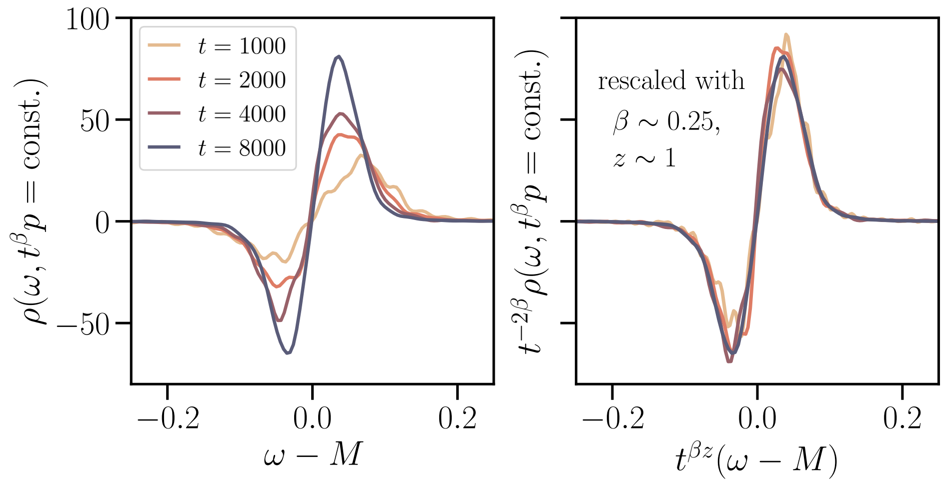

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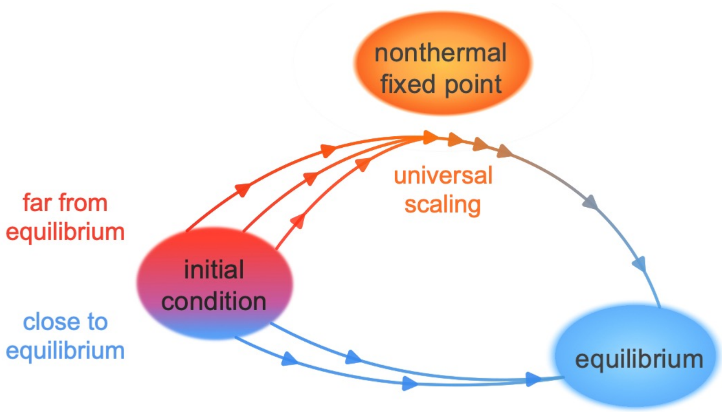
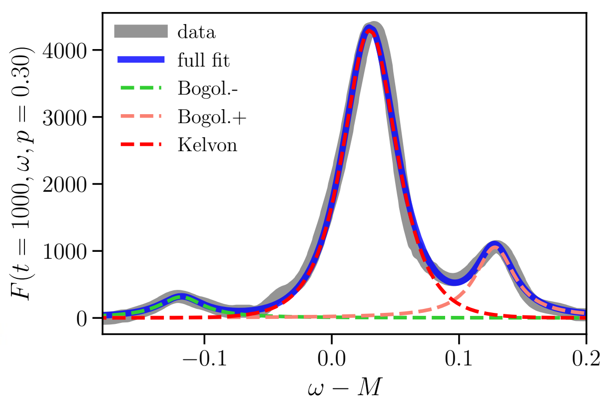
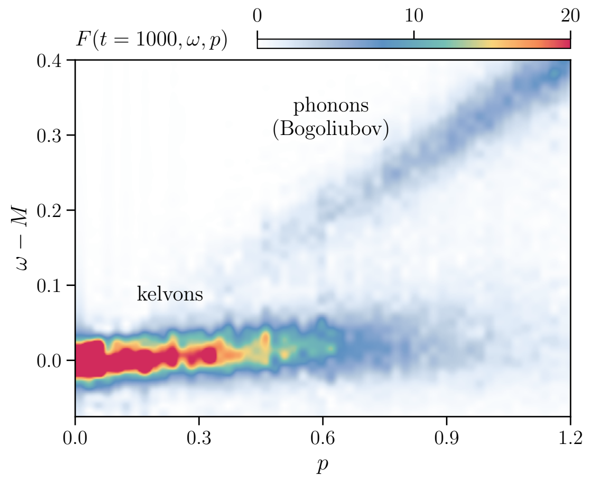
Summary & Outlook

