Can we make a statement on the QCD critical point by lattice data from just one temperature?

Probabilities in certain rings in the complex plane

Region $\mathcal{R}$ (MeV)	$P(\mathcal{R})$ lattice	$P(\mathcal{R})$ HRG mock
387–502	8.8%	9.7%
420–750	1.9%	4.6%
420–632	1.3%	4.1%
540.0–664.2	0.41%	0.49%

With yc about zero, $|y_c| \leq 0.1$ MeV: $P_{\mathrm{LAT}} = 0$

Our analyses provide direct evidence for smooth, analytic thermodynamics at T=108 MeV

Slide by H.-T. Ding

What is the best trial fit function for the analysis of the low temperature small-volume QCD data?

Analyses using critical ansatz in the complex plane

Critical ansatz: Ansatz III: $f(\mu_B) = A\cosh(\mu_B) + C(|\mu_B - \mu_{B,c}|^\alpha + |\mu_B + \mu_{B,c}|^\alpha)$

$\alpha = 1 + \sigma_{\mathrm{LYE}} = 1.085$

Gliozzi and Rago, JHEP 10 (2014) 042 [1403.6003]

In the complex plane to fit the LQCD data:

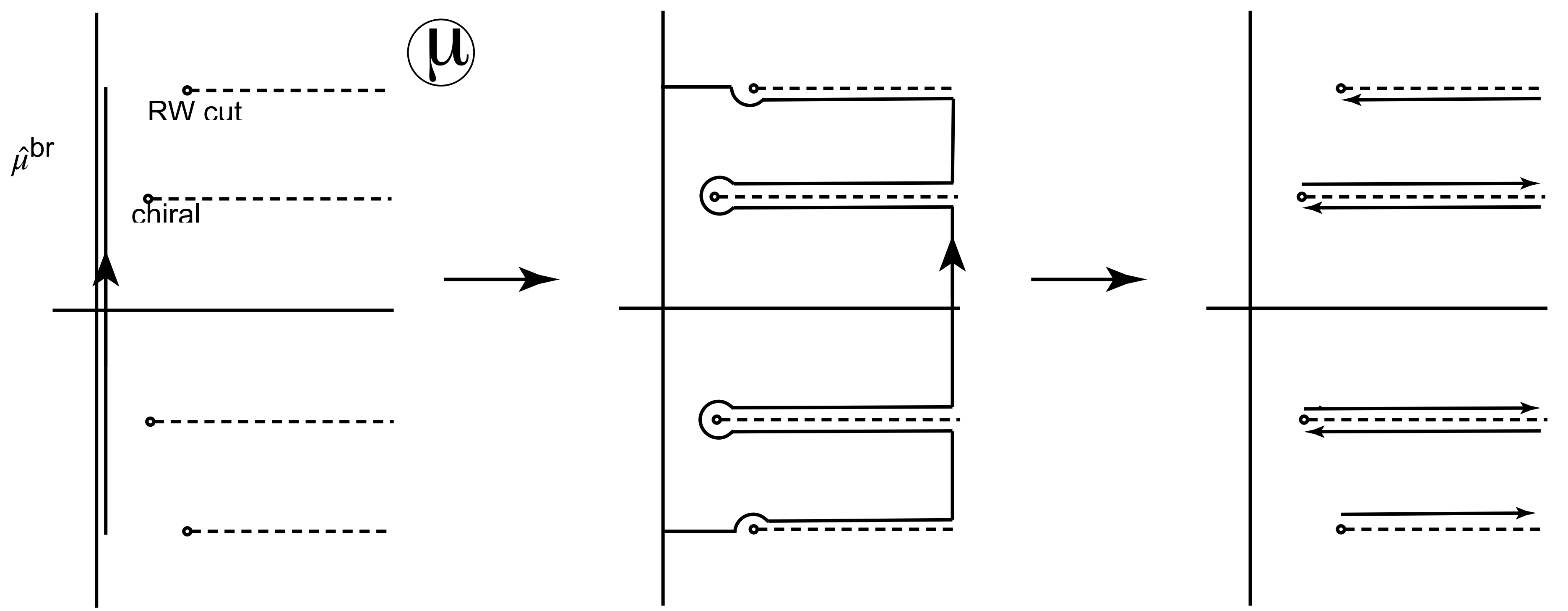
$$f(i\theta_B) = A(T)\ \cos(\theta_B) + C\left[\left(x_c^2 + (\theta_B - y_c)^2\right)^{\alpha/2} + \left(x_c^2 + (\theta_B + y_c)^2\right)^{\alpha/2}\right]$$

