Nuclear Science Computing Center at CCNU

Search for CEP at a low temperature of 108 MeV from Lattice QCD

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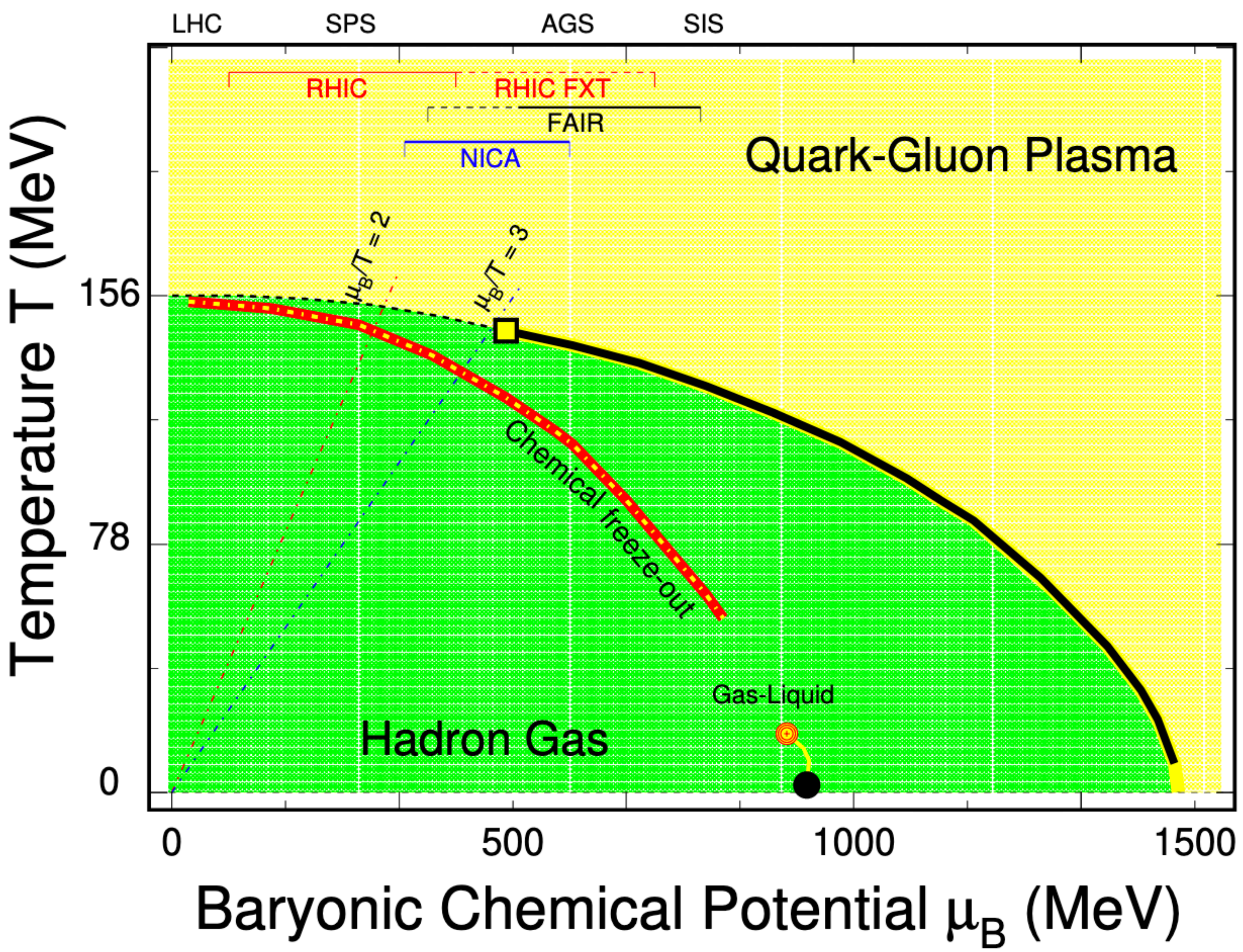
Central China Normal University