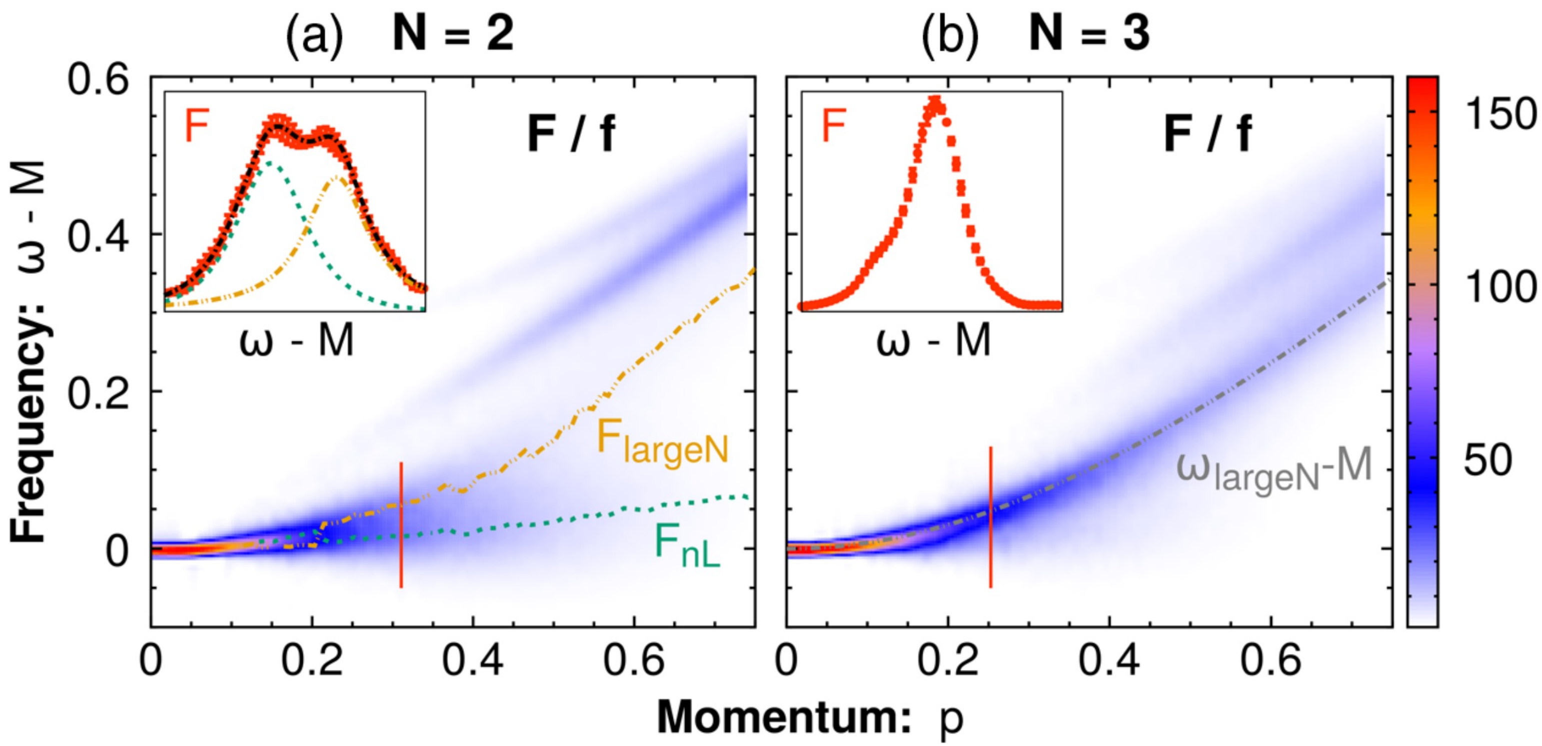
- NTFP: different effective descriptions
  → kinetic theory & low energy EFT: scattering of quasiparticles
  → coarsening dynamics: topological defects
- Unequal time correlators:
  → large-*N* scalar fields have a single p^2 excitation
  → dominant infrared excitation in single component scalar fields are kelvons
- Technique applicable in other systems to disentangle phenomena
- Towards cold atom experiments:
  → Kelvin waves pictured directly before
  V Bretin, P Rosenbusch, F Chevy, GV Shlyapnikov, J Dalibard, PRL 90 (2003)
  → Measuring commutators & anticommutators of fields
  KT Geier, P Hauke, PRX Quantum 3 (2022)