- Lee-Yang edge singularity (x_c, y_c)
- Radius of convergence $|\mu_{B,c}|_{min} = \left(\sqrt{x_c^2 + y_c^2}\right)_{min}$

Slide by H.-T. Ding

Do we know anything about LYZ beyond YLE and RW?

- We can deform the integration contour to integrate along the cuts



Only these integrals remain

- Assume that we can express the density along the cuts as

edge coefficient, $\sigma > -1$

$$n_B(\hat{\mu}) = \underbrace{A(\hat{\mu} - \hat{\mu}^{\mathrm{br}})^{\sigma}}_{\text{Leading order non-analytic part}}(1 + B(\hat{\mu} - \hat{\mu}^{\mathrm{br}})^{\theta_c} + \ldots) + \underbrace{\sum_{n=0}^{\infty} a_n(\hat{\mu} - \hat{\mu}^{\mathrm{br}})^n}_{\text{analytic part}}$$

Leading order non-analytic part

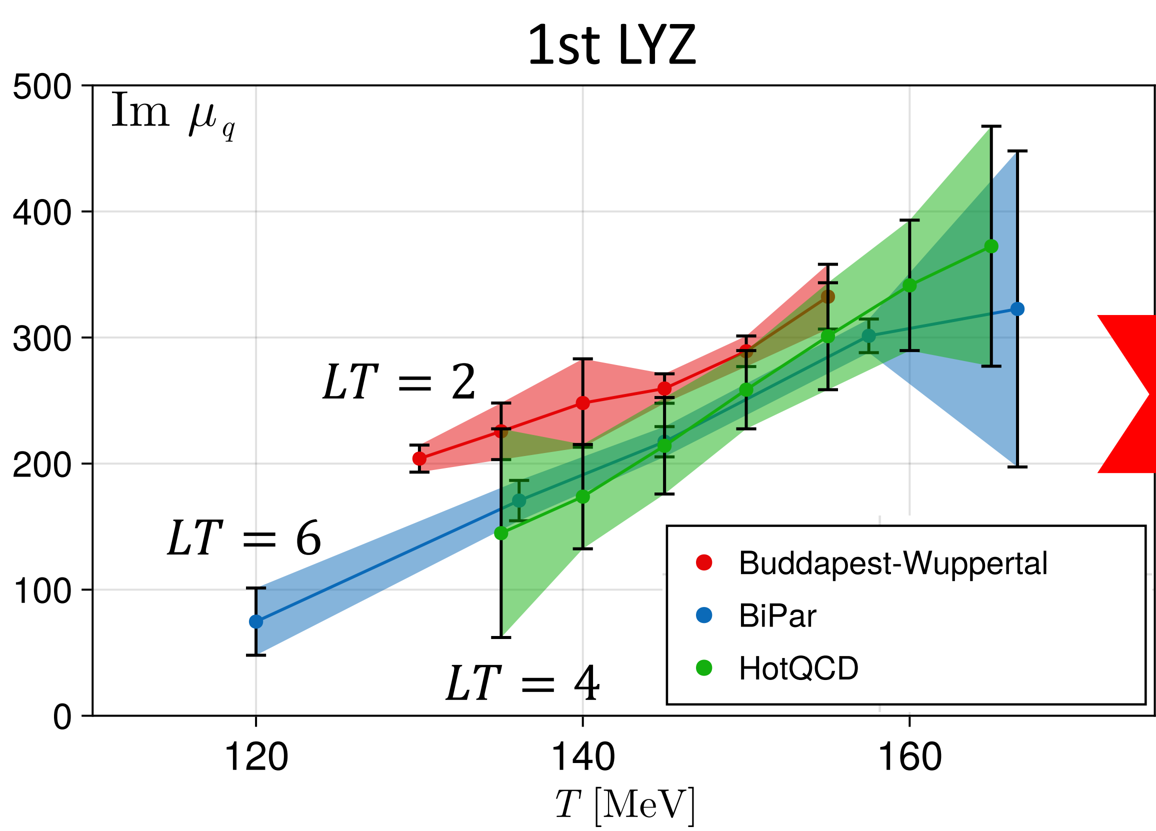
analytic part

Slide by C.Schmidt

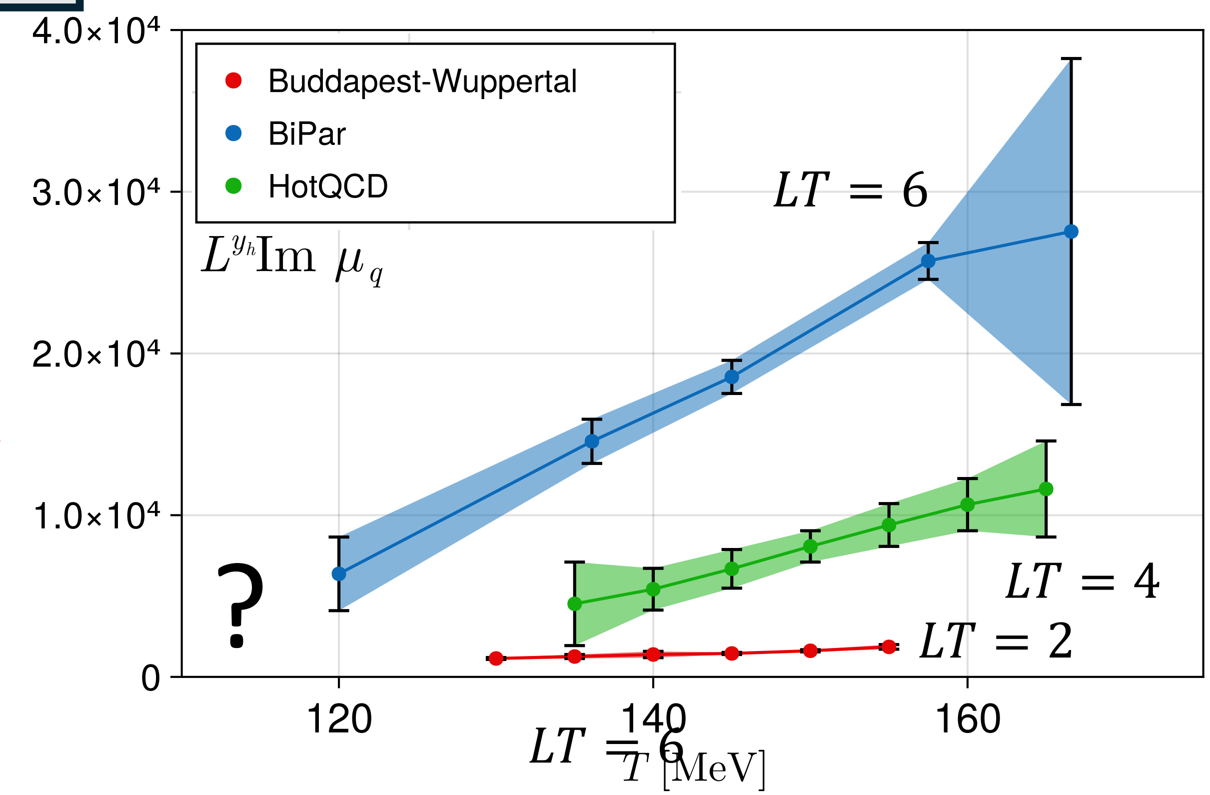
Can we perform Wada/Kitazawa analysis of LYZ in QCD?

