UNIVERSALITY OF THE YANG-LEE EDGE SINGULARITY & ITS APPLICATIONS

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CRC-TR 211
Strong-interaction matter under extreme conditions

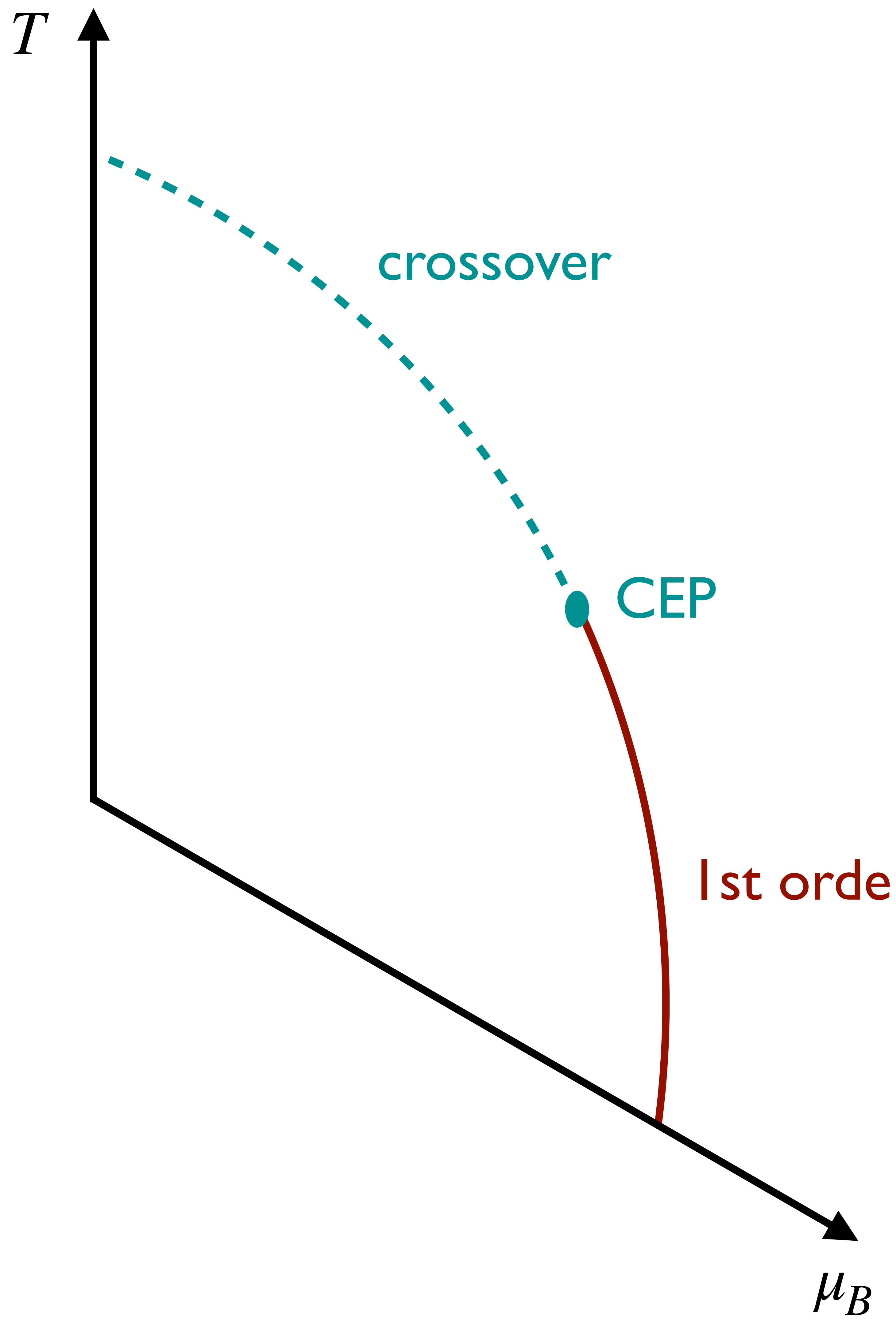
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HFHF

