Hydrodynamic Attractors: Theoretical Overview

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FMF | UNIVERSITY OF LJUBLJANA
Faculty of Mathematics and Physics

Funded by the European Union

Overview

Motivation

Recap of hydro attractors

Attractors in MIS, RTA kinetic theory, holography

Outlook

Hydrodynamics' unreasonable effectiveness

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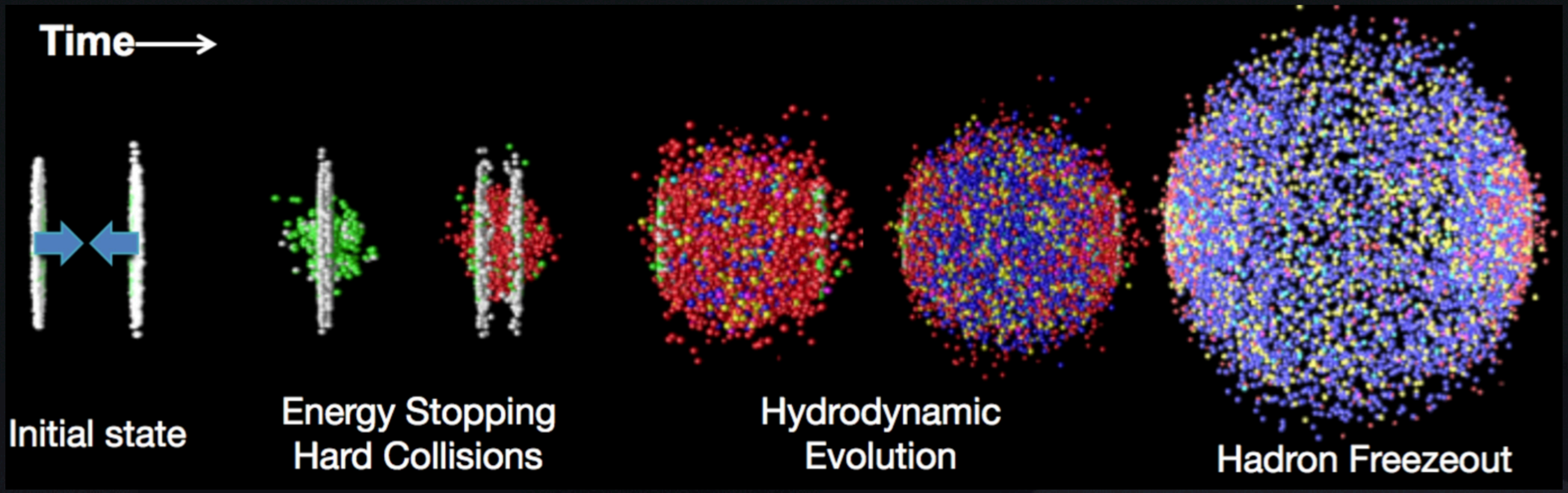

Nayak 2012

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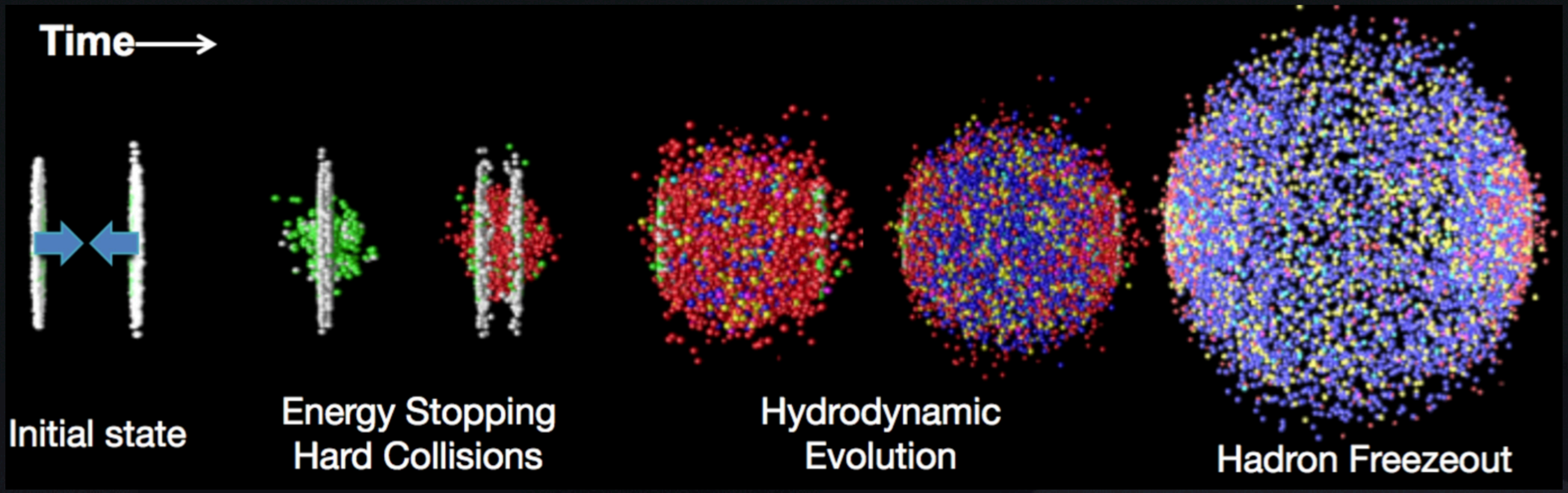

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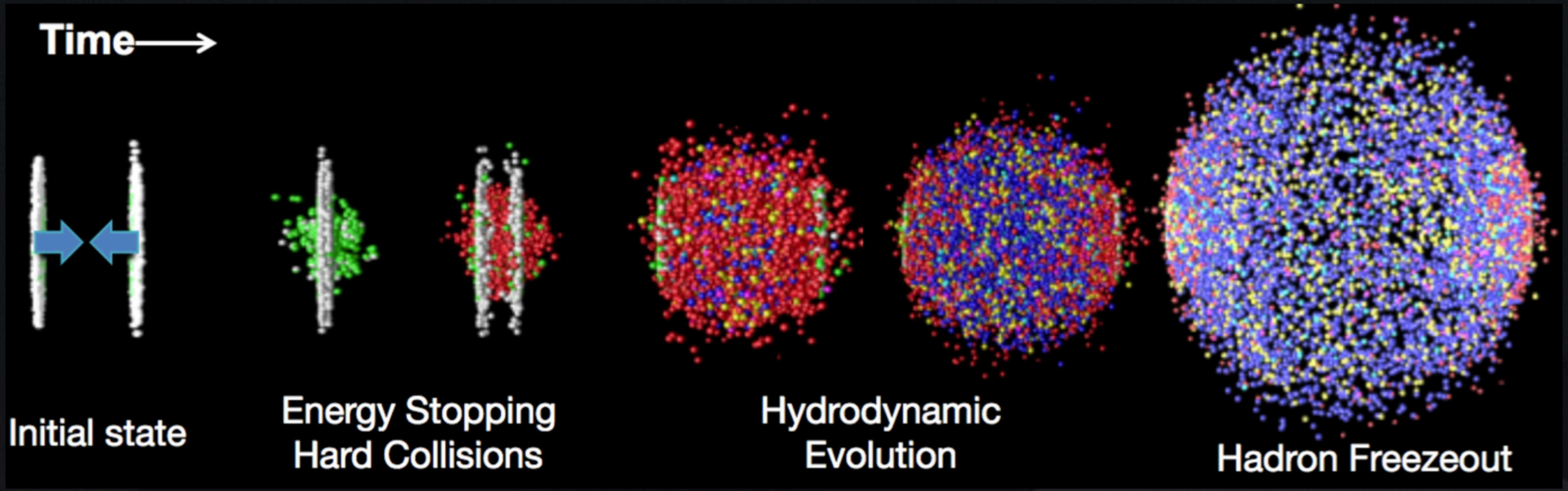

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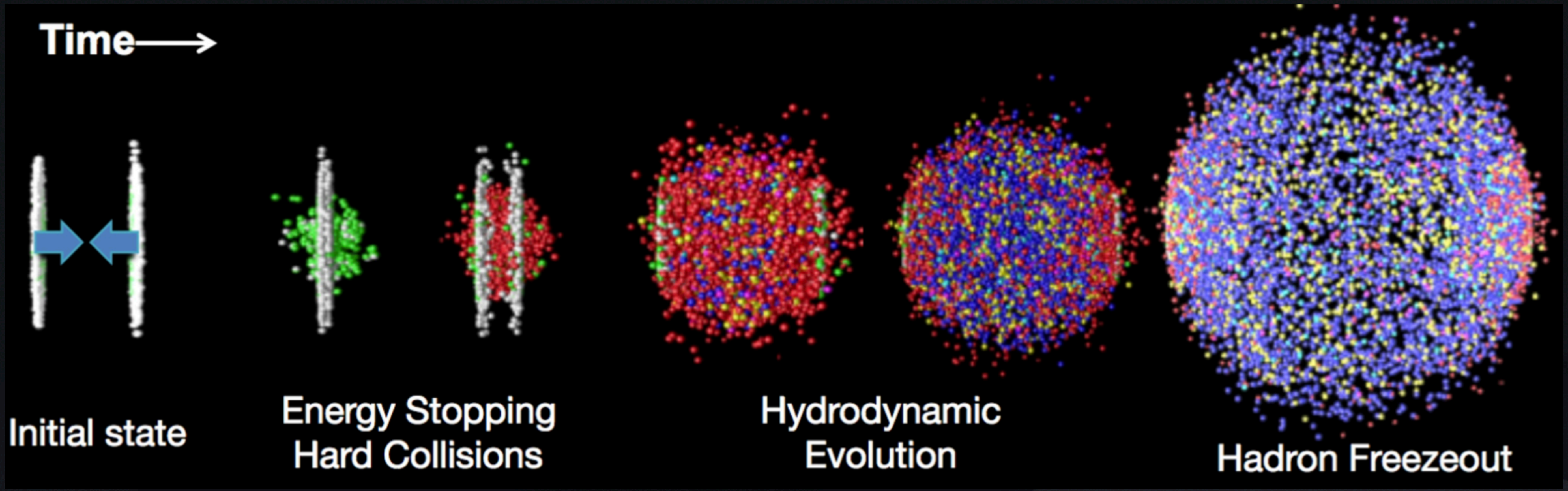

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- Essential ingredients for attractor: competition between expansion and interaction

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- Alternatively, general frame approach (BDNK) remains causal at 1st order

Bemfica, Disconzi, Noronha 2018, Kovtun 2019, Gavassino, Antonelli Haskel 2020, Gavassino 2021

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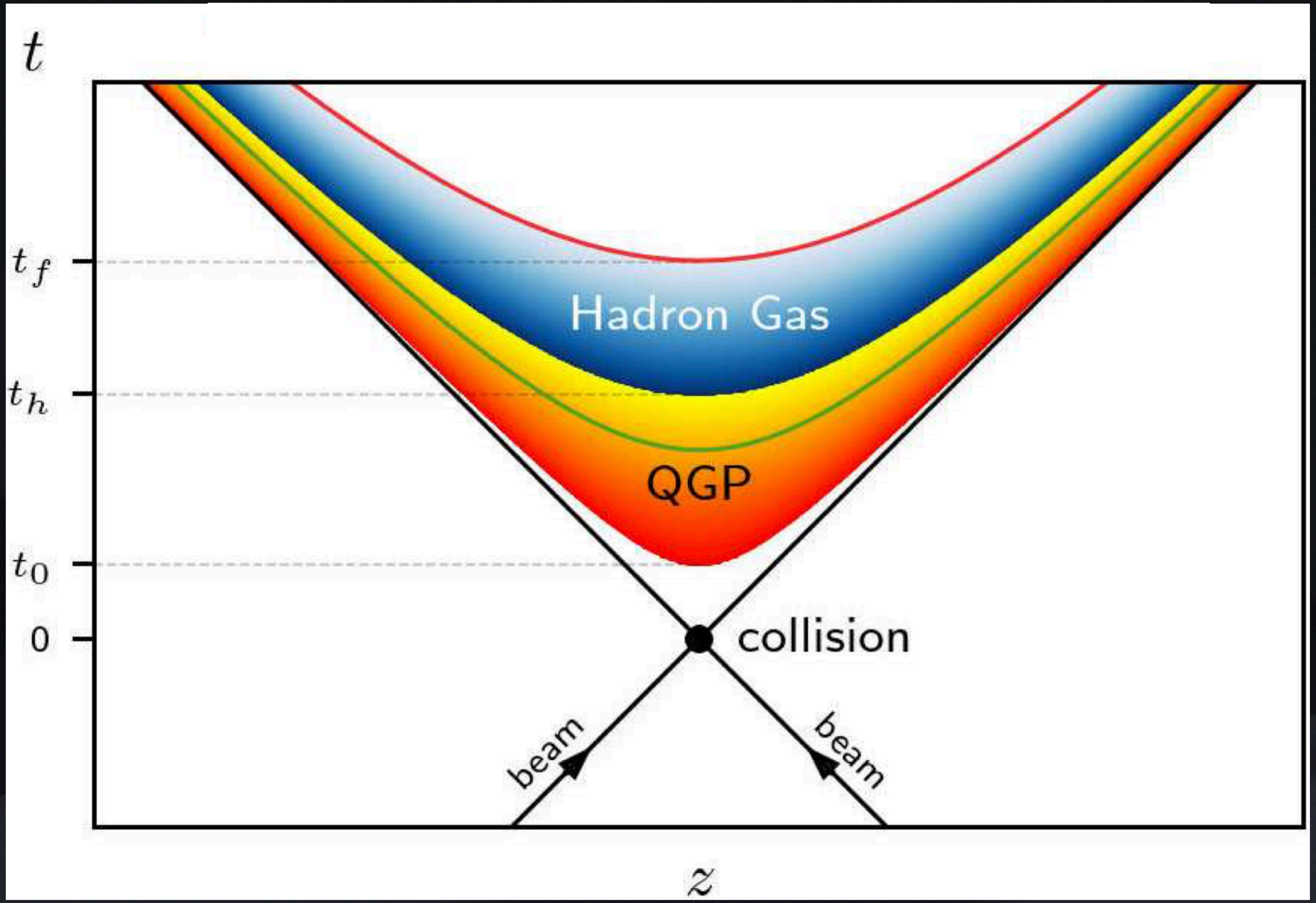

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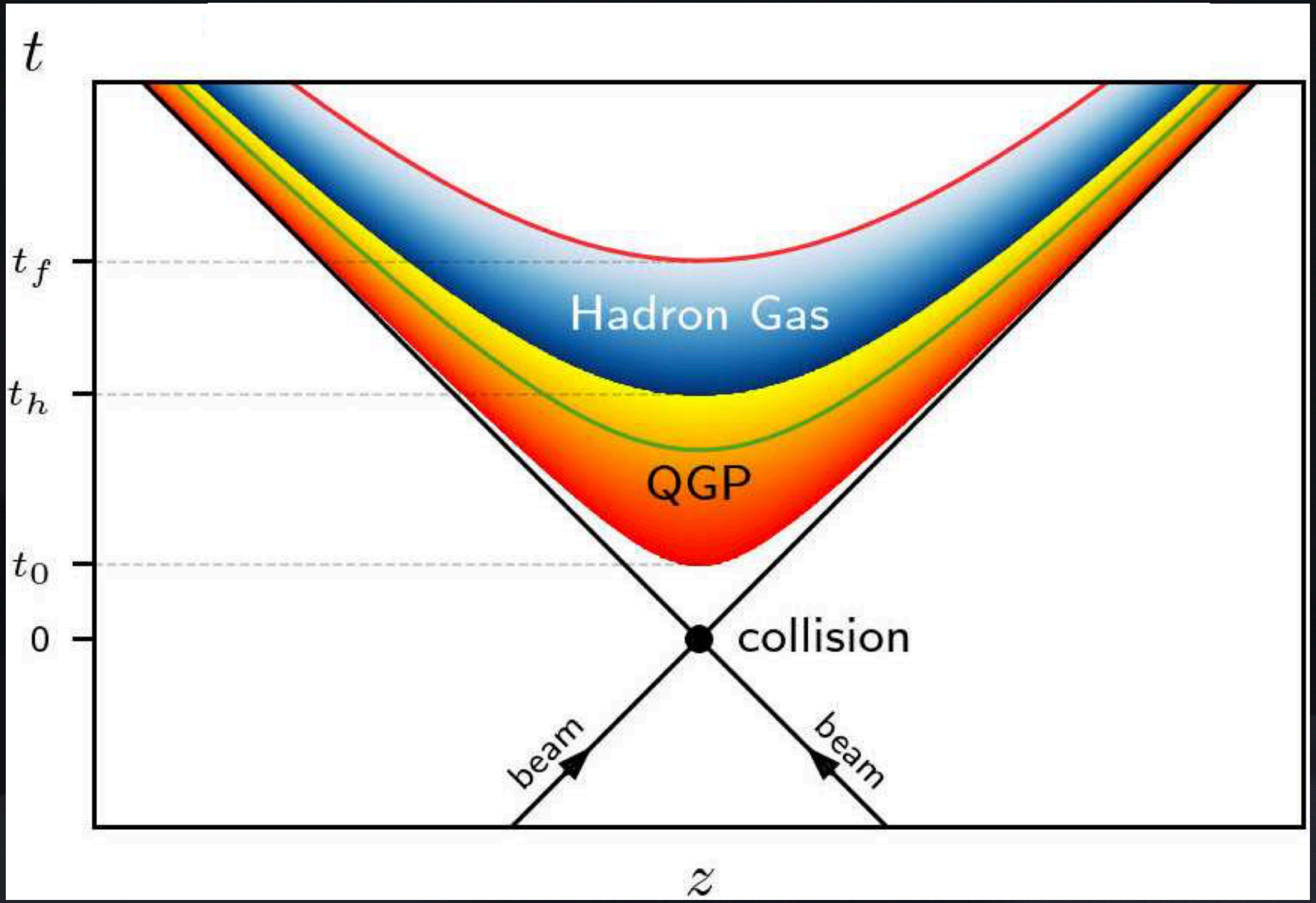

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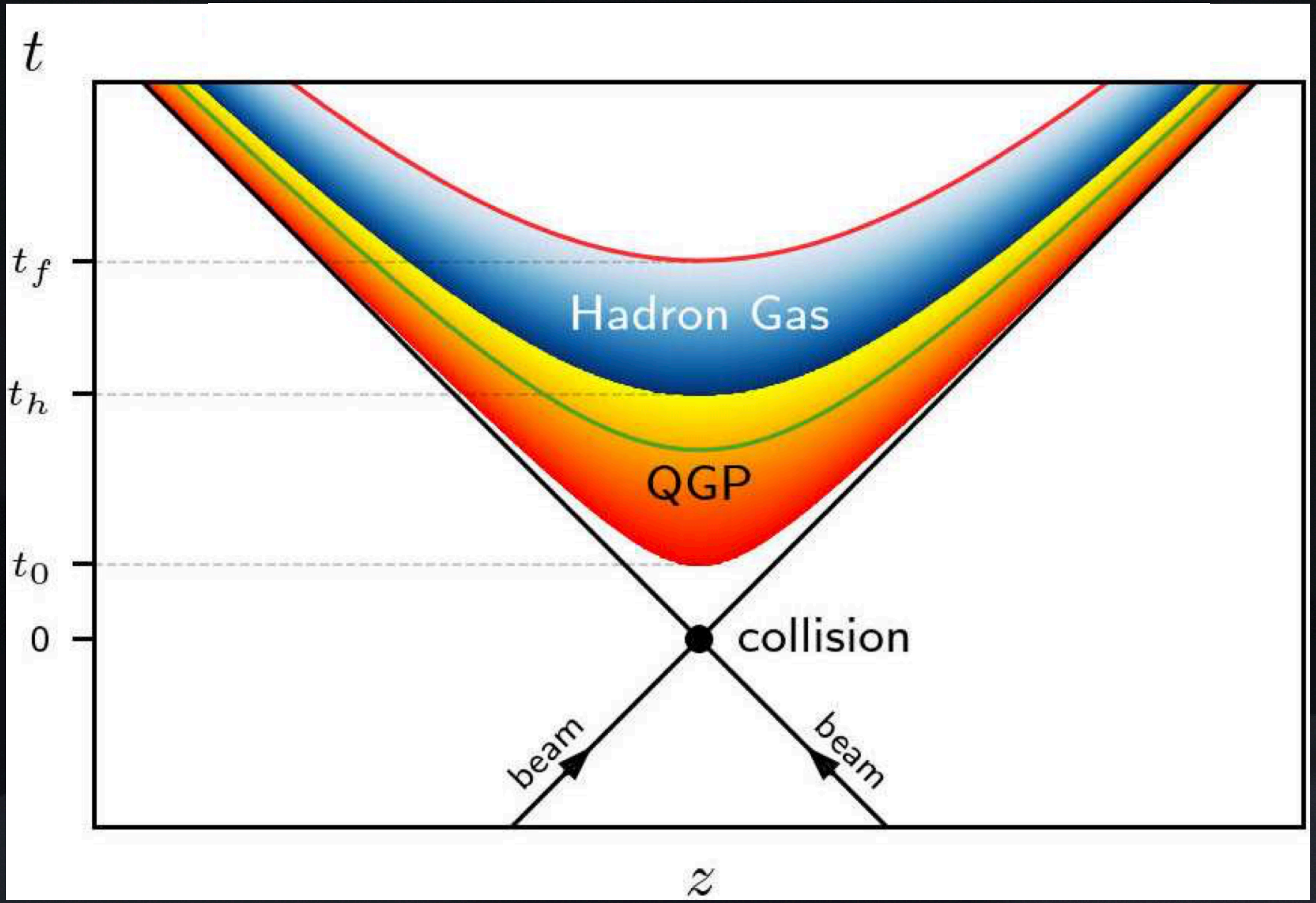

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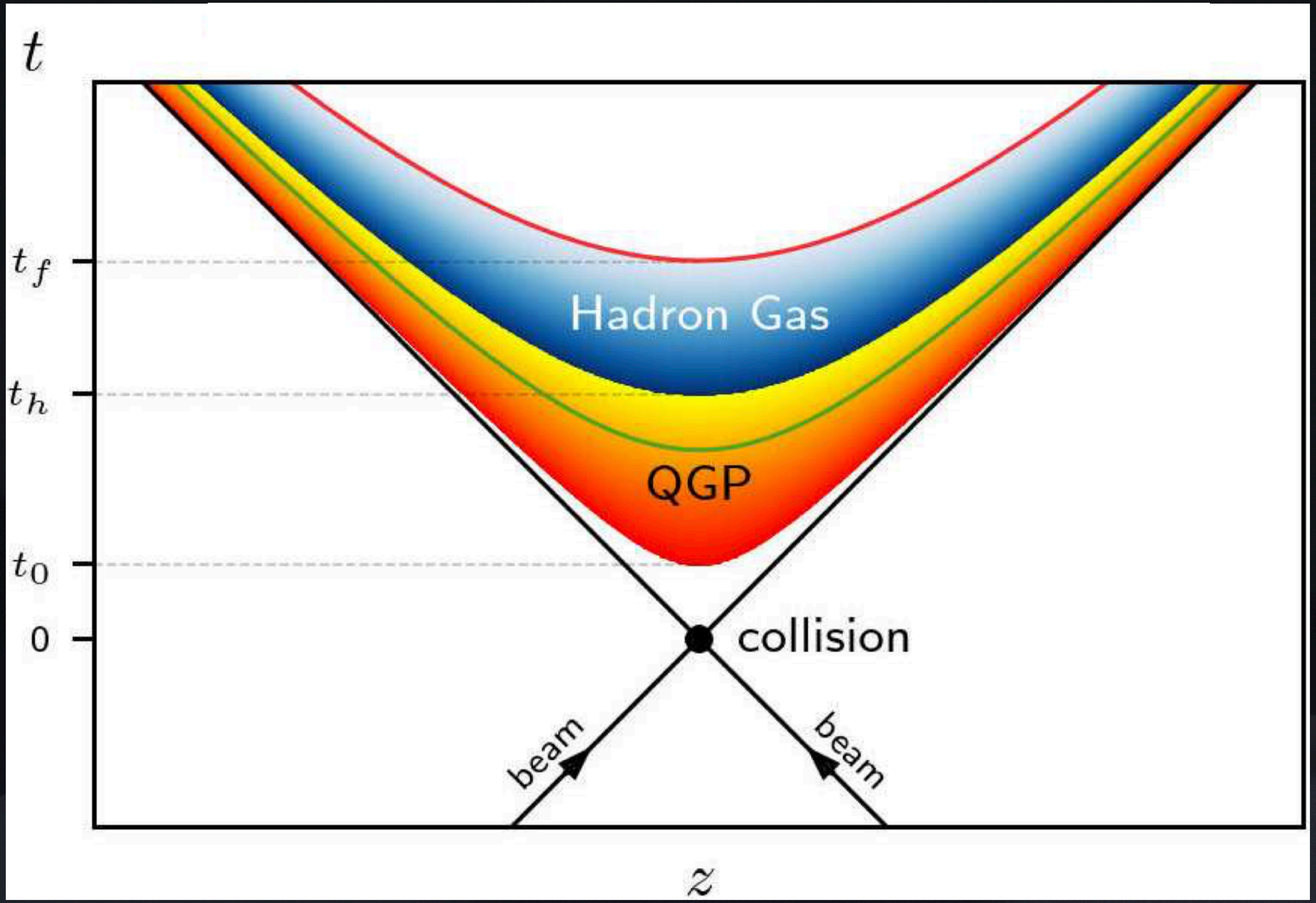

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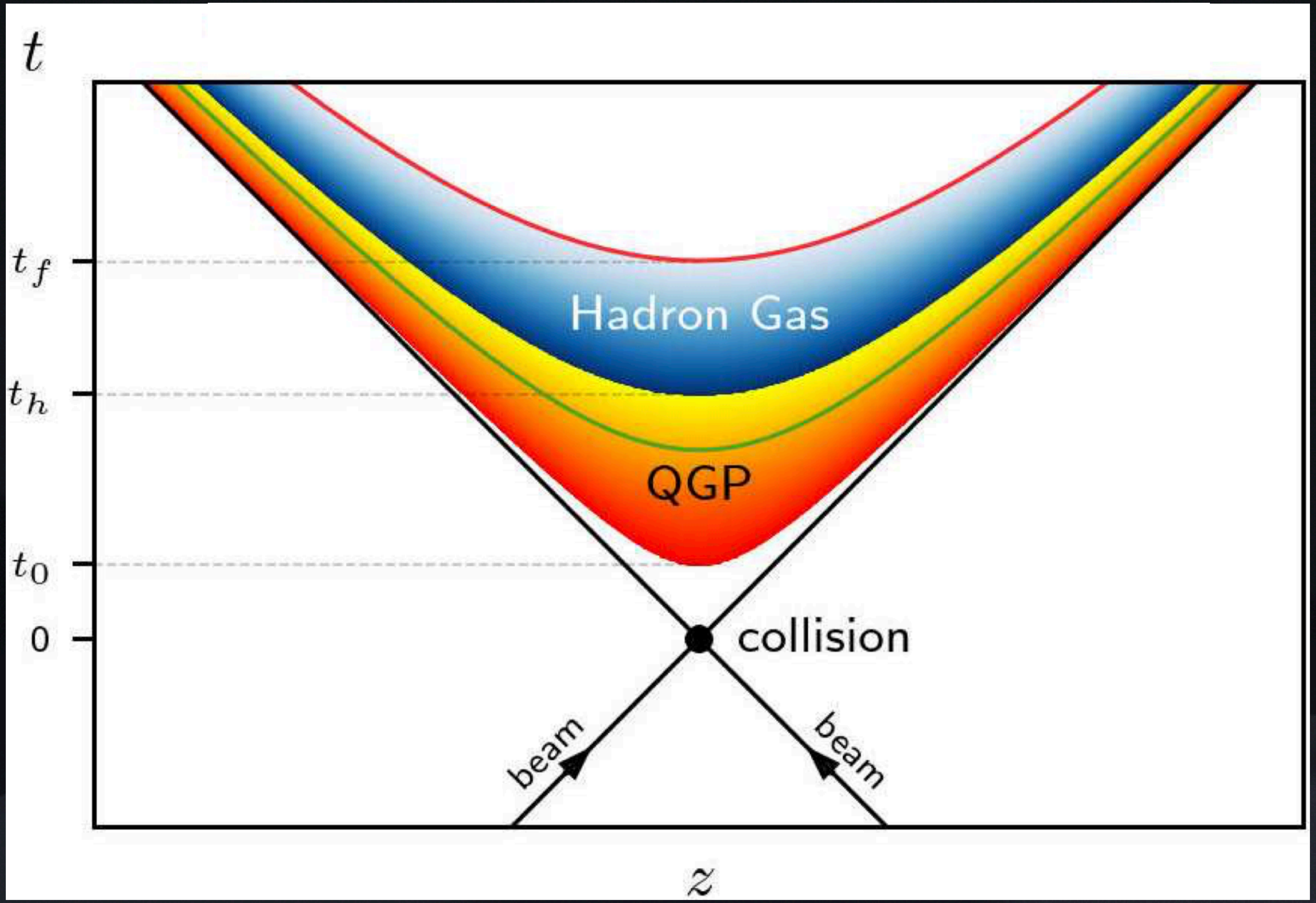