Single LYZ method in N_f=2+1 QCD

$$L^{y_h} h_{\mathrm{LY}}^{(n)}(L^{y_t} t) = X_n + Y_n L^{y_t} t + \mathcal{O}(t^2)$$



Pure imaginary chemical potential
+ Pade approximation



Lattice result is still high temperature.
$T_c = 90 - 100$ MeV?

Slide by T.Wada

What is the size of the scaling region? Is there a mean-field region?

YLE OF THE CEP

Sometimes (e.g. for lattice and experiment) $(T_{\rm CEP}, \mu_{\rm CEP})$ not directly accessible. Then:

- reconstruct YLE location for available T and μ
- extrapolate to $\mathrm{Im}\,\mu_{\rm YLE} = 0$ to locate CEP

(see talks of Adam, Basar, Goswami, Schmidt, Zambello, ...)

How to extrapolate?

- if data is in scaling region of CEP: use universality
- but how large is scaling region?

$O(4)$-scaling of light chiral limit (physical m_s):

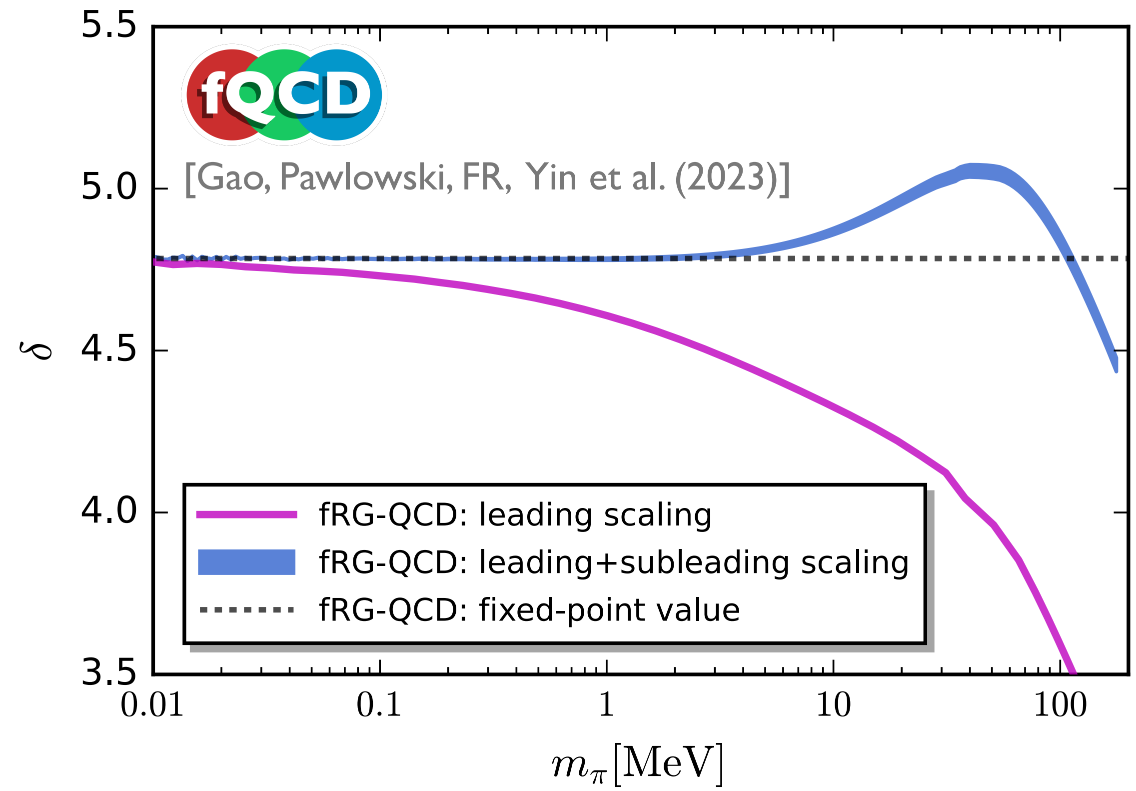
- $m_\pi \lesssim 5\,\mathrm{MeV}$ at $T = T_c$
- $T_c - T \lesssim 7\,\mathrm{MeV}$ at $m_\pi = 0$

