Hydrodynamic transport in ultracold atoms

probing the boundaries of hydrodynamics

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Attractor workshop, ECT* Trento, 22 September 2025

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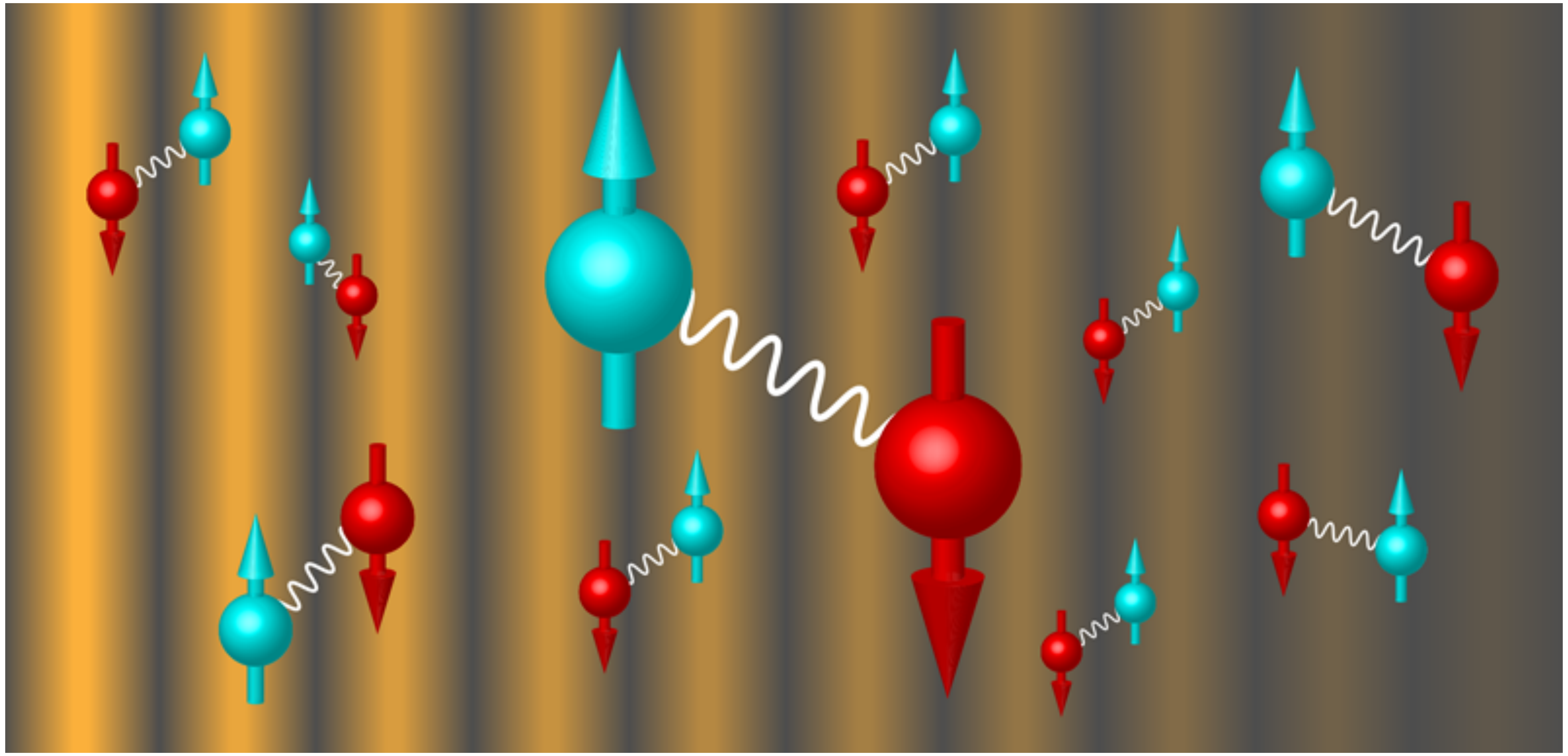
strongly interacting Fermi gas

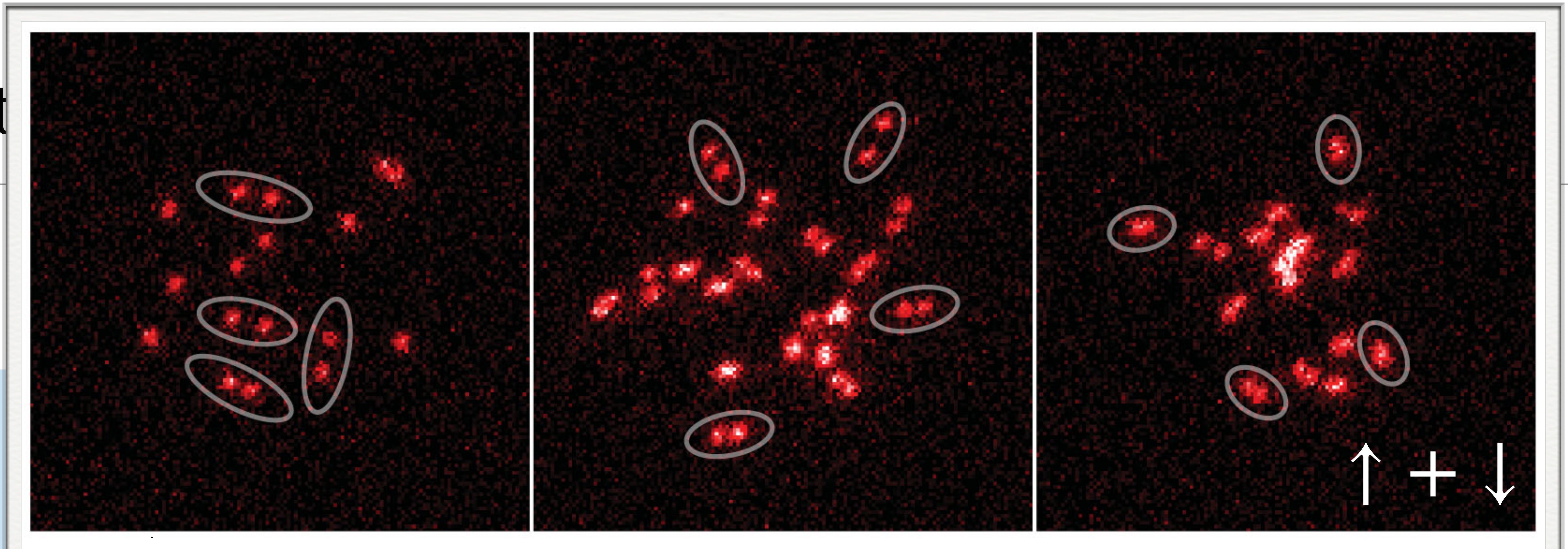


dilute gas of ↑ and ↓ fermions with contact interaction

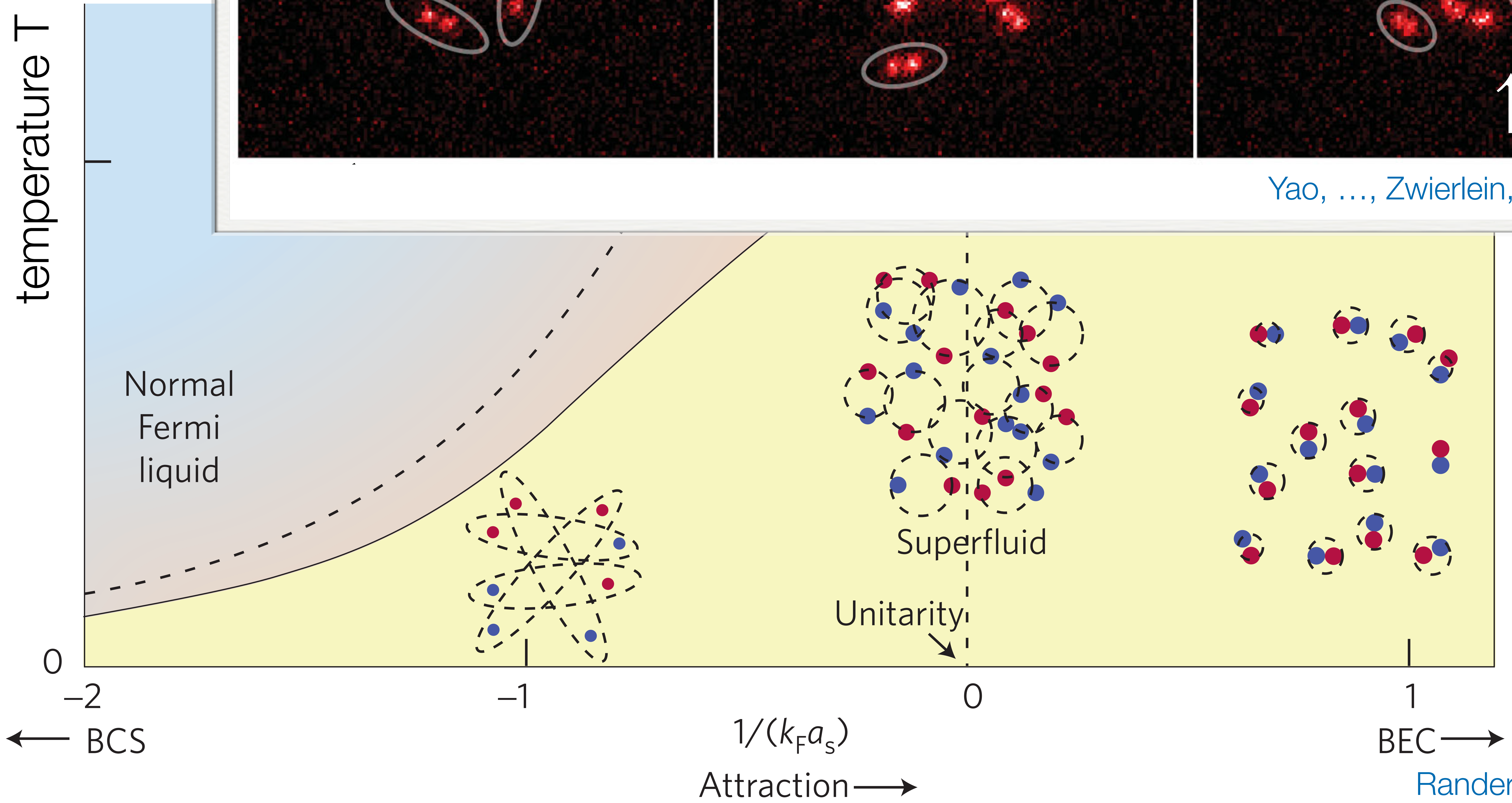
$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

Enss, Physics 2020

strongly int



Yao, …, Zwierlein, PRL 2025



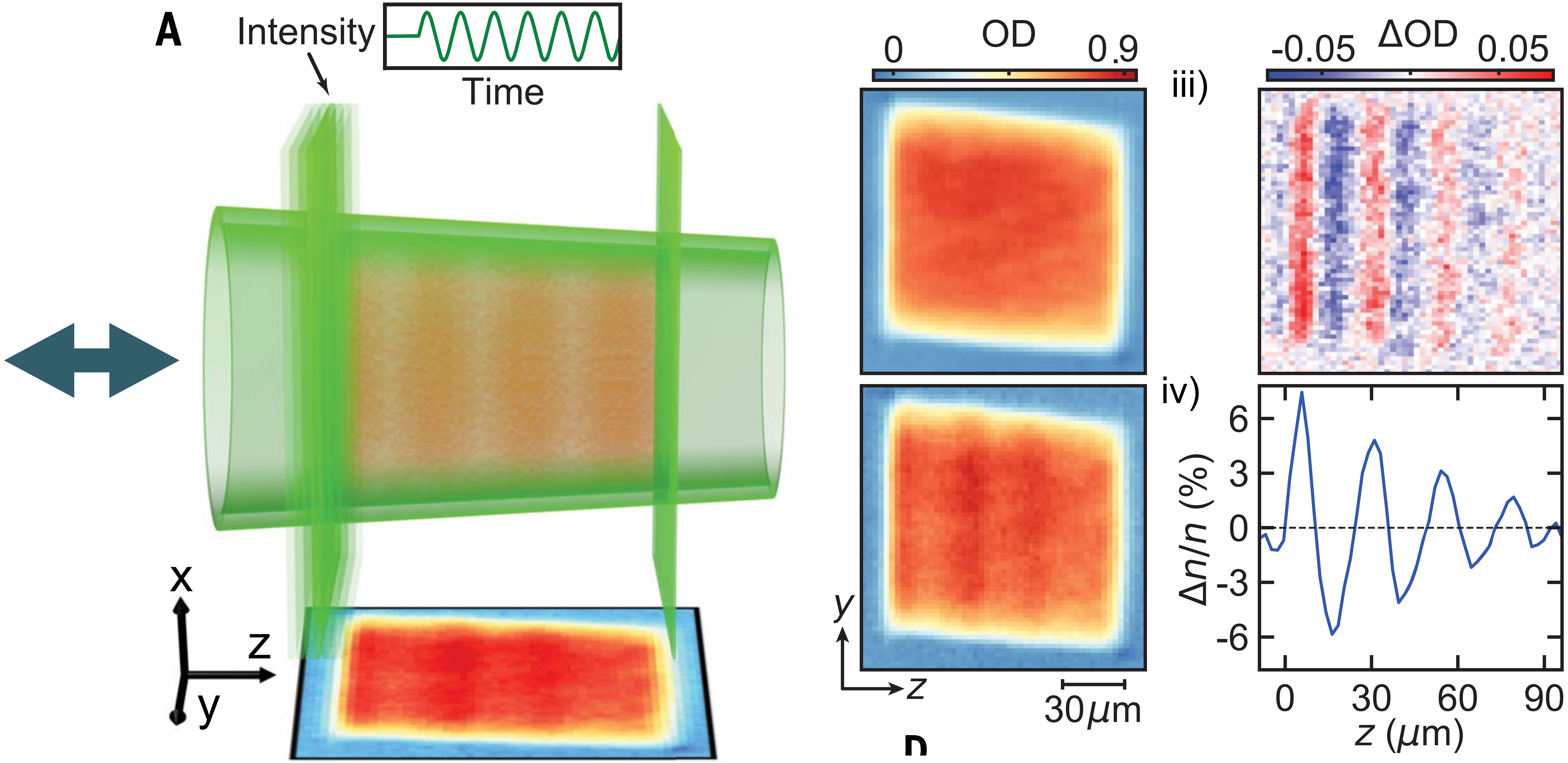
Randeria, Zwerger & Zwierlein 2012

sound waves in a quantum gas

gas of ^{6}Li atoms

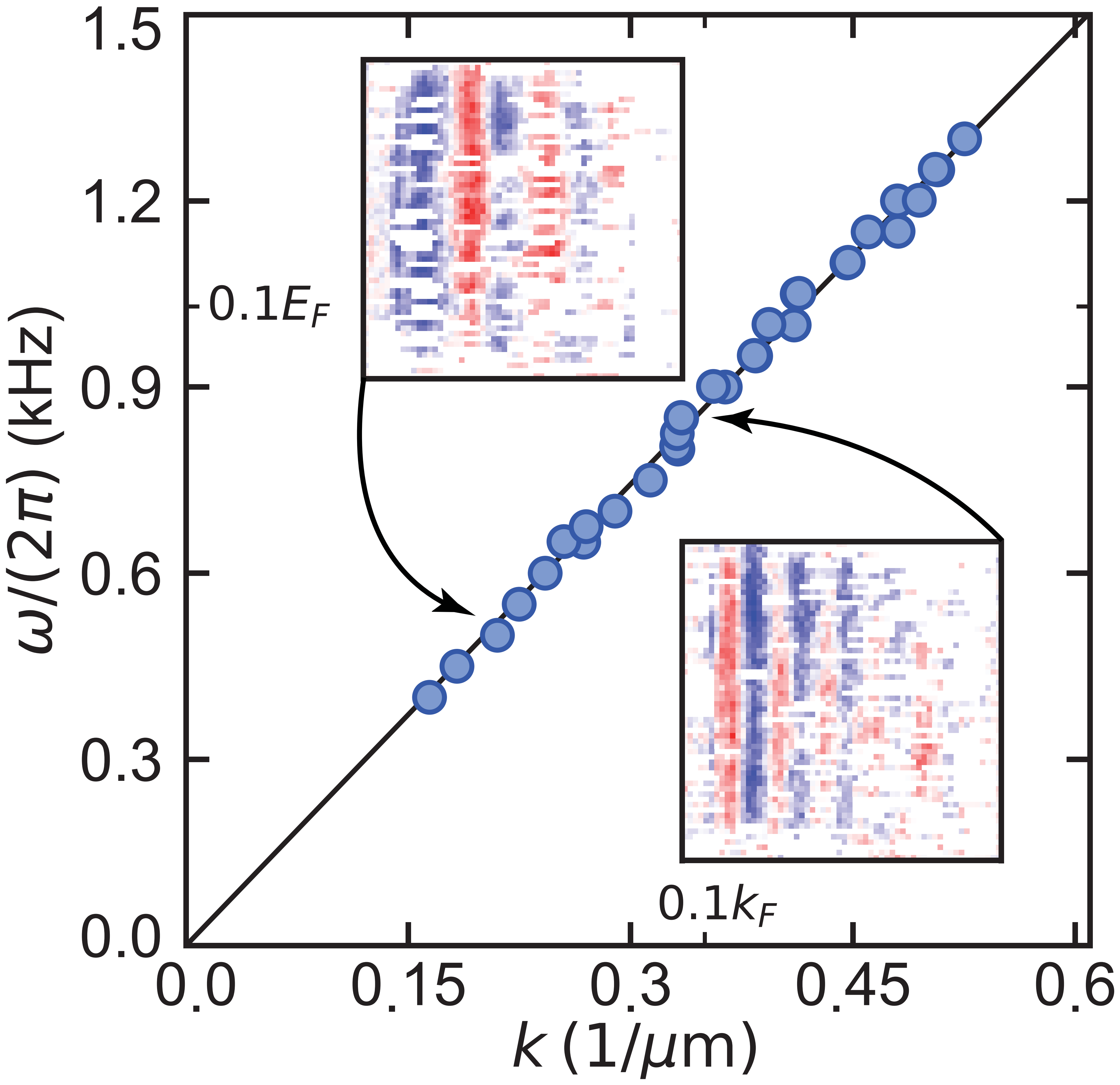
length 100μm

density ~1μm^{-3}



Patel et al., Science 2020 (Zwierlein group, MIT)

how quickly does sound propagate?