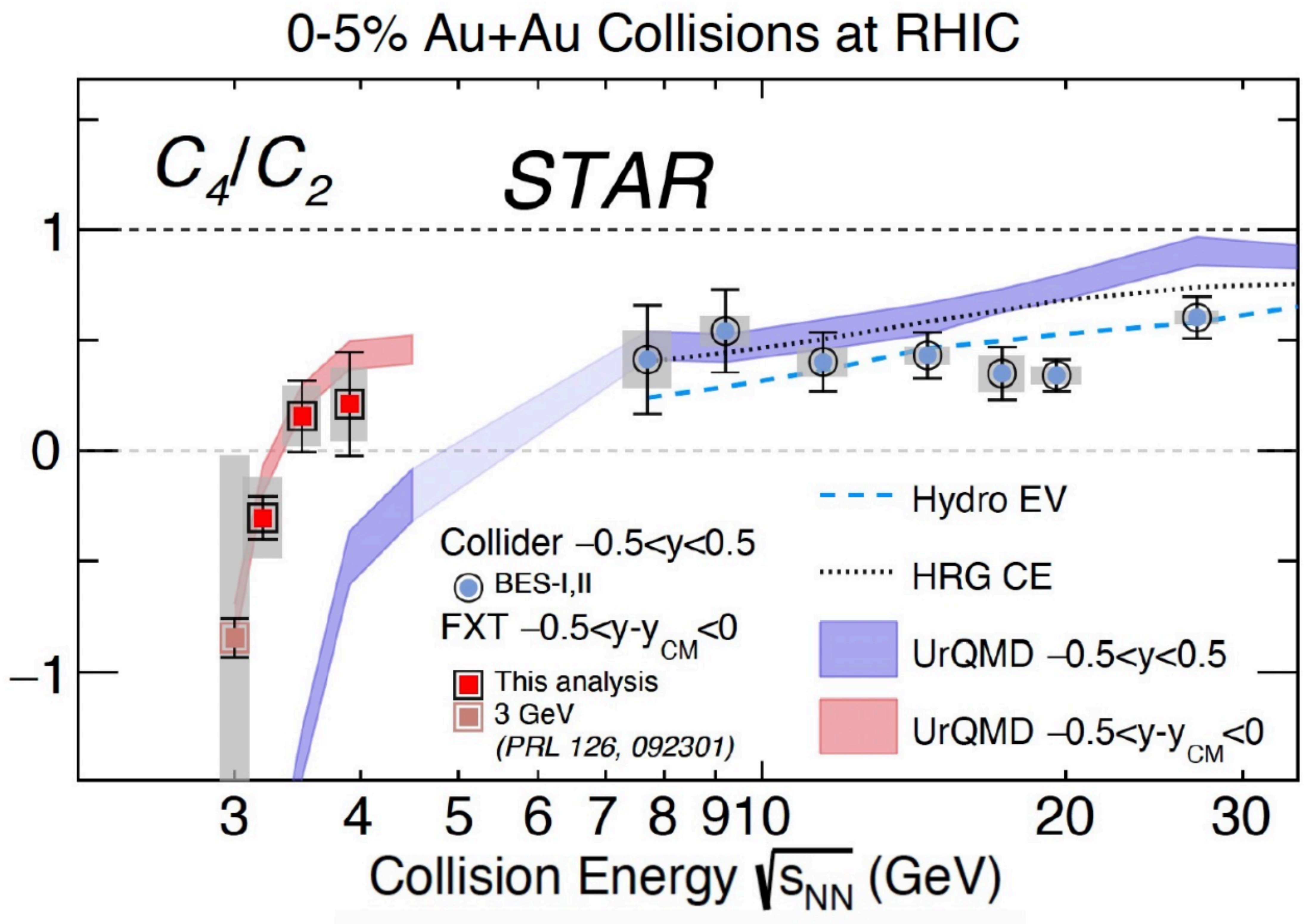
Kai-Fan Ye et al., arXiv:2509.xxxx

Analytic structure of QCD and Yang-Lee edge singularity

Sep. 8-12, 2025 @ ECT*

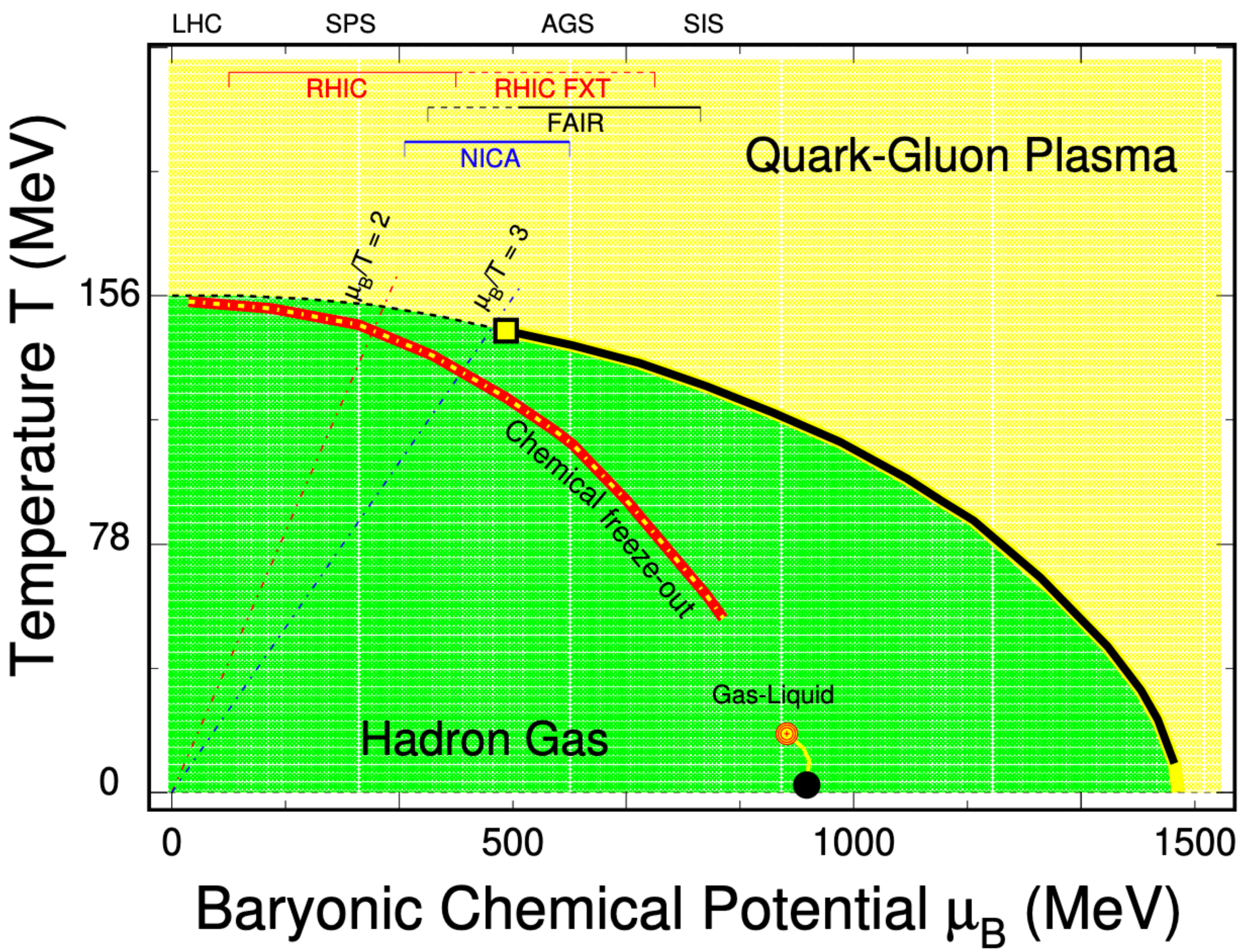
Search for the critical end point & criticalities