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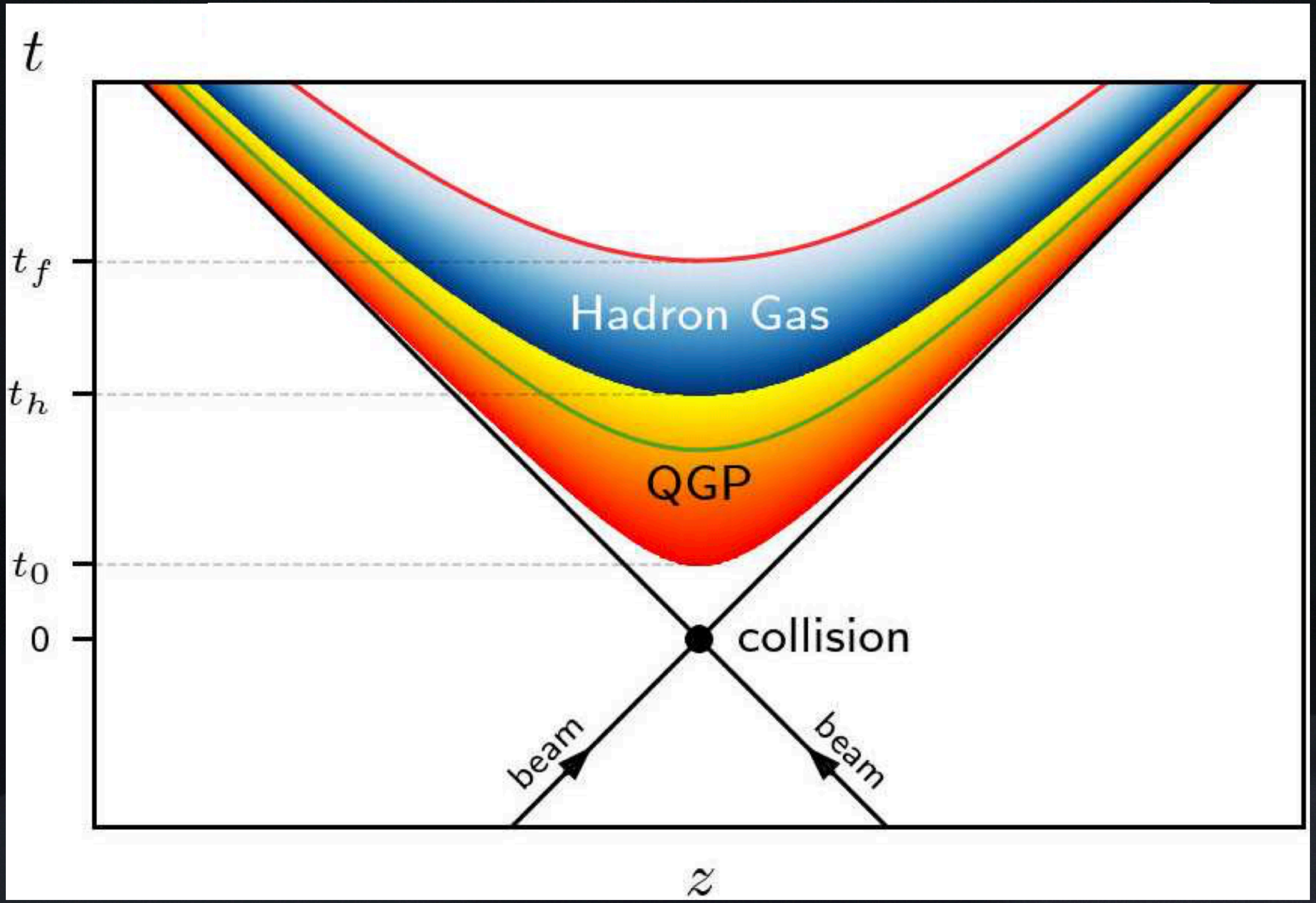

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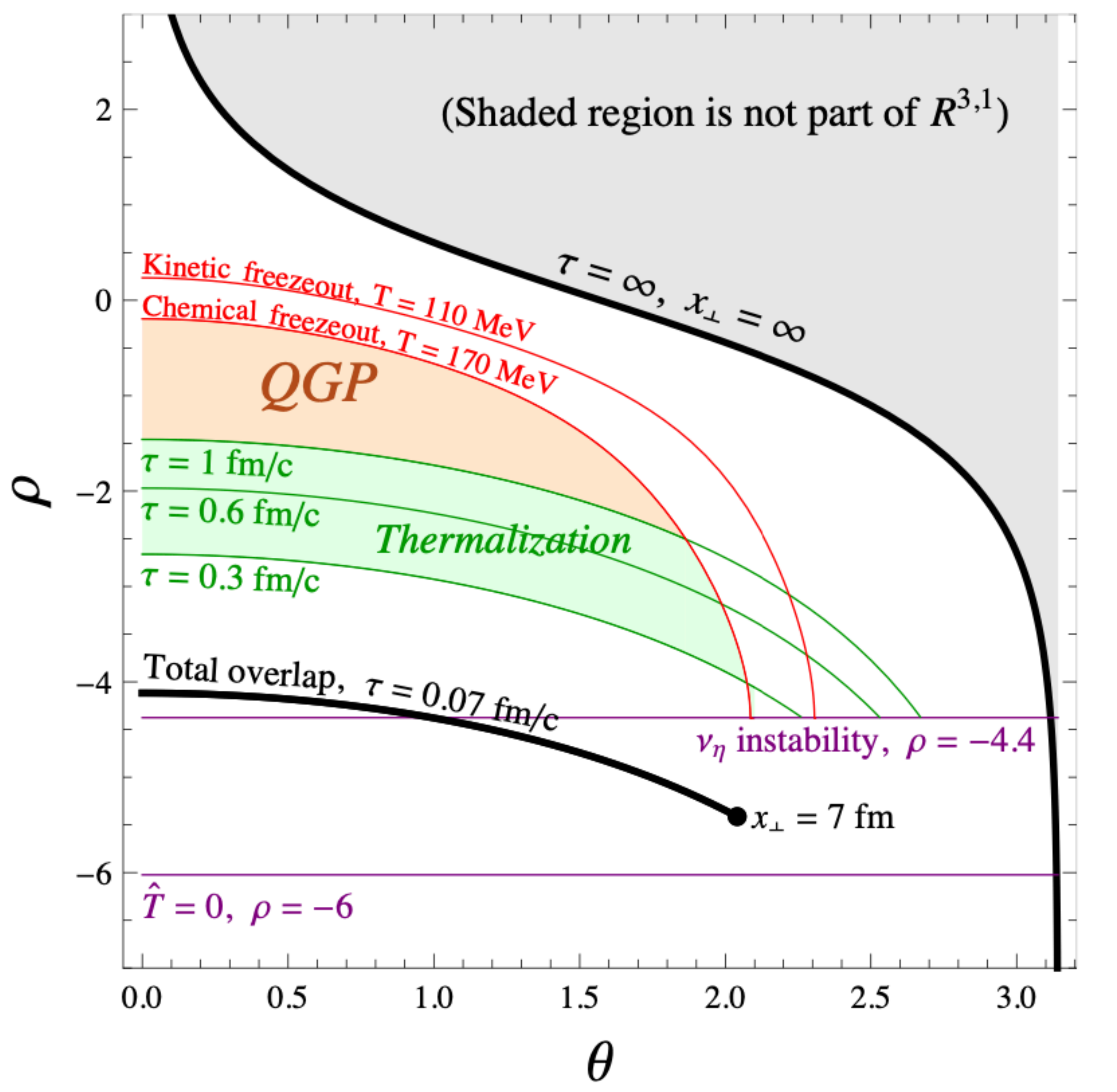

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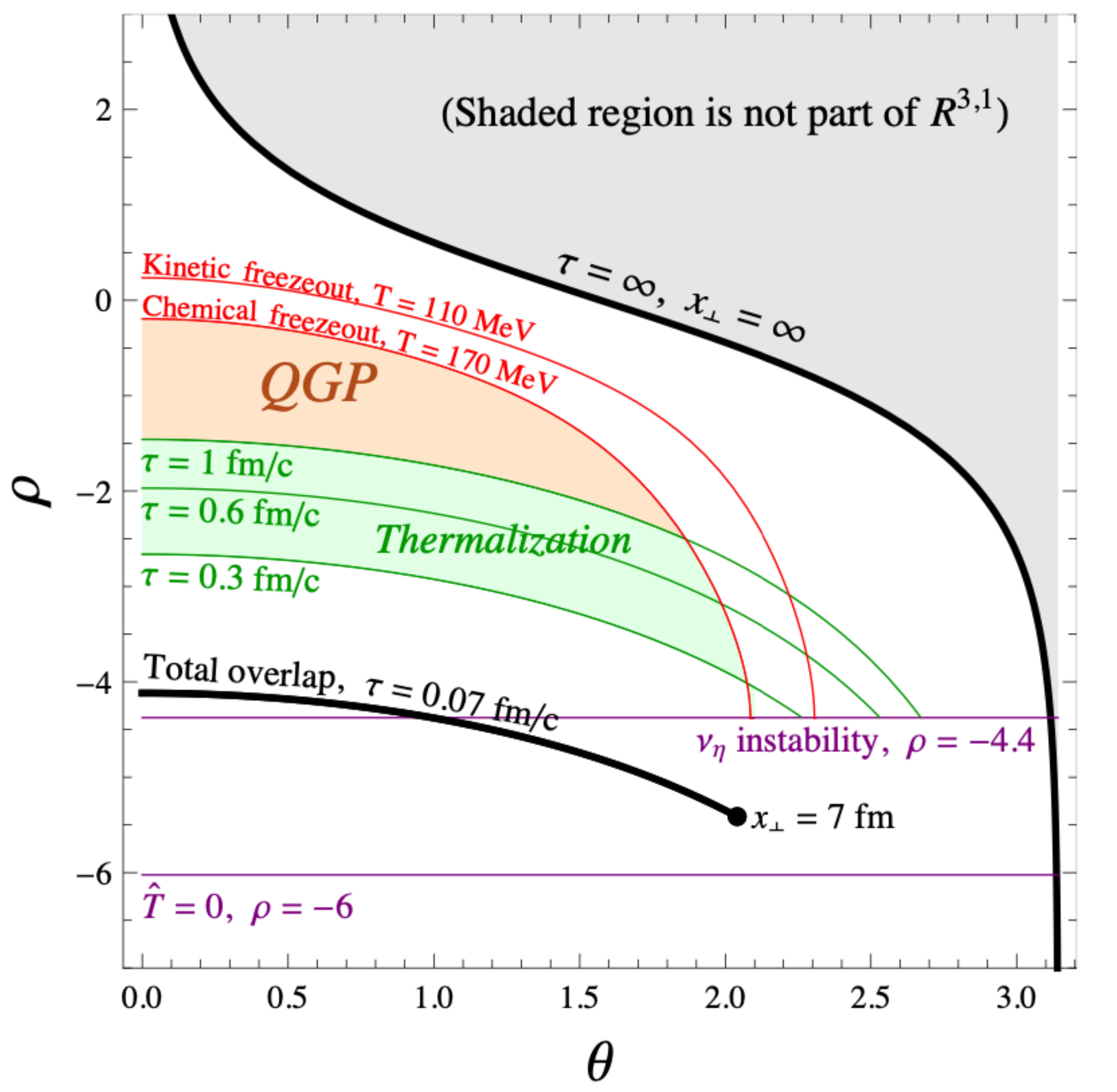

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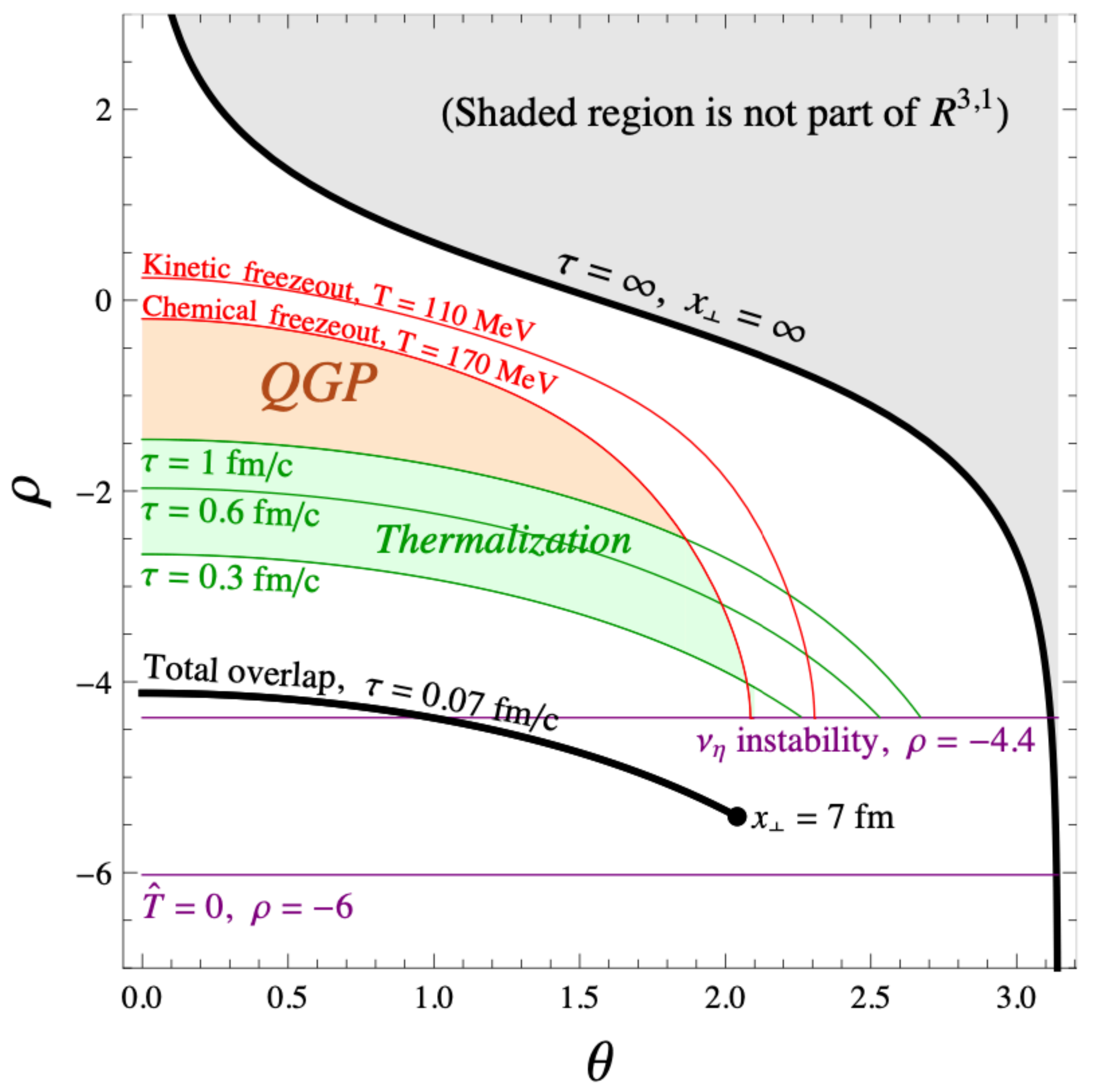

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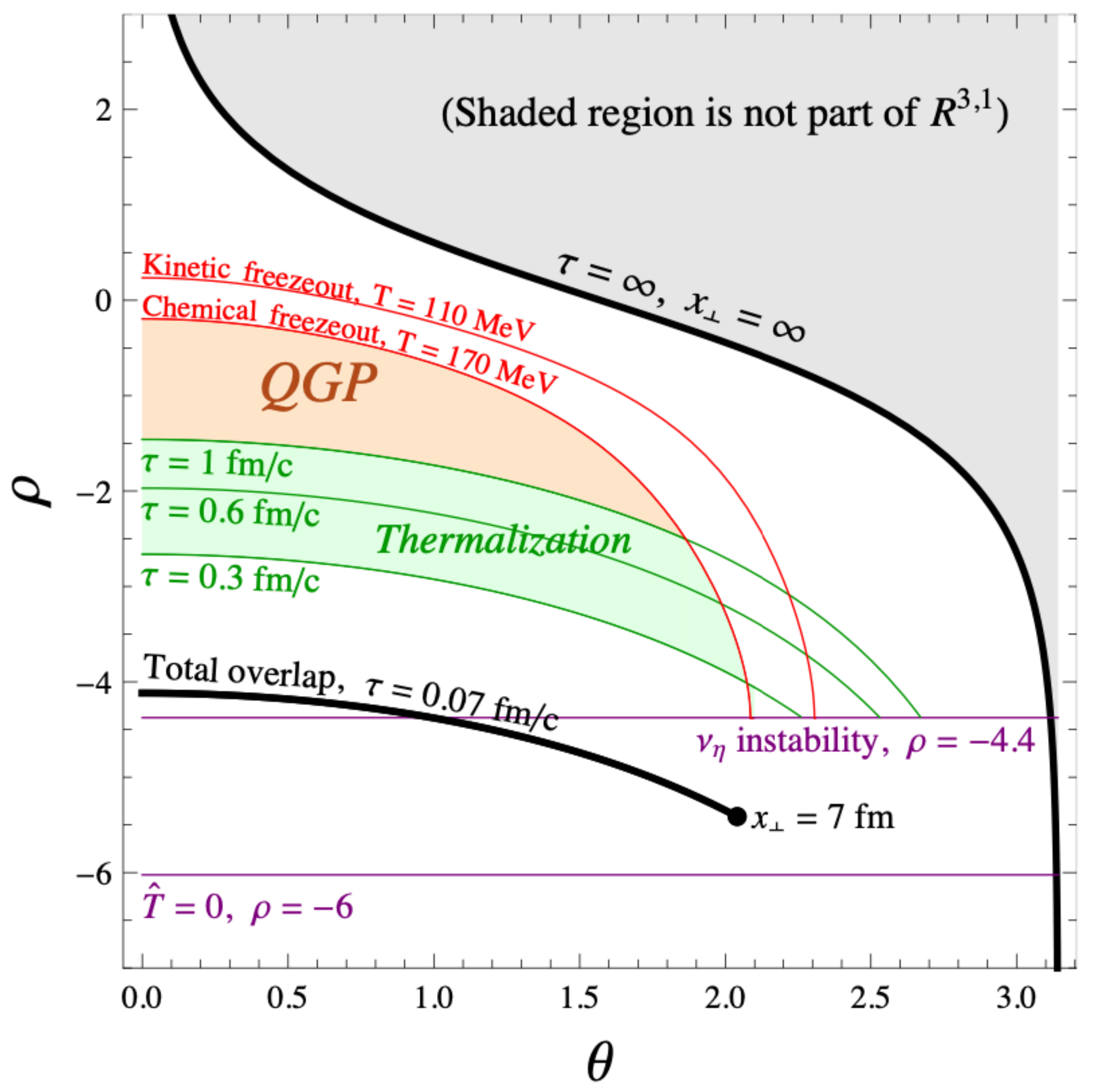

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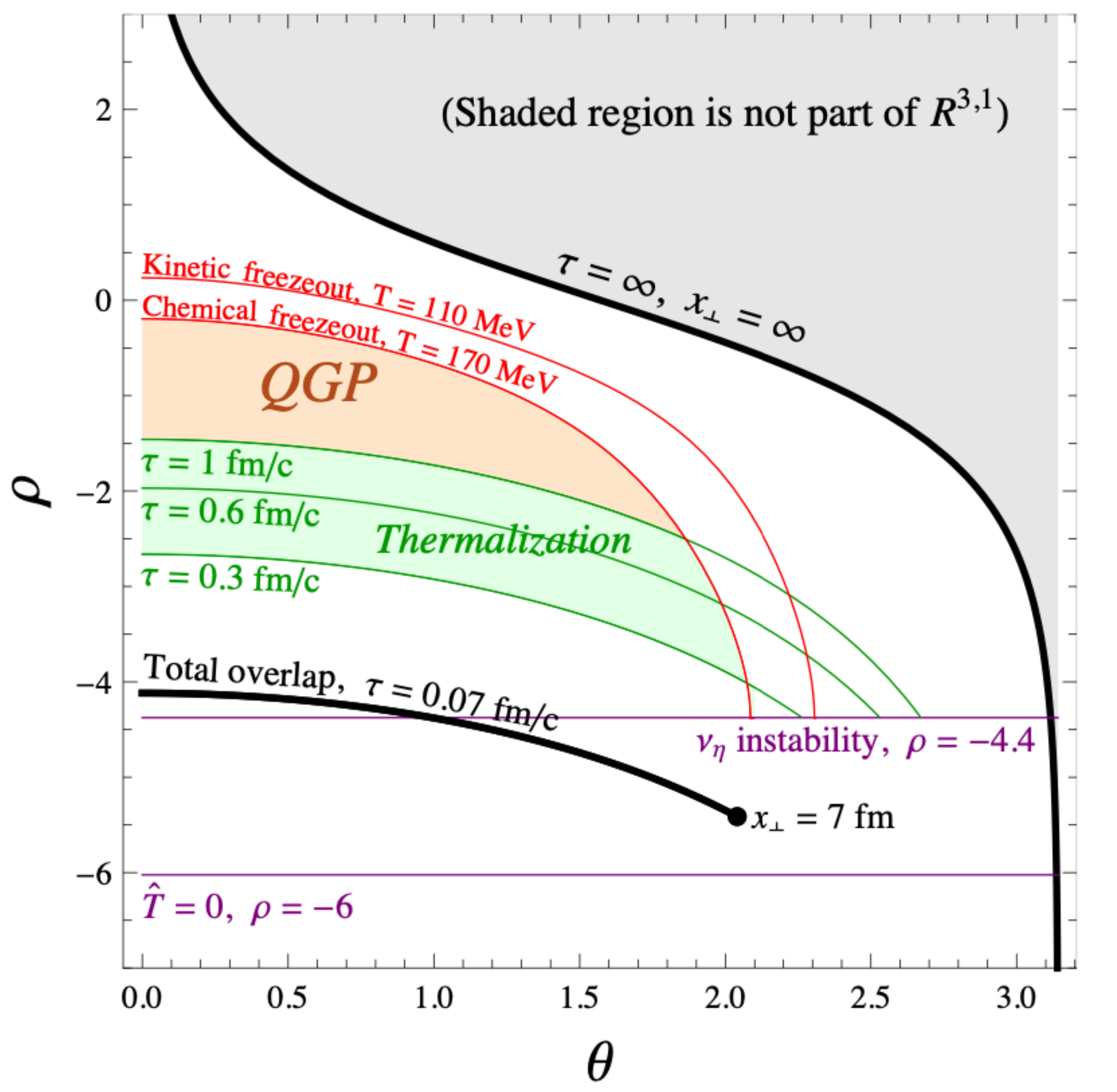

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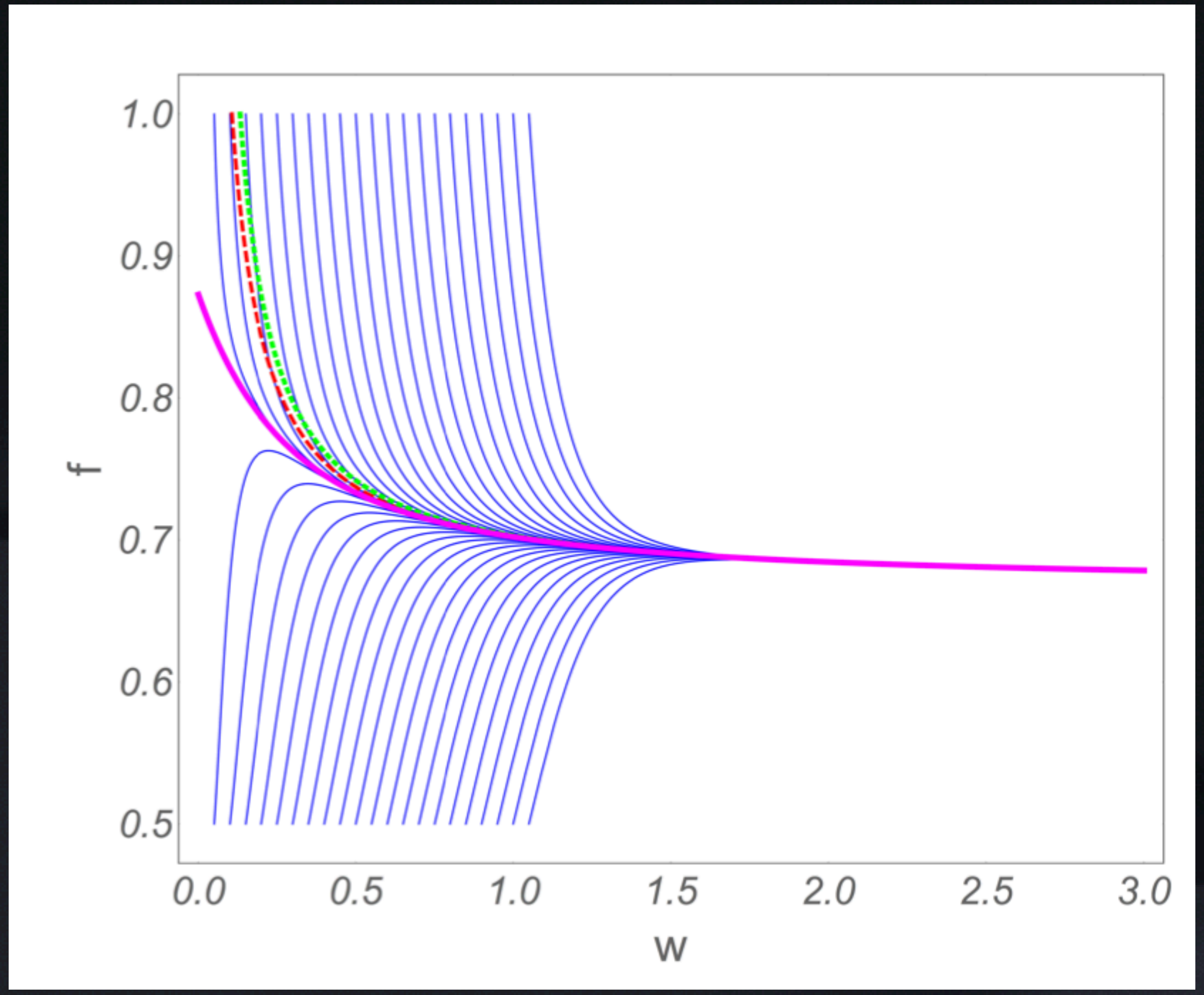

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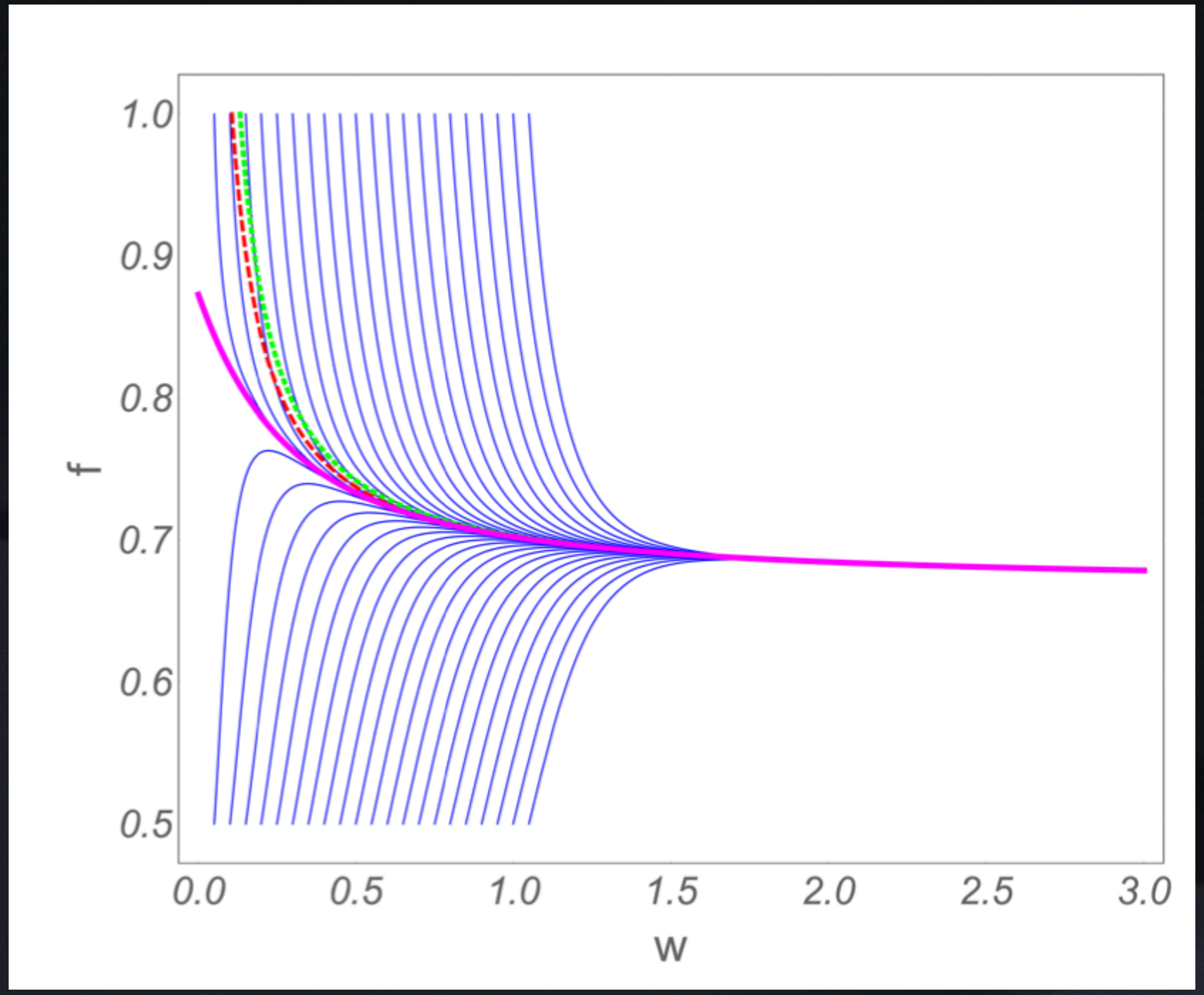