ANALYTIC STRUCTURE OF QCD AND YANG-LEE EDGE SINGULARITY
ECT* TRENTO - 09/09/2025

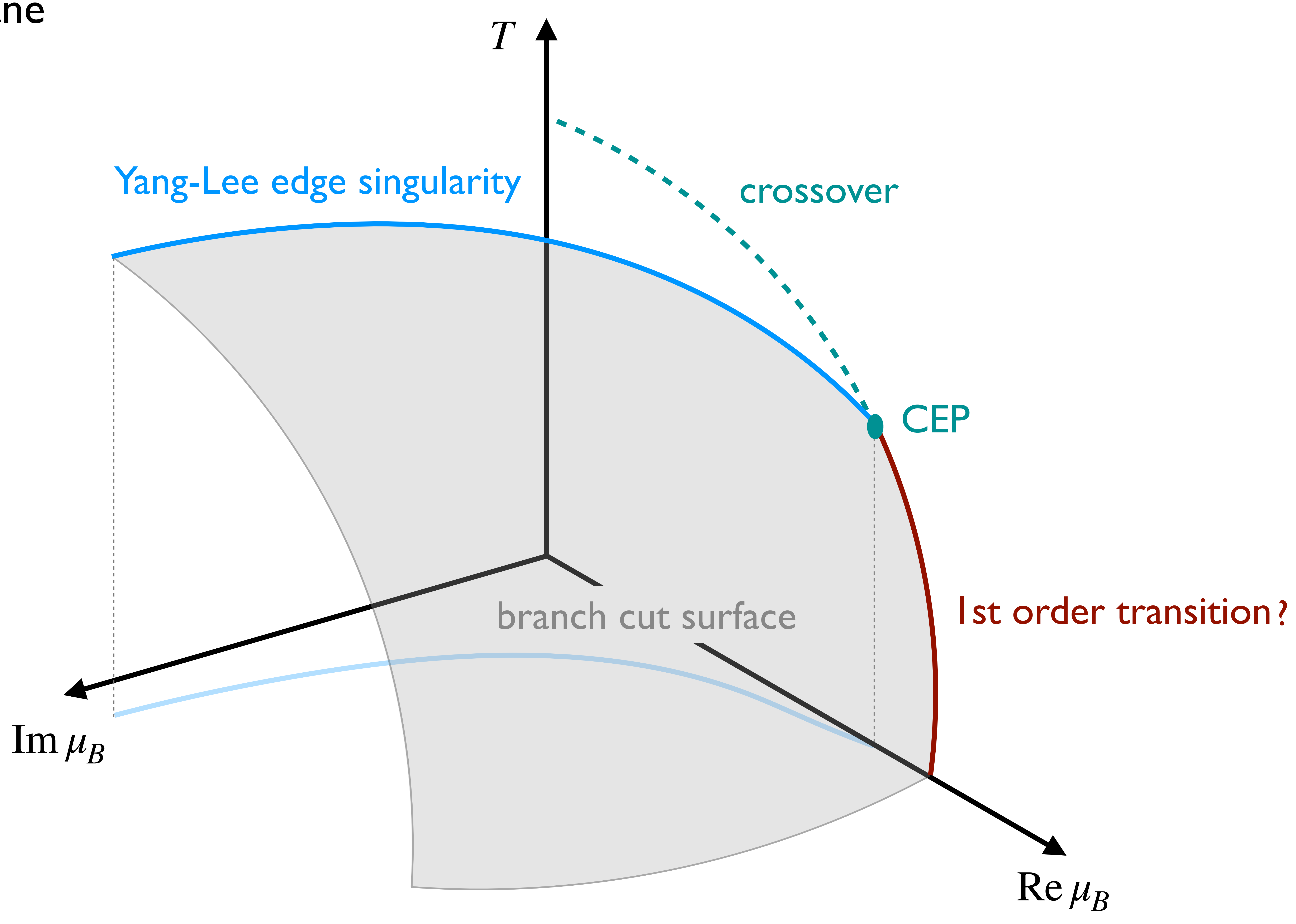
THE CHIRAL PHASE TRANSITION

in the (T, μ_B) plane



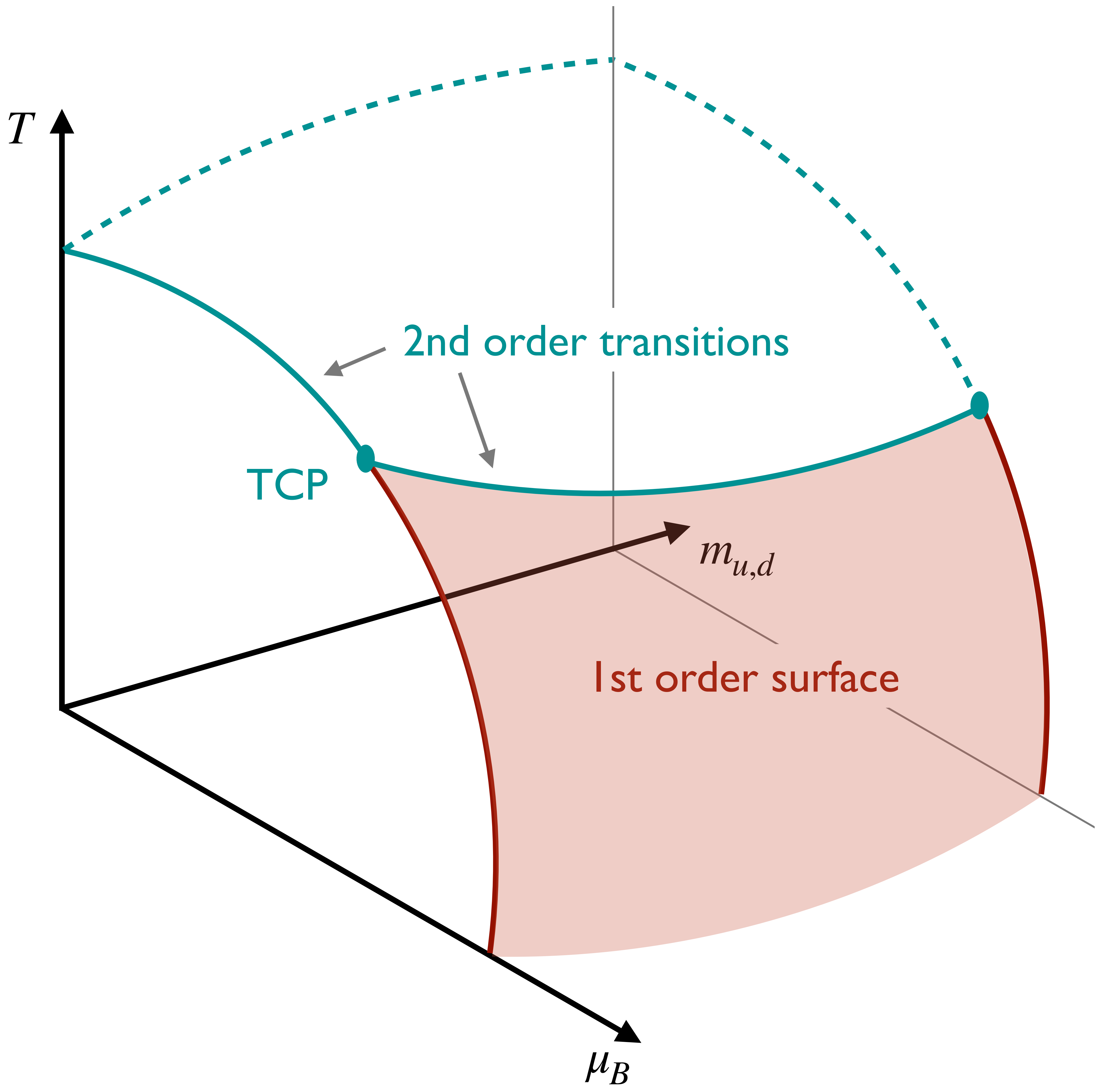
THE CHIRAL PHASE TRANSITION

in the $(T, \mathrm{Re}\,\mu_B, \mathrm{Im}\,\mu_B)$ plane



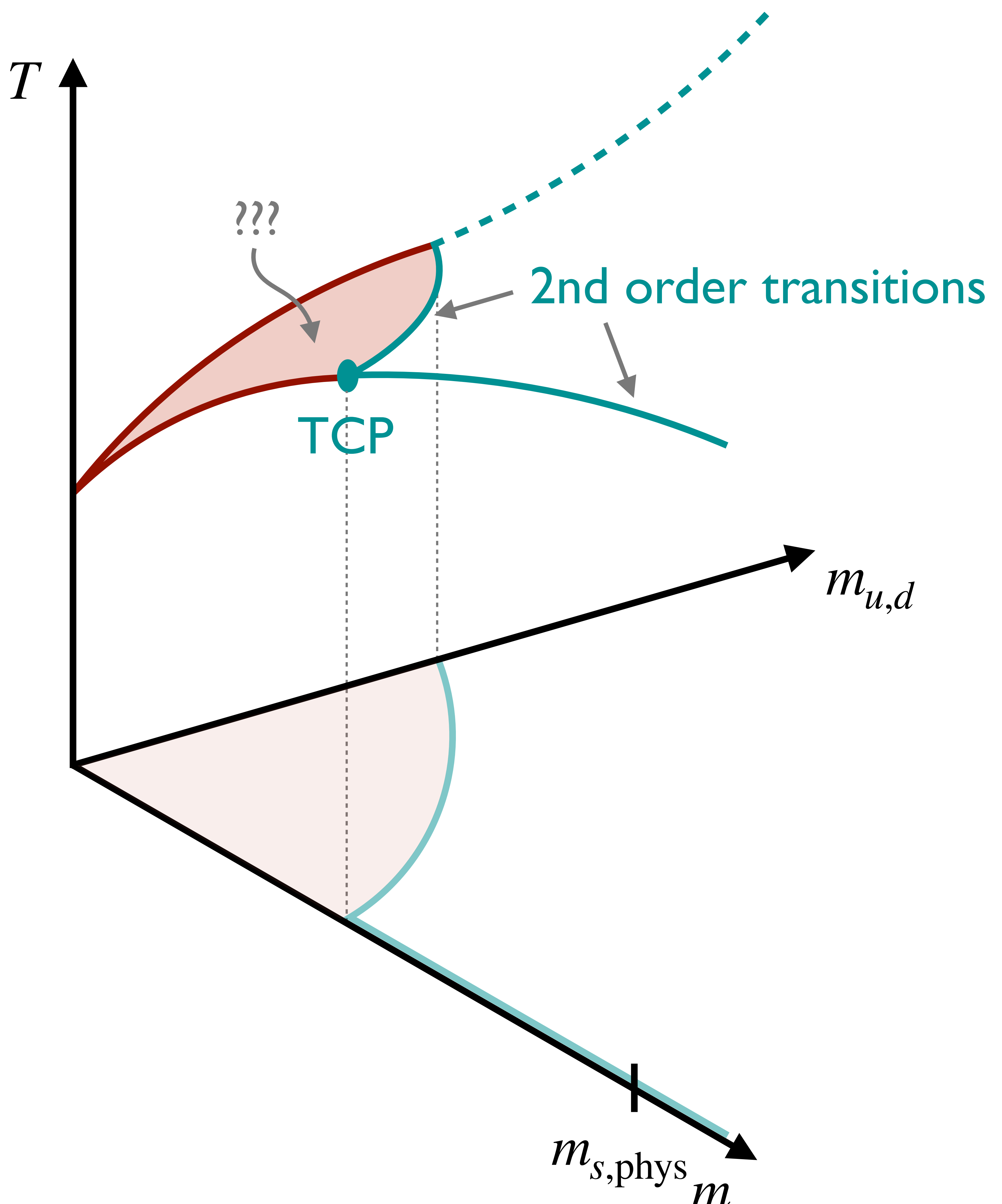
THE CHIRAL PHASE TRANSITION

in the $(T, \mu_B, m_{u,d})$ plane



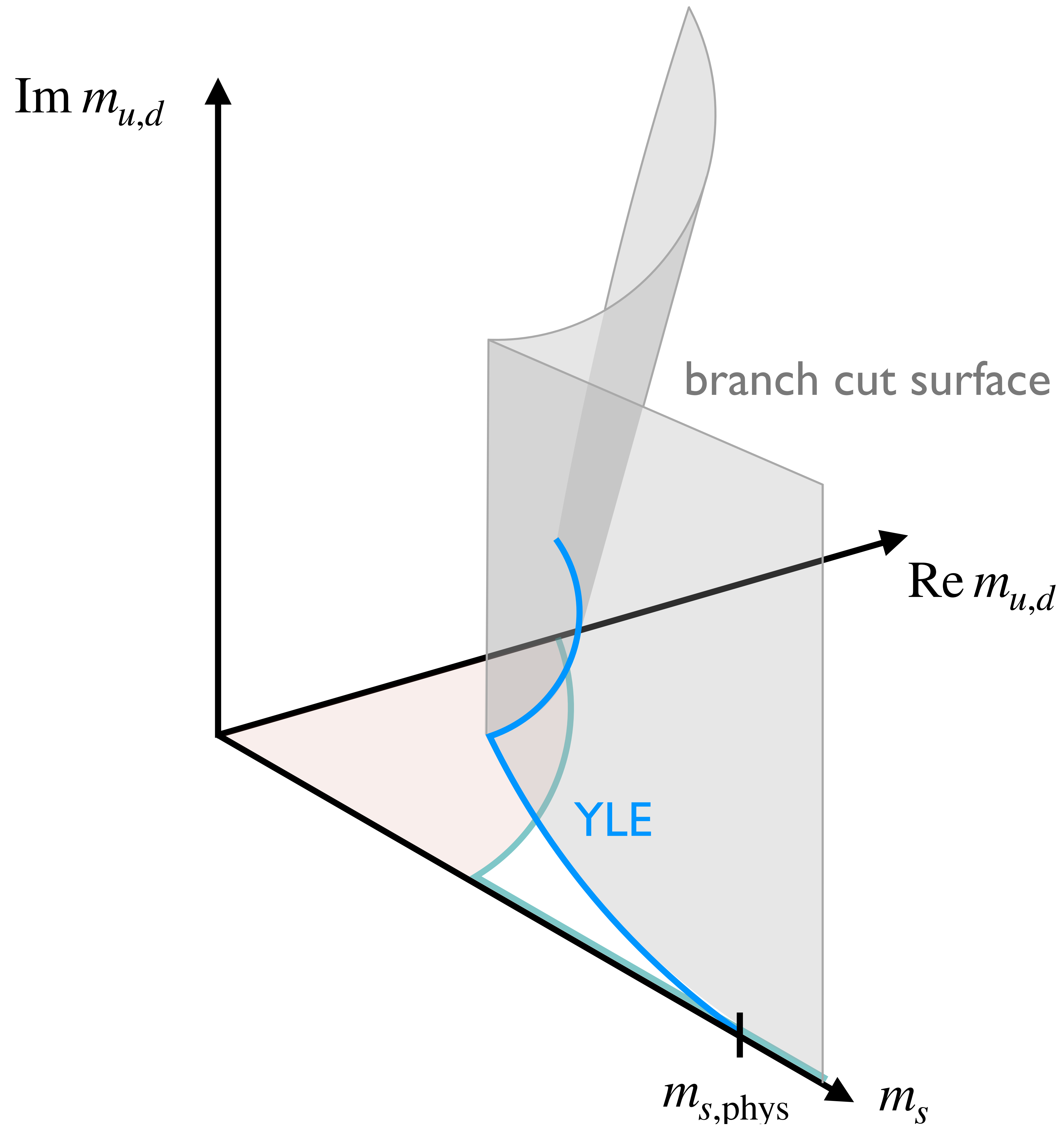
THE CHIRAL PHASE TRANSITION

in the $(T, m_s, m_{u,d})$ plane at $\mu_B = 0$

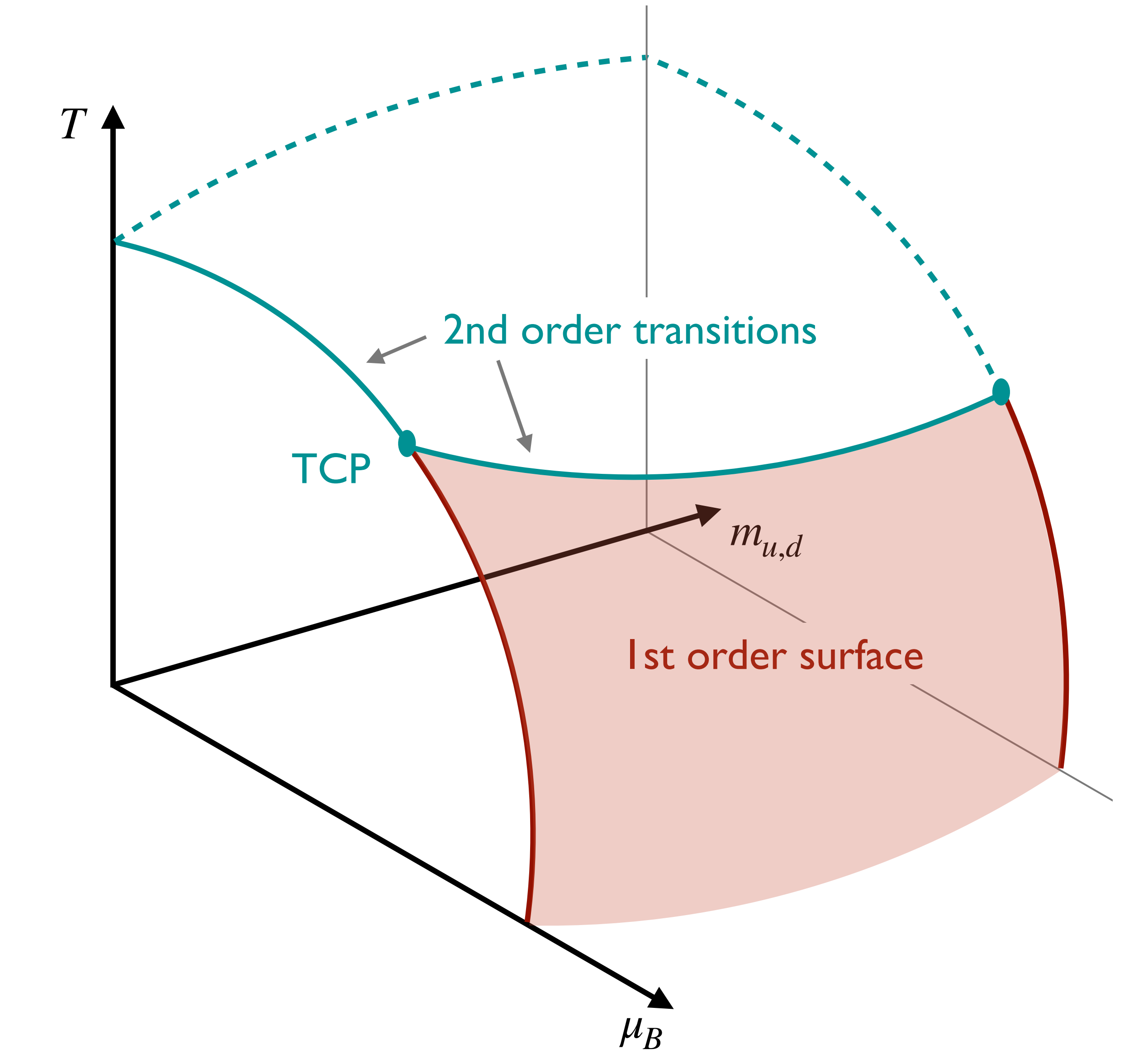


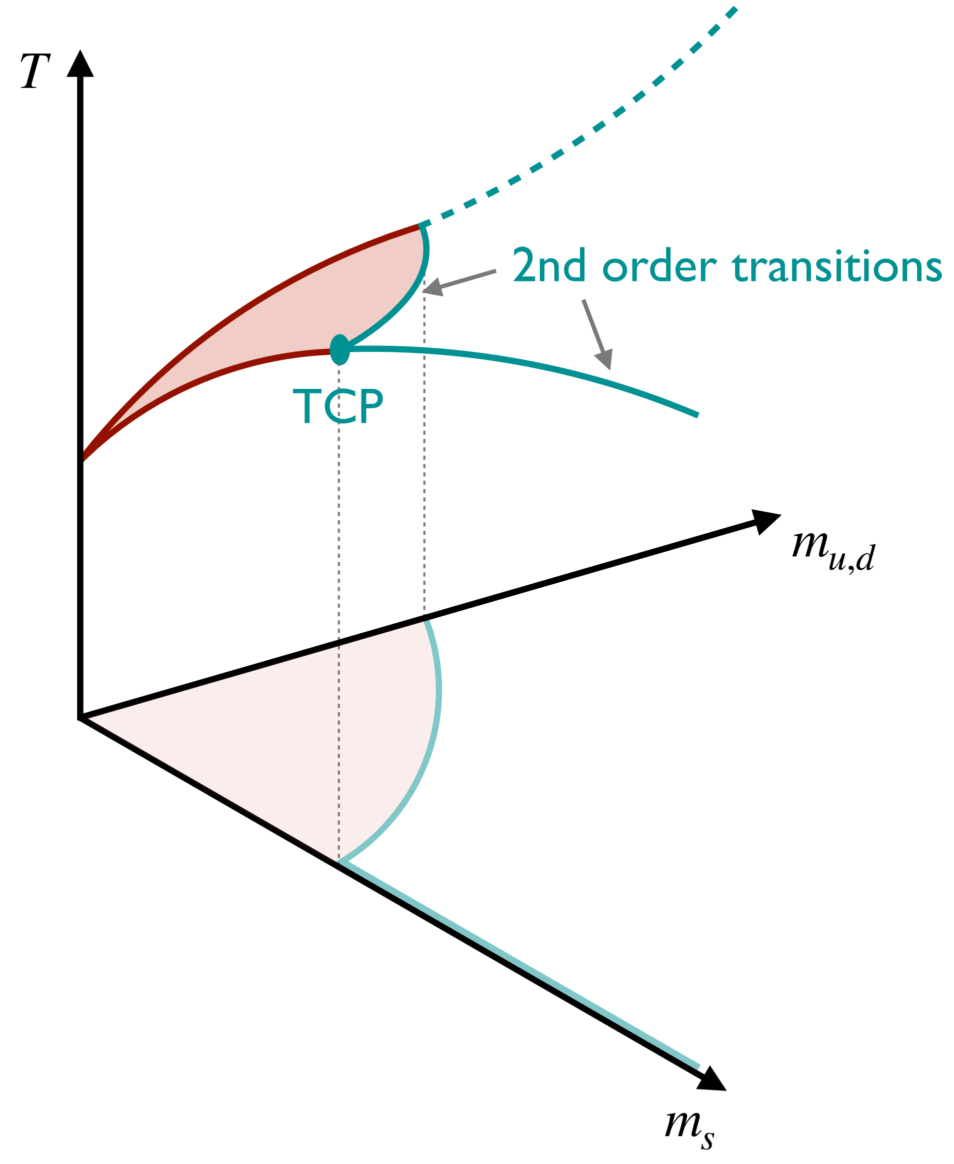
THE CHIRAL PHASE TRANSITION

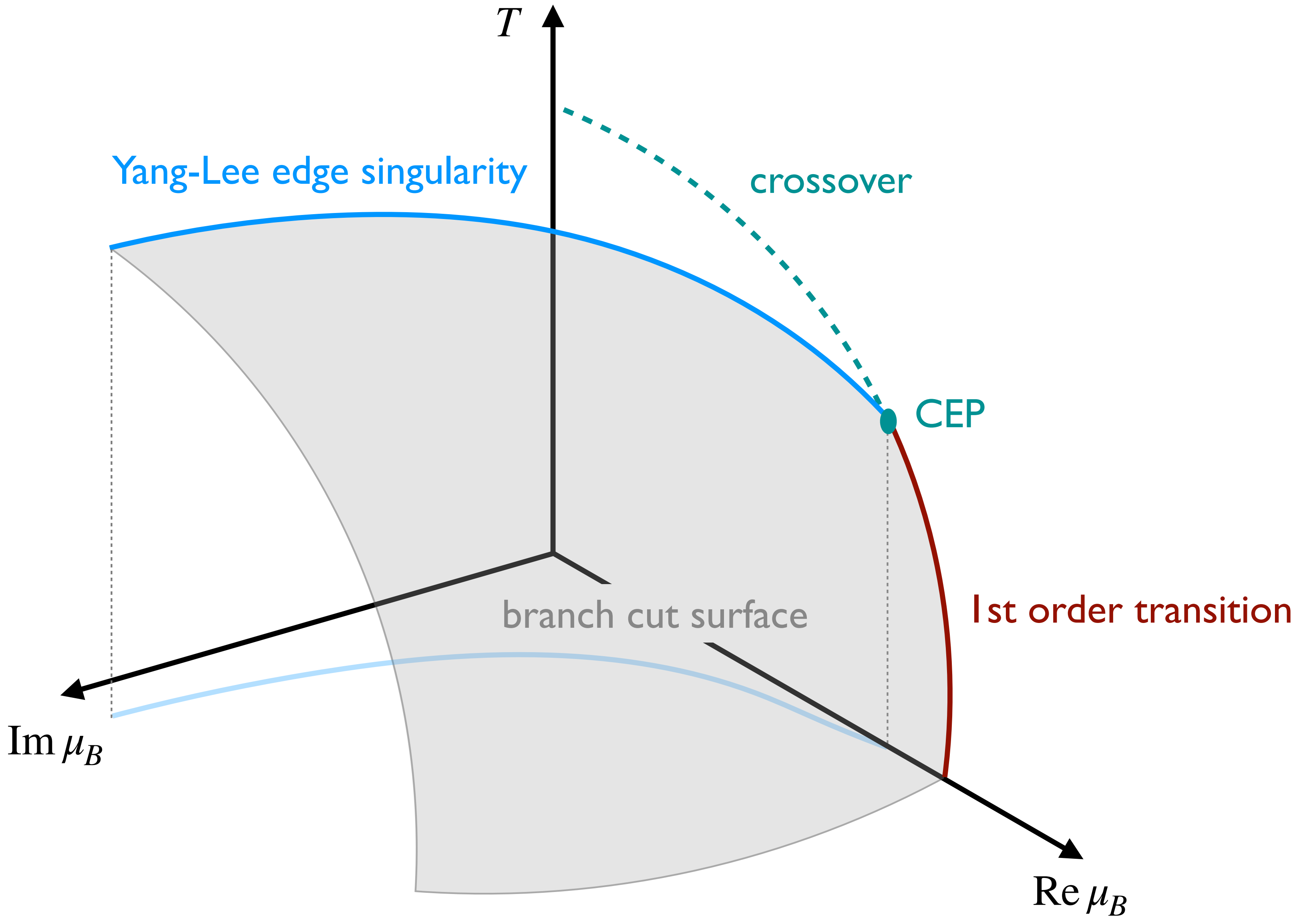
in the $(\mathrm{Im}\, m_{u,d}, m_s\,, \mathrm{Re}\, m_{u,d})$ plane at $T = T_c|_{\mathrm{m_{s,phys}}}$

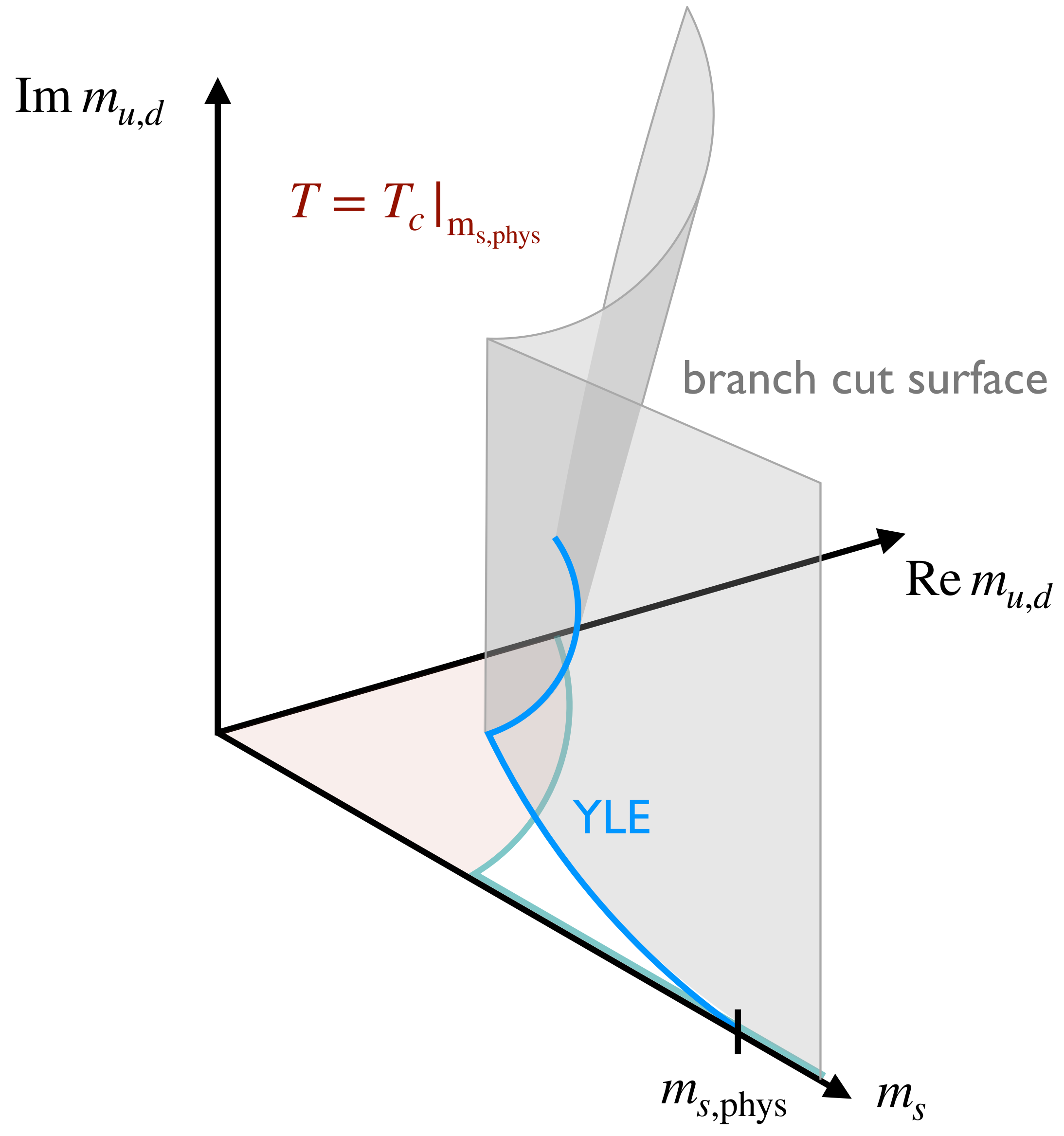


MANY FACES OF THE PHASE DIAGRAM





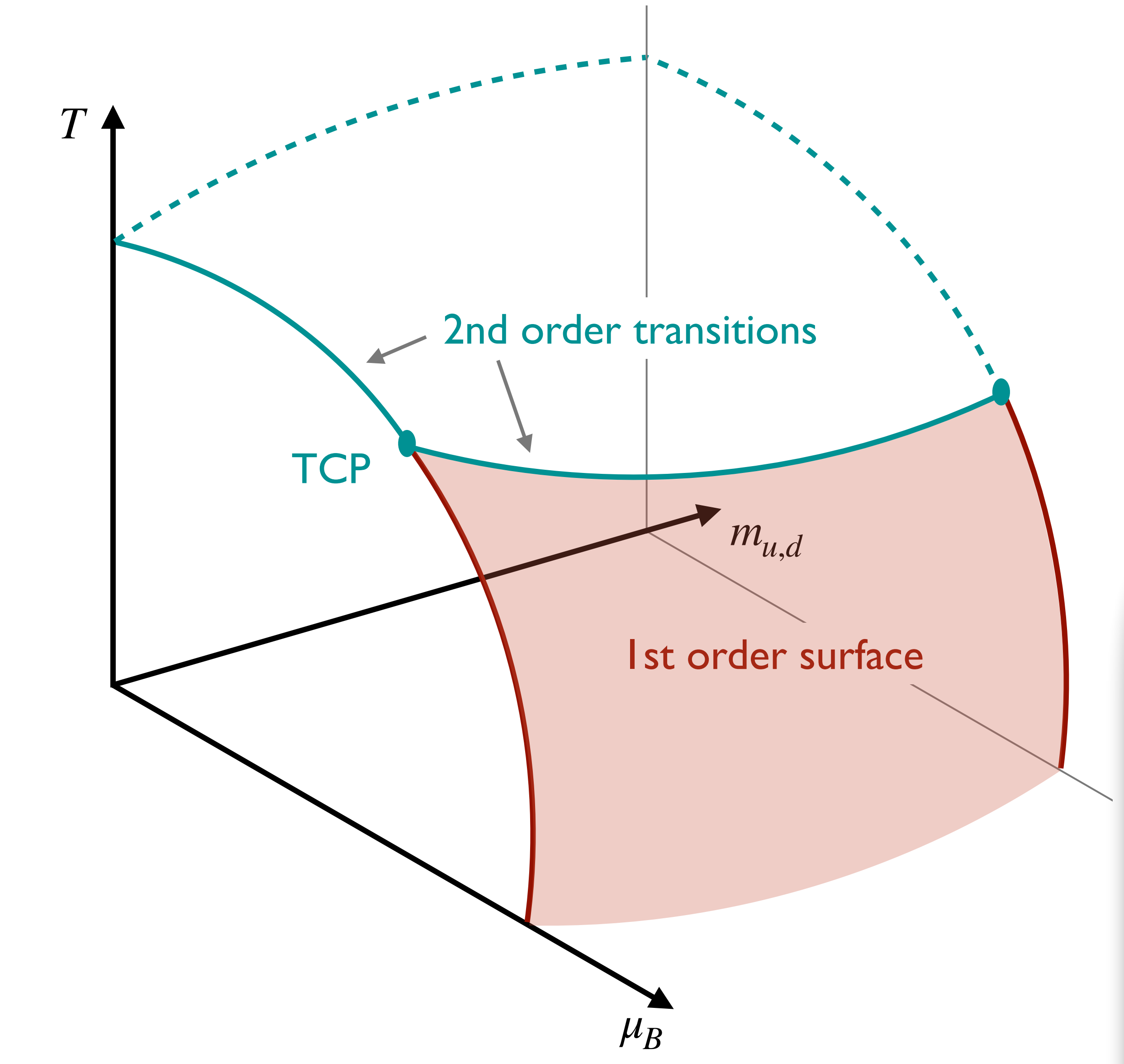


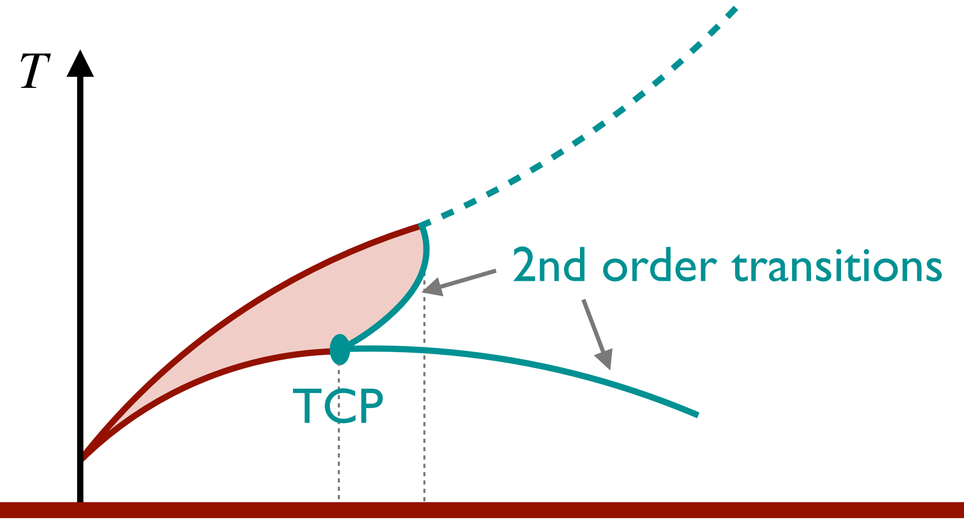


- direct detection of phase transitions challenging
- powerful constraints from universality and analytic structure near 2nd-order transitions

leverage this to understand QCD phase diagram

MANY FACE OF THE PHASE DIAGRAM





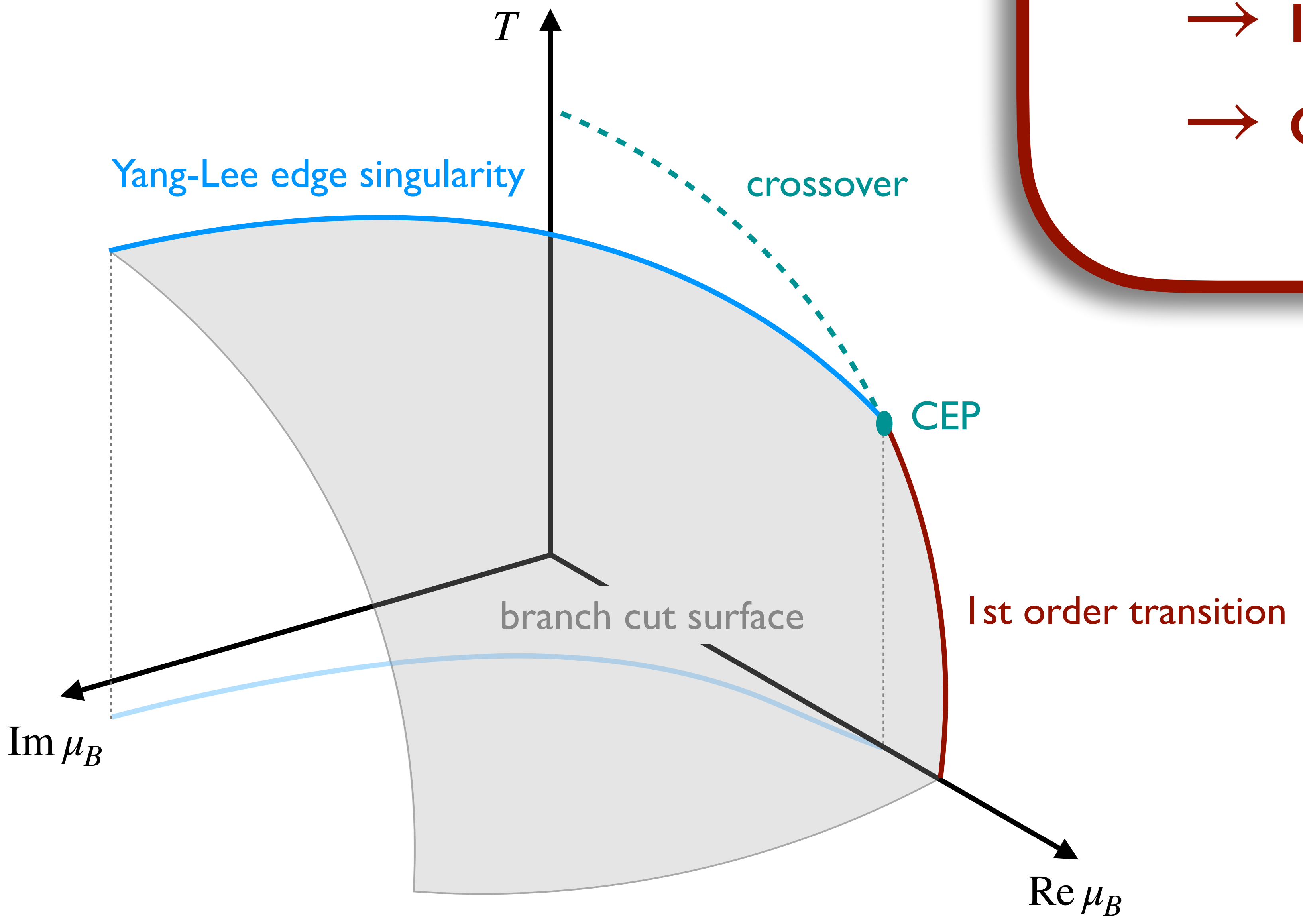
This talk:

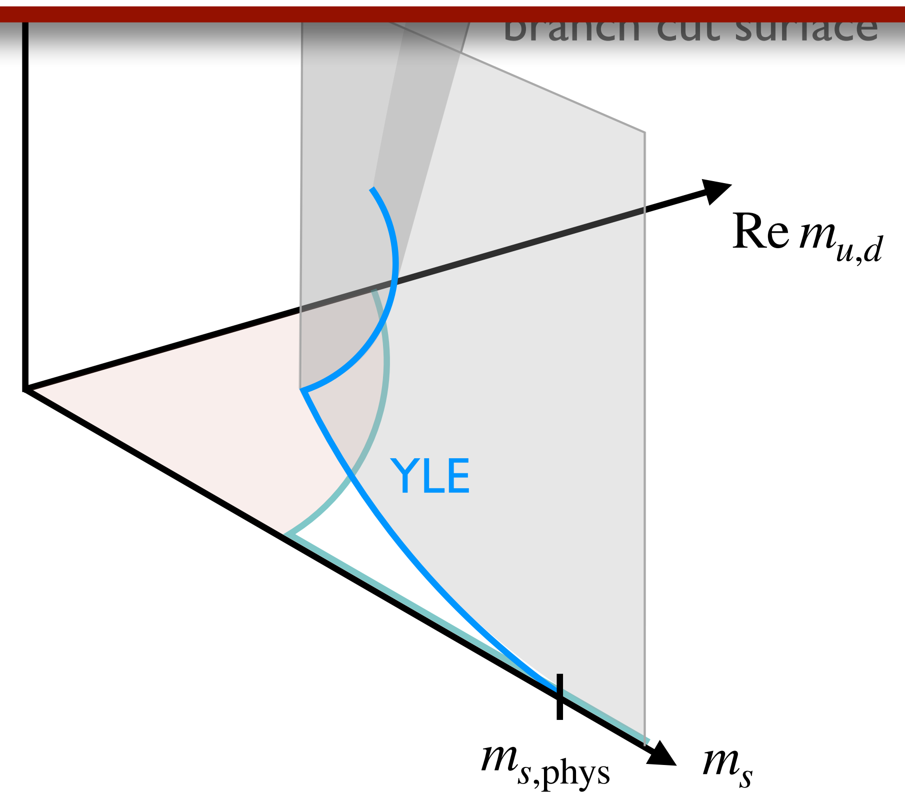
- **universality of YLEs**
  → basics & state-of-the-art
- **application to the phase diagram**
  → importance of non-universal corrections
  → order of the chiral phase transition

- direct detection of phase transitions challenging
- ...erful constraints from ...ersality and analytic structure ... 2nd-order transitions

↓

...erage this to understand QCD phase diagram





UNIVERSALITY OF THE YLE

LEE-YANG THEOREMS

phase structure $\longleftrightarrow$ analytic structure in the complex plane

Consider system of N atoms: grand canonical partition function is polynomial of degree N in a finite volume V,

fugacity $z \sim e^{\mu/T}$,
μ: chemical potential, magnetic field, source,...

$$Z_V = \sum_{i=1}^{N} \mathcal{Z}_i(T)\, z^i = \prod_{i=1}^{N} (z - z_i)$$

canonical partition function with i particles

Lee-Yang zeros

LY theorem
if Z_V has no zeros in a region R in $\mathbb{C}$, then all thermodynamic quantities are analytic for $z \in R$ for $V \to \infty$

[Yang, Lee (1952)]

