Yang-Lee zeros from the quark number density and the anaytic structure of the universal scaling function

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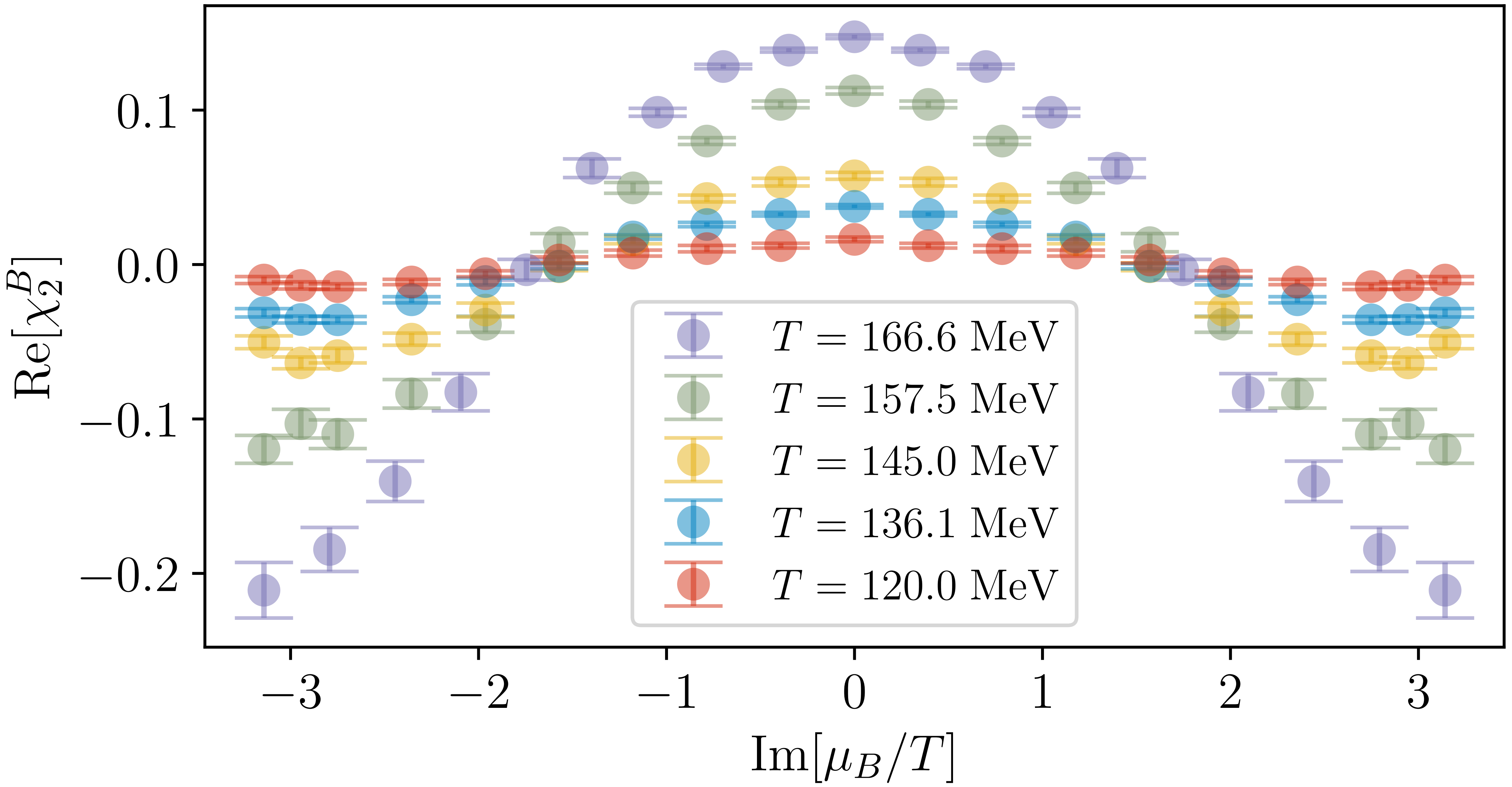
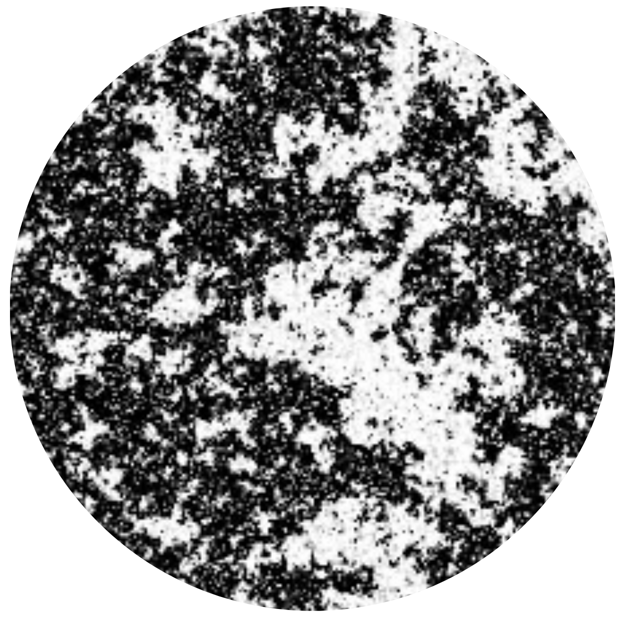
- The analytic structure of the universal scaling function using the Schofield parameterization → **Karsch, Schmidt, Singh, PRD 109 (2024) 1**
- Learning the quark number density with complex valued periodic neural networks (sinh-net) → **Brandt, Schmidt (work in progress)**
  - ➡ Fourier coefficients of the density and their asymptotic behaviour from universal scaling → **Bryant, Schmidt, Skokov, PRD 109 (2024) 7**
  - ➡ Canonical partition functions and the fugacity representation of the grand canonical partition function → **Outlook**



Data from:
Clarke, Dimopoulos, Di Renzo, Goswami, Schmidt, Singh, Zambello, arXiv: 24.0510196



Scaling hypotheses:

Free energy: $f_s(t, h, L) = b^{-d} f_s(b^{y_t} t, b^{y_h} h, L^{-1} b)$

Effective model O(4)/O(2)/Z(2):

Scaling fields

t reduced temperature

h reduced symmetry breaking field

L^{-1} inverse system size

map QCD to the effective model

controlled by non-universal parameters:

t_0, h_0, l_0
T_c, H_c

(2+1)-flavor QCD:

Scaling fields

$$t = \frac{1}{t_0}\left(\Delta T + \kappa_2^l \hat{\mu}_l^2 + \kappa_2^s \hat{\mu}_s^2 + 2\kappa_{11}^{ls} \hat{\mu}_l \hat{\mu}_s\right)$$

$$\Delta T = \frac{T - T_c}{T_c}$$

$$h = \frac{1}{h_0}(H - H_c), \quad H = \frac{m_l}{m_s}$$

$$l = l_0 L^{-1}$$

- Establish universal scaling is important for locating and classifying critical points.
- Determinations of the non-universal parameters, including $\kappa_2^l, \kappa_2^s, \kappa_{11}^{ls}$ are available

HotQCD, PRD 109 (2024) 11

Order parameter and (infinite size) scaling functions

Ansatz for the free energy density

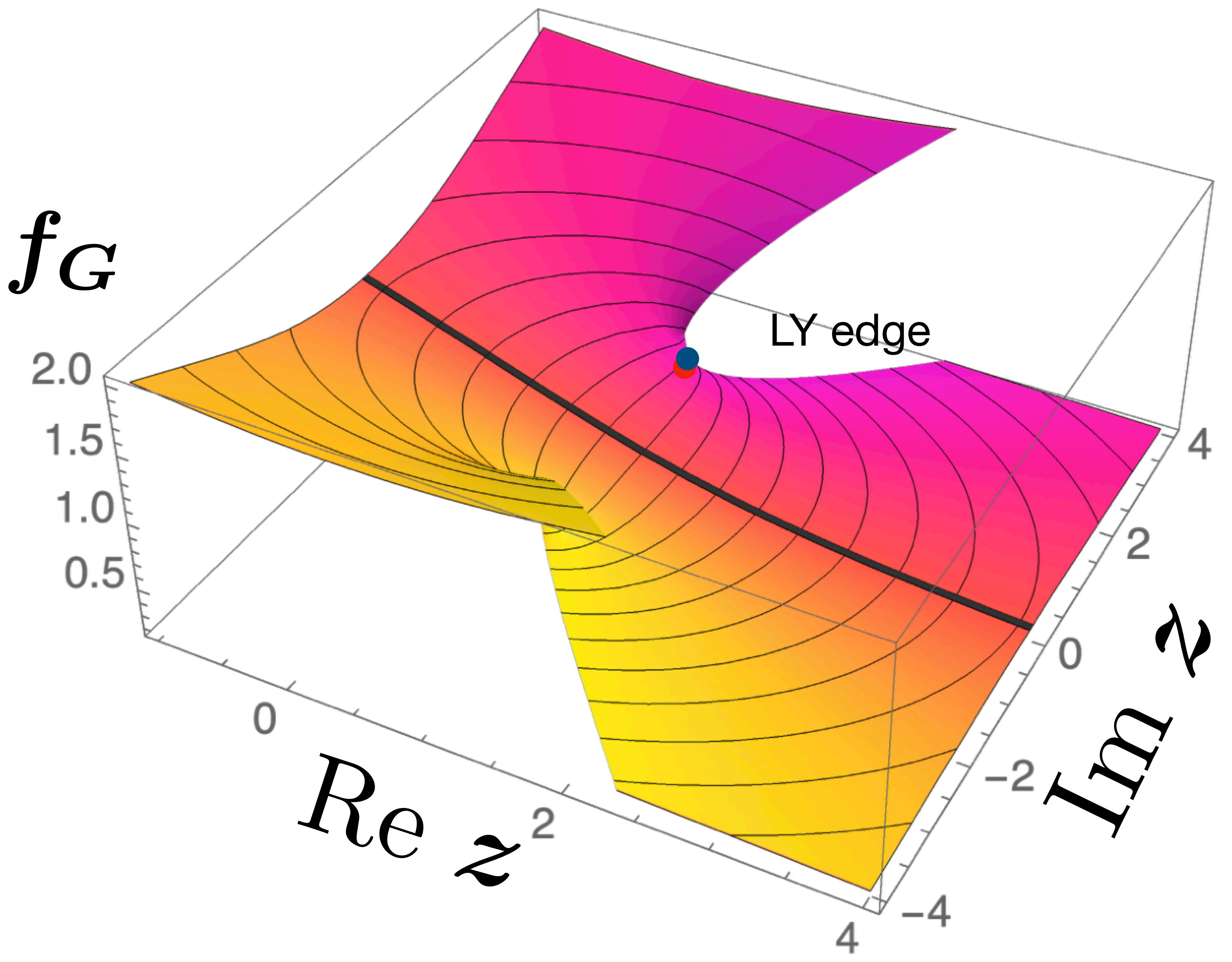
$$\frac{f(T,H)}{T} \equiv -\frac{1}{V}\ln Z(T,H) = H_0 h^{1+1/\delta} f_f(z) + reg.$$

with $z = \frac{t}{h^{1/\beta\delta}}$, $h \equiv H/H_0$, $t = t_0^{-1}\frac{T-T_c}{T_c}$

Order parameter

$$M = -\frac{\partial f}{\partial H} \equiv h^{1/\delta} f_G(z)$$

with $f_G(z) = -\left(1+\frac{1}{\delta}\right) f_f(z) + \frac{z}{\beta\delta} f_f'(z)$



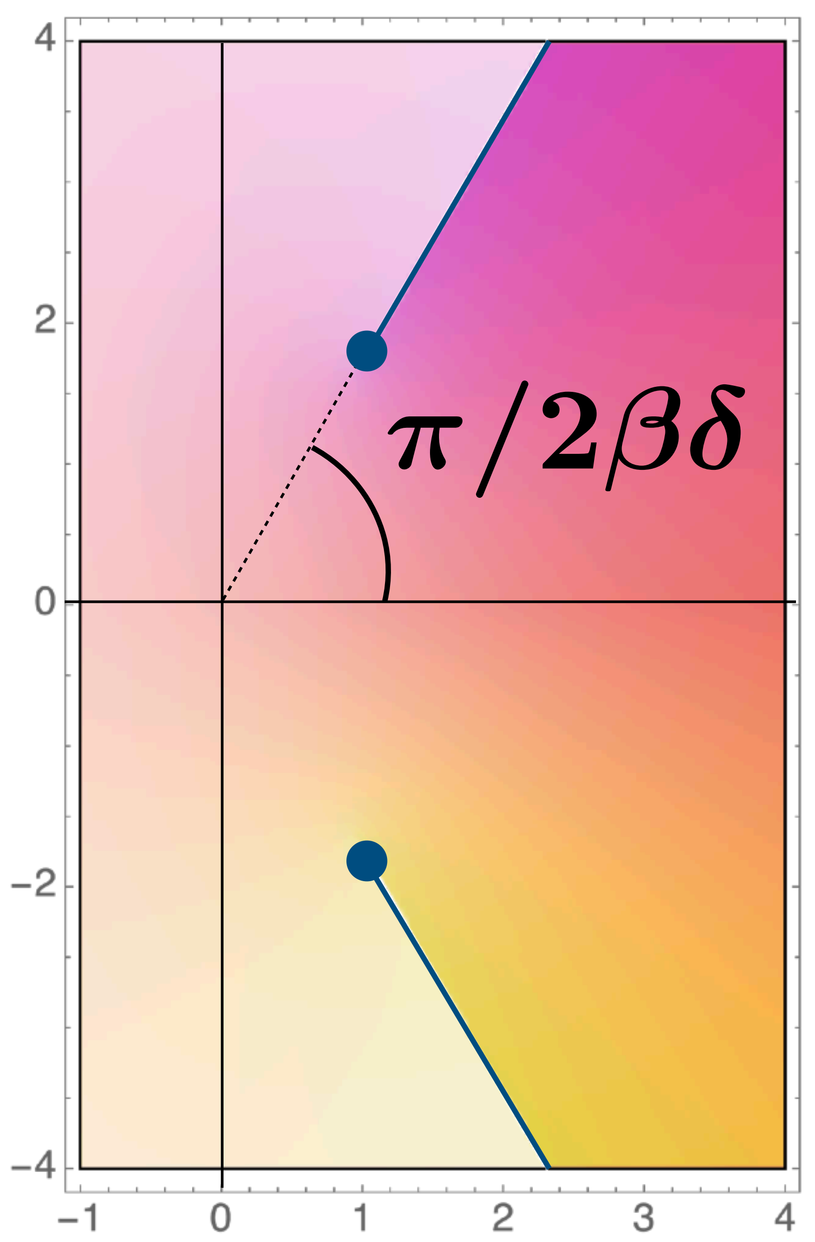
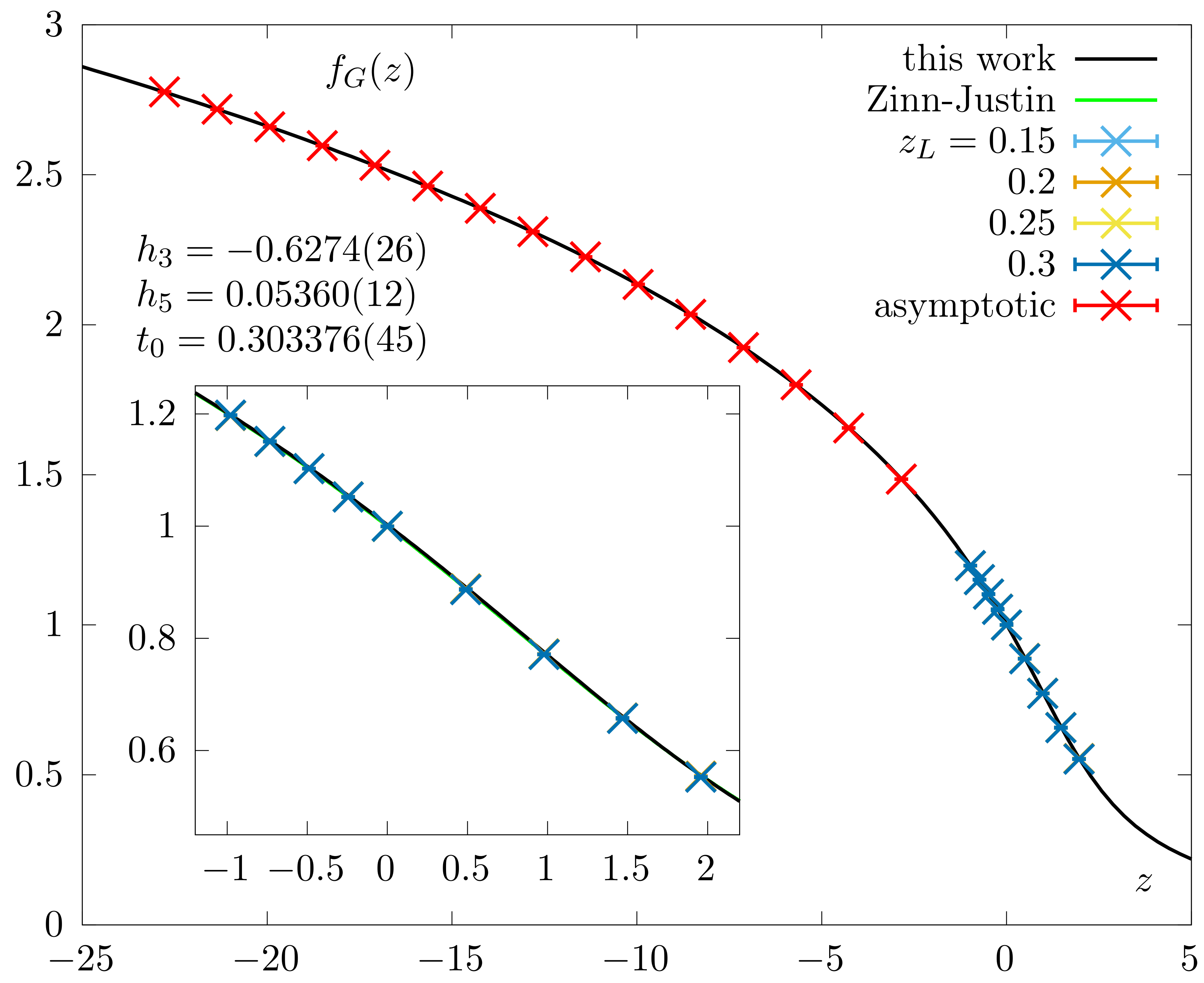


Fig: complex scaling function $f_G(z)$ over the complex z pane, courtesy to V. Skokov



[Karsch et al. PRD 108 (2023) 014505]

- f_G of $O(N)$ models has been determined analytically (ε-expansion for $N=1$ and $N=\infty$) and by Monte Carlo simulations.