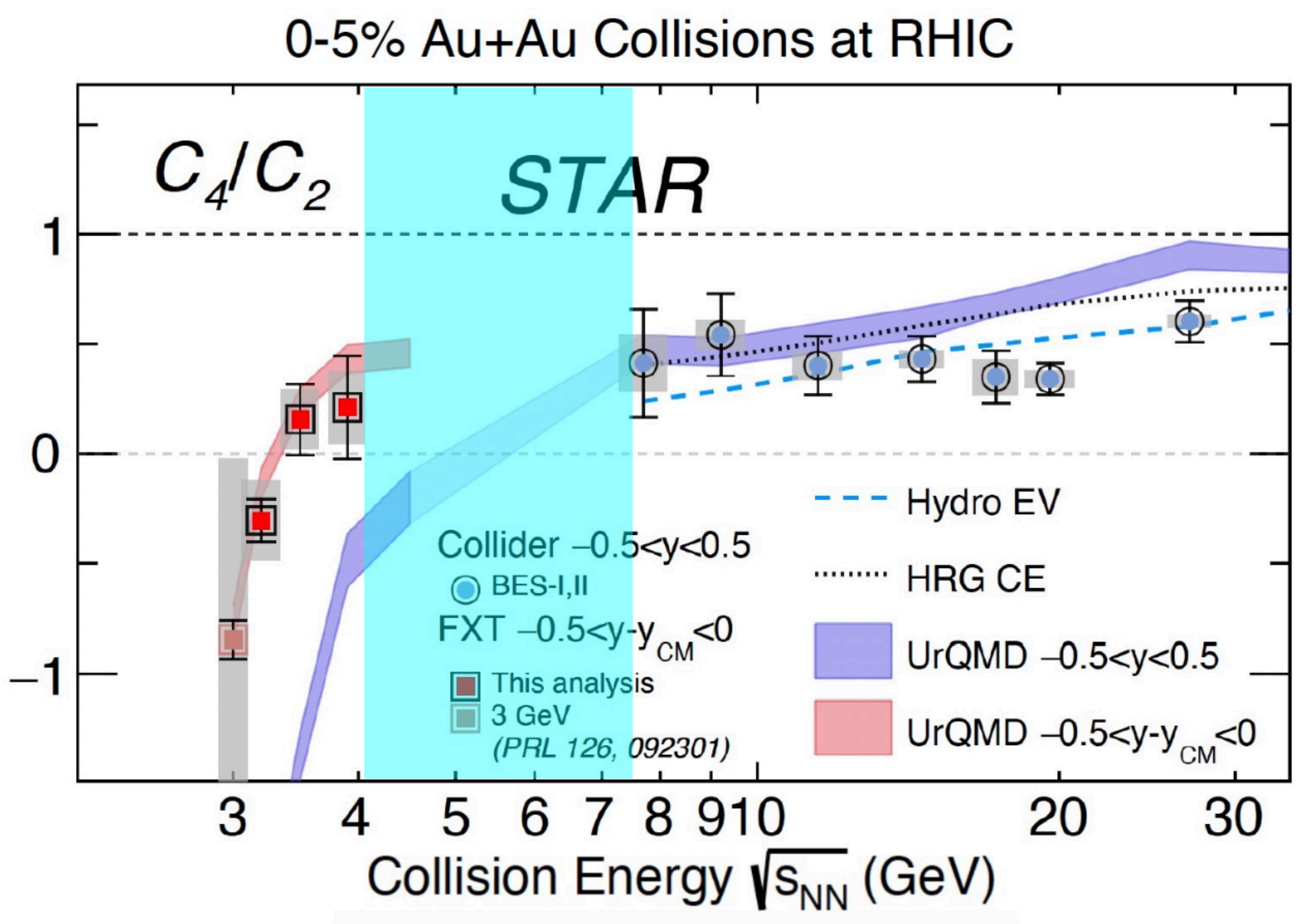




STAR 7.7-27 GeV results from arXiv: 2504.00817
New results at 3.2,3.5 and 3.9 GeV from Zachary Sweger(QM2025)

Search for the critical end point & criticalities





STAR 7.7-27 GeV results from arXiv: 2504.00817
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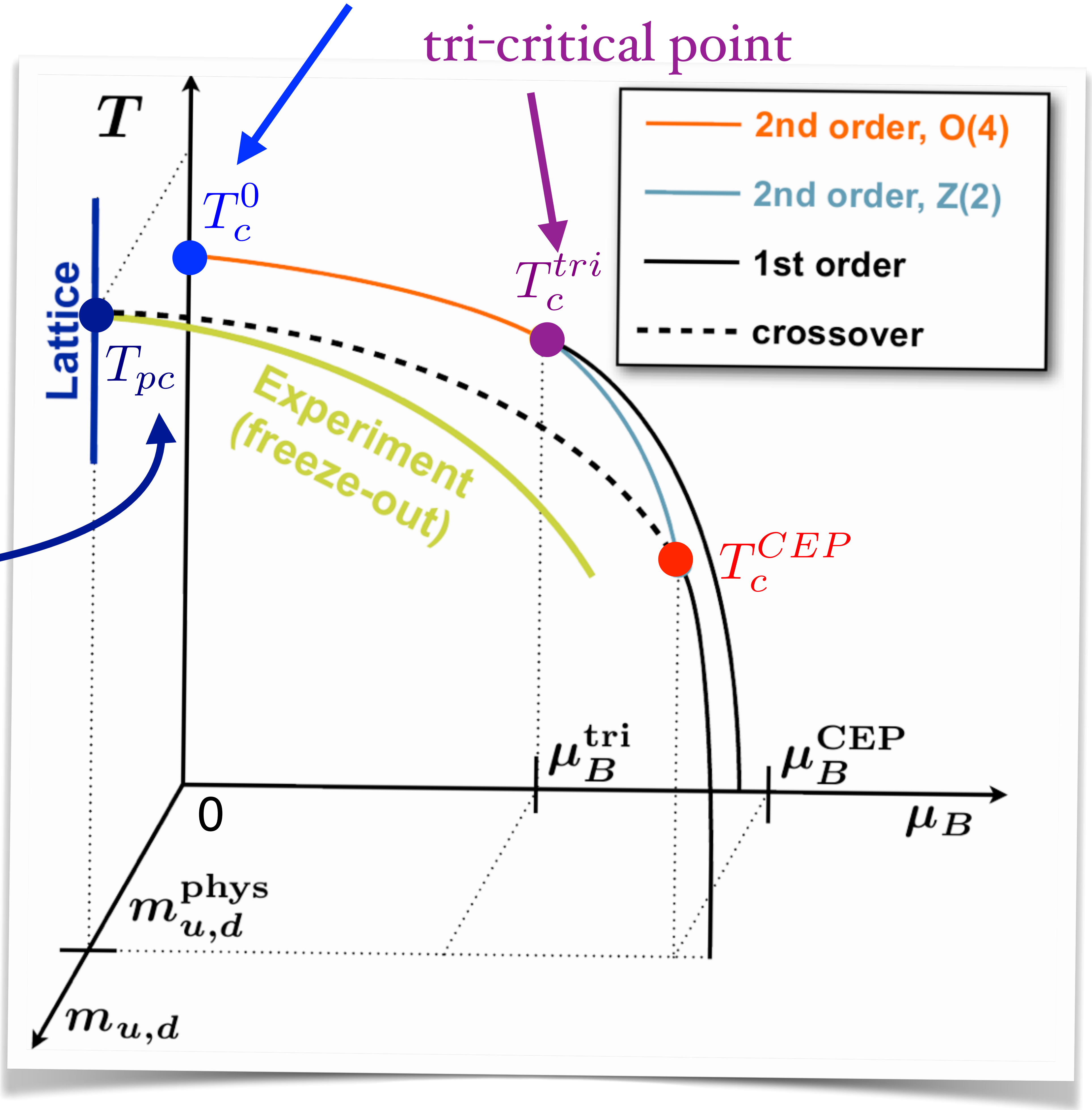
if the CEP exits, most likely, it maybe found in $\sqrt{s_{\mathrm{NN}}} \in (3 - 3.9,\ 7.7)$ GeV