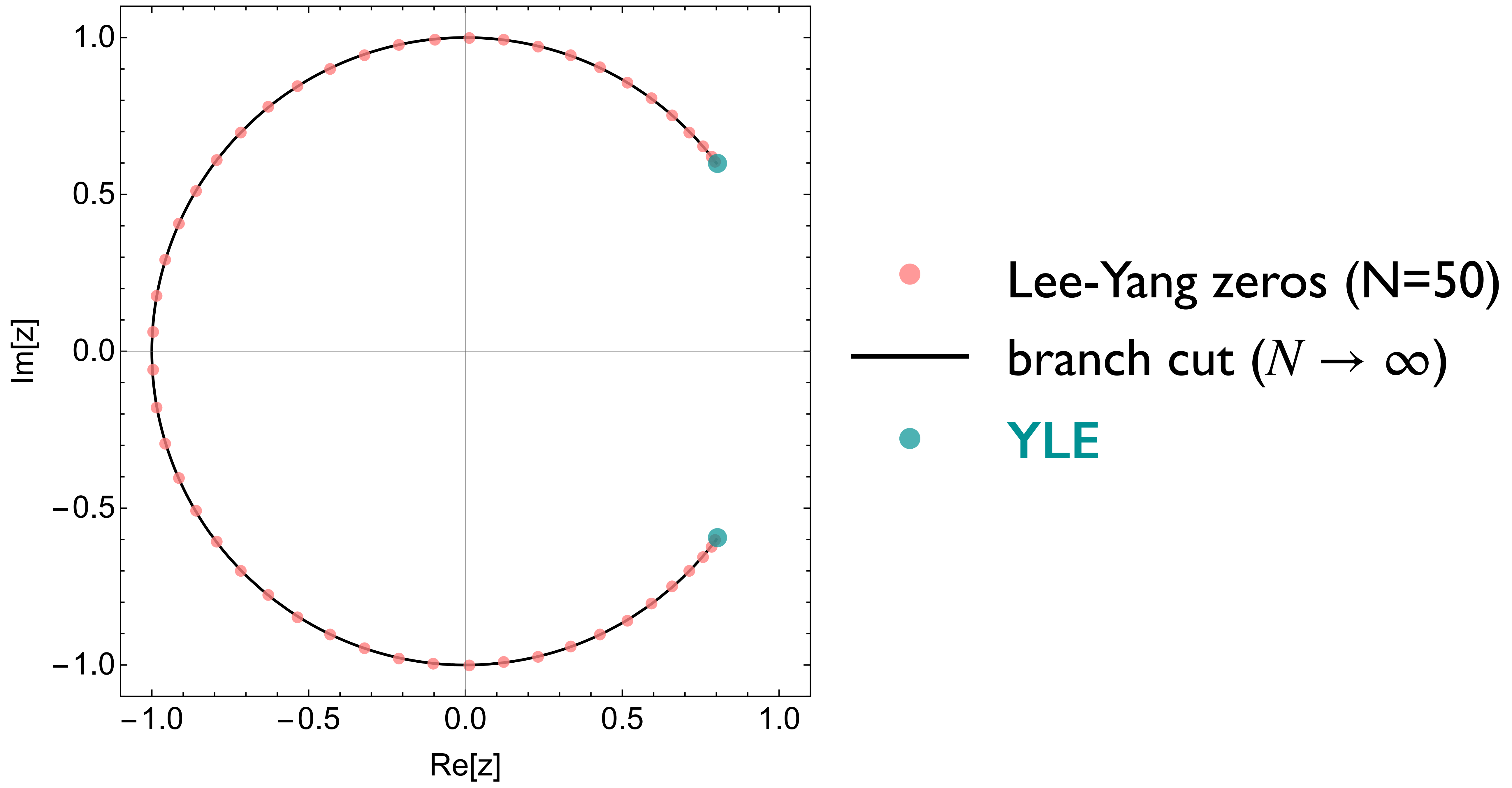
At a phase transition thermodynamic functions can't be analytic

- Lee-Yang zeros are poles of the free energy $f(T,z) = -\lim_{V\to\infty} \frac{T}{V} \ln Z_V(T,z)$
- for $V \to \infty$ they coalesce into branch cuts in the complex z-plane
- branch points: **Yang-Lee edge singularities (YLE)**

$\longrightarrow$ **Lee-Yang zeros/cuts & YLEs encode phase structure**

YANG-LEE EDGE SINGULARITY

Example: analytic structure of the free energy density of the 1d Ising model, $z = e^{2h/T}$



- no thermal phase transition in 1d Ising: YLE never touches the real, positive axis
- zeros/cut on the unit circle/at purely imaginary h: **Lee-Yang circle theorem**

All zeros/cuts/YLEs are at **imaginary magnetic fields**

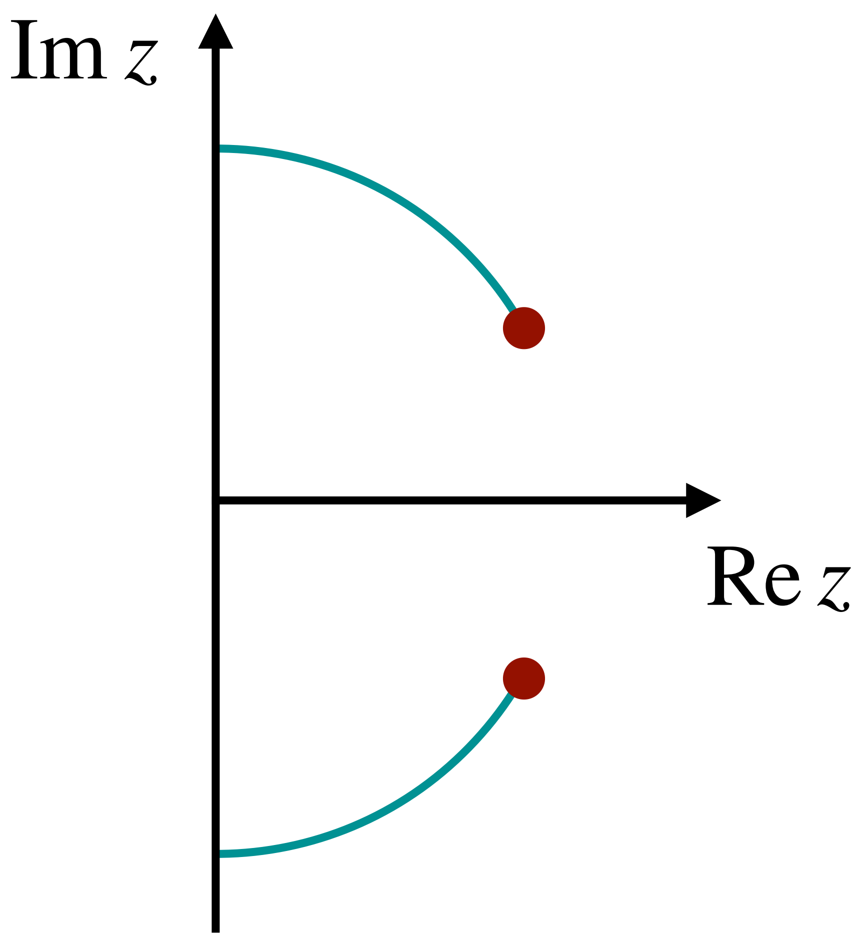
- rigorously proven for ferromagnetic spin-1/2 systems and $O(N = 1,2,3,\infty)$
- systematic results suggest that it holds for all N

[Lee, Yang (1952); Simon, Griffith (1972); Dunlop, Newman (1975); Lieb, Sokal (1981); Kurtze, Fisher (1978); Johnson, FR, Skokov (2020-2022)]

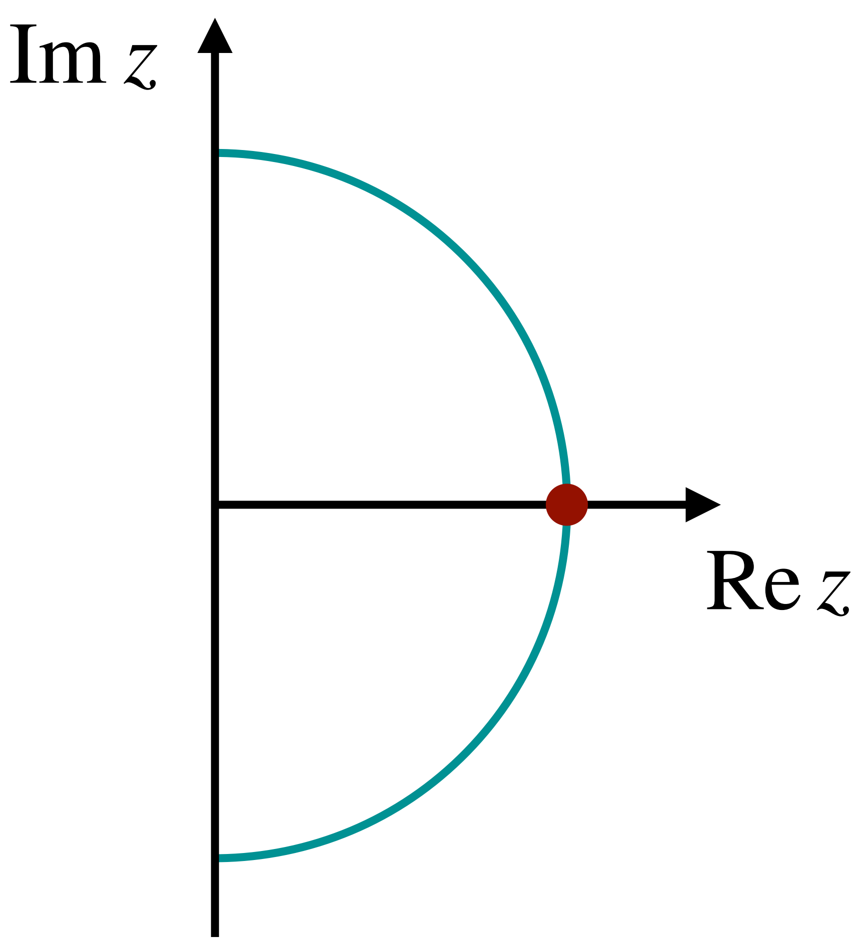
YLE & PHASE TRANSITIONS

Phase transitions can be understood from the location of the YLE:

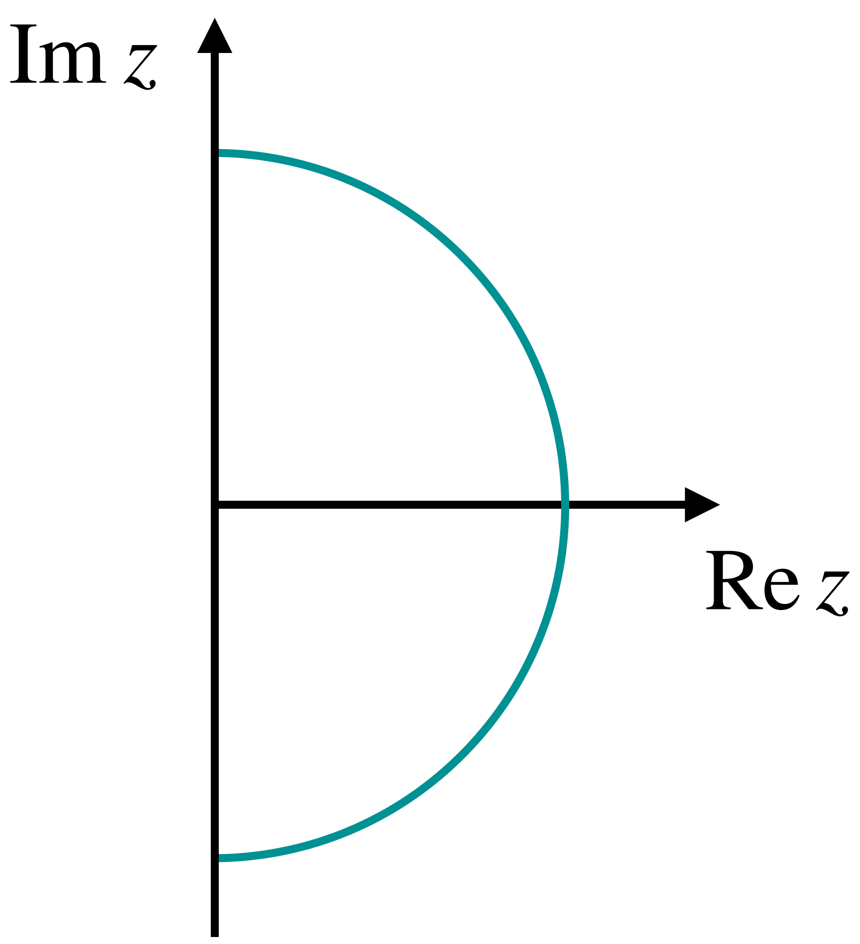
no phase transition
YLE in the complex plane



2nd order transition
YLE pinches real axis



1st order transition
cut cuts real axis



IDENTIFYING THE YLE

Free energy from the effective action, (h: explicit symmetry breaking/magnetic field)

$$f(T,h) = -\,\Omega\big(\bar\phi(T,h)\big) \equiv -\,\frac{T}{V}\,\Gamma\big[\bar\phi(T,h)\big]$$

effective potential — effective action

$$\Gamma[\phi] = \sup_J \left\{ \int_x J\cdot\phi - \ln Z[J] \right\}$$

$$\ln Z[J] = \int \mathcal{D}\phi\, e^{-S[\phi]+\int_x J\cdot\phi}\,,$$

Order parameter field/magnetization $\bar\phi(T,h) = -\,\partial_h f(T,h)$ determined by EoM,

$$\left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\bar\phi} = h$$

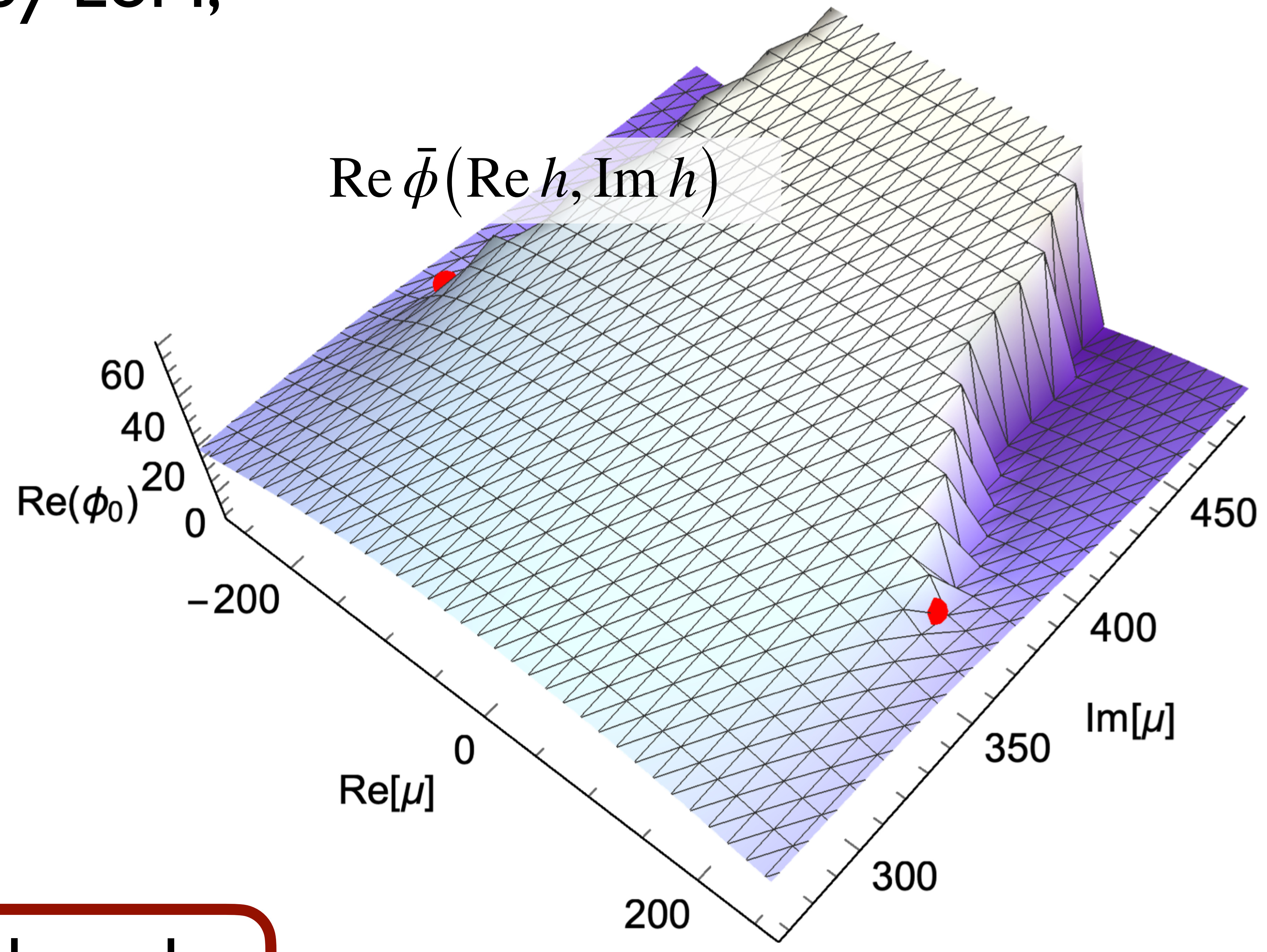
YLE: branch point $h = h_c \in \mathbb{C}$ of $f(T,h)$ for $T \geq T_c$ ($T < T_c$: spinodals)

$\longrightarrow$ edge singularity encoded in magnetization

$\bar\phi(T,h)$ implicitly defined: use implicit function theorem to identify h_{YLE} from Hessian:

$$\det H = \det\left(\left.\frac{\delta^2\Gamma[\phi]}{\delta\phi_i\delta\phi_j}\right|_{\phi=\bar\phi(T,h_c)} \right) = 0$$

zero eigenvalue ↔ critical mode:
YLE is a critical point!

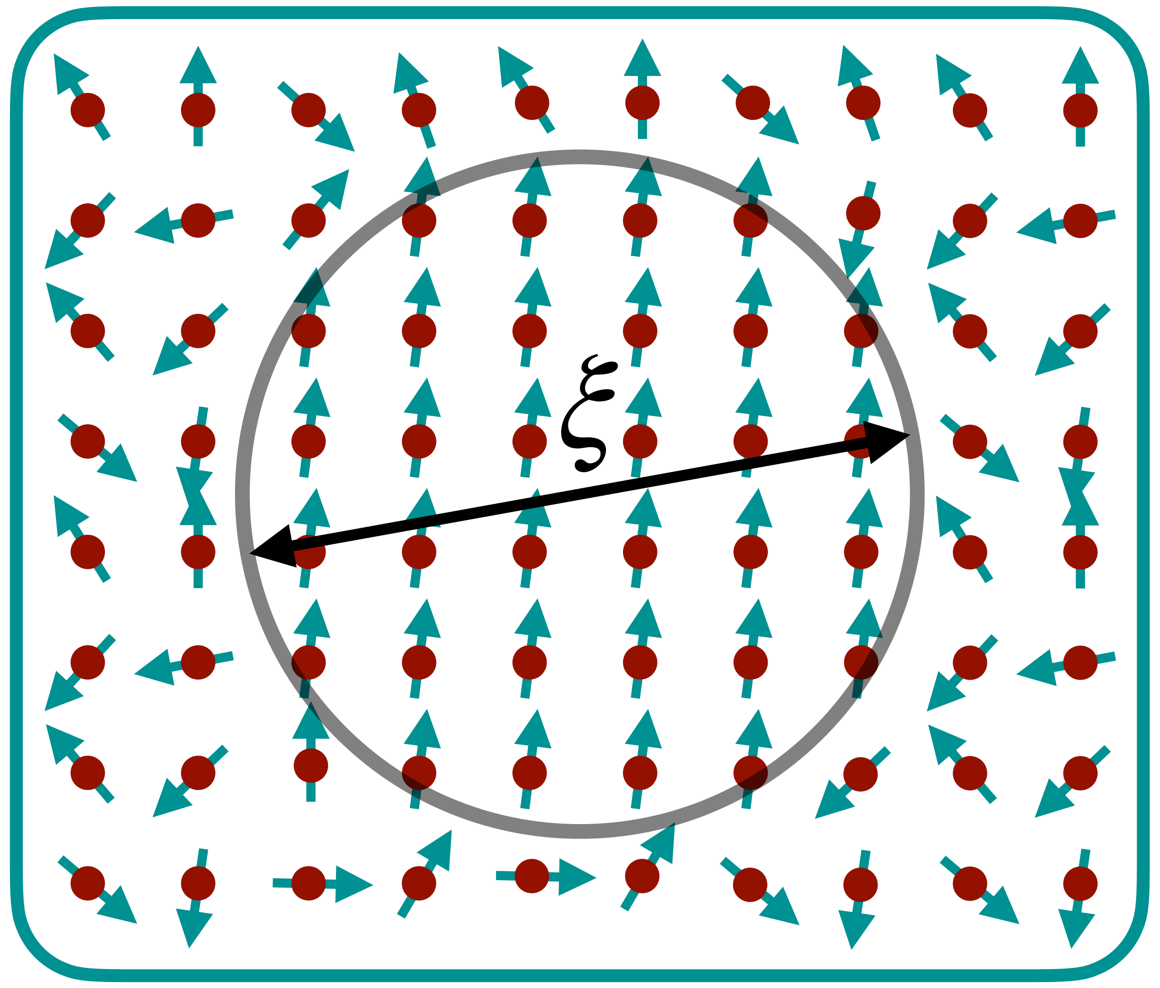


[Mukherjee, FR, Skokov (2021)]

[Haensch, FR, von Smekal (2023)]