An alternative coordinate frame introduced by Schofield:

$$(t,h) \to (R,\theta)$$

$$M = m_0 R^\beta \theta$$
$$t = R(1-\theta^2)$$
$$h = h_0 R^{\beta\delta} h(\theta)$$

$$f_G(z) \equiv f_G(\theta(z)) = \theta \left(\frac{h(\theta)}{h(1)}\right)^{-1/\delta}$$

$$z(\theta) = \frac{1-\theta^2}{\theta_0^2-1}\theta_0^{1/\beta}\left(\frac{h(\theta)}{h(1)}\right)^{-1/\beta\delta}$$

- Asymptotic behaviour of f_G by construction
- All non-trivial information in $h(\theta)$

Find singular points of magnetic susceptibility

$$\chi_h = \frac{\partial M}{\partial H} = \frac{h^{1/\delta-1}}{H_0} f_\chi(z)$$

$$f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f'_G(z)\right)$$

using Schofield's parametrisation

$$f_\chi(z) = \frac{\left((2\beta-1)\theta^2+1\right)h(1)}{2\beta\delta\theta h(\theta)-(\theta^2-1)h'(\theta)} \left(\frac{h(\theta)}{h(1)}\right)^{1-1/\delta}$$

Condition for singular points:

$$0 = 2\beta\delta\theta h(\theta) - \left(\theta^2 - 1\right) h'(\theta)$$

$$\Leftrightarrow \quad 0 = \frac{\mathrm{d}z(\theta)}{\mathrm{d}\theta}$$

Form of $h(\theta)$ in the LPM approximation:

- $h(\theta)$ is an odd function of θ
- $h(\theta)$ has roots at $0,\ 1,\ \theta_0$
- (Generalised) linear parametric model approximation:

$$h(\theta) = \theta(1 - (\theta/\theta_0)^2)^g \quad , \quad g = 1,\ 2$$

with $$g = \begin{cases} 1 \ , \mathrm{mean-field} \\ 2 \ , N = \infty \end{cases}$$

- LPM approximation is kept exact for epsilon expansion up to $O(\varepsilon^2)$
- Beyond LPM we define

$$h(\theta) = \theta\left(1 - \left(\frac{\theta}{\theta_0}\right)^2\right)^g \left(1 + c_2\theta^2 + c_4\theta^4 + \mathcal{O}(\theta^6)\right)$$

$$\equiv \theta(1 + h_3\theta^2 + h_5\theta^4 + h_7\theta^6 + \mathcal{O}(\theta^8))$$

Eos:

$$f_G(z + f_G^2)^g = 1$$

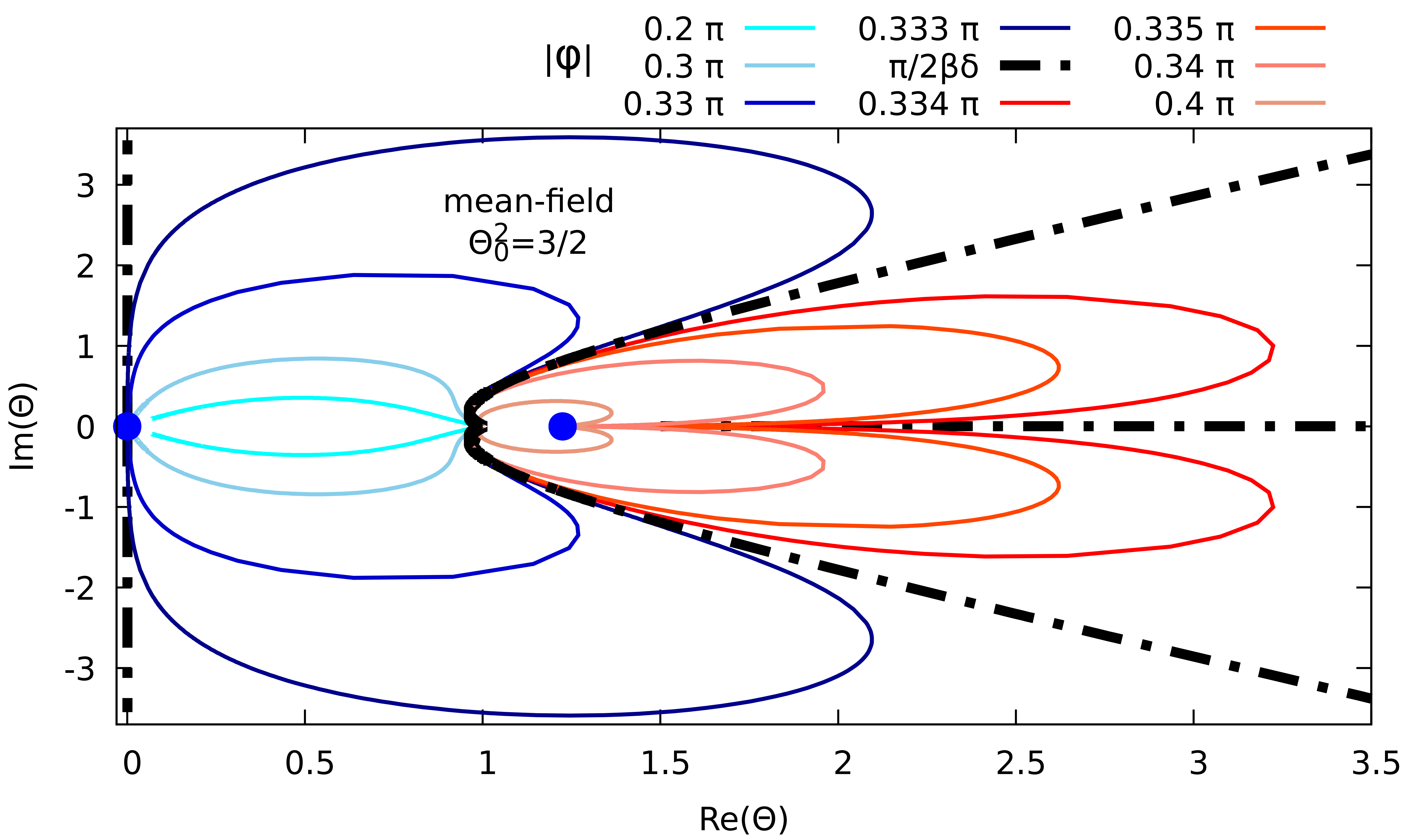
Critical exponents:

$$\beta = \frac{1}{2} \quad , \quad \delta = \begin{cases} 3 \, , \text{mean} - \text{field} \\ 5 \, , N = \infty \end{cases}$$

YLE

$$z_{LY} = \begin{cases} 3 \cdot 2^{-2/3} \, e^{\pm i\pi/3} \, , & \text{mean} - \text{field} \\ 5 \cdot 2^{-8/5} \, e^{\pm i\pi/5} \, , & N = \infty \end{cases}$$

Lines of constant $\arg(z)$ for a particular choice of $z(\theta)$ in the mean — field case



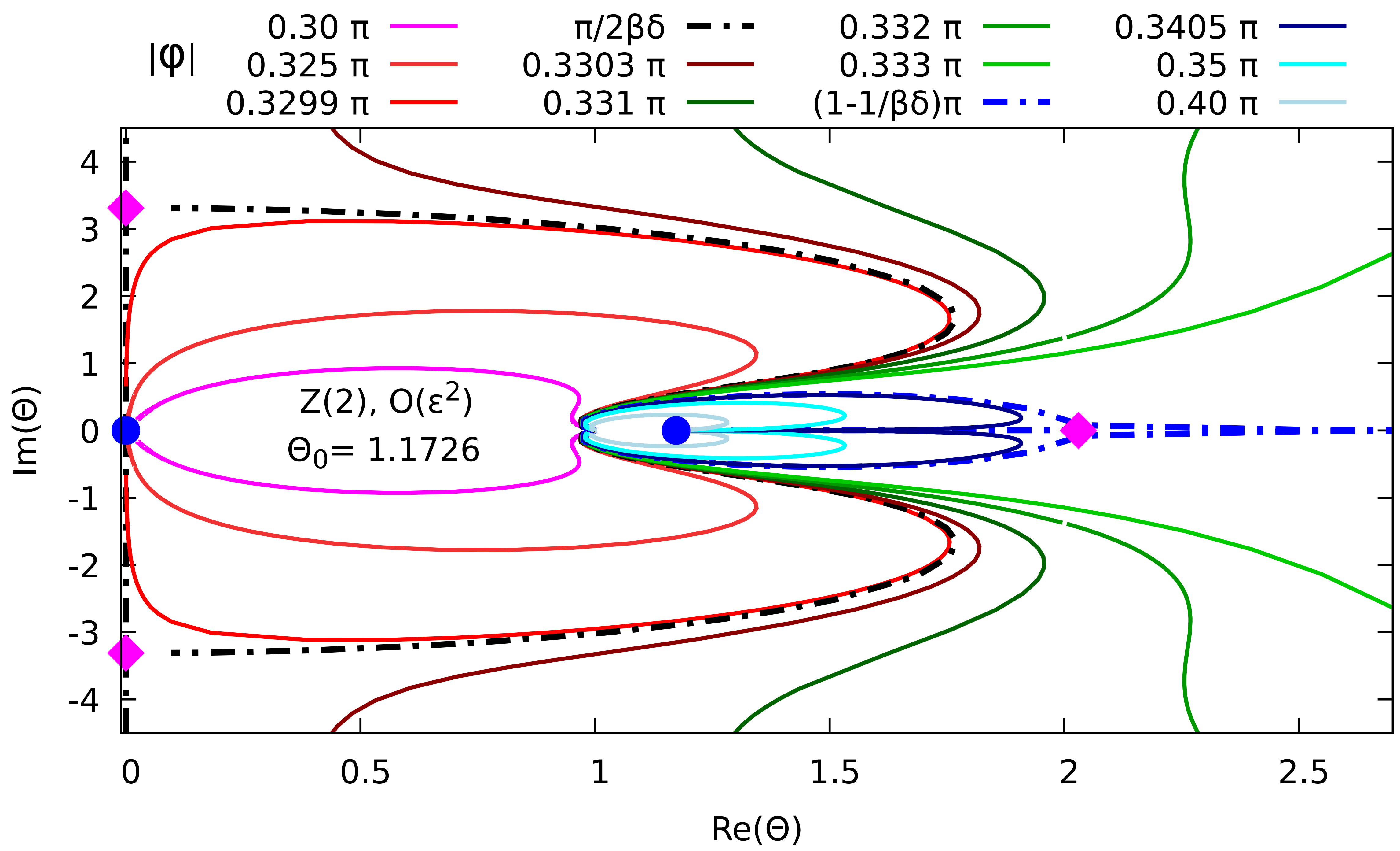
- θ_{LY} is θ_0 dependent
- For $\theta_0 = 3/2$ we have $z(\theta) : \mathbb{C}^+ \to \mathbb{C}$ and $\theta_{LY} = \infty$

- LPM approximation is kept exact for epsilon expansion up to $O(\varepsilon^2)$

Critical exponents:

$$\beta = \frac{1}{2} - \frac{1}{6}\epsilon + \frac{1}{162}\epsilon^2 + \mathcal{O}(\epsilon^3)$$
$$\delta = 3 + \epsilon + \frac{25}{54}\epsilon^2 + \mathcal{O}(\epsilon^3)$$
$$\beta\delta = \frac{3}{2} + \frac{1}{12}\epsilon^2 + \mathcal{O}(\epsilon^3)$$

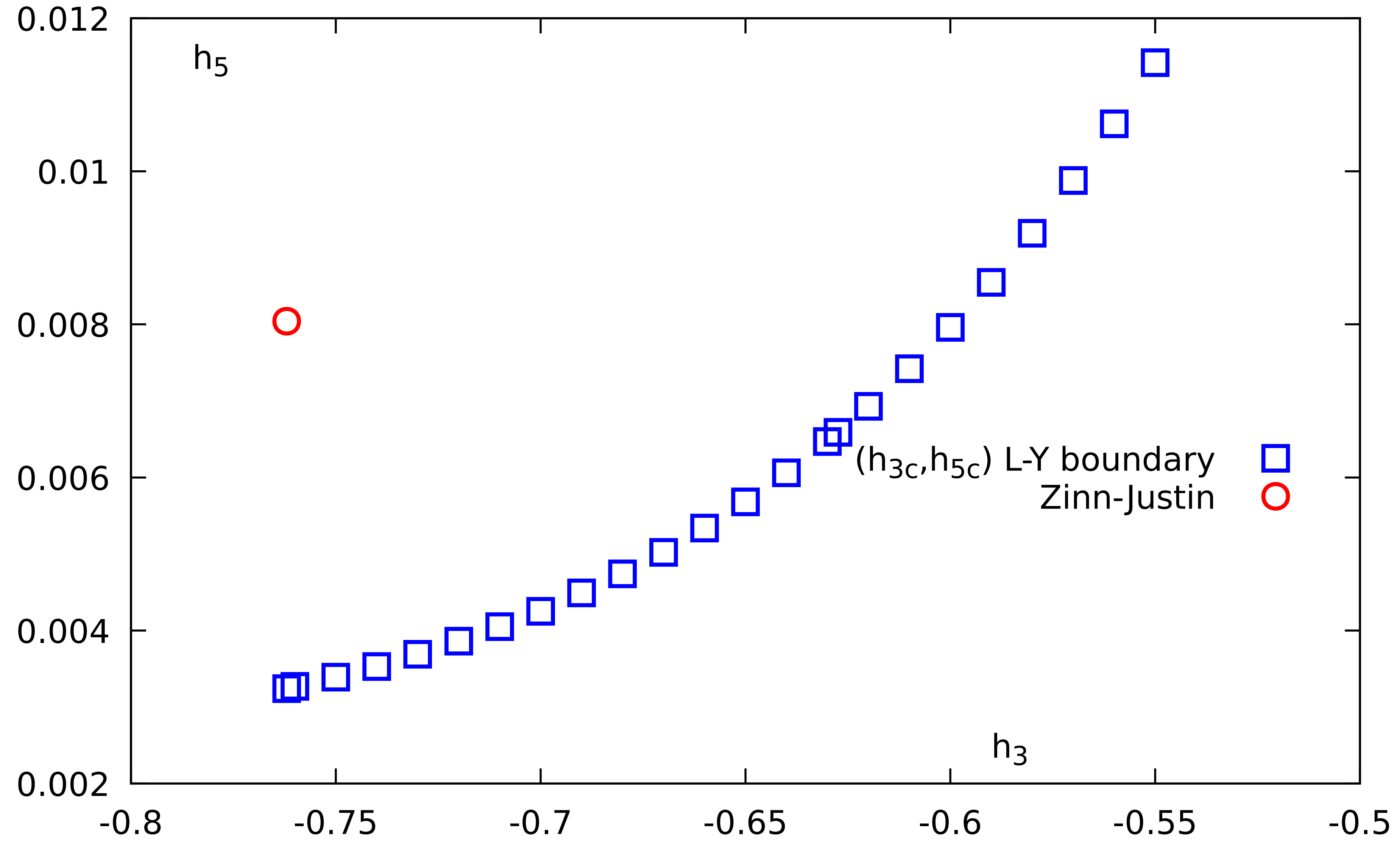
Lines of constant $\arg(z)$

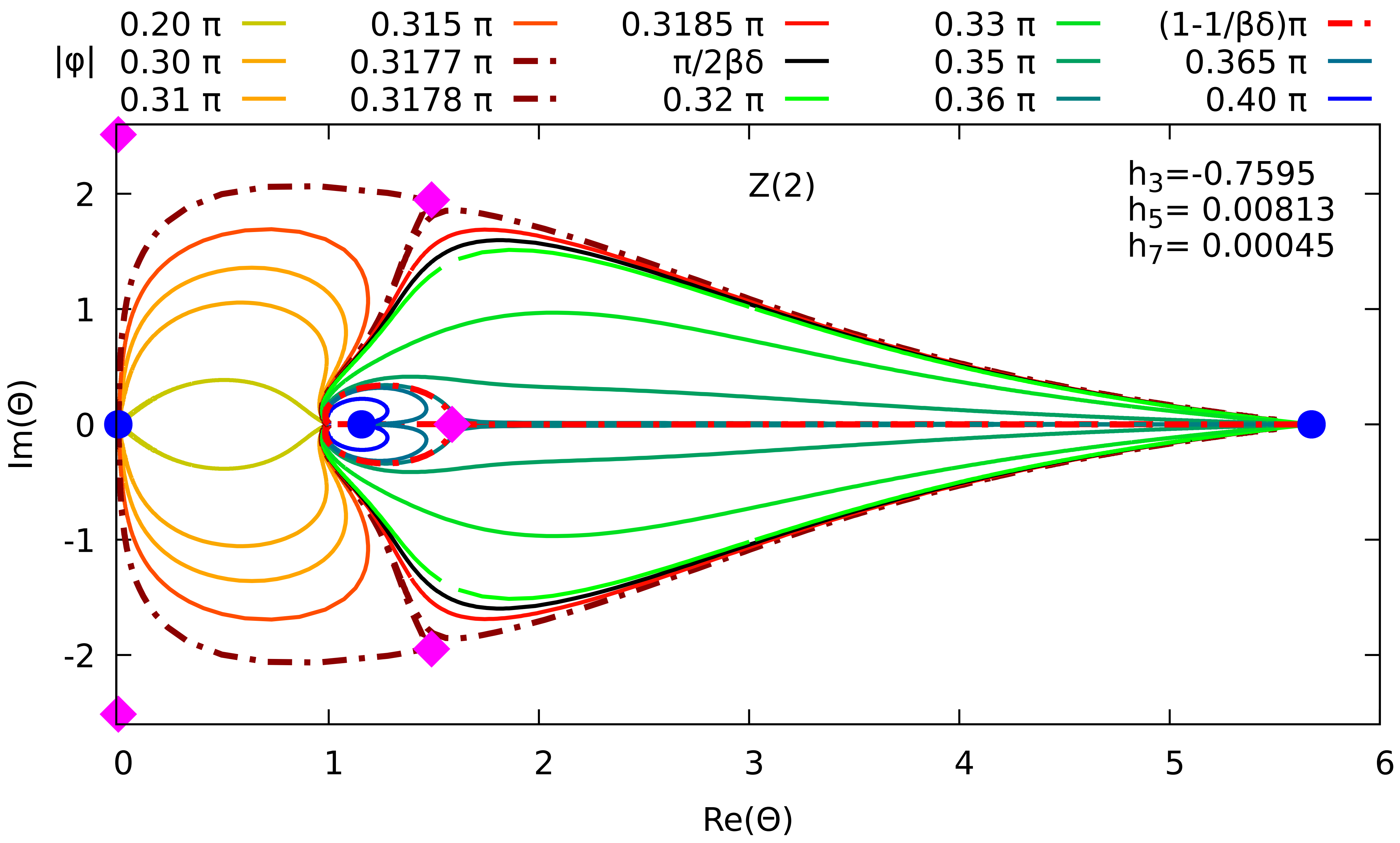


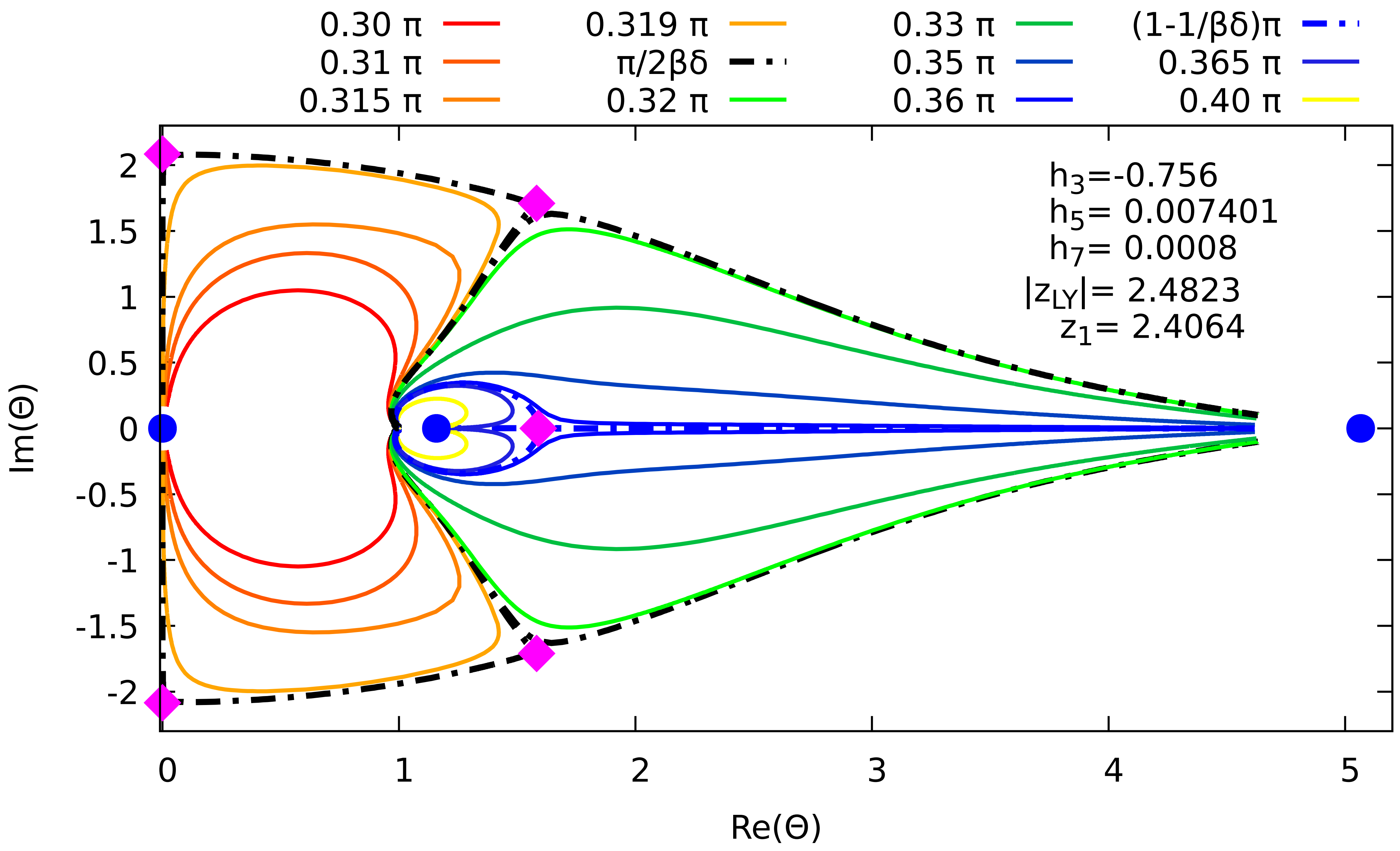
YLE

$$\theta_{Lan,+} = 2.03038\ , \qquad z_{Lan} = 2.24047 e^{i\pi(1-1/\beta\delta)}$$
$$\theta_{LY,\pm} = \pm 3.31198\ i \qquad z_{LY,\pm} = 2.30674\ e^{\pm i\pi/2\beta\delta}$$