find linear dispersion relation

$$\omega = ck$$

speed of sound in ultracold gas:

$$c \approx 15\,\text{mm/s}$$

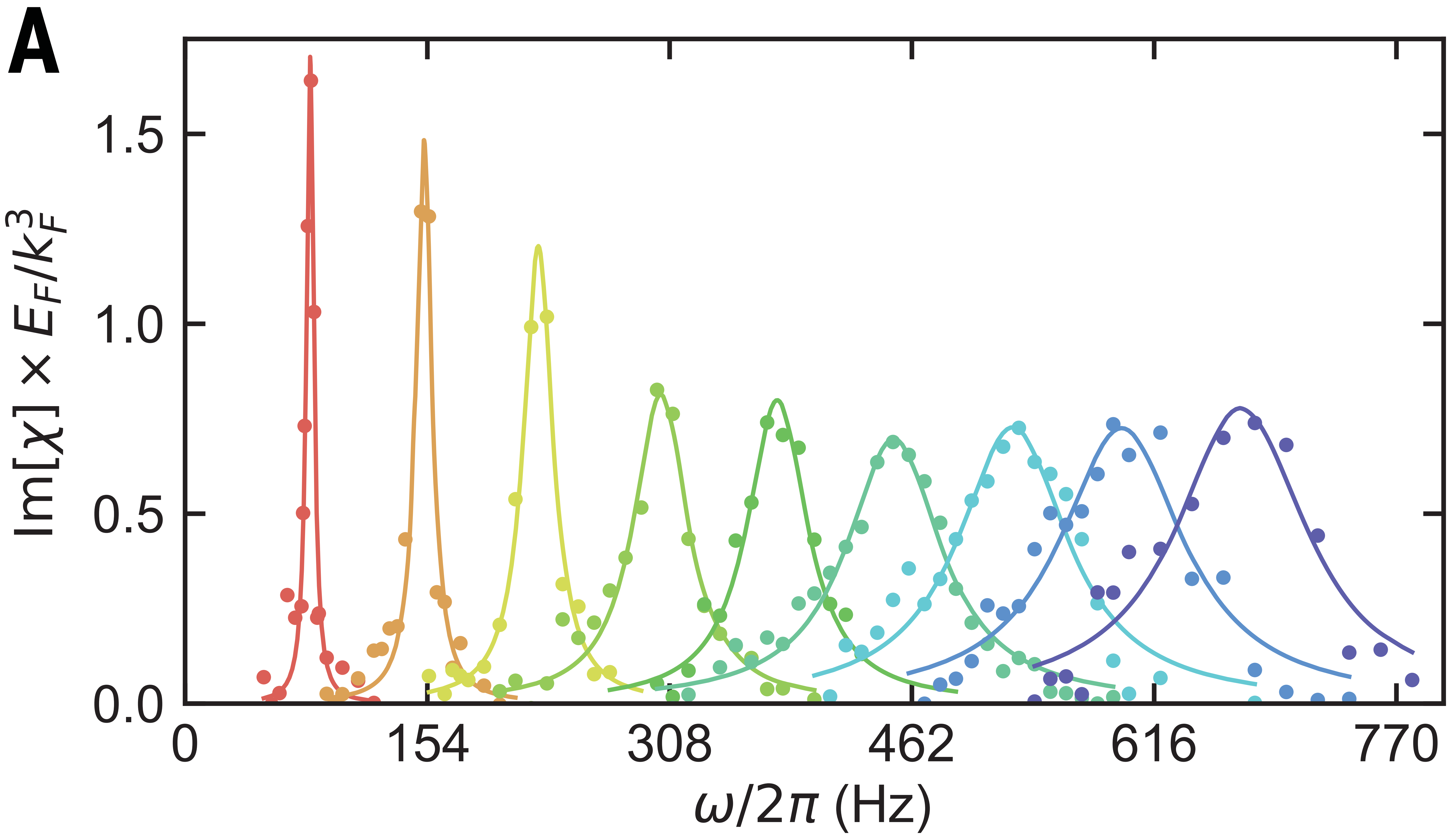
$$mc^2 = \left(\frac{\partial P}{\partial n}\right)_S = \frac{V^2}{N}\left(\frac{\partial^2 E}{\partial V^2}\right)_S$$

Patel et al., Science 2020

density response: sound attenuation

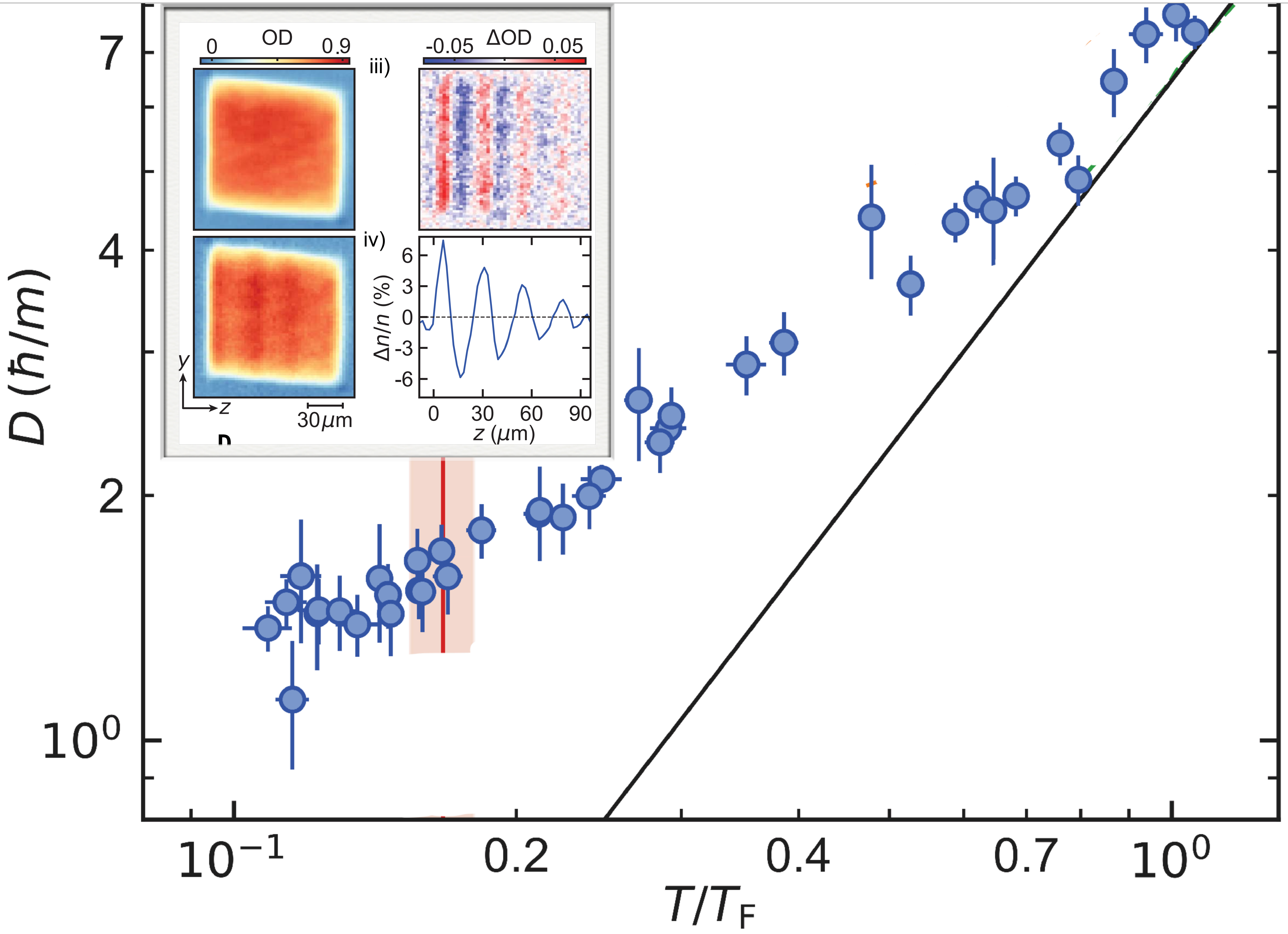
larger damping for
higher momenta

$\Gamma = Dk^2$ at low momenta:
hydrodynamic regime,
D sound diffusion



Patel et al., Science 2020

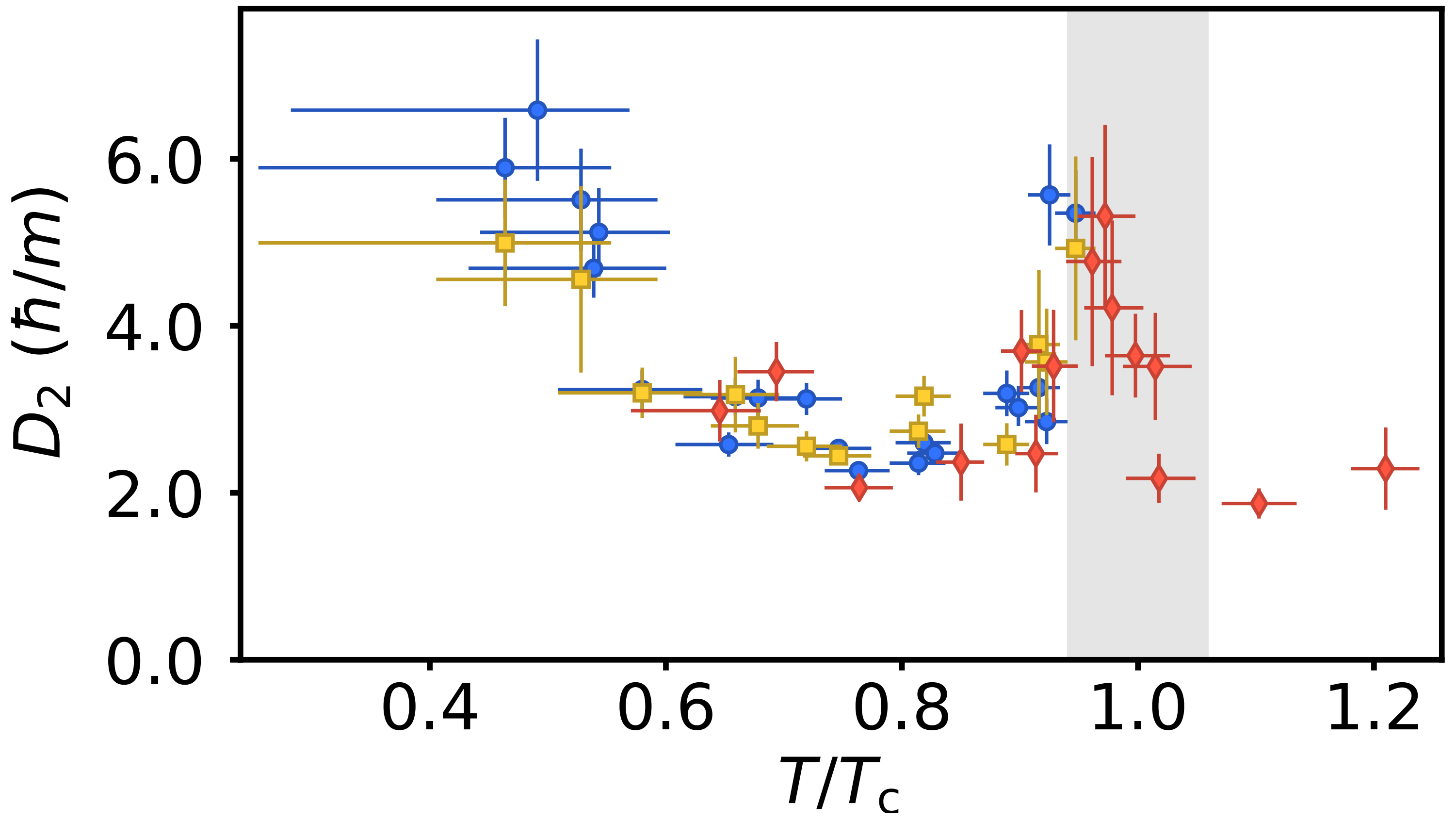
dissipative hydrodynamics: quantum limited diffusion



$$D = \frac{4}{3}\frac{\eta}{mn} + \frac{\zeta}{mn} + \frac{c_p - c_v}{c_v}\frac{\kappa}{c_p}$$

almost perfect fluidity

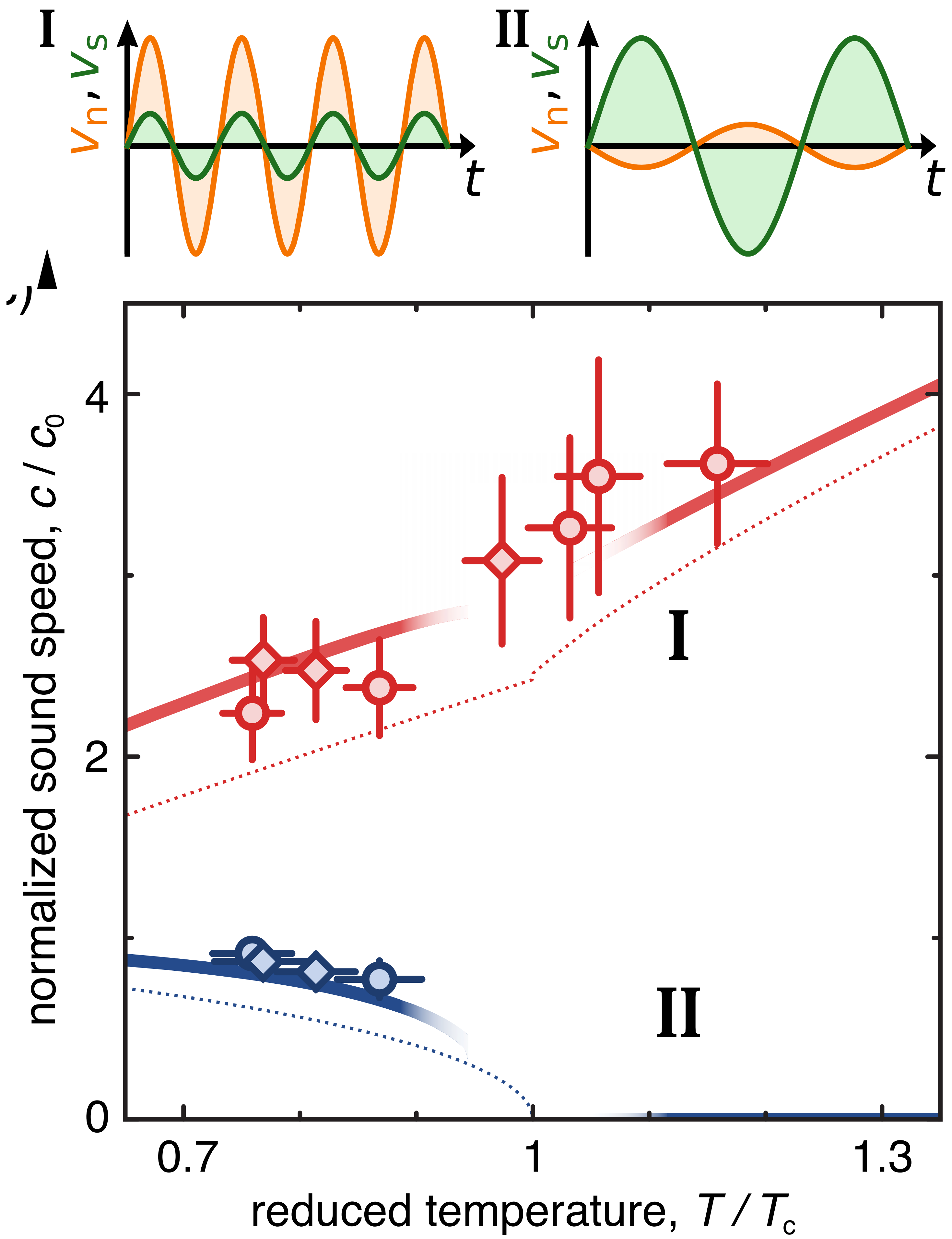
$$\frac{\eta}{s} \simeq 0.5\frac{\hbar}{k_B}$$