or $\mu_B \in (420,\ 632 - 750)$ MeV

QCD phase diagram in 3D: quark mass, μ_B, T

$M_\pi \to 0$, O(4)
Chiral phase transition T at $\mu_B=0$

Chiral crossover transition T at $\mu_B=0$ and M_π=140 MeV



- $T_c^0(\mu_B)$ decreases as μ_B up to NLO from LQCD

O. Kaczmarek et al., PRD83 (2011) 014504
P. Hegde & HTD, PoS LATTICE2015 (2016) 141
HTD et al., PRD 109 (2024) 114516

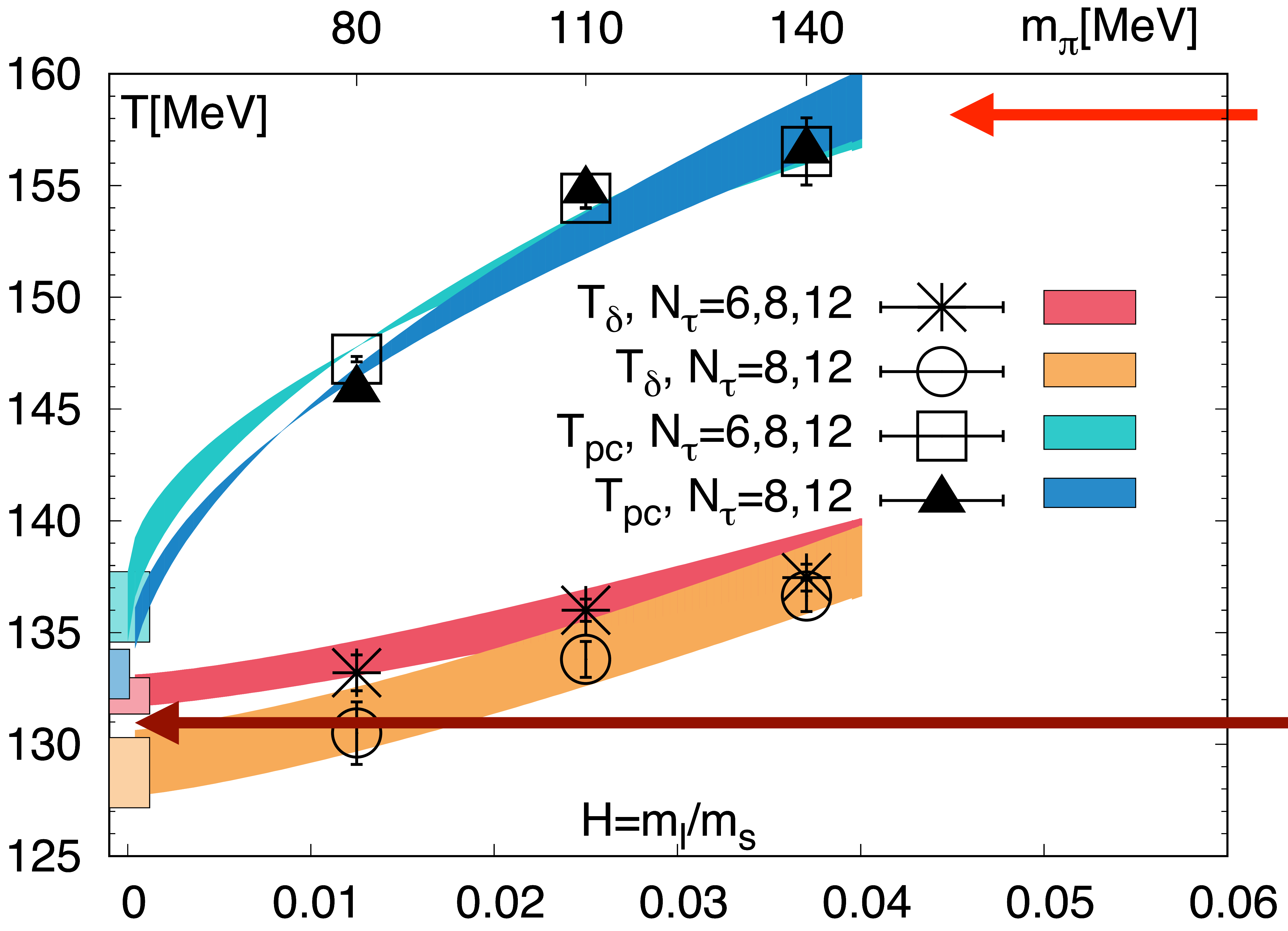
- Random Matrix Model & NJL suggests:

M. A. Halasz et al, PRD 58 (1998) 096007
M. Buballa, S. Carignano, PLB791(2019)361
Y. Hatta & T. Ikeda, PRD67 (2003) 014028

$$T_c^{tri} - T_c^{CEP}(m_q) \propto m_q^{2/5}$$

Indications:

$$T_{pc}^{phys} > T_c^0 > T_c^{tri} > T_c^{CEP}$$



HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], PRL 123 (2019) 062002
HTD, arXiv:2002.11957

Chiral crossover transition temperature

$$T_{pc}^{\rm phys} = 155 - 158.6 \text{ MeV}$$

HotQCD, PLB 19'
WB, PRL 20'

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

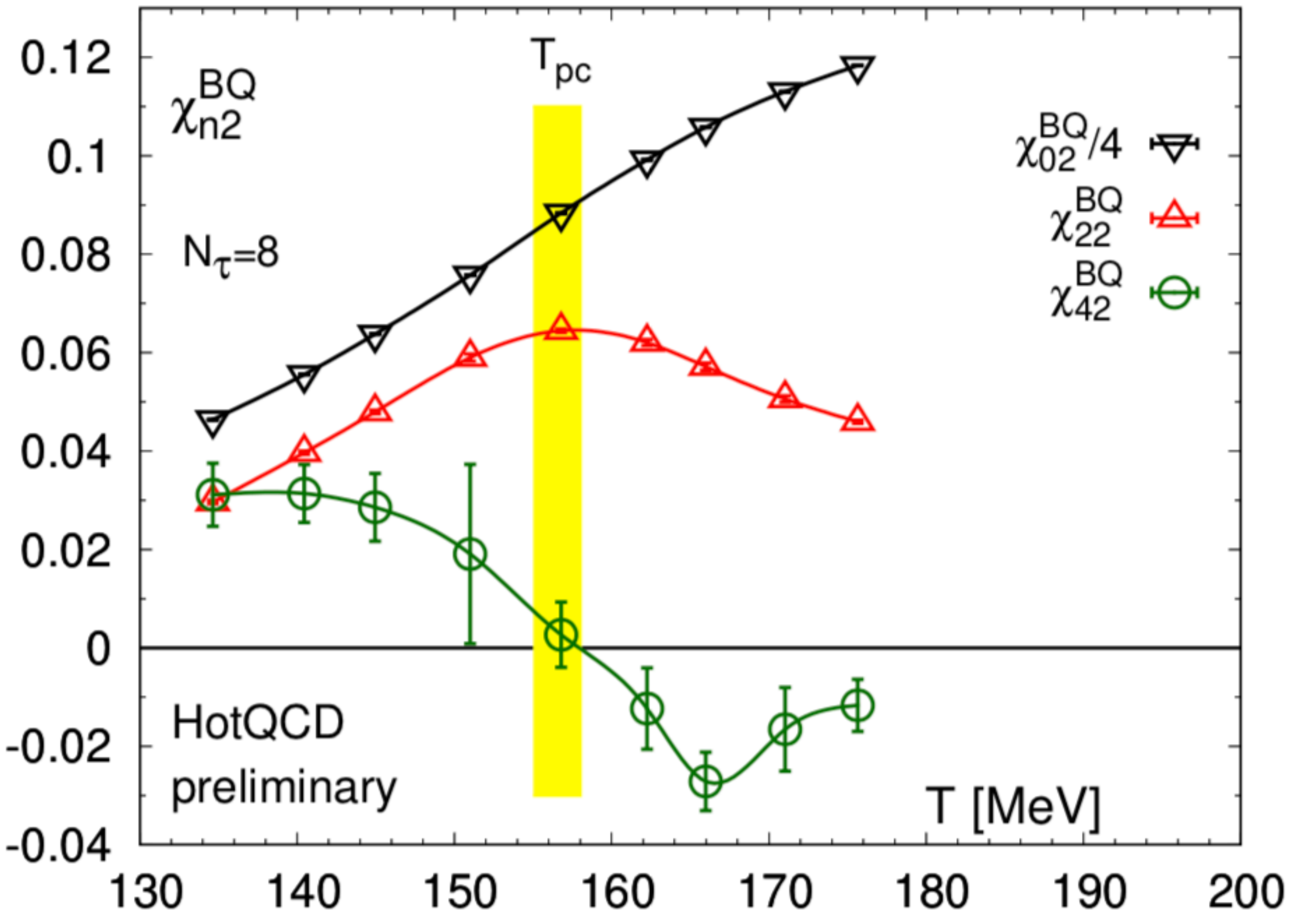
Chiral phase transition T

$$T_c^0 = 132^{+3}_{-6} \text{ MeV}$$

See also in QCD-inspired model calculations:
e.g. J. Berges, D. U. Jungnickel and C. Wetterich, Phys. Rev.D59, 034010 (1999)
J. Braun, B. Klein, H.-J. Pirner and A. H. Rezaeian,Phys. Rev. D73, 074010 (2006)
And consistent results using Wilson fermions from Kotov et al , Phys.Lett. B 823(2021) 136749