- Perturbative estimate of h_3, h_5, h_7 by Zinn-Justin, Phys.Rept.344:159-178,2001
- Only finite region of the θ-plane is mapped to the z-plane.
- Find that the LYE has wrong phase
- Parameters can be tuned to correct phase







- In O(2) and O(4) models the coefficients for h_3, h_5, h_7 are obtained from MC calculations
  [Karsch et al. PRD 108 (2023) 014505]
- Again, coefficients can be tuned for the correct LY phase
- Universal z_{LY} for Z(2) and O(2) are in agreement with FRG calculations
  [Johnson, Rennecke, Skokov, PRD 107, 116013 (2023)]

	$Z(2)$: $\mathcal{O}(\epsilon^2)$	$Z(2)$	$O(2)$	$O(4)$
$\|z_{LY,b}\|$	–	–	$2.281(72)$	$1.977(73)$
$\|z_{Lan}\|$	2.240	$2.374(36)$	–	–
$\|z_{LY}\|$	2.307	$2.429(56)$	$1.95(7)$	$1.47(3)$
$\|z_{LY}\|$ $[FRG]$		$2.43(4)$	$2.04(8)$	$1.69(3)$

Karsch, Schmidt, Singh, PRD 109 (2024) 1

Definition:

$$b_k(T) = \frac{1}{\pi}\int_0^{2\pi} \mathrm{d}\theta_B \, \mathrm{Im}\,\chi_1^B(T, i\theta_B)\sin(k\theta_B)$$

$\mu_B/T = i\theta_B$, with $\theta_B \in \mathbb{R}$

highly oscillatory for large k

Data only defined on a discrete set of points

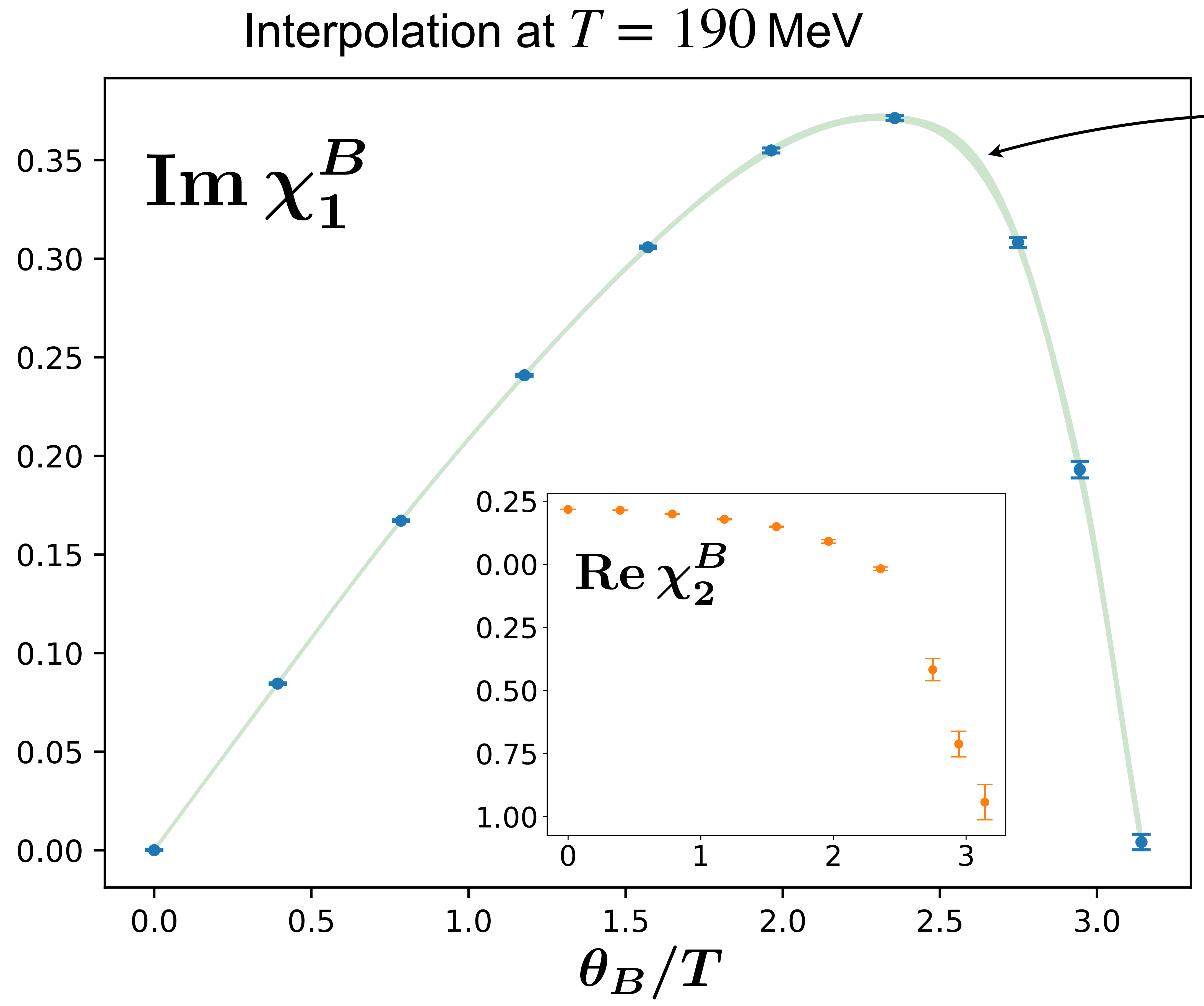
A Fourier interpolation of the data is periodic by construction!

Method:

- Interpolate $\mathrm{Im}\,\chi_1^B$, take also it's derivative $\mathrm{Re}\,\chi_2^B$ and eventually higher derivatives up to order s into account
  $\rightarrow$ Hermite-interpolation (spline)
- Piecewise integration can be done analytically

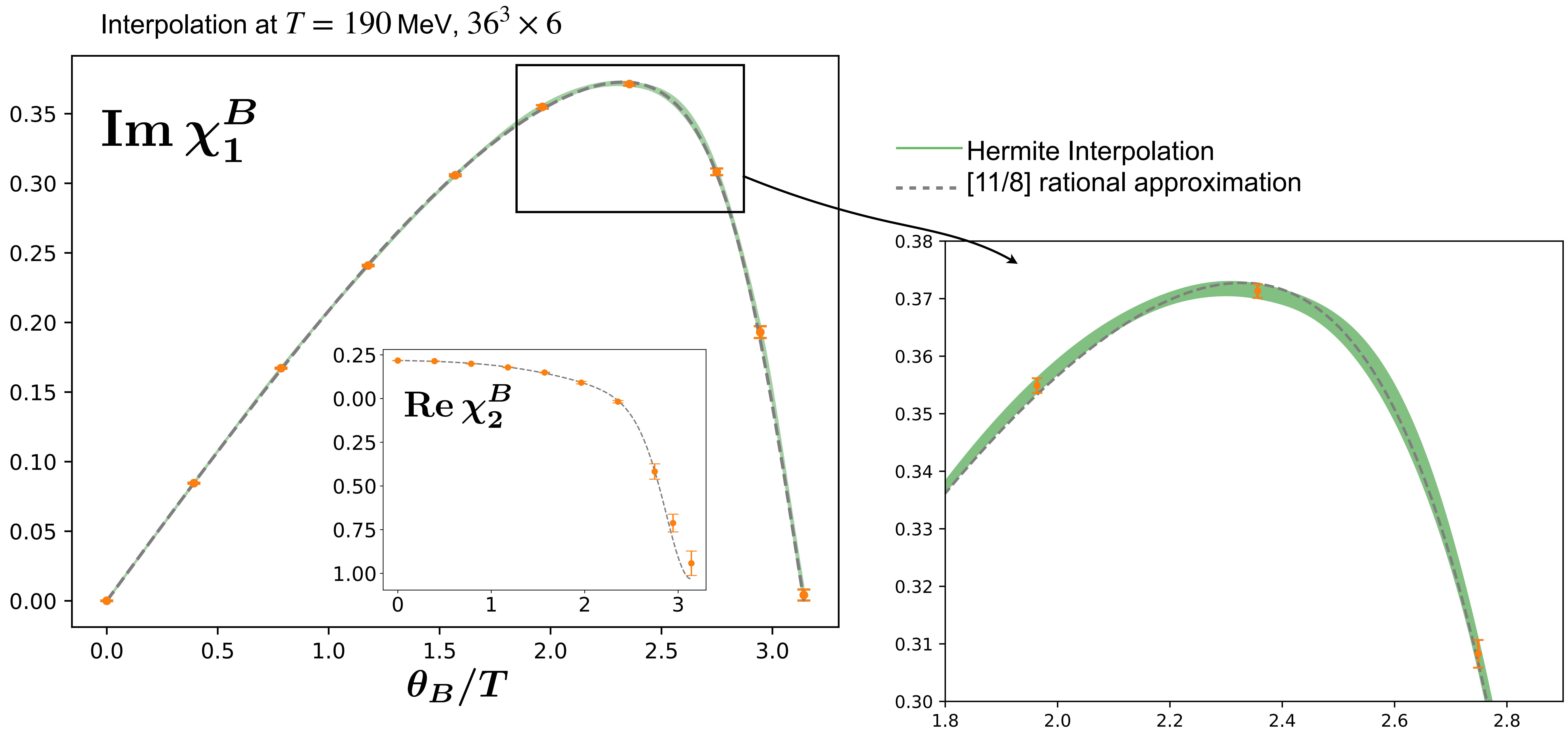
$$b_k = \frac{2}{\pi}\sum_{i=0}^{N-1}\int_{\theta_B^{(i)}}^{\theta_B^{(i+1)}} \mathrm{d}\theta_B \, p(\theta_B)\sin(k\theta_B) \quad \text{with} \quad 0 = \theta_B^{(0)} < \theta_B^{(1)} < \dots < \theta_B^{(N)} = \pi$$

  $\rightarrow$ variant of a Filon-type quadrature: error decreases as $\mathcal{O}(k^{-s-2})$ (for exact data)
- Statistical error is estimated by bootstrapping over the error of $\mathrm{Im}\,\chi_1^B$ and $\mathrm{Re}\,\chi_2^B$.



Error band of the Hermite-Interpolation (piecewise cubic, taking Im χ_1^B and Re χ_2^B into account)

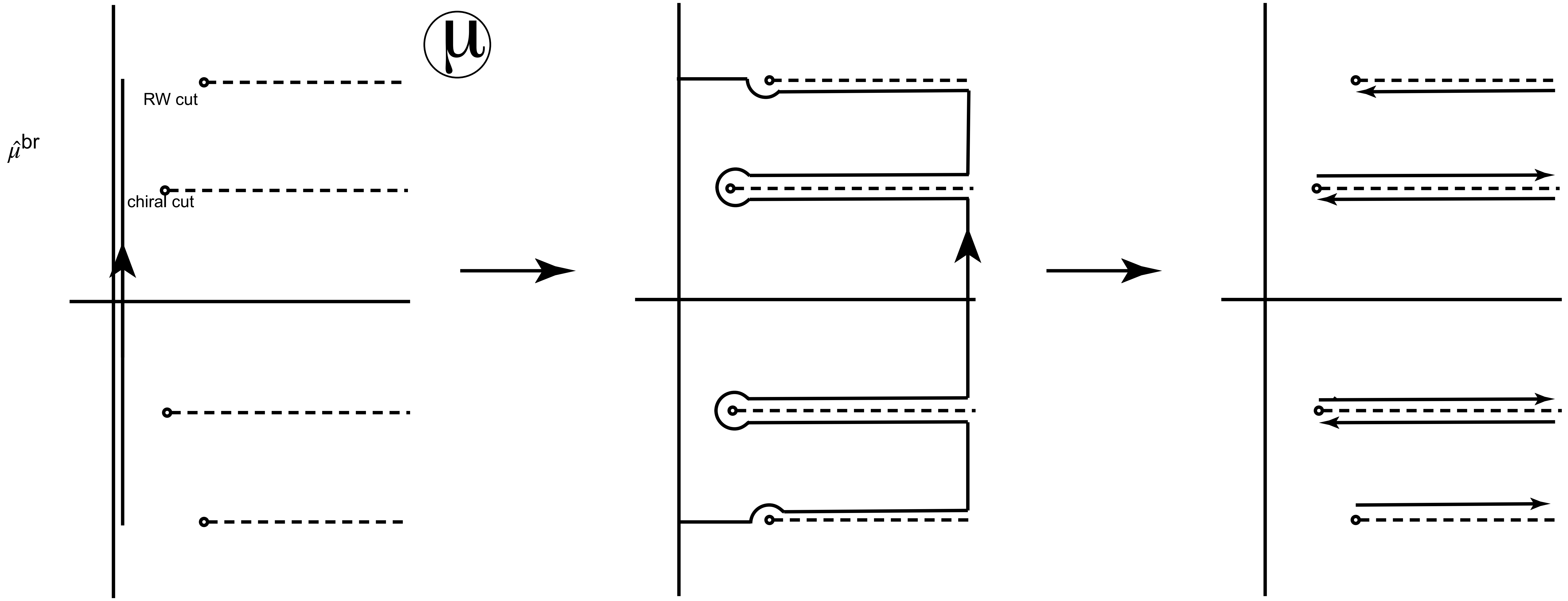
- Note asymmetry of the data, w.r.t a sin function: data can be described by $\mathcal{O}(10)$ Fourier-coefficients



- Note asymmetry of the data, w.r.t a sin function: need $\mathcal{O}(10)$ Fourier-coefficients to describe the data

- Both interpolations agree within error.

- We can deform the integration contour to integrate along the cuts



Only these integrals remain

- Assume that we can express the density along the cuts as

edge coefficient, $\sigma > -1$

$$n_B(\hat{\mu}) = \underbrace{A(\hat{\mu}-\hat{\mu}^{\rm br})^{\sigma}}_{\text{Leading order non-analytic part}}(1 + B(\hat{\mu}-\hat{\mu}^{\rm br})^{\theta_c} + \ldots) + \underbrace{\sum_{n=0}^{\infty} a_n(\hat{\mu}-\hat{\mu}^{\rm br})^n}_{\text{analytic part}}$$

Leading order non-analytic part

analytic part

- The final result for one cut is

$$b_k = \frac{e^{-\mu^{\text{br}}k}}{i\pi} A \frac{\Gamma(1+\sigma)}{k^{1+\sigma}} \left(1 - e^{i2\pi\sigma} + \frac{B}{k^{\theta_c}}\left[1 - e^{i2\pi(\sigma+\theta_c)}\right] \frac{\Gamma(1+\sigma+\theta_c)}{\Gamma(1+\sigma)} + \dots\right)$$

Absorbing k-independent factors into A and B we get

$$b_k = \tilde{A} \frac{e^{-\hat{\mu}^{\text{br}}k}}{k^{1+\sigma}} \left(1 + \frac{\tilde{B}}{k^{\theta_c}} + \dots\right)$$

Note that the regular part cancels completely

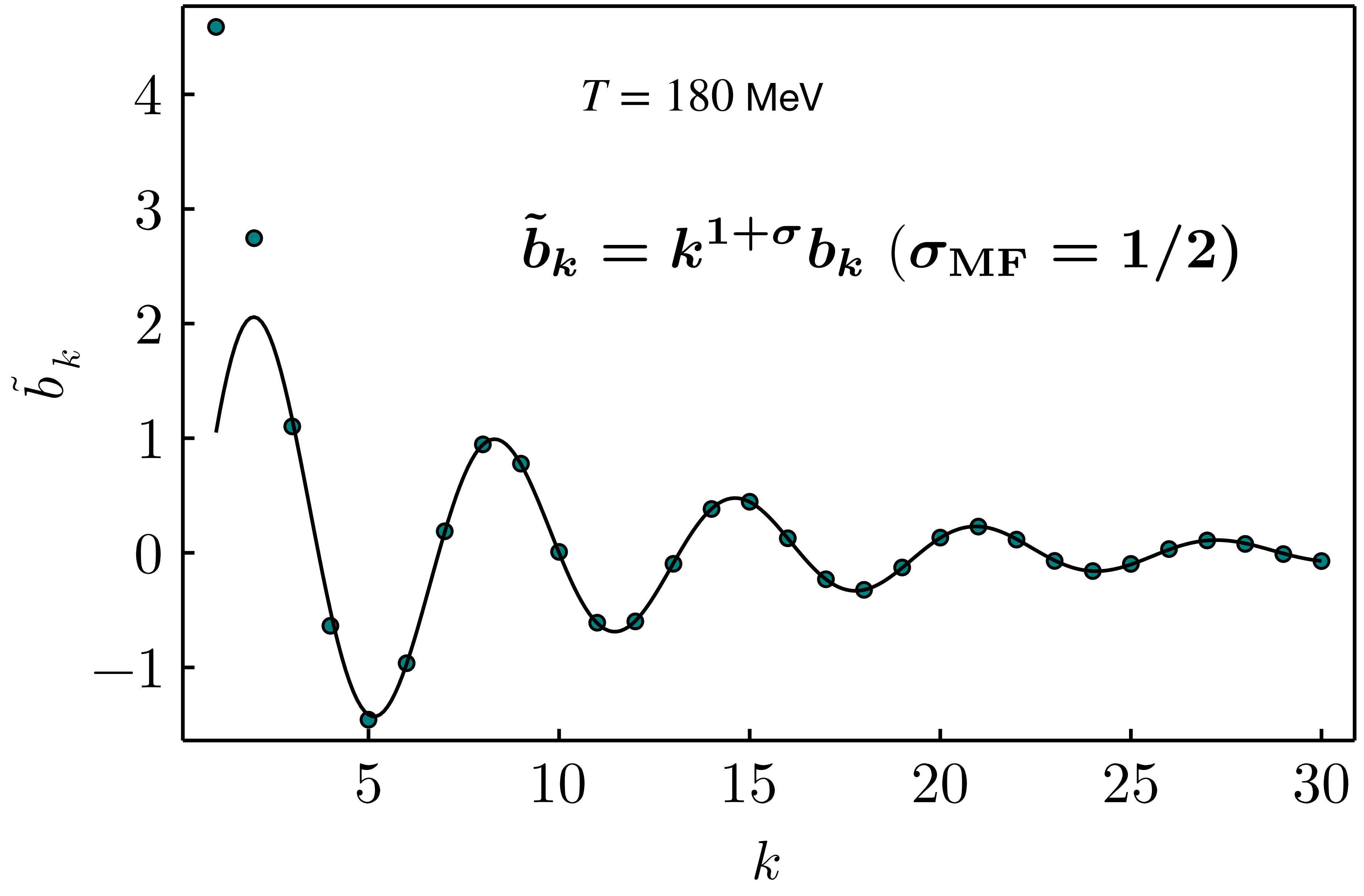
- The final result for both cuts is (dropping NLO)

$$b_k = |\tilde{A}_{\text{YLE}}| \frac{e^{-\hat{\mu}_r^{\text{YLE}}k}}{k^{1+\sigma}} \cos(\hat{\mu}_i^{\text{YLE}}k + \phi_a^{\text{YLE}}) + |\hat{A}_{\text{RW}}|(-1)^k \frac{e^{-\hat{\mu}_r^{\text{RW}}k}}{k^{1+\sigma}}$$