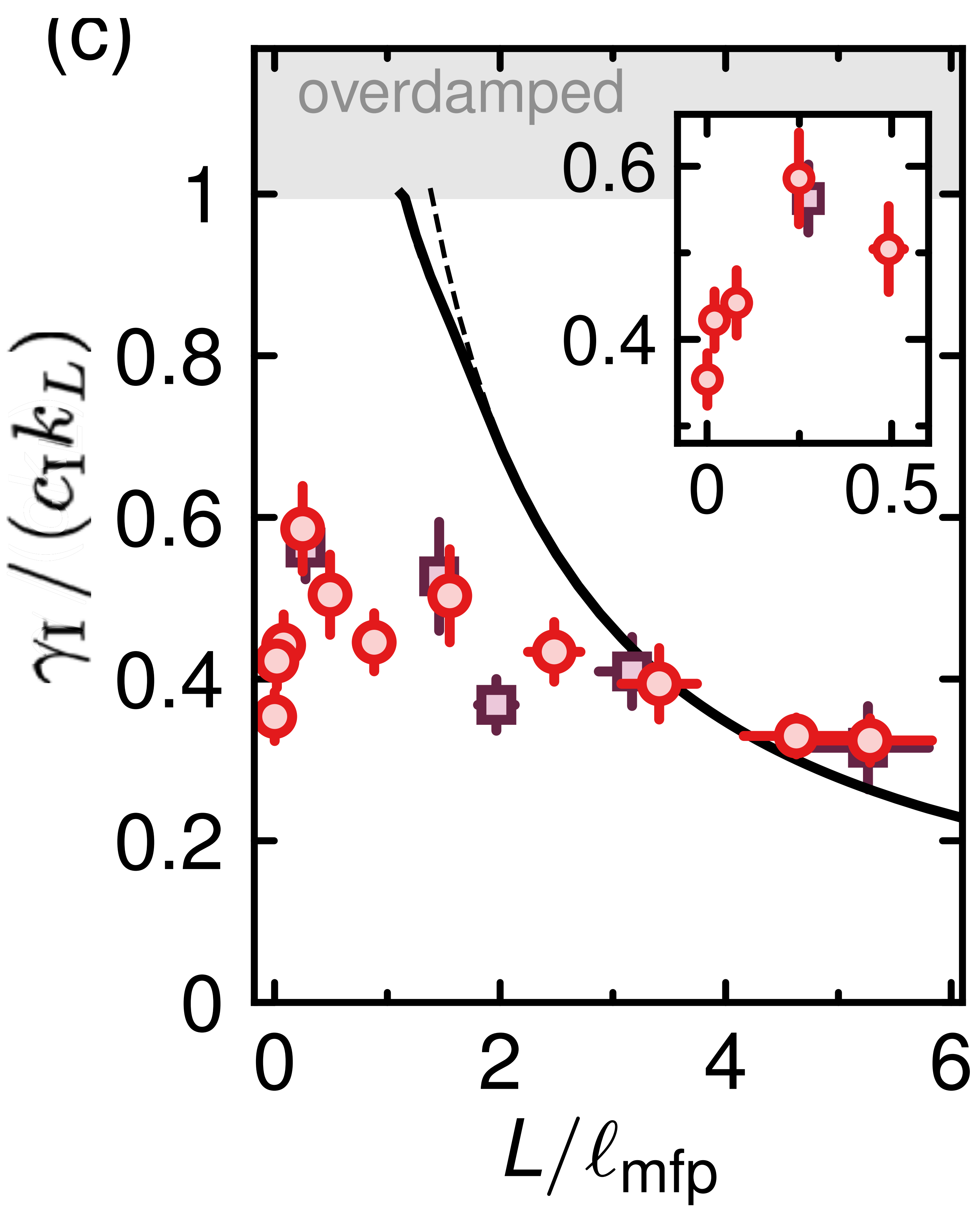


Patel et al., Science 2020; Li et al., Science 2022; Yan et al., Science 2024

superfluid hydrodynamics in a Bose gas

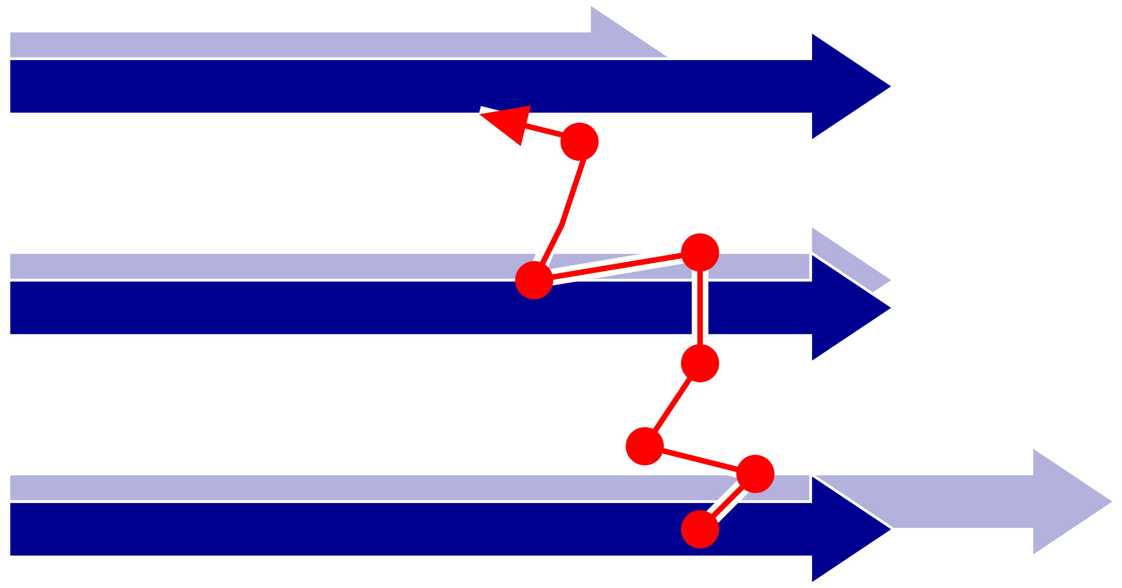
- first and second sound: two-fluid hydro Hilker, …, Hadzibabic, PRL 2022

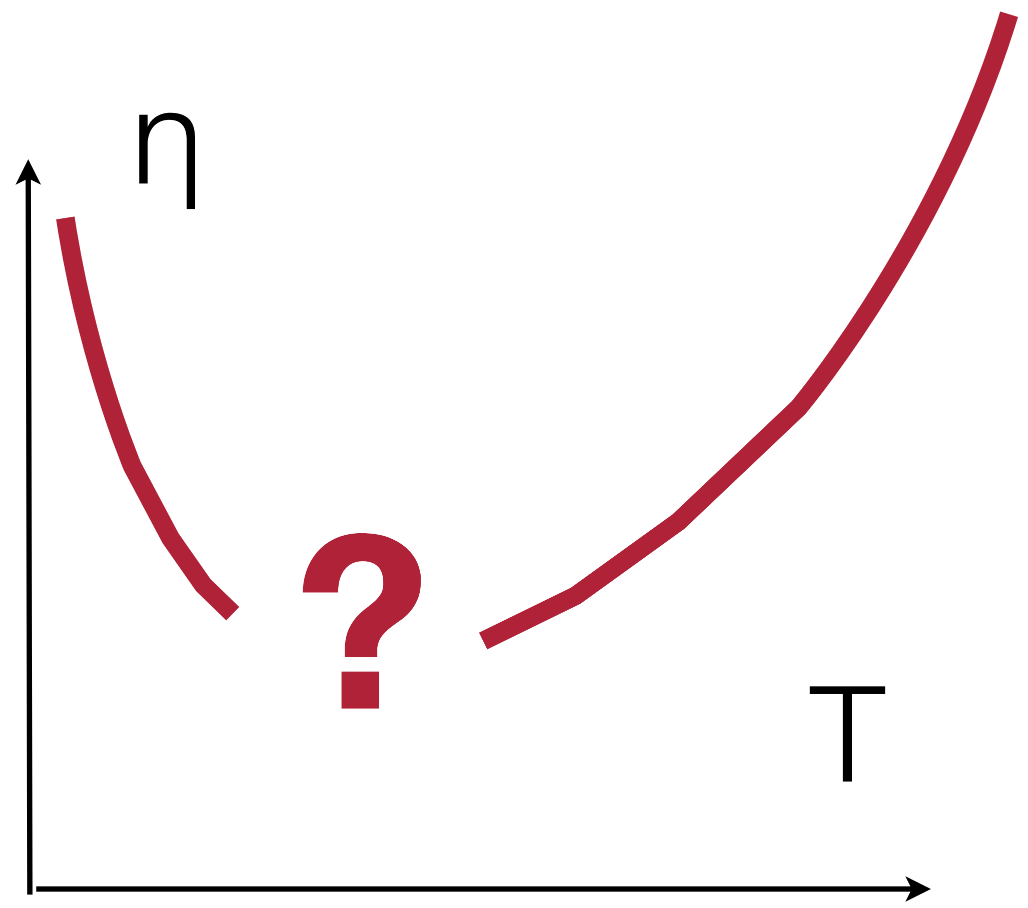




how does friction arise?

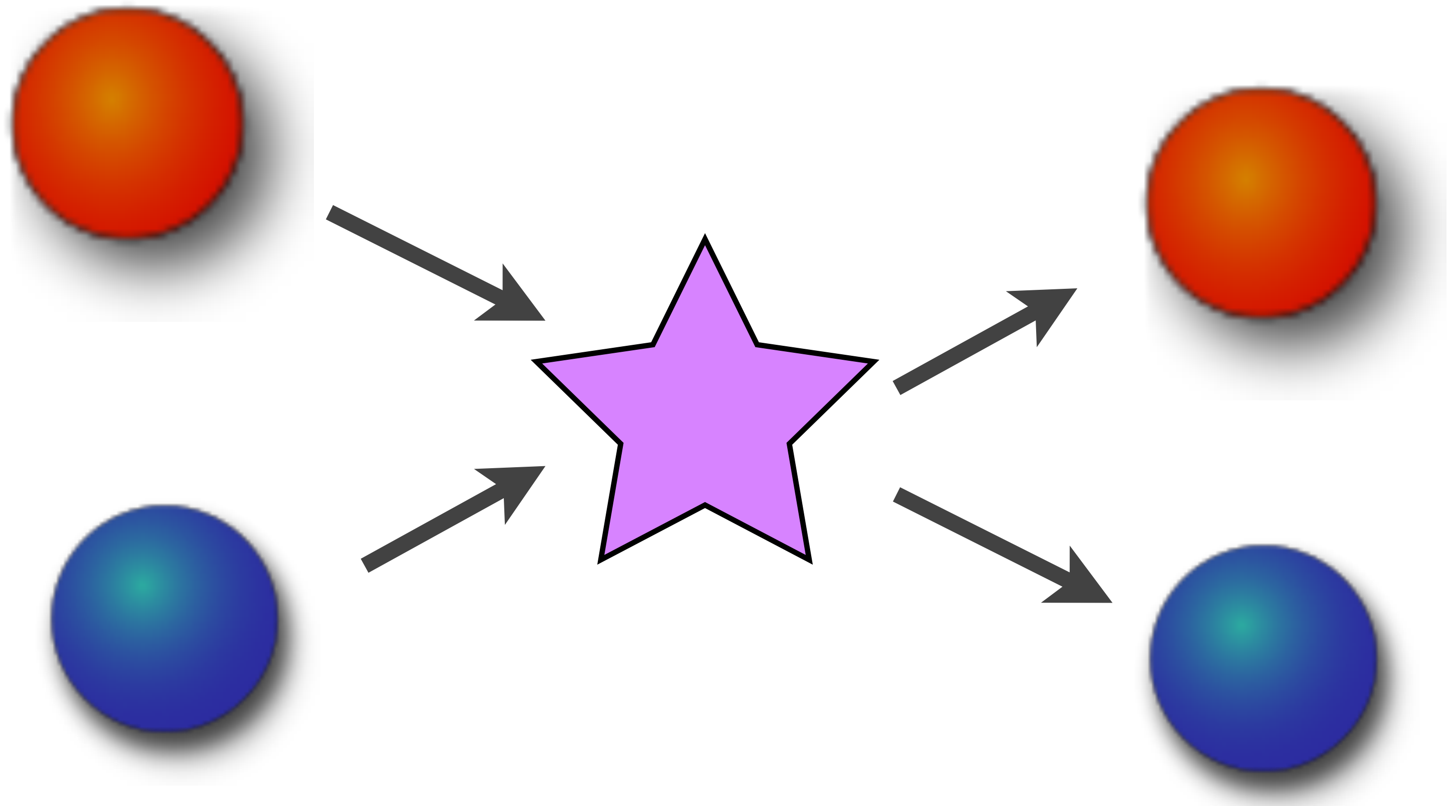
shear viscosity measures momentum transport





kinetic theory for dilute gas (Boltzmann equation):

diffusion $D \simeq \frac{\bar{p}}{m} \ell_{\mathrm{mfp}}$



Fermi gas: $D \simeq \frac{\hbar}{m} \frac{\ell_{\mathrm{mfp}}}{\ell} \gtrsim \frac{\hbar}{m}$

Boltzmann kinetic theory

- single-particle distribution $f(r, p, t)$:

$$\frac{\partial f}{\partial t} + \boldsymbol{v_p} \cdot \nabla_{\boldsymbol{r}} f + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

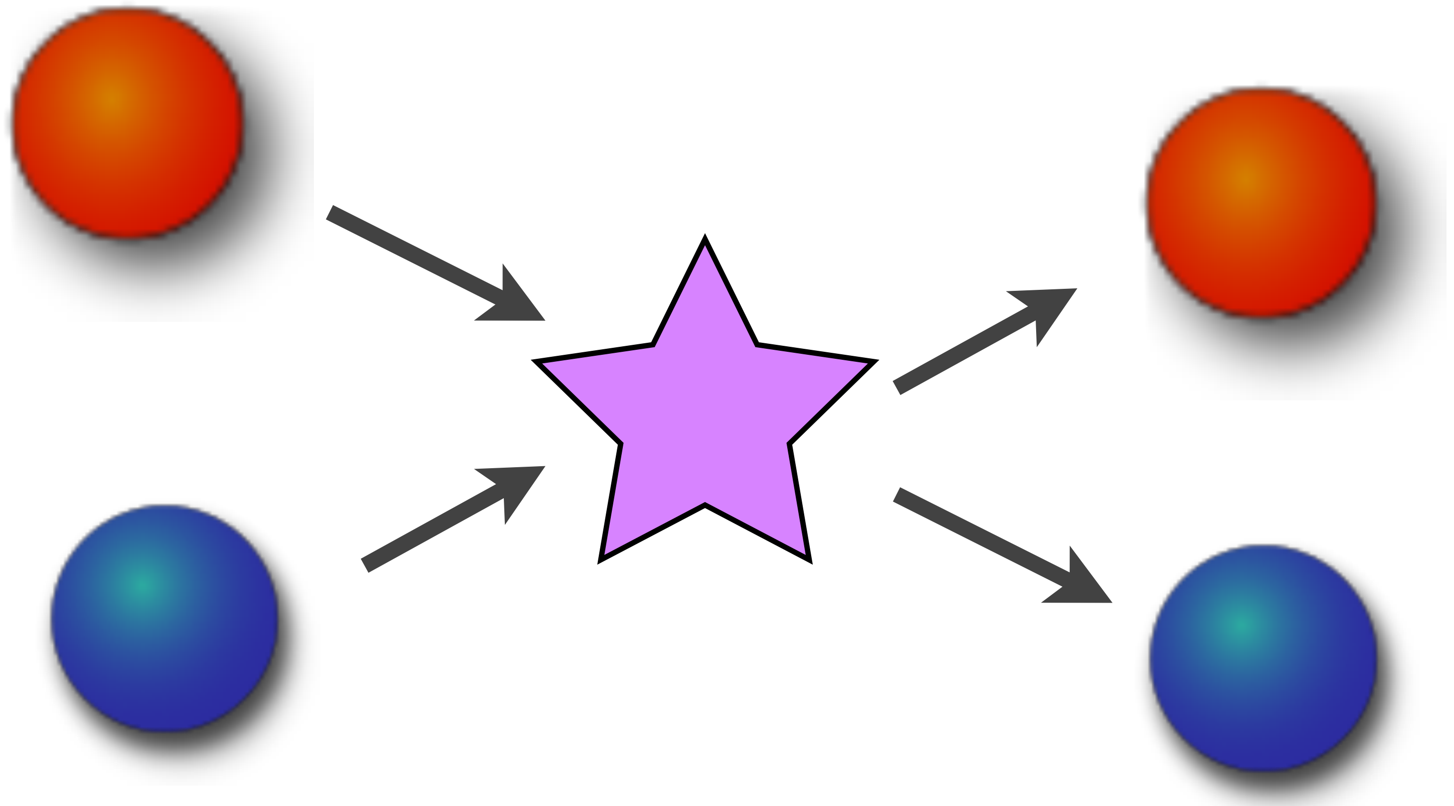
assumes molecular chaos

- collision integral

$$\left(\frac{\partial f_1}{\partial t}\right)_{\mathrm{coll}} \simeq -\int \mathrm{d}\boldsymbol{p}_2\, \mathrm{d}\Omega\, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} |\boldsymbol{v}_1 - \boldsymbol{v}_2| [f_1 f_2 (1 - f_{1'})(1 - f_{2'}) - (1 - f_1)(1 - f_2) f_{1'} f_{2'}]$$

- 2-body scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2 + \frac{1}{a^2}} + \text{ medium corr.}$$



☑ shear / thermal transport at high T

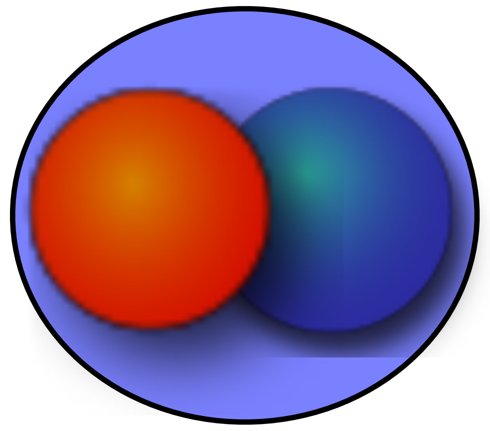
strong fermion correlations: contact

„Contact“ (many-body) → $\mathbf{C}$ **few-body** → $\frac{1}{a}$

pair correlation $g^{(2)}(r) = \langle \hat{n}_\uparrow(r)\hat{n}_\downarrow(0)\rangle \simeq \mathbf{C}\left(\frac{1}{r} - \frac{1}{a}\right)^2 + \dots$ $\quad r_0 \lesssim r \lesssim \ell$

contact operator: $\hat{C}(x) = g_0^2\, \hat{n}_\uparrow(x)\, \hat{n}_\downarrow(x) = \hat{\Delta}^\dagger(x)\hat{\Delta}(x)$

local pair $\hat{\Delta}(x) = g_0 \hat{\psi}_\downarrow(x)\hat{\psi}_\uparrow(x)$