Indication of $T_{CEP} \lesssim 135$ MeV

Critical behavior and higher order cumulants



$$\chi_{ijk}^{BQS}(T) = \frac{\partial^{i+j+k} P(T,\hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}=0}$$

$$t \sim \frac{T - T_c^0}{T_c^0} + \kappa_2^{B,0} \left(\frac{\mu_B}{T}\right)^2$$

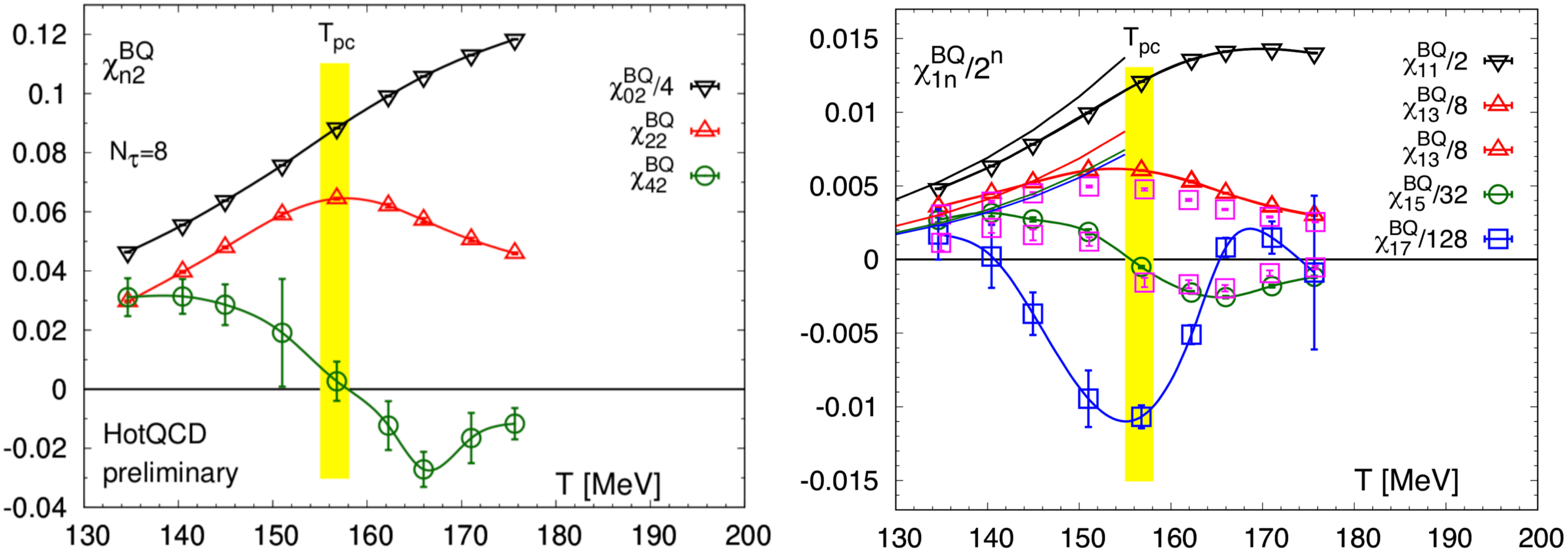
In the scaling regime: $\frac{\partial}{\partial T} \simeq \frac{\partial^2}{\partial \mu_B^2}$

F. Karsch, PoS CORFU2018 (2019) 163

Irregular sign change seen at T>T$_{pc}$ in χ_{42}^{BQ}

Irregular sign change expected at T≳ 135 MeV in χ_{62}^{BQ}

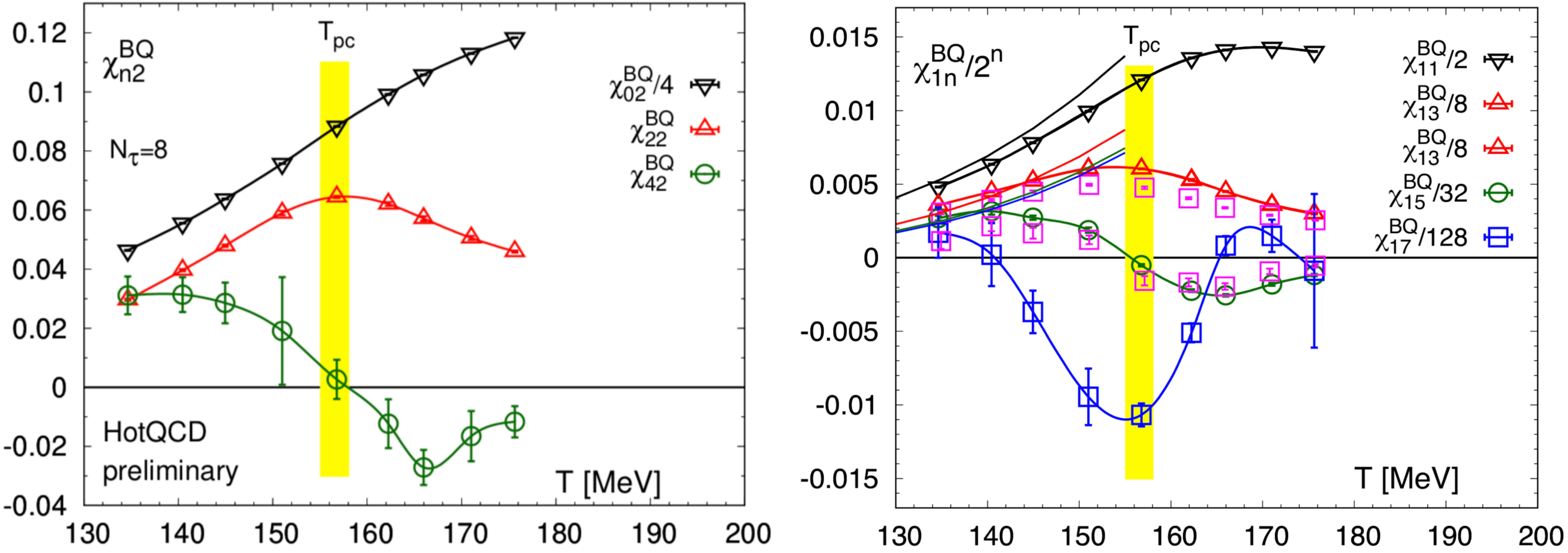
Critical behavior and higher order cumulants