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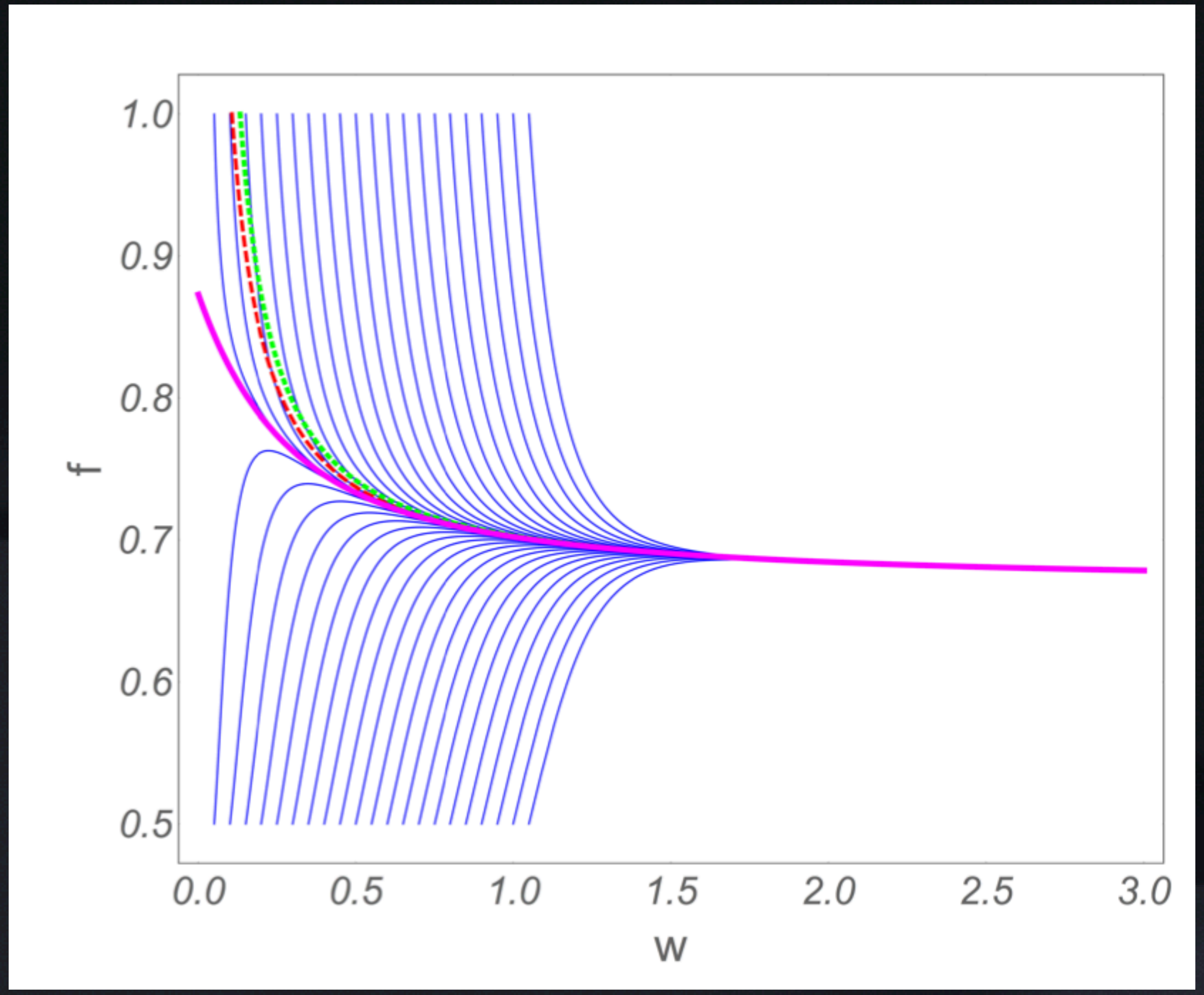

- See that initial conditions decay rapidly to attractor curve

$$f(w) = \frac{2}{3} + \frac{4C_\eta}{9w} + \frac{8C_\eta C_{\tau\Pi}}{27w^2} + O\left(\frac{1}{w^3}\right).$$

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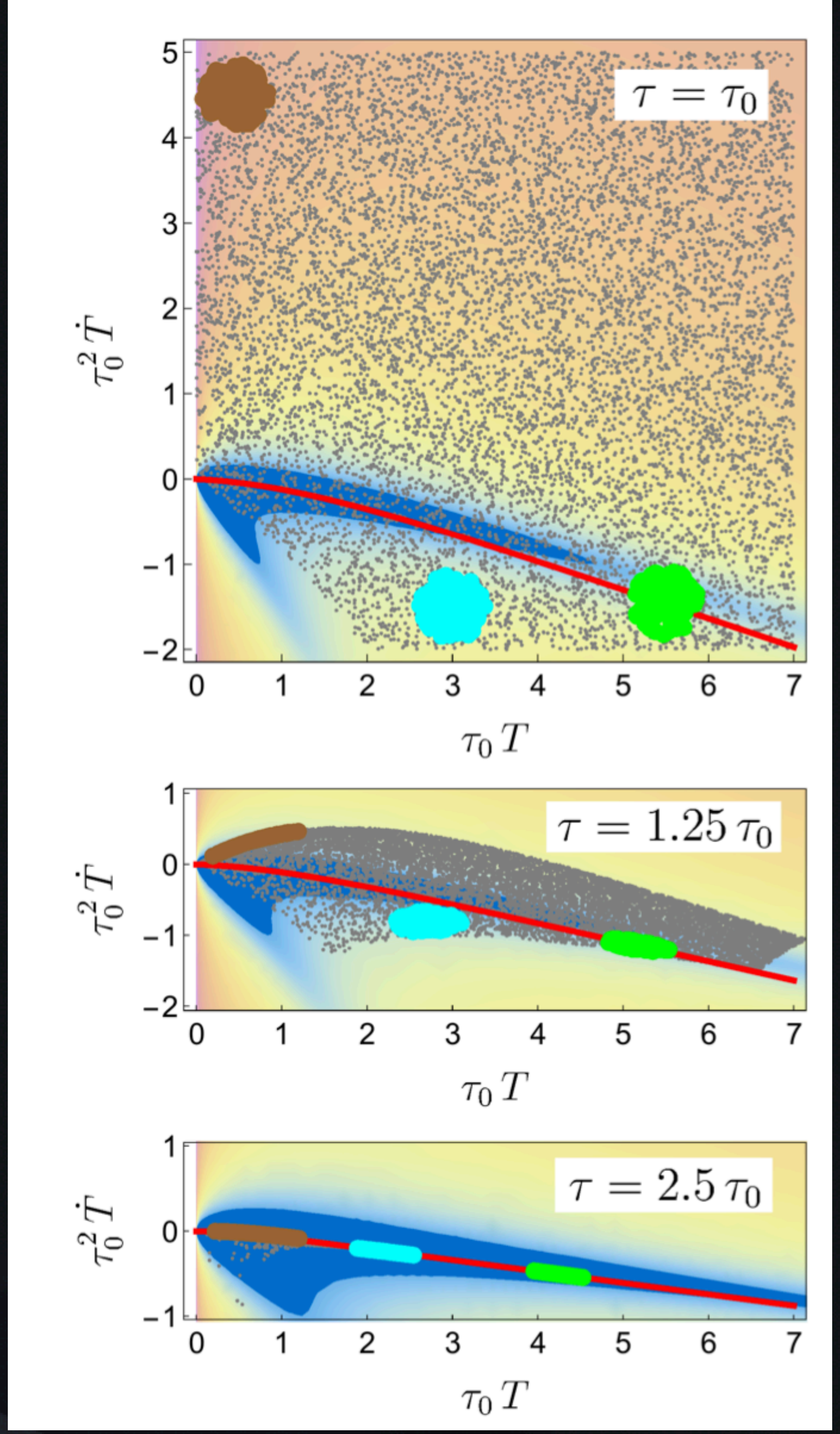

Heller, Jefferson, Spałinski, Svensson 2020

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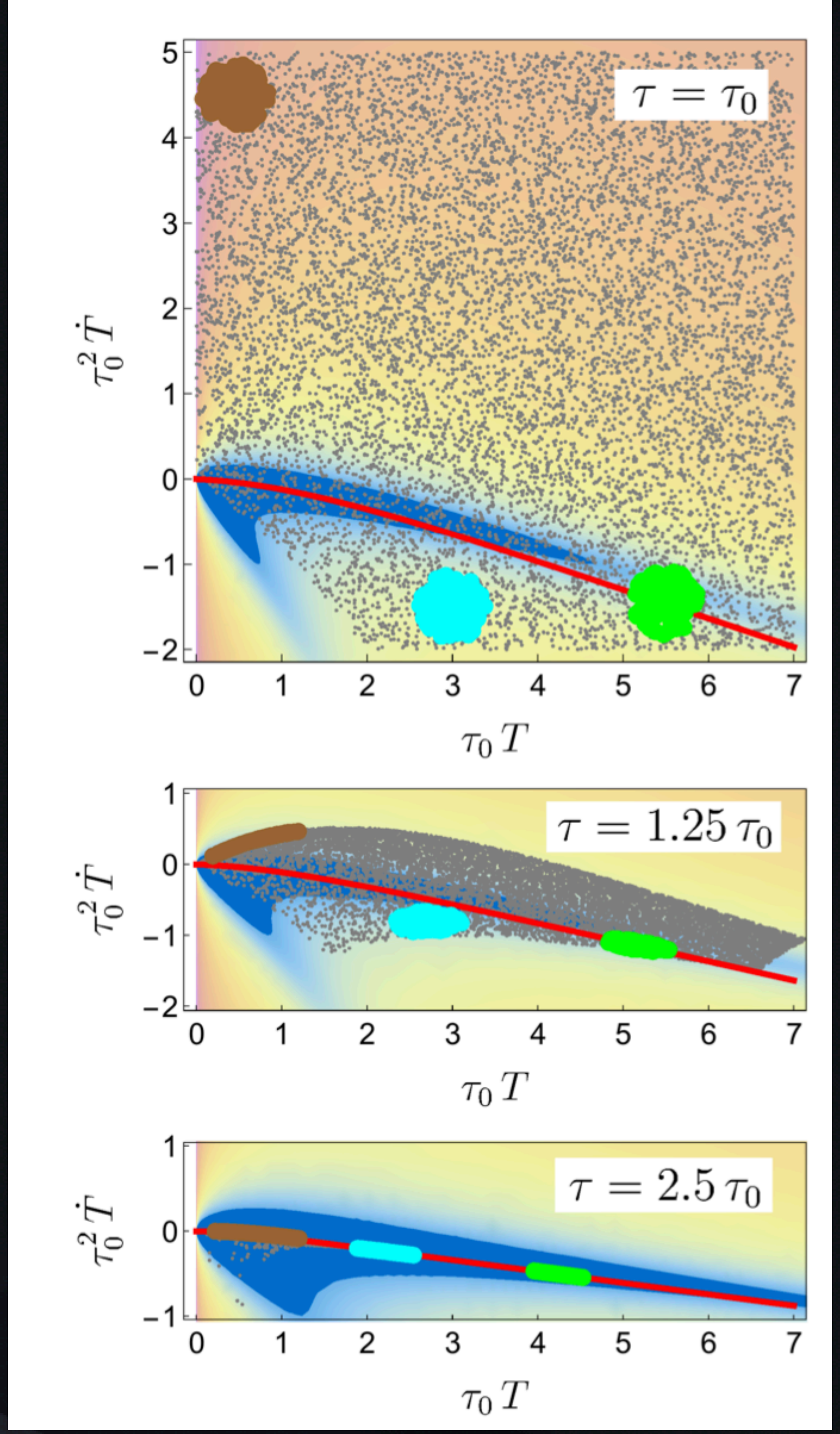

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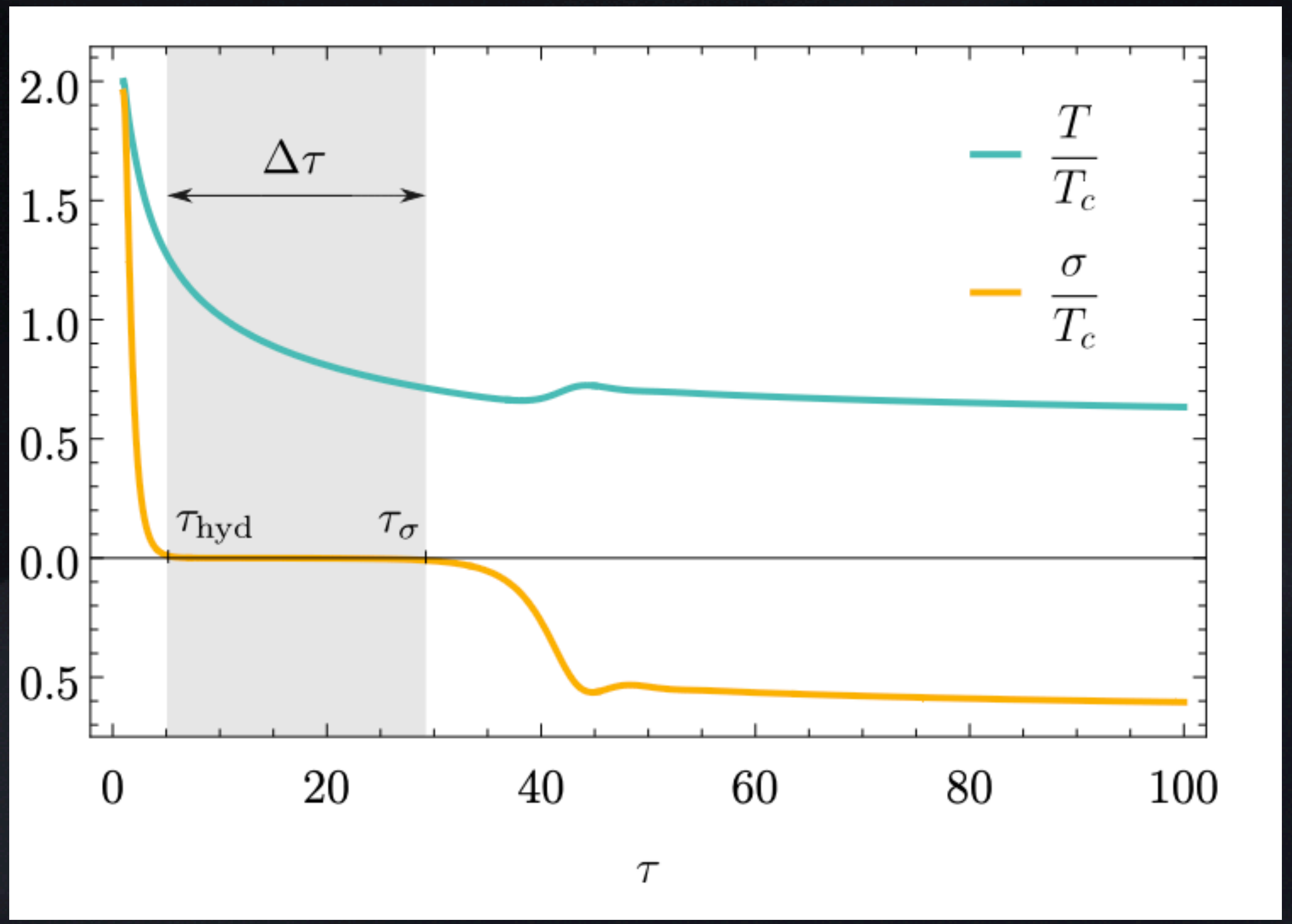

Buza, Mitra, AS 2024

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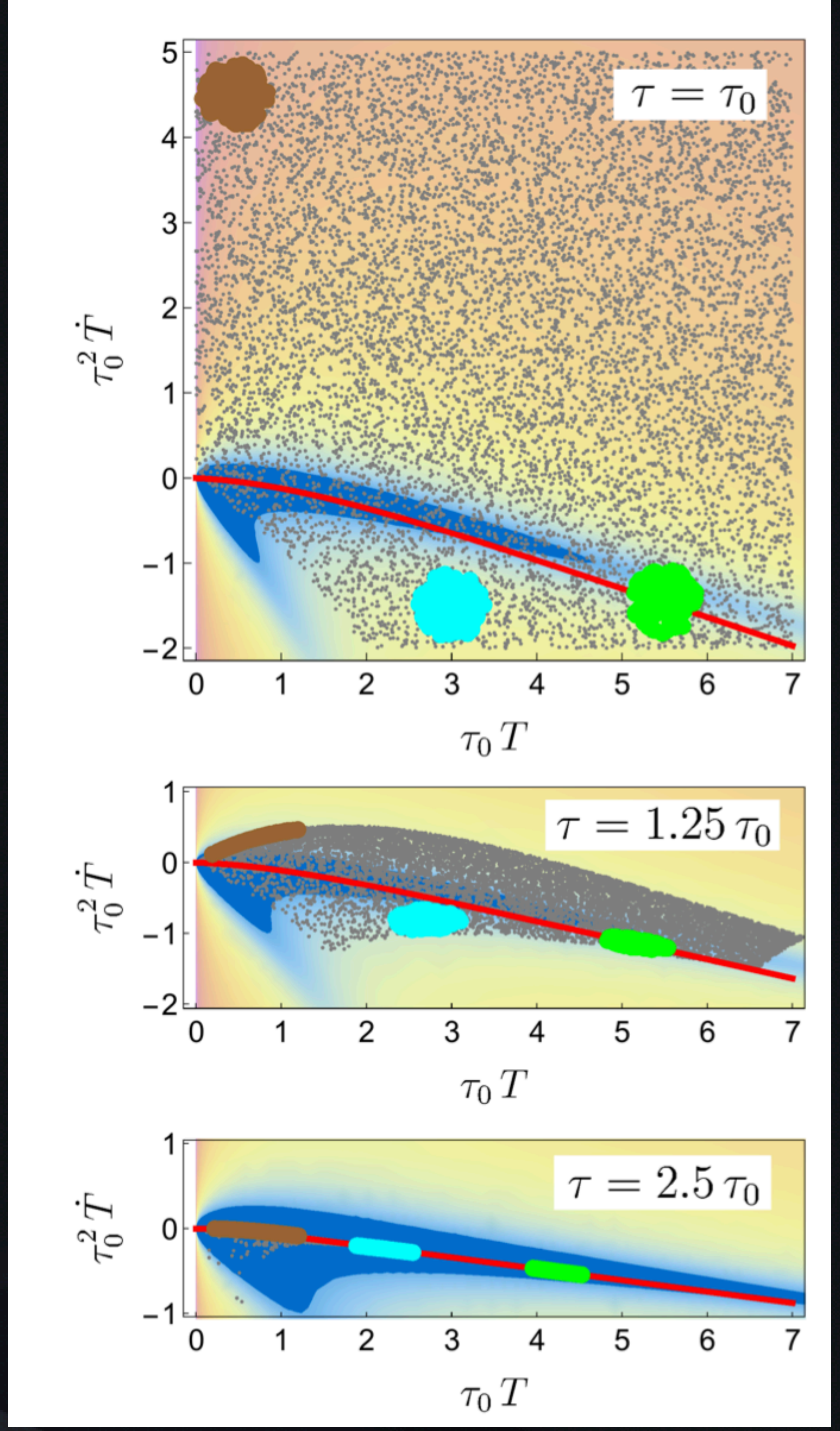

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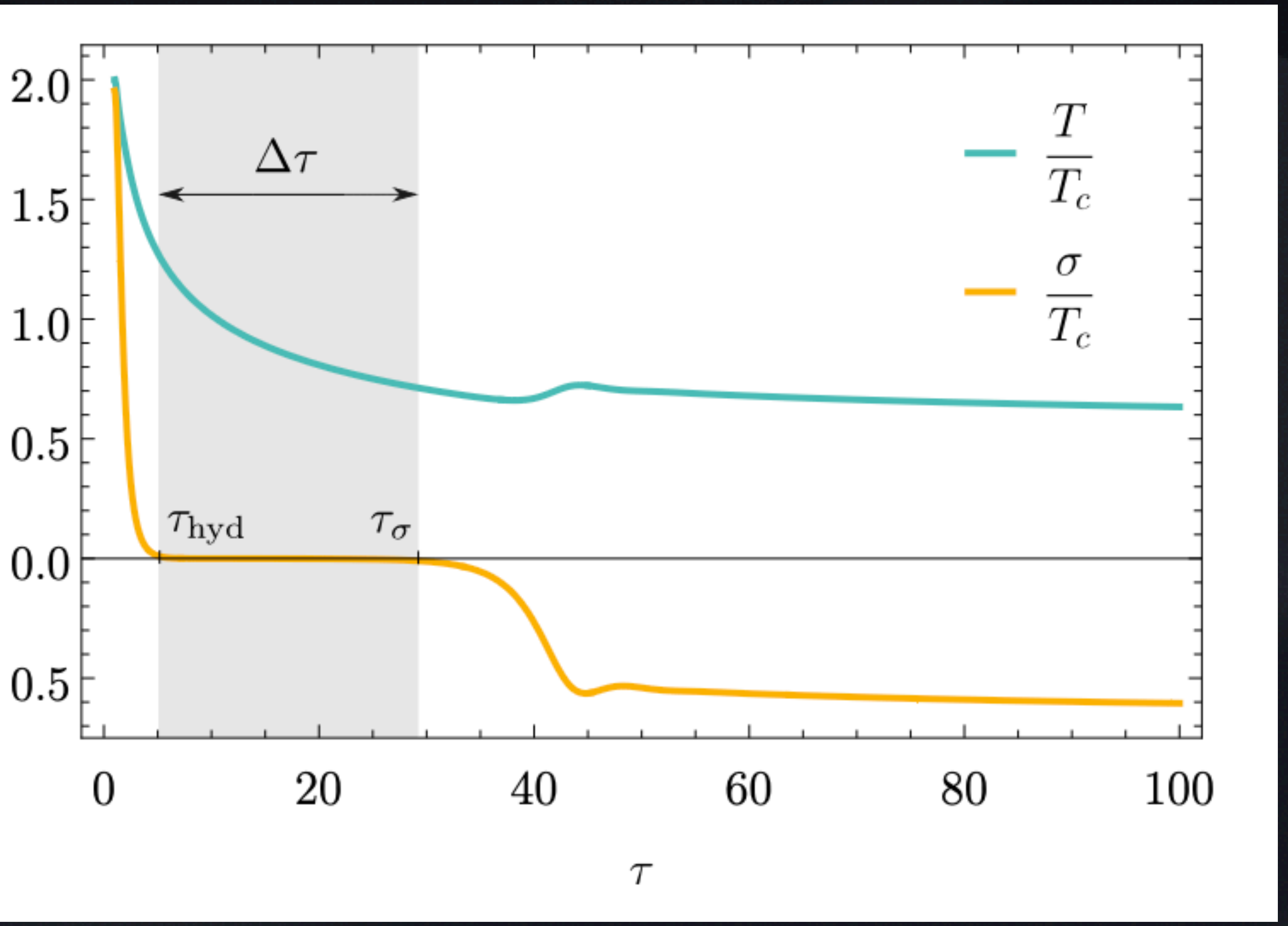

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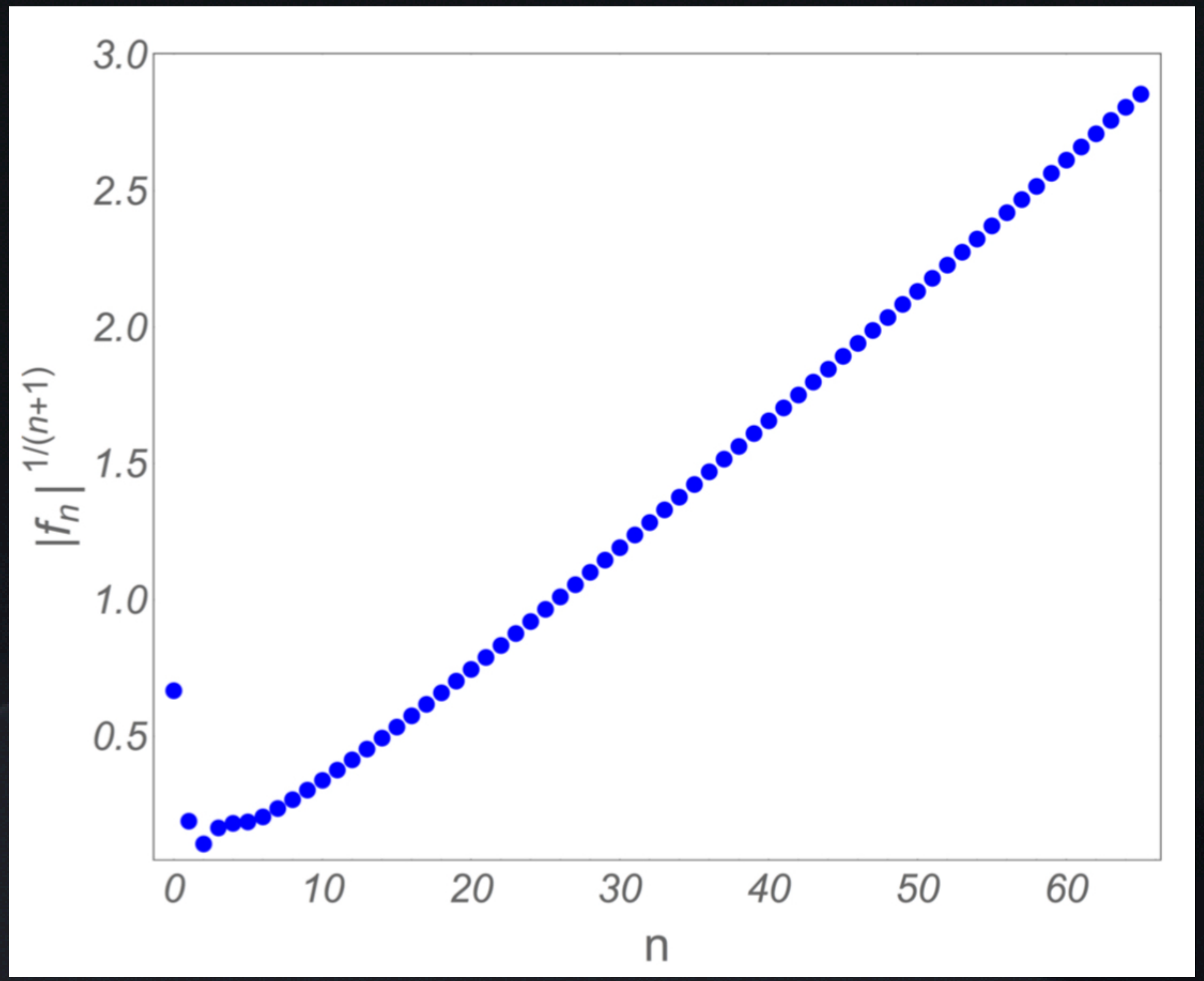