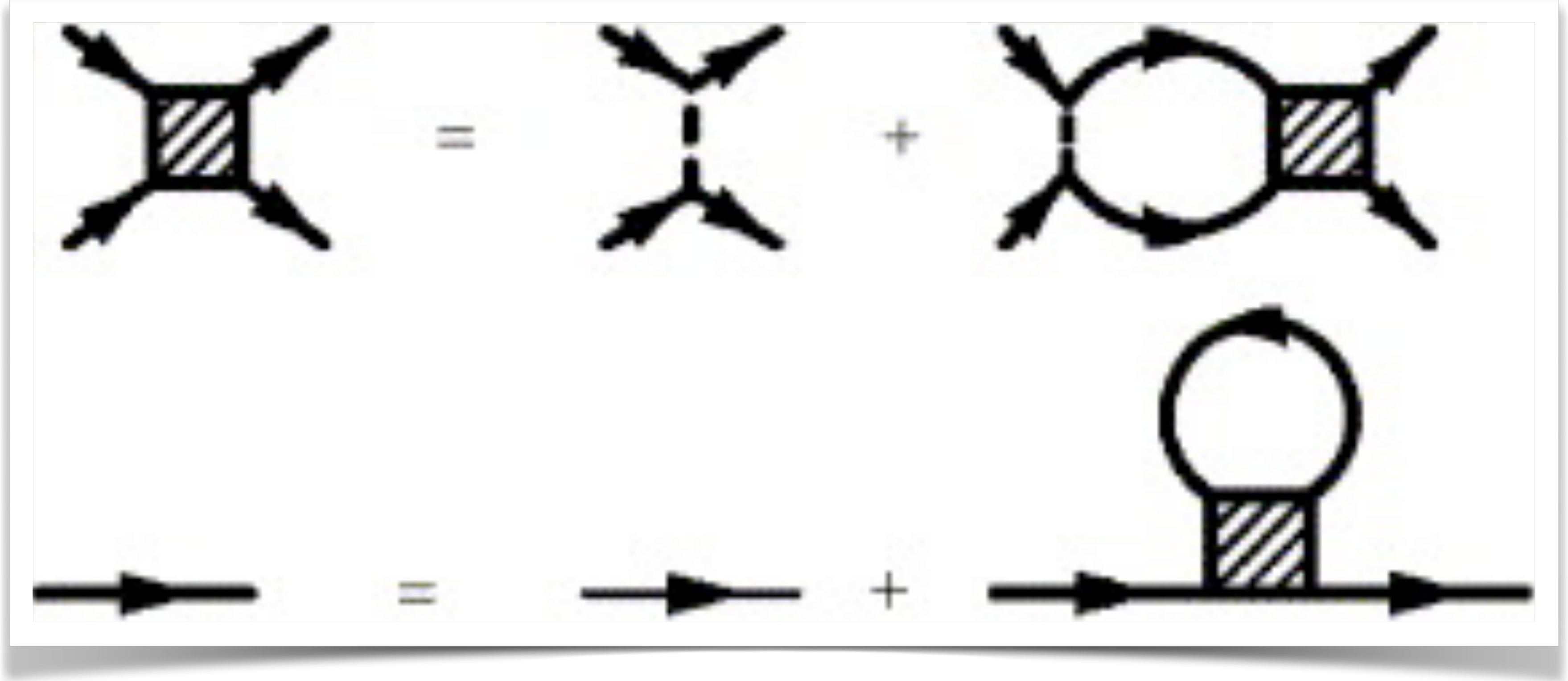


Hamiltonian $\hat{H} = \hat{H}_{\text{kin}} + \frac{\hat{C}}{g_0} = \hat{H}_{\text{unitary}} + \frac{\hat{C}}{4\pi m a}$ breaks scale invariance for $\frac{1}{a} \neq 0$

Tan 2005

quantum many-body theory

Luttinger-Ward approach (2PI)

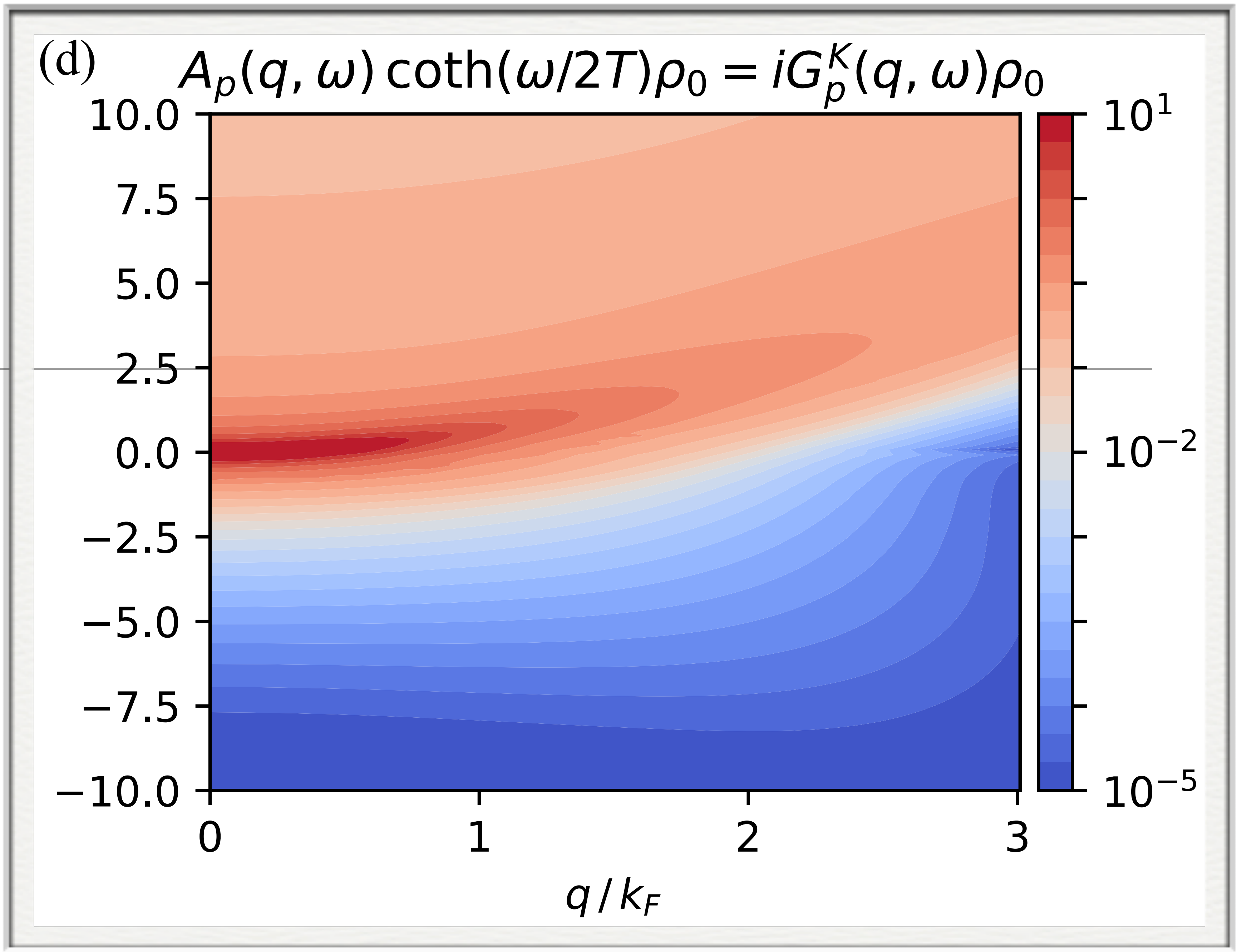


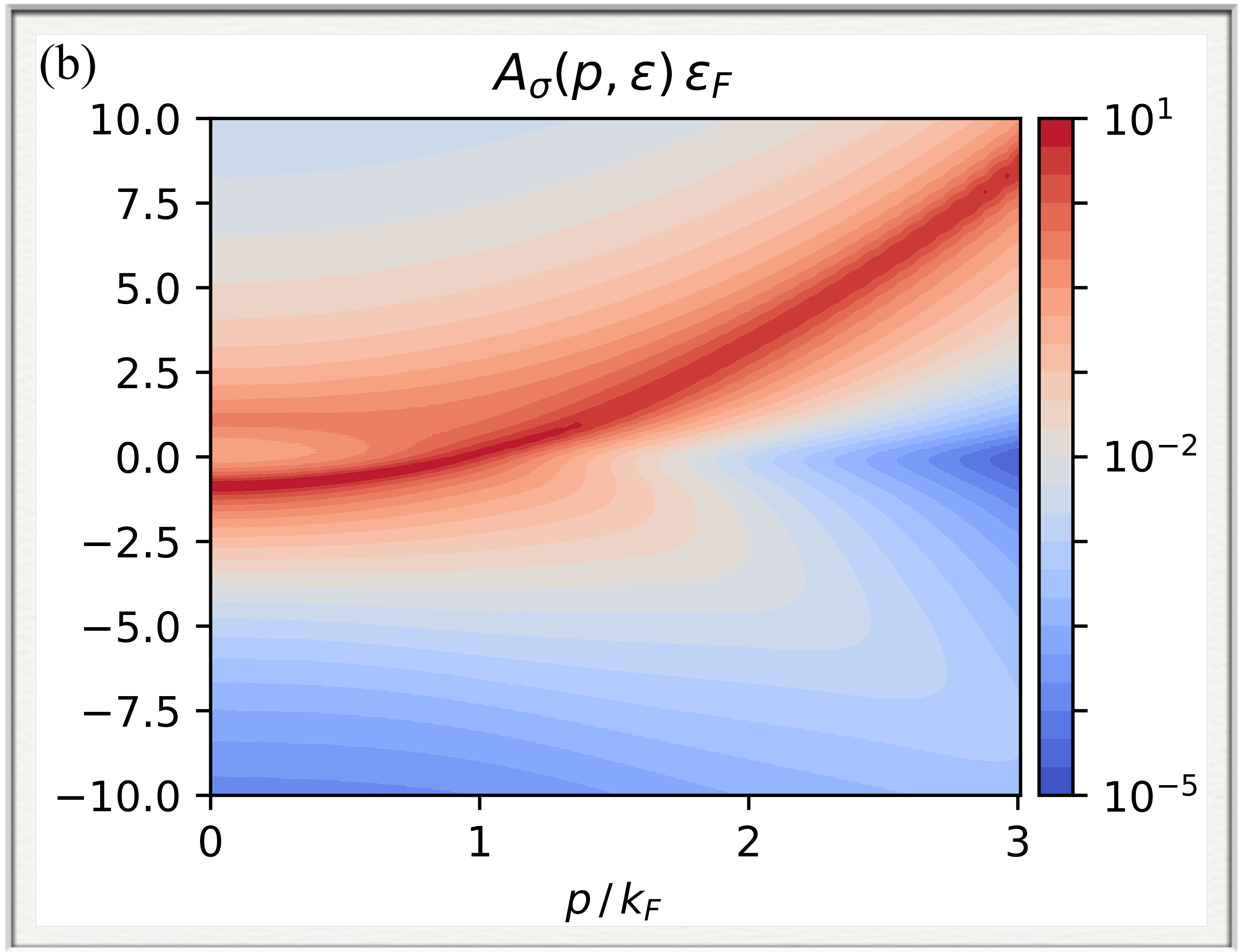
repeated scattering between particles

fermion spectra in medium

Keldysh computation in real frequency (avoid analytical continuation of num. data)

Haussmann et al. 2009; Johansen+ 2024; Enss 2024; Dizer+ 2024
Enss, Haussmann & Zwerger 2011, 2012, 2019

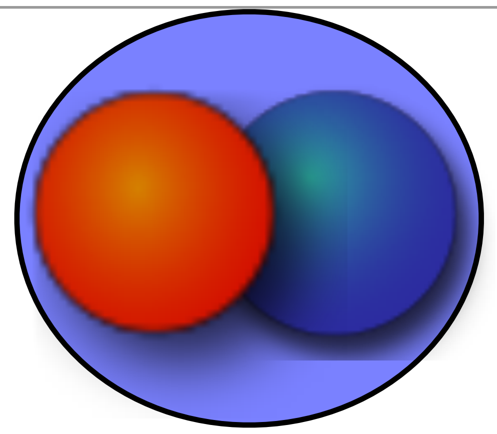




Enss 2024

transport in linear response

- no assumption of „molecular chaos“
  inelastic scattering

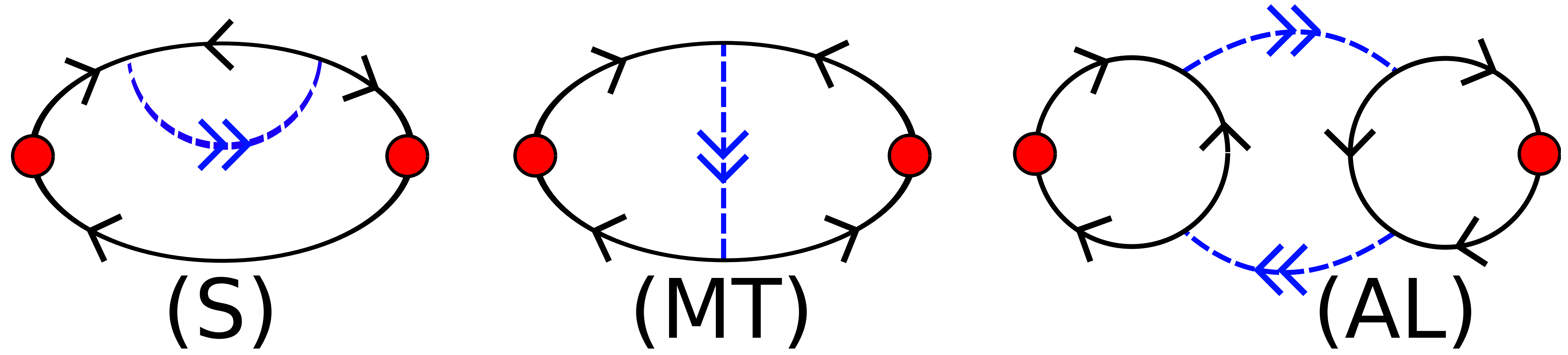


- shear viscosity from stress response function (Kubo formula)

$$\eta(\omega) = \int \mathrm{d}^d x\, \mathrm{d}t\, \frac{e^{i(\omega+i0)t} - 1}{i(\omega + i0)}\, i\theta(t) \langle [\hat{\Pi}_{xy}(\boldsymbol{x}, t), \hat{\Pi}_{xy}(0,0)] \rangle$$

- physical ingredients:

$\eta(\omega) =$

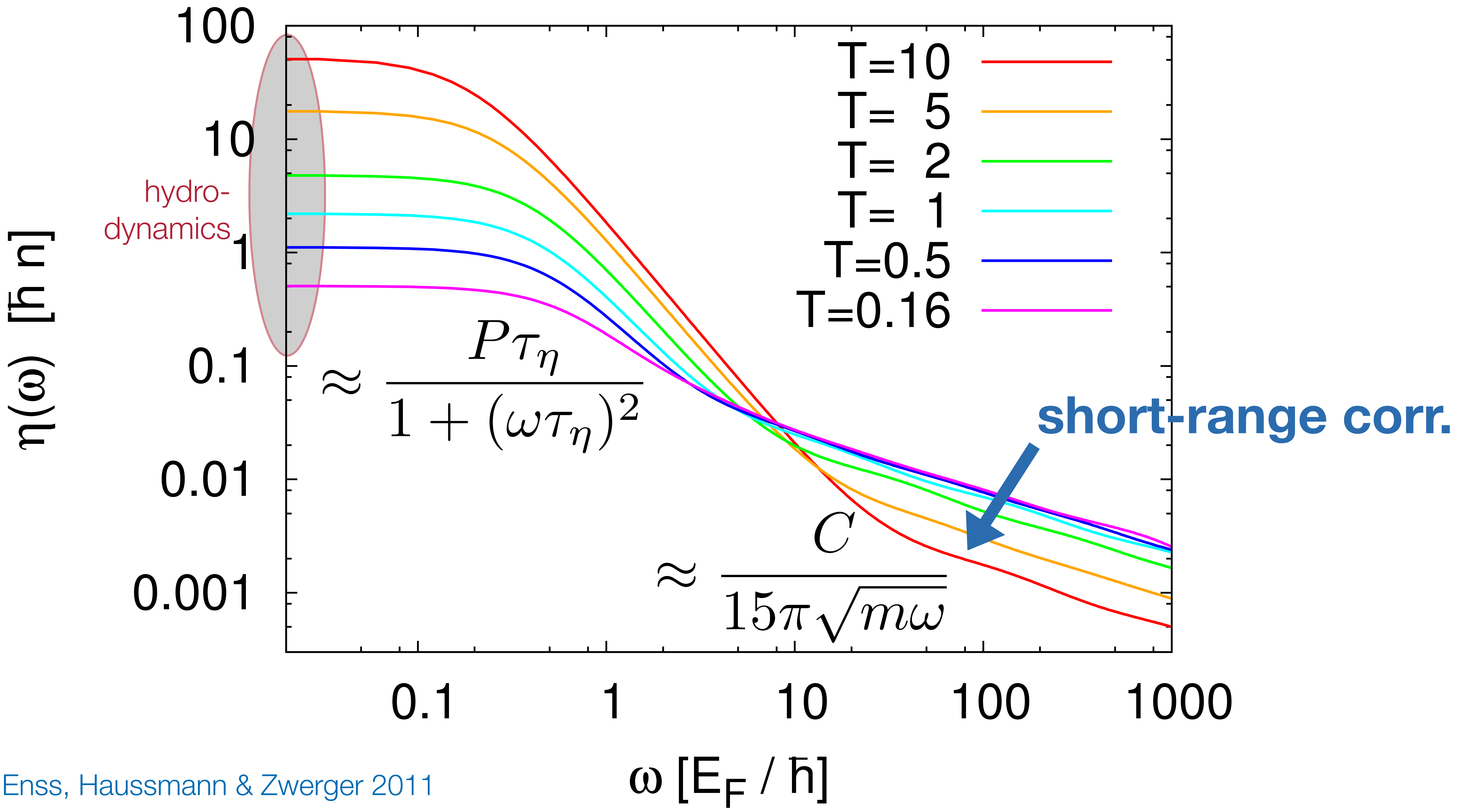


(resummed to infinite order)