F. Karsch, PoS CORFU2018 (2019) 163

Many 8th order fluctuations turn to be negative at T ≳ 135-140 MeV

Critical behavior and higher order cumulants



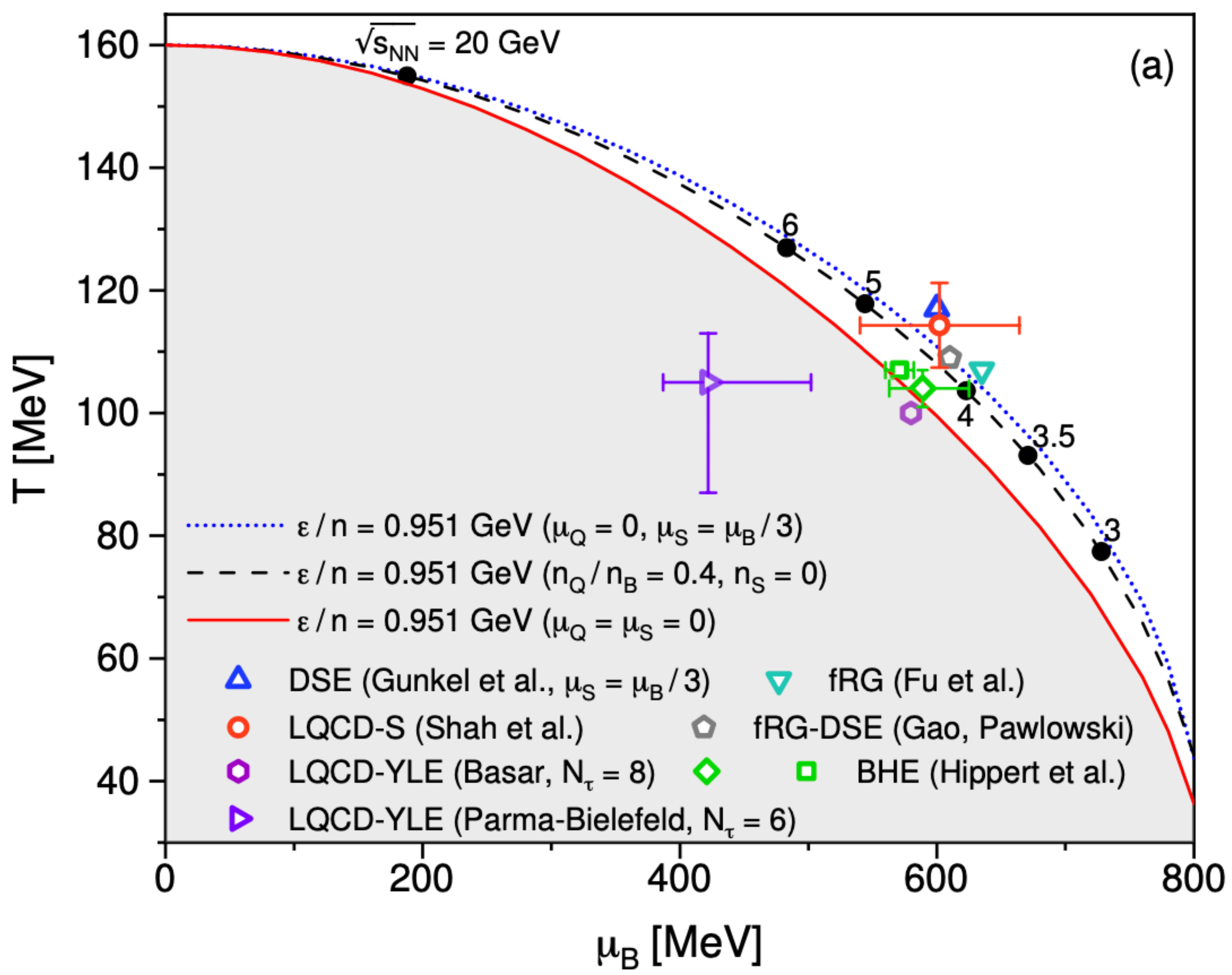
F. Karsch, PoS CORFU2018 (2019) 163

Many 8th order fluctuations turn to be negative at T ⩾ 135-140 MeV

Suggests singularity in the complex plane at T>135-140 MeV

More support for $T_{CEP} < T_c^0$

Recent theoretical predictions/estimates of CEP



- Recent predictions/estimates:
  - $T_c^{CEP} \approx 90 - 120$ MeV
    $\mu_{B,c}^{CEP} \approx 400 - 650$ MeV
  - or CEP does not exist
- The region hope to find to CEP in EXP

$$\sqrt{s_{NN}} \approx 3.9 - 7 \text{ GeV}$$

$$\mu_{B,c}^{CEP} \approx 420 - 632 \text{ MeV}$$

Lysenko et al., arXiv:2408.06473, *Phys.Rev.C* 111 (2025) 5, 054903

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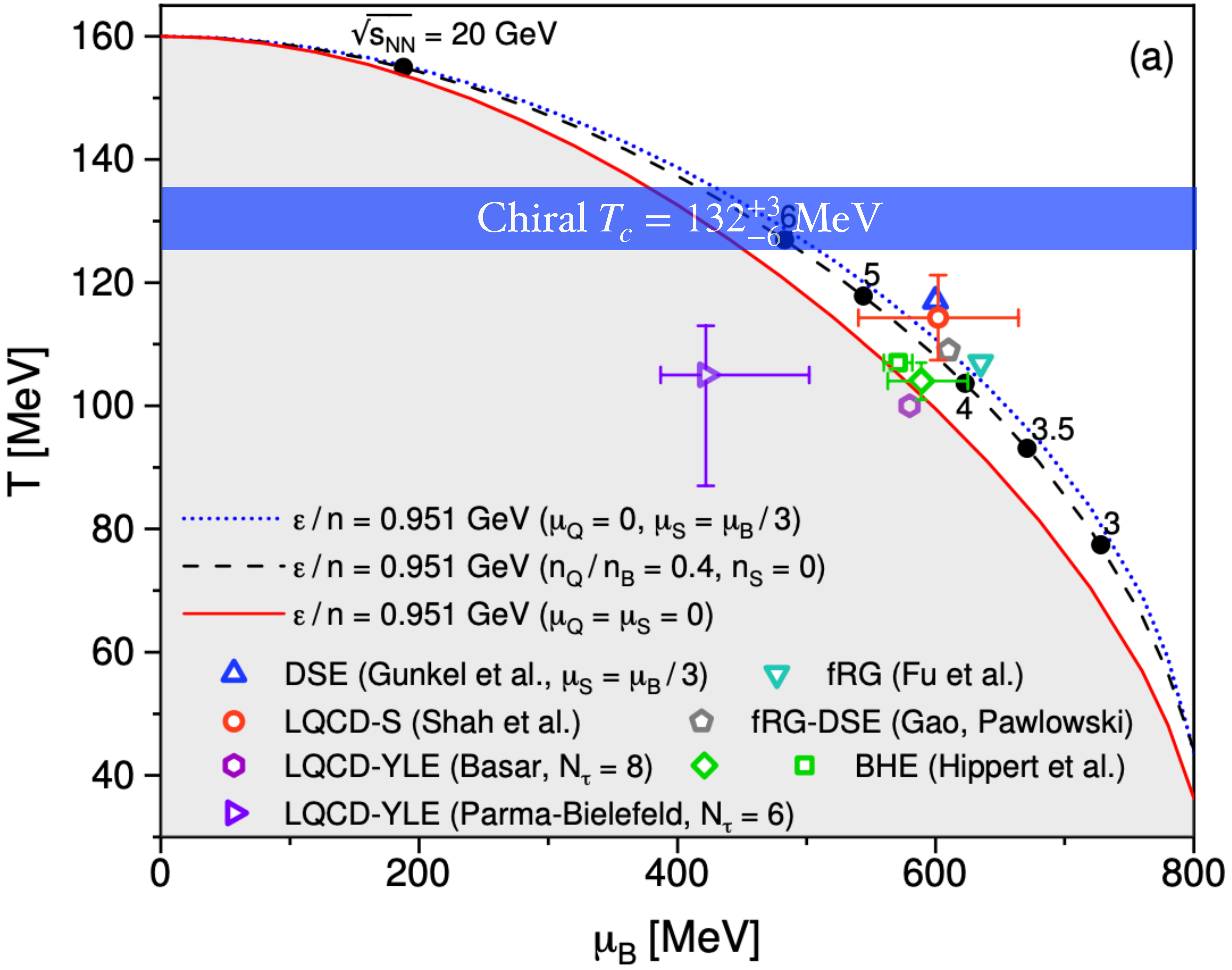

