Chiral phase transition and universality class

- Behavior of the free energy close to critical lines

$$f(m,T)=f_s(z)+f_{reg}\,, \qquad z=t/h^{1/\beta\delta}$$

$h=\frac{1}{h_0}\frac{m_l}{m_s}$ $\quad t=\frac{1}{t_0}\frac{T-T_c}{T_c}$

- Universality scaling behavior of

Order parameter:

$$M=-\partial f_s(t,h)/\partial H=h^{1/\delta}f_1(z)$$

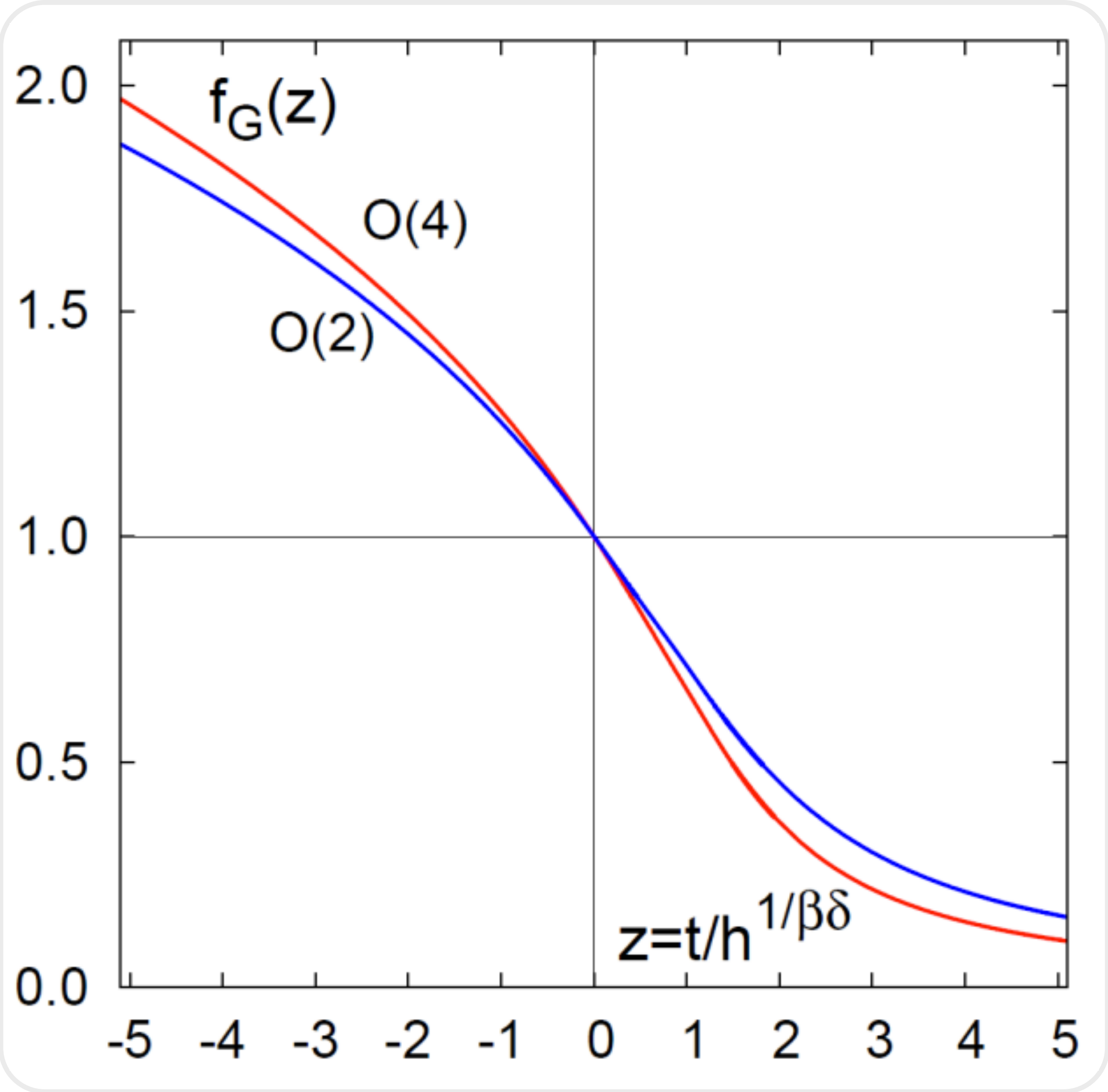
Order parameter susceptibility:

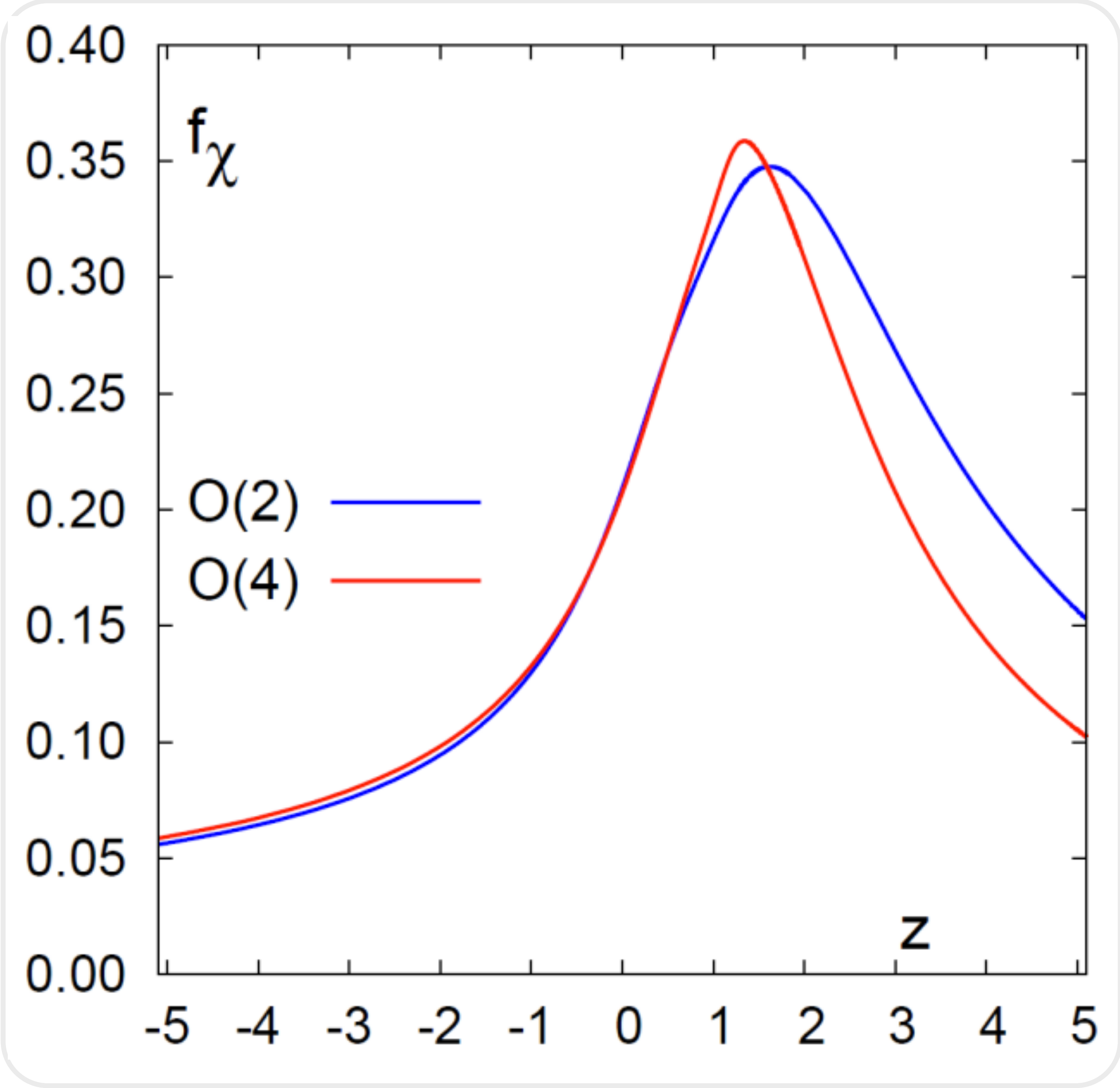
$$\chi_M=-\partial M/\partial H=\frac{1}{h_0}h^{1/\delta-1}f_2(z)$$