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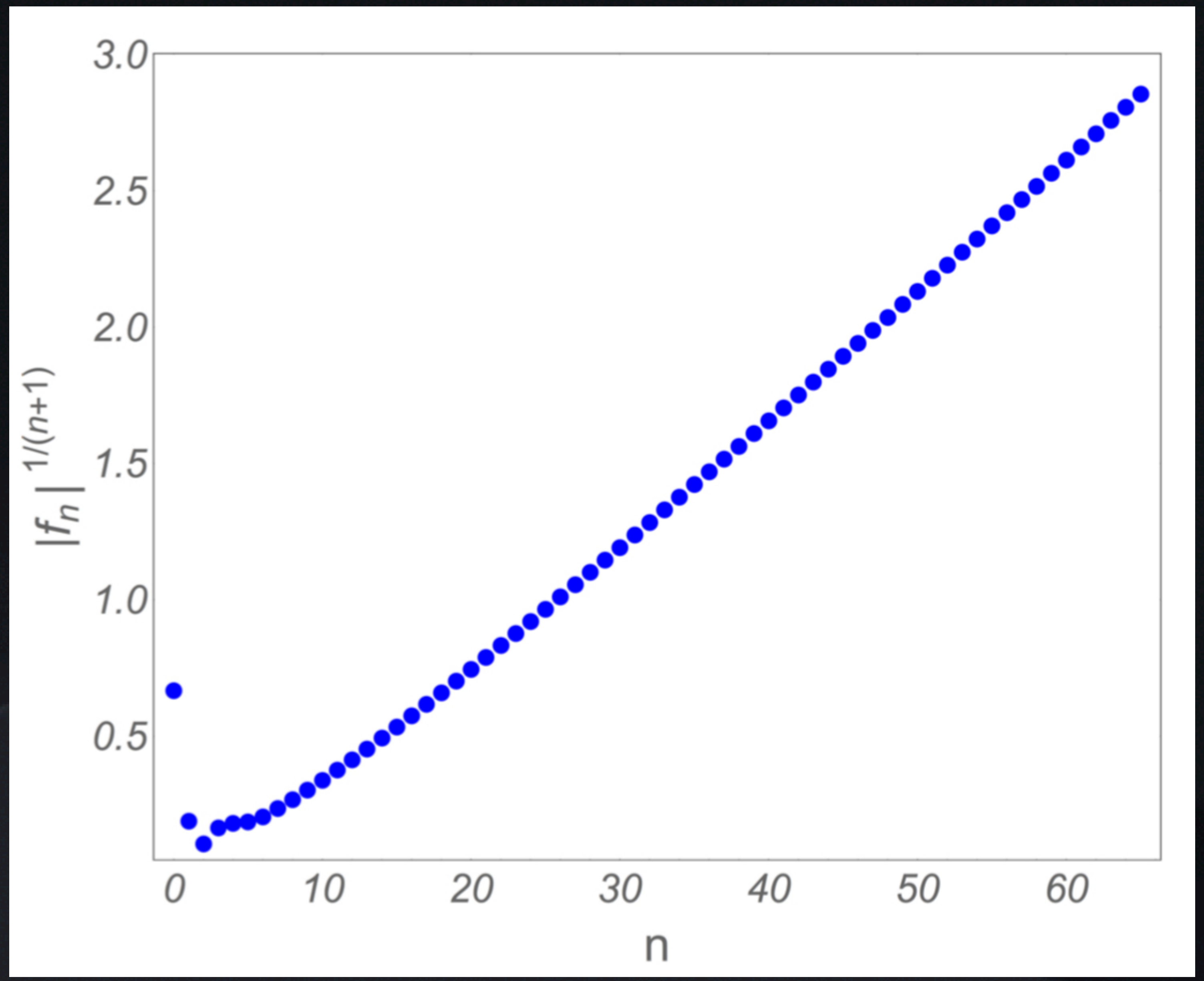

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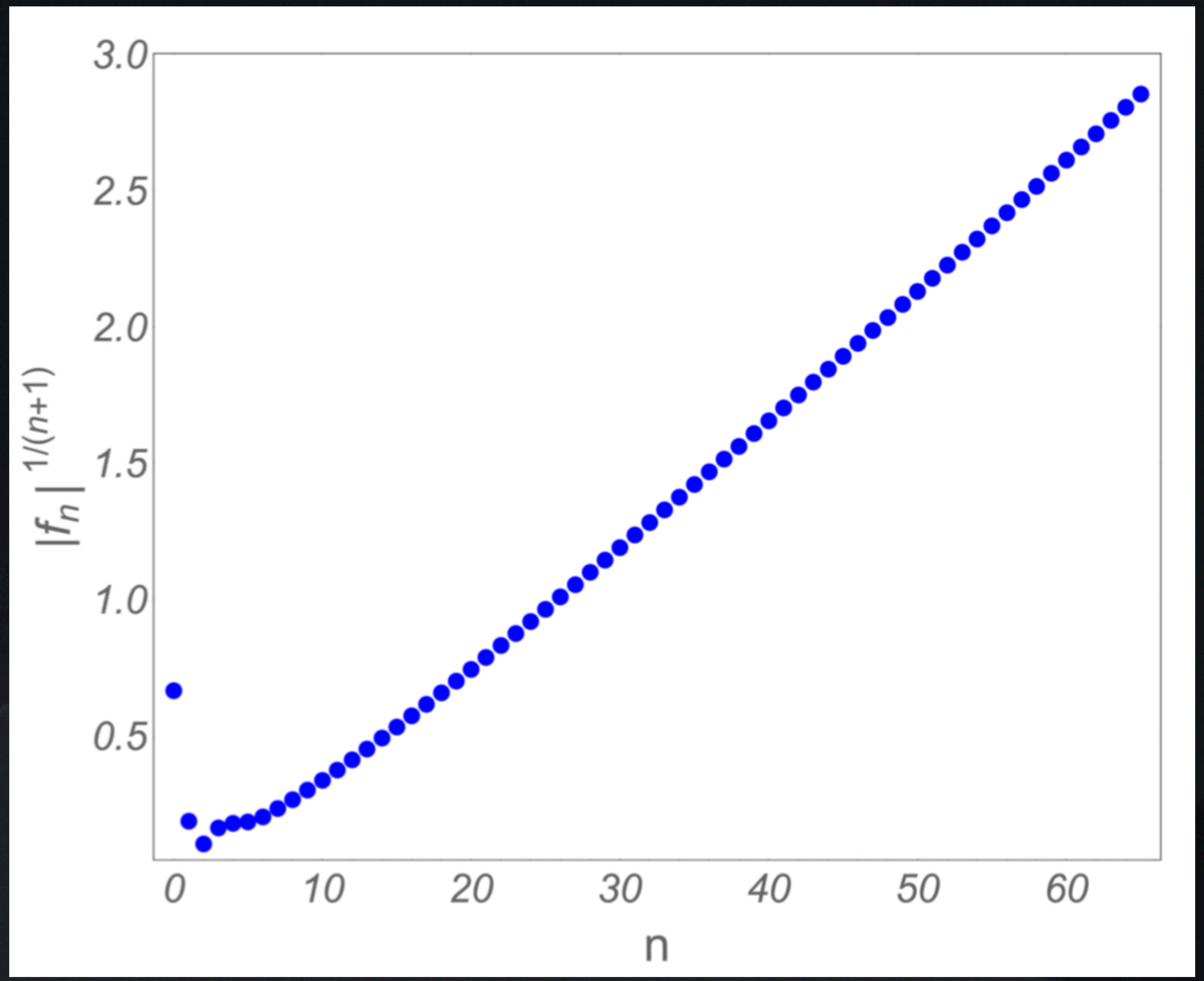

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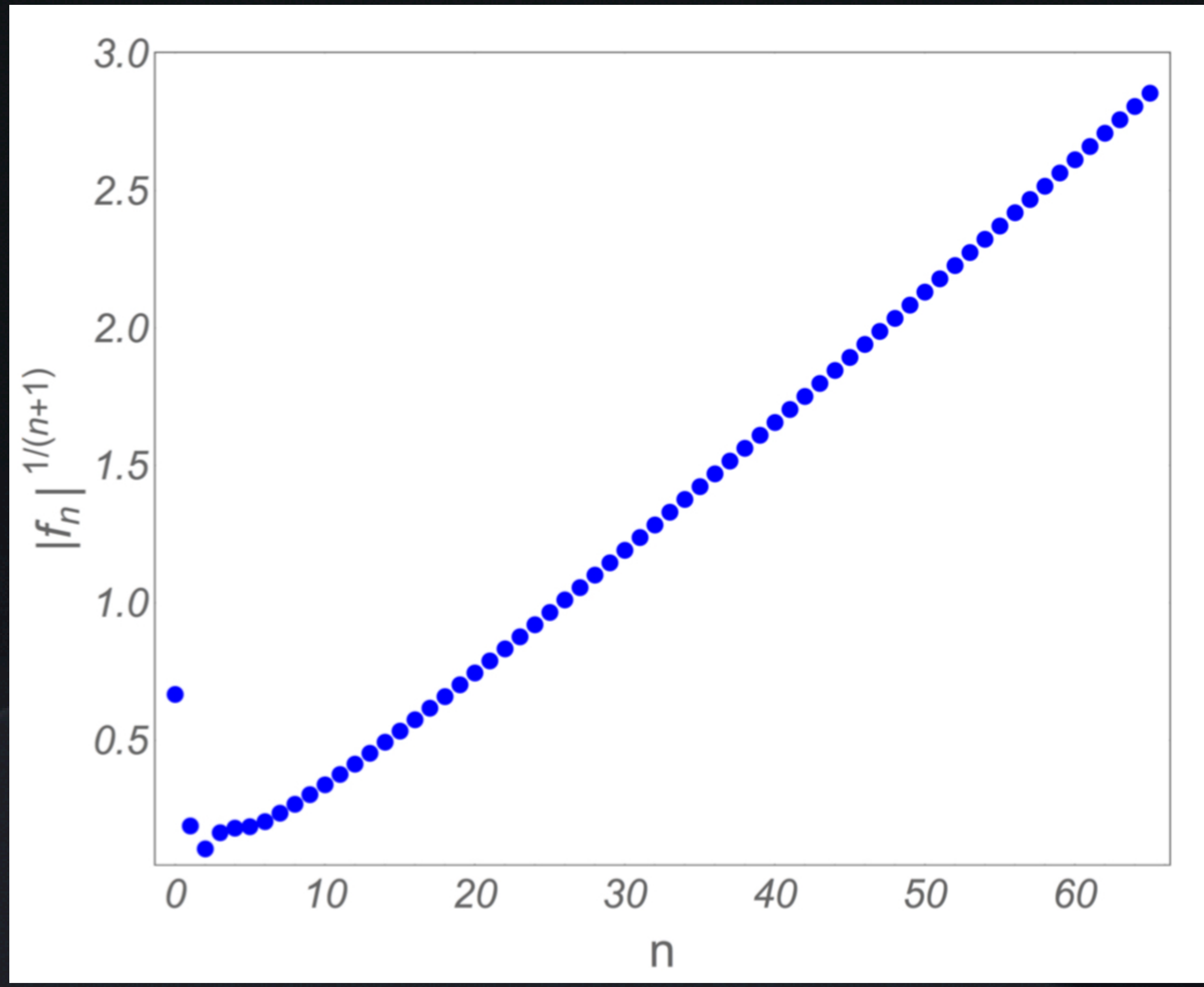
- Hydro as a trans-series

$$f(w) = \sum_{m=0}^{\infty} c^m \Omega(w)^m \sum_{n=0}^{\infty} a_{m,n} w^{-n}$$

$$\Omega \equiv w^{-\gamma} \exp(-w\xi_0)$$



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Attractors are everywhere

Hydro via MIS/BRSSS

Kinetic theory

Holography

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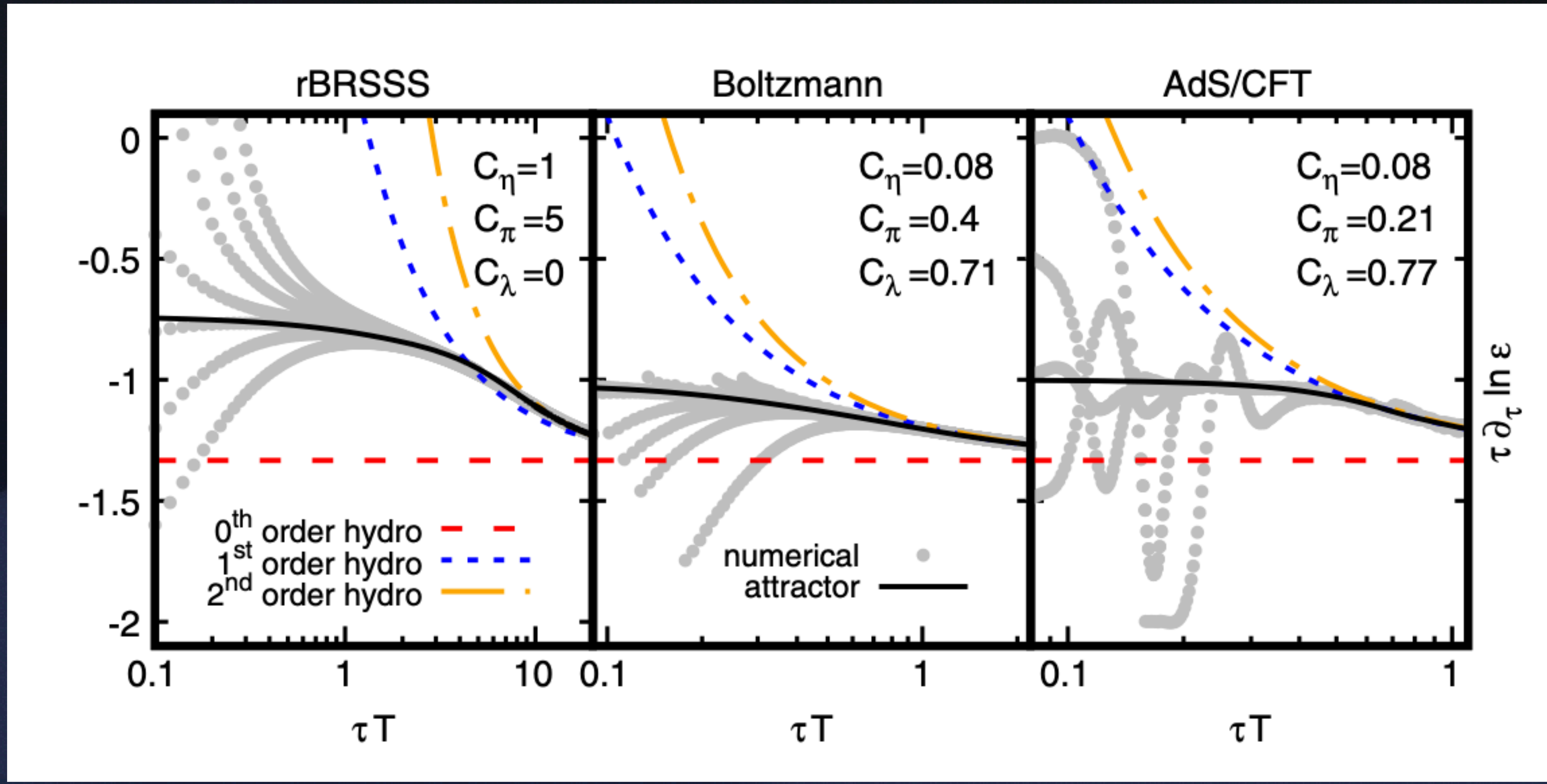
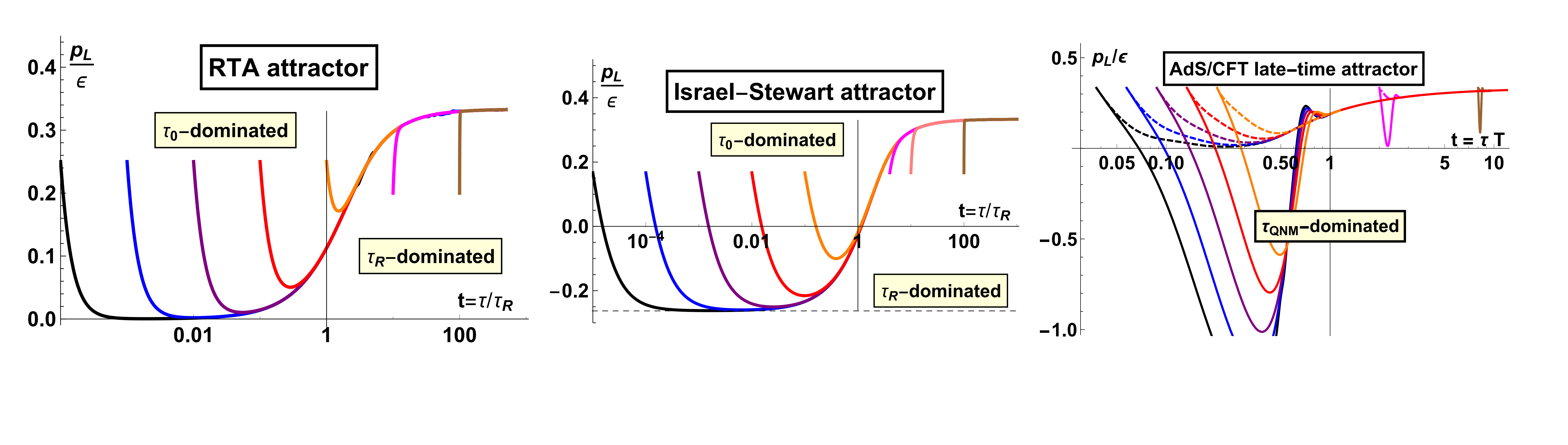
Hydro via MIS/BRSSS

Kinetic theory

Holography





Kurkela, van der Schee, Wiedemann, Wu 2019

Attractors are everywhere

Romatschke 2018

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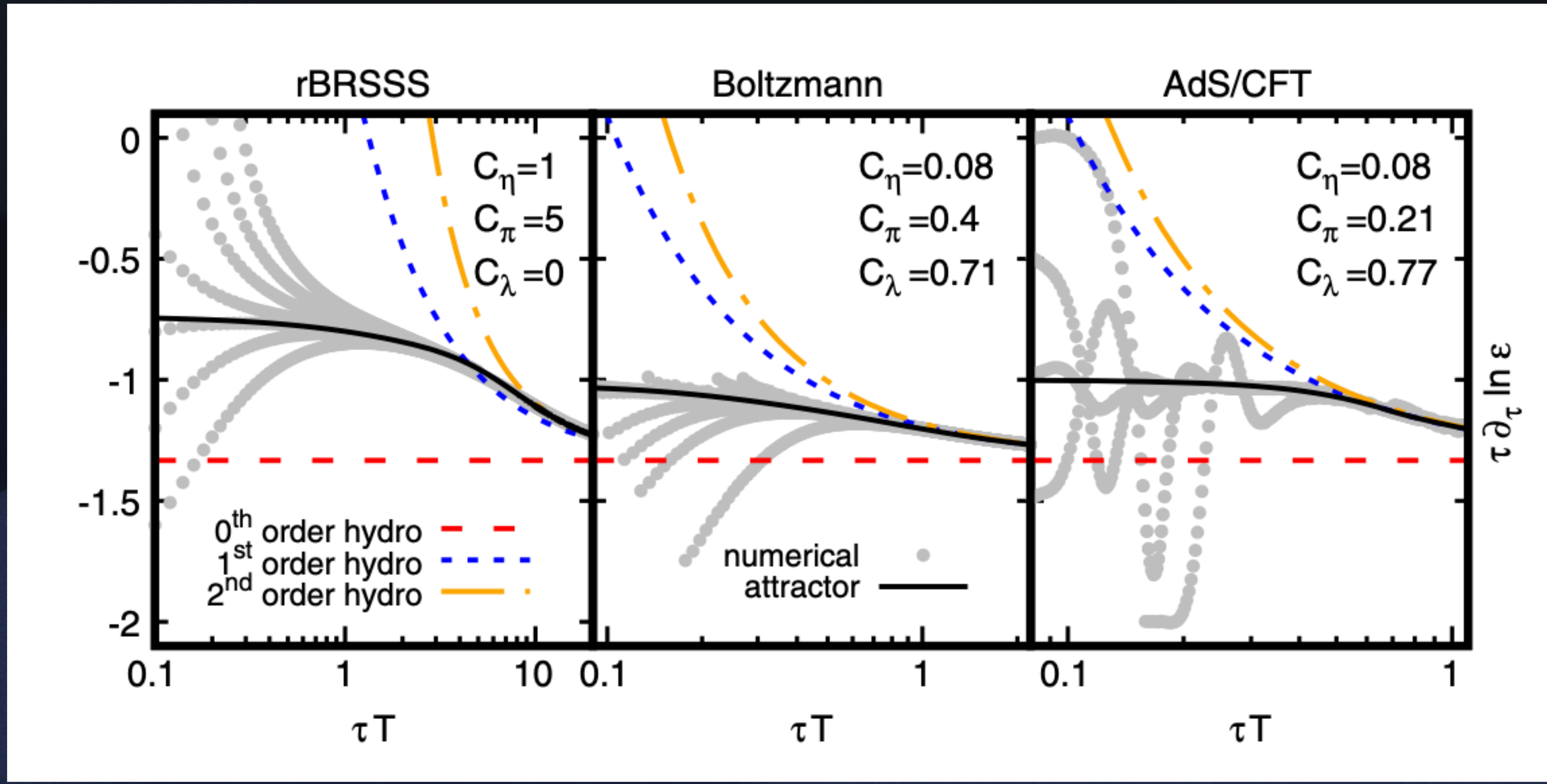

MIS/BRSSS attractors

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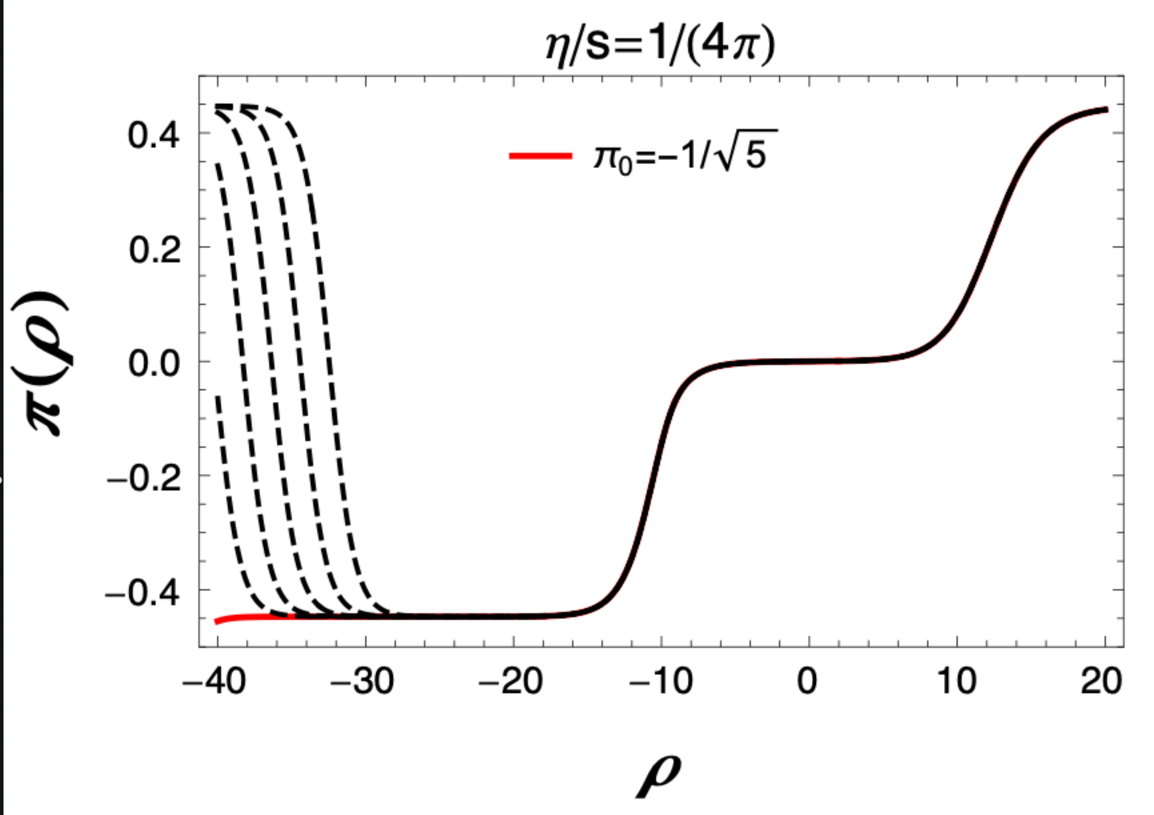

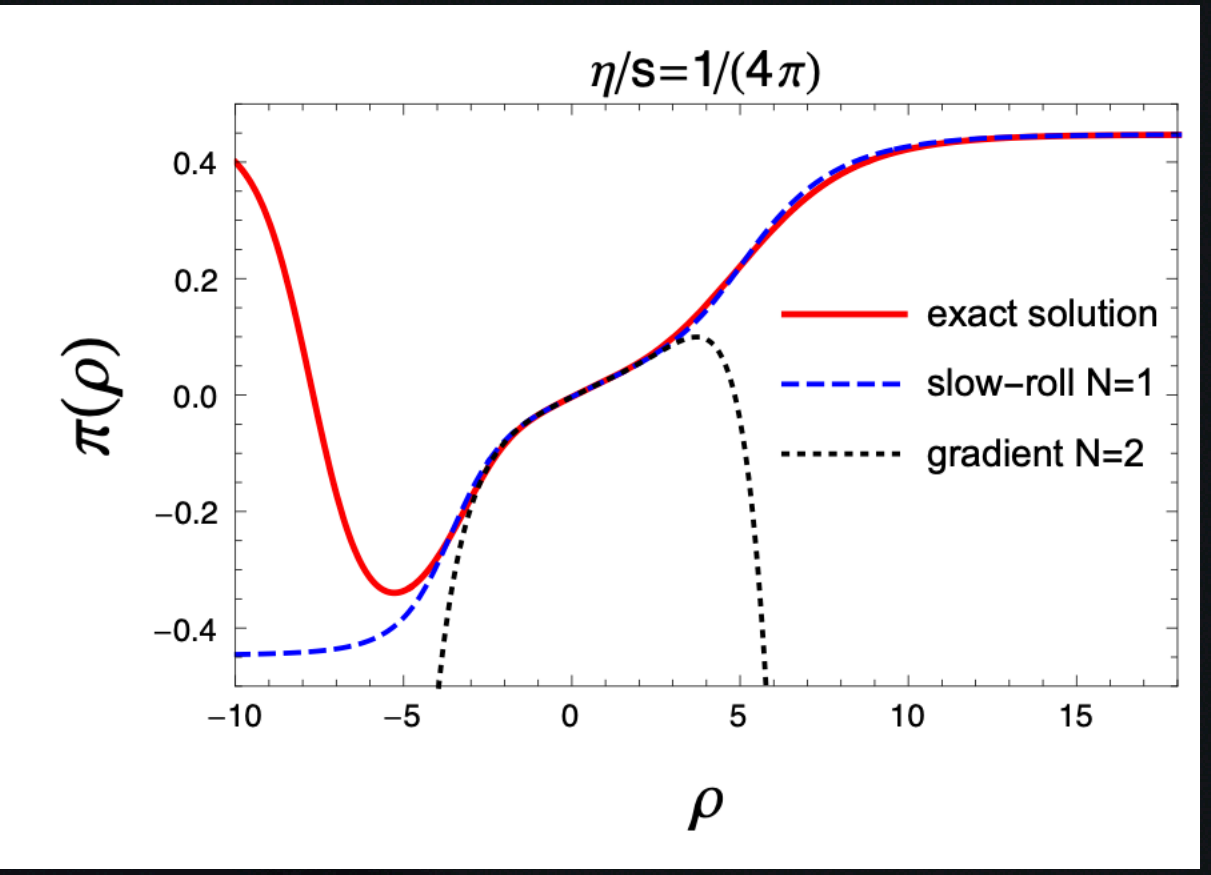

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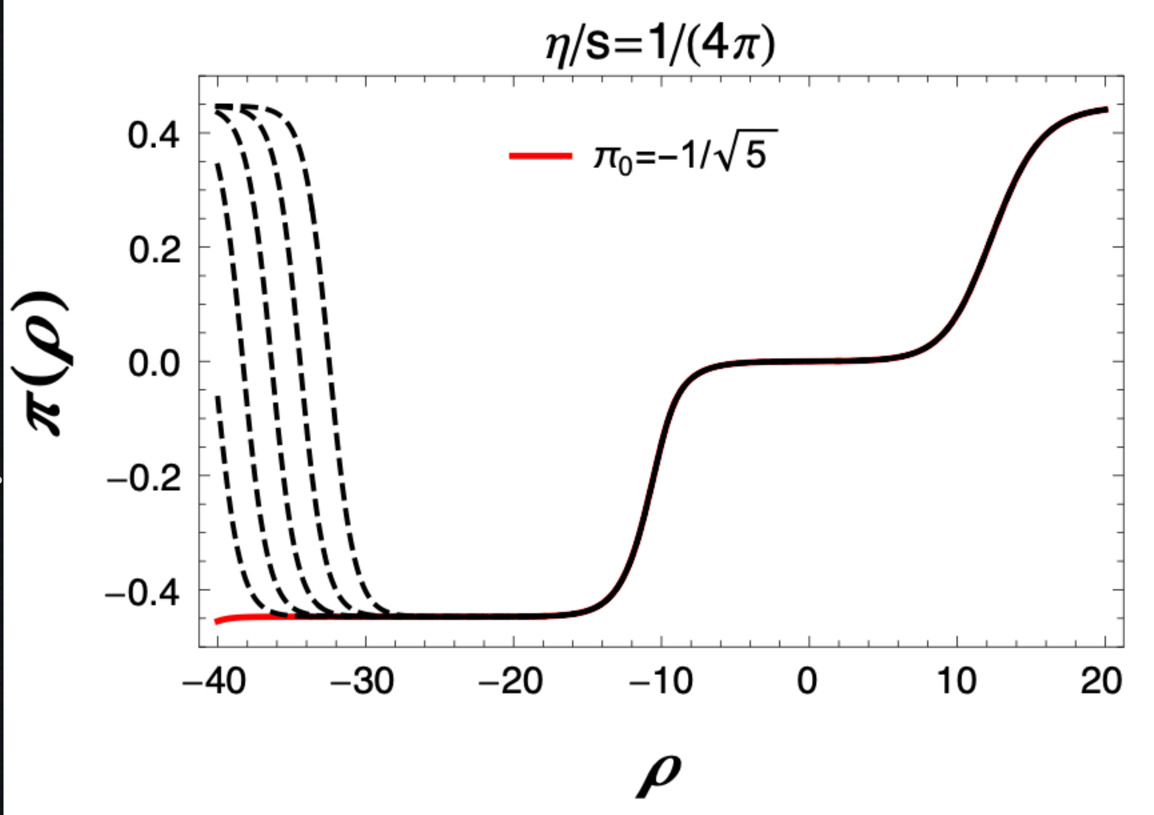