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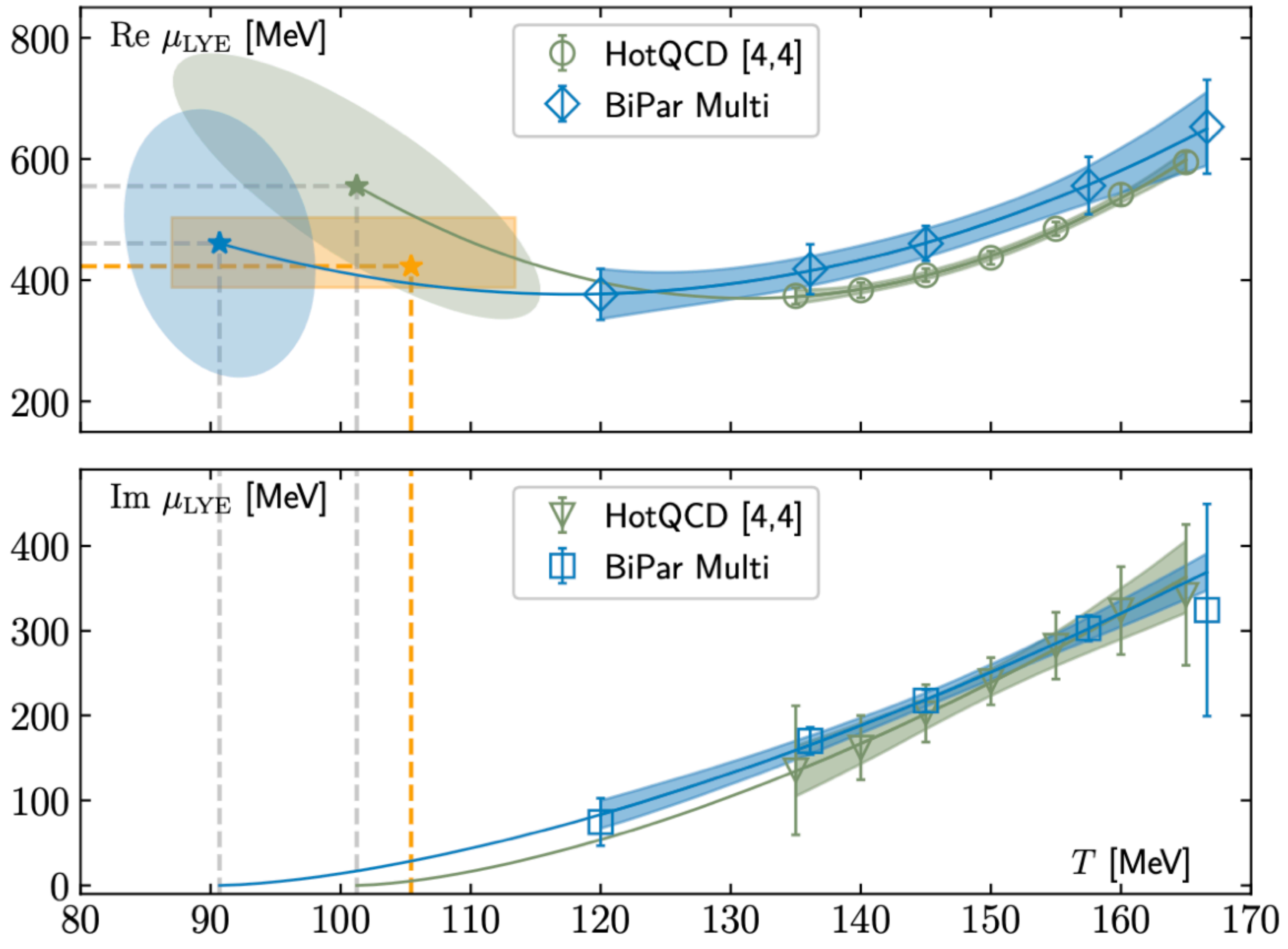
Recent results on the CEP search in Lattice QCD