CEP scaling region probably also small

[Fu, Luo, Pawlowski, FR, Yin (2021, 2023)]

→ **non-universal information necessary**

(see talks of Gao and Pawlowski)



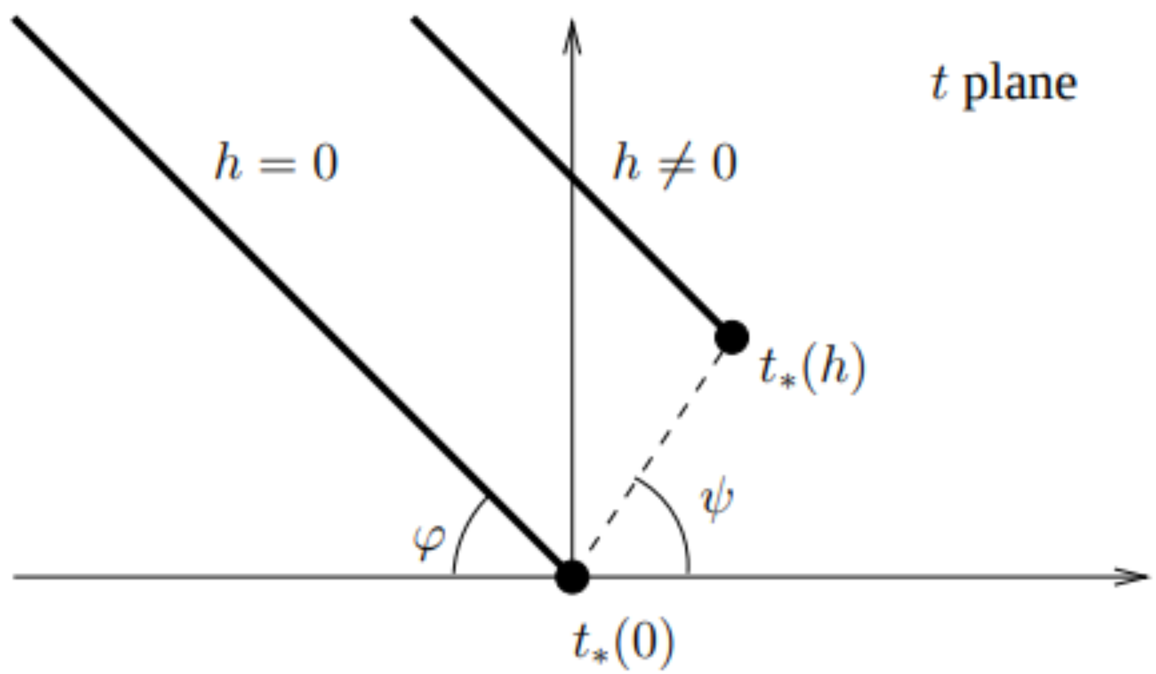
fits of the form

$\bar{\Delta}_l(m_\pi) = B_c\, m_\pi^{2/\delta}\big(1 + a_m m_\pi^{2\theta_H}\big) + c_1\, m_\pi^2 + c_2\, m_\pi^4$

break down for $m_\pi \gtrsim 25\,\mathrm{MeV}$

Slide by F. Rennecke

See attached figure from the Misha's paper; is the line $h \neq 0$ parallel to the $h = 0$ line? The derivation for the angles φ and ψ is based on the continuity of Omega and is given by the critical properties of the underlying universality class. It does not seem to translate to $h \neq 0$. So do we actually know the direction of the cut?



M. Stephanov, *Phys.Rev.D* 73 (2006) 094508

How do we transition from the exp(-V) to power law in performing the extrapolation of the Taylor series expansion. We need factorial divergence of the coefficience. Do we see this in the data?