n-th order susceptibility:

$$\chi_M^n=-\partial^n M/\partial H^n=\frac{1}{h_0}h^{1/\delta-n+1}f_n(z)$$

Universal scaling functions





Cumulants of order parameter

- n-th order cumulant of chiral condensate:

$$\mathbb{K}_n(\bar\psi\psi) = \frac{T}{V}(-1)^n \frac{\partial^n \mathbb{G}(m_l;\epsilon)}{\partial m_l^n}\Bigg|_{\epsilon=m_l}$$

$$\mathbb{G}(m_l;\epsilon) = \ln\left\langle \exp\left\{-m_l\bar\psi\psi(\epsilon)\right\}\right\rangle_0$$

$$\bar\psi\psi(\epsilon) = 2\ \mathrm{Tr}\ (\not{D}(U)+\epsilon)^{-1} \equiv \frac{4\epsilon}{\lambda^2+\epsilon^2}$$

$$\mathbb{K}_1\left[\bar\psi\psi\right] = \frac{T}{V}\langle\bar\psi\psi(m_l)\rangle,\ \ \mathbb{K}_2\left[\bar\psi\psi\right] = \frac{T}{V}\left\langle\left(\bar\psi\psi(m_l)\ -\langle\bar\psi\psi(m_l)\rangle\right)^2\right\rangle, \cdots$$

- Connection to correlation of Dirac Eigenvalues

$$\rho_U(\lambda) = \sum_j \delta(\lambda-\lambda_j)\ , P_U(\lambda;\epsilon) = \frac{4\epsilon\rho_U(\lambda)}{\lambda^2+\epsilon^2}$$

NEW

$$\mathbb{K}_n(\bar\psi\psi) = \int_0^\infty P_n(\lambda)\ \mathrm{d}\ \lambda$$

$$n=1,\ P_1(\lambda) = K_1[P_U(\lambda,m_l)]$$

$$n\geq 2,\ P_n(\lambda) = \int_0^\infty K_1[P_U(\lambda;m_l), P_U(\lambda_2;m_l),\cdots,P_U(\lambda_n;m_l)]\prod_{i=2}^{n}\mathrm{d}\lambda_i$$

A generalized Banks-Casher relation: $\lim_{m\to 0}\mathbb{K}_n(\bar\psi\psi) = (2\pi)^n\mathbb{K}_n[\rho_U(0)]$

HTD, W.-P. Huang, S. Mukherjee and P. Petreczky, arXiv:2305.10916, PRL 131 (2023) 161903

Scaling behavior of $\mathbb{K}_n(\bar{\psi}\psi)$

In the proximity of T_c

- n-th order susceptibility of chiral order parameter:

$$\chi_M^n = -\,\partial^n M/\partial H^n = \frac{1}{h_0} m^{1/\delta - n+1} f_n(z) = \mathbb{K}_n(\bar{\psi}\psi) + \text{ other singular parts}$$

