Early time hydrodynamic attractor in a nearly-unitary Fermi gas

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in collaboration with
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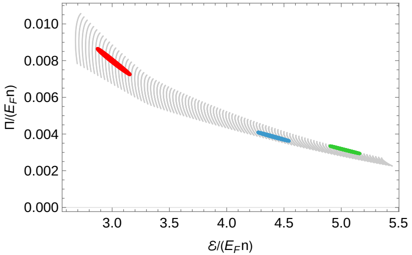
based on 2507.02838



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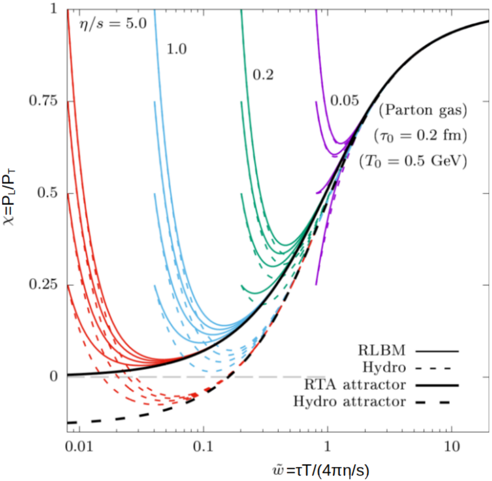
What is a hydrodynamic attractor?

Convergence to universal time evolution long before equilibrium

prototypical example:
Bjorken flow pressure anisotropy (P_L/P_T), energy density and other ratios

interpretation:
expansion provides another information loss mechanism

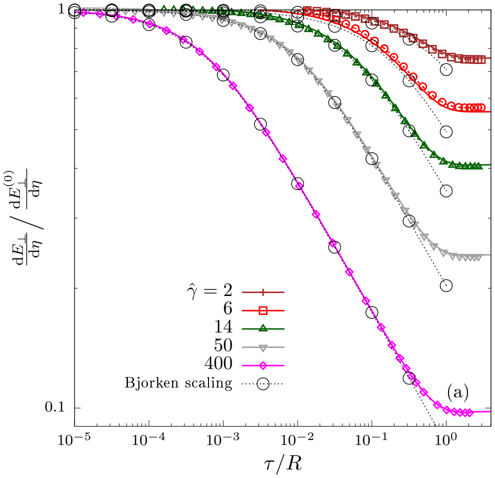


Victor Ambrus, Lorenzo Bazzanini, Alessandro Gabbana, Daniele Simeoni, Raffaele Tripiccione, Sauro Succi, Nature Computat.Sci. 2 (2022) 641-654

even in 2+1D simulations, at early times $\tau \ll R$, transverse dynamics can be neglected
$\rightarrow$ local Bjorken flow attractor

transverse collection of attractor curves can predict behavior of dynamical quantities: $u_\perp$, ϵ_2, $\mathrm{d}E_\perp/\mathrm{d}\eta$, ...

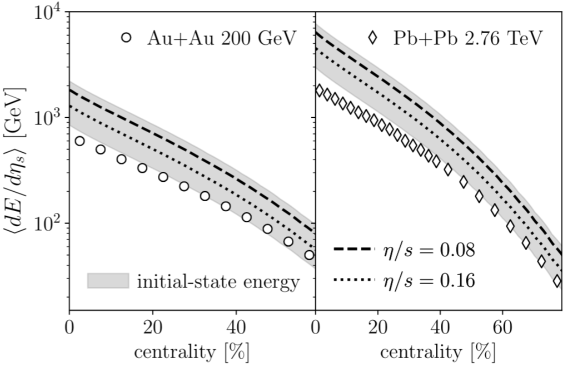
Victor Ambruș, Sören Schlichting, Clemens Werthmann, PRD 105 (2022) 014031 and PRD 107 (2023) 094013



Practical applications

final state Multiplicity $\sim$ Entropy
$\rightarrow$ attractor gives link between Multiplicity and initial energy!

$$\frac{dN_{ch}}{d\eta} = \frac{4}{3}\frac{N_{ch}}{S}C_\infty^{\frac{3}{4}}\left(4\pi\frac{\eta}{s}\right)^{\frac{1}{3}}\left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{\frac{1}{3}}\int d^2\mathbf{x}\,[e(\mathbf{x})\tau]_0^{\frac{2}{3}}$$



Giuliano Giacalone, Aleksas Mazeliauskas, Sören Schlichting, PRL 123 (2019) 262301