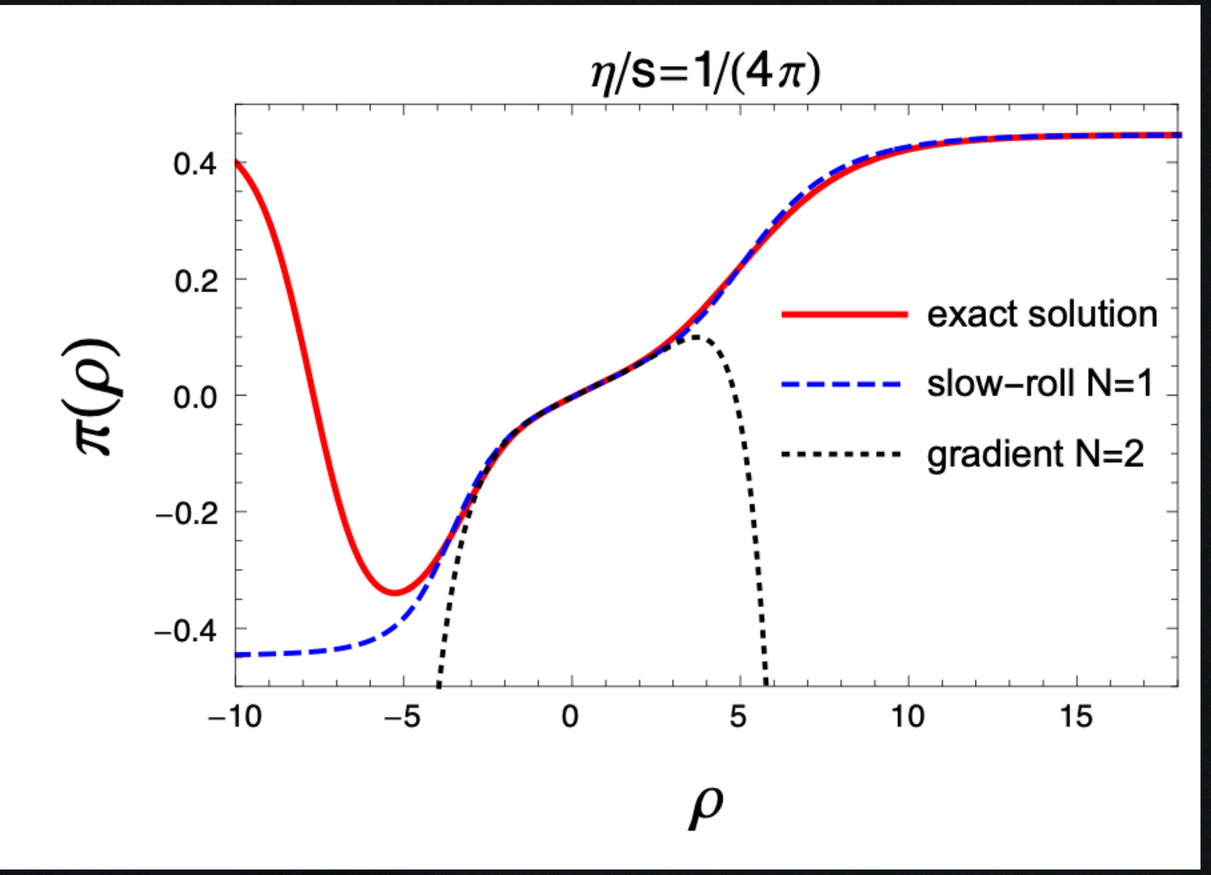

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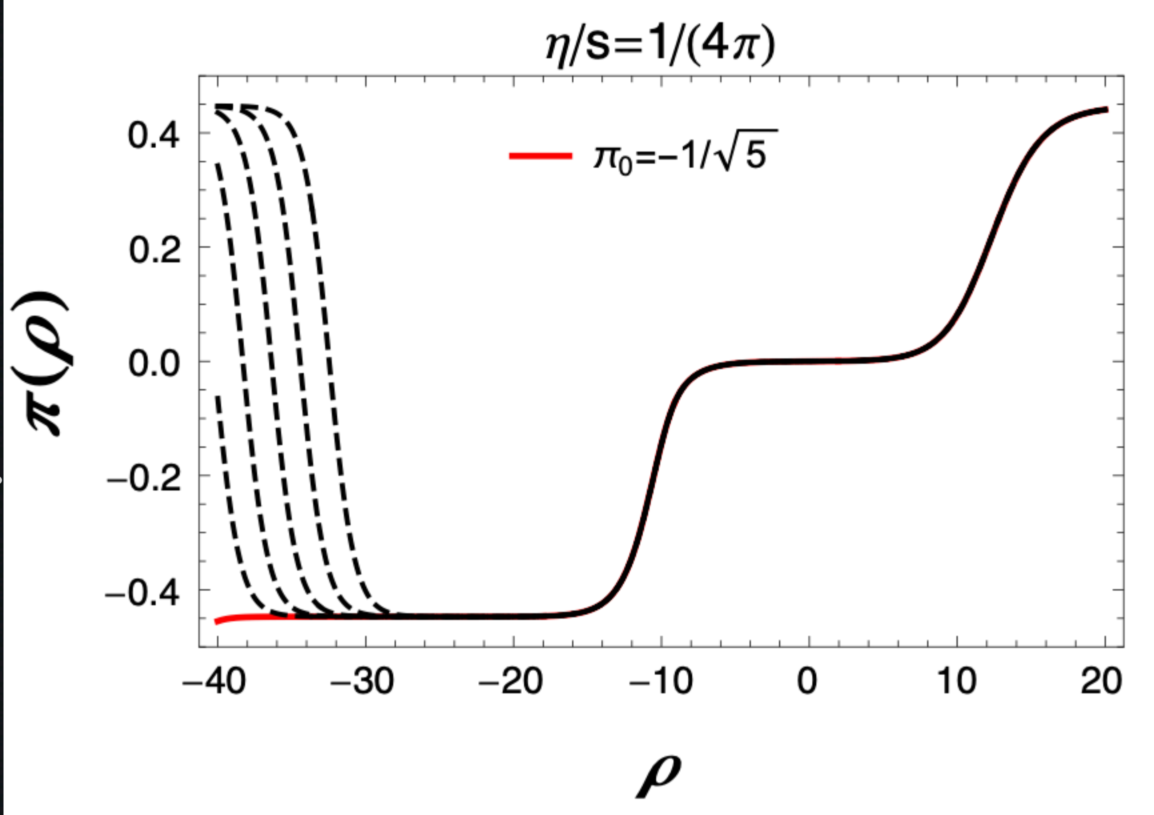
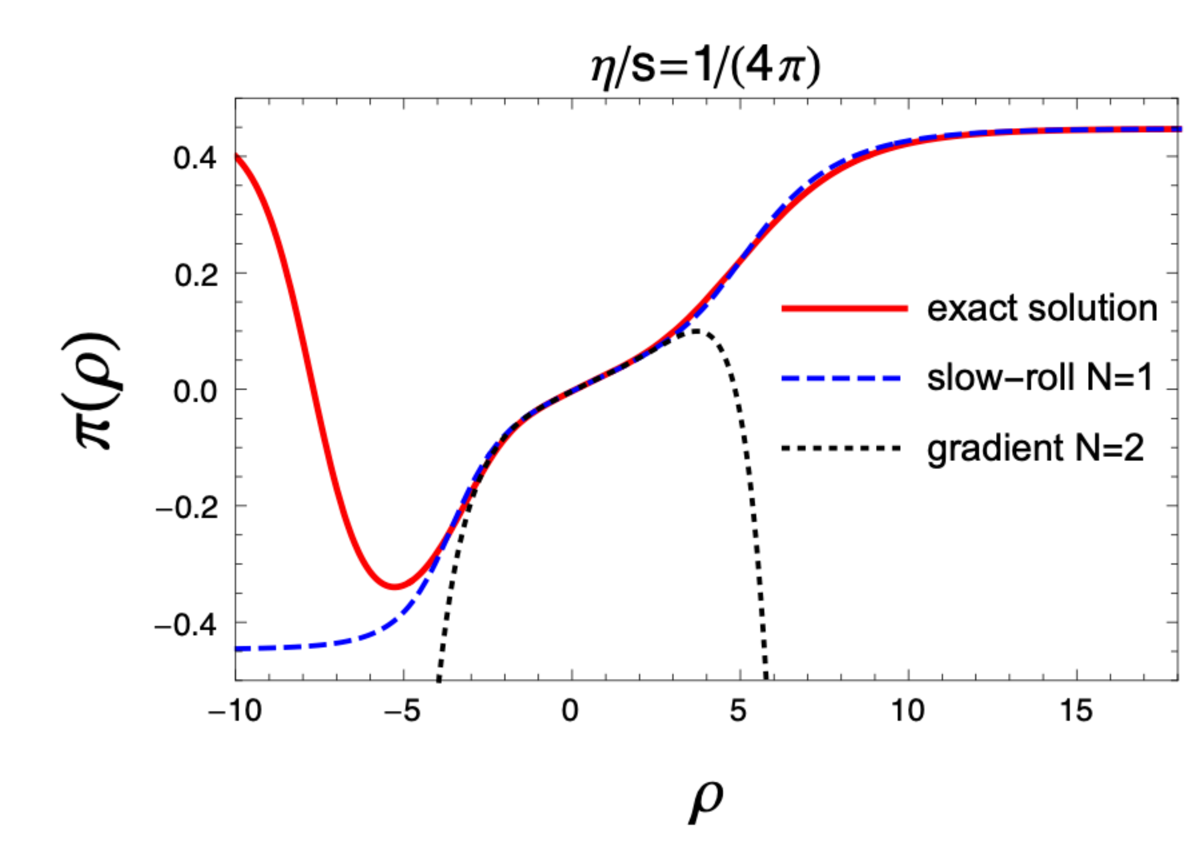
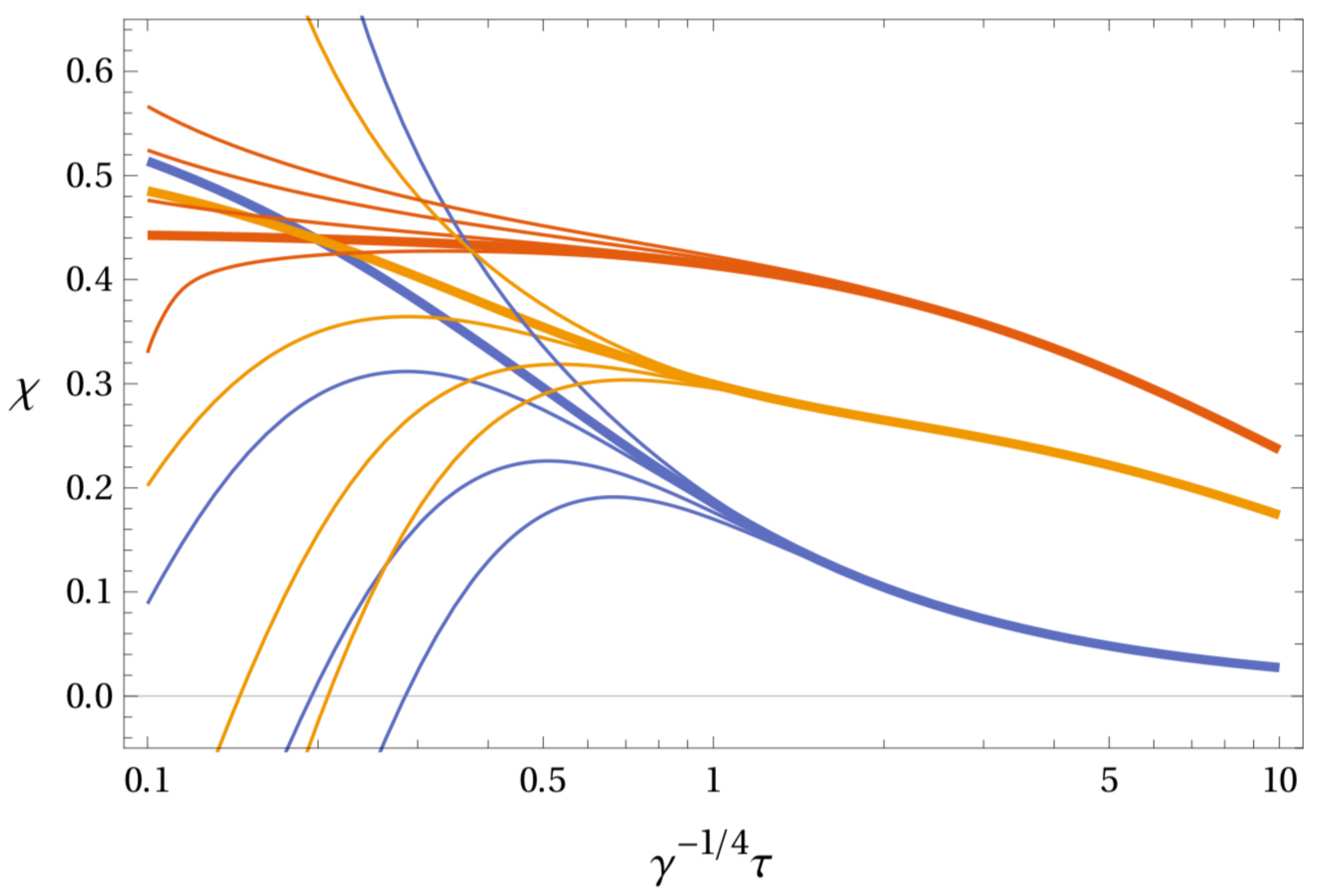




Mitra, Mondkar, Mukhopadhyay, Rebhan, AS 2023, 2024



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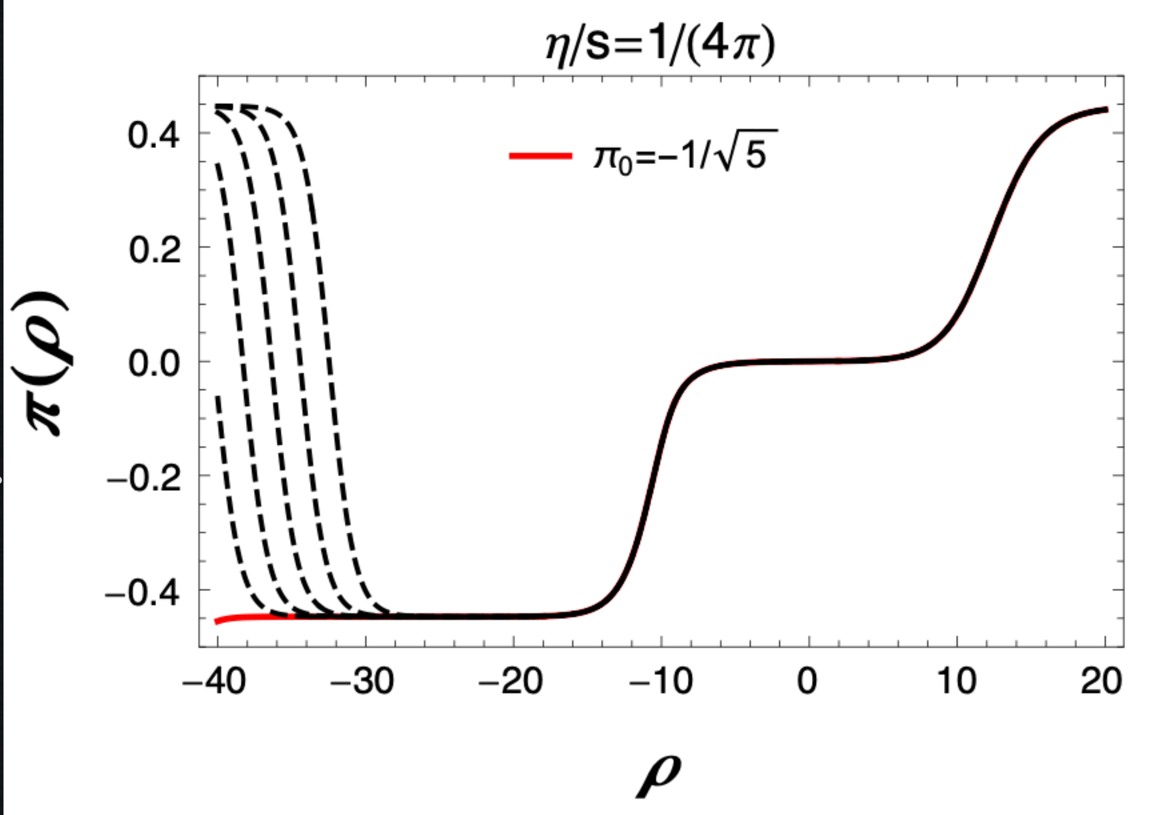

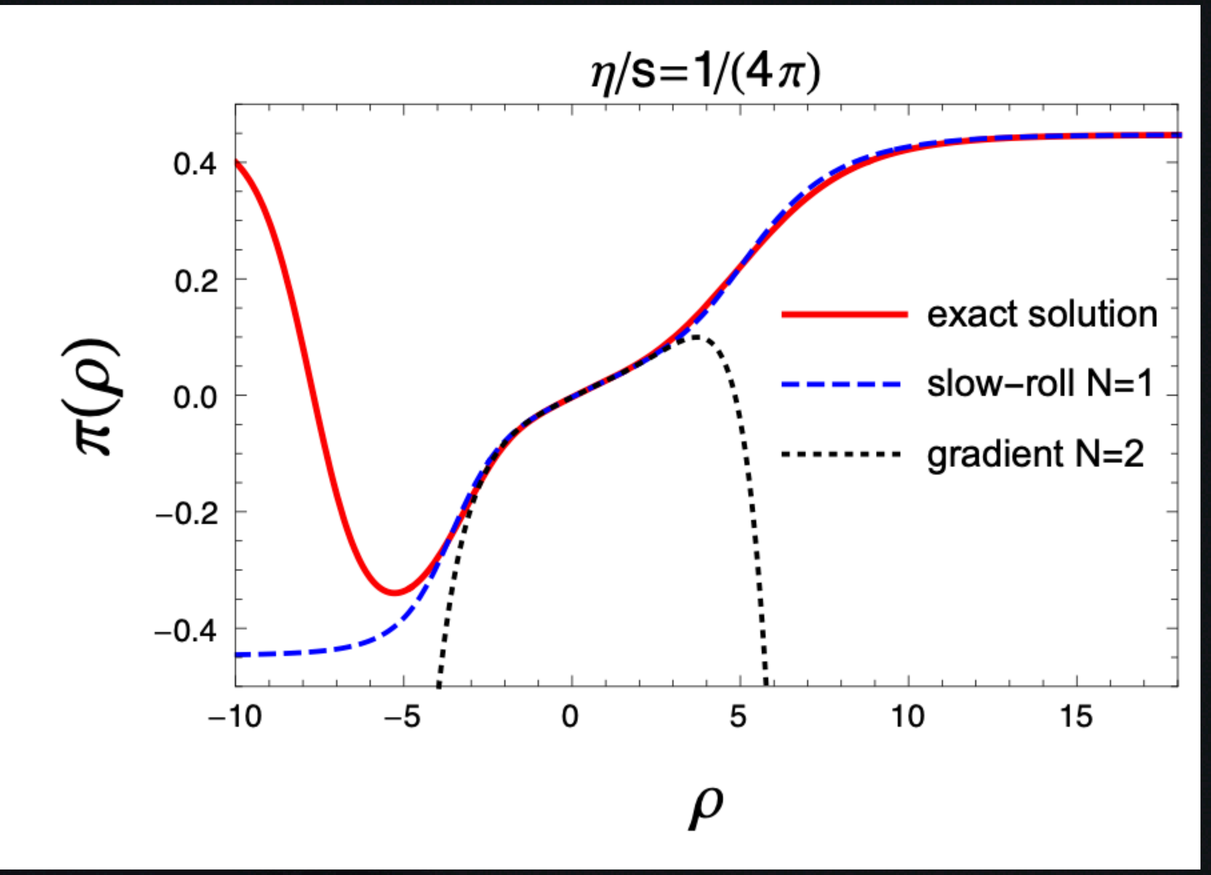

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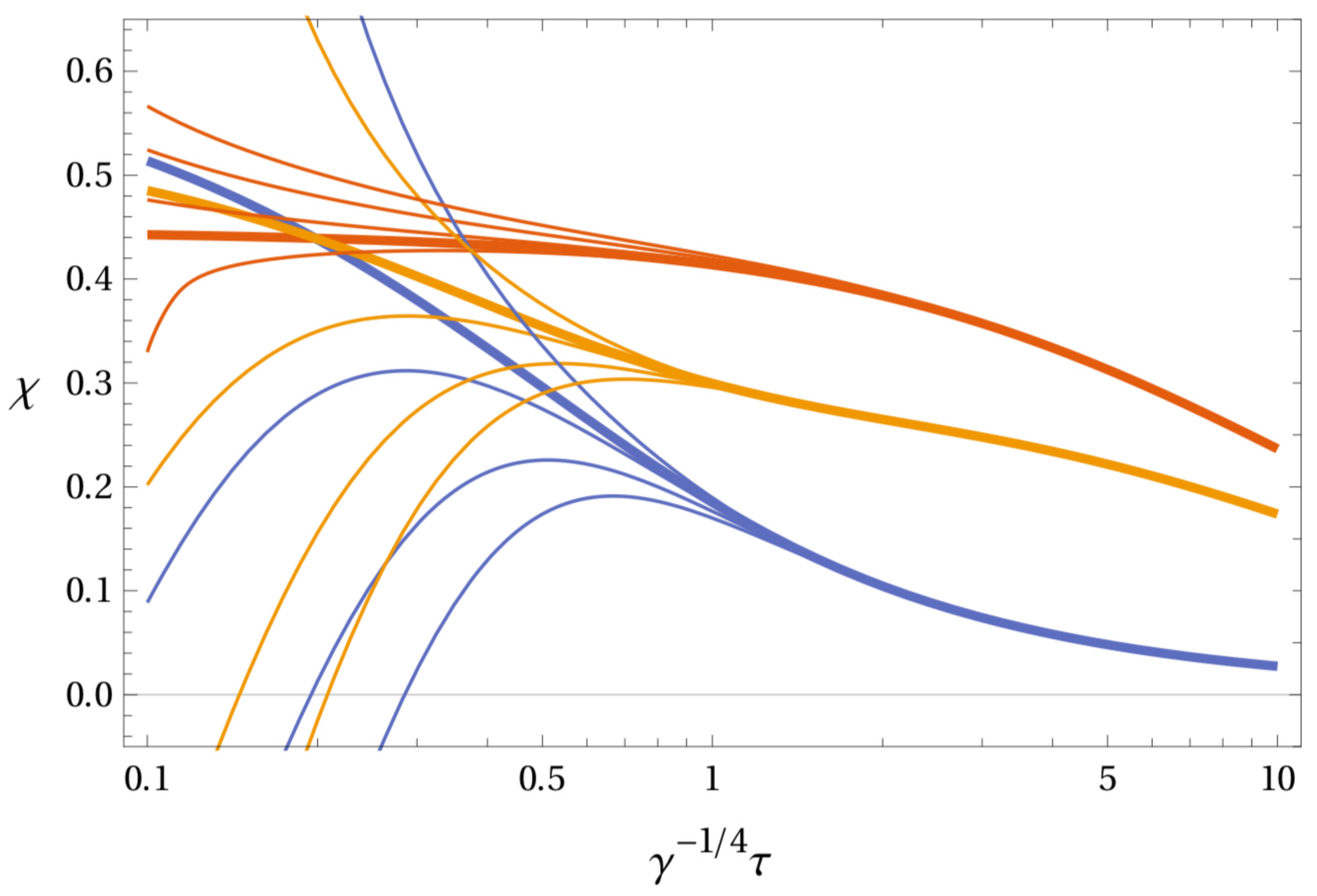

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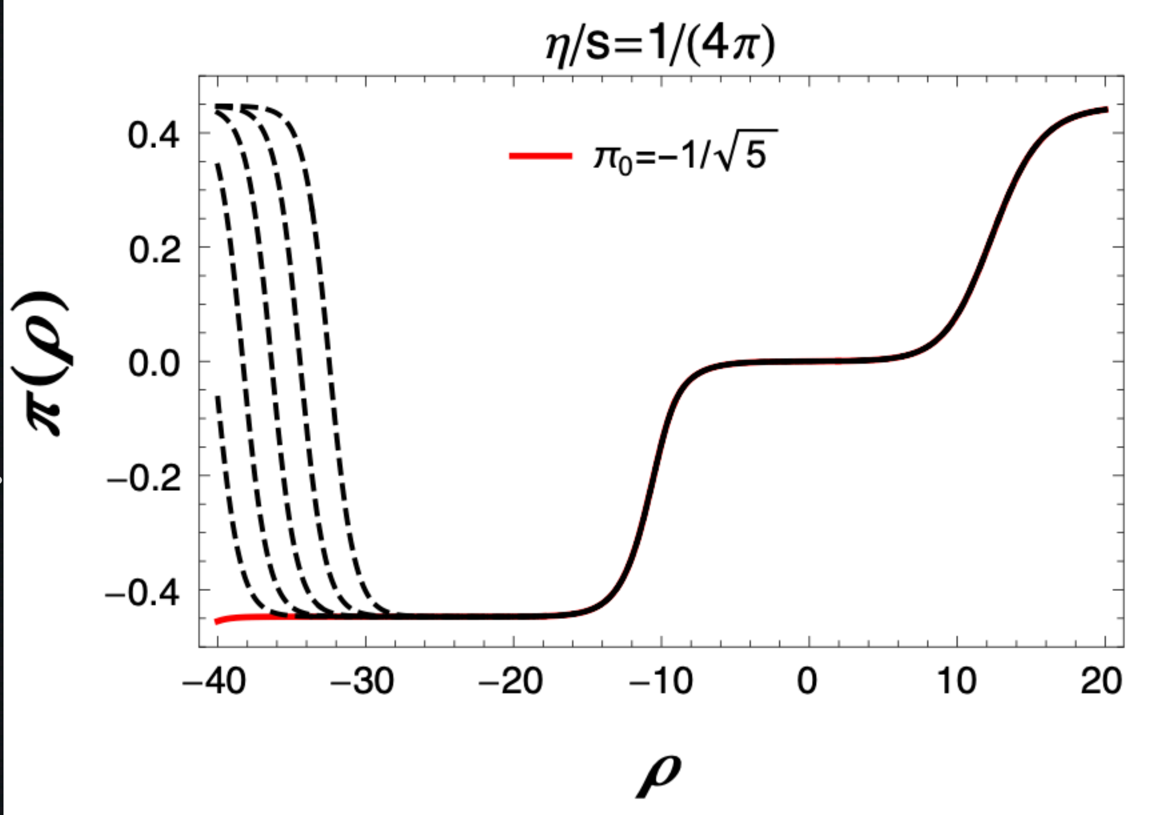

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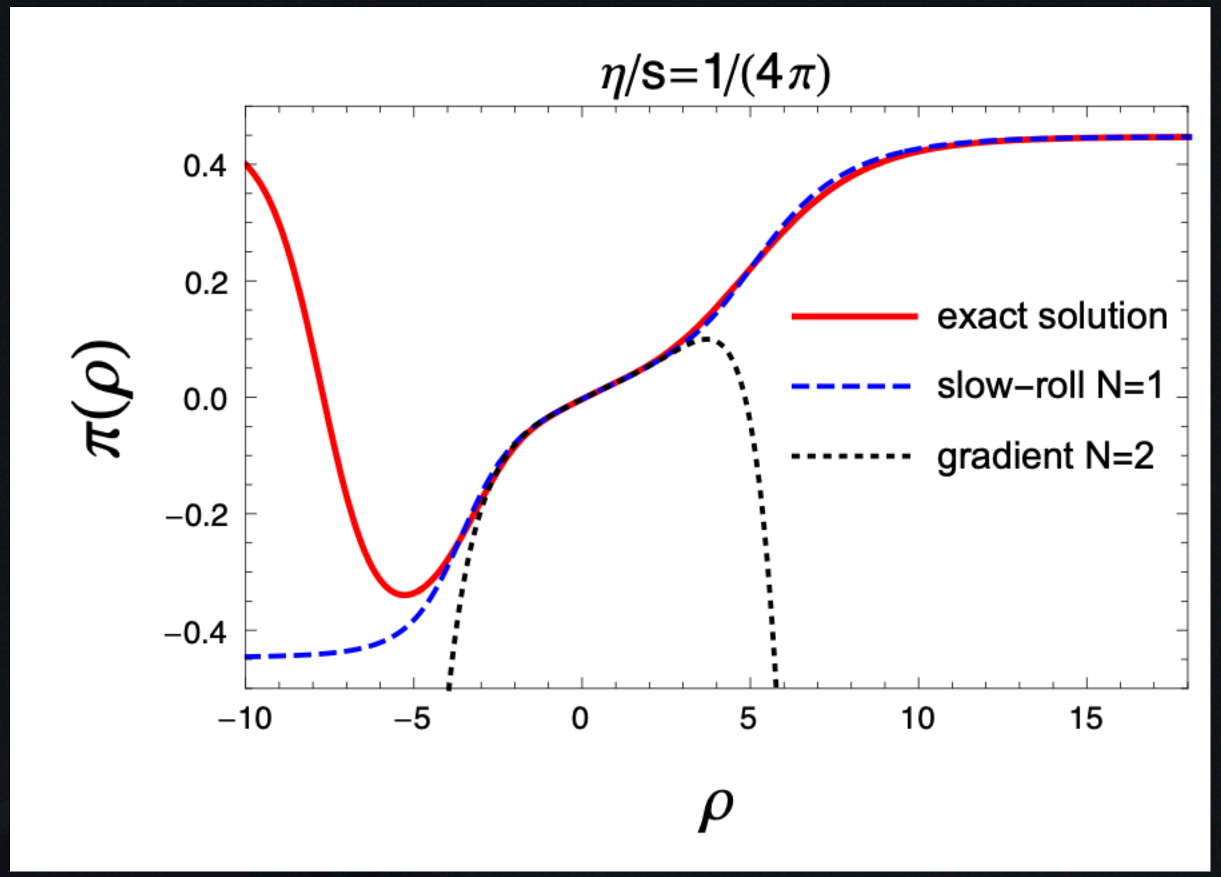