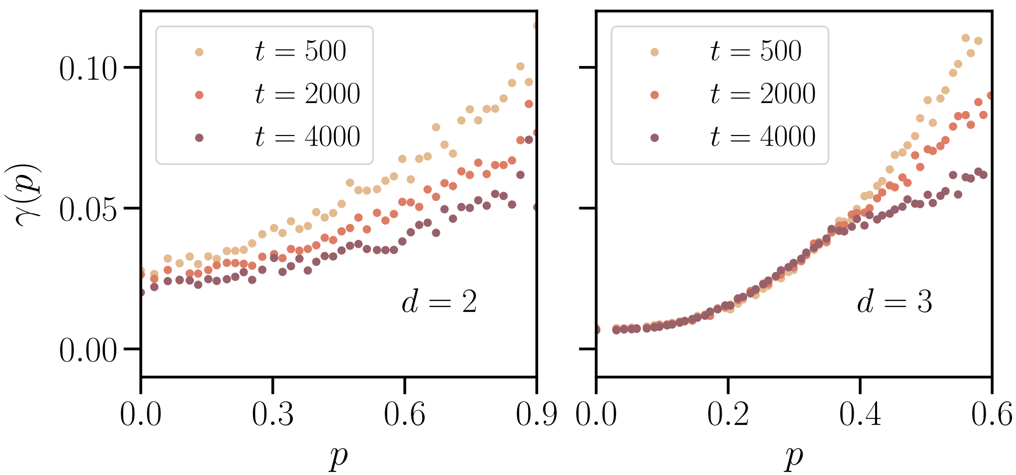


O(2), O(3) excitations in *F*

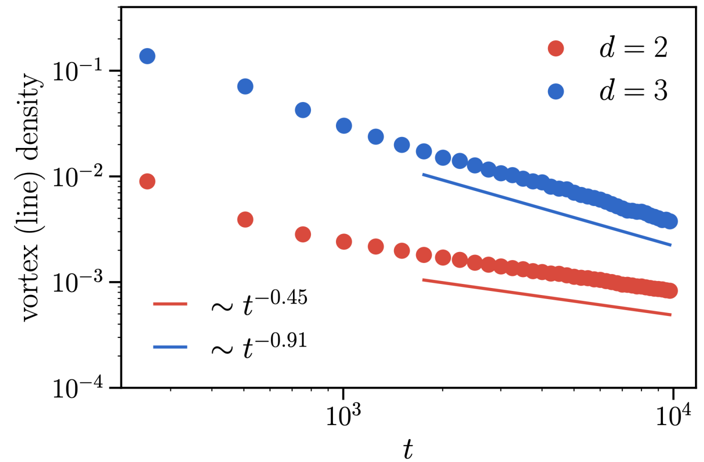


K Boguslavski, A P Orioli, PRD 101 (2020)