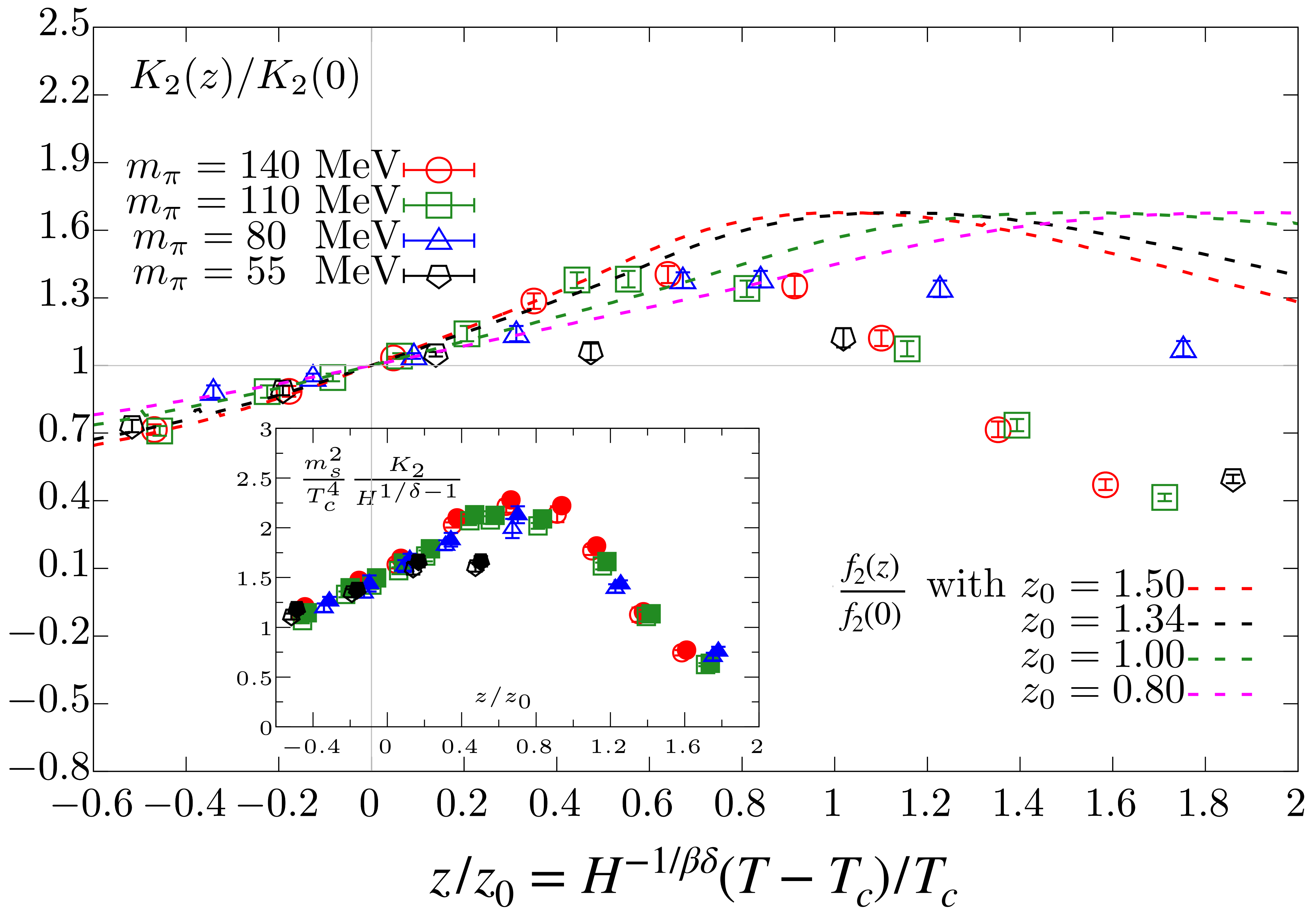
$f_n(z)$: scaling function, $\quad z = z_0(T - T_c)/T_c H^{-1/\beta\delta}$: scaling variable