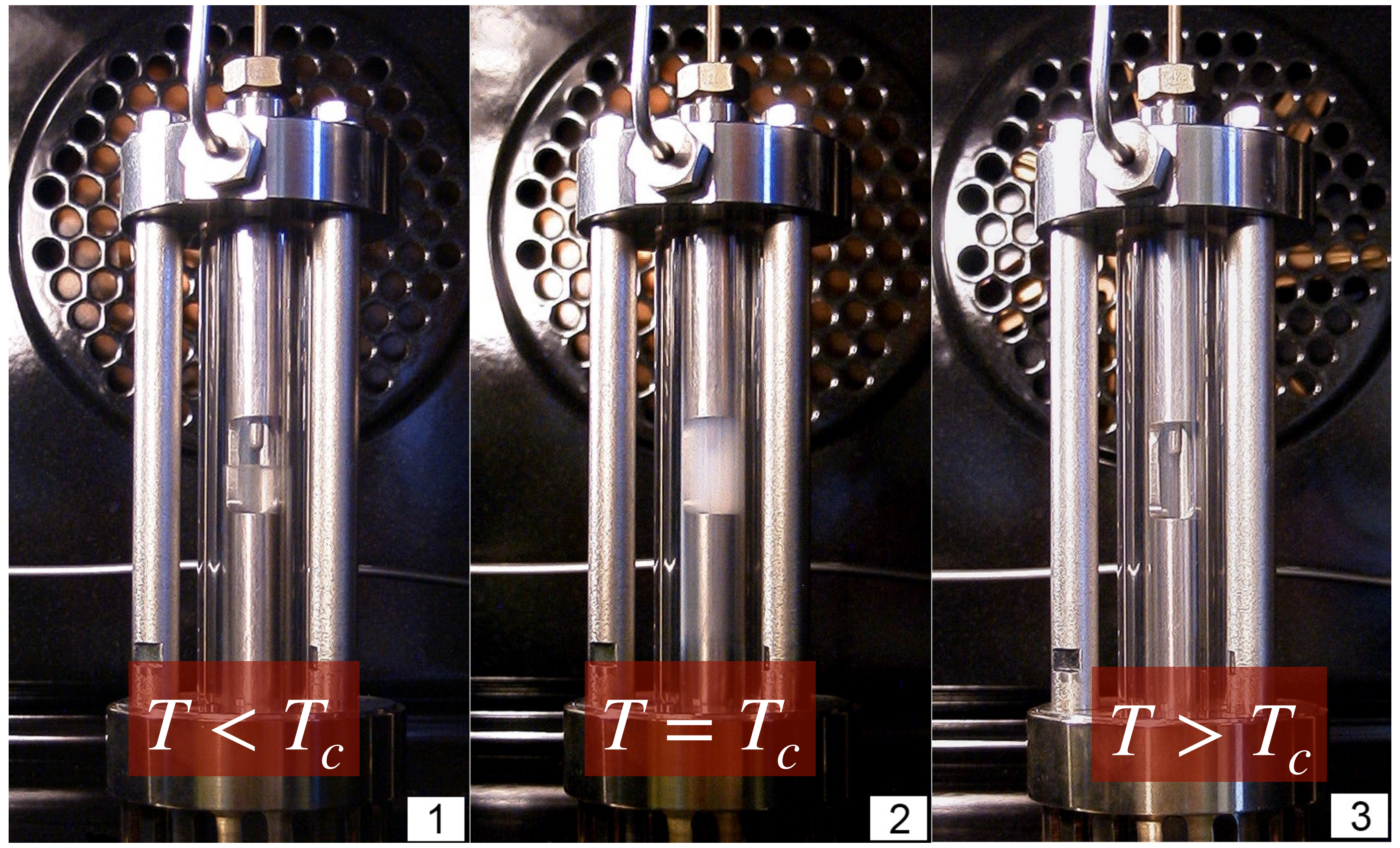
CRITICAL PHENOMENA & UNIVERSALITY

correlation length



2nd order transition:
$\xi \to \infty$

fluctuations on all length scales

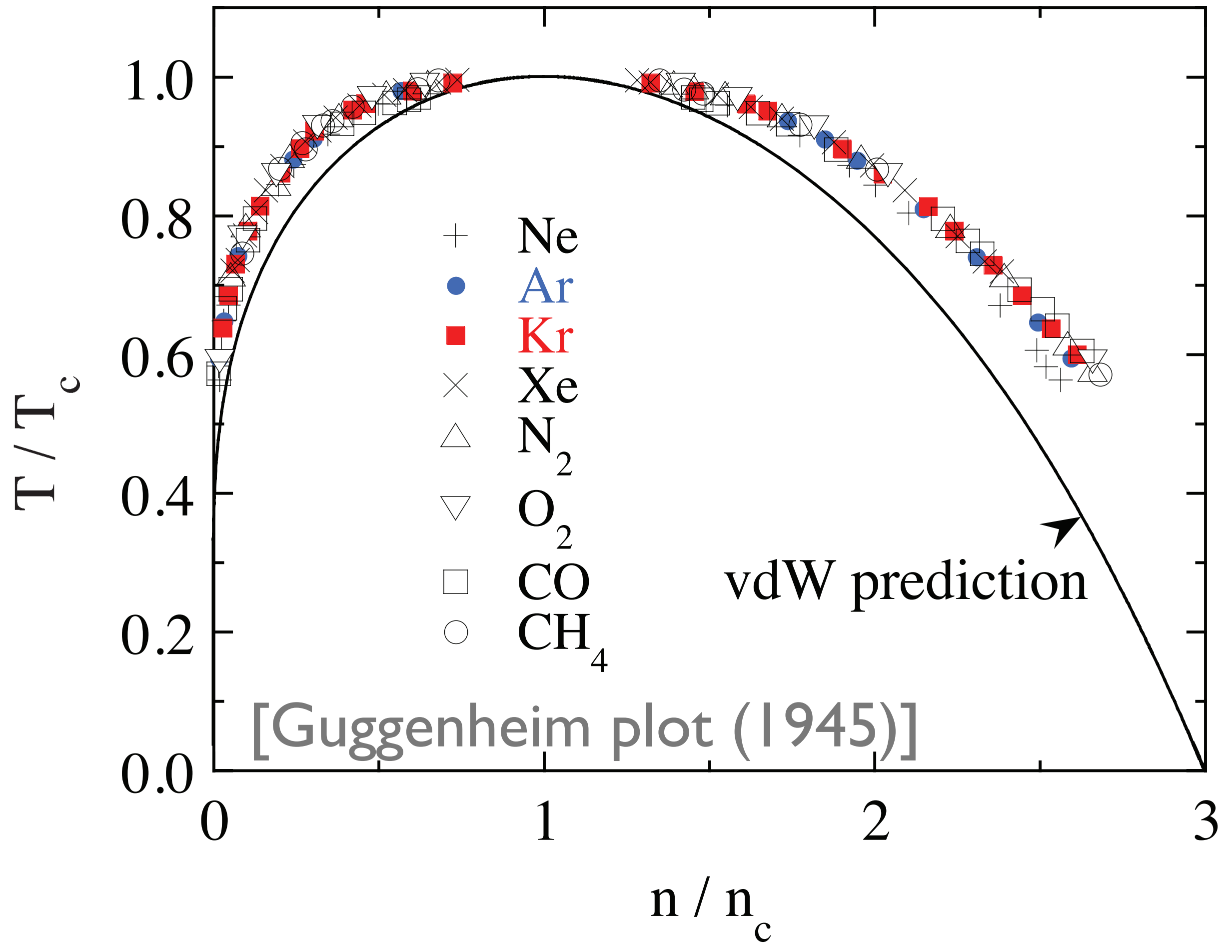


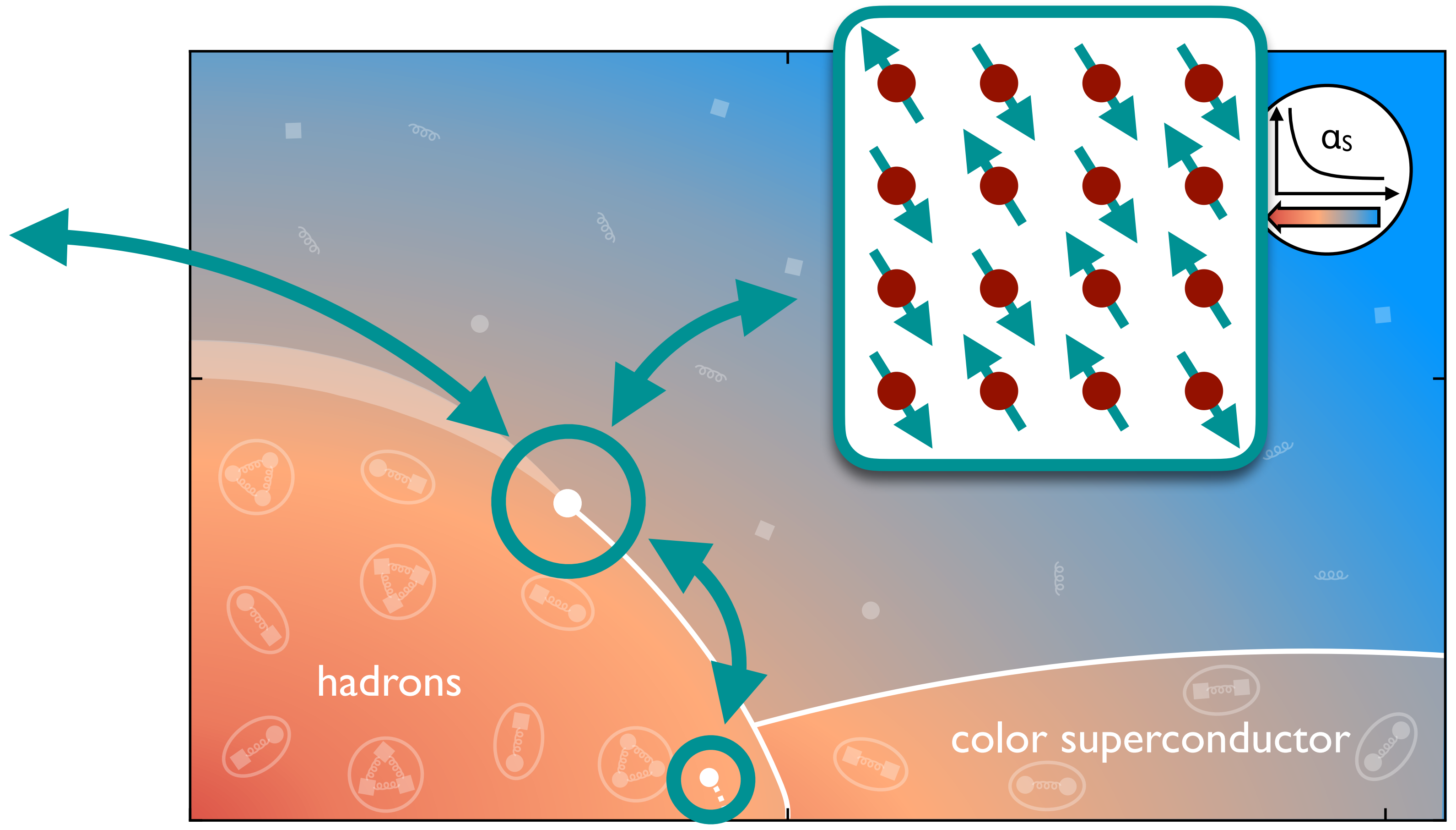
critical opalescence of ethane [Wikipedia]

Near the critical point the system is scale invariant and microscopic details are irrelevant

Universality: main features of the system are described by universal critical exponents, e.g., $\xi \sim (T - T_c)^{-\nu}$

example:
liquid-gas transition
=
3d Ising
=
QCD CEP





SCALING HYPOTHESIS AND THE EDGE SINGULARITY

Scale invariance near 2nd order transition: free energy is homogeneous function (for two relevant directions)

$$\boxed{f(t,h) = b^{-d} f_f\left(t\, b^{\frac{1}{\nu}}, h\, b^{\frac{\beta\delta}{\nu}}\right) + f_{\mathrm{reg}}(t,h)}$$

$t = \frac{T - T_c}{T_c}$ dim.less rescaling

f_f is a universal scaling function and β, δ, ν are universal critical exponents

$\longrightarrow$ **YLE is universal at criticality**

Consider magnetic scaling: $b = |h|^{-\frac{\nu}{\beta\delta}} \longrightarrow f(t,h) = h^{\frac{d\nu}{\beta\delta}} f_f(z) + \ldots$ with scaling variable $z = \frac{t}{h^{\frac{1}{\beta\delta}}}$

Order parameter described by universal magnetic equation of state $f_G(z)$ (using $\bar{\phi}(t,h) = -\partial_h f(t,h)$)

circle theorem $h_c \in i\mathbb{R}$:

$$\boxed{\bar{\phi}(t,h) = h^{\frac{1}{\delta}} f_G(z) + \bar{\phi}_{\mathrm{reg}}(t,h)}$$

$\longrightarrow$ **YLE is branch point of $f_G(z)$**

$$\boxed{z_c = |z_c|\, e^{\pm \frac{i\pi}{2\beta\delta}}}$$

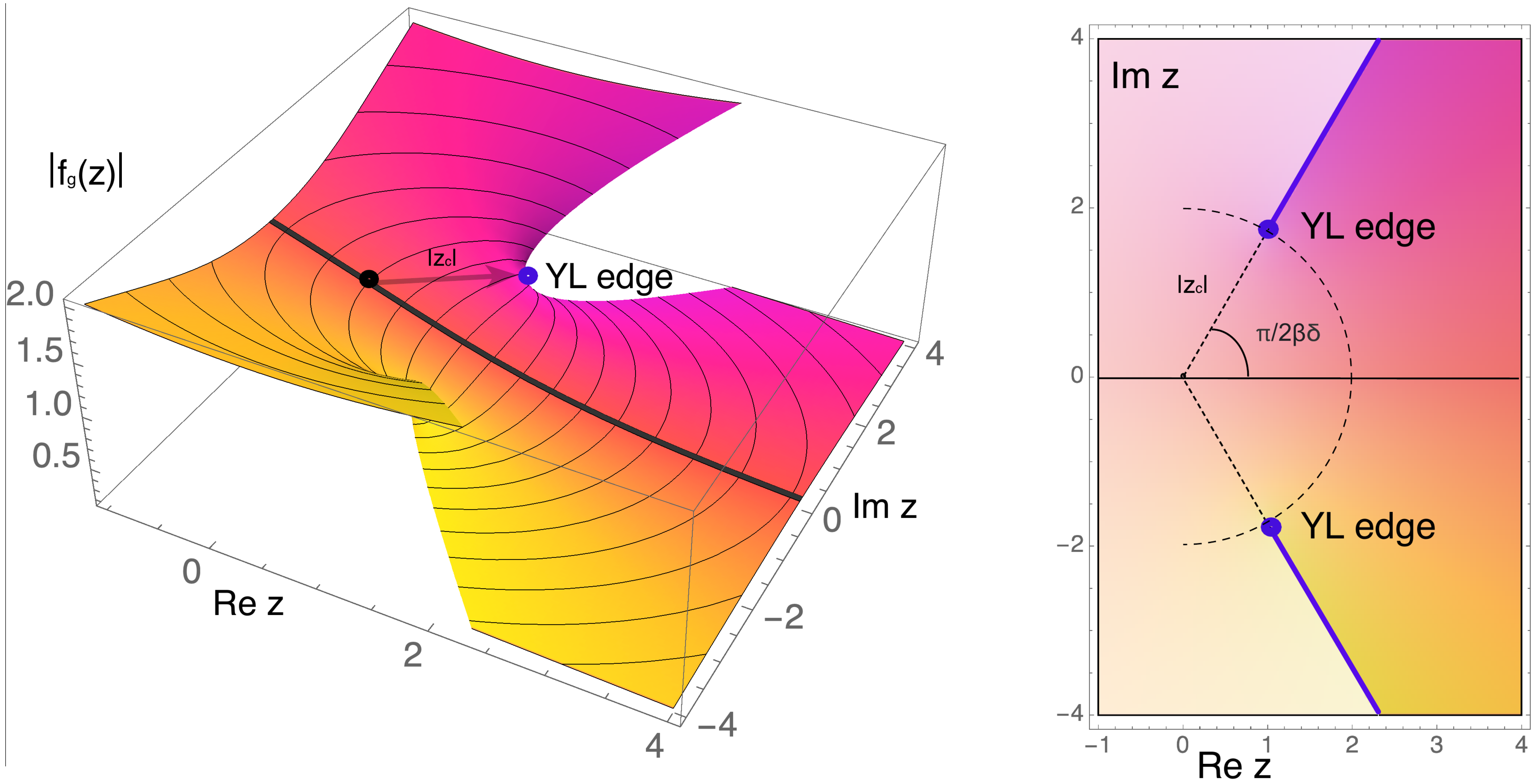
MEAN FIELD INTUITION

Relevant features can be illustrated by simple mean-field analysis of ϕ^4-theory/$O(N)$-model

$$\Omega(\phi) = \frac{1}{2} t\,\phi^2 + \frac{1}{4}\phi^4 - h\phi$$

- EoM at $t=0$: $\bar{\phi} = h^{\frac{1}{3}} \;\Rightarrow\; \delta_{\mathrm{MF}} = 3$
- EoM at $h=0$: $\bar{\phi} = (-t)^{\frac{1}{2}} \;\Rightarrow\; \beta_{\mathrm{MF}} = \dfrac{1}{2}$
- YLE from $\Omega'(\bar{\phi}_c) = 0 = \Omega''(\bar{\phi}_c)$:

$$h_c = \pm\, 2i\,(t/3)^{3/2}\,, \qquad z_c = \frac{3}{2^{2/3}}\, e^{\pm\frac{i\pi}{3}}$$



credit: V. Skokov

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$$\Omega(\phi) = \frac{1}{2}t\,\phi^2 + \frac{1}{4}\phi^4 - h\phi$$

- $\Omega''(\bar{\phi}_c) = 0 \;\Rightarrow\; \bar{\phi}_c = \pm i\sqrt{t/3}$: nonzero imaginary magnetization at $t > 0$
- expand Ω about $\bar{\phi}_c$, $\phi = \varphi + \bar{\phi}_c$

$\longrightarrow$ $\Omega(\varphi) \approx i\bar{\lambda}\,\varphi^3 - \bar{h}\varphi$ $\longrightarrow$ **YLE has its "own" universality class** (LY-theory), [Fisher (1978)] independent of underlying $O(N)$ universality of ϕ^4-theory

$\bar{\lambda} = \sqrt{t/3}$

- $O(N)$ model at $t, h \neq 0$: 2 relevant directions ($\leftrightarrow$ 2 independent crit. exponents), upper crit. dimension $d_c = 4$
- LY-theory at $t, h \neq 0$: 1 relevant direction, $d_c = 6$

$\delta_{d=3}$	$O(1)$	$O(4)$	LY
MF	3	3	2
best*	$4.78984(1)$	$4.7915(67)$	$11.7(1)$

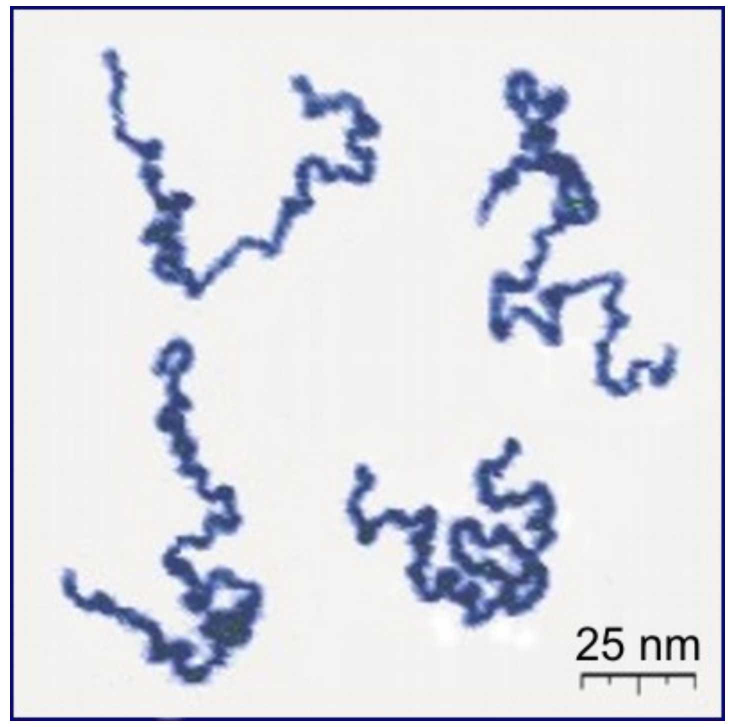
$\delta = \dfrac{d+2-\eta}{d-2+\eta}$ $\longrightarrow$ $\eta \approx -0.53$ at YLE!

*conformal bootstrap & FRG:
[Gliozzi, Rago (2014); Kos et al. (2014, 2015)]
[Balog et al. (2019), De Polsi et al. (2020)]

UNIVERSALITY OF THE EDGE SINGULARITY

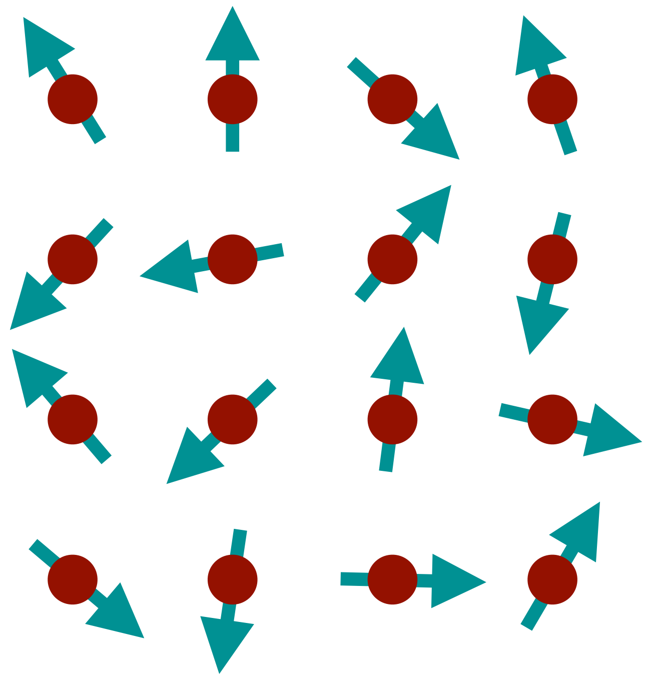
Consider $O(N)$ model as relevant example

$N = 0$: polymer chains

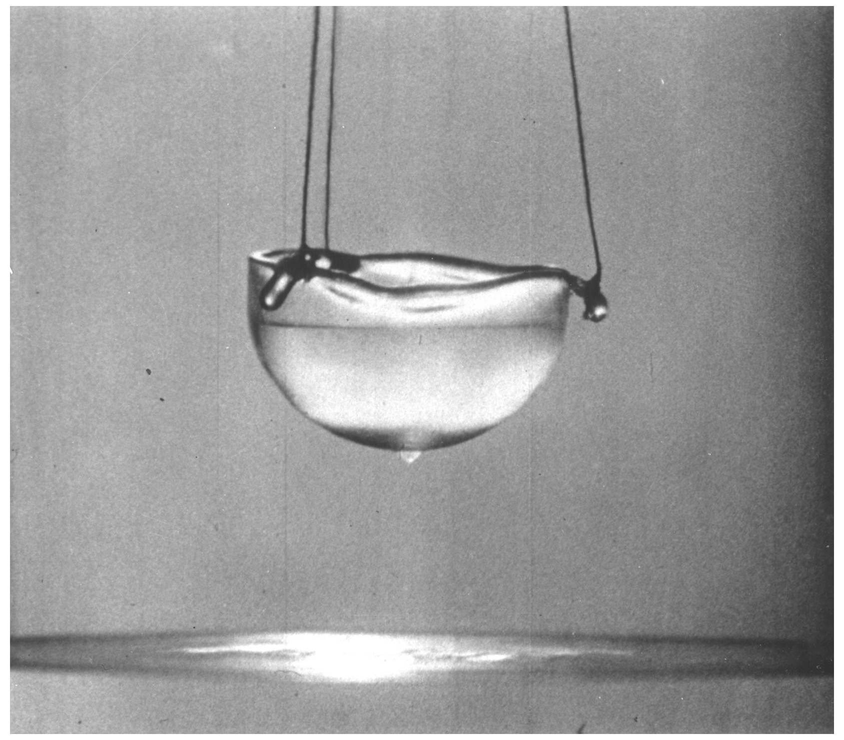


[Wikipedia]

$N = 1$: Ising model

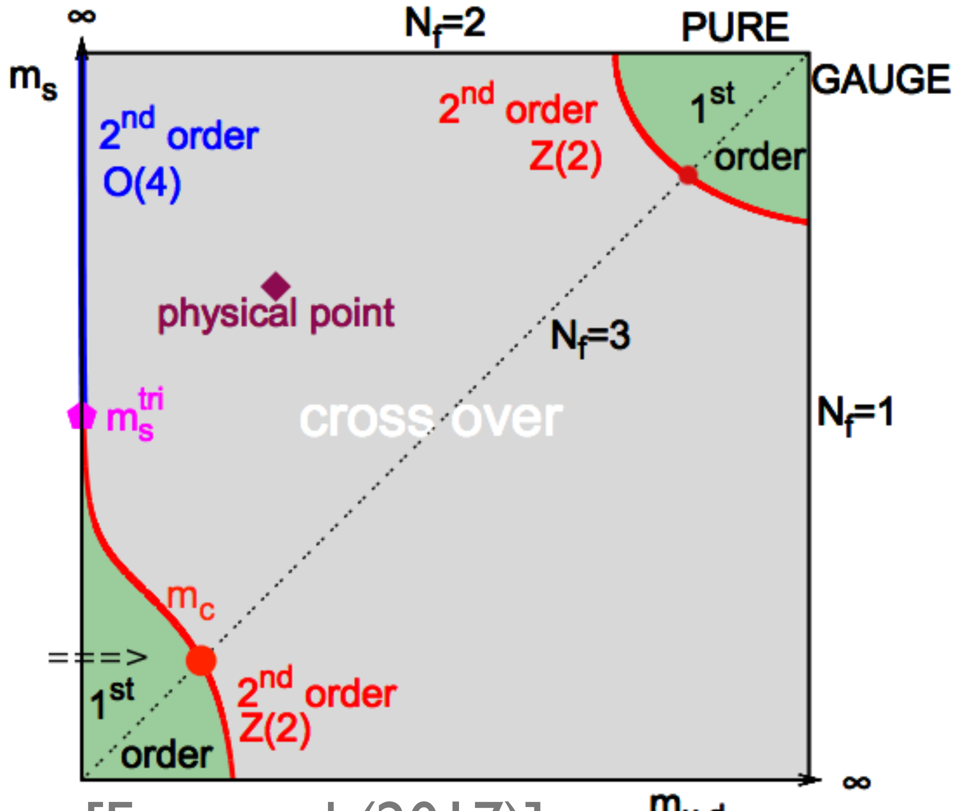


$N = 2$: (XY model) superfluidity



[Wikipedia]

$N = 4$: QCD in the chiral limit



[Forcrand (2017)]

$z_c = |z_c| e^{\pm \frac{i\pi}{2\beta\delta}}$ is universal in scaling region of Wilson-Fisher fixed point.

Phase is well known from various methods: CB, Monte Carlo, $d_c - \epsilon$ expansion, FRG, ...

But what about $|z_c|$?

- $d_c = 6$: non-perturbative for $d < 6$ → ~~$4 - \epsilon$ expansion~~
- complex parameters: sign problem → ~~Monte Carlo~~
- analytic structure of scaling function → ~~CB~~

systematic & direct (i.e. no reconstruction, extrapolation, ...)