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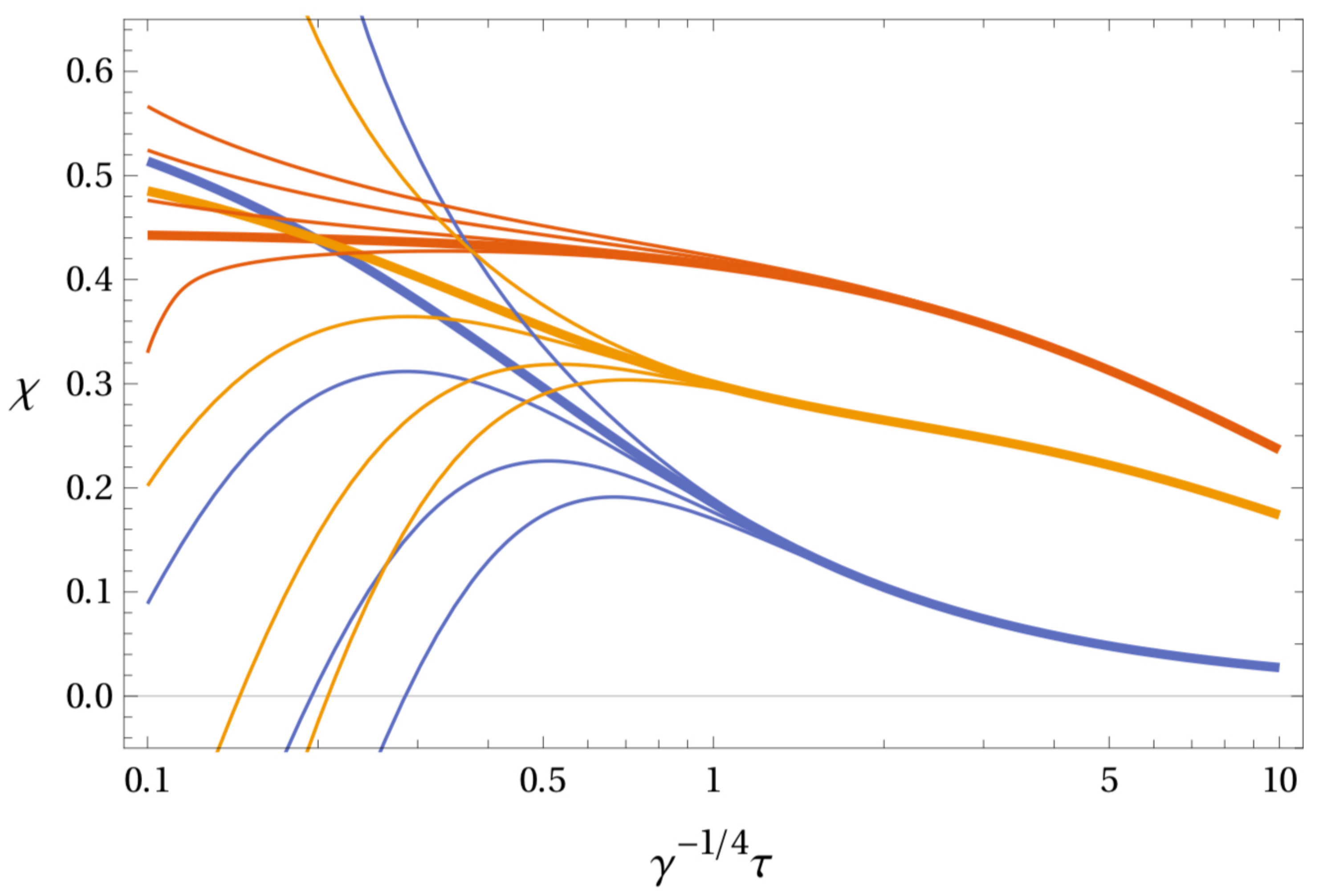

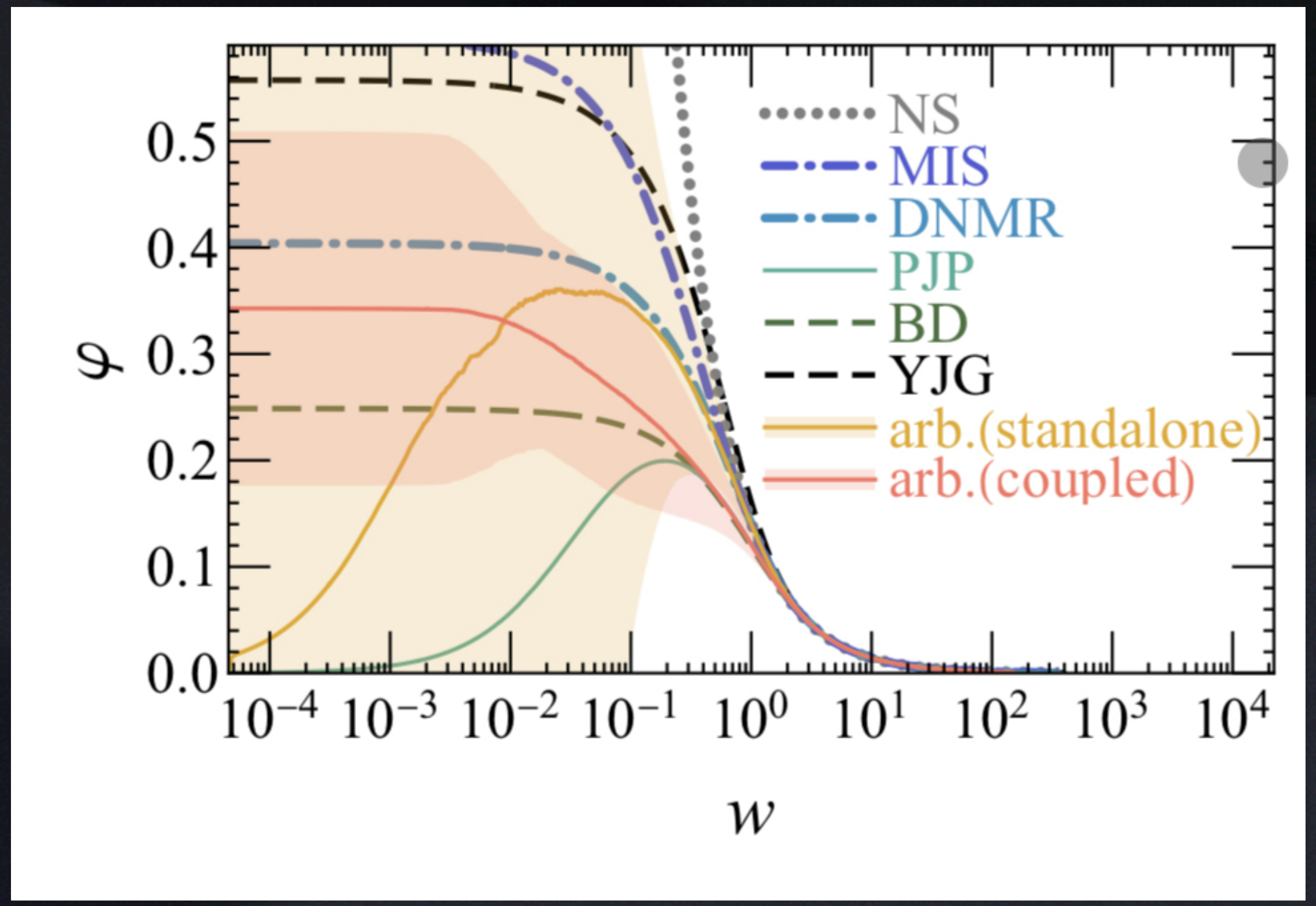

Attractors in kinetic theory

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- Main object is the 1 particle distribution function $f = f(t, x^i, p_i)$

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$$f(\tau, \omega, p_\perp) = D(\tau, \tau_0) f_0(\omega, p_\perp) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_R(\tau')} D(\tau, \tau') f_{\mathrm{eq}}(\tau', \omega, p_\perp)$$

Florkowski, Ryblewski, Strickland 2013, Denicol, Heinz, Martinez, Noronha, Strickland 2014

- Damping function, $D(\tau_2, \tau_1) = e^{-\int_{\tau_1}^{\tau_2} d\tau'' \tau_R^{-1}}$ $\omega = t p_L - z E$

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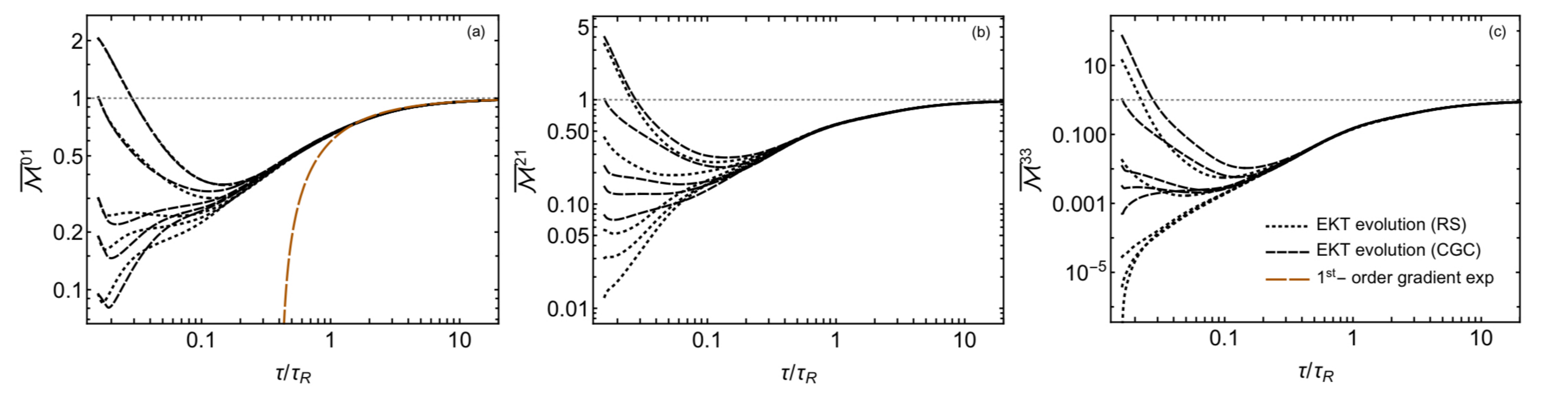
Almaalol, Kurkela, Strickland 2020, Alawi, Strickland 2023 ...



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Almaalol, Kurkela, Strickland 2020

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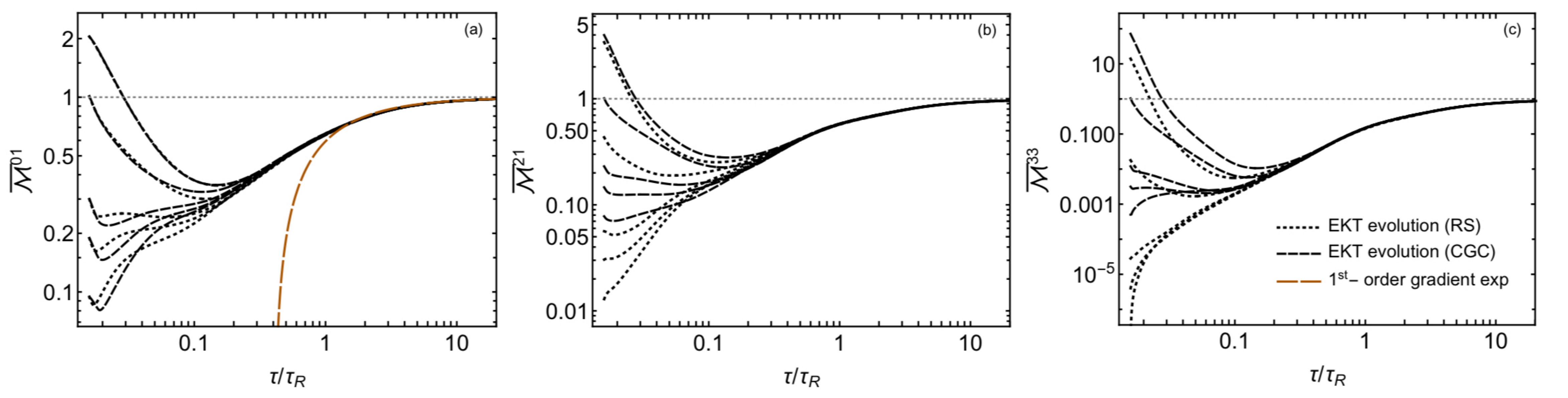

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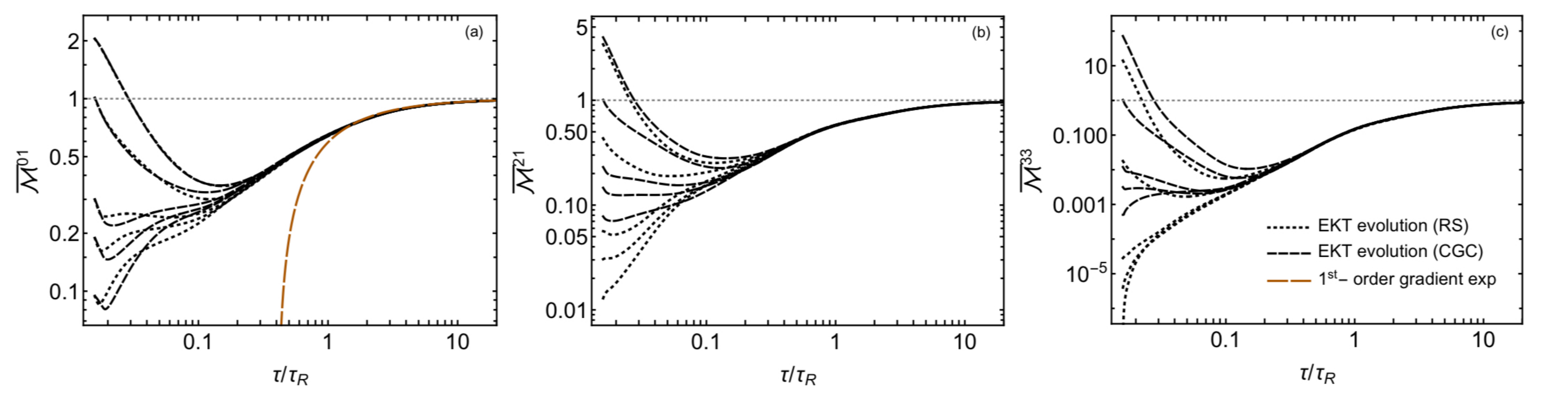

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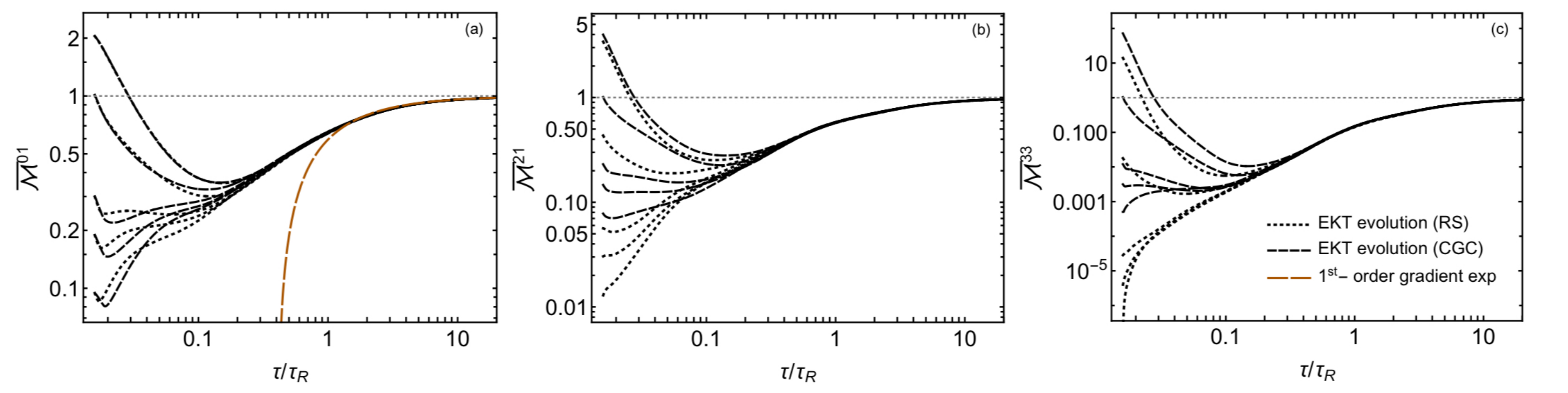

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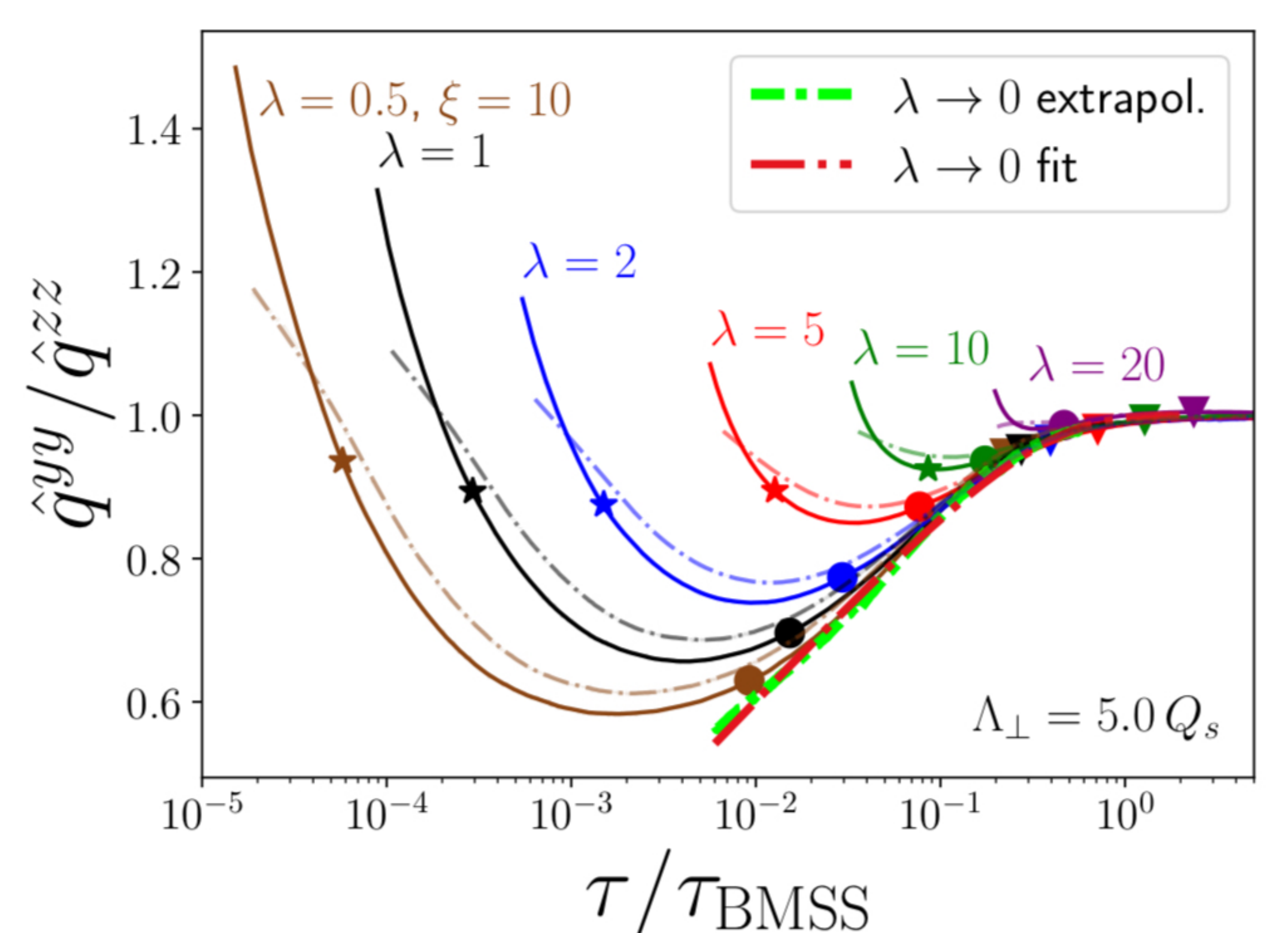
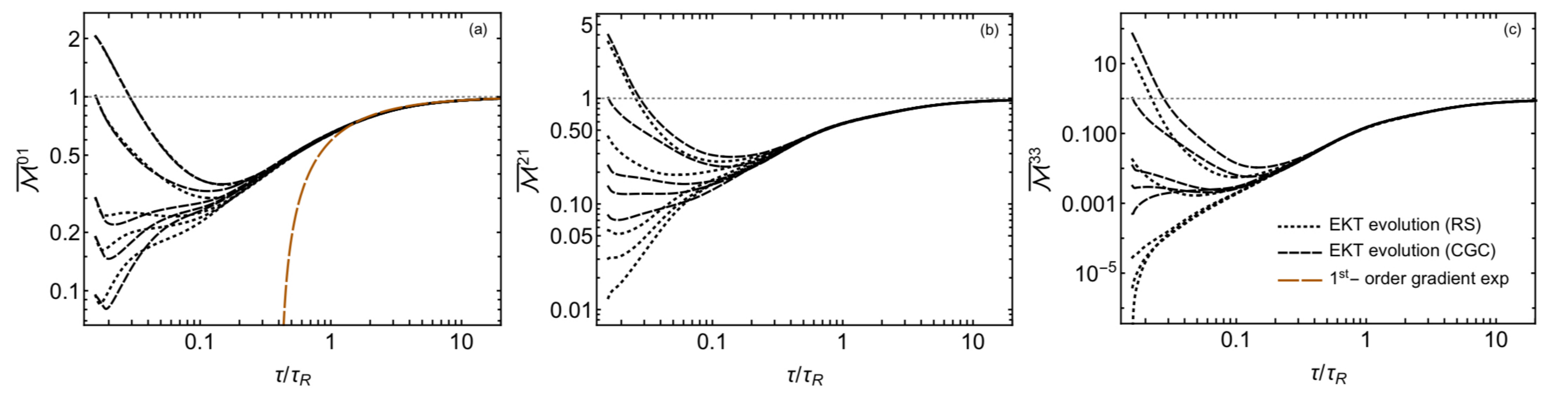
- Limiting attractors F. Lindenbauer Tue. 17:00



Boguslavski, Kurkela, Lappi, Lindenbauer, Perron 2023



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Pre-equilibrium Bjorken attractor

Aniceto, Noronha, Spalinski 2024

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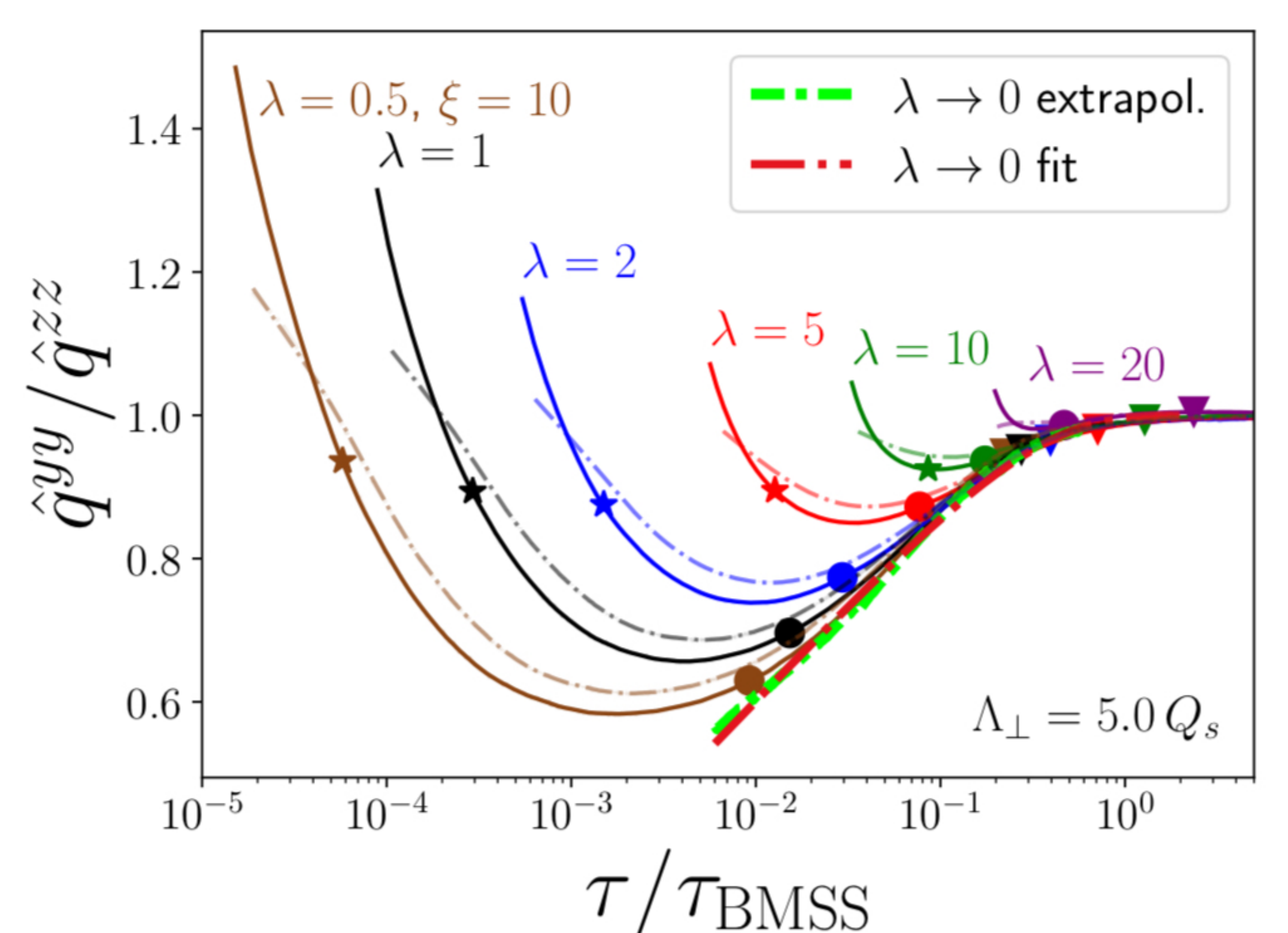

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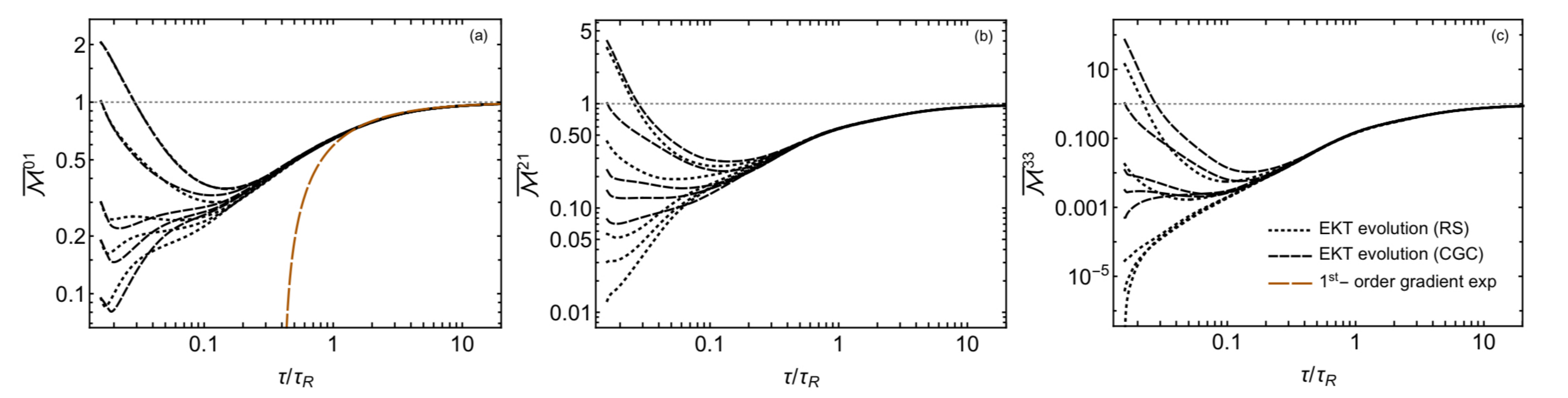