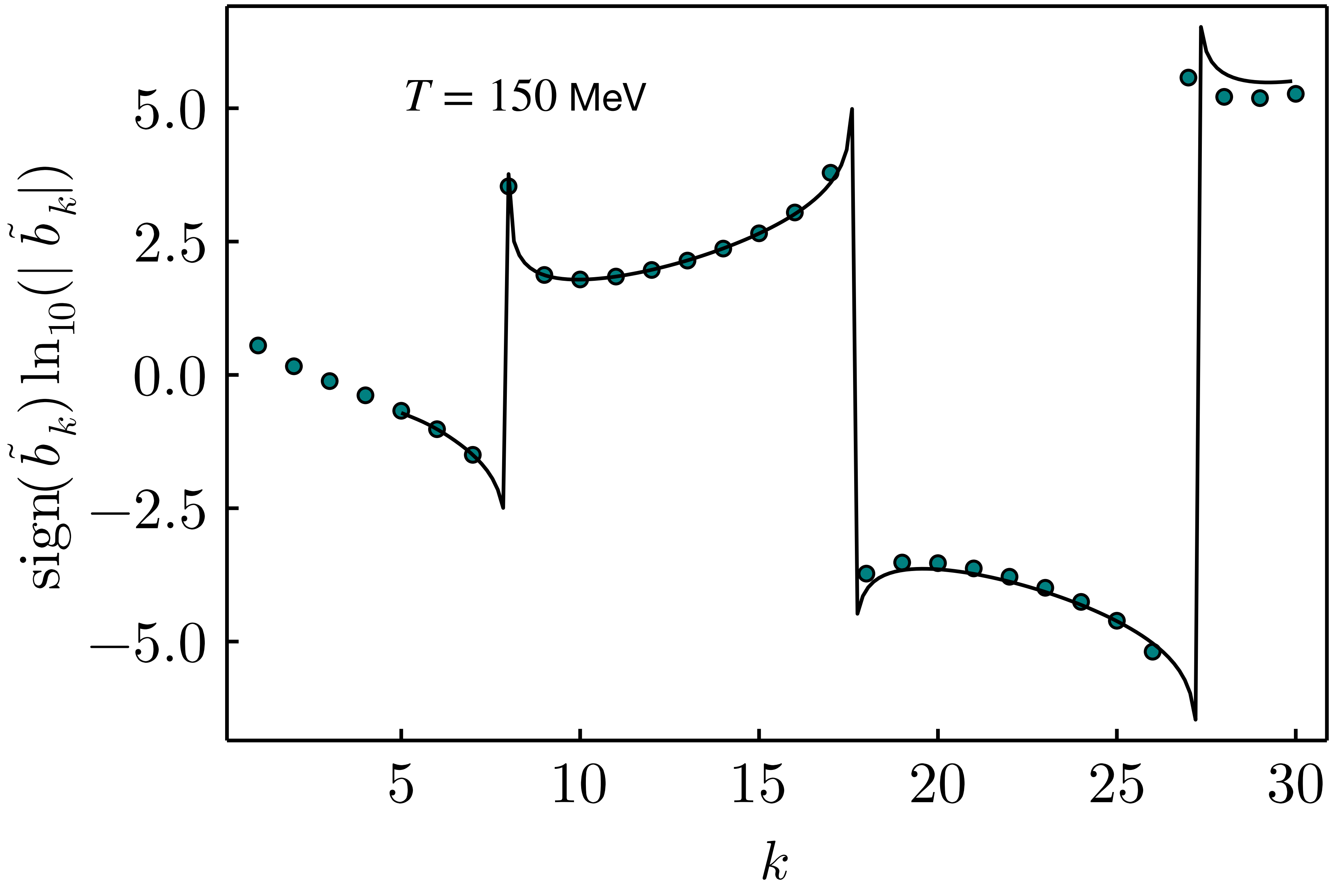
$\hat{\mu}_i^{RW} = \pi$

- The analytic form fits the Fourier coefficients from the quark-meson model well. Details of the Model can be found here [Skokov et al., PRD 82 (2010) 034029]

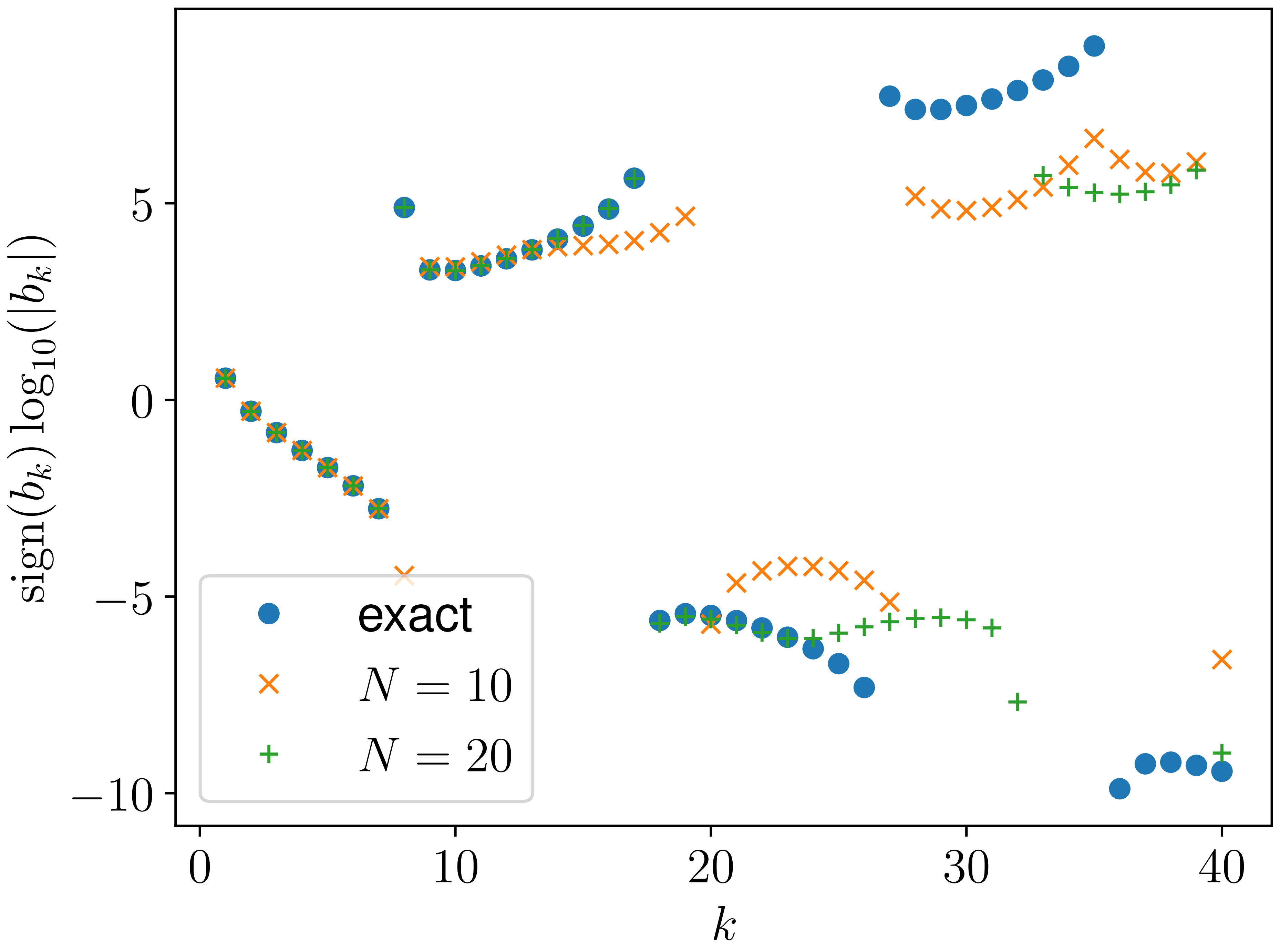
- In Mean-Field and LPA approximation fits to the Fourier coefficients reproduce the correct location of the LY edge up to (5 – 7)%.

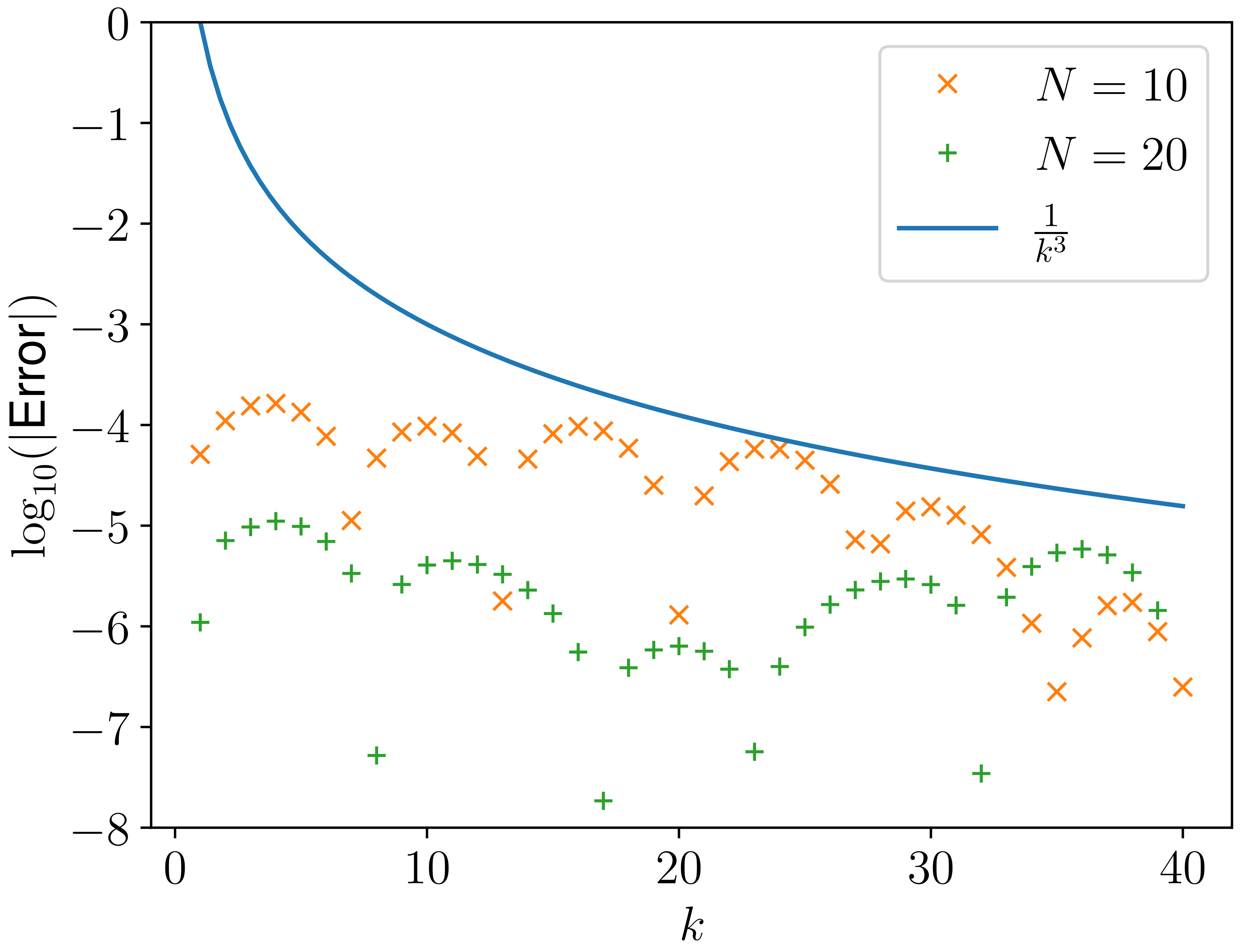


$$\hat{\mu}^{\mathrm{fit}}_{\mathrm{YLE}} = 0.1156(6) + i\,0.9952(5)$$
$$\hat{\mu}_{\mathrm{YLE}} = 0.118657 + i\,1.00256$$



$$\hat{\mu}^{\mathrm{fit}}_{\mathrm{YLE}} = 0.441(2) + i\,0.325(3)$$
$$\hat{\mu}_{\mathrm{YLE}} = 0.412884 + i\,0.342187$$

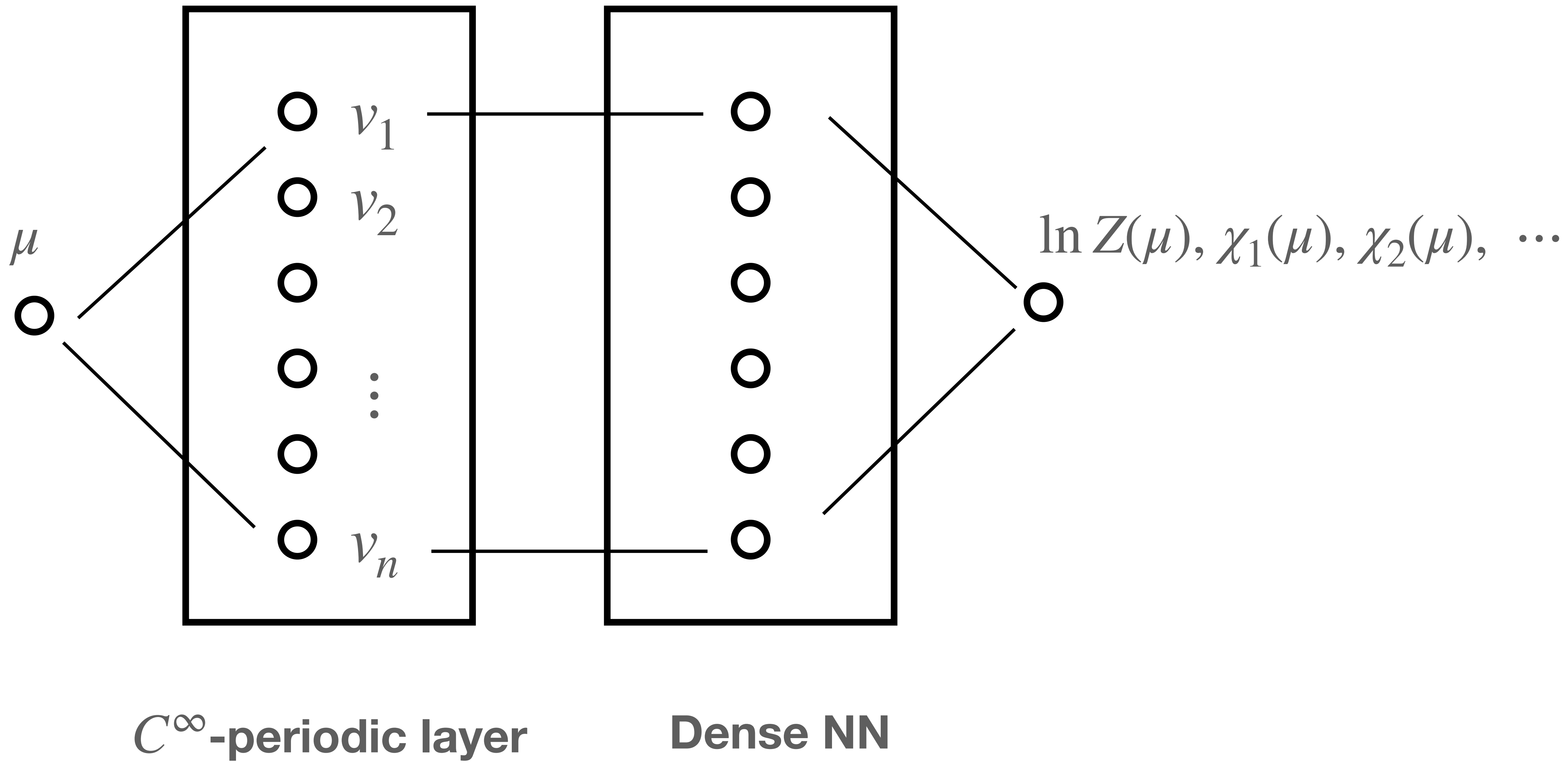




- Determination of Fourier coefficients from lattice data is very hard
- Need precision of $10^{-3} - 10^{-4}$ for $\text{Im}[\chi_1]$
- Need to calculate $\gtrsim 30$ coefficients, i.e. $\gtrsim 30$ μ-values when using FFT

Idea:

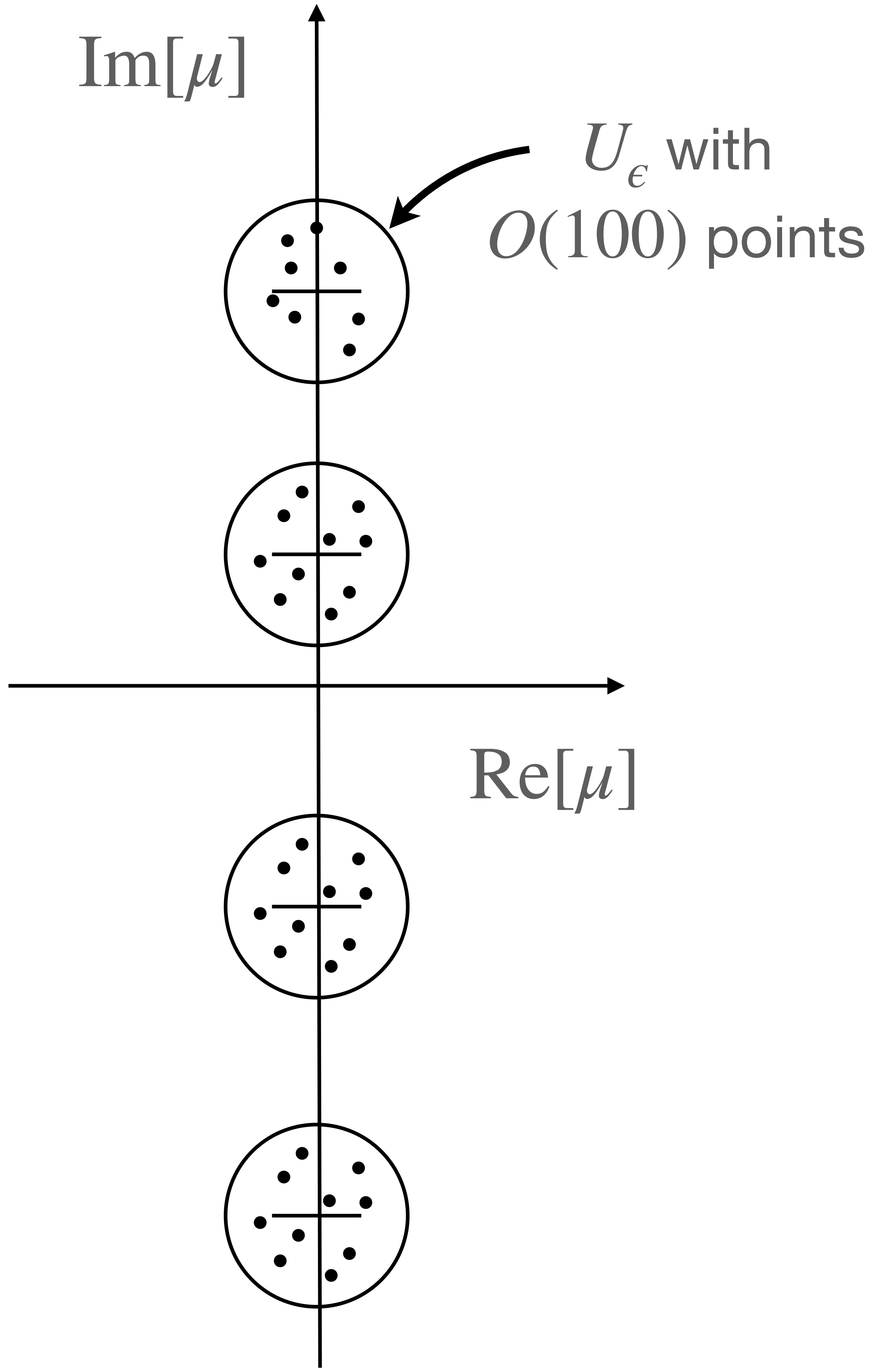
- Use complex ML model with build in periodicity for $\ln Z$
- Use automatic differentiation to learn from $\chi_1, \chi_2, \chi_3, \ldots$



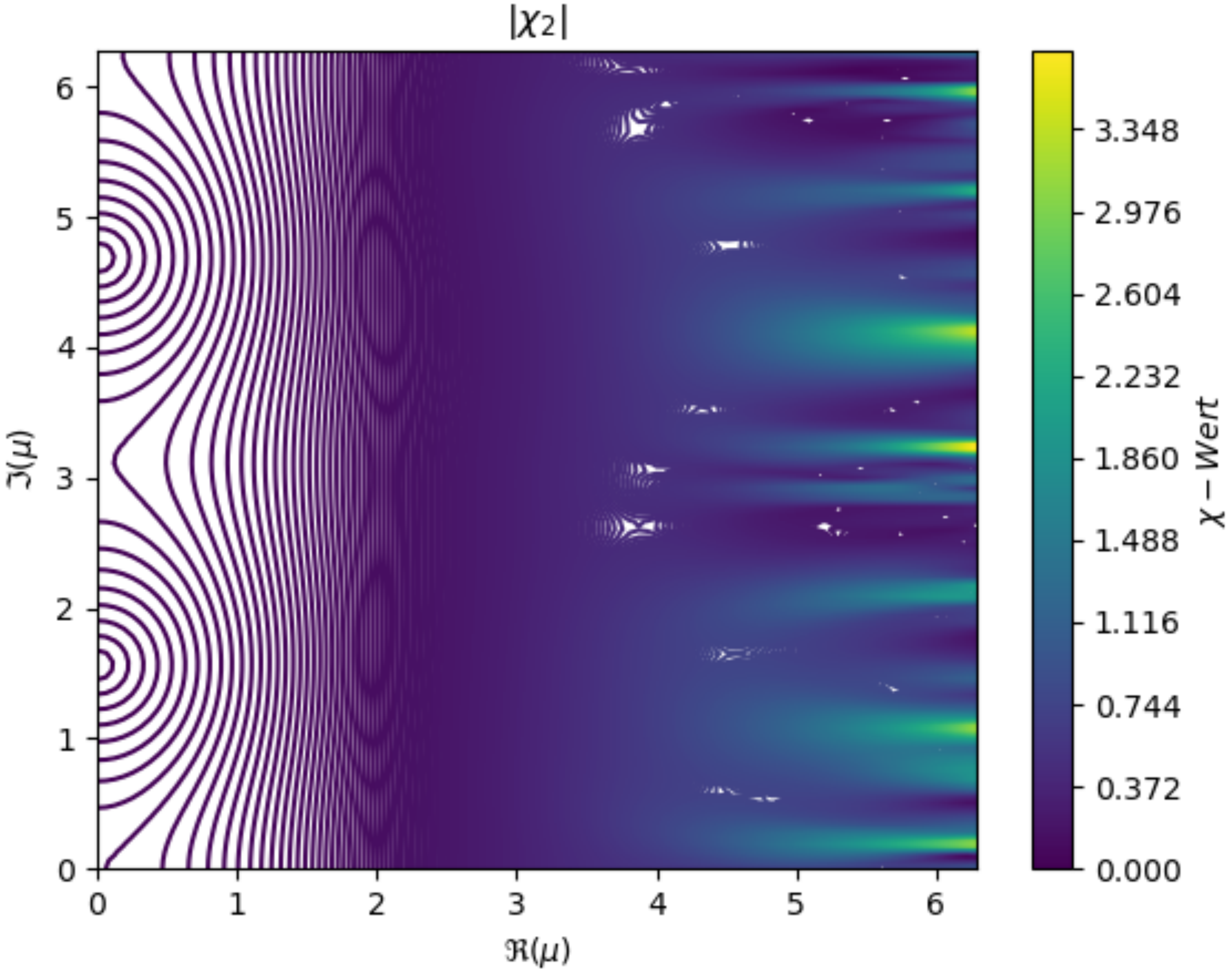
$$v_k = \sigma(A_k \cosh(\omega\mu + \phi_k) + c_k),\ \omega = \frac{2\pi}{L} \quad \Rightarrow \quad v^{(n)}(\mu) = v^{(n)}(\mu + L)$$

- Non-linear activation function generates higher harmonics
- We generalise the C^∞-periodic layer from [Dong, Ni, J. Comp. Phys 435 (2021) 110242] to complex numbers and replace cos by cosh functions.