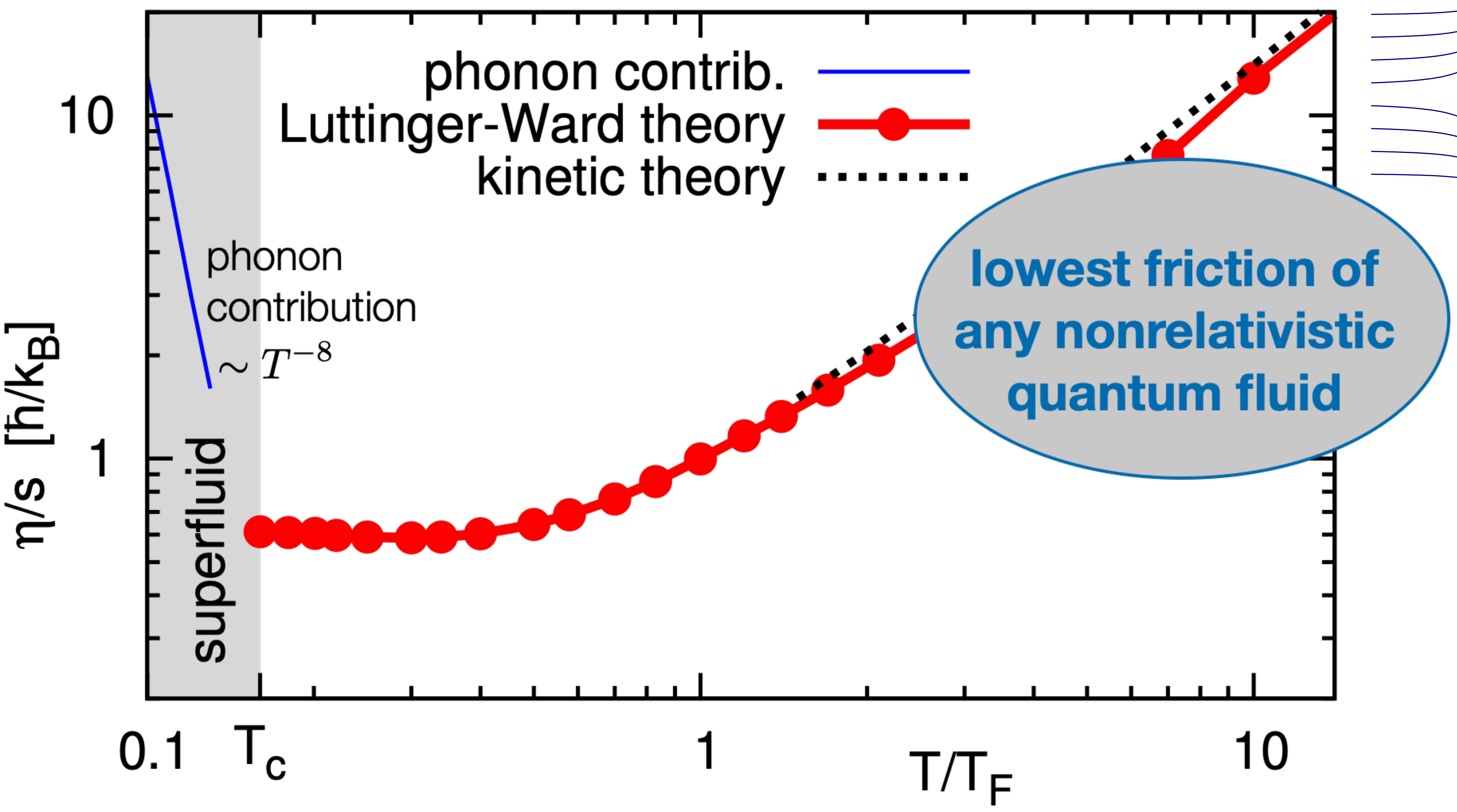
- transport via fermions and **pairs (superfluid fluctuations)**

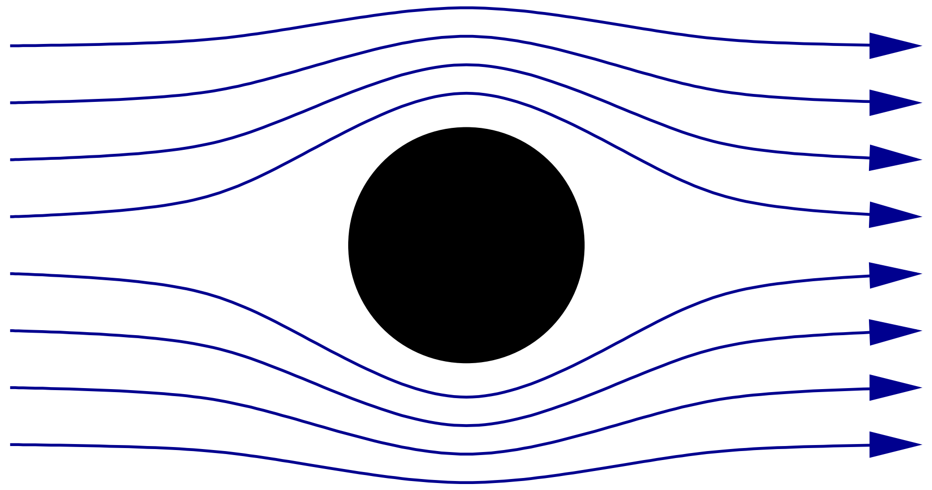
Enss, Haussmann & Zwerger, Annals Physics 2011

dynamical stress correlations (shear viscosity)



Enss, Haussmann & Zwerger 2011



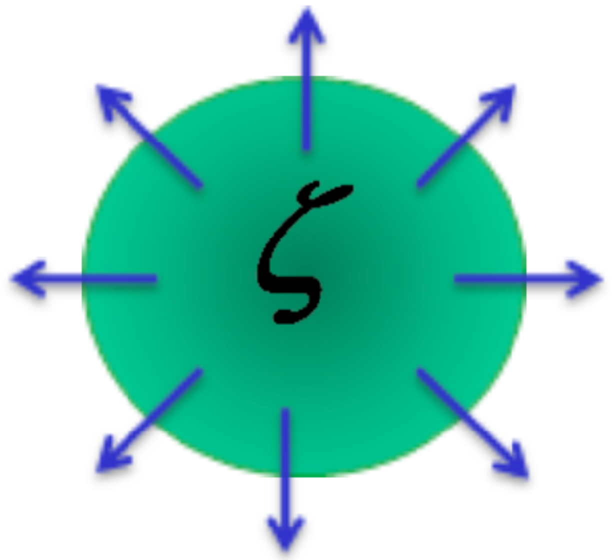


Shear viscosity/entropy
of the unitary Fermi gas

Enss, Haussmann & Zwerger 2011

bulk viscosity probes scaling violation

Kubo formula: pressure correlation function cf. Fujii & Nishida PRA 2020



$$\zeta(\omega) = \int \mathrm{d}^d x\, \mathrm{d}t\, \frac{\mathrm{e}^{\mathrm{i}(\omega+\mathrm{i}0)t} - 1}{\mathrm{i}(\omega + \mathrm{i}0)}\, \mathrm{i}\theta(t) \langle [\delta\hat{p}(\boldsymbol{x}, t), \delta\hat{p}(0,0)] \rangle$$

dilute quantum gas: pressure fluctuations

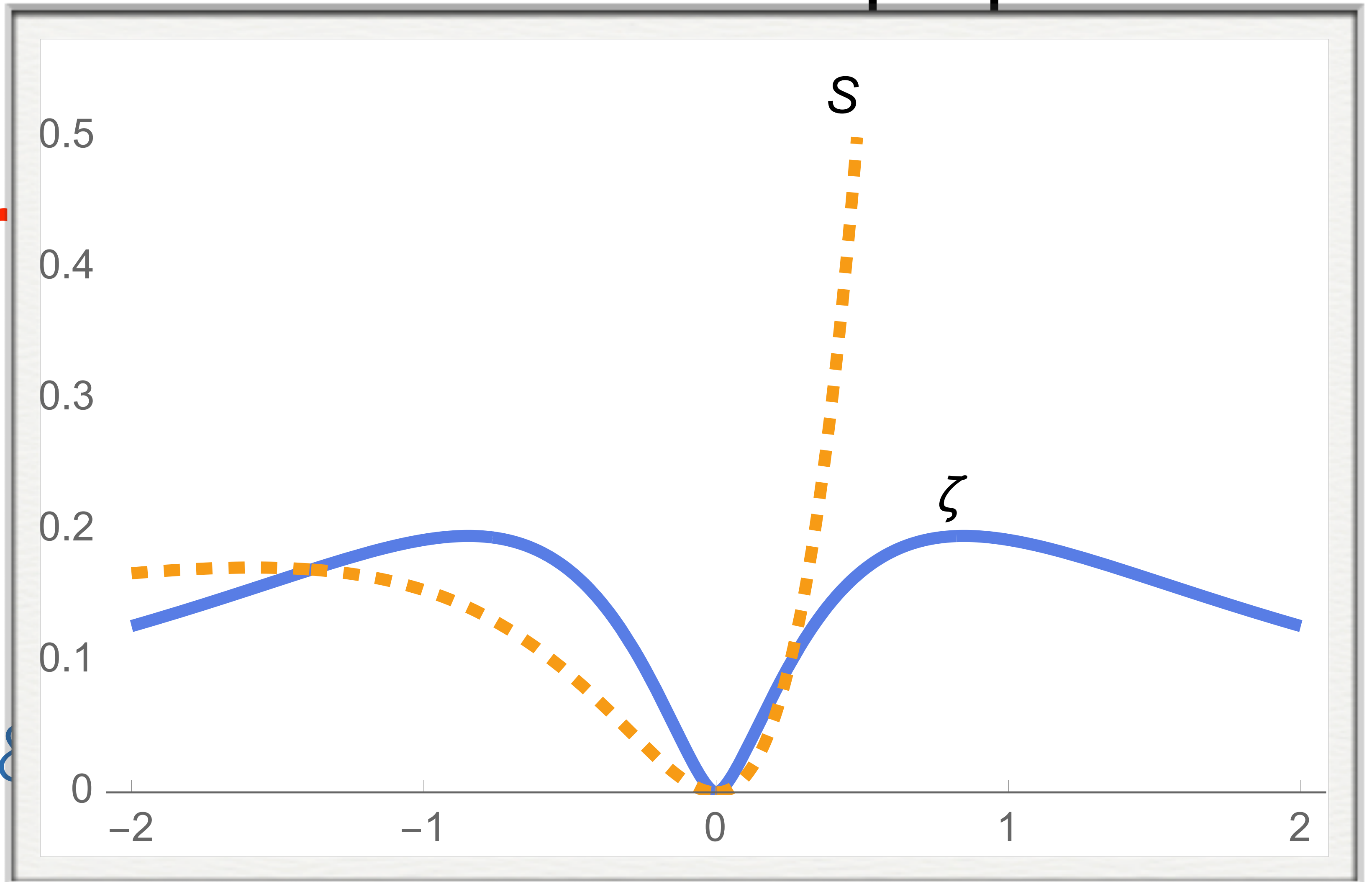
$$\delta\hat{p} = \frac{2}{3}\hat{H} + \frac{\hat{C}}{12\pi m a} - \left(\frac{\partial p}{\partial E}\right)_n \hat{H} - \left(\frac{\partial p}{\partial n}\right)_E \hat{n}$$

(β function $\frac{\partial H_{\text{int}}}{\partial \ln |a|}$)

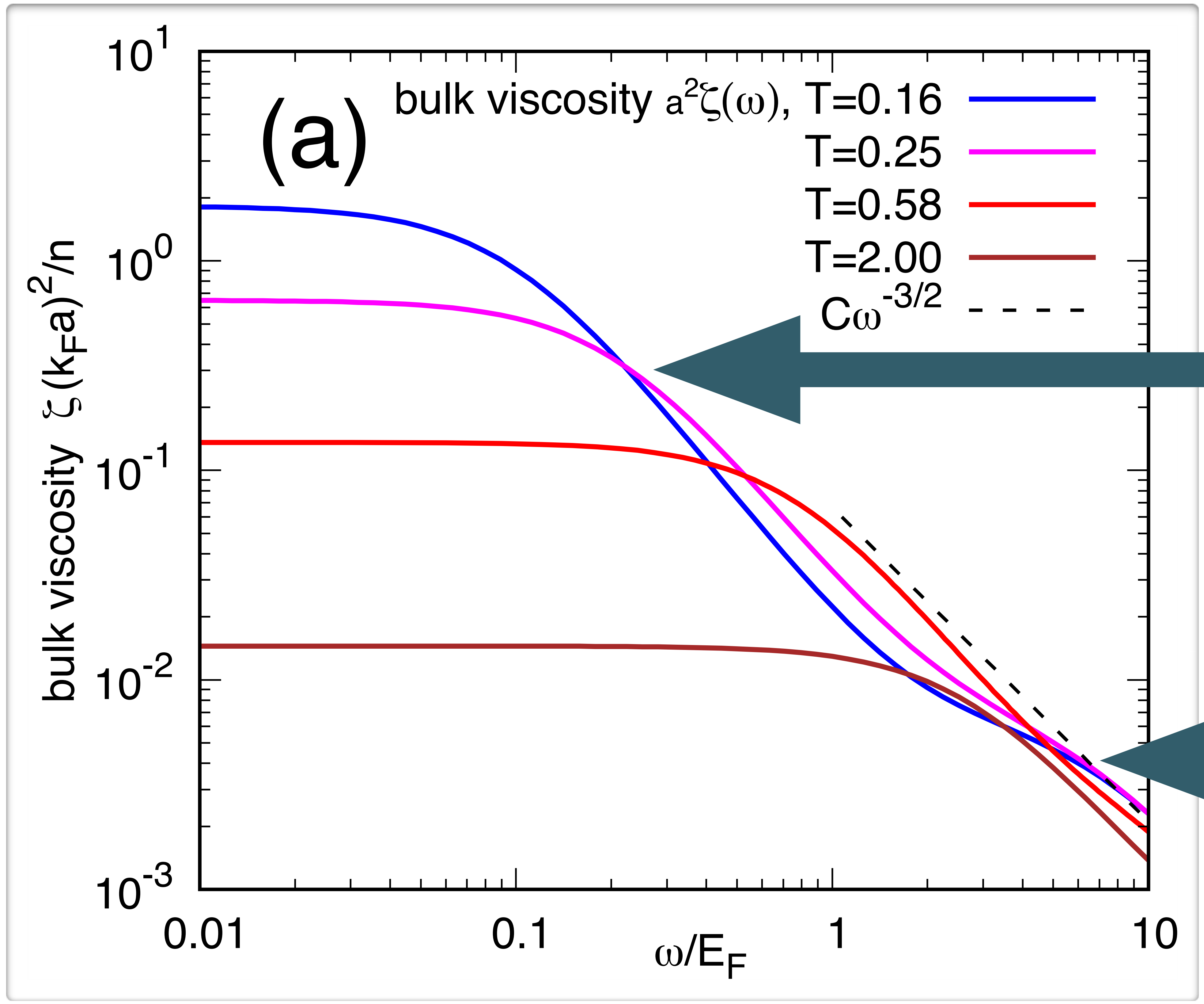
bulk viscosity probes **contact correlation** **(local pair**

$$\zeta(\omega > 0) = \int d\boldsymbol{x}\, dt\, \frac{e^{i\omega t} - 1}{i\omega}\, i\theta(t) \left\langle \left[\frac{\hat{C}(\boldsymbol{x}, t)}{12\pi m a}, \frac{\hat{C}(0,0)}{12\pi m a} \right] \right\rangle$$

Enss PRL 2019, Nishida AoP 2019, Hofmann PRA 2020; cf. Fujii &



dynamical bulk viscosity (Luttinger-Ward theory)



transport peak (Drude form)

$$\zeta(\omega) = \frac{\chi\tau_\zeta}{1+(\omega\tau_\zeta)^2}$$

width $\tau_\zeta^{-1} \simeq 0.7\, k_B T/\hbar$:

T-linear scaling of scattering rate

independent of density!

contact tail $C/\omega^{3/2}$

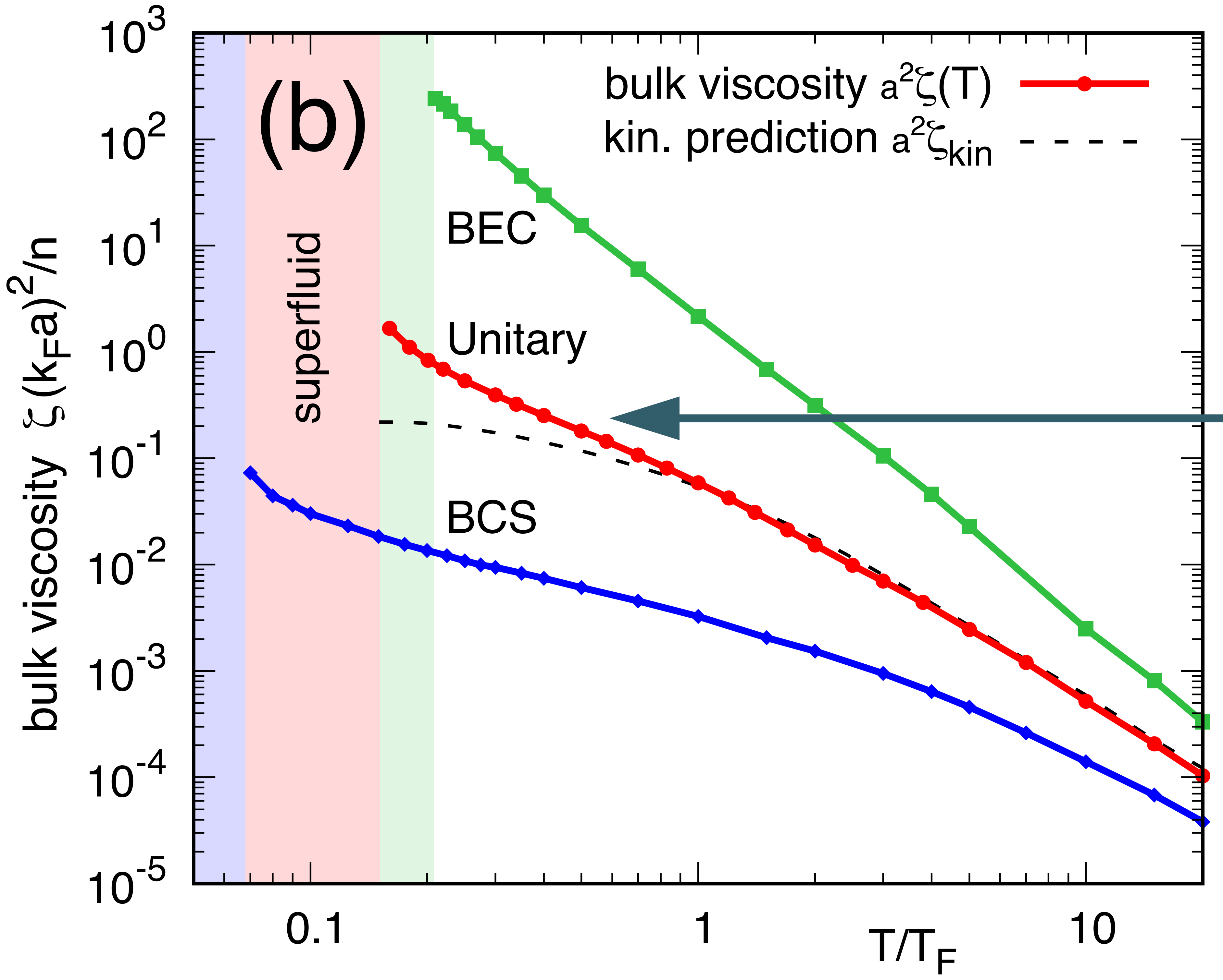
at high frequency

Enss PRL 2019; Enss C.R.Phys 2025

cf. Bulgac et al. PRR 2024: unitary Fermi gas has no (semi-)classical limit, thermalizes 1000x slower than predicted by ETH

quantum degenerate regime (Luttinger-Ward theory)

strong enhancement in quantum degenerate regime ($\zeta > \eta$)