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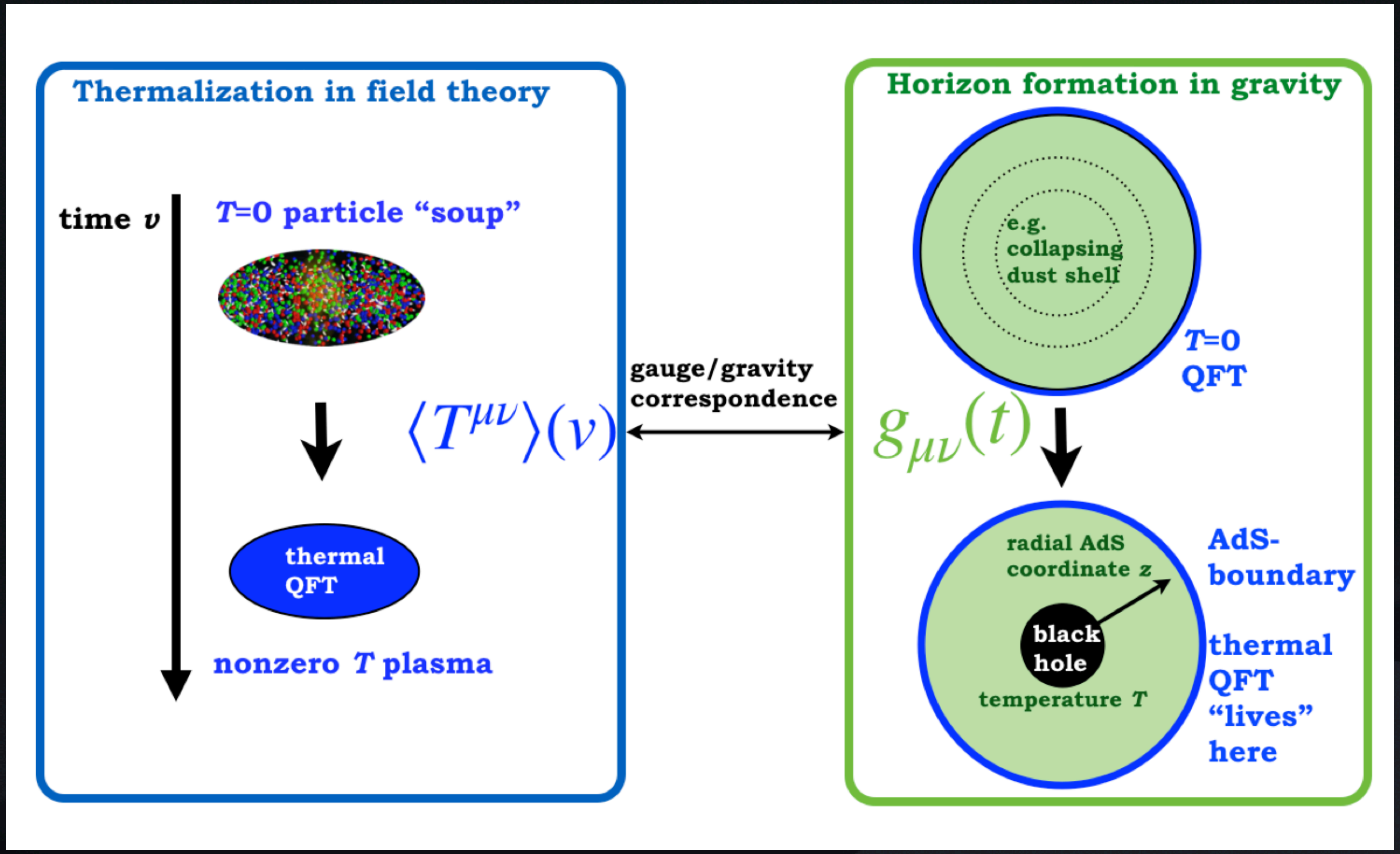

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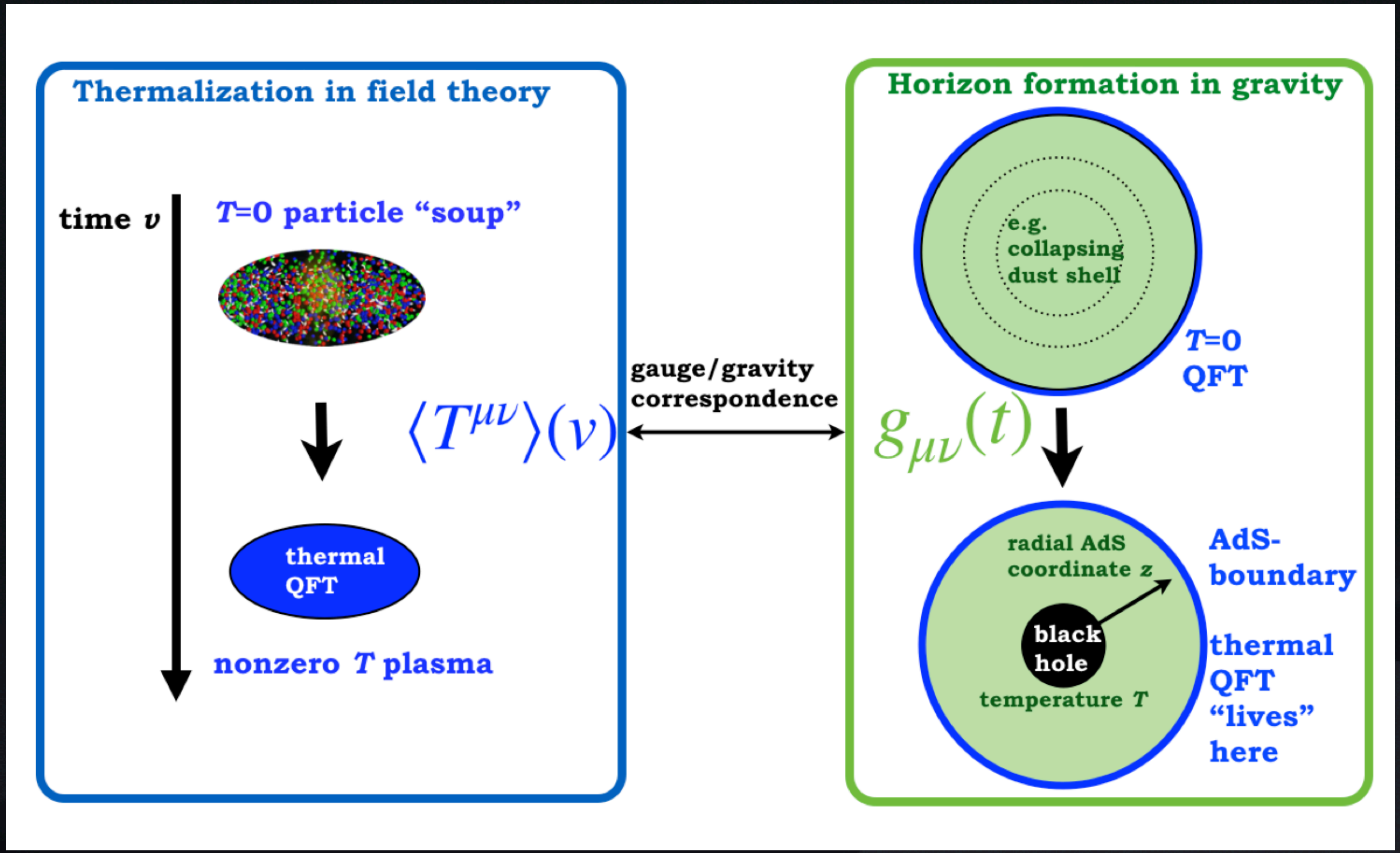

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- Same physical theory

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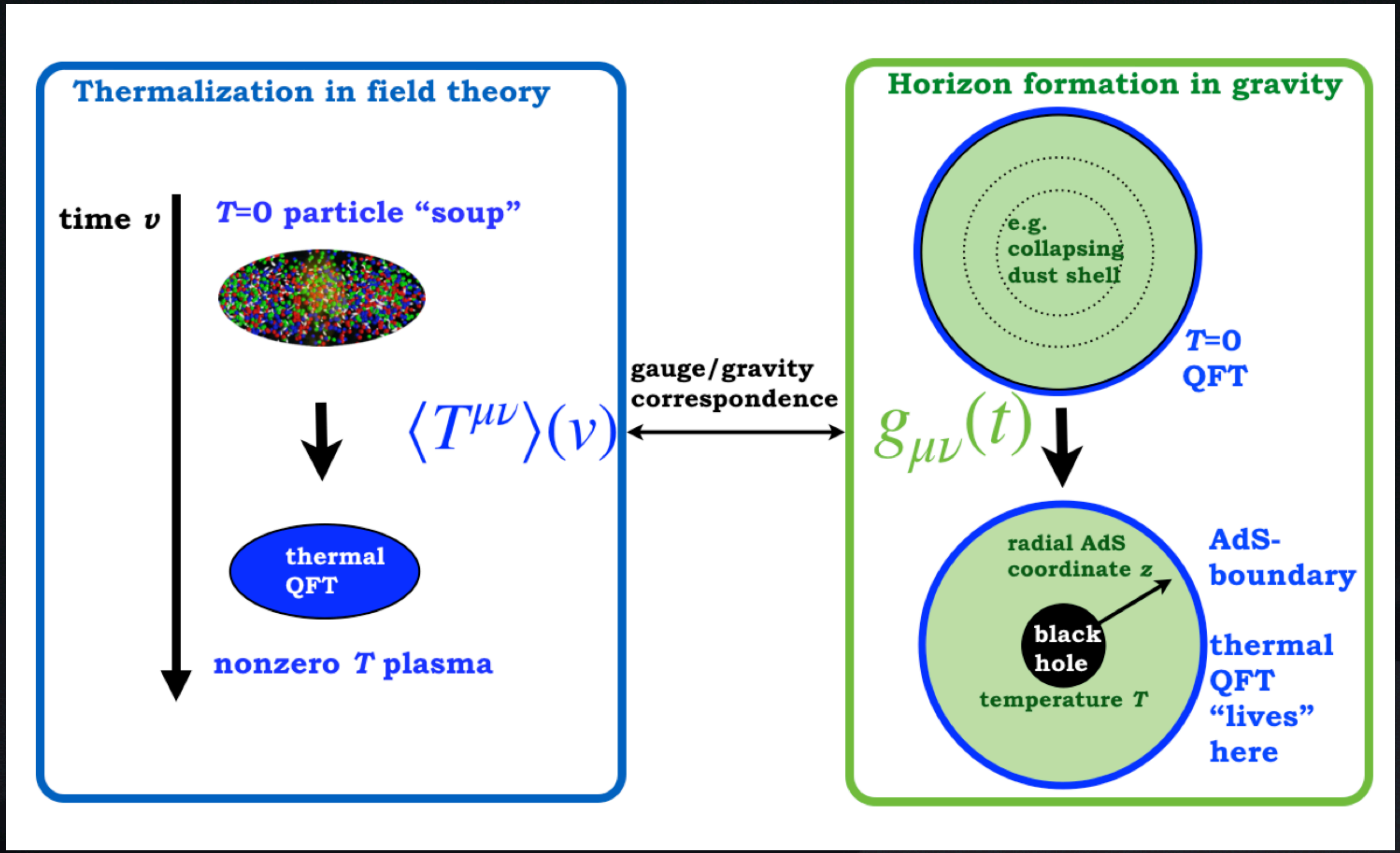

Kaminski, Cartwright, Knipfer, Wondrak, Schenke, Bleicher 2023

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- Two equivalent formulation: strong ↔ weak correspondence

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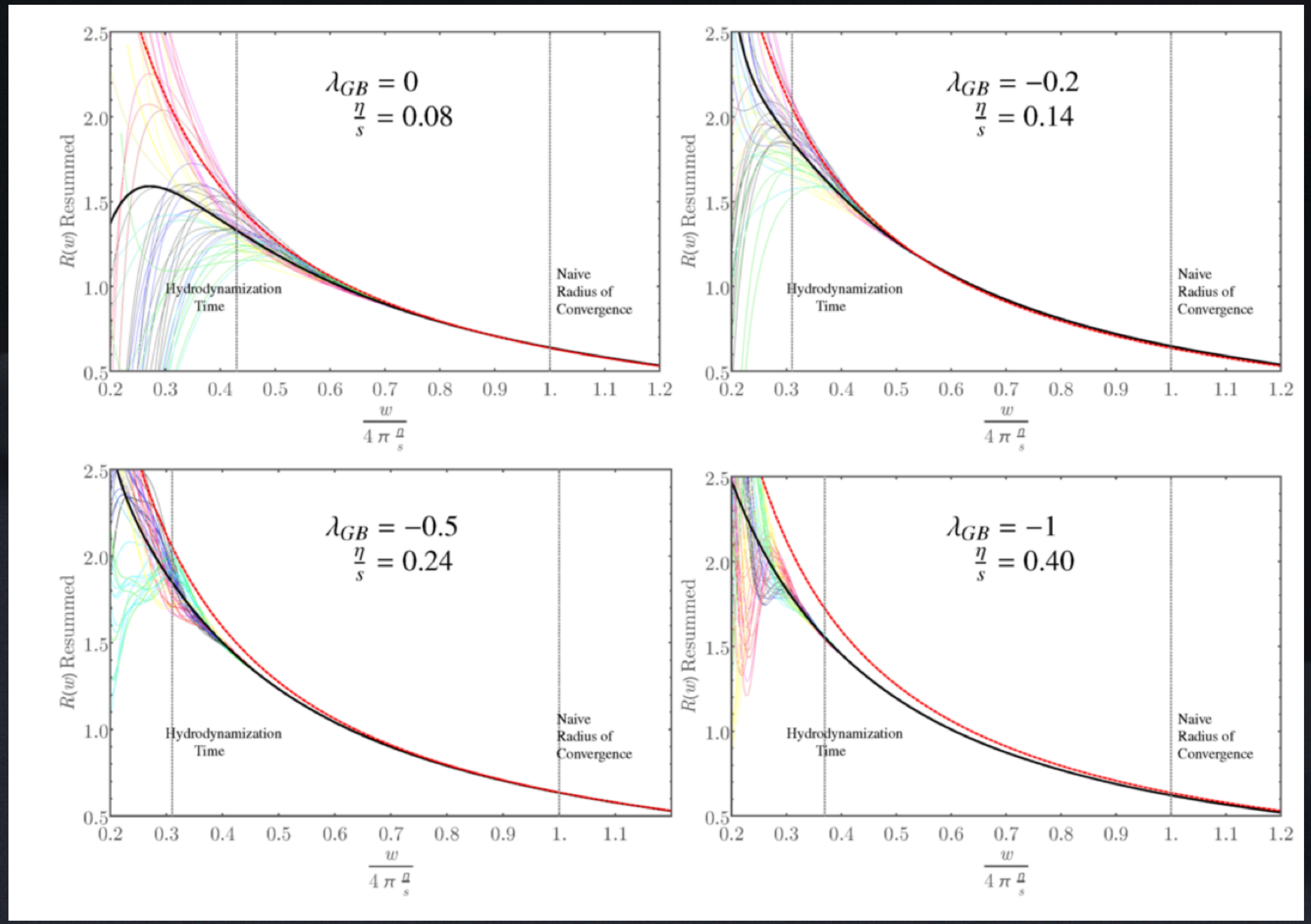

Holographic attractors

Spalinski 2018

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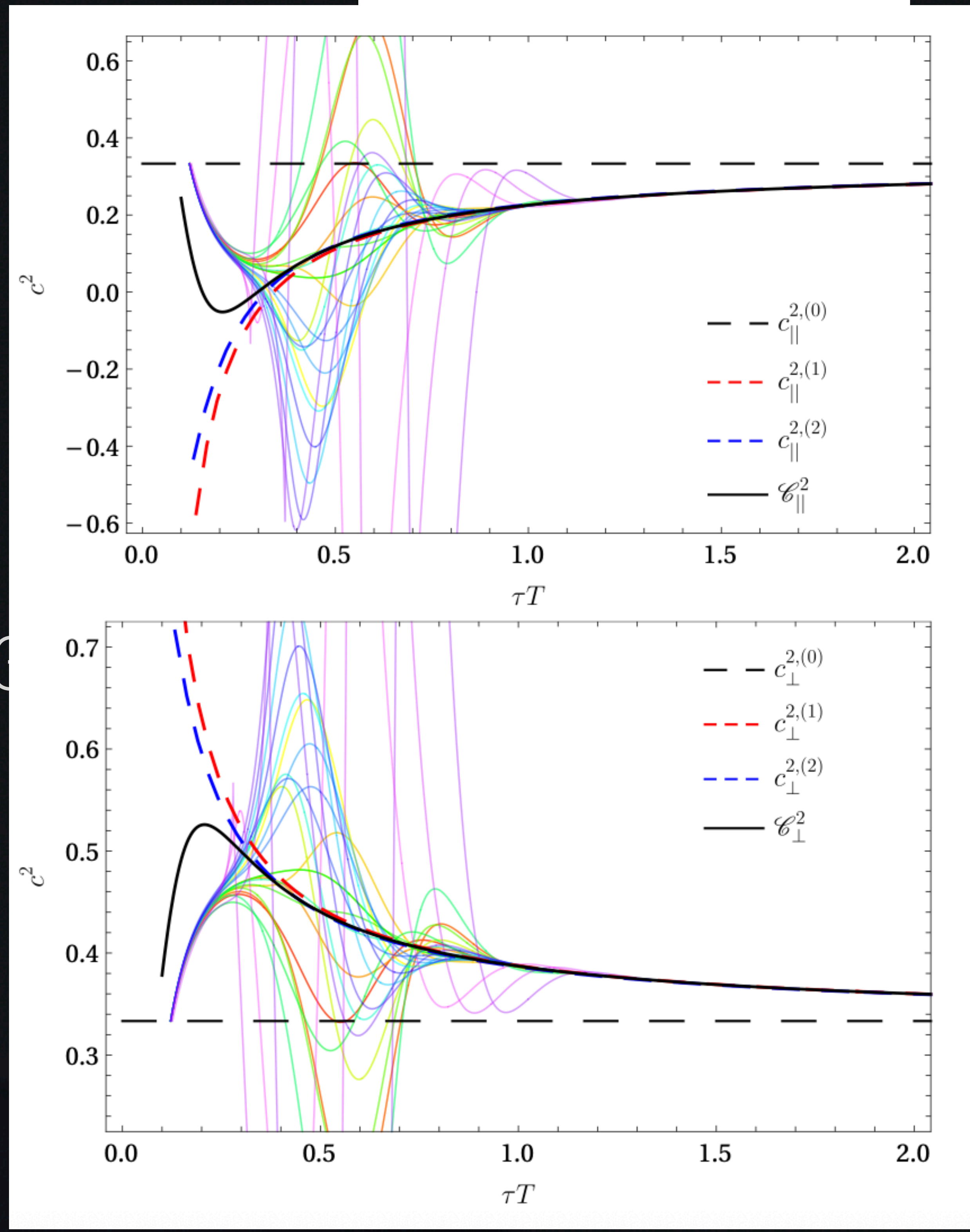

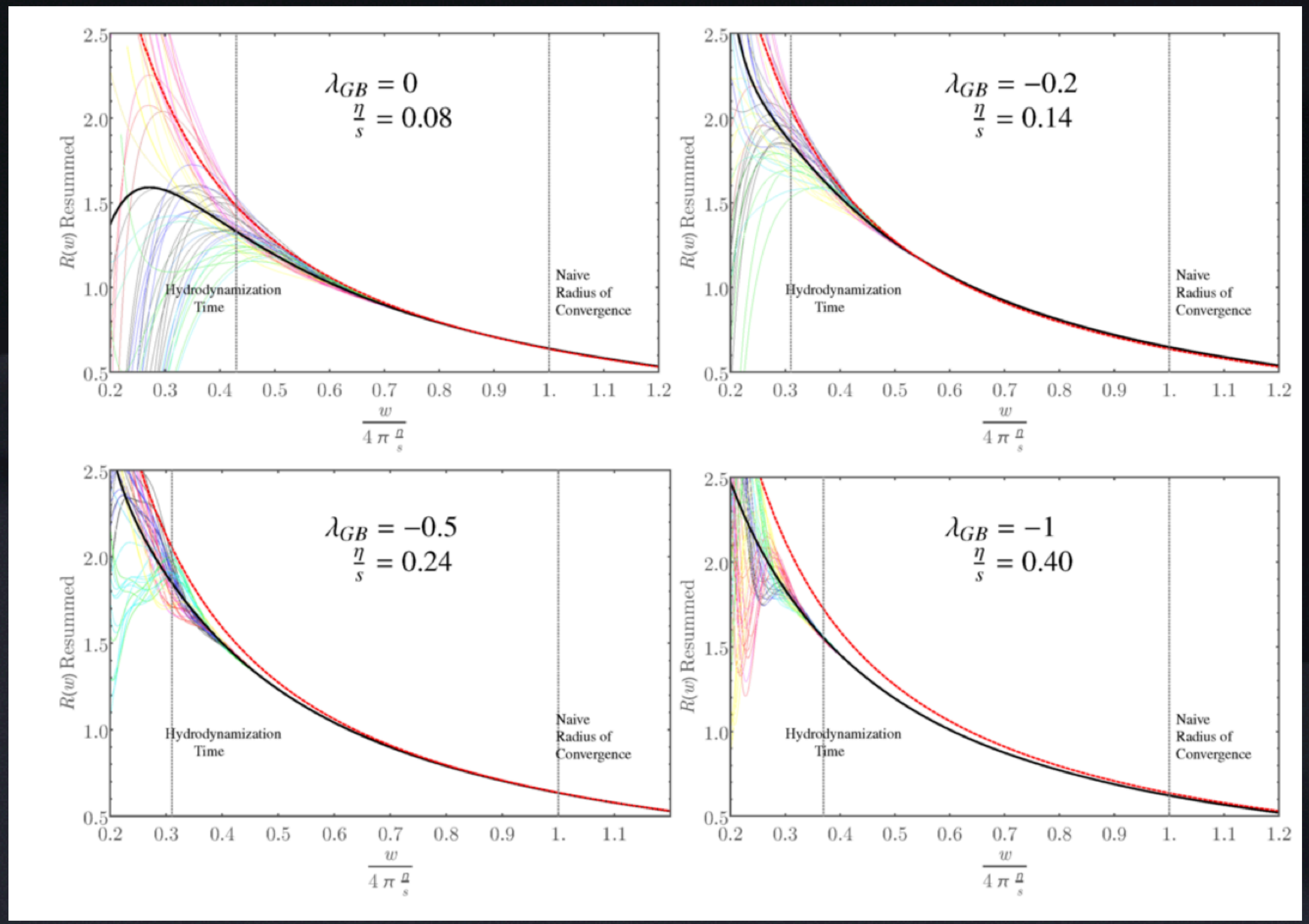

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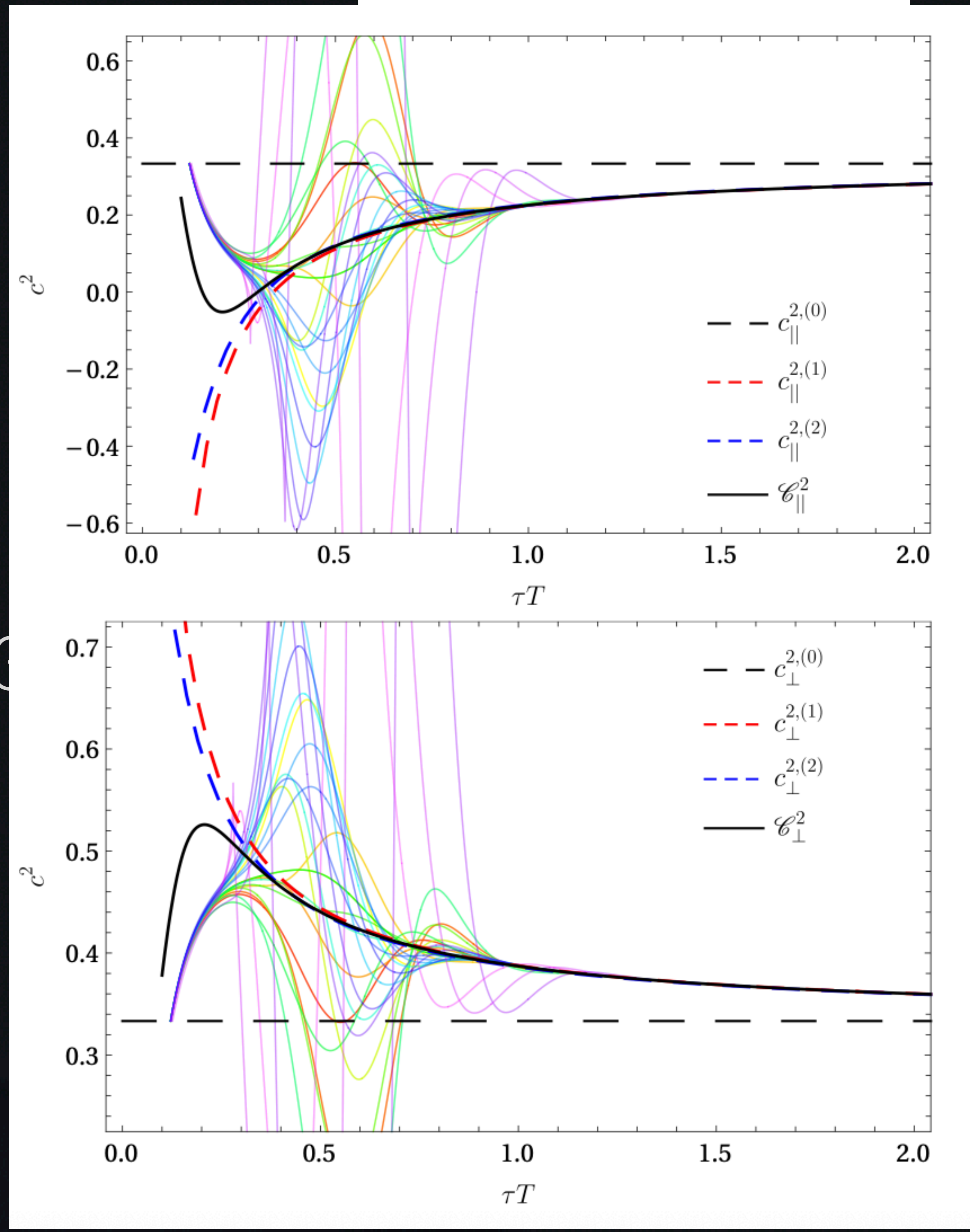