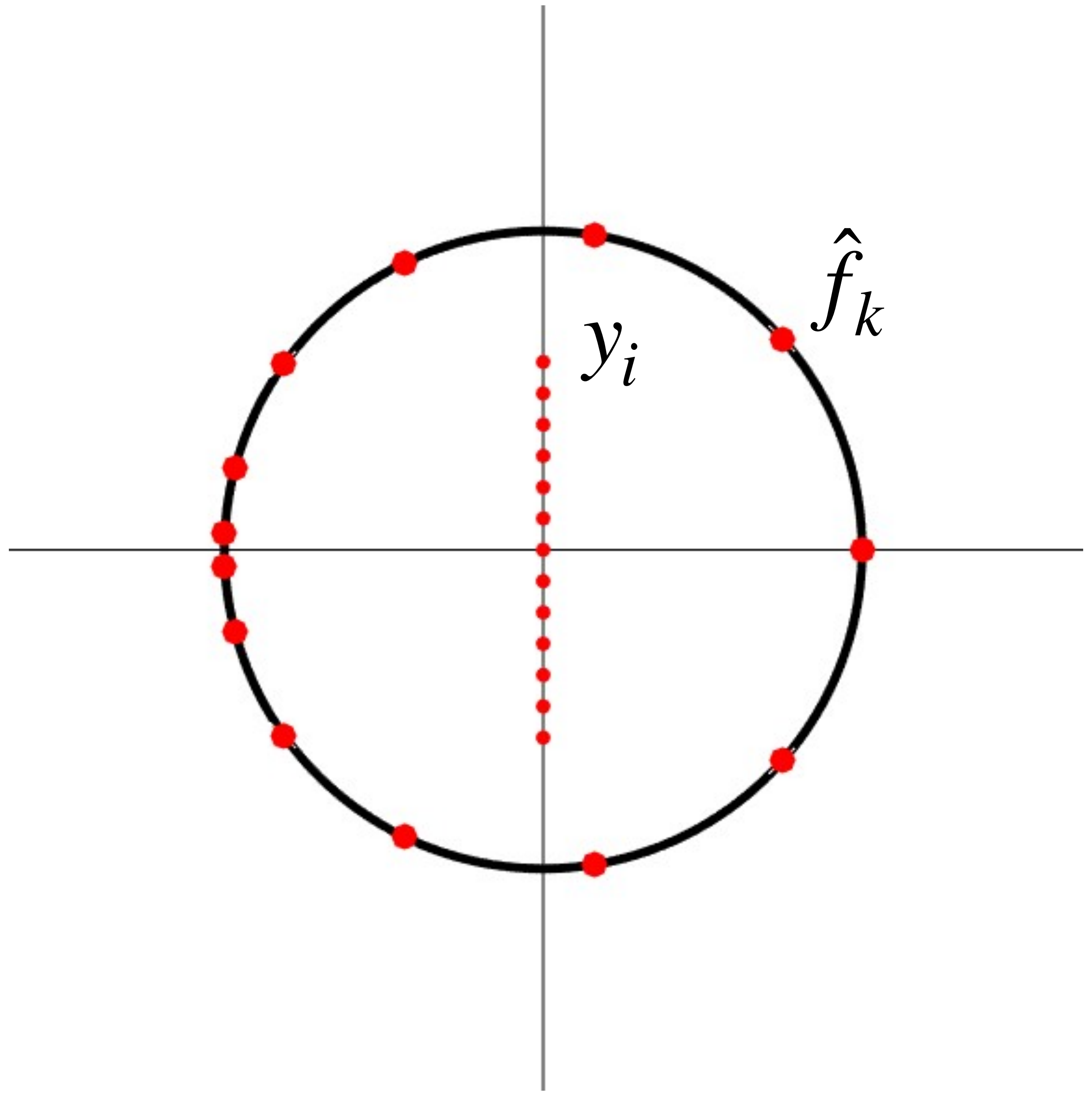
- Learning data: $\chi_1, \chi_2, \chi_3, \cdots$
- In order to break the symmetry we generate a number of complex points, normal distributed around the simulations points Taylor expand the data to that points



- Analytic continuation of χ_2, can we identify LYZ?
- Work in progress: use $\ln Z$ to calculate canonical partition functions



➡ **Talk by F. Di Renzo on Friday on analytic continuations from imaginary μ data**



- New method to calculate Taylor coefficients and analytic continuations from discreet data at imaginary μ, based on Cauchy's formula

$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0} dz$$

- Once the function on the contour is known, every point in the interior is known
  - ➡ Replace the integral by a Quadrature rule
  - ➡ Use Legendre weights
  - ➡ Set up a linear system and solve for the function values on the contour

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} \frac{f(R\,e^{i\theta})\,R\,e^{i\theta}}{R\,e^{i\theta} - z_0} d\theta \simeq \frac{1}{2\pi} \sum_{k=1}^{n} w_k \frac{f(R\,e^{i\theta_k})\,R\,e^{i\theta_k}}{R\,e^{i\theta_k} - z_0}$$

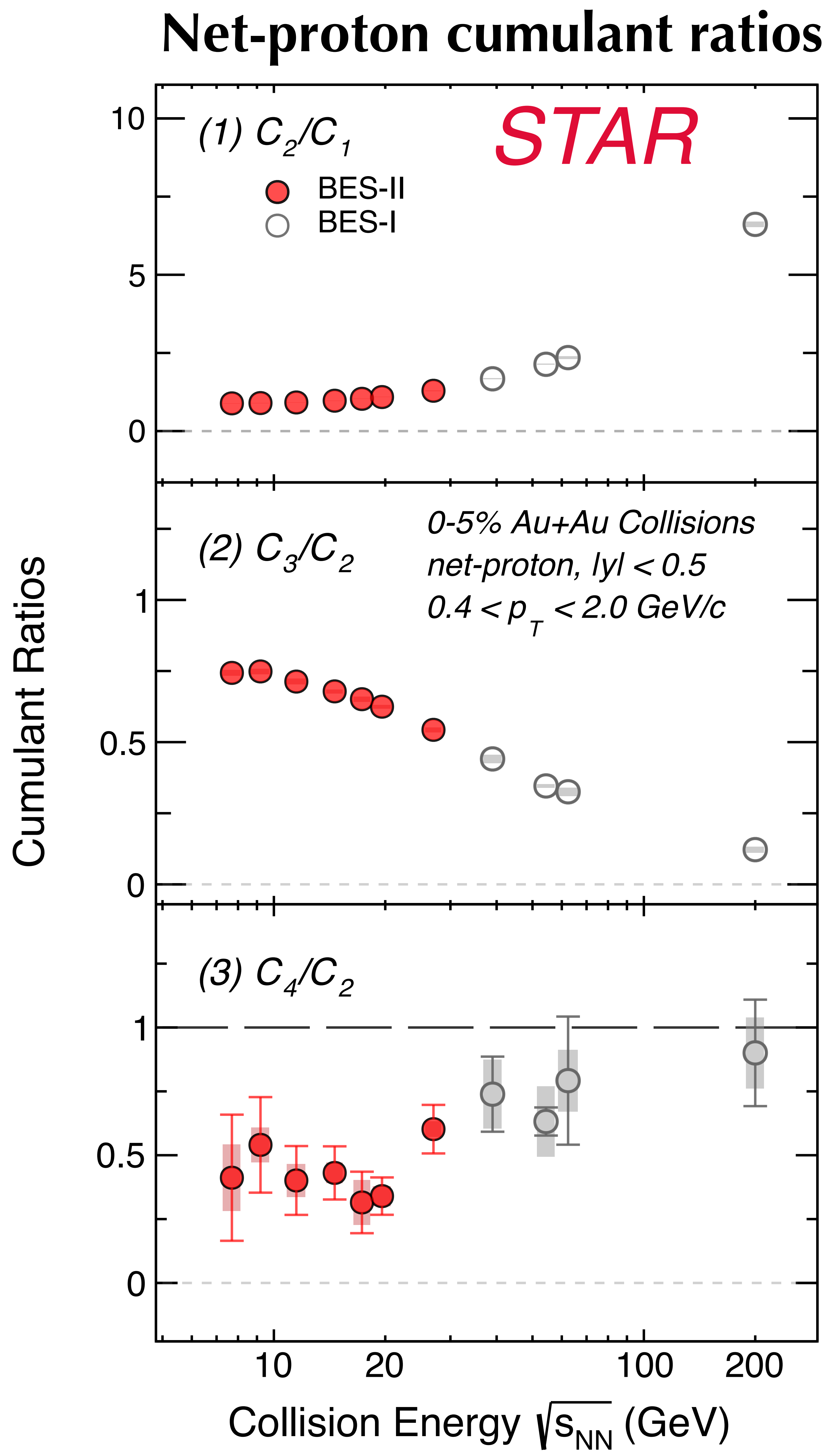
$$y_i = \frac{1}{2\pi} \sum_{k=1}^{n} w_k \frac{R\,e^{i\theta_k}}{R\,e^{i\theta_k} - z_i} \hat{f}_k\,, \quad i = 1, 2, \ldots, n$$

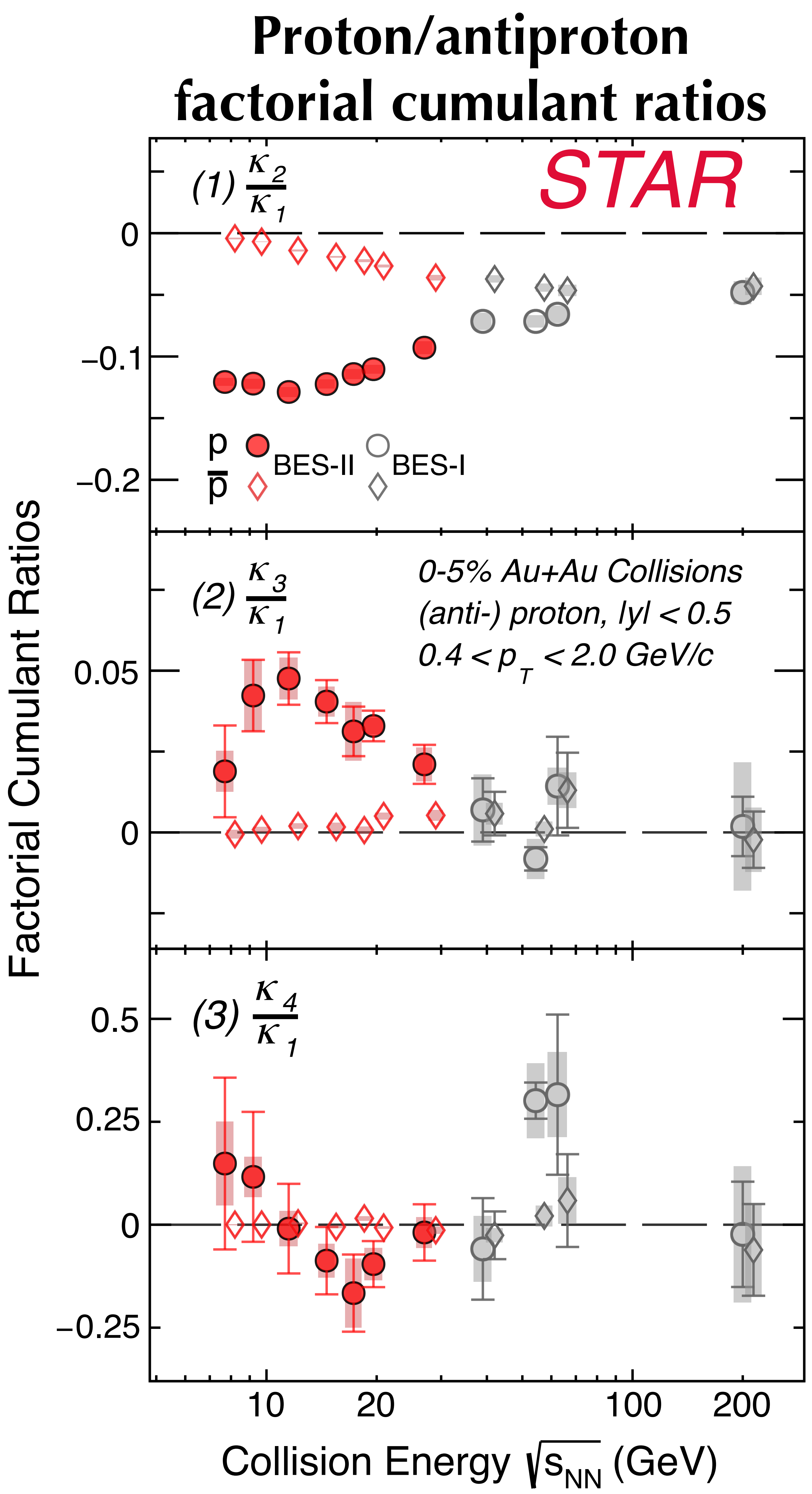
- Need to understand the analytic structure of the universal scaling function better. Identify features to that are most sensitive to LYE

- Need more precise data on the imaginary quark number density

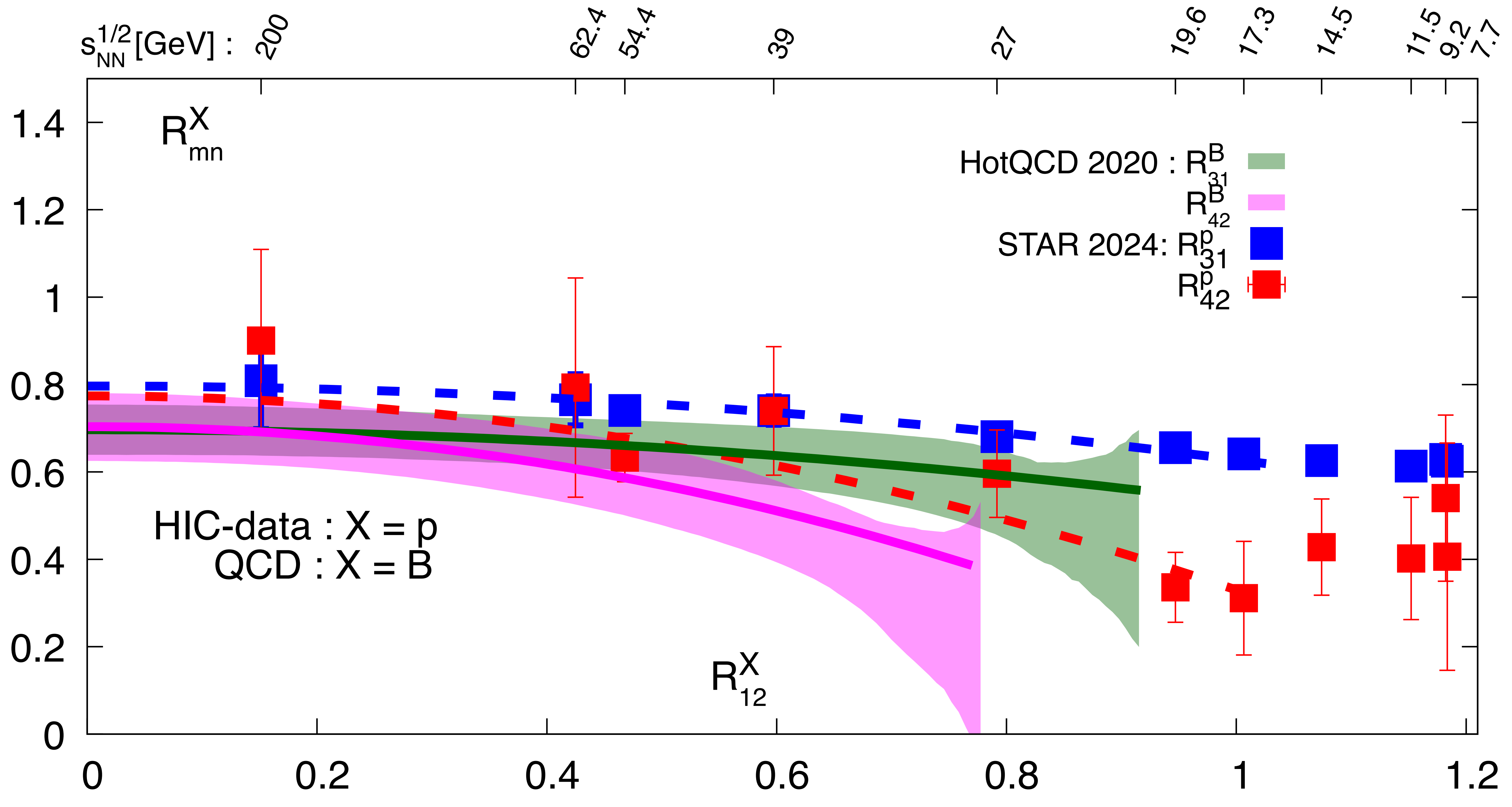
  ⇒ Develop new stable numerical methods to calculate Fourier coefficients

  ⇒ Possibly explore power of AI for analytic continuation and Fourier transformation

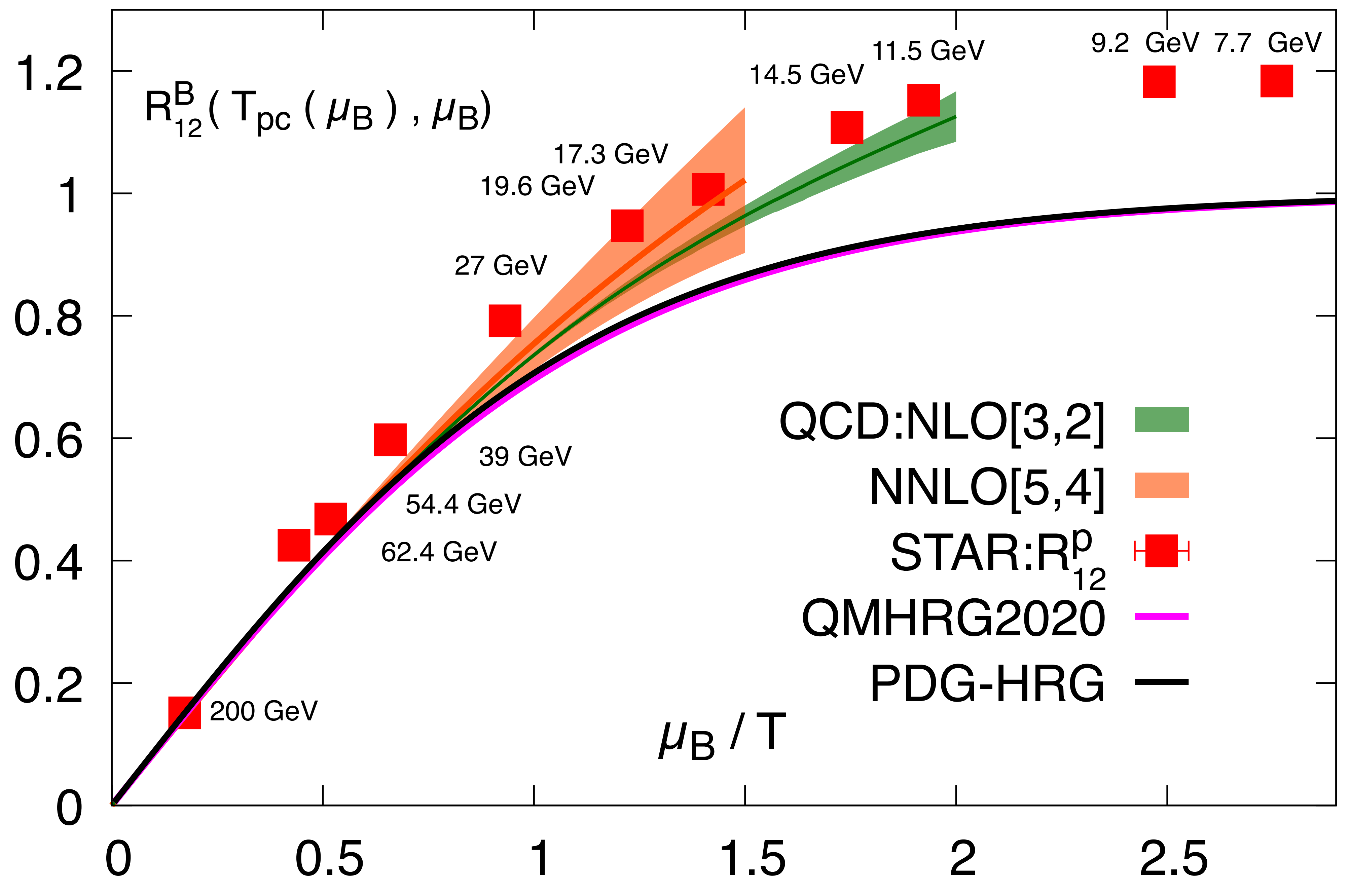




- Final BES-II data on proton number cumulants (in collider mode) was presented in CPOD '24 [A. Pandav, CPDOD '24]
- Error is significant reduced compared to the BES-I results
- Non-monotonic behaviour in C_4/C_2 is considered a signal for the QCD critical point [Stephanov et al. '99]
- Significance of non-monotonicity still needs to be estimated
- General structure is in agreement with the expectation: first a dip then a bump [Stephanov '11]