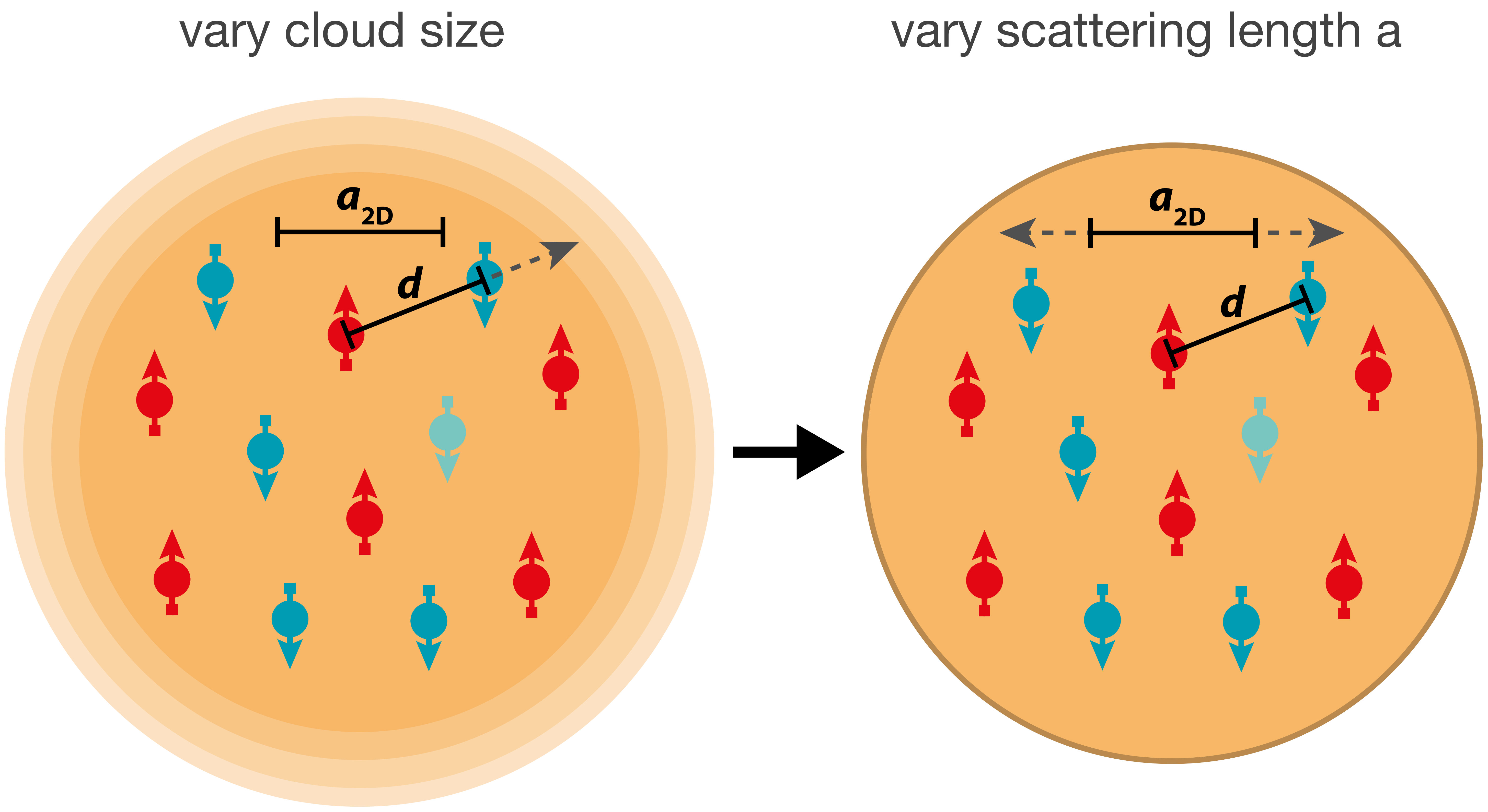


larger than kinetic theory prediction for $T < T_F$

$$\frac{\zeta}{\eta} \simeq \left(\frac{P - 2E/3}{P}\right)^2 \simeq \left(\frac{C/a}{P}\right)^2$$

Enss PRL 2019

novel transport measurement technique

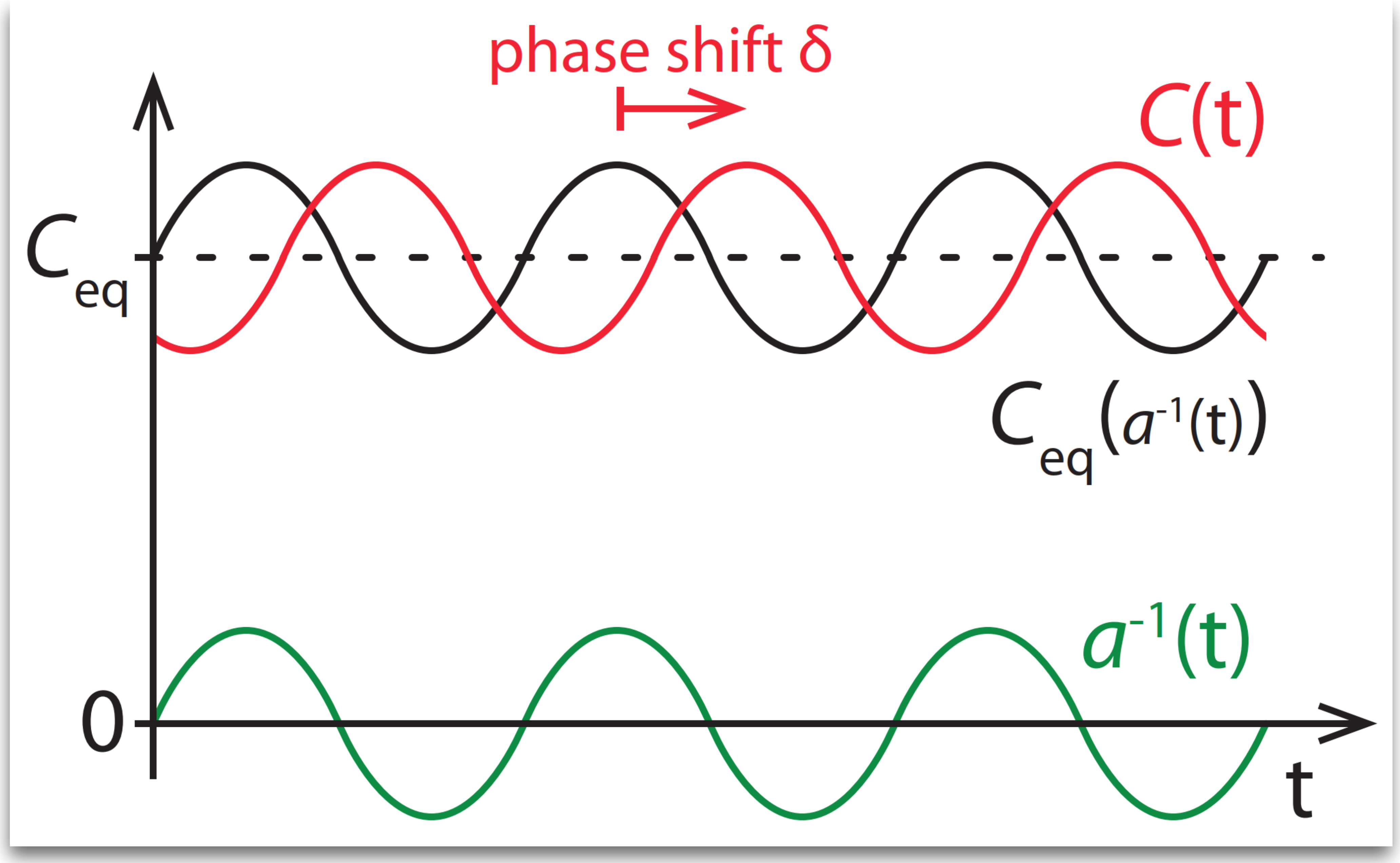


Fujii & Nishida PRA 2018

measuring bulk viscosity

response of contact to change of scattering length:

$$\frac{\partial \langle C(t) \rangle}{\partial a^{-1}(t')} \simeq i\theta(t - t') \langle [C(t), C(t')] \rangle \underset{\sim}{\text{lin.resp.}} a^2 \zeta \, e^{-(t-t')/\tau_\zeta} \, \theta(t - t')$$

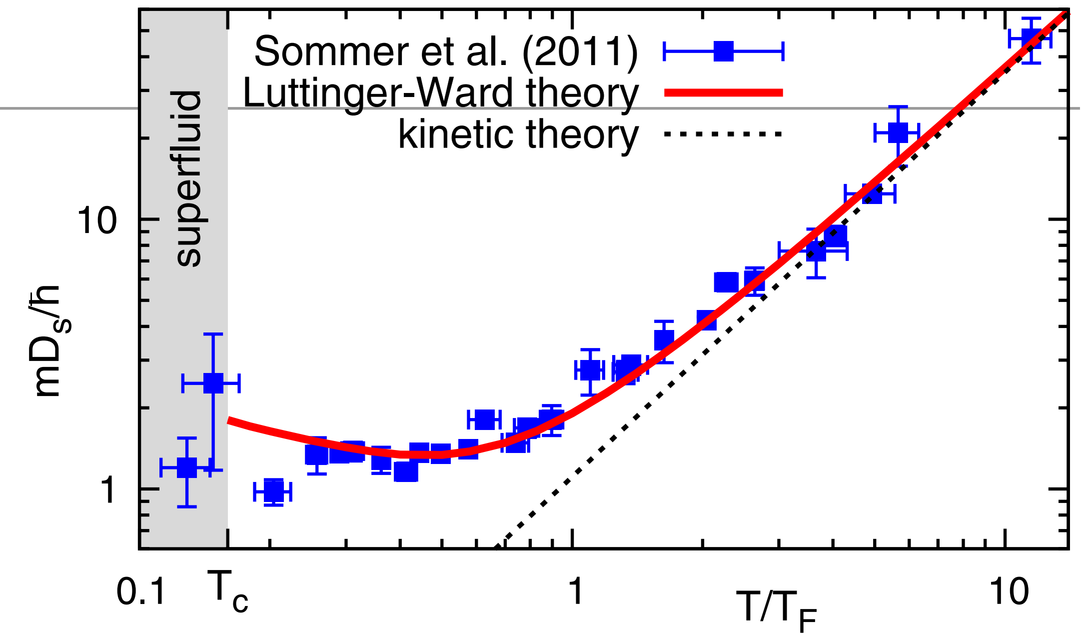


microscopic derivation of MIS!

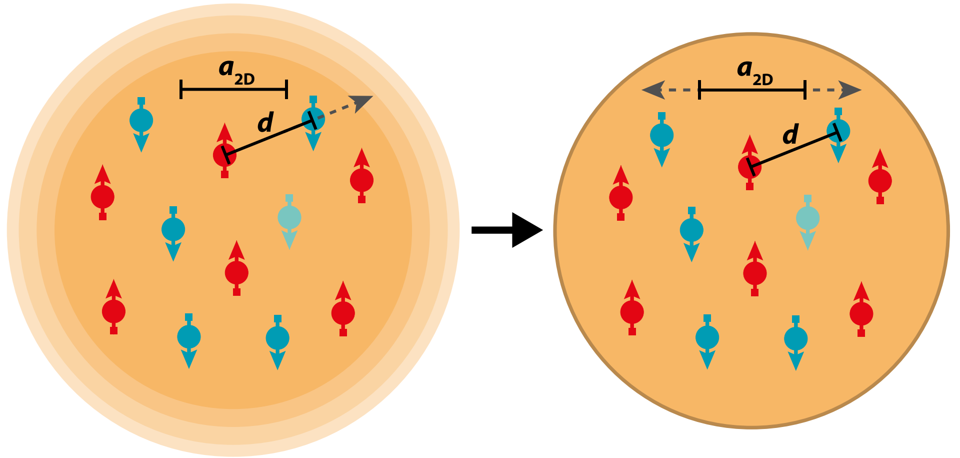
$$\delta C(t) = \int_{-\infty}^{t} dt' \, \frac{\partial C(t)}{\partial a^{-1}(t')} \, \delta a^{-1}(t')$$

new, fast measurement of contact:
Xie, …, Enss, Julienne, Yu, Thywissen, 2506.13707

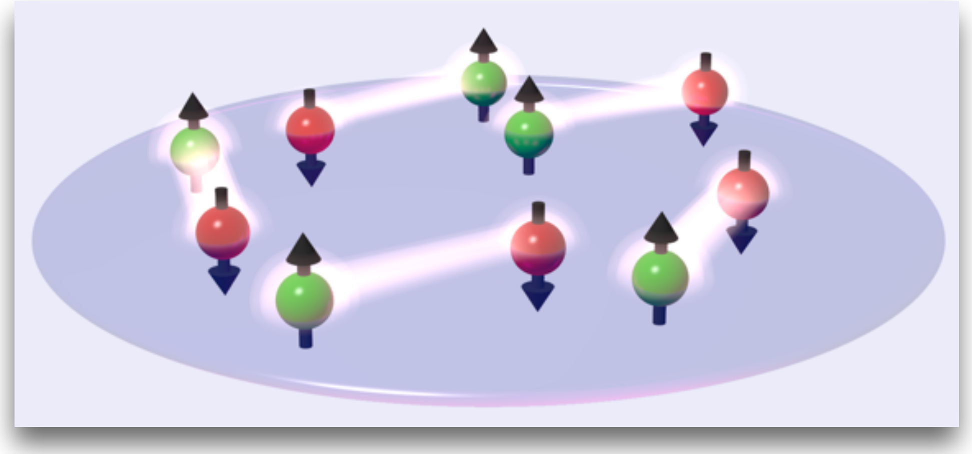
Outlook



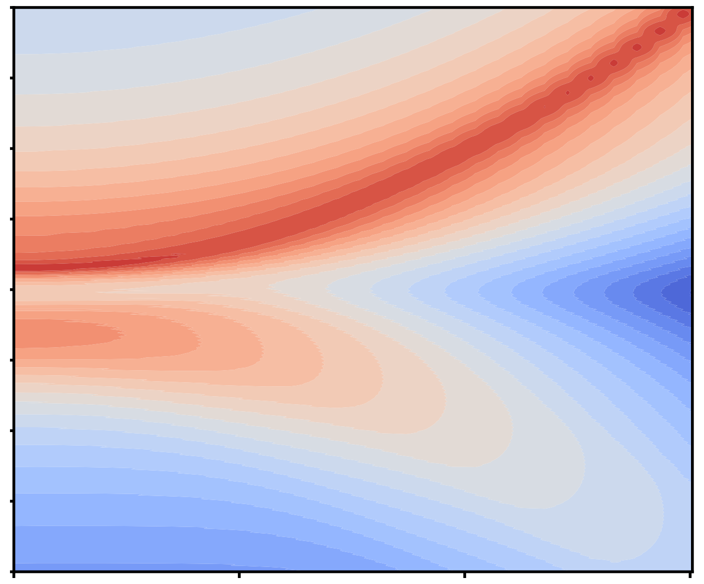
- **perfect fluidity** at strong scattering: slowest dissipation/diffusion consistent with QM



- cold atom experiment can probe **local dissipation,** hydrodynamics beyond Navier-Stokes in **real time**

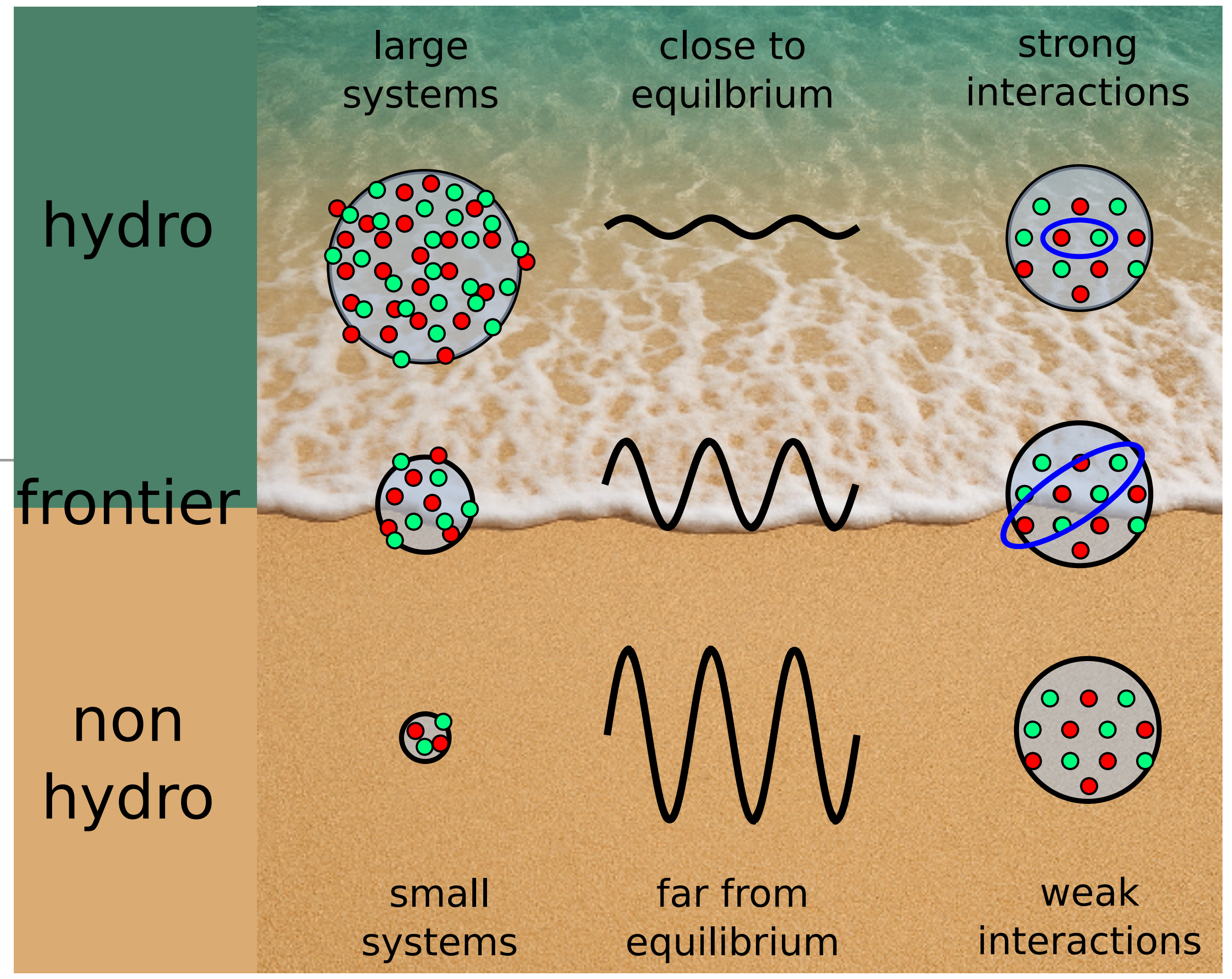


- theory: **quantum transport** of correlated particles $\tau^{-1} \sim T$, kinetic theory for fermions + pairs Fujii & Enss, Ann. Phys. 2023
slow modes (symmetries, critical fluct.)