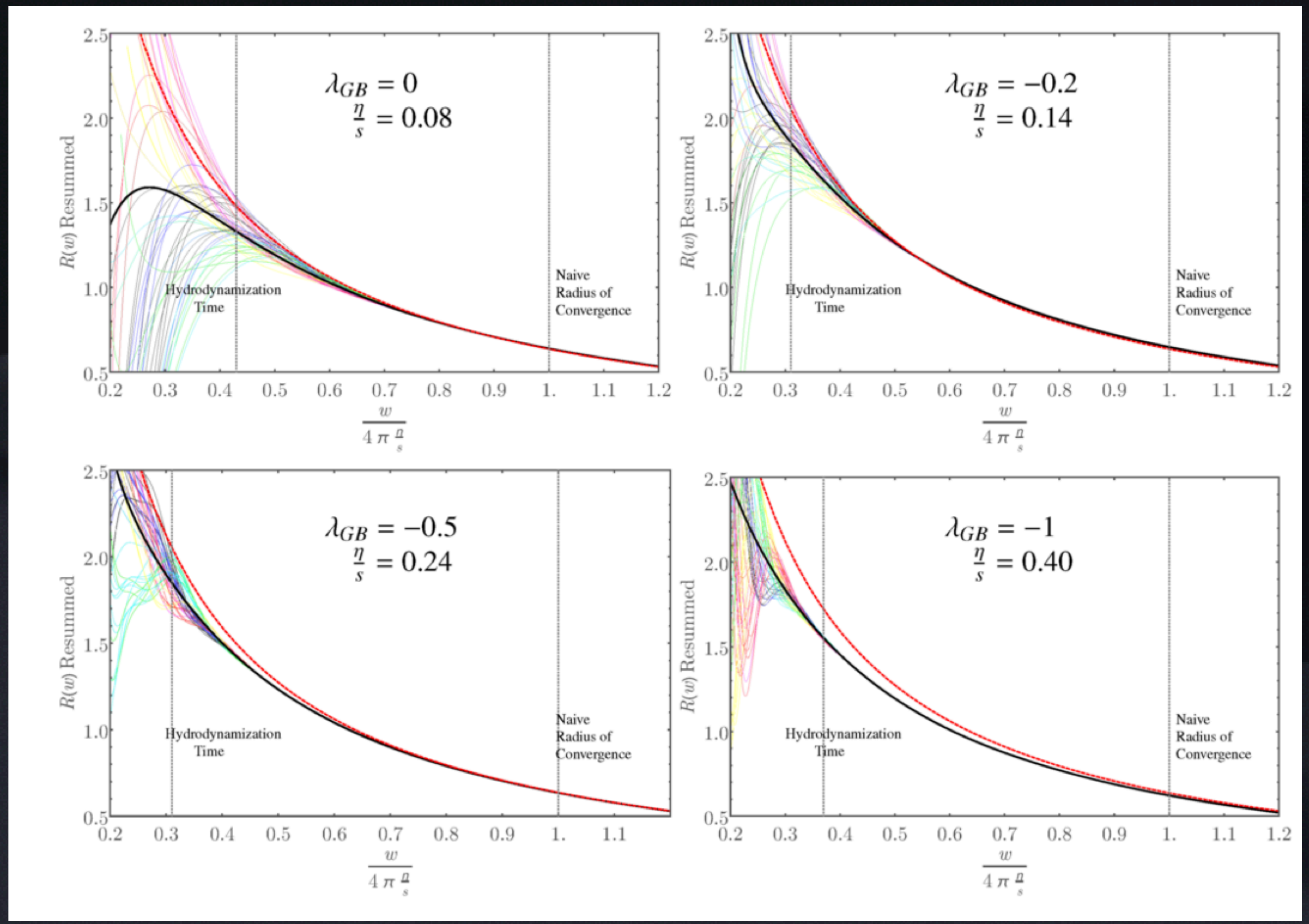

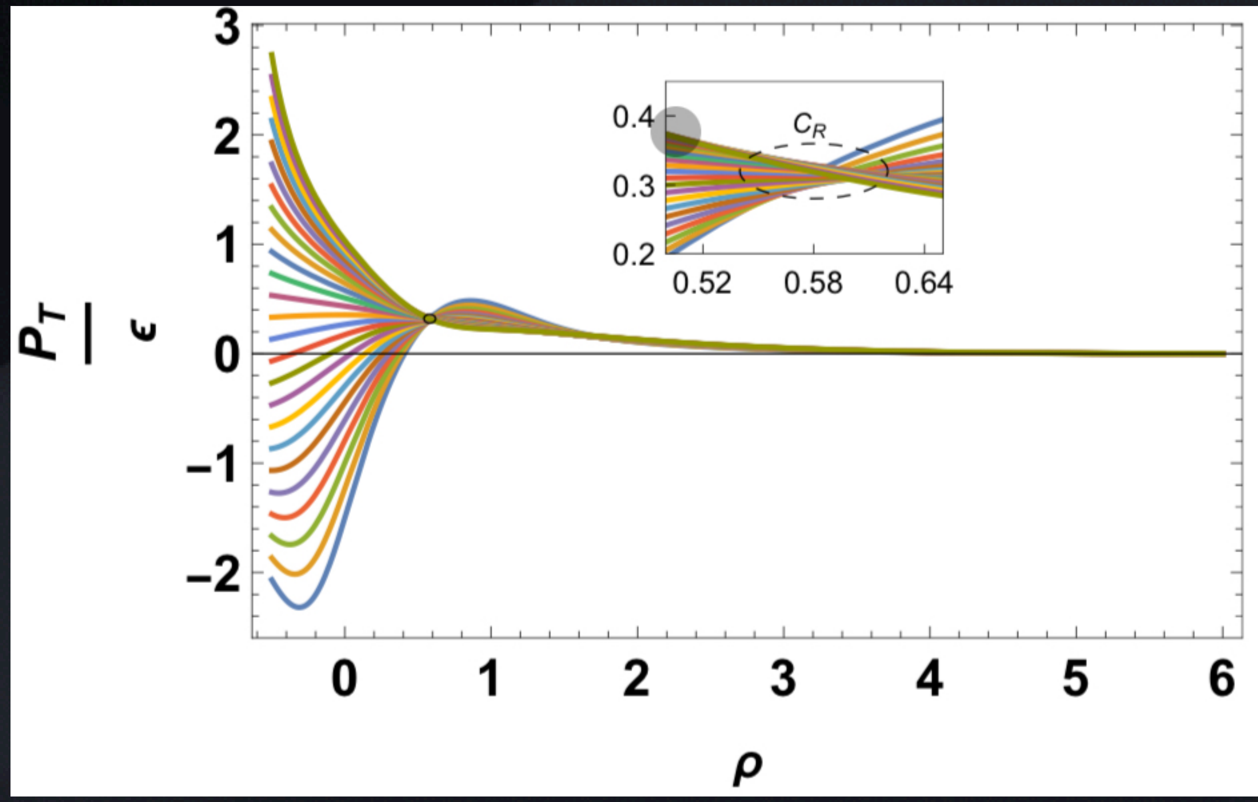

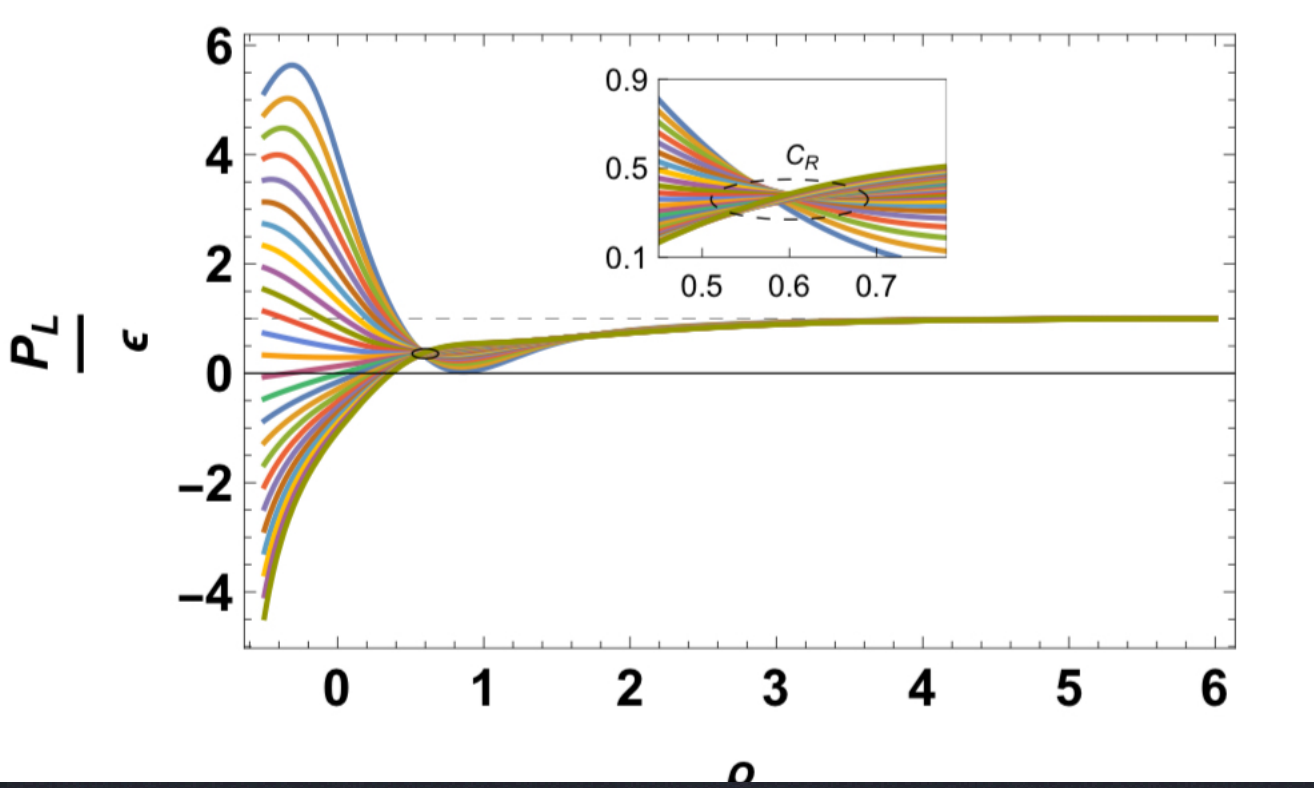

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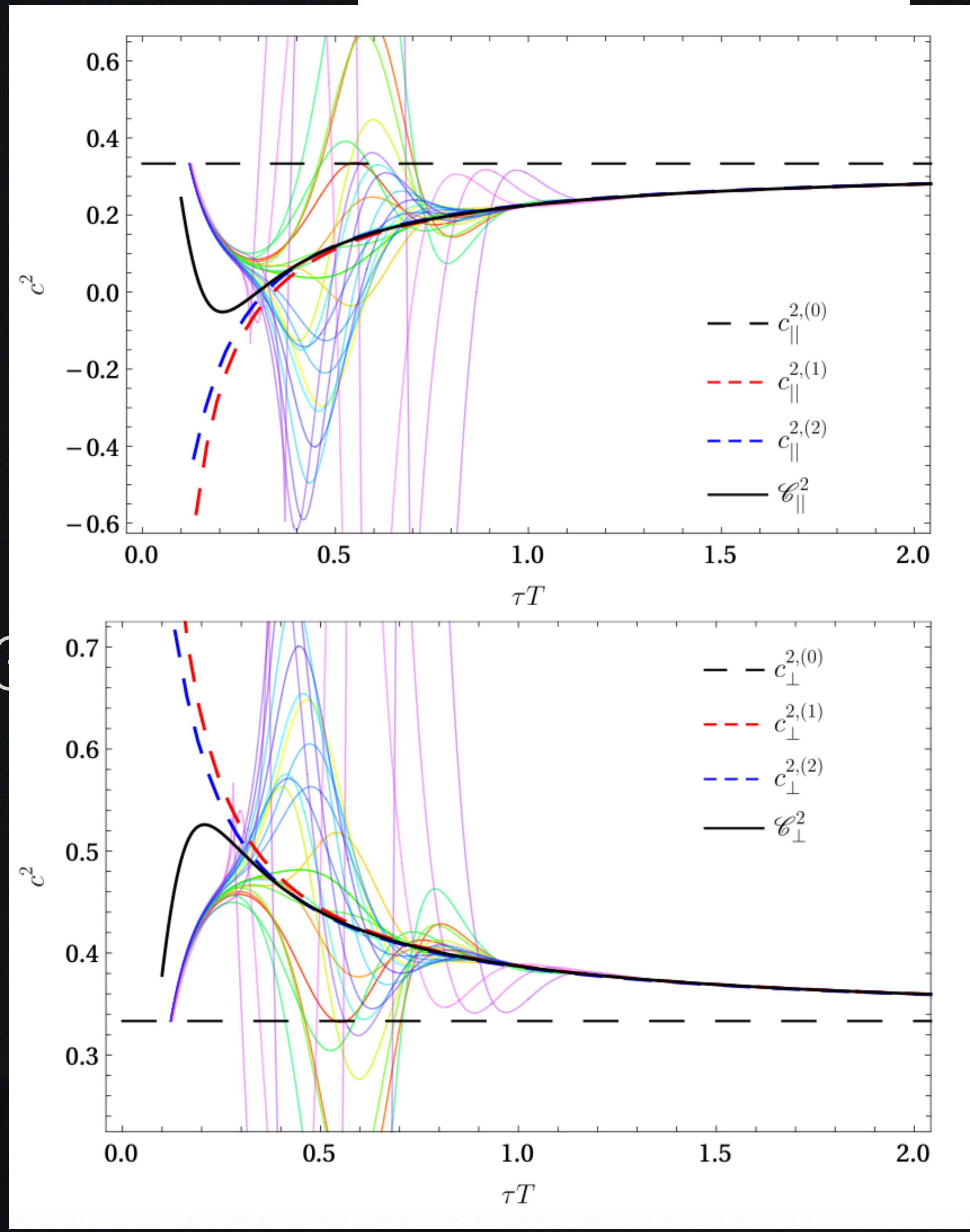
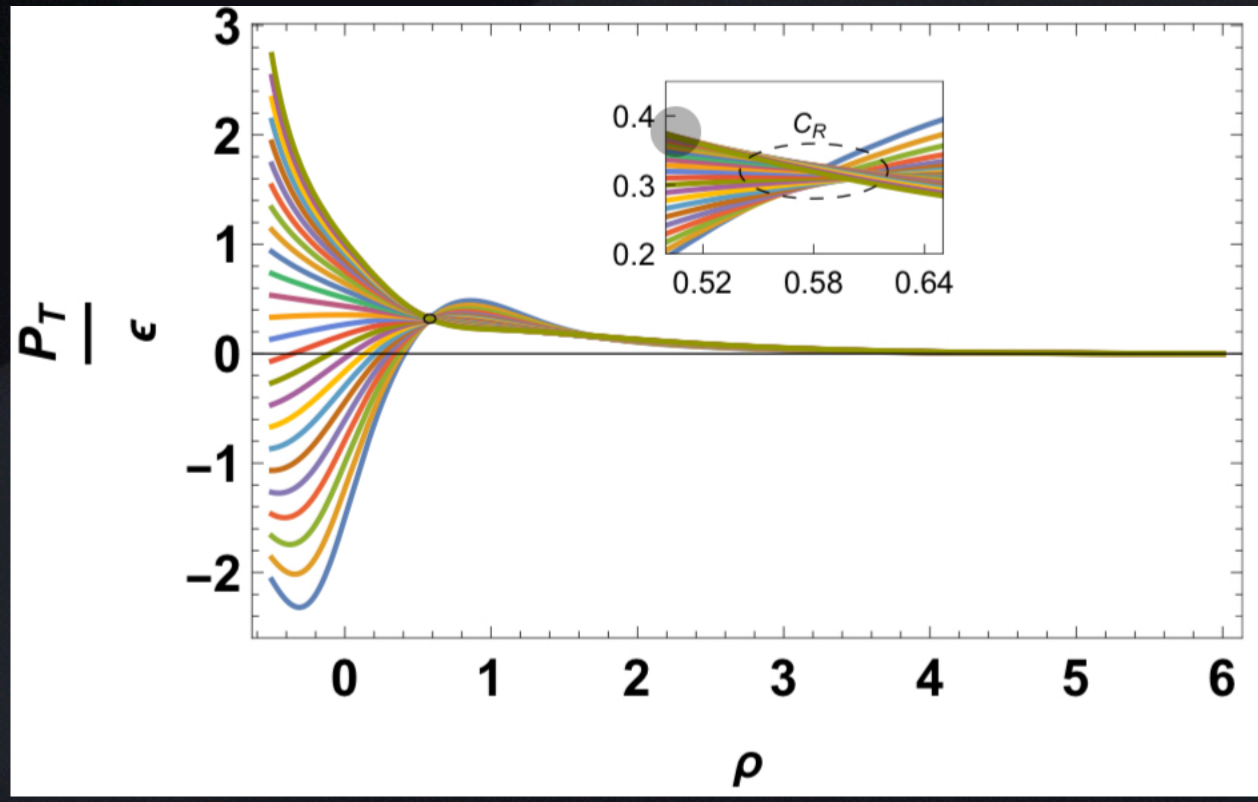
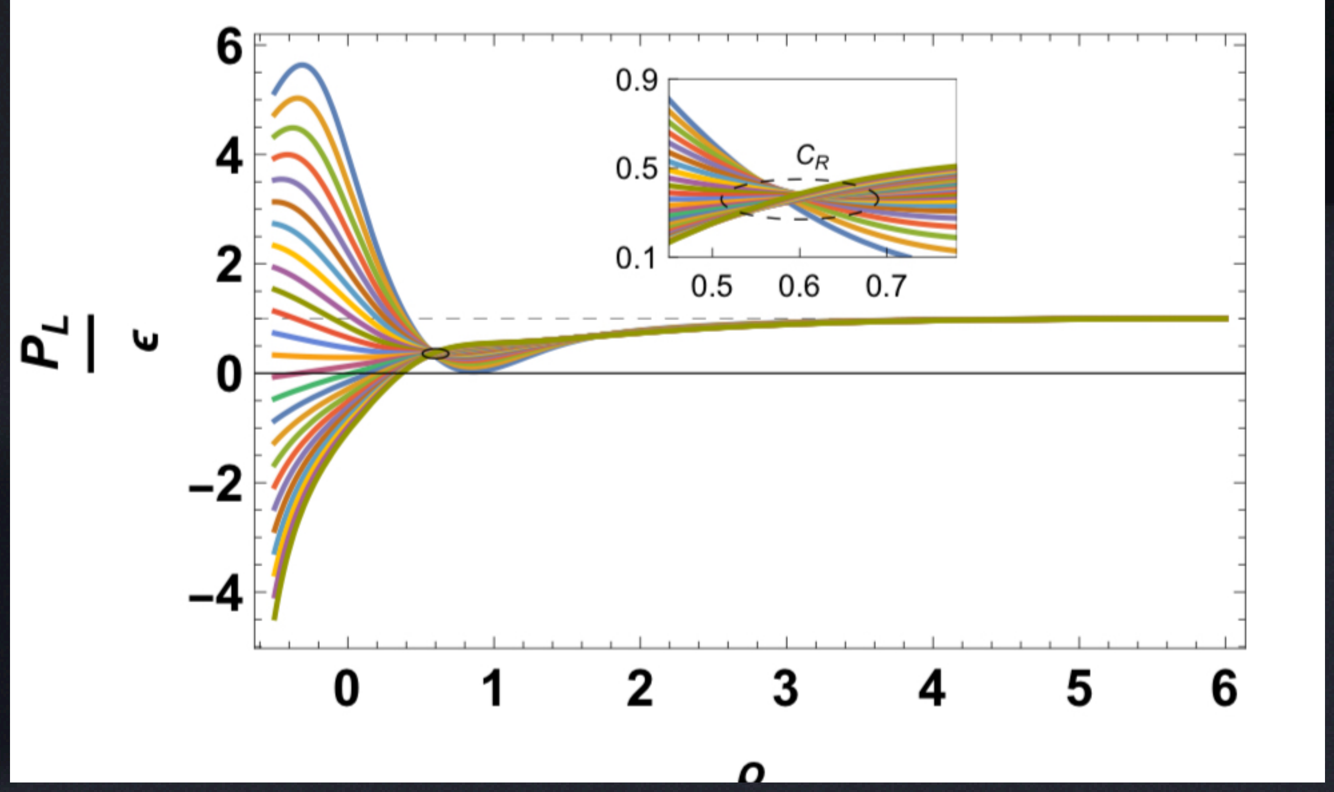






Holographic attractors

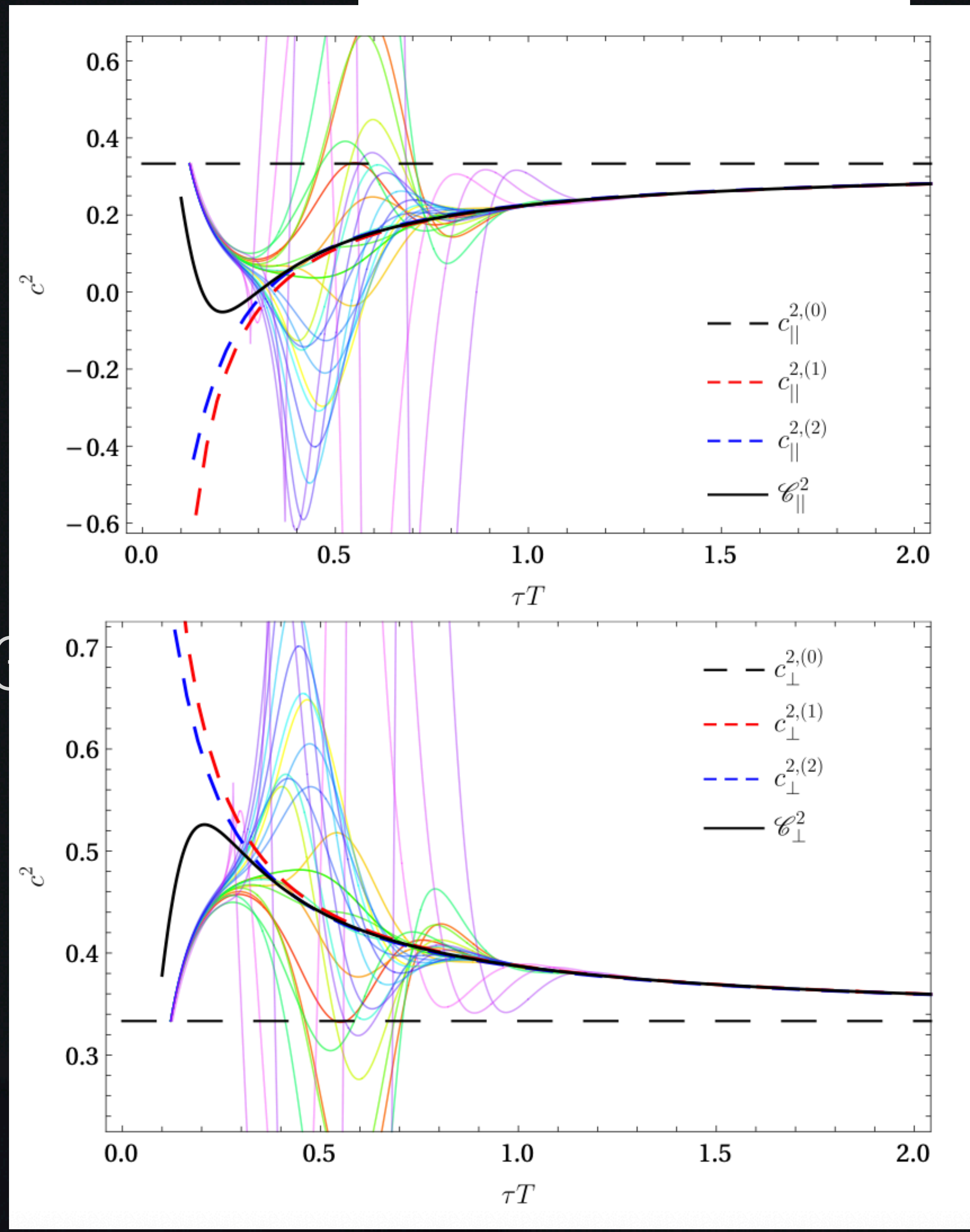
Spalinski 2018

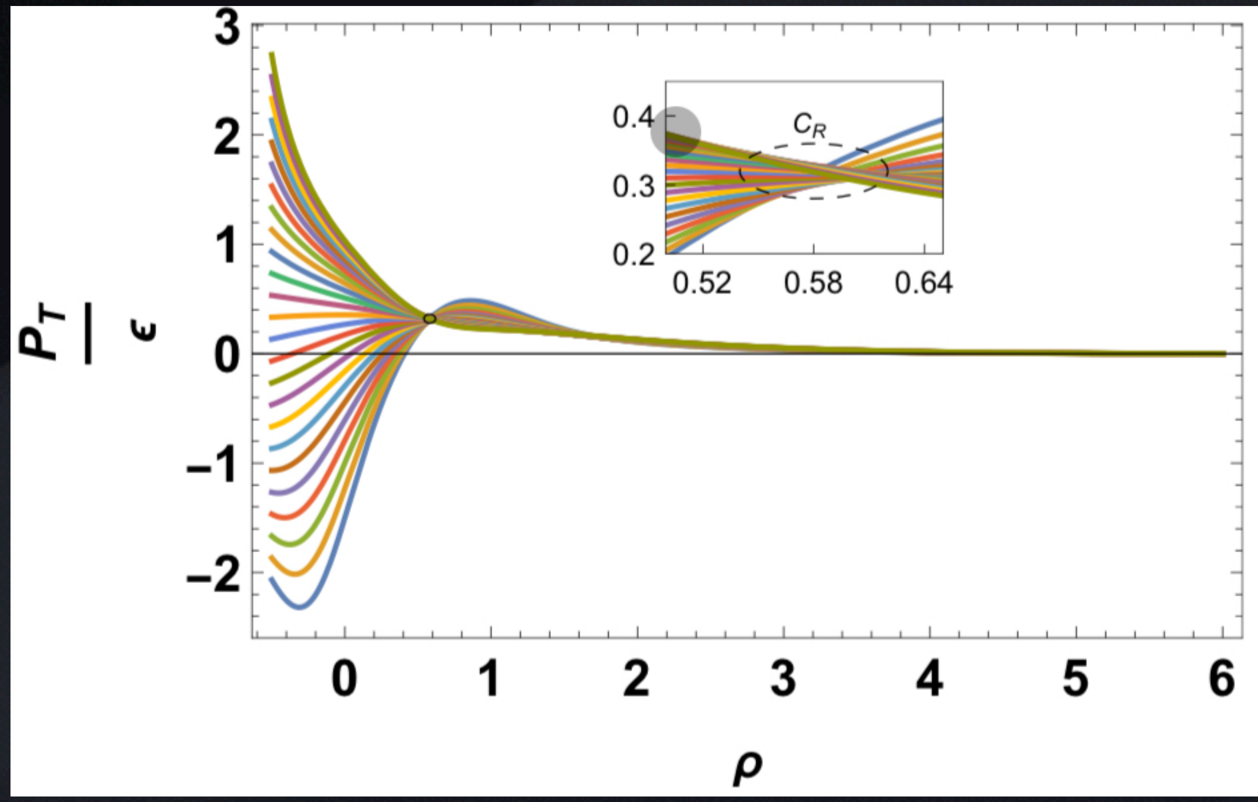
$$\mathscr{A}_0(w) = \frac{2530w - 276}{3975w^2 - 570w + 120}$$

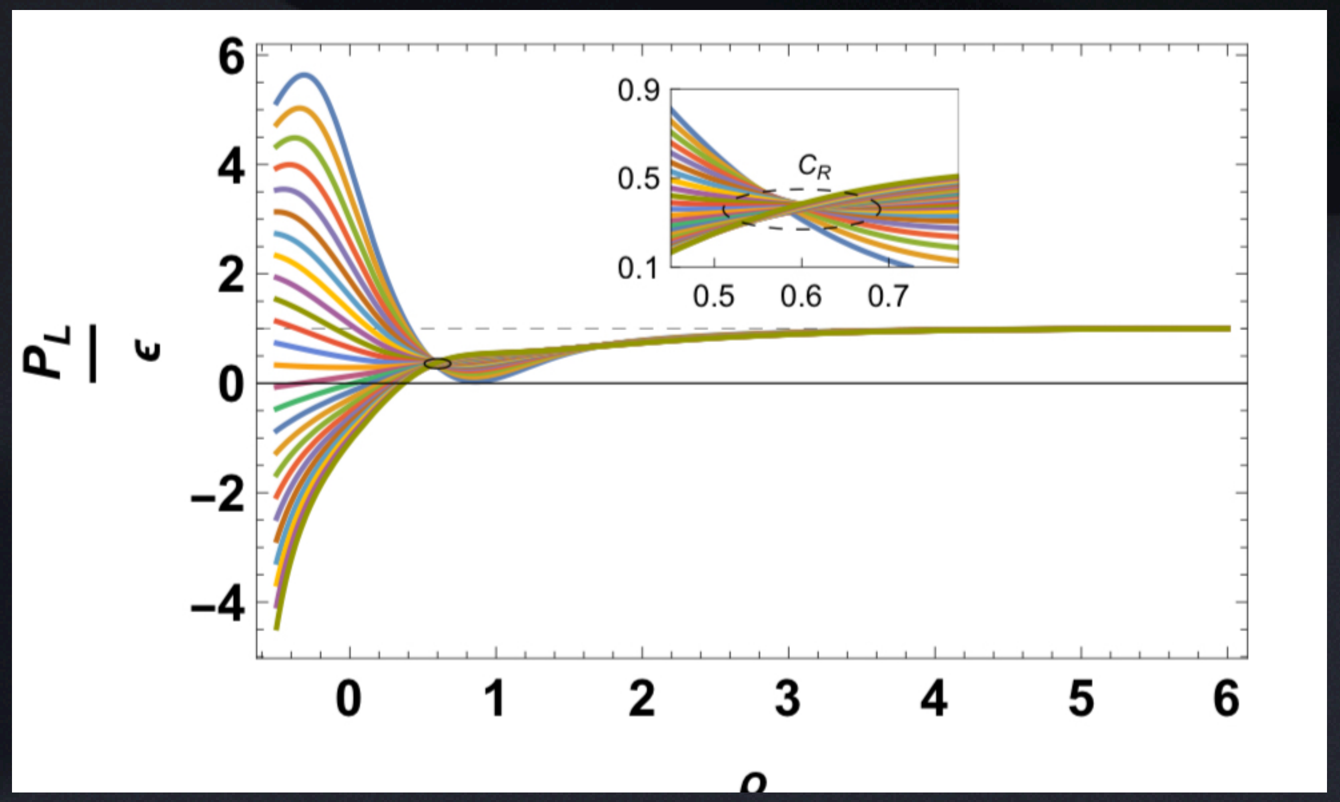
$$\mathscr{C}_\perp^2 = \frac{1}{3} + \frac{1}{9}\left(\mathscr{A}_0(w) + \frac{w}{4}\frac{\partial \mathscr{A}_0(w)}{\partial w}\right)$$

$$\mathscr{C}_{||}^2 = \frac{1}{3} - \frac{2}{9}\left(\mathscr{A}_0(w) + \frac{w}{4}\frac{\partial \mathscr{A}_0(w)}{\partial w}\right)$$

- Holographic Bjorken flow — Janik, Peschanski 2005, Kinoshita, Mukhohyama, Nakamura, Oda, 2008 ...
- Gauss-Bonnet — Casalderrey-Solana, Gushterov, Meiring 2018
- Speed of sound attractors — Cartwright, Kaminski, Knipfer 2022 Kaminski, Cartwright, Knipfer, Wondrak, Schenke, Bleicher 2023
- Holographic Gubser flow — Banerjee, Mitra, Mukhopadhyay, AS 2023; Mitra, Mondkar, Mukhopadhyay, AS 2024
- Large D-results — Casalderrey-Solana, Herzog, Meiring 2018
- Convergence of hydro modes — Heller, Serantes, Spalinski, Svensson, Withers 2021...







Outlook

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Hydrodynamic attractors reveal **universal features** far-from-equilibrium across diverse theories

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- Role of fluctuations, phase transitions, …