[Karsch, Goswami, XQCD '24]



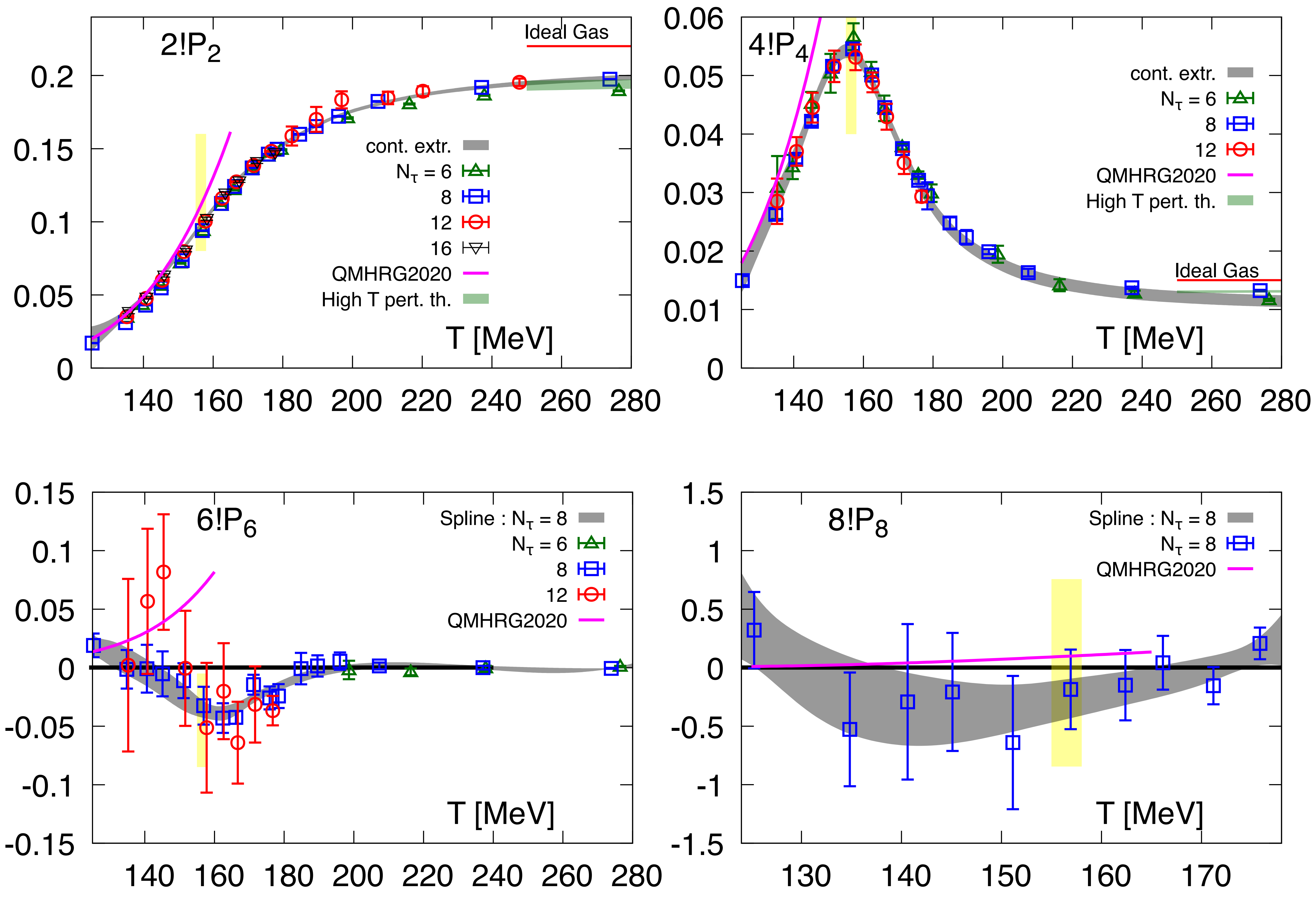
[Bollweg et al. '24]

- QCD and STAR results are in good agreement for $\sqrt{s_{NN}} < 19.6\ GeV$
- Slight vertical shift might suggest that freeze-out temperature is slightly below T_{pc}

- HRG model calculations based on noninteracting, point-like hadrons will always lead to $R^B_{12} < 1$
  ➡ HRG can not reproduce STAR data

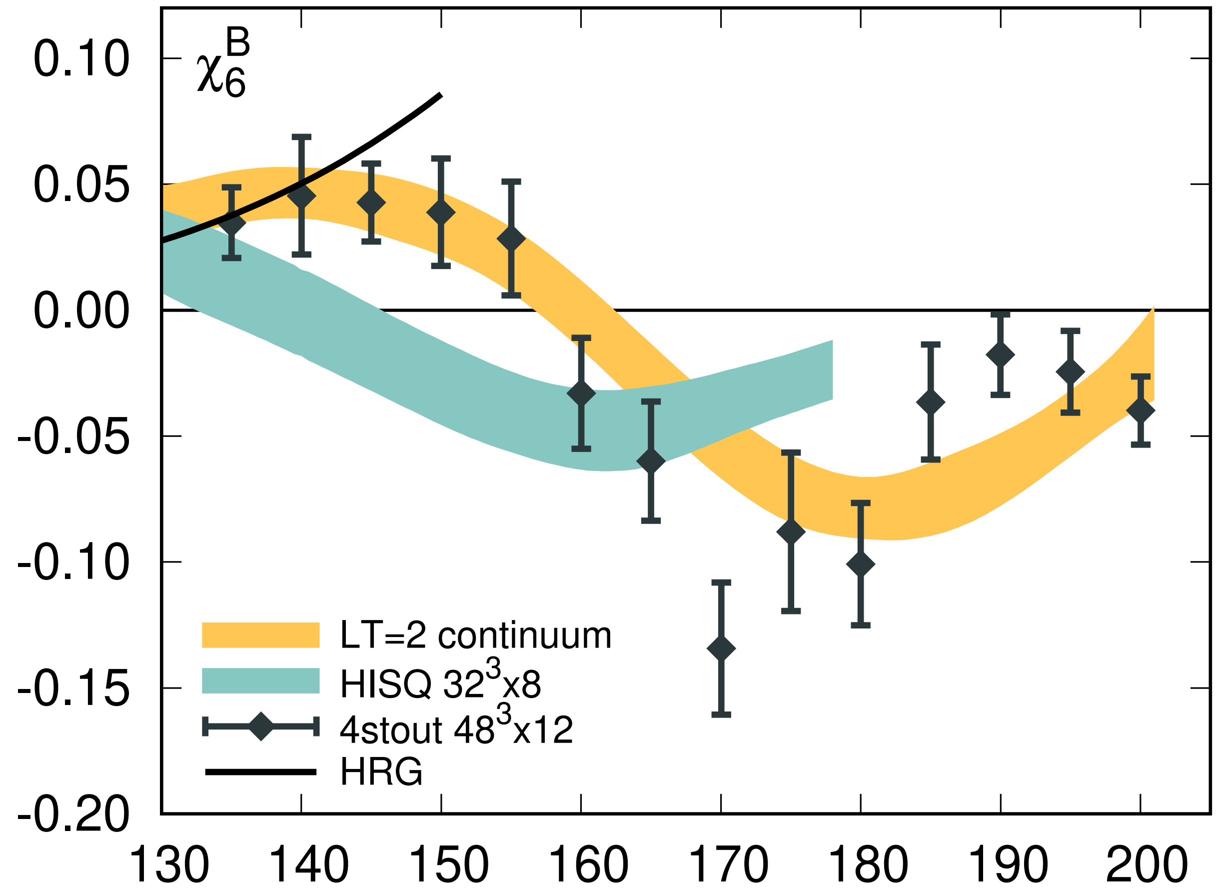
- Detecting phase transitions via Padé and post-Padé approximants has a long history in statistical and high energy physics
- They are often used in combination with perturbation theory
- QCD is non-perturbative in the vicinity of the phase
- The numerical calculation of the pressure series in μ_B is difficult

$$\Delta\hat{p} \equiv \frac{p(T,\mu_B)}{T^4} - \frac{p(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\hat{\mu}_B^{2k}$$



[HotQCD, PRD 108 (2023) 1, 014510, arXiv: 2212.09043]

➡ Talk by J. Guenther on continuum extrapolated χ_6^B and χ_8^B at LT=2



[Borsanyi et al. '24]

- Discrepancy between HotQCD and BW of unclear origin

➡ Talk by L. Pirelli on finite volume effects of chiral and deconfinement observables

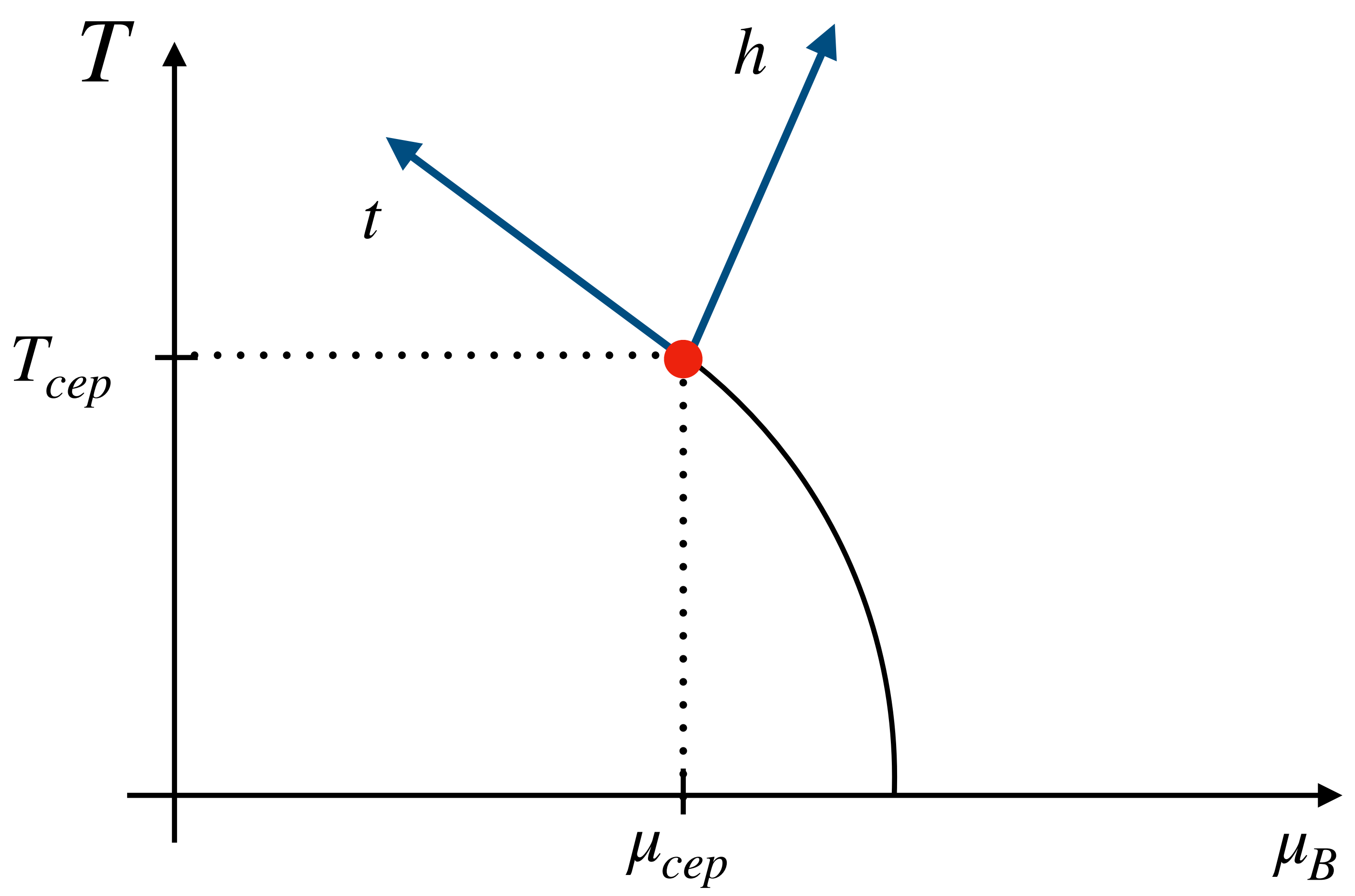
➡ Talk by J. Goswami on electric charge fluctuations with MDWF

Mixing of scaling fields:

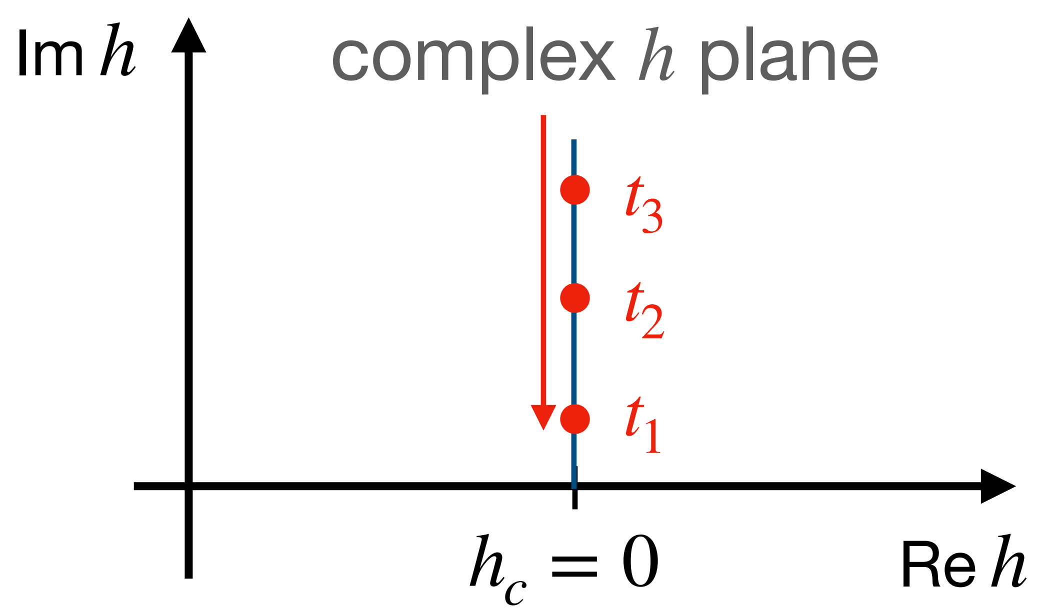
- Scaling fields are unknown, a frequently used ansatz is given by a linear mixing of T, μ_B

$$t = A_t \Delta T + B_t \Delta \mu_B,$$
$$h = A_h \Delta T + B_h \Delta \mu_B,$$

with $\Delta T = T - T^{\mathrm{CEP}}$ and $\Delta \mu_B = \mu_B - \mu_B^{\mathrm{CEP}}$



Lee-Yang edge:



- Poles approach critical point along imaginary h-axis **[Yang,Lee'59]**
- $t/h^{1/\beta\delta} = z_c$ is const. and universal

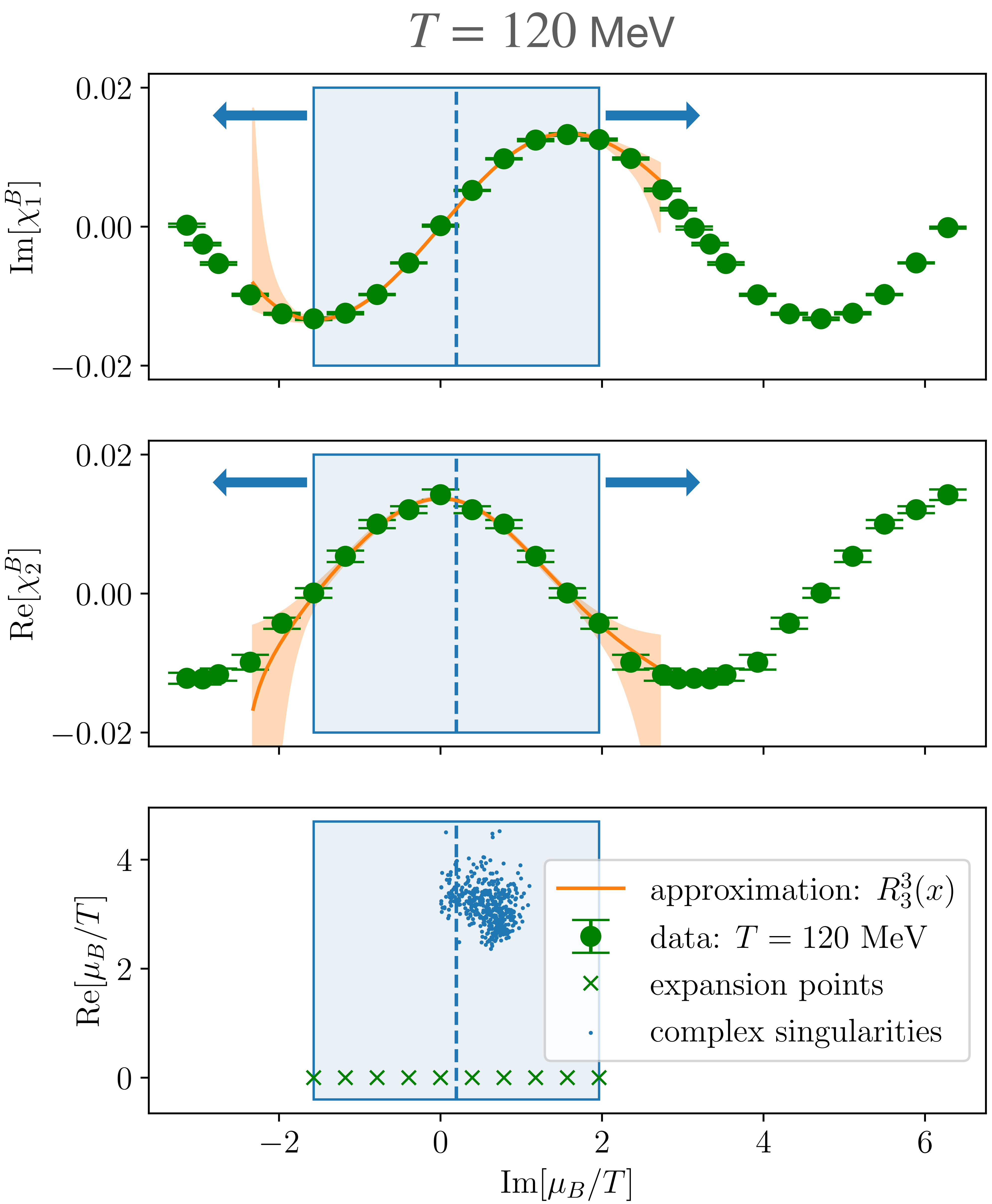
Fit Ansatz:

- For a constant $z = z_c$ we obtain

$$\mathrm{Re}[\mu_{\mathrm{LYE}}] = \mu_B^{\mathrm{CEP}} + c_1 \Delta T + c_2 \Delta T^2 + O(\Delta T^3)$$
$$\mathrm{Im}[\mu_{\mathrm{LYE}}] = c_3 \Delta T^{\beta\delta},$$