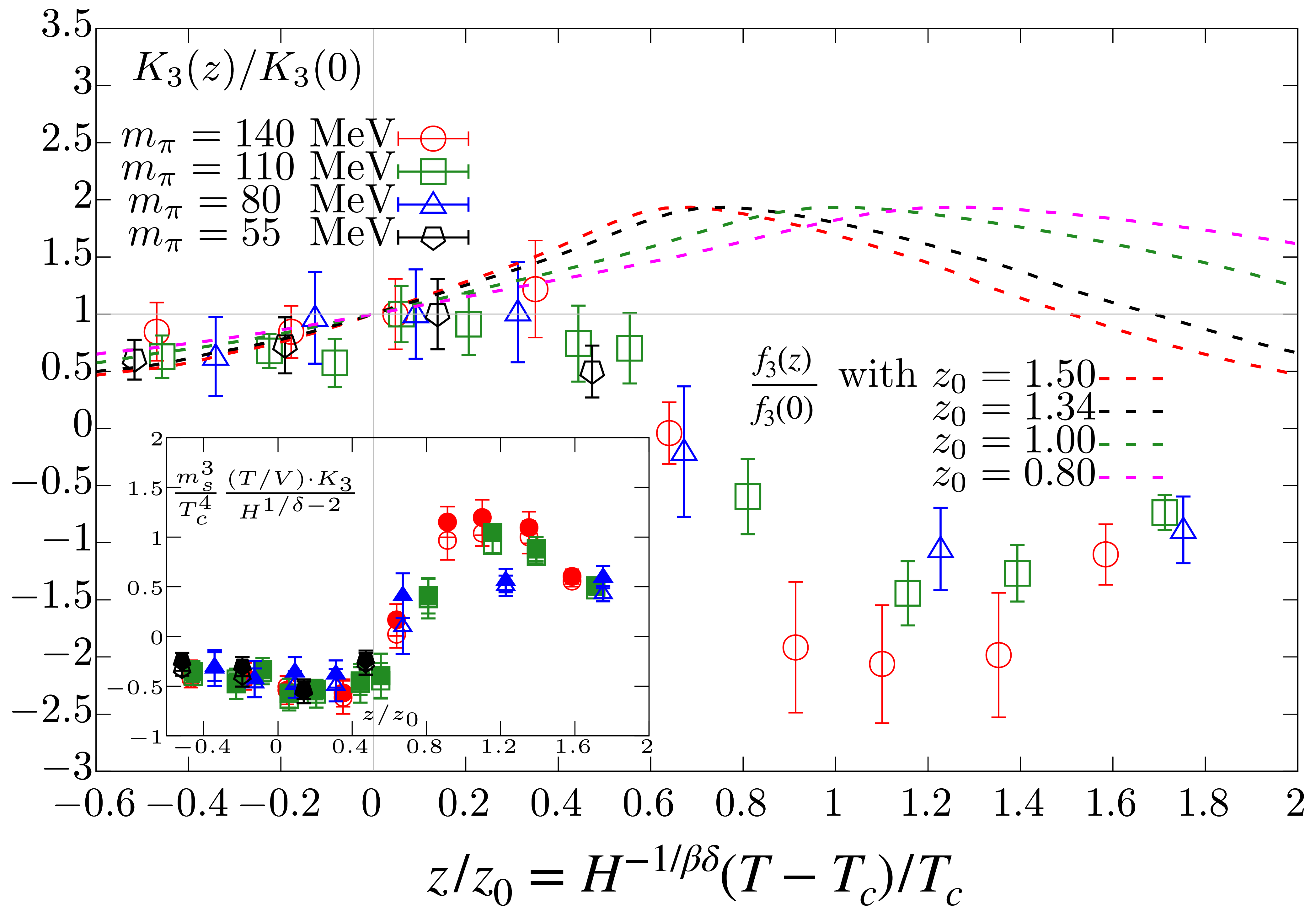
- n-th order cumulant of chiral condensate:

$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\, \mathrm{d}\,\lambda \quad \sim m^{1/\delta - n+1} f_n(z)\,?$$

Scaling behavior of $\mathbb{K}_n(\bar{\psi}\psi)$

HISQ, Nt=8, pion mass ranging from 140 to 55 MeV





$$\mathbb{K}_2(\bar{\psi}\psi) \sim m_l^{1/\delta-1} f_2(z) \qquad \mathbb{K}_3(\bar{\psi}\psi) \sim m_l^{1/\delta-2} f_3(z)$$

$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\,\mathrm{d}\,\lambda \quad \sim m^{1/\delta-n+1} f_n(z)\,!$$