⟶ **FRG is the only way** for $2 < d < 6$

$d = 1$ trivial; CFT methods at $d = 2$

FRG: [Johnson, FR, Skokov (2020-2022)]

CFT: [Fonseca, Zamolodchikov (2001); Xu, Zamolodchikov (2022)]

FUNCTIONAL RENORMALIZATION GROUP

Successively integrate-out fluctuations

- start with bare action $\Gamma_\Lambda = S$ at small distance/large momentum scale Λ
- gradually include fluctuations of larger size by integrating out modes with increasingly small momenta: Wilson RG (Nobel Prize in 1982)

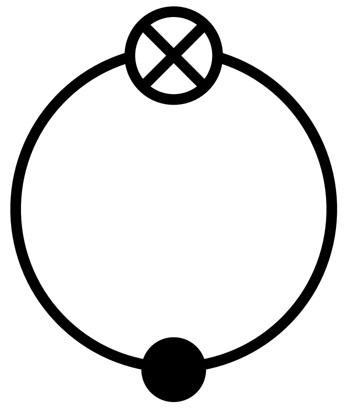
→ running couplings with RG scale k
→ new couplings are generated dynamically

→ **Scale dependent effective action Γ_k:** incorporates all fluctuations down to scale k

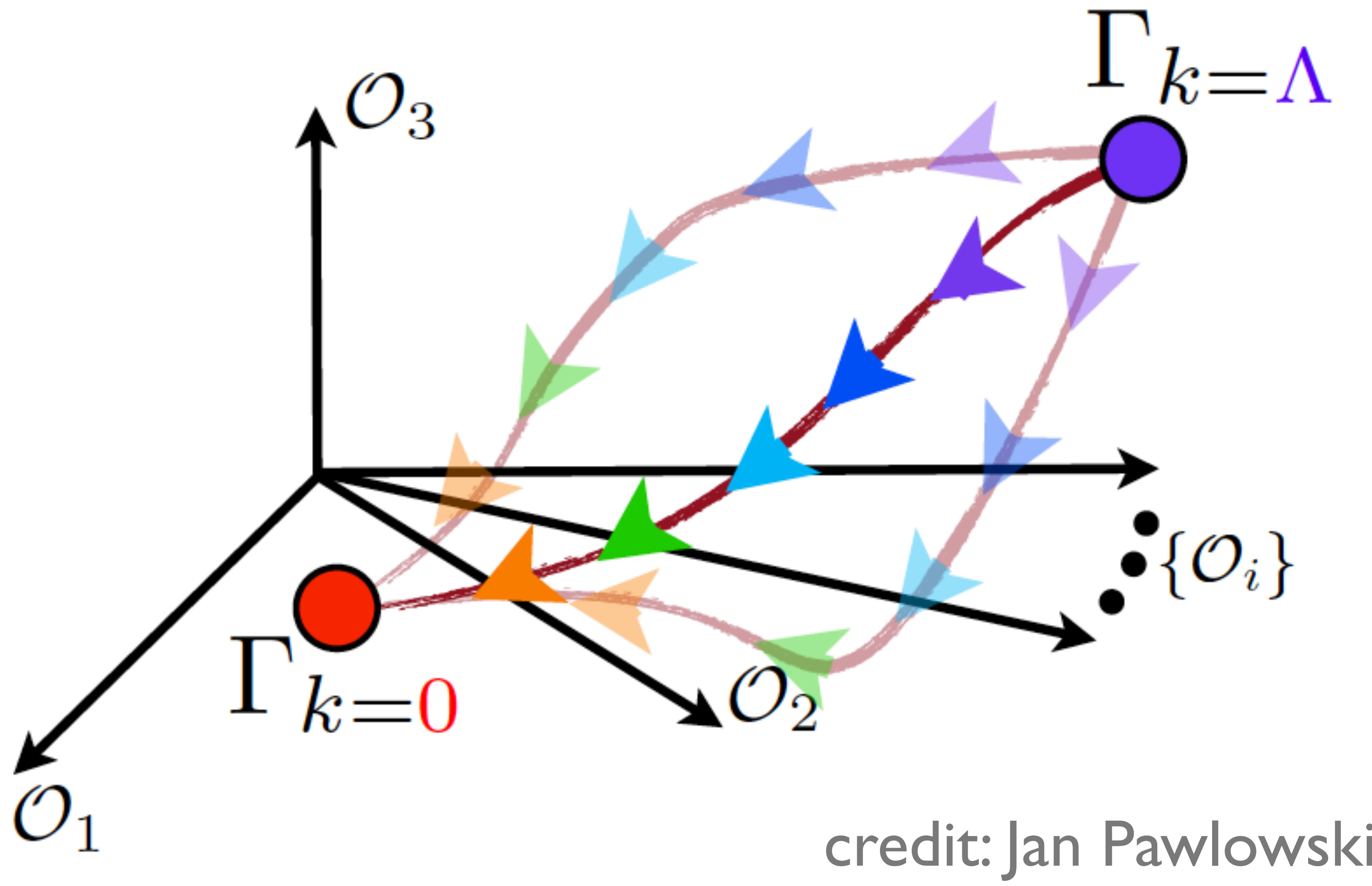
$\Gamma_\Lambda = S$ → Γ_k → $\Gamma_0 = \Gamma$

Practical implementation: Wetterich equation [Wetterich (1993)]

$$\boxed{\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1} \cdot \partial_k R_k\right] = \frac{1}{2}}$$



- pros: one-loop exact, non-perturbative, no sign problem
- cons: requires truncation



credit: Jan Pawlowski

FRG FOR CRITICAL PHENOMENA

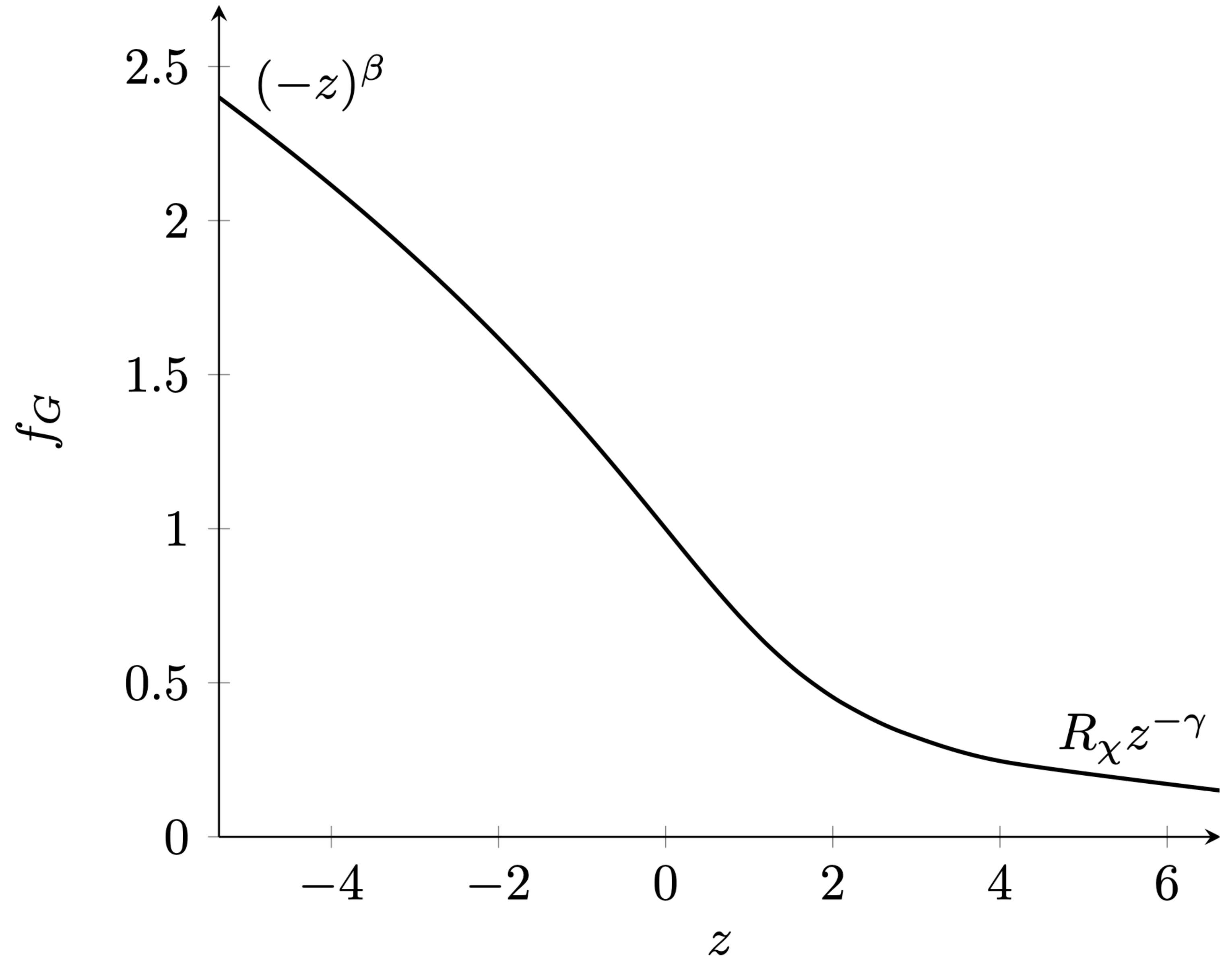
Consider $O(N)$-model with $\phi = (\phi_1, \ldots, \phi_N)$

- systematic truncation for critical phenomena: derivative expansion in $\boldsymbol{p}^2/k^2$ [Balog et al. (2019), De Polsi et al. (2020)]

$$\Gamma_k = \int d^d x \left\{ U_k(\phi) + \frac{1}{2} Z_k(\phi) \left(\partial_\mu \phi\right)^2 + \frac{1}{4} Y_k(\phi) \left(\partial_\mu \rho\right)^2 \right\} + \mathcal{O}\left(\partial^4\right)$$

- Taylor expand U_k, Z_k, Y_k around $\phi = \bar{\phi}_k = (\bar{\sigma}_k, 0, \ldots, 0)$ (σ: critical radial mode)
  - conventional choice: $\partial_\sigma U|_{\bar{\phi}_k} = h = \text{const}. \to 0$ ⟵ requires fine-tuned initial conditions to find FP
  - convenient for Yang-Lee FP: $\partial_\sigma^2 U|_{\bar{\phi}_k} = m_\sigma^2 = \text{const}. \to 0$

    disadvantage: numerically expensive in broken phase as expansion point lies in flat part of the convex potential for any $k > 0$



⟶ follow RG flow of critical point in symmetric phase & read-off h_c

⟶ convenient scaling variable: $\zeta = \dfrac{z}{R_\chi^{1/\delta}}$

- use optimized regulator for NLO derivative expansion: $R_k(q^2) = a\, Z_{\sigma,k}\,(k^2 - q^2)\,\theta(k^2 - q^2)$ [Litim (2001)]

a: free parameter to estimate regulator dependence (truncation error)

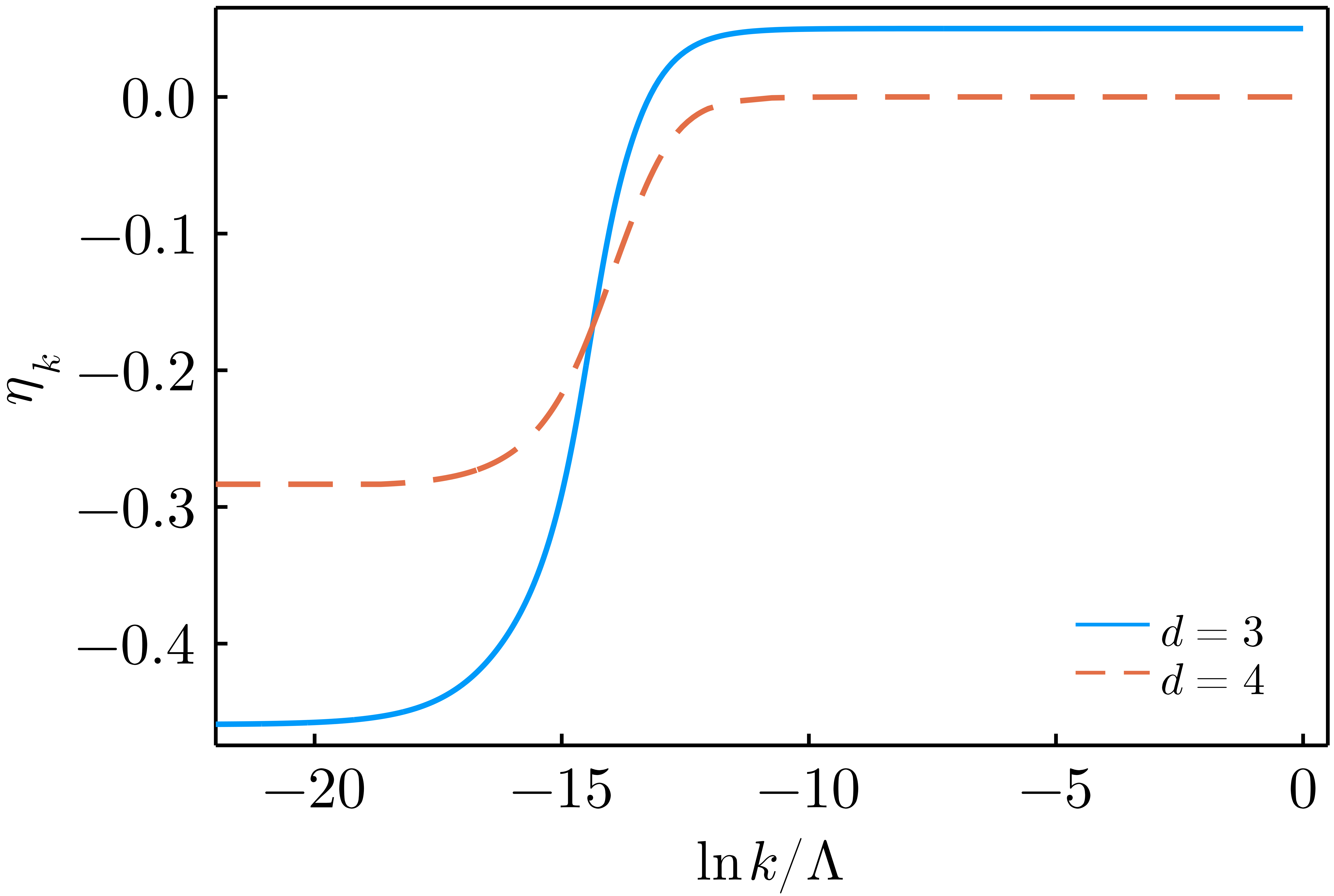
RG FLOW FROM WF TO LY

Initialize system close to Wilson-Fisher and follow RG flow to Lee-Yang fixed point (here: Ising ($O(1)$) model)

[FR, Skokov (2022)]

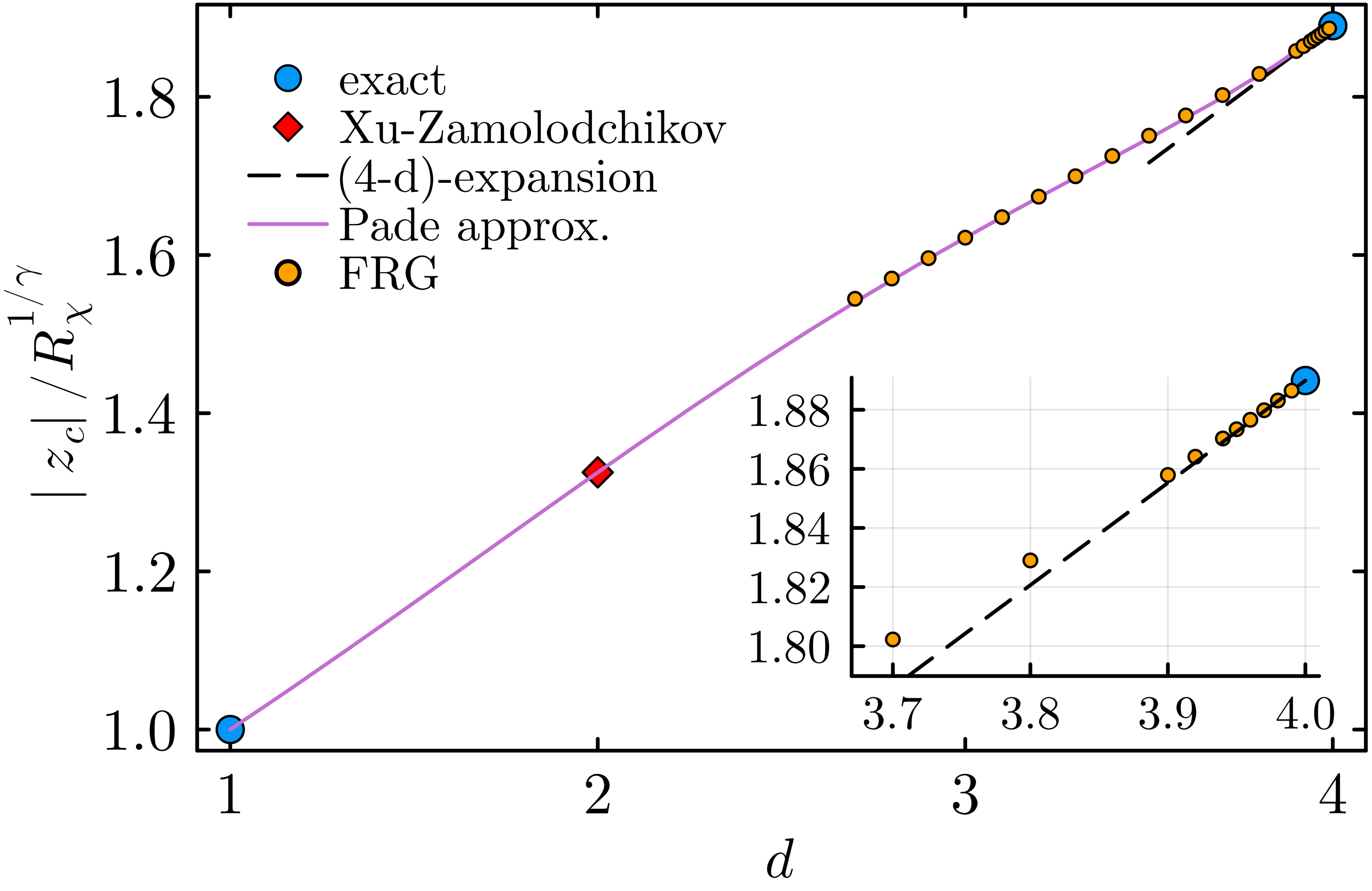
anomalous dimension

$$\eta_k = -\frac{\partial_t Z_k}{Z_k}$$



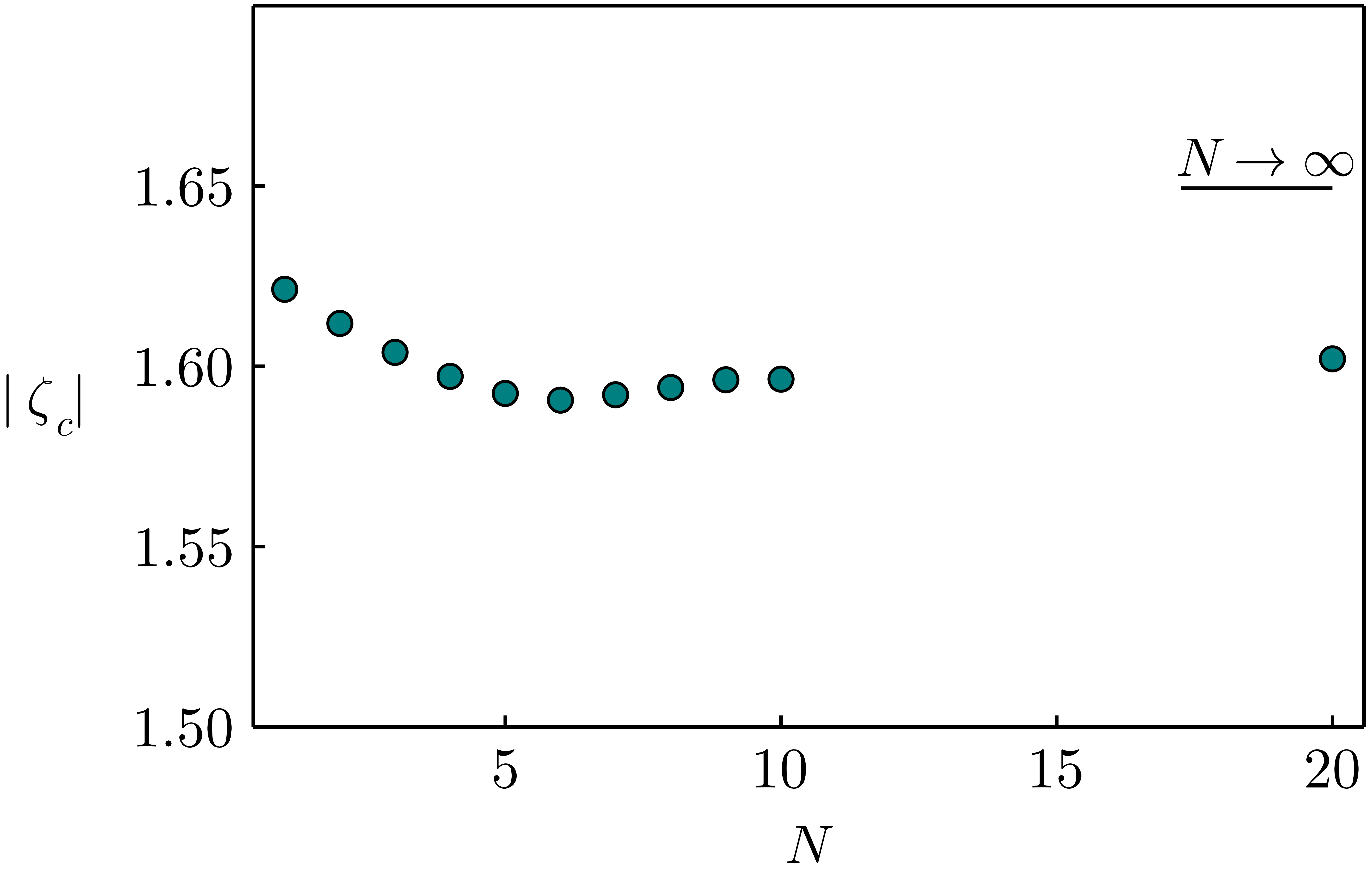
ISING YLE

Edge singularity in the Ising model [FR, Skokov (2022)]



D=3 YLE

Edge singularity for various N in $3d$ [Johnson, FR, Skokov (2022)]



using known values of R_χ:

N	1	2	3	4	5
$\lvert z_c \rvert$	2.43(4)	2.04(8)	1.83(6)	1.69(3)	1.55(4)

→ universal YLE location known for all $O(N)$ models!