[Stephanov, Phys. Rev. D, 73.9, 094508 (2006)]

- The fit parameter c_1 gives the (inverse) slope of the 1st order line at the critical point: $c_1 = -A_h/B_h$

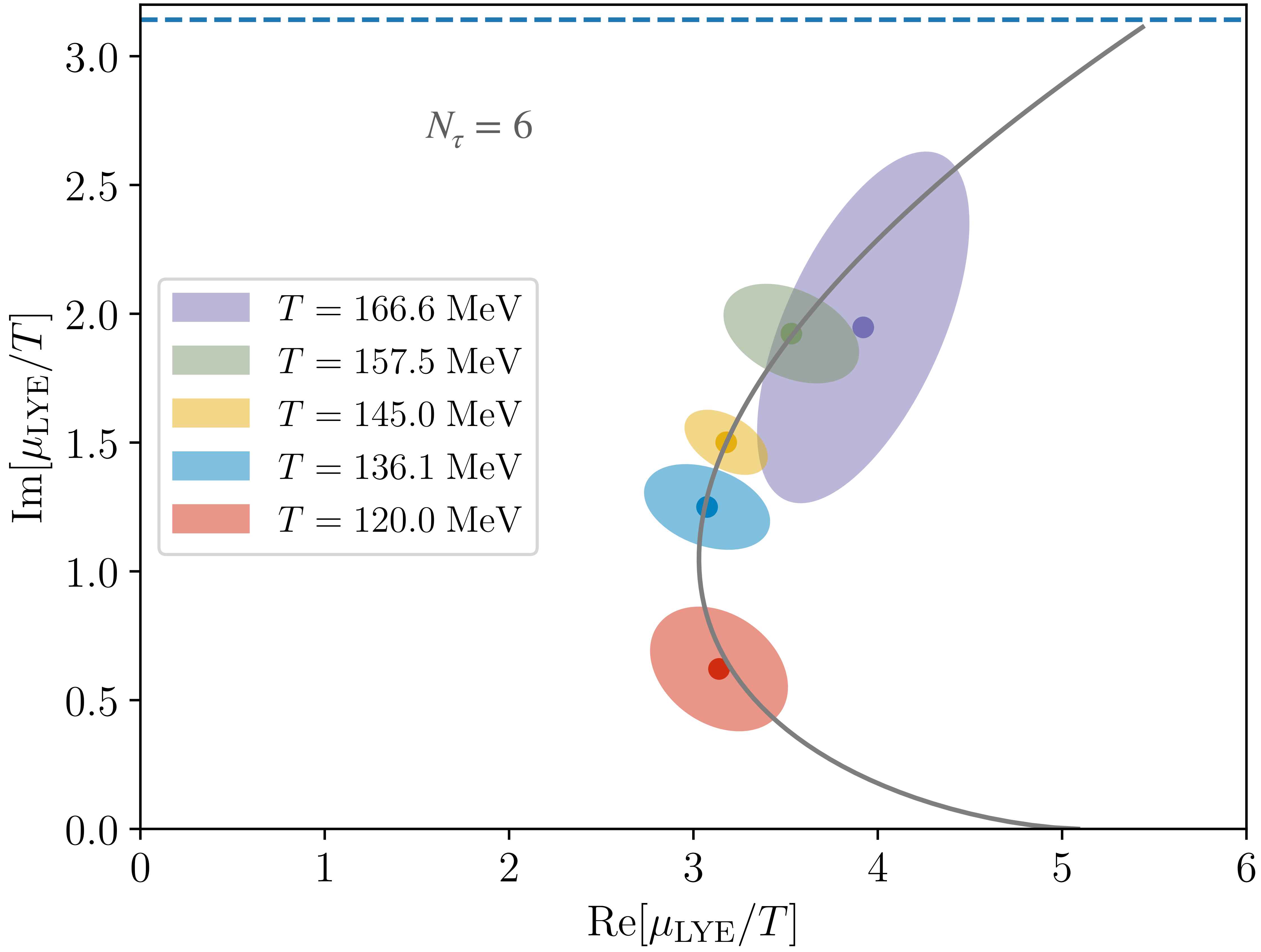


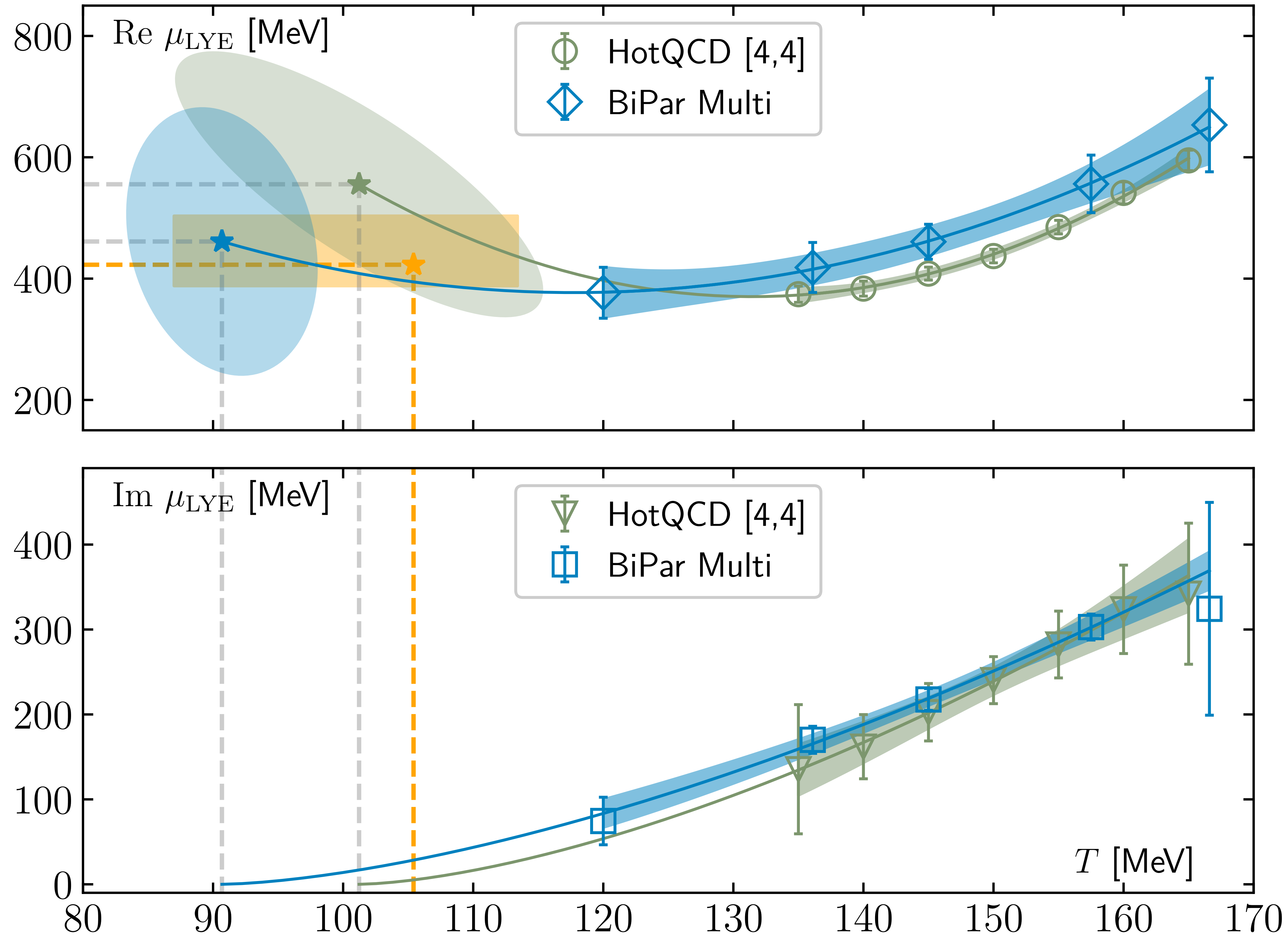
[Clark et al. (BiePar), arXiv: 2405.10196]

Procedure:

- Perform simultaneous fits to χ_1^B and χ_2^B for each temperature
- Use [3,3]-Padé
- Varry length of the fit interval in $[\pi, 2\pi]$ and the center of the interval in $[-\pi/2, +\pi/2]$
- bootstrap over the data by assuming independent and normal distributed errors
- Calculate roots of the denominator and keep only roots in the first quadrant
- Collect all the results for Lee-Yang scaling fits. We have 55 different intervals per temperature.

- Perform one fit for $N_\tau = 8$ (LT=4) and $\mathcal{O}(10^5)$ fits for $N_\tau = 6$ (LT=6)

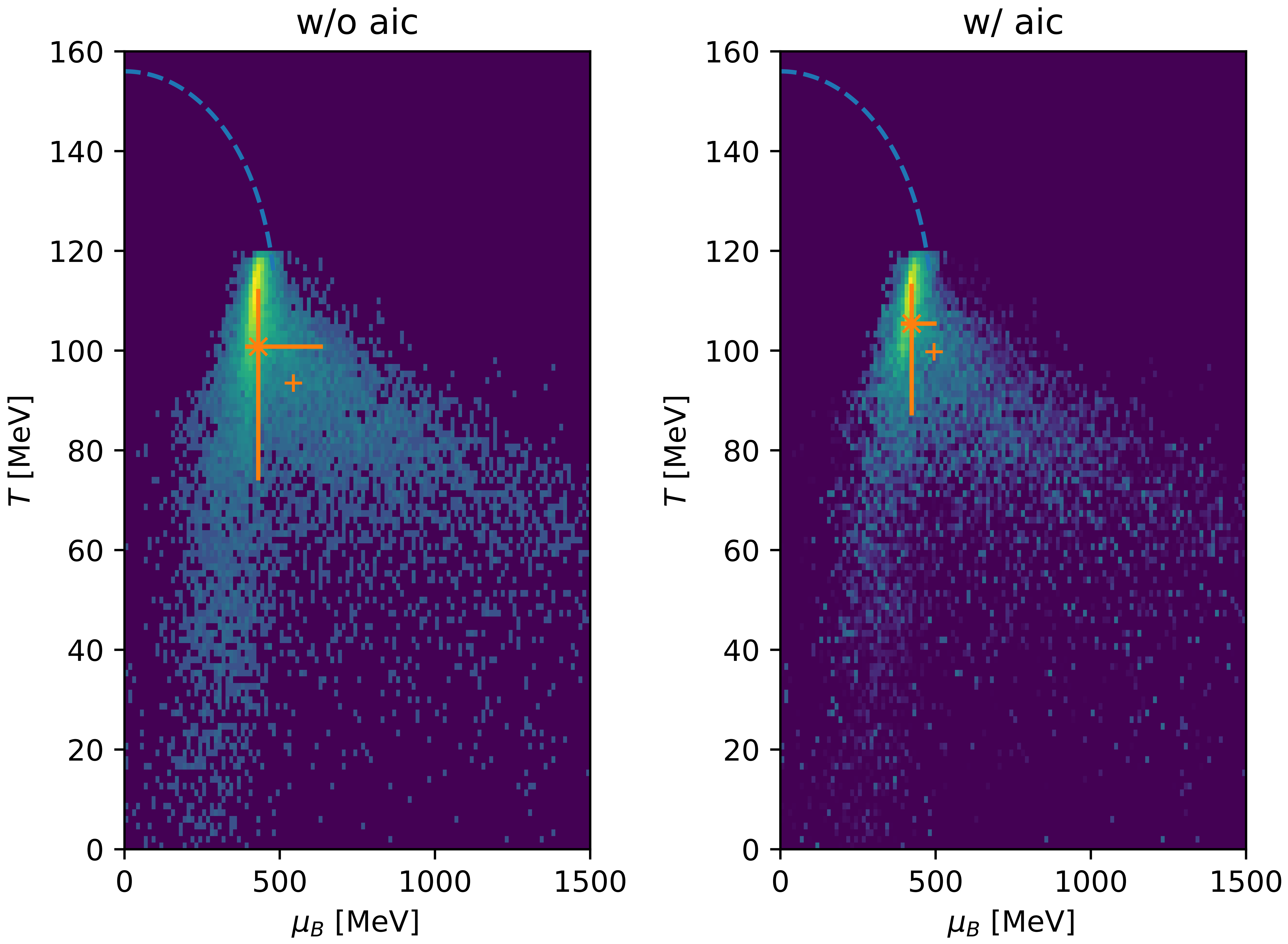




$$\mathrm{Re}[\mu_{\rm LYE}] = \mu_B^{\rm CEP} + c_1 \Delta T + c_2 \Delta T^2$$

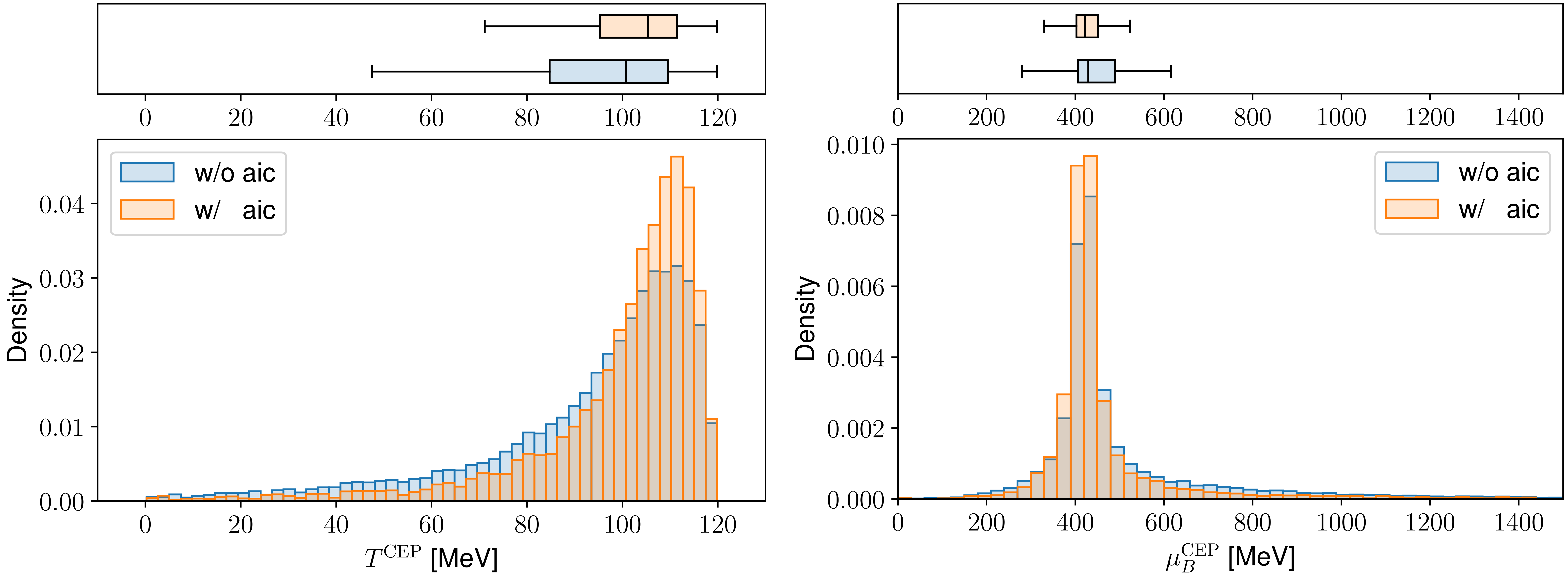
$$\mathrm{Im}[\mu_{\rm LYE}] = c_3 \Delta T^{\beta\delta},$$

- Ellipses show 1σ confidence region, using the Pearson correlation coefficient
- $N_\tau = 6$ singularities show here are chosen on the basis of the χ^2 of the scaling fit ("best fit")
- Orange box shows the AIC weighted result for $N_\tau = 6$, based on $\mathcal{O}(10^5)$ scaling fits



- Histogram over the T^{CEP} and μ_B^{CEP} from the $\mathcal{O}(10^5)$ fits
- Error bars are based on the inner 68-percentile
- Observe interesting structure
- Dashed line indicates the continuum extrapolated crossover line

[Clark et al. (BiePar), arXiv: 2405.10196]



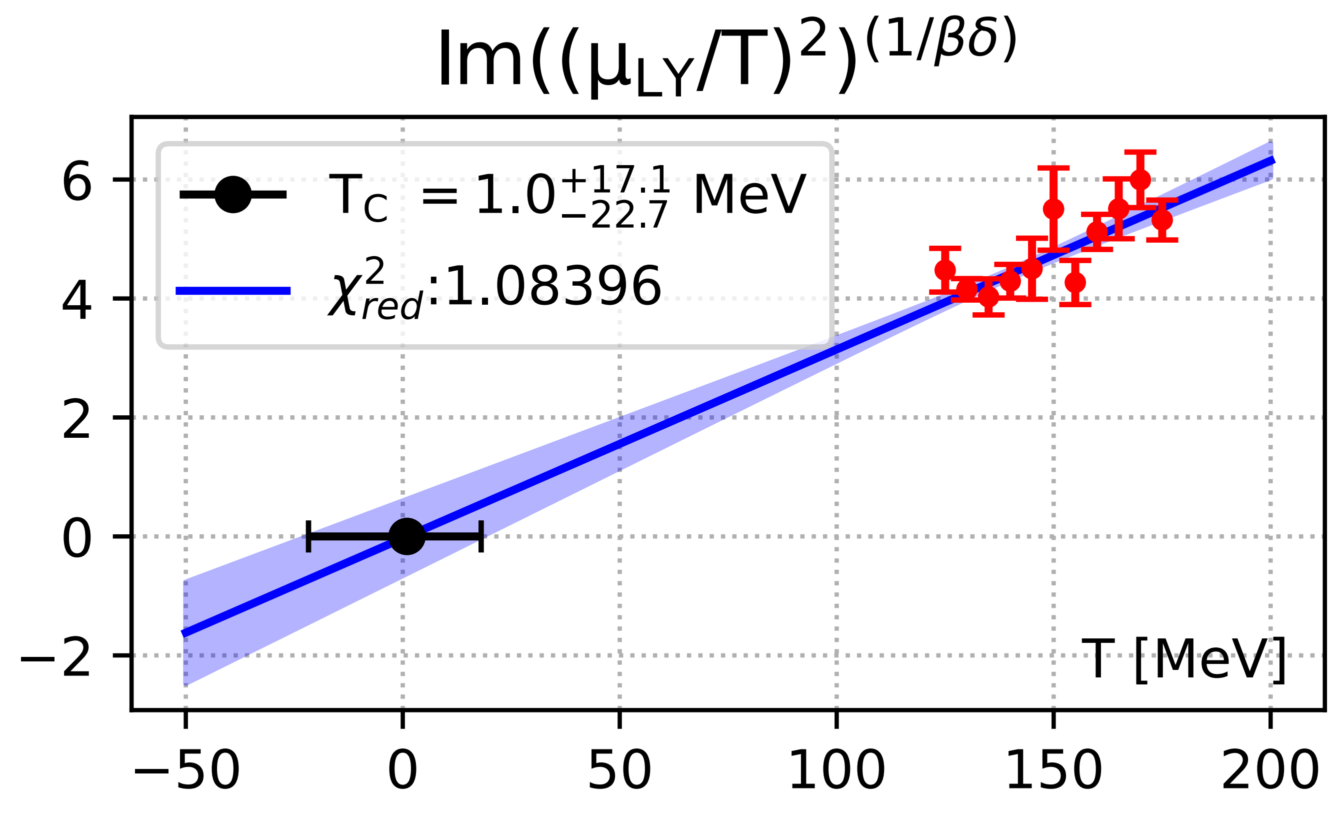
	$N_\tau = 6$ multi-point Padé			$N_\tau = 8$ [4,4]-Padé		
	T^{CEP} [MeV]	μ_B^{CEP} [MeV]	μ_B/T	T^{CEP} [MeV]	μ_B^{CEP} [MeV]	μ_B/T
best fit	90.7 ± 7.7	461.2 ± 220	5.09 ± 0.68	101 ± 15	560 ± 140	5.5 ± 1.7
weight-1	$105.4 + 8.0 - 18.4$	$422.9 + 80.5 - 34.9$	$3.92 + 1.52 - 0.24$			
weight-2	$100.8 + 11.6 - 26.8$	$430.9 + 208.2 - 42.2$	$4.20 + 4.13 - 0.47$			
	c_1	c_2	c_3	c_1	c_2	c_3
best fit	-6.2 ± 9.2	0.115 ± 0.090	0.424 ± 0.086	-12.3 ± 8.1	0.203 ± 0.059	0.55 ± 0.25

- For $N_\tau = 8$: similar results by Basar, based on the same HotQCD data **[Basar, arXiv: 2312.06952]**