Long term goal

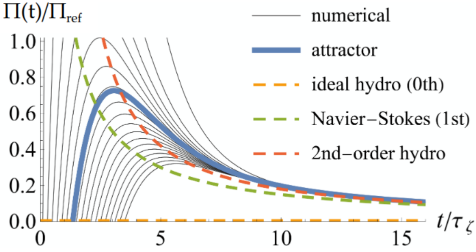
new effective theory for attractor behaviour, superceding hydrodynamics

Paul Romatschke, PRL 120 (2018) 1, 012301

to arrive at this, improve understanding of emergence: examine attractor in more generality

recent discovery: attractor in ultracold quantum gases

Keisuke Fujii, Tilman Enss, PRL 133 (2024) 173402



Bjorken flow vs. Cold atom attractor

Bjorken flow	cold atoms
ultrarelativistic	nonrelativistic
longitudinal expansion	mimicked isotropic contraction
evolution of shear	evolution of bulk
experimentally inaccessible	**directly accessible in experiment**

Attractor in cold atoms

numerical observation: bulk viscosity is of Drude form

Tilmann Enss, PRL 123 (2019) 205301

$$\zeta(\omega) = \frac{i\chi}{\omega + i\tau_\Pi^{-1}} \quad (\zeta = \chi\tau_\Pi)$$

$\Rightarrow$ promote Π-equation to MIS type, look for attractor

Keisuke Fujii, Tilman Enss, PRL 133 (2024) 173402

$$\tau_\Pi \dot{\Pi} + \Pi = -3\zeta\, a^{-1}\partial_t a$$

$a(t)$: "scattering length" quantifies interaction strength

simple t-dep. via unitary limit $\zeta \overset{a\to\infty}{\sim} a^{-2}$, power law drive $a \sim t^\alpha$

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Tilmann Enss, PRL 123 (2019) 205301

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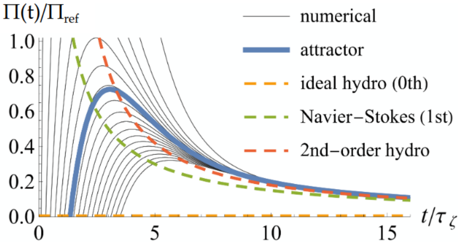
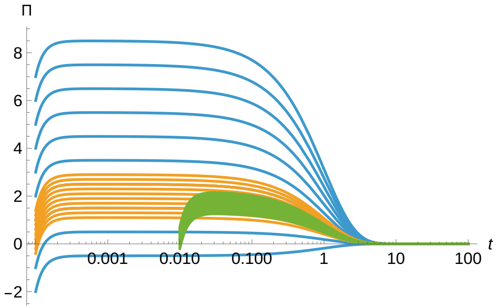


But is there an early time attractor?

Early time behaviour of bulk pressure

With $\tau_\Pi \dot{\Pi} + \Pi = -3\zeta\ a^{-1}\partial_t a$, $\zeta = \zeta^{(2)} a^{-2}$: no early time attractor!



Late time attractor behaviour: competition of

- excitation via drive ($\dot{\Pi} \sim -3\tau_\Pi^{-1}\zeta\ a^{-1}\partial_t a$)
- relaxation ($\dot{\Pi} \sim -\tau_\Pi^{-1}\Pi$)

starting at early times: timescale separation!

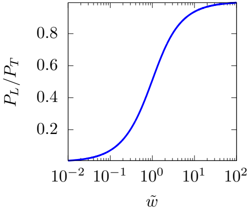
How does it work in Bjorken flow?

Attractor curve transitions between early and late time "fixed points"

Jean-Paul Blaizot, Li Yan, Annals Phys. 412 (2020) 167993 and PRC 104 (2021) 5, 055201

for $f_\pi = (P_L - P)/\mathcal{E}$: $\quad \partial_\tau f_\pi = -\tau_{\text{int}}^{-1} f_\pi$

vs. $\quad \partial_\tau f_\pi = \tau^{-1}(f_\pi^2 - \lambda f_\pi - 64/15)$



Generally, when specific t-dep. dominates evolution equation:

$$\partial_t f = G(f,t) \quad \Rightarrow \text{hopefully in some } t\text{−range} \quad \partial_t f \approx T(t)F(f)$$

then we can read off attractor behaviour

$F(f) = \text{const.}$: f changes by $\text{const.} \cdot \int^t \mathrm{d}t' \; T(t')$ (no attractor)

$F(f_0) = 0$: f_0 is a fixed point that is

- attractive if $F'(f_0) < 0$
- repulsive if $F'(f_0) > 0$
- mixed if $F'(f_0) = 0$