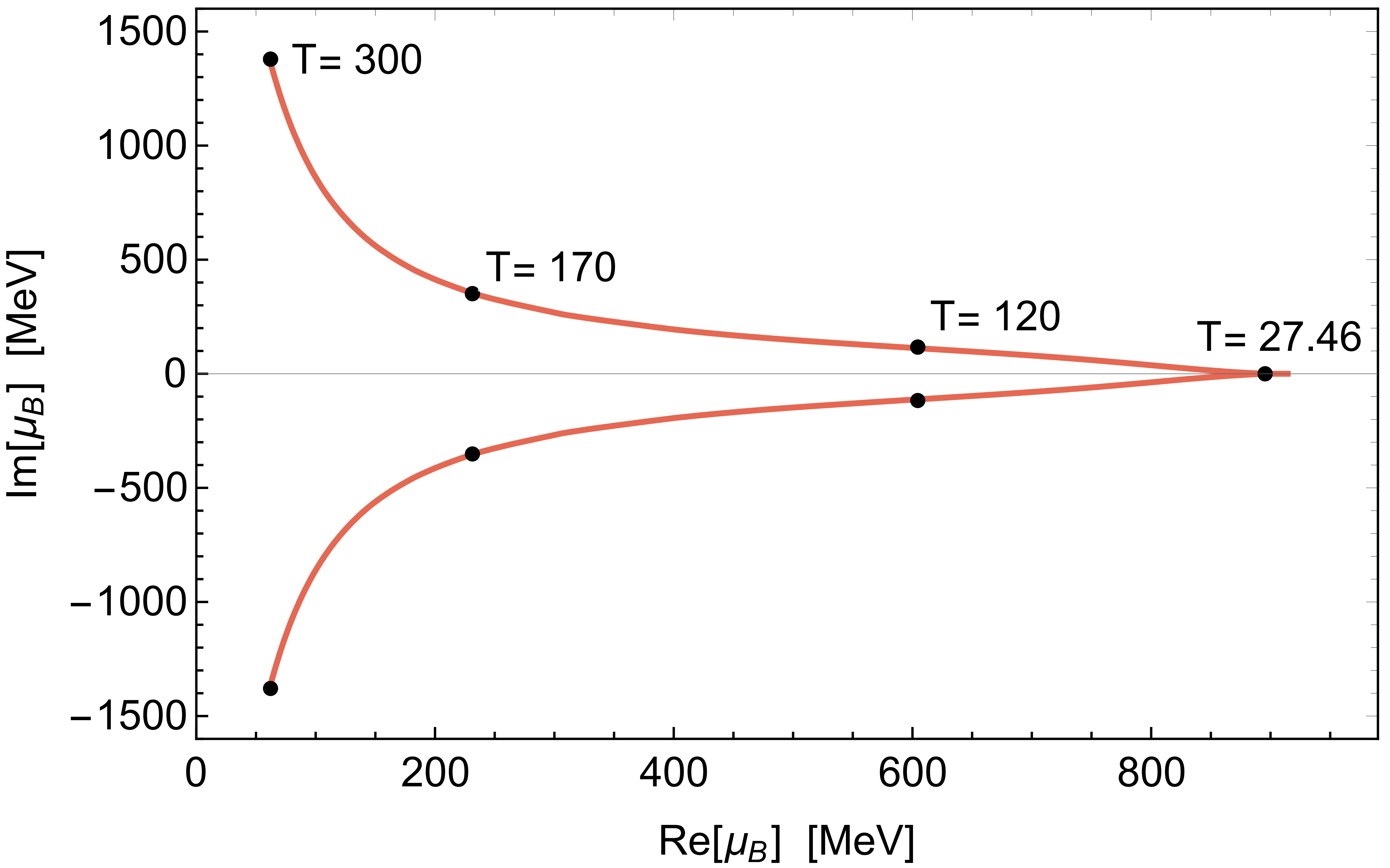
To do for QCD:
$U(N) \times U(N)$ and $SU(N) \times SU(N)$

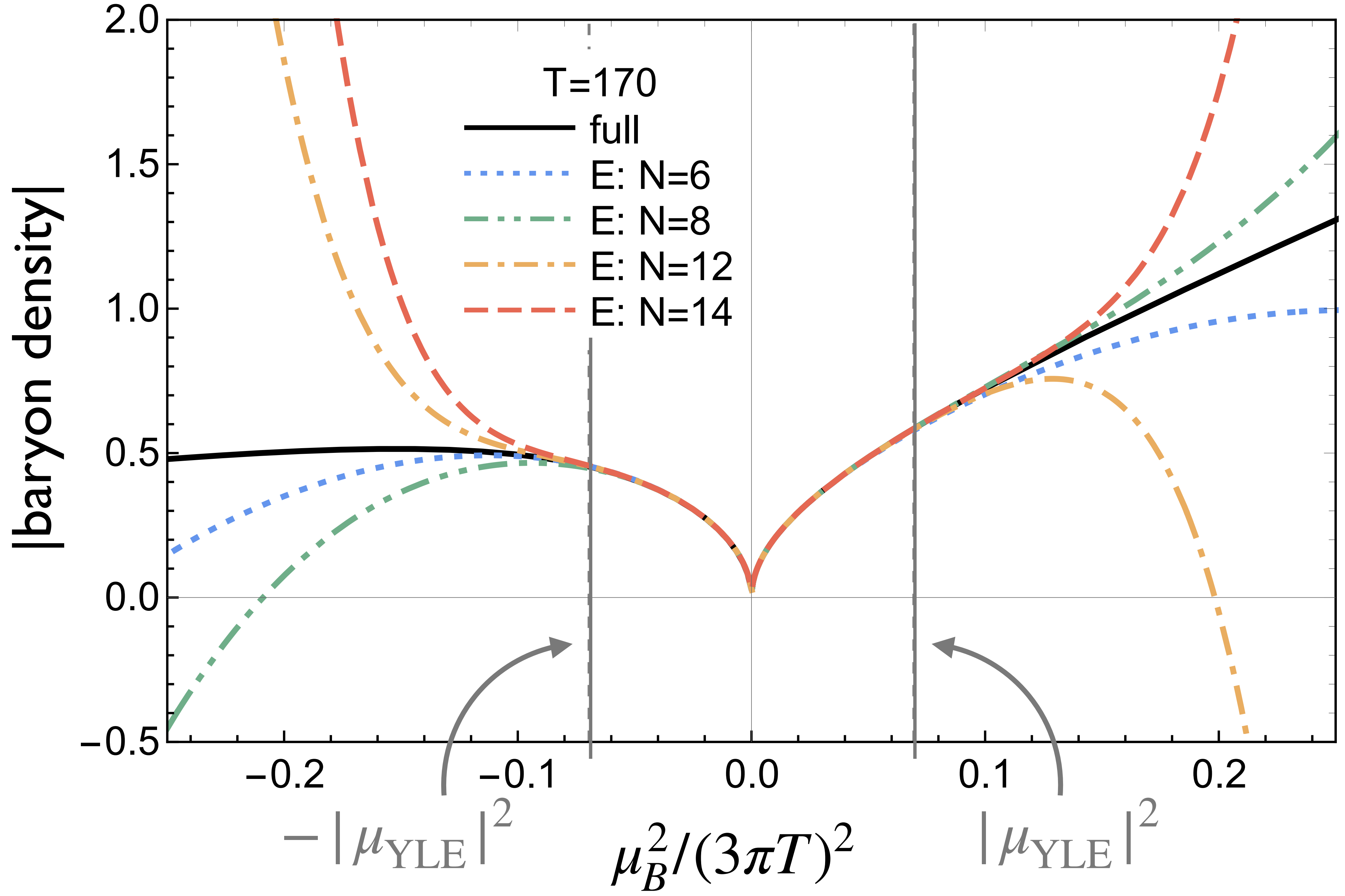
APPLICATION TO THE PHASE DIAGRAM(S)

YLE OF THE CEP

Consider system with a CEP at $(T_{\mathrm{CEP}}, \mu_{\mathrm{CEP}})$ in the complex μ plane

MF quark-meson model [Mukherjee, FR, Skokov (2021)]





- $T = T_{\mathrm{CEP}} : \mu_{\mathrm{YLE}} = \mu_{\mathrm{CEP}} \in \mathbb{R}$
- $T > T_{\mathrm{CEP}} : \mu_{\mathrm{YLE}} \in \mathbb{C}$

(see Fei Gao's talk for QCD results)

At $\mu = 0$ the YLE is the nearest singularity

→ determines radius of convergence for expansions around $\mu = 0$

YLE OF THE CEP

Sometimes (e.g. for lattice and experiment) $(T_{\rm CEP}, \mu_{\rm CEP})$ not directly accessible. Then:

- reconstruct YLE location for available T and μ
- extrapolate to $\mathrm{Im}\,\mu_{\rm YLE} = 0$ to locate CEP

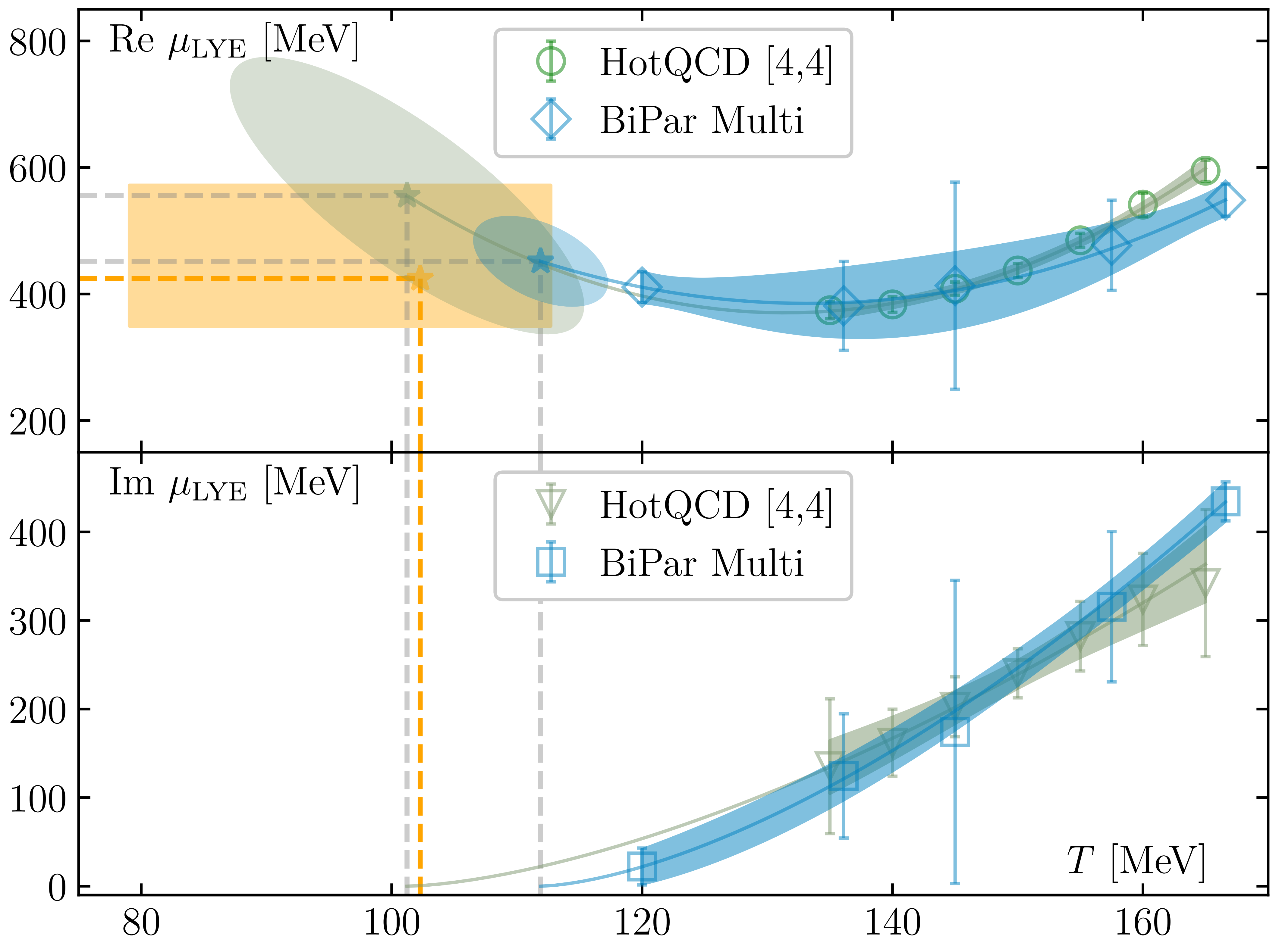
  (see talks of Adam, Basar, Goswami, Schmidt, Zambello, ...)

How to extrapolate?

- if data is in scaling region of CEP: use universality

$$\mu_{\rm YLE} = \mu_{\rm CEP} + c_1(T - T_{\rm CEP}) \pm i c_2 \left(\frac{T - T_{\rm CEP}}{|z_c|} \right)^{\beta\delta}$$

[Stephanov (2006)]



[Clarke et al. (2024)]

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(see talks of Adam, Basar, Goswami, Schmidt, Zambello, ...)

How to extrapolate?

- if data is in scaling region of CEP: use universality
- but how large is scaling region?

$O(4)$-scaling of light chiral limit (physical m_s):

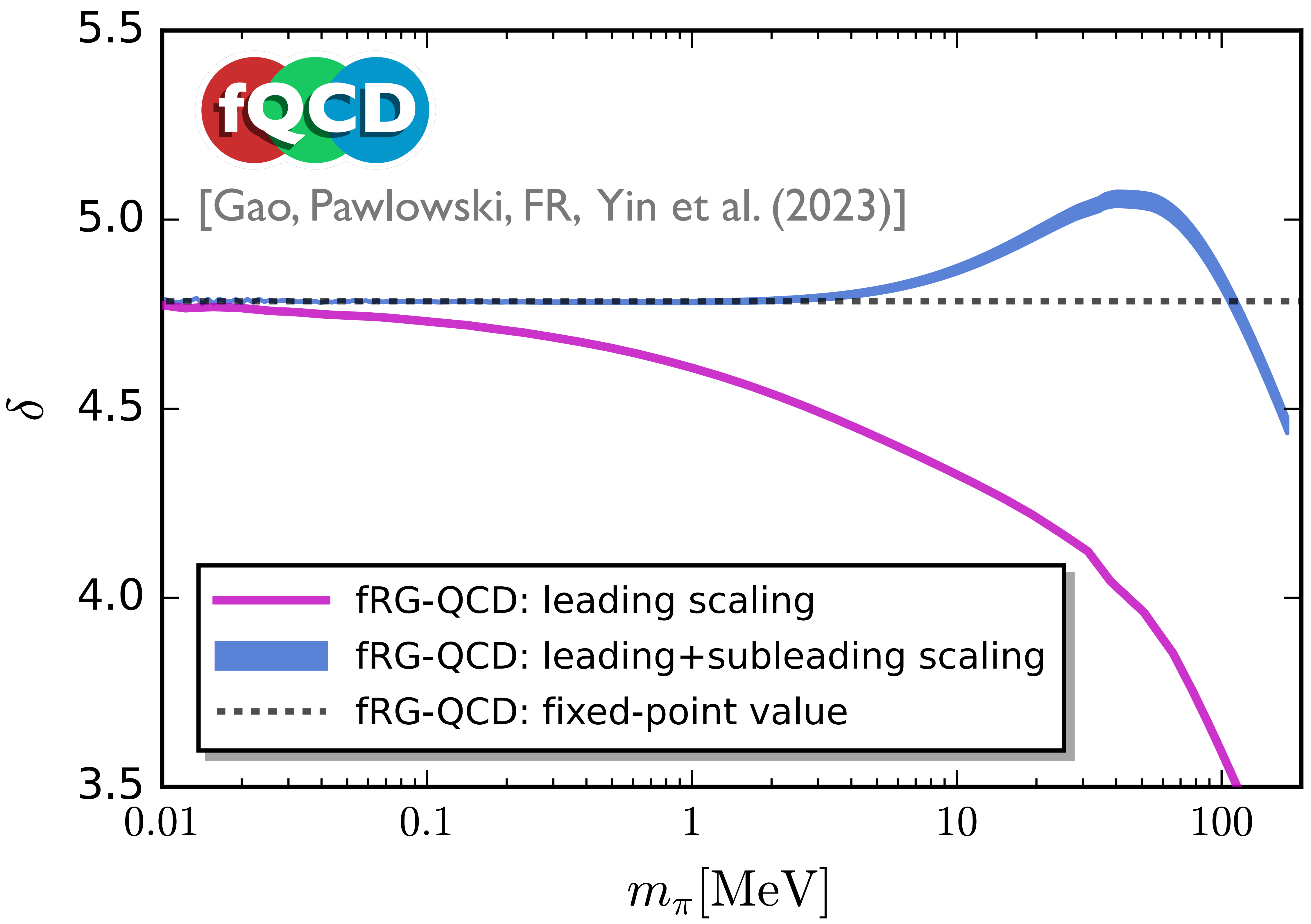
- $m_\pi \lesssim 5\,\mathrm{MeV}$ at $T = T_c$
- $T_c - T \lesssim 7\,\mathrm{MeV}$ at $m_\pi = 0$

CEP scaling region probably also small

[Fu, Luo, Pawlowski, FR, Yin (2021, 2023)]

→ **non-universal information necessary**

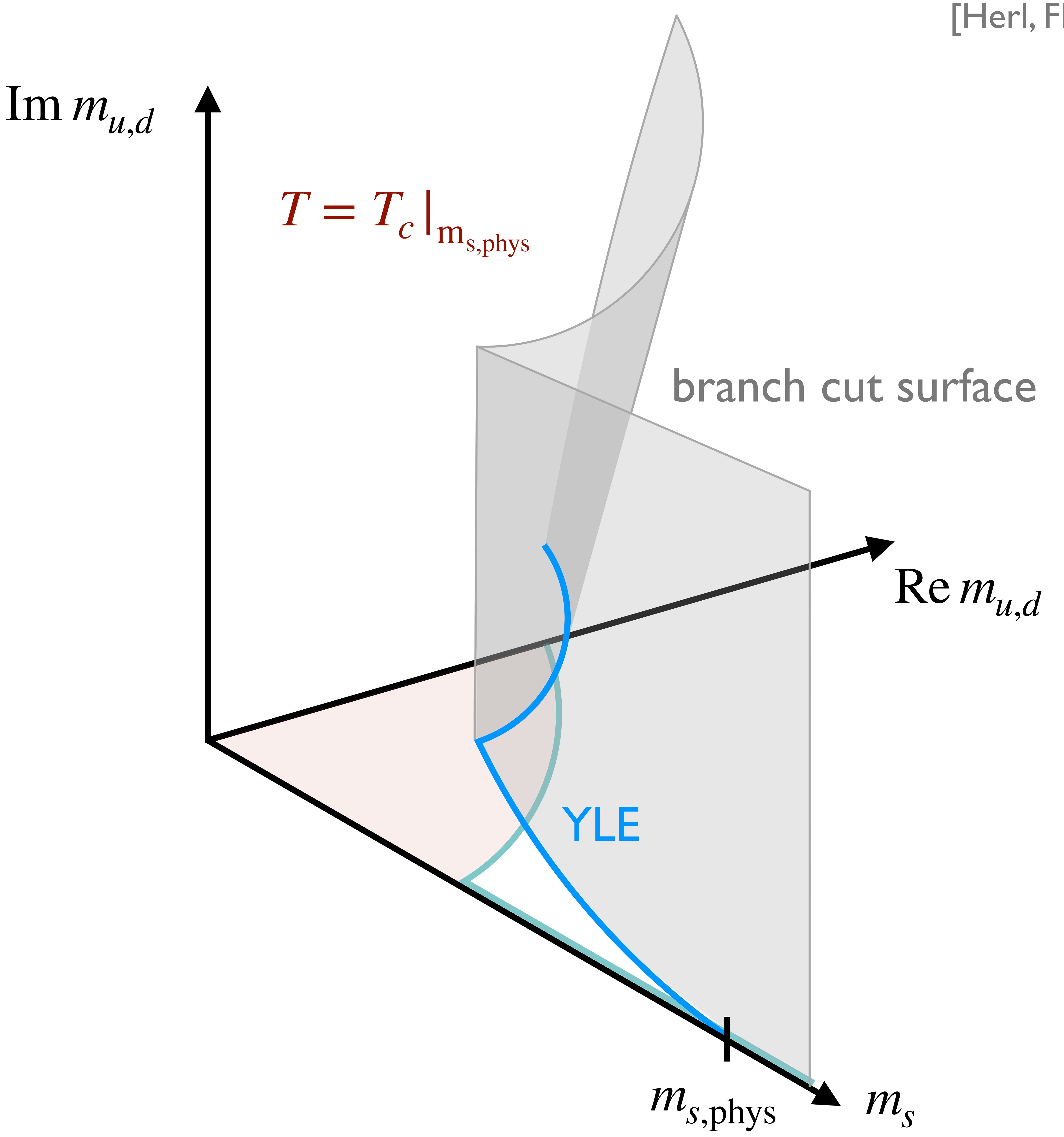
(see talks of Gao and Pawlowski)



fits of the form
$$\bar\Delta_l(m_\pi) = B_c\, m_\pi^{2/\delta}\left(1 + a_m m_\pi^{2\theta_H}\right) + c_1\, m_\pi^2 + c_2\, m_\pi^4$$
break down for $m_\pi \gtrsim 25\,\mathrm{MeV}$

THE COLUMBIA PLOT AND EDGE SINGULARITIES

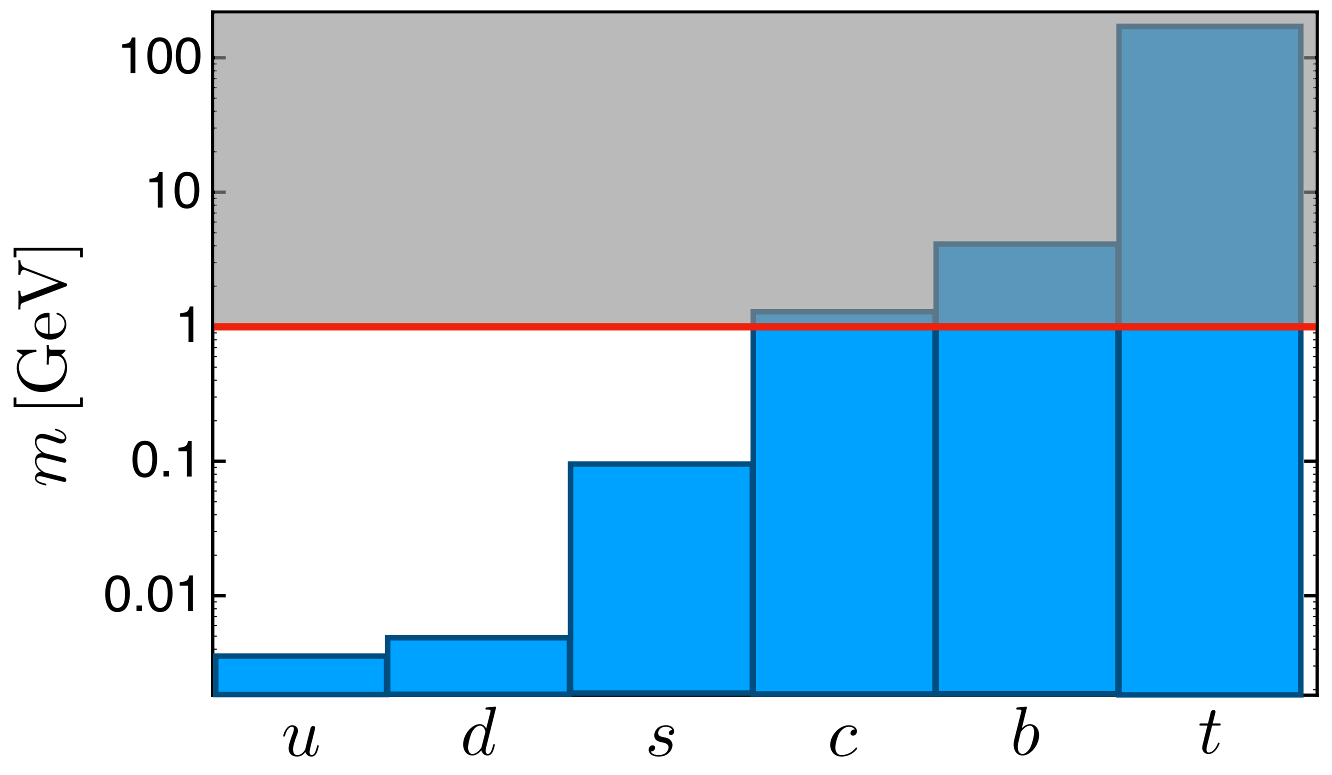
[Herl, FR, Schmidt, von Smekal (in preparation)]



THE COLUMBIA PLOT

How does the order of the chiral phase transition depend on the quark mass?

- distinct mass hierarchy of quarks ($2\pi T_c \approx 1\,\mathrm{GeV}$)



→ what if u, d were even lighter?

- relevant flavor symmetry:

$$U(3)_L \times U(3)_R \approx SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

↓ axial anomaly

$$SU(3)_V \times SU(3)_A \times U(1)_V$$

↓ chubby strange quark

$$\underbrace{SU(2)_V \times SU(2)_A}_{\sim O(4)} \times U(1)_V$$

↓ light quark masses

$$SU(2)_V \times U(1)_V$$

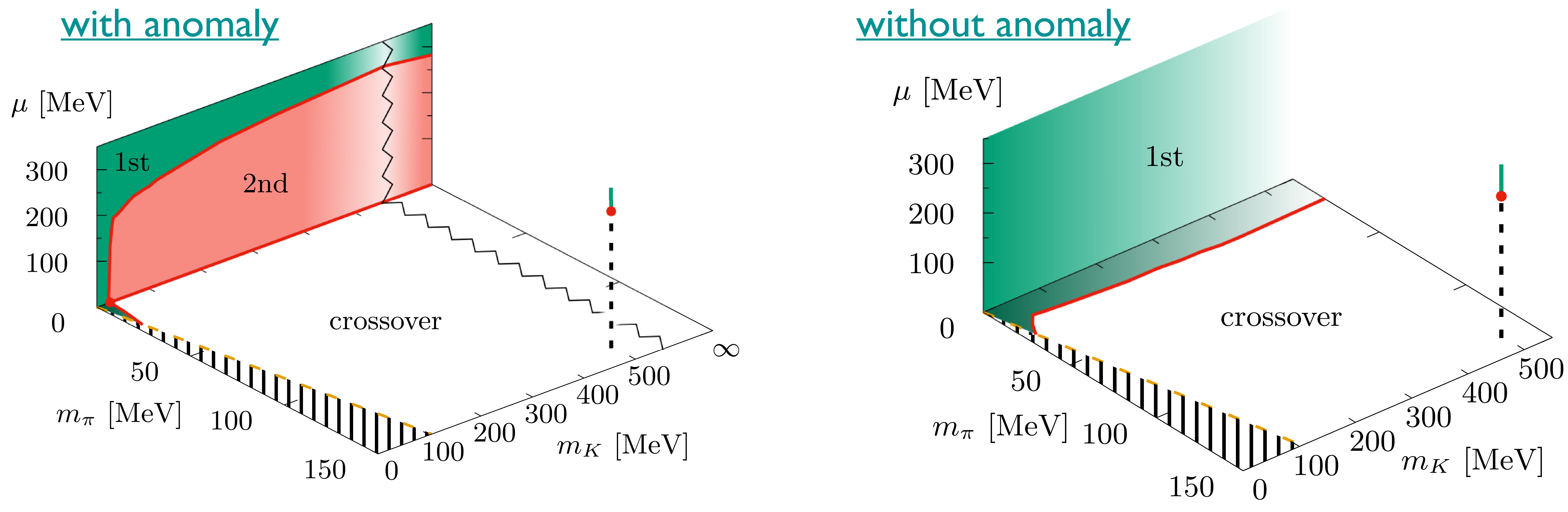
→ any "remnants" at physical quark masses?