HTD, W.-P. Huang, S. Mukherjee and P. Petreczky, PRL 131 (2023) 161903

Microscopic encoding of Macroscopic criticality

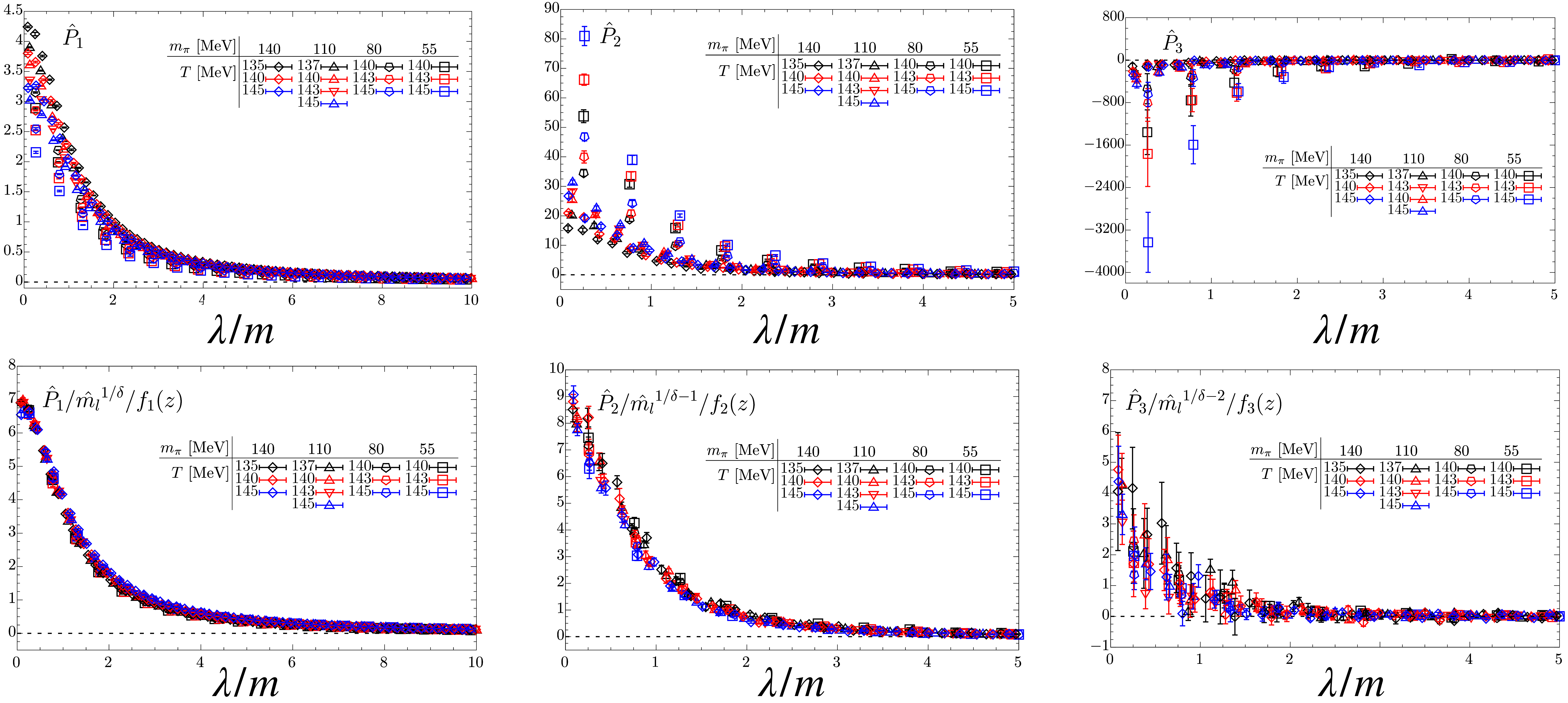
$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\, \mathrm{d}\,\lambda \;\; \sim m^{1/\delta - n + 1} f_n(z)$$

$$\lim_{m\to 0} \mathbb{K}_n(\bar{\psi}\psi) = (2\pi)^n \mathbb{K}_n[\rho_U(0)]$$

Critical behavior in $\lim_{m\to 0} \mathbb{K}_n(\bar{\psi}\psi)$: must arise from universal behaviors of λ-independent $\mathbb{K}_n[\rho_U(0)]$

Conjecture: $P_n(\lambda) = m^{1/\delta - n + 1} f_n(z)\; g(\lambda/m)$

Microscopic encoding of Macroscopic criticality



$$P_n(\lambda) = m^{1/\delta-n+1} f_n(z)\ g(\lambda/m)$$