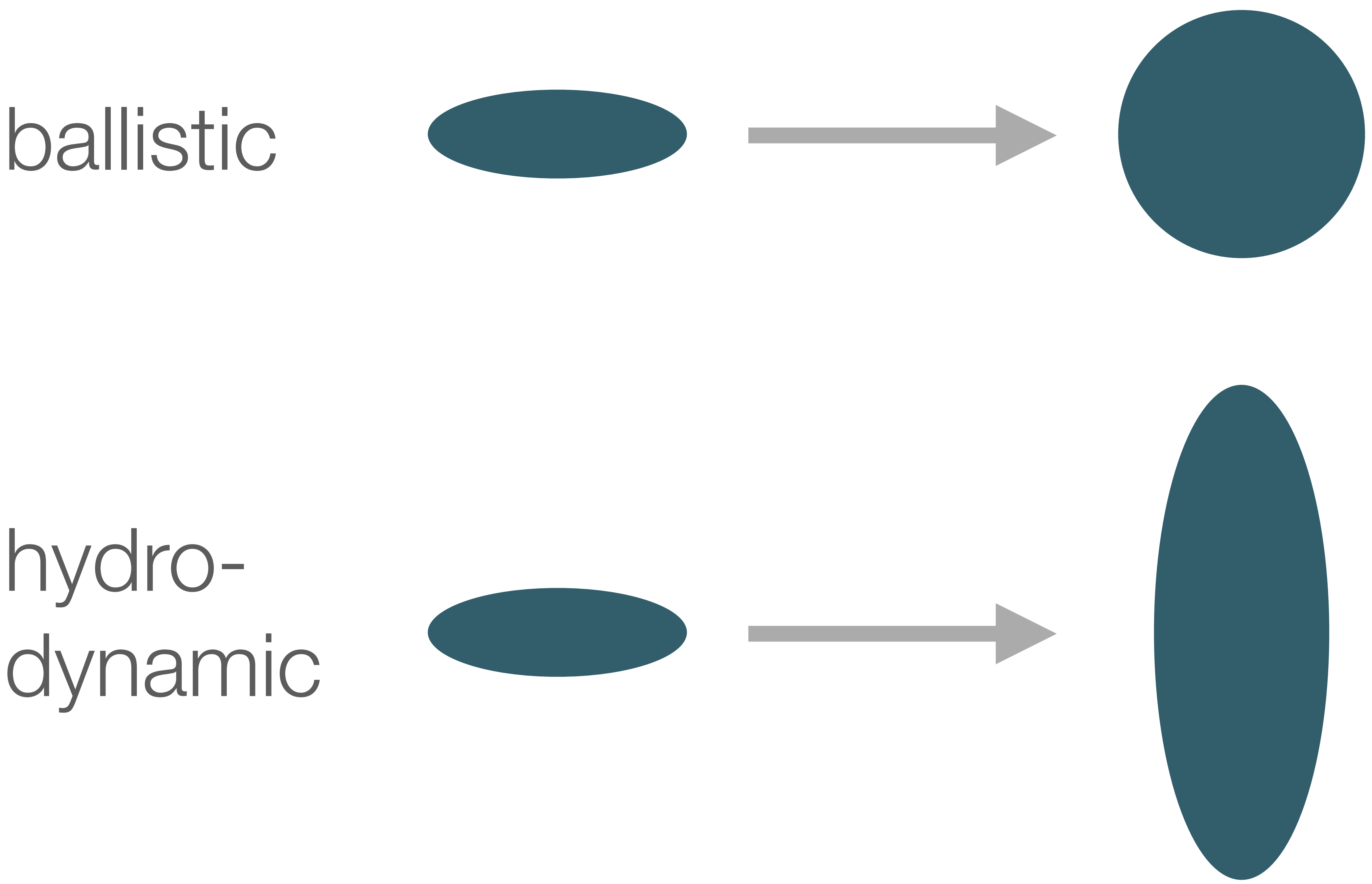
- dynamical response in real time, **far from equilibrium**

boundaries of hydrodynamics

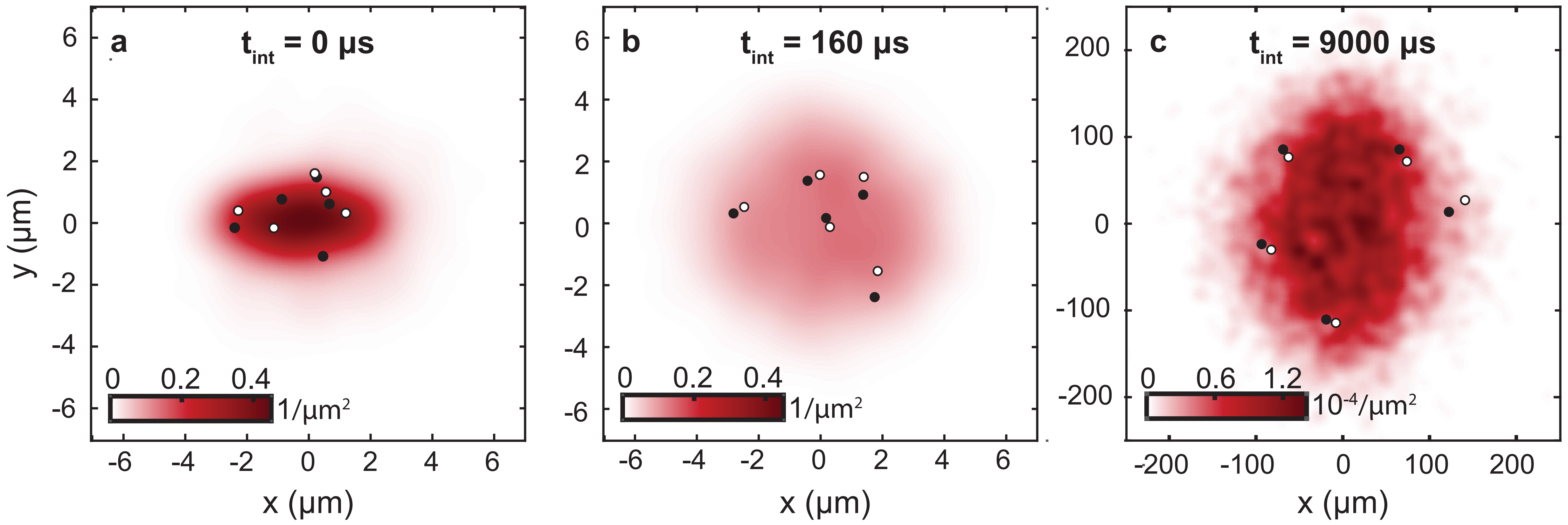


does fluid dynamics work for **very small systems?**

5 ↑ + 5 ↓ fermions in 2D (Jochim group, Heidelberg)



theory beyond hydro: SLDA *(Uri Sharell)*
-> wave fcts: quantum, small, full EOS



new perspectives article: 2509.05049

Brandstetter et al., Nature Phys. 2025

Extra material

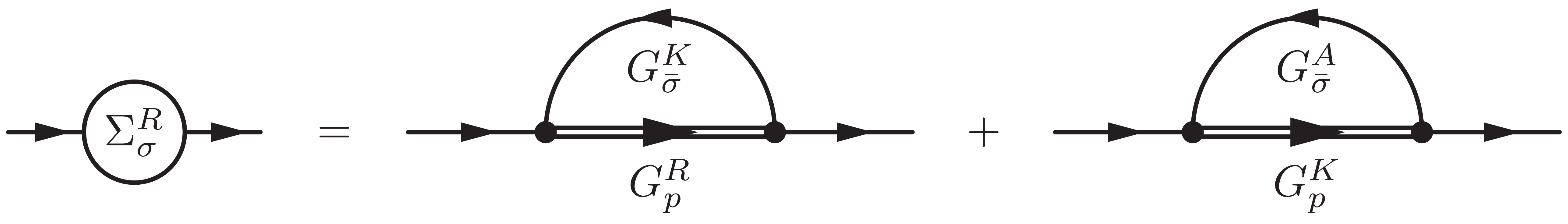
solving the Luttinger-Ward equations in real frequency

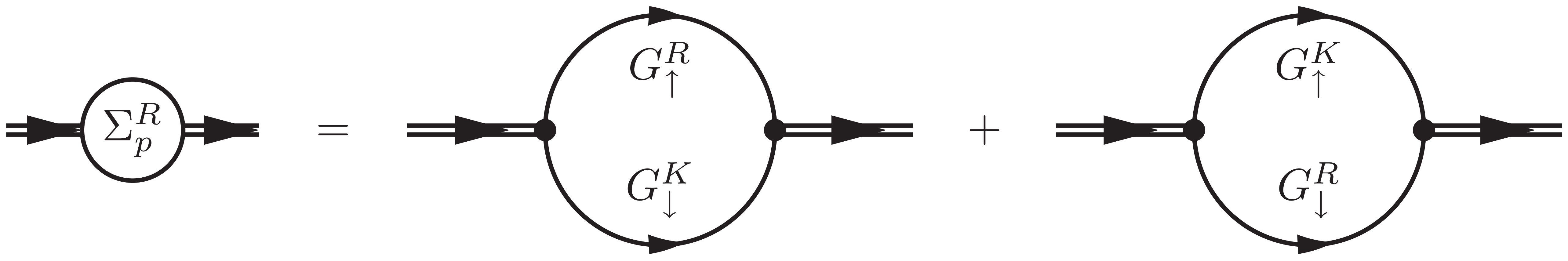
$$H = \sum_\sigma \int d\boldsymbol{r}\, \psi^\dagger_\sigma(\boldsymbol{r})\Big(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma(\boldsymbol{r}) + g_0 \int d\boldsymbol{r}\, \psi^\dagger_\uparrow(\boldsymbol{r})\psi^\dagger_\downarrow(\boldsymbol{r})\psi_\downarrow(\boldsymbol{r})\psi_\uparrow(\boldsymbol{r}).$$

$$S = \int dr \int_0^\beta d\tau \Big[\sum_\sigma \psi^*_\sigma\Big(\partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma - \frac{1}{g_0}|\Delta|^2 - \psi^*_\uparrow\psi^*_\downarrow\Delta - \Delta^*\psi_\downarrow\psi_\uparrow\Big],$$

imaginary frequency: continue analytically (=> E. Gull, ERG 2022)

directly in real frequency (Keldysh in equilibrium):





$$\mathrm{Im}\,\Sigma^R_\sigma(\boldsymbol{p},\varepsilon) = -\pi \int_{\boldsymbol{p}',\varepsilon'} [f(\varepsilon') + b(\varepsilon+\varepsilon')] \times A_p(\boldsymbol{p}+\boldsymbol{p}', \varepsilon+\varepsilon')A_{\bar\sigma}(\boldsymbol{p}',\varepsilon').$$

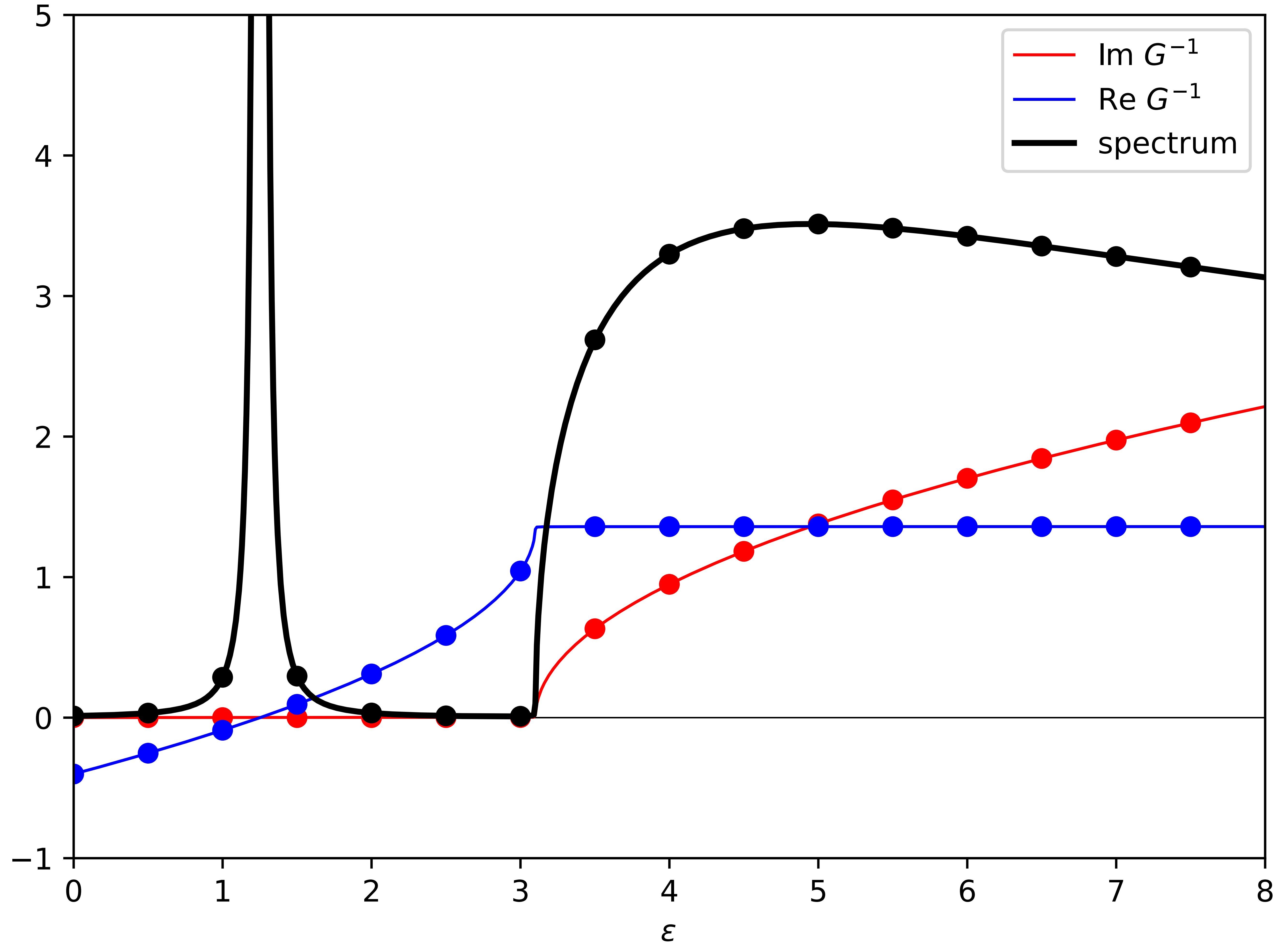
$$\mathrm{Im}\,\Sigma^R_p(\boldsymbol{q},\omega) = -\pi \int_{\boldsymbol{p},\varepsilon} [1-2f(\varepsilon)]A_\uparrow(\boldsymbol{p},\varepsilon) \times A_\downarrow(\boldsymbol{q}-\boldsymbol{p}, \omega-\varepsilon).$$