Decay widths

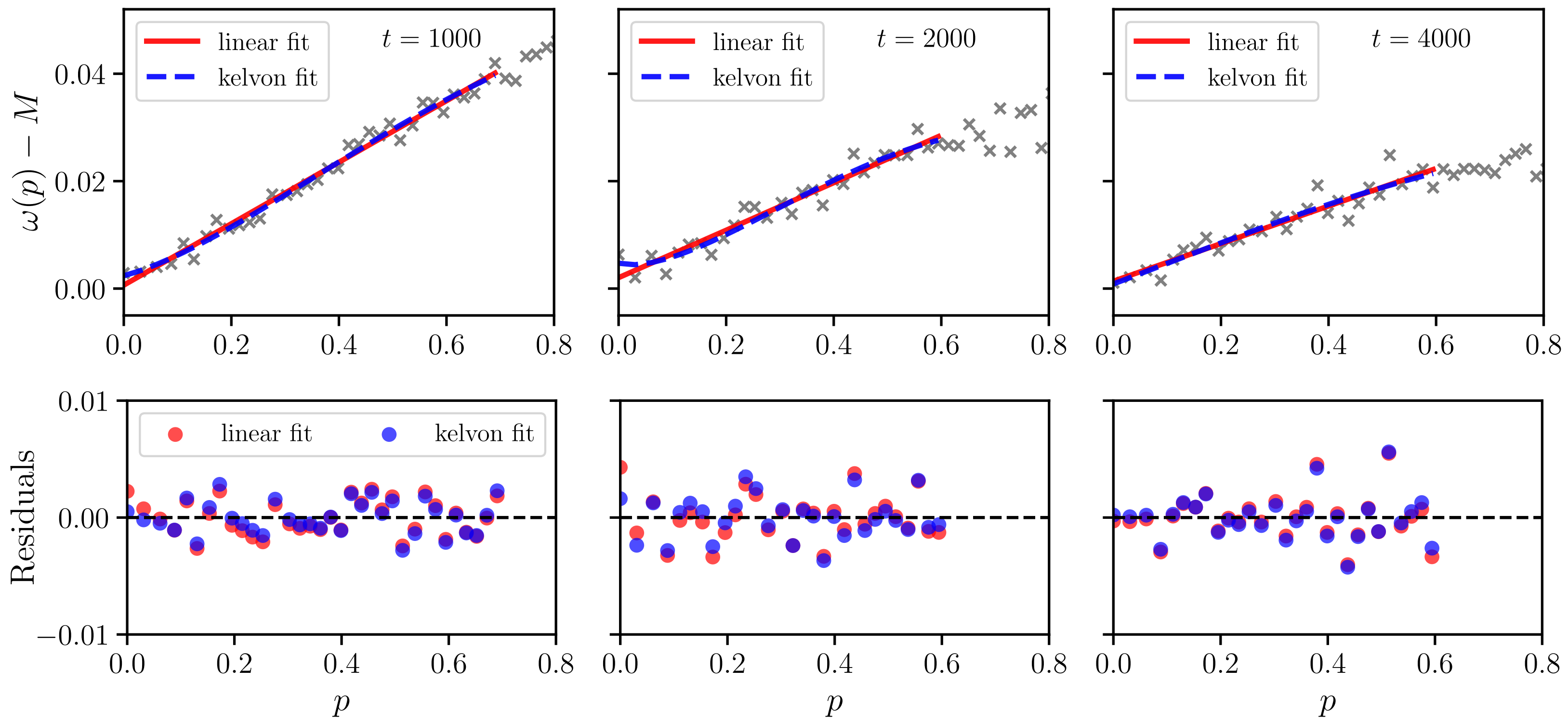


Coarsening exponents



V Noel, T Gasenzer, K Boguslavski, PRR 7 (2025)

Kelvon dispersion residues (2D)