THE COLUMBIA PLOT

Expectation from Pisarski & Wilczek (1983) (perturbative RG analysis of a linear sigma model):

- $N_f = 3$ chiral quarks: 1st order transition
- $N_f = 2$ chiral quarks: depends on the fate of the axial anomaly

FRG analysis: [Resch, FR, Schaefer (2017)]



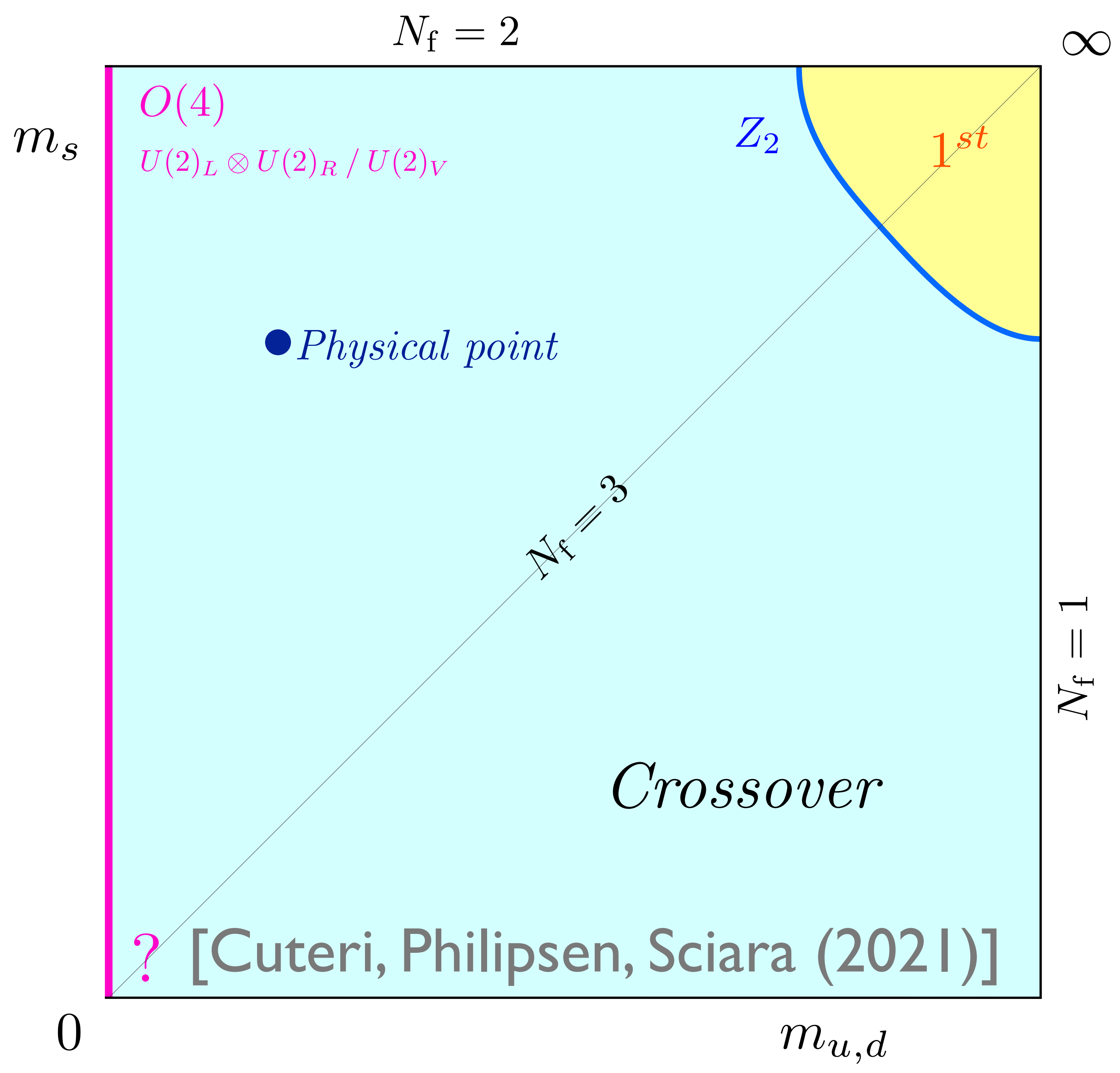
→ suggests very small 1st order region in the 3-flavor chiral limit

(triggered by bosonic fluctuations - large corrections to mean-field)

Also: no **stable** fixed point for $N_F = 3$ from recent FRG analysis in the 3-flavor chiral limit [Fejos (2022)]

THE COLUMBIA PLOT

Could there even be a 2nd order transition in the 3-flavor chiral limit?

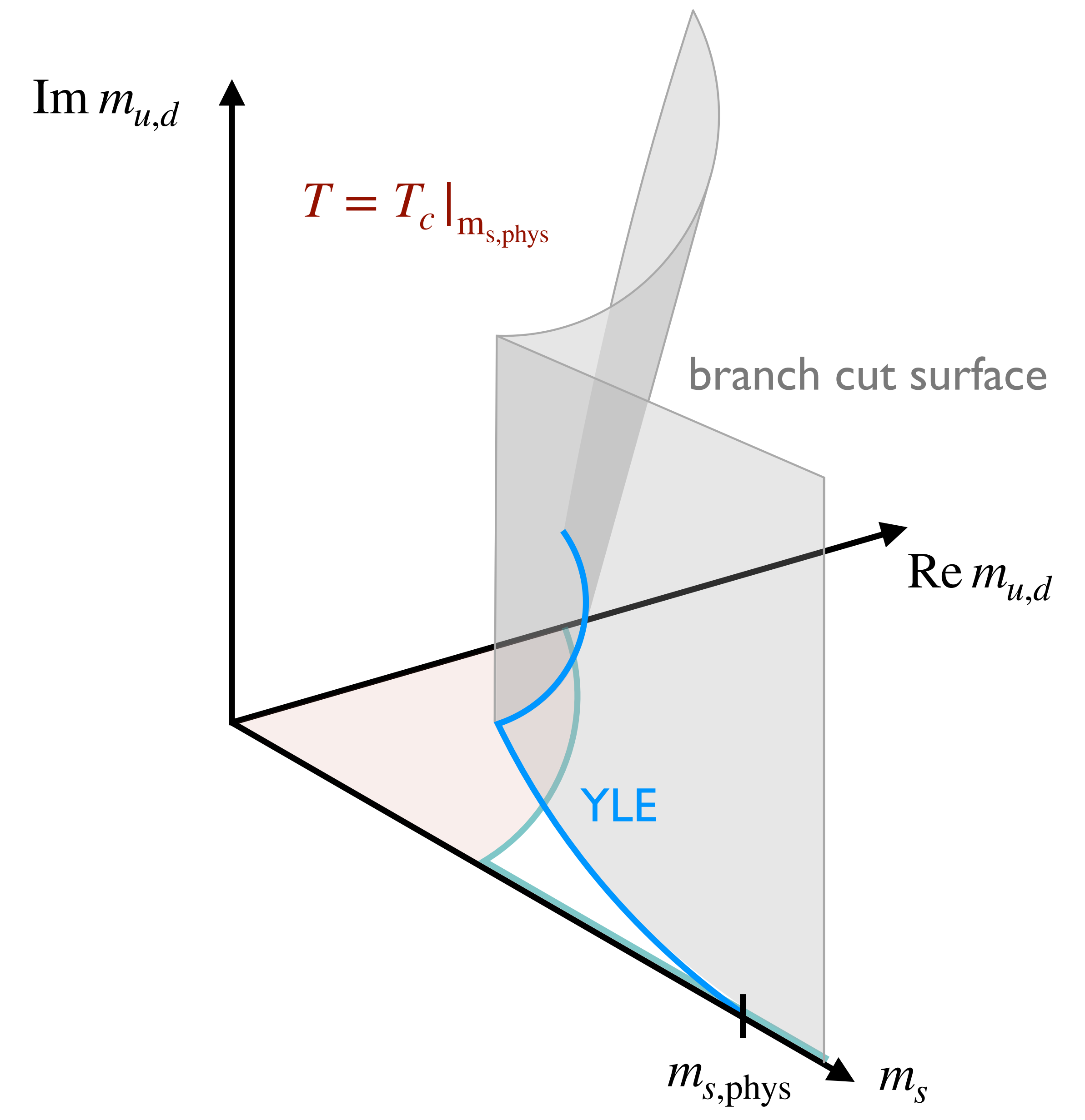


- generic prediction of mean-field studies of models without 't Hooft determinant [e.g. Resch, FR, Schaefer (2017)]
- detailed lattice study suggests 2nd order transition even for $N_f \leq 6$ massless quarks [Cuteri, Philipsen, Sciara (2021)]
- fixed-point analyses: only possible if $U(1)_A$ is restored at T_c? [Fejos (2022), Kousvos and Stergiou (2023)]
- cannot be excluded from lattice computations [Aarts et al. (2023) & references therein]
- suggested by recent DSE study [Bernhardt, Fischer (2023)]
- conjecture: dominance of higher topological charges at $T \lesssim T_c$ necessary for this scenario [Pisarski, FR (2024)]

Can YLEs help us here?

YLE AND THE COLUMBIA PLOT

- consider quark mass as thermodynamic control parameter
  (acts like magnetic field in $O(N)$ models)
- search for 2nd order transition at some (T_c, m_c)
- YLE for $m \in \mathbb{C}$ at $T > T_c$



There are in general 3 different scenarios

[Herl, FR, Schmidt, von Smekal (in preparation)]

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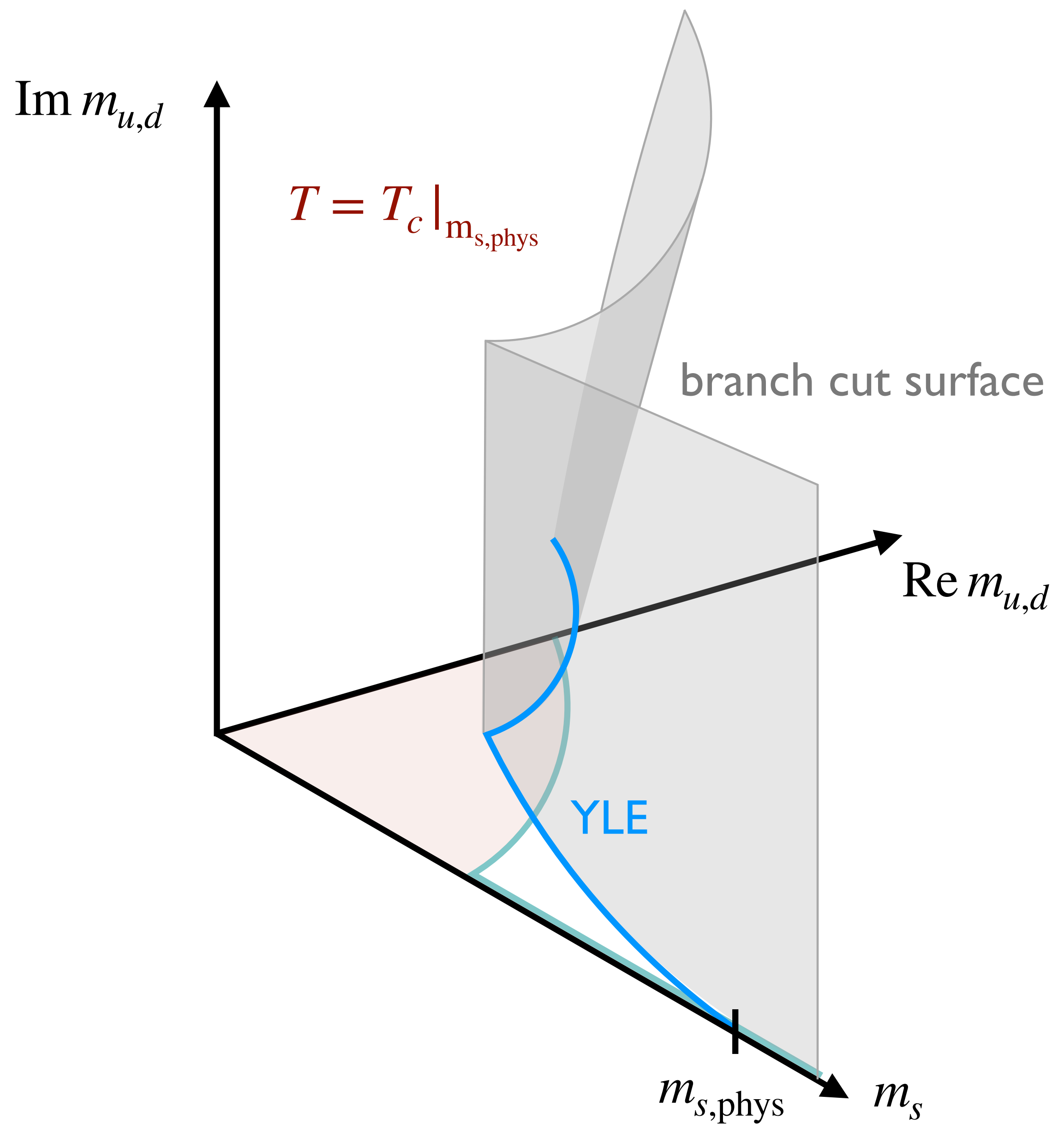
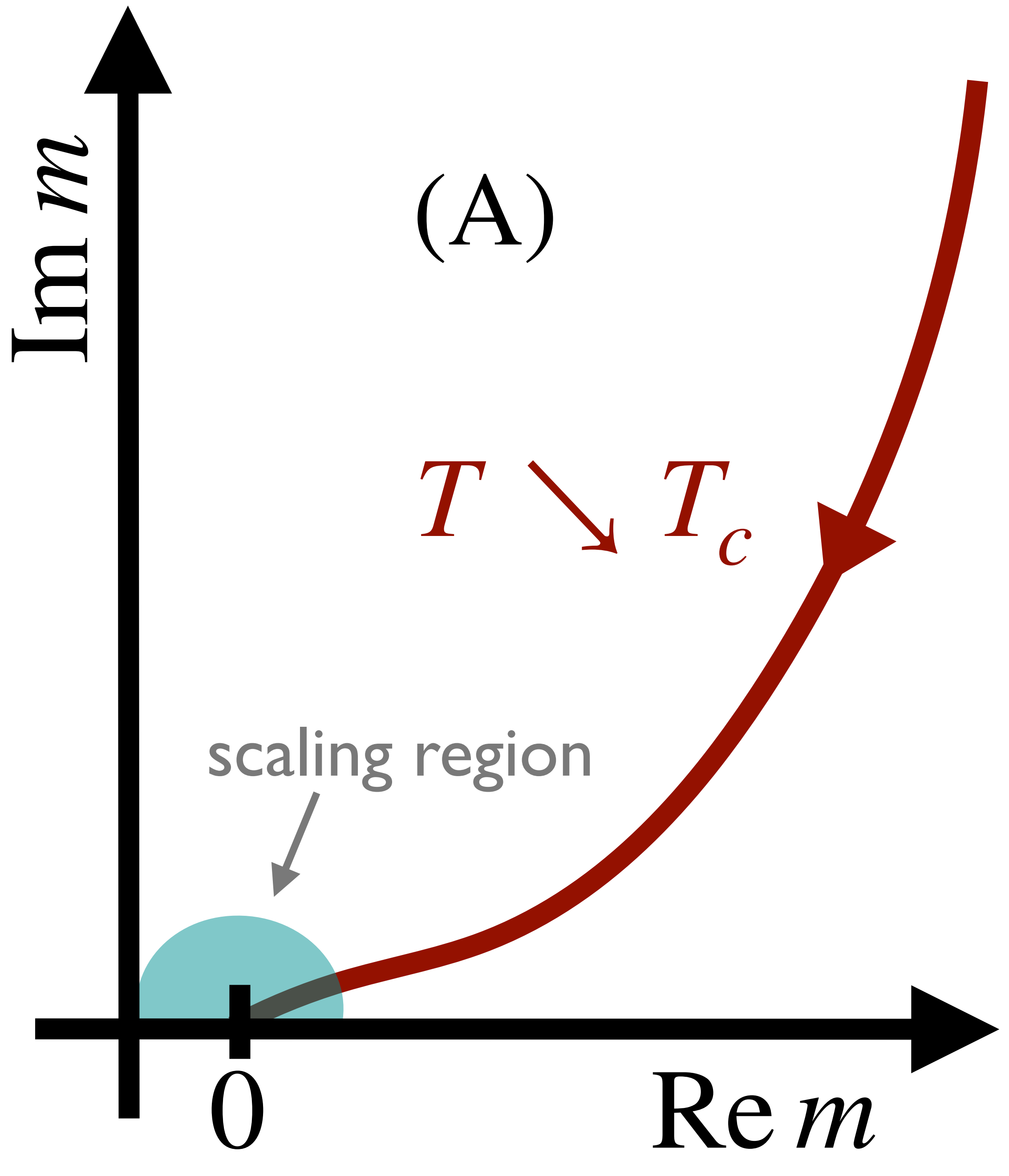


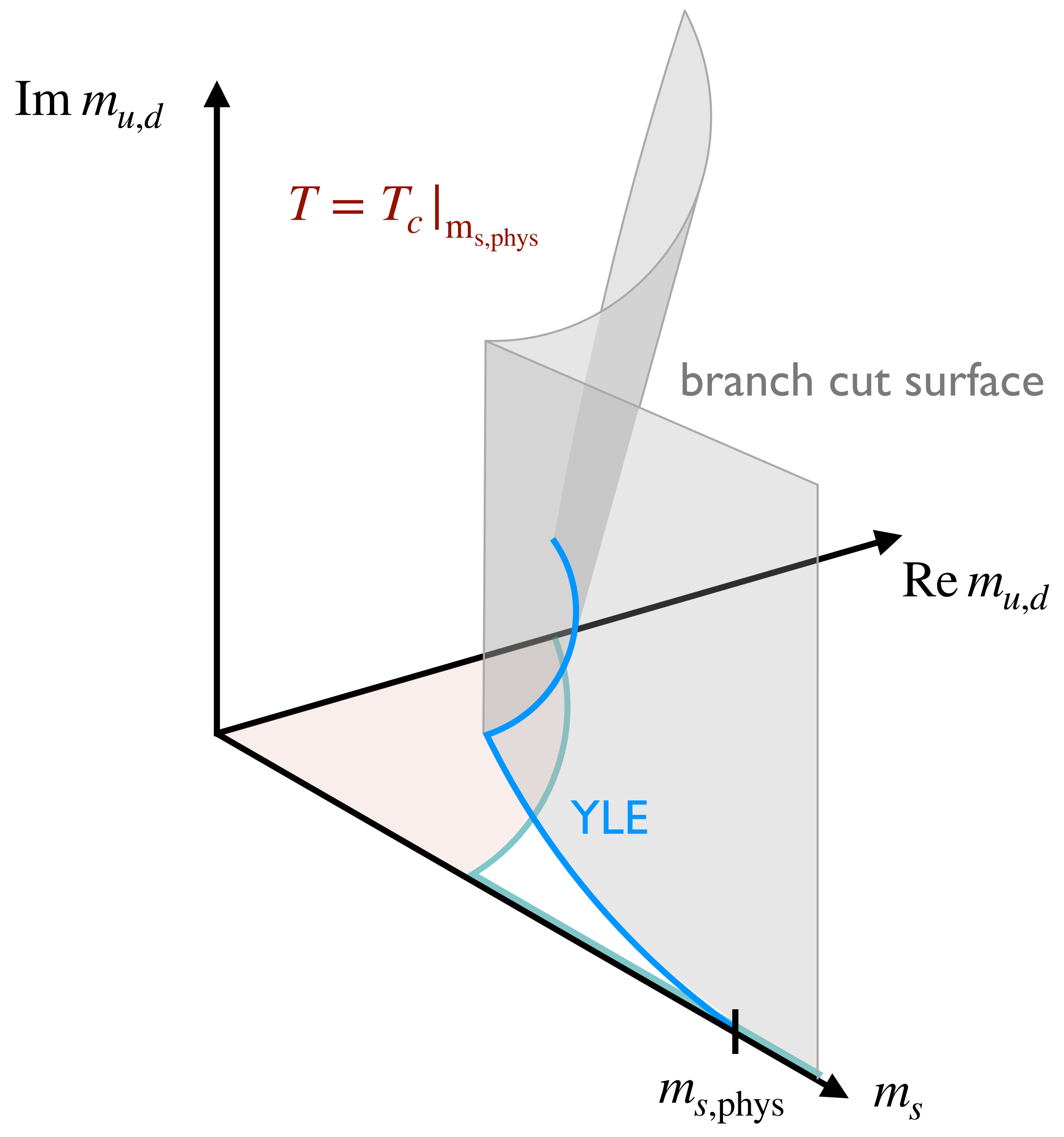
There are in general 3 different scenarios, **A**:



- 2nd order transition at zero mass
- no further restriction on the transition
- requires reconstruction + extrapolation for various T in the continuum limit

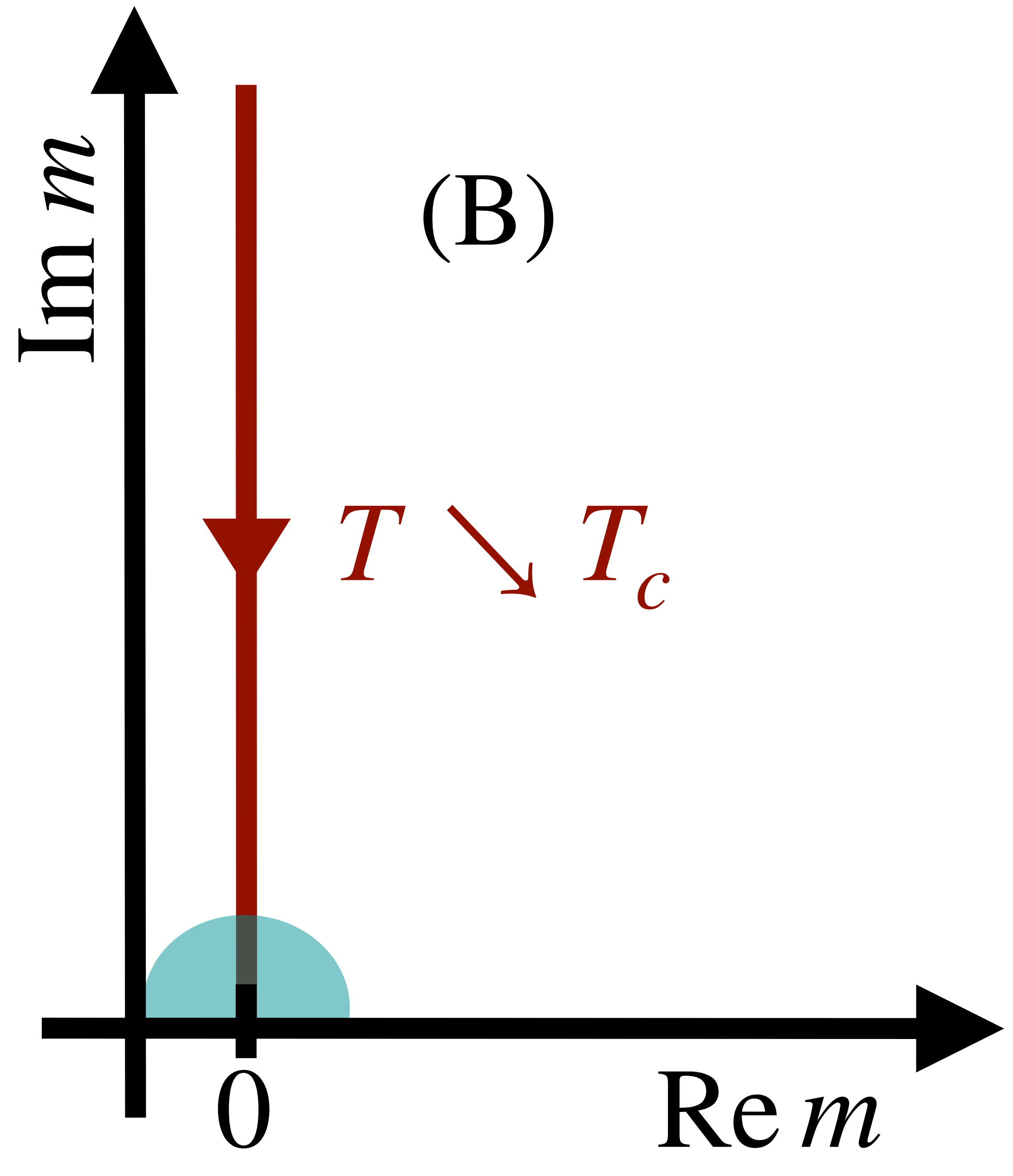
[Herl, FR, Schmidt, von Smekal (in preparation)]