How to get early time attractor?

idea: like in Bjorken, look at ratio $\Pi/\mathcal{E}$
energy density evolves according to
Shina Tan, AoP 323 (2008) 2971

$$\partial_t \mathcal{E} = -3\Pi \, a\partial_t a^{-1} + \frac{C_{\rm eq}}{4\pi m}\partial_t a^{-1}$$

$\Rightarrow \partial_t(\Pi/\mathcal{E})$ gets extra term from evolution of $\mathcal{E}$

$$\partial_t(\Pi/\mathcal{E}) = \left[3(\Pi/\mathcal{E})^2 - 3(\chi/\mathcal{E})\right] \, a\partial_t a^{-1} - \tau_\Pi^{-1}(\Pi/\mathcal{E})$$

$(\chi = \tau_\Pi^{-1}\zeta)$

$\Rightarrow$ if $\chi/\mathcal{E} \sim$ const., same form as in the Bjorken case!
expect early time attractor and repulsor at

$$\Pi/\mathcal{E} = \pm\sqrt{\chi/\mathcal{E}}$$

1st caveat: a-dep. of χ

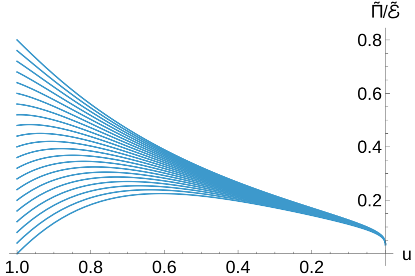
calculations show (close to unitarity): $\chi \sim a^{-2}$
K.Dusling, T.Schäfer, PRL 111 (2013) 120603   T.Enss, PRL 123 (2019) 205301

$$\partial_t(\Pi/\mathcal{E}) = \left[3(\Pi/\mathcal{E})^2 a - (\chi a^2/\mathcal{E})\ a^{-1}\right] \partial_t a^{-1} - \tau_\Pi^{-1}(\Pi/\mathcal{E})$$

$\Rightarrow$ terms compete if $a \sim \mathcal{O}(1)$, but no "fixed point": $\partial_t(\Pi/\mathcal{E}) \neq 0$

Early attractor behaviour independent of form of $a^{-1}(t)$:
$u = a^{-1}(t)$, $X(t) = \tilde{X}[u(t)]$, dropping $\tau_\Pi^{-1}(\Pi/e)$:

$$\partial_u(\tilde{\Pi}/\tilde{\mathcal{E}}) = 3(\tilde{\Pi}/\tilde{\mathcal{E}})^2 u^{-1} - (\chi a^2/\tilde{\mathcal{E}})\ u$$

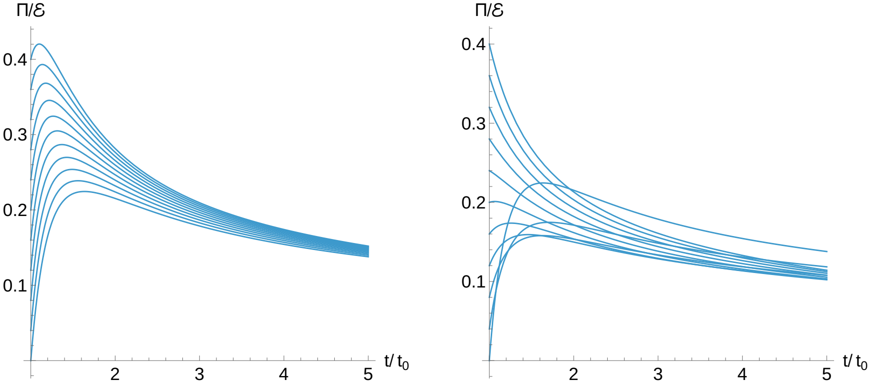


2nd caveat: $\mathcal{E}$-dep. of χ

$\chi/\mathcal{E} \sim \mathcal{E}^{-5/2}$ $(T \gg T_F)$ $\Rightarrow$ nontrivial coupling of two equations

$$\partial_t(\Pi/\mathcal{E}) = \left[3(\Pi/\mathcal{E})^2 a - \chi_0 a^{-1}\mathcal{E}^{5/2}\right]\partial_t a^{-1} - \tau_\Pi^{-1}(\Pi/e)$$

$$\partial_t \mathcal{E} = -3\Pi a \partial_t a^{-1}$$



$\Rightarrow$ attractor is a curved 2D surface in $(\Pi, \mathcal{E}, t)$-space!