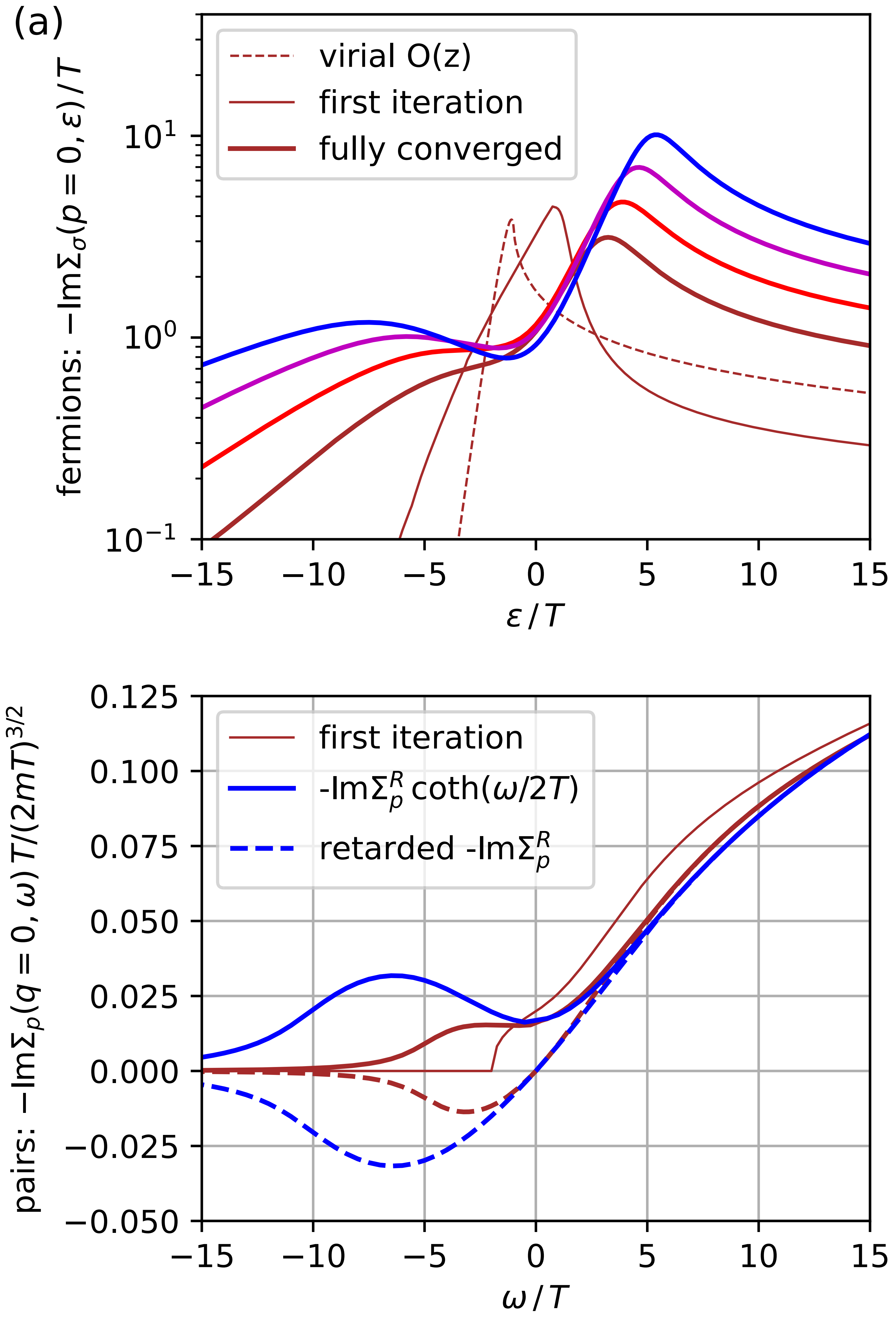
Johansen+ 2024; Enss 2024; Dizer+ 2024

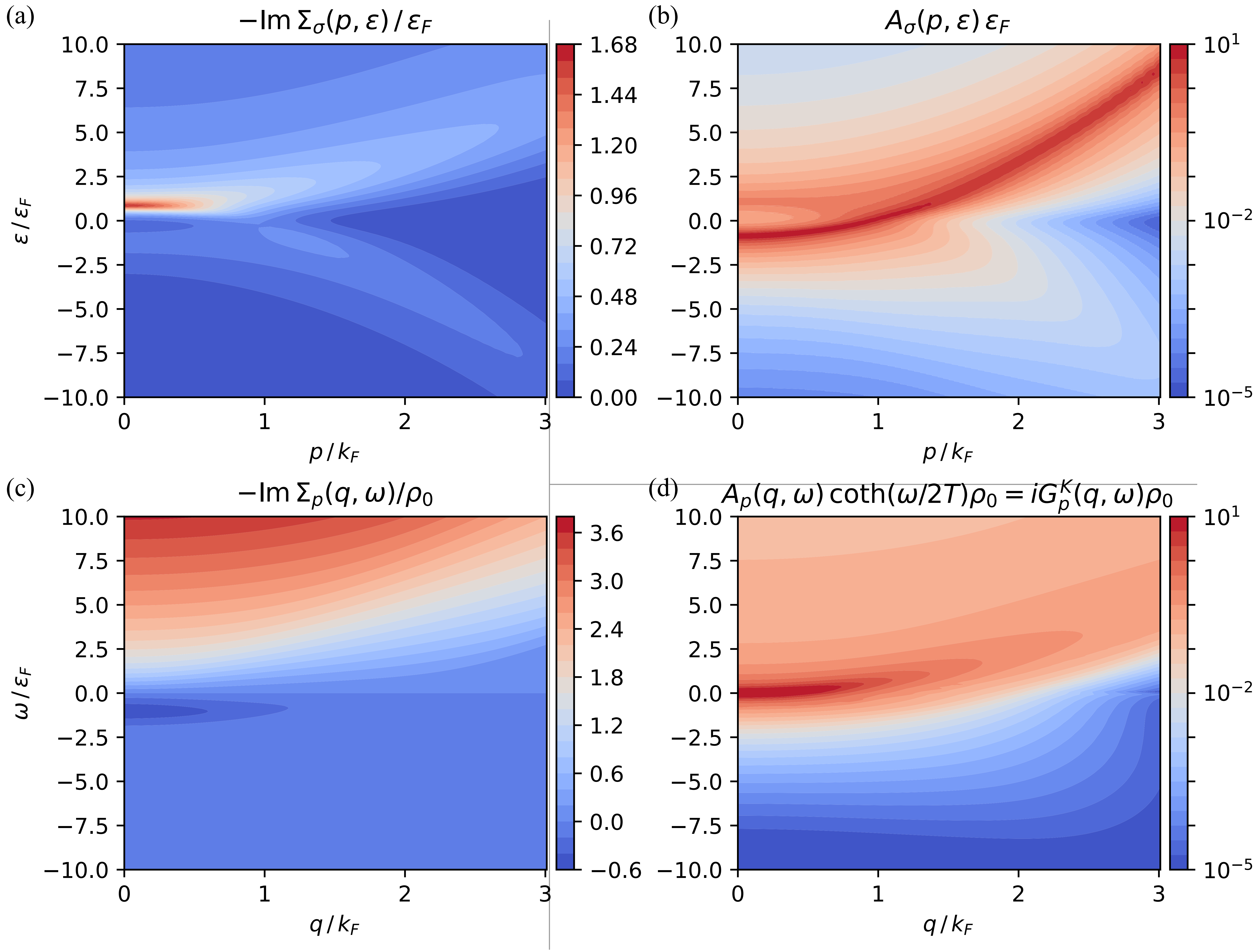
pair spectrum

sharp peaks in real frequency:

- convolution by Fourier transform
  Johansen, Frank, Lang 2024
- adaptive mesh to resolve peak
  Dizer, Horak, Pawlowski 2024
- linearize inverse propagator between grid points
  Enss 2024

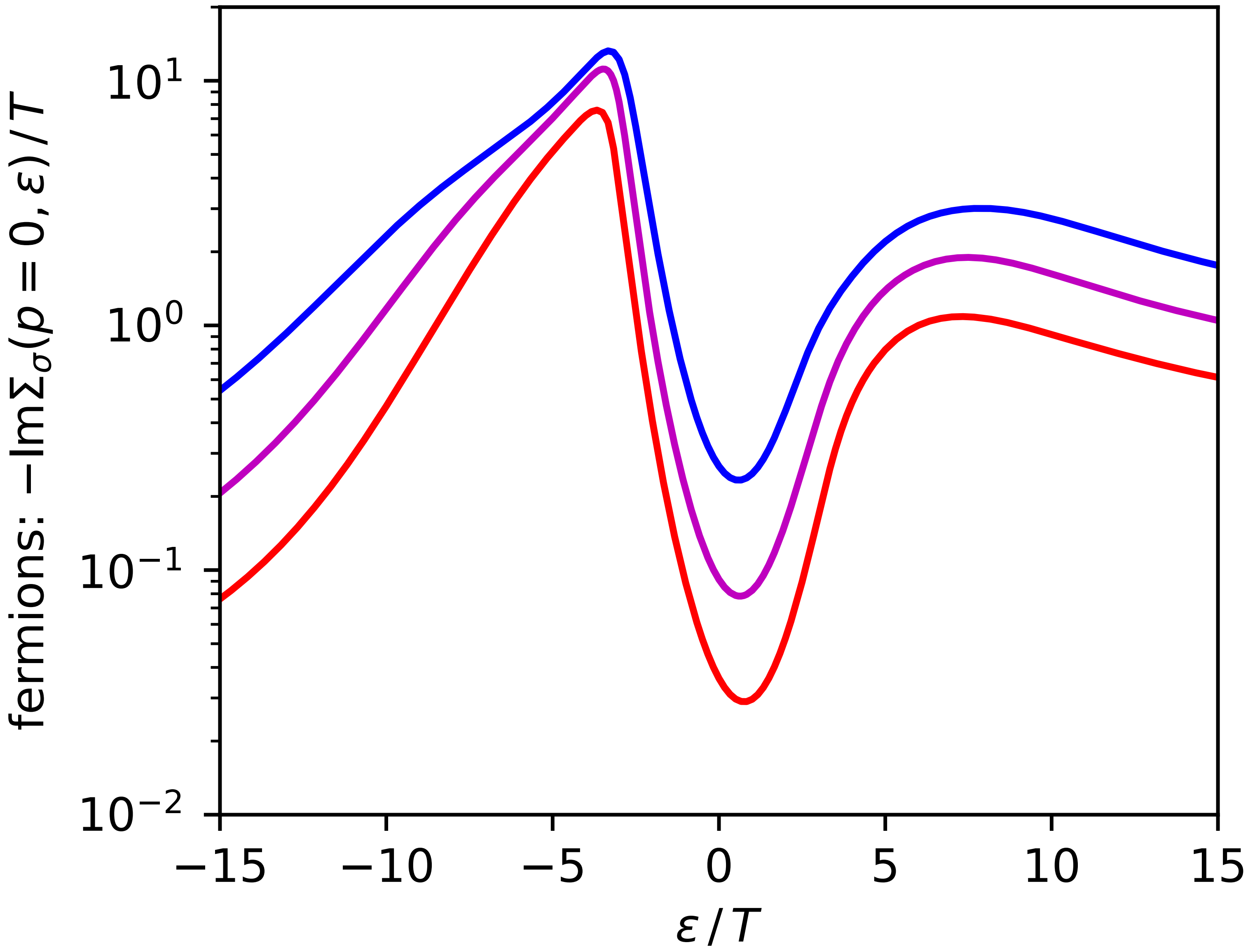


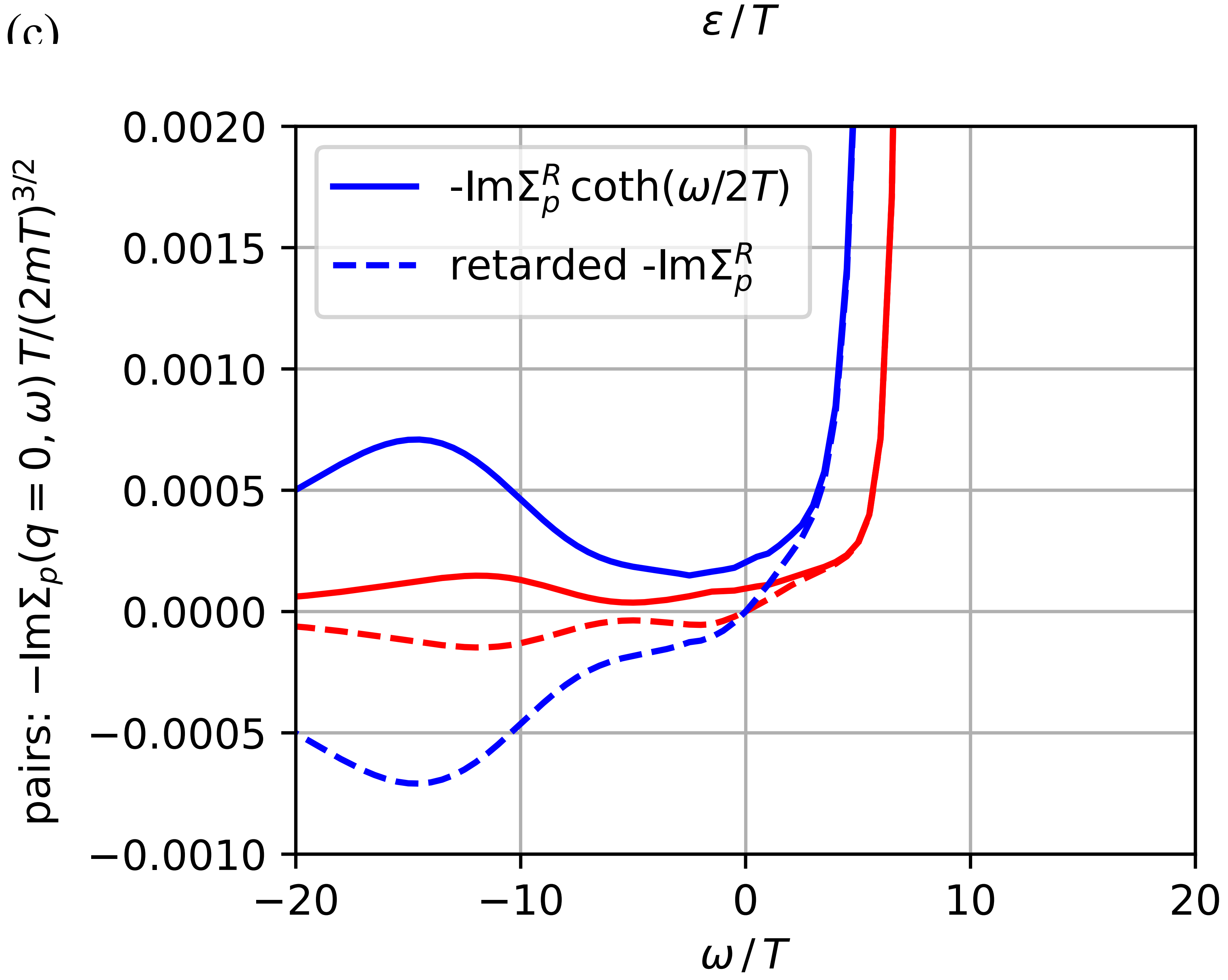


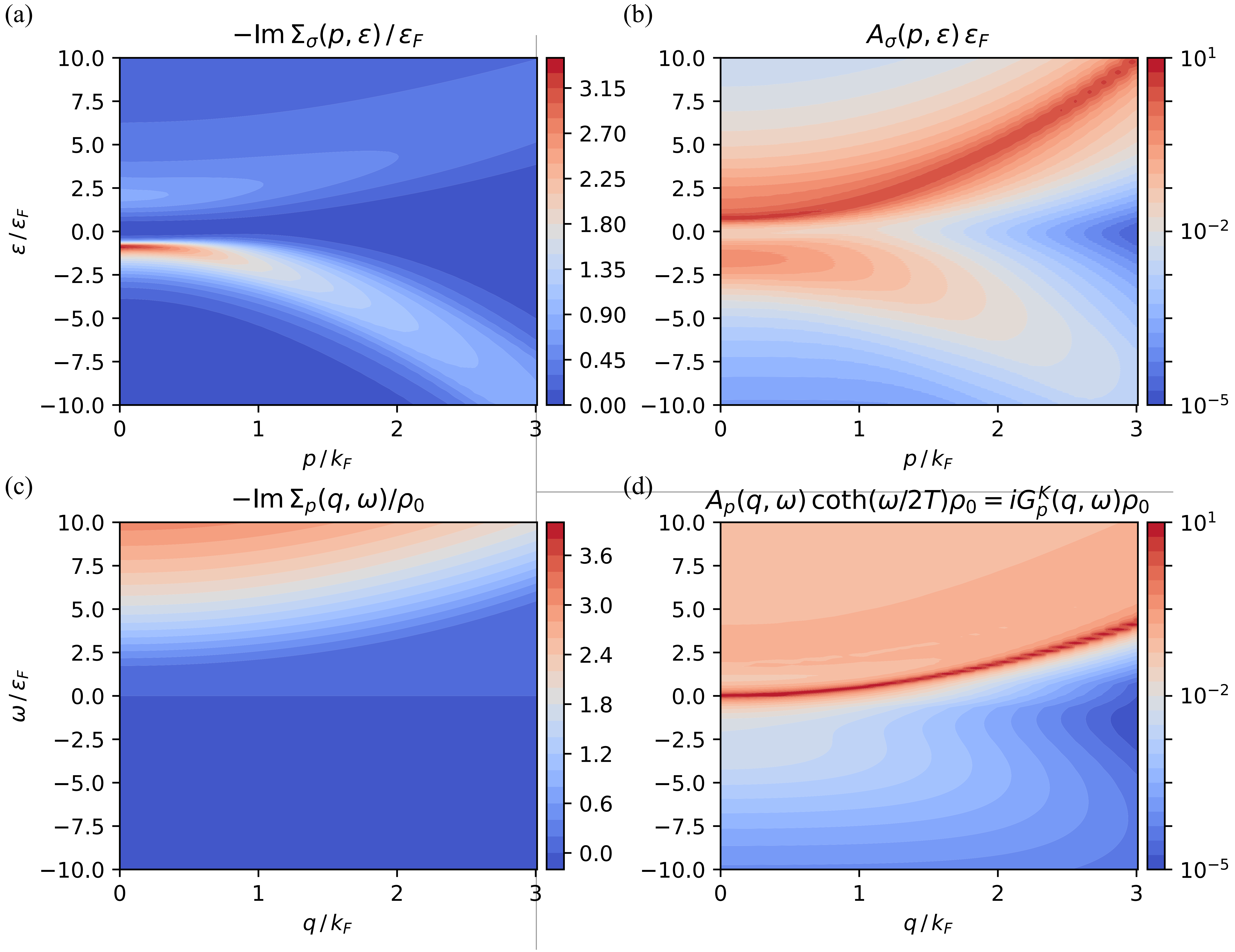


Fermions and pairs at unitarity

Enss PRA 2024







Fermions and pairs at strong binding (BEC side)

Enss PRA 2024