YLE AND THE COLUMBIA PLOT



- consider quark mass as thermodynamic control parameter
  (acts like magnetic field in $O(N)$ models)
- search for 2nd order transition at some (T_c, m_c)
- YLE for $m \in \mathbb{C}$ at $T > T_c$

There are in general 3 different scenarios, **B**:



- 2nd order transition at zero mass
- **Lee-Yang circle theorem applies**
- YLE and LY zeros must lie on the imaginary mass axis

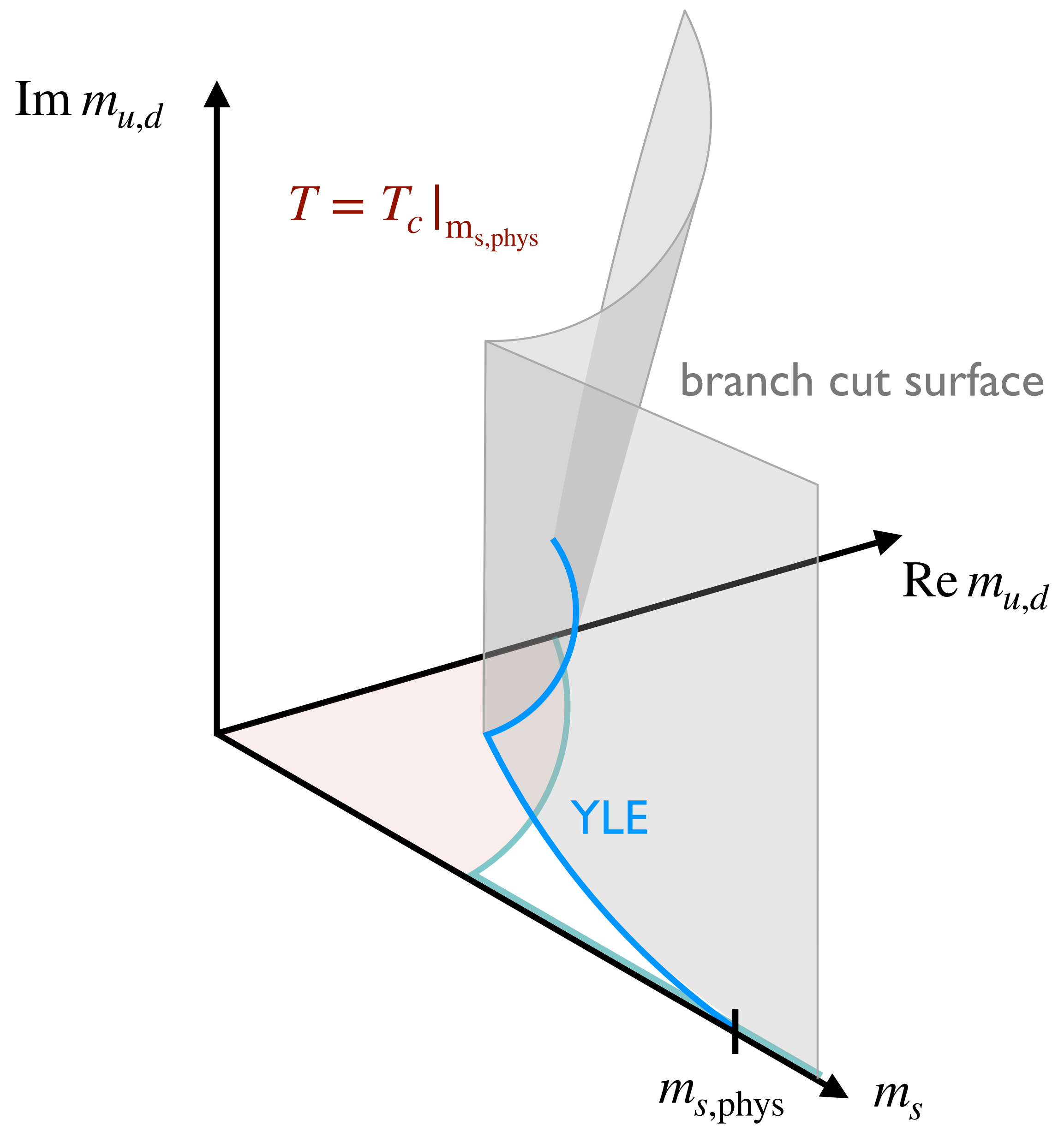
→ **infer that transition must be at zero mass without any extrapolation, neither to small T, m or the continuum**

- reconstruction of YLE still necessary

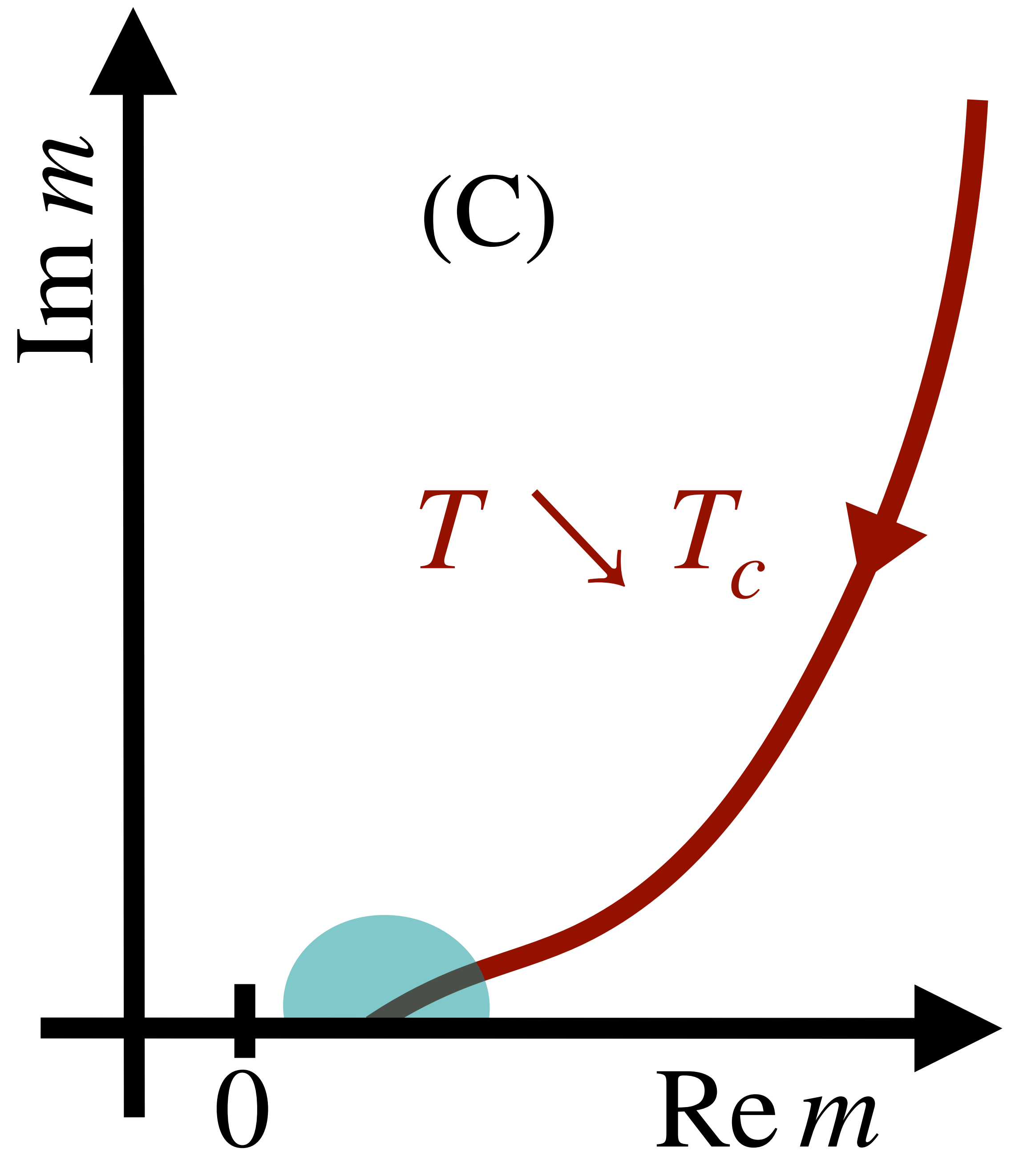
[Herl, FR, Schmidt, von Smekal (in preparation)]

YLE AND THE COLUMBIA PLOT

- consider quark mass as thermodynamic control parameter
  (acts like magnetic field in $O(N)$ models)
- search for 2nd order transition at some (T_c, m_c)
- YLE for $m \in \mathbb{C}$ at $T > T_c$



There are in general 3 different scenarios, **C**:



- 2nd order transition at nonzero mass
- circle theorem irrelevant, as map from m to critical magnetic field is nontrivial
- requires reconstruction + extrapolation for various T in the continuum limit

[Herl, FR, Schmidt, von Smekal (in preparation)]

RECONSTRUCTING THE YLE

Adapt the strategy used for finite μ in [Dimopoulos et al. (2022)] to finite m:

$\longrightarrow$ multi-point Padé reconstruction

- assume that analytic structure of the free energy is captured by a rational function

$$f(z) \approx R_n^m(z) = \frac{P_m(z)}{1 + Q_n(z)} = \frac{\sum_{i=0}^m a_i\, z^i}{1 + \sum_{j=1}^n b_j\, z^j}$$

- consider $f(z)$ at N nodes z_k $(k = 1, \ldots, N)$ and assume we know its derivatives up to order L_k at each node

$\longrightarrow$ we can fix $n + m + 1 = \sum_{k=1}^{N} (L_k + 1)$ Padé coefficients

$$\begin{aligned}
P_m(z_1) - f(z_1)\, Q_n(z_1) &= f(z_1) \\
P'_m(z_1) - f'(z_1)\, Q_n(z_1) - f(z_1)\, Q'_n(z_1) &= f'(z_1) \\
&\vdots \\
P_m(z_N) - f(z_N)\, Q_n(z_N) &= f(z_N) \\
P'_m(z_N) - f'(z_N)\, Q_n(z_N) - f(z_N)\, Q'_n(z_N) &= f'(z_N) \\
&\vdots
\end{aligned}$$

(see talks of Adam, Basar, Goswami, Schmidt, Zambello, ...)

RECONSTRUCTING THE YLE

- rational functions can only have isolated poles (zeros of the denominator)
- branch cuts are indicated by arcs of poles, accumulating at branch points for large N, [Stahl (1997)]
- identify YLE as closest pole to real axis that is stable under variation of the Padé order [m/n]

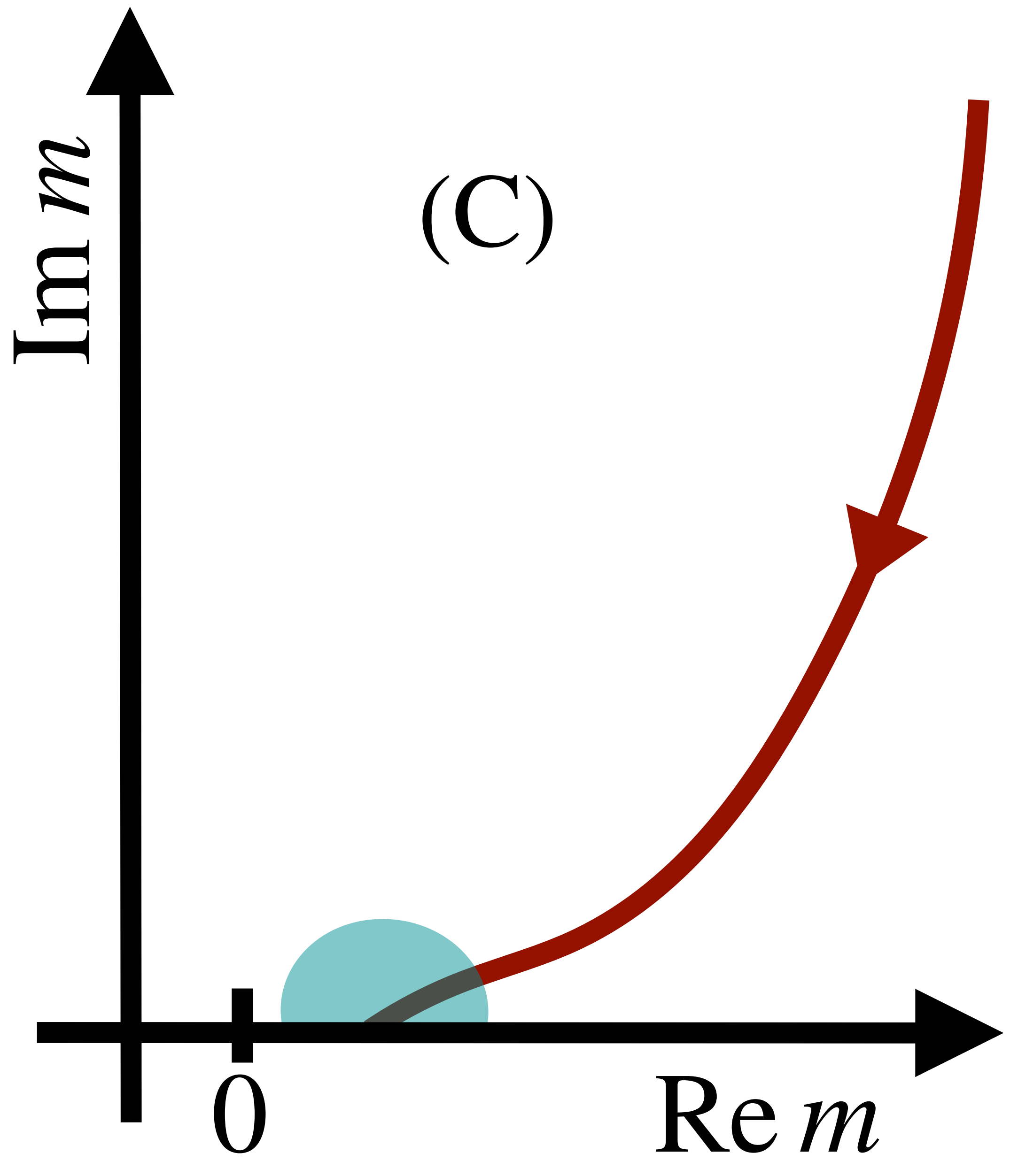
Proof of concept: $N_f = 2$ QM model, where scenario (B) and (C) can be realized (depends on choice of parameters).

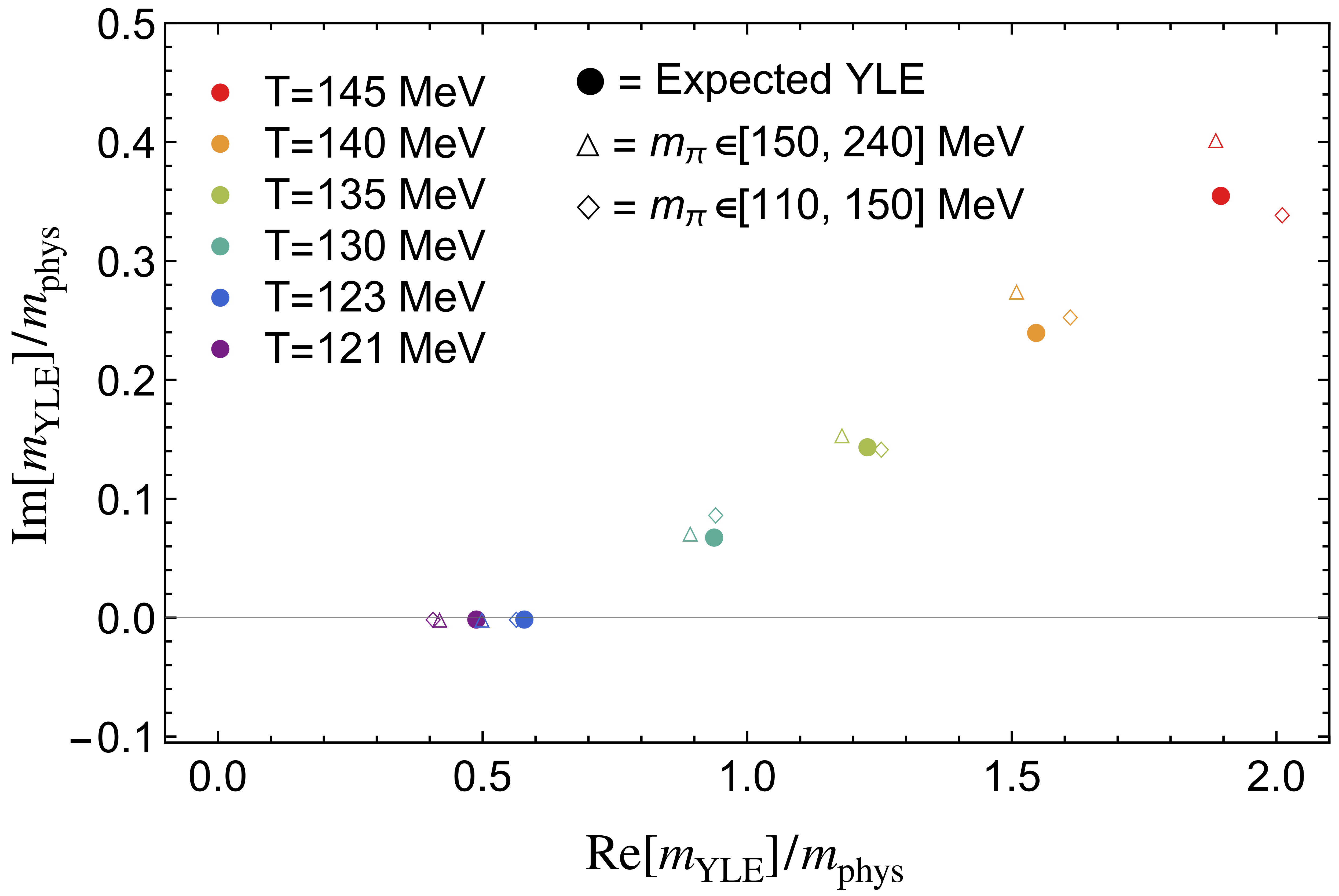
- use 6 nodes for the chiral susceptibility $\chi_m \sim \dfrac{\delta\sigma}{\delta m}$
- 2 known derivatives at each node
- susceptibility is an even function of m

$\longrightarrow$ use [16/18] Padé in m

SCENARIO C

In this model: Ising transition at $m > 0$



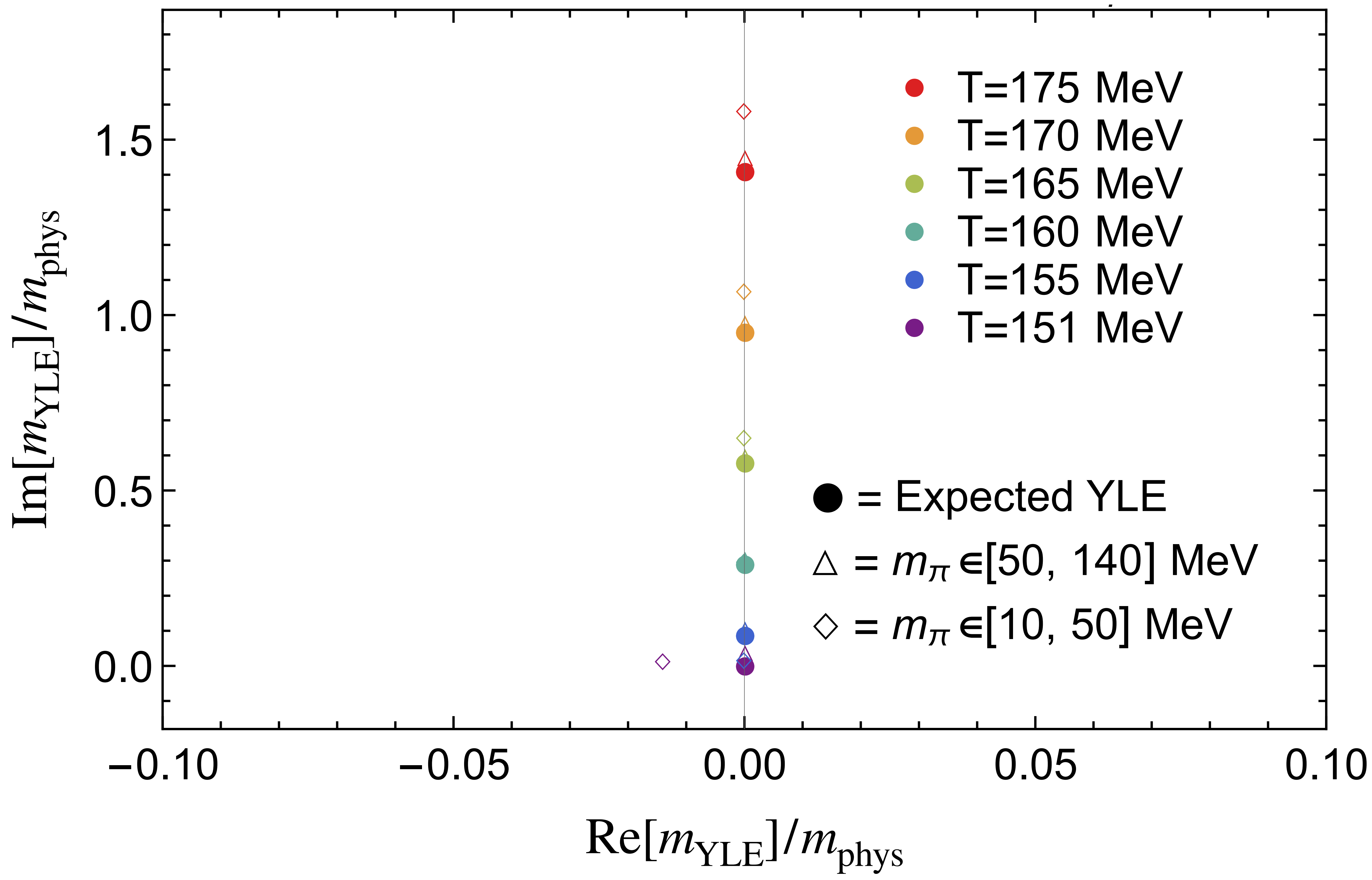


→ reconstruction works well, but extrapolation is required if data at smaller T not available

[Herl, FR, Schmidt, von Smekal (in preparation)]

SCENARIO C

In this model: $O(4)$ phase transition at $m = 0$



→ reconstruction works well, **no extrapolation required to infer m_c**

To do: apply to lattice data!

[Herl, FR, Schmidt, von Smekal (in preparation)]

CONCLUSIONS

We can learn a lot from YLEs

Their location is universal.

It has been established using FRG (for relevant systems).

Universality only in the scaling regime of Wilson-Fisher fixed point.

But this is likely to be small → non-universal information needed.

Also: circle theorem can provide shortcut to solve Columbia plot puzzle.