What about the Bjorken case?

$(f_\pi = (P_L - P)/\mathcal{E})$

$$\tau\partial_\tau f_\pi + \left(\frac{\delta_{\pi\pi}}{\tau_\pi} + \frac{\tau_{\pi\pi}}{3\tau_\pi} - \frac{4}{3} + \frac{\tau}{\tau_\pi} + \frac{\tau}{\tau_\pi}\phi_7 e f_\pi - f_\pi\right) f_\pi + \frac{16}{45} = 0$$

In (conformal) Bjorken flow, coefficients also depend on $\mathcal{E}$, but:

1. all coefficients are either constant or $\propto \tau T$
2. variable transformation $\tilde{w} = \text{const.} \cdot \tau T$
   comes with Jacobian $\tau\partial_\tau = (2/3 - f_\pi/4)\tilde{w}\partial_{\tilde{w}}$,
   which does not introduce another $\mathcal{E}$-dep.

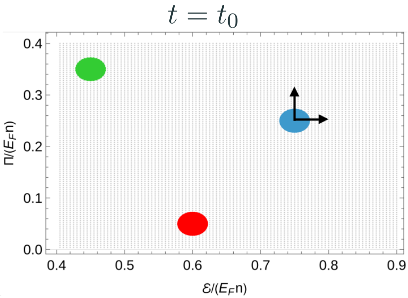
neither statement holds in the cold atom case!

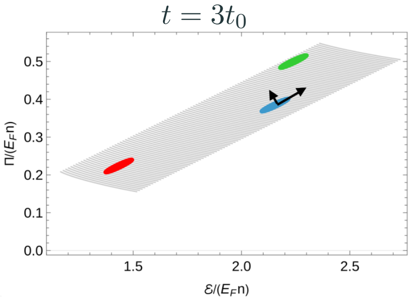
$$0 = (2/3 - f_\pi/4)\tilde{w}\partial_{\tilde{w}}\mathcal{M}_n + a(n)\mathcal{M}_n + b(n)\mathcal{M}_{n-1} + c(n)\mathcal{M}_{n+1}$$

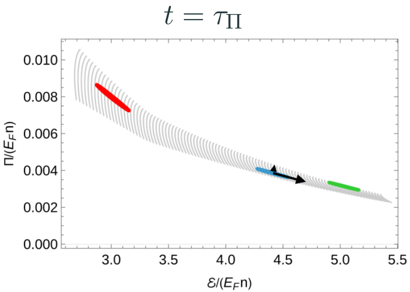
kinetic theory: also coupled equations, but ratios of moments $\mathcal{M}_n = M_n/M_{n,\text{eq}}$ fixed on attractor, while $\mathcal{E}$ is not.

Consider curves in 2D state space: contract to 1D curve!
$\Rightarrow$ to track convergence & attractor evolution:
Principal component analysis

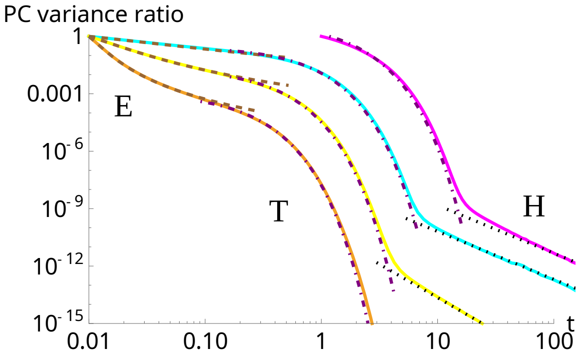
Michał P. Heller, Ro Jefferson, Michał Spaliński, Viktor Svensson, PRL 125 (2020) 132301







Principal component variance ratio



recover all expected behaviour (squared because of variance):

E) expansion driven convergence, same as in absence of interaction

T) thermalization driven convergence $\sim [\exp(-t/\tau_\Pi)]^2$

H) hydrodynamic Navier-Stokes tail $\sim [a^{-1}(t)\partial_t a^{-1}(t)]^2$

- early time attractor in ultracold quantum gases by including evolution of energy density
- problem 1: balance between expansion terms depends on time $\Rightarrow$ no early time fixed point
- problem 2: nontrivial coupling of evolution equations for $\mathcal{E}$ & Π $\Rightarrow$ no 1D attractor curve

Solution

1.Pick $a \sim \mathcal{O}(1)$. 2.All expected attractor behaviour can be recovered via PCA in higher dimensional state space!