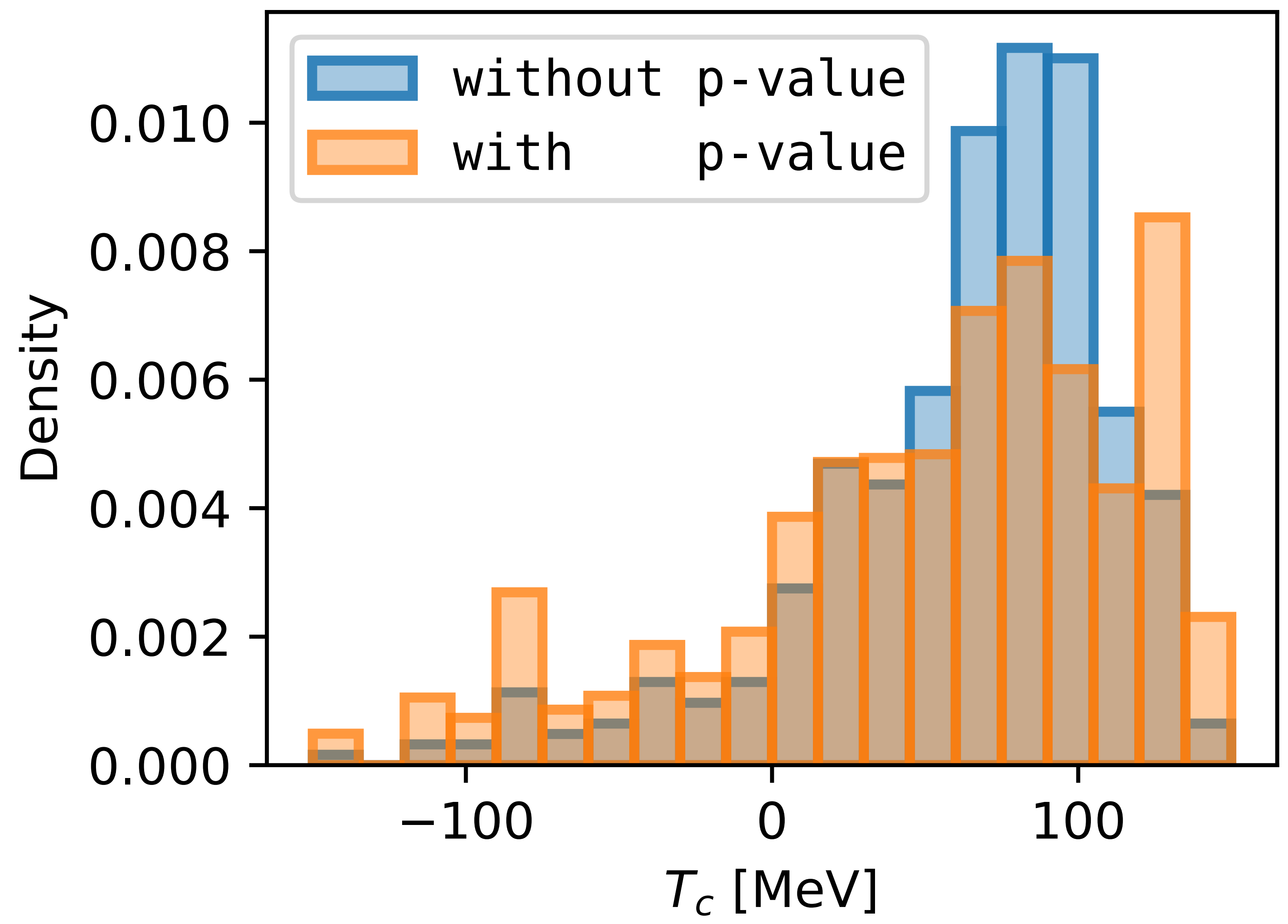
➡ **Talk by A. Adam@Lattice24: single point Padé approximation of the pressure based on $\chi_2^B, \chi_4^B, \chi_6^B, \chi_8^B$ (LT=2)**

- Interesting to check results with the Budapest-Wuppertal data
- Preliminary results for single Point Padé analysis on $16^3 \times 8$ lattices, multi-Point is work in progress
- Preliminary result on the transition temperature based on extrapolations 432 on different approximations and fit ranges
- T^{CEP} around 90 MeV, in agreement with **[BiePar, 2405.10196]**
- Results are very sensitiv to noise

One example of the extrapolations



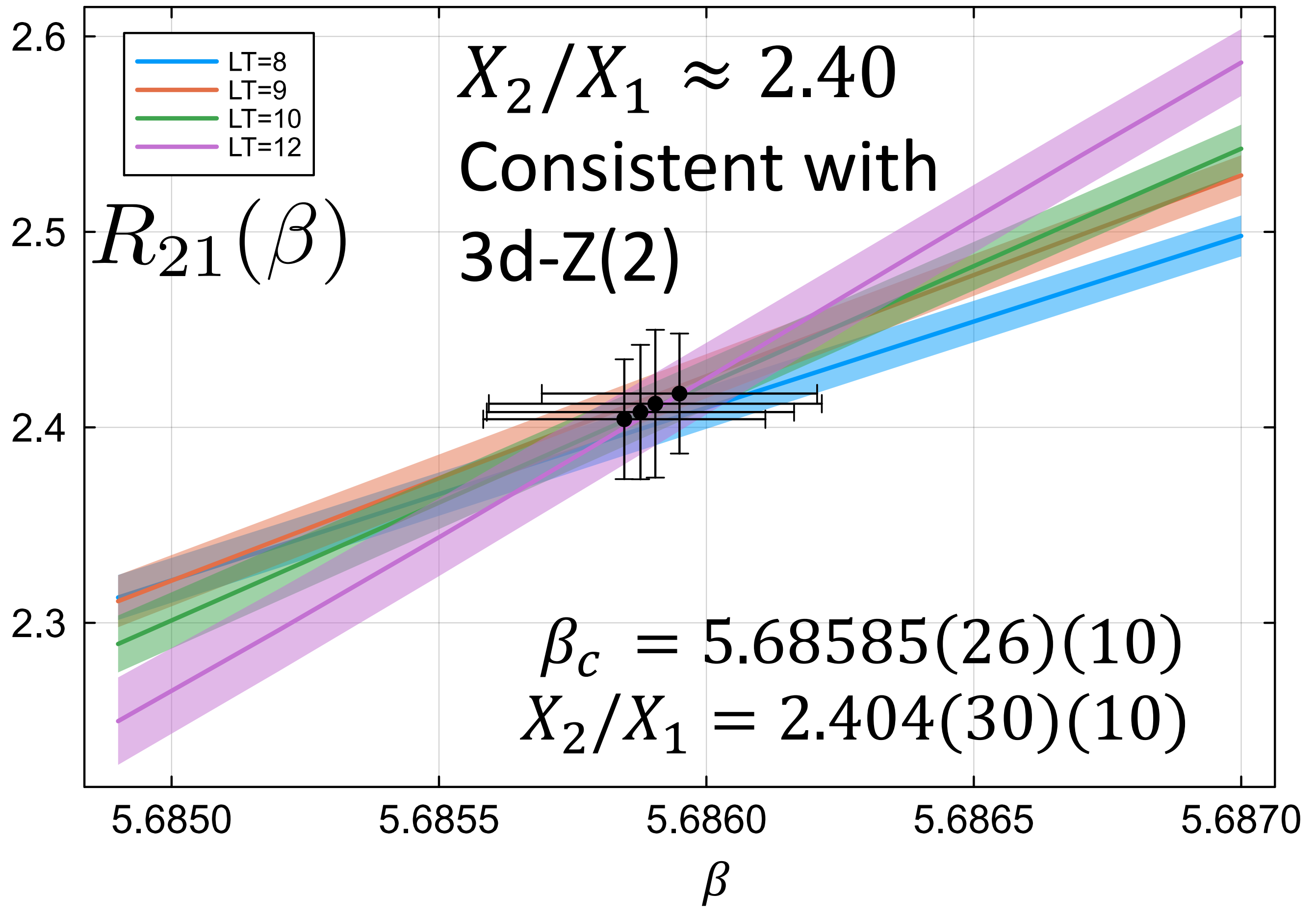
Histogram of the T^{CEP} results



➡ **[Talk by T. Wada:]() Finite size scaling of Lee-Yang zeros in 3d Pots model and heavy-quark QCD**

- Construct ratios of the LYZ locations
- Scaling is in accordance with a ratio of scaling function: non-universal pre-factors cancel, intersection point of different volumes is universal.
- Ratios show reduced corrections to scaling and regular parts
- T^{CEP} is shifted to higher values, results from extrapolation of first LYZ can serve as a lower bound

$2^{nd}/1^{st}$ LYZ Ratio



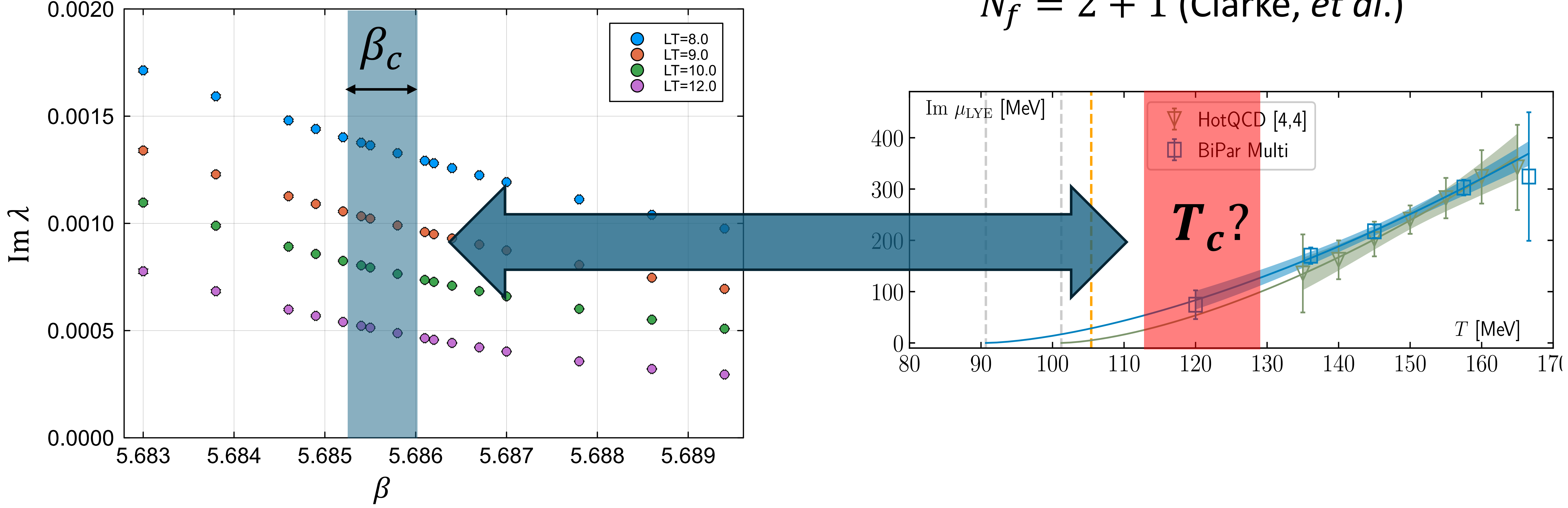
$$R_{12}(t) = \frac{h_{LY}^{(2)}(t)}{h_{LY}^{(1)}(t)}$$

$$= \frac{X_1}{X_2}(1 + C(tL^{y_t}))\underbrace{(1 + D(tL^{2(y_t - y_h)}))}_{\text{mixing with temperature like}}$$

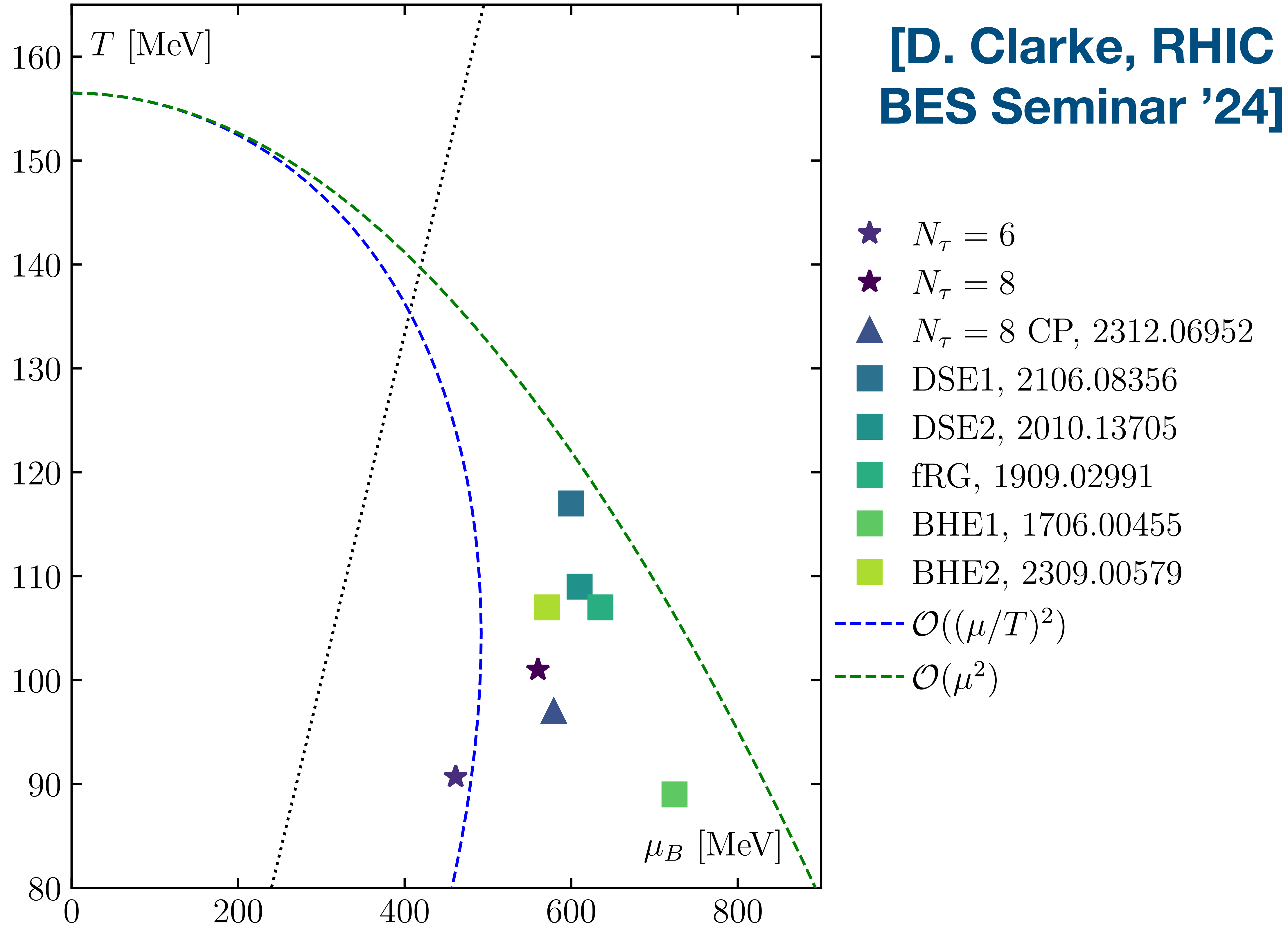
+ higher orders

1st LYZ data (HQ-QCD)

$N_f = 2 + 1$ (Clarke, *et al.*)



- No continuum estimate of the Lee-Yang zero extrapolation yet (possible large systematic errors)
  ➡ Finite volume and finite cut-off effects will increase both T^{CEP} and μ_B^{CEP}
- Results from other approaches seam to cluster in a narrow region (DSE, fRG, BHE)



[D. Clarke, RHIC BES Seminar '24]

Parametrizations of the crossover line:

1.) $$T_{\rm pc}(\mu_B) = T_{\rm pc}(0)\left[1+\kappa_2^B\left(\frac{\mu_B}{T}\right)^2+\kappa_4^B\left(\frac{\mu_B}{T}\right)^4\right]$$

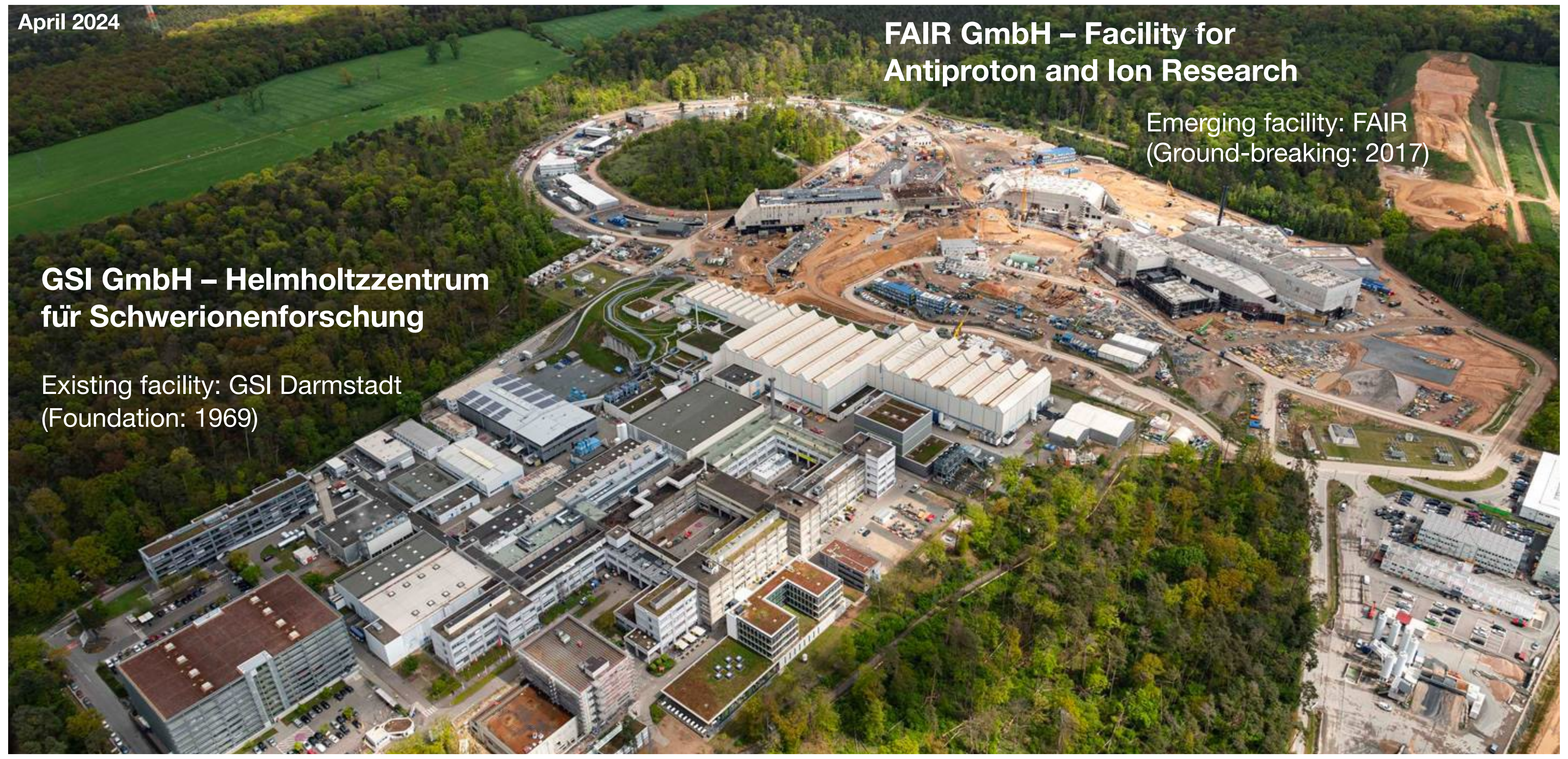
2.) $$T_{\rm pc}(\mu_B) = T_{\rm pc}(0)\left[1+\bar{\kappa}_2^B\left(\frac{\mu_B}{T_{\rm pc}(0)}\right)^2+\bar{\kappa}_4^B\left(\frac{\mu_B}{T_{\rm pc}(0)}\right)^4\right]$$

- $\kappa_2 = \bar{\kappa}_2 = -0.015(1)$ **[HotQCD, 2403.09390]**
- Many results seem to favour a small $\bar{\kappa}_4 \approx -0.0002(1)$

- New experimental results are expected to arrive BES-II (fixed target), CMB@FAIR, …,
- We start to calculate multi-dimensional phase diagrams
- Universal scaling and Lee-Yang zeros are powerful tools to explore the QCD phase diagram
  ➡ New techniques are still being designed
- However, for finite μ_B we still rely on extrapolatory methods
  ➡ we will need very precise data

installation of SIS100 dipoles Apr'24

CBM cave, Jun'24



[T. Galatyu, Criticality II '24]

Apologies to all speakers of the finite temperature and finite density sessions, how's talks I could not highlight

Thank you for your attention!