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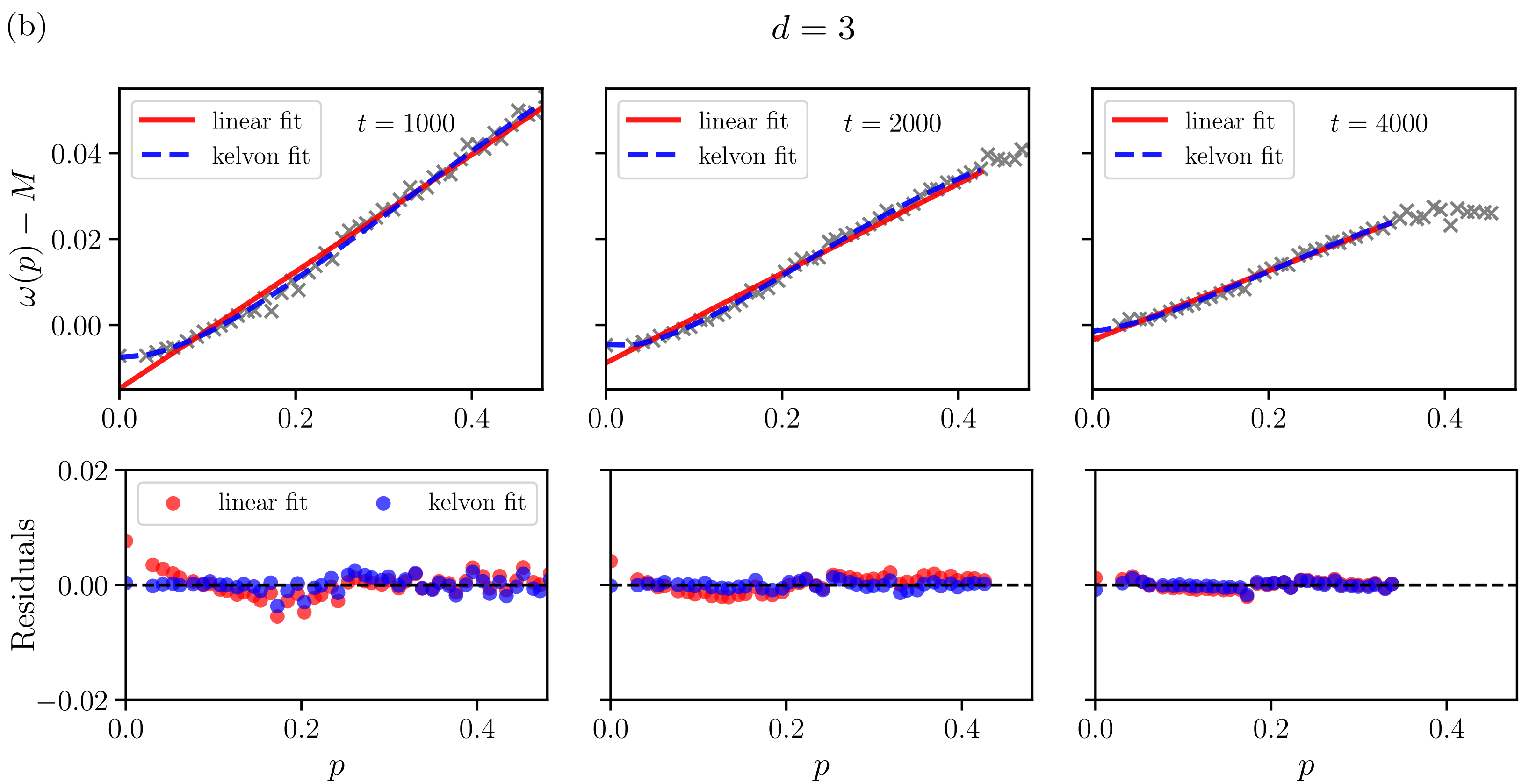
Kelvon dispersion residues (3D)



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Persistent homology observables