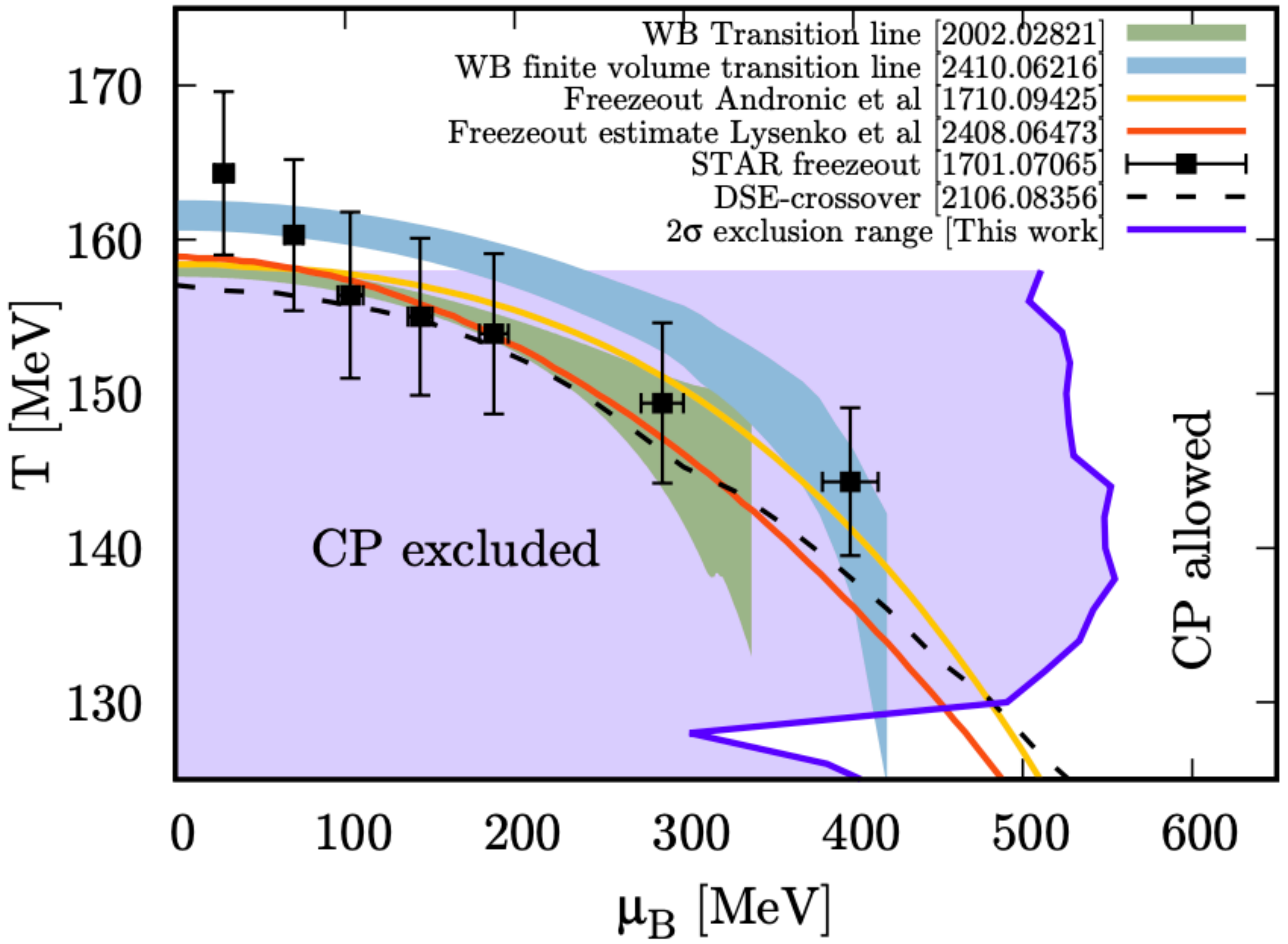
Lee-Yang edge singularity



$$T^{\rm CEP} = 105^{+8}_{-18}\ \text{MeV}$$
$$\mu_B^{\rm CEP} = 422^{+80}_{-35}\ \text{MeV}$$

Bielefeld-Parma, arXiv:2405.10196

Contour of constant entropy



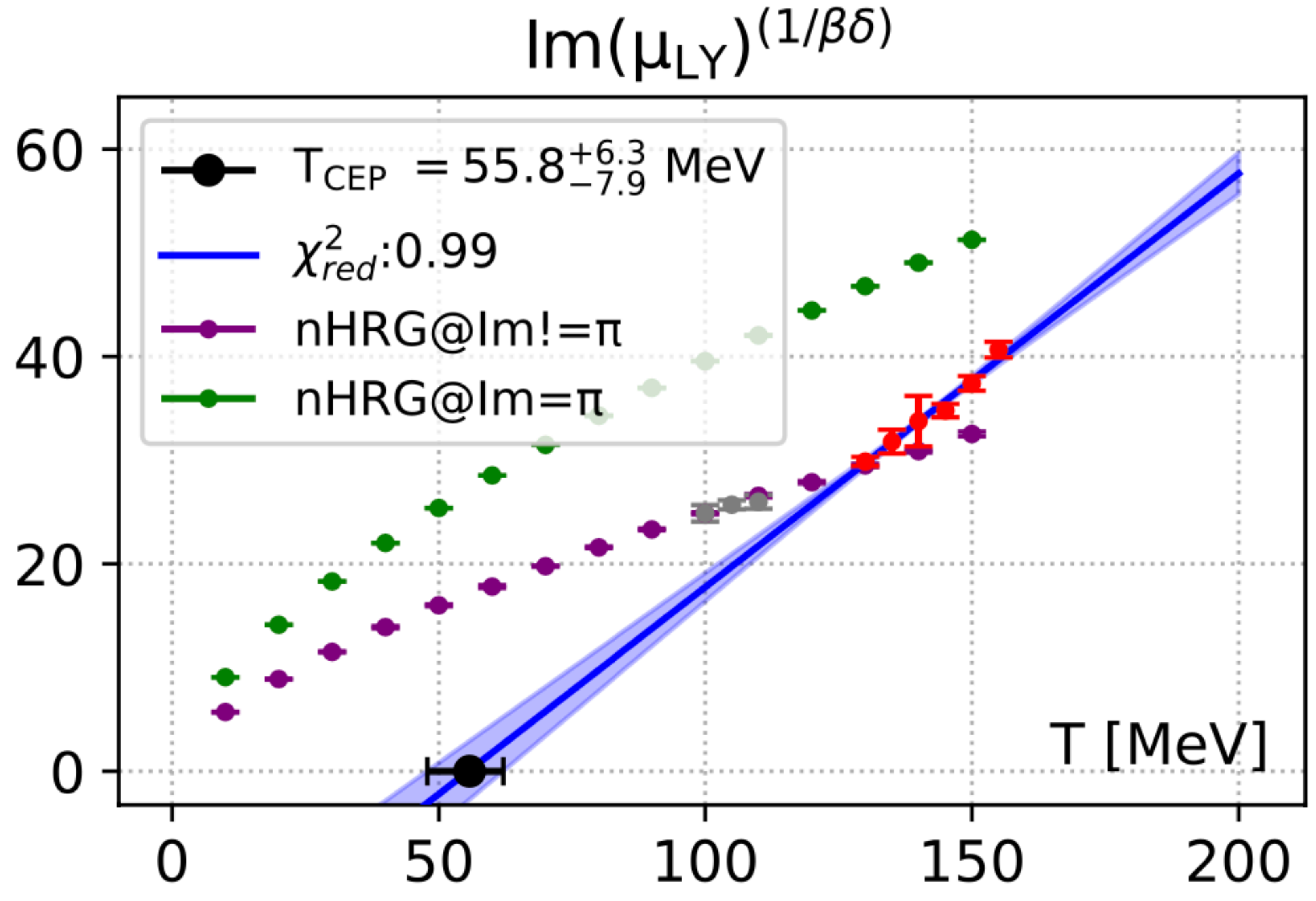
No CEP at $\mu_B < 450$ MeV at the 2σ level

Borsányi et al., arXiv:2502.10267

$$T^{\rm CEP} = 114.3 \pm 6.9\ \text{MeV}$$
$$\mu_B^{\rm CEP} = 602.1 \pm 62.1\ \text{MeV}$$

Shah et al., arXiv:2410.16206

Lee-Yang edge singularity



$T^{\rm CEP} < 103$ MeV
with 84% probability or
CEP does not exit

Adam et al., arXiv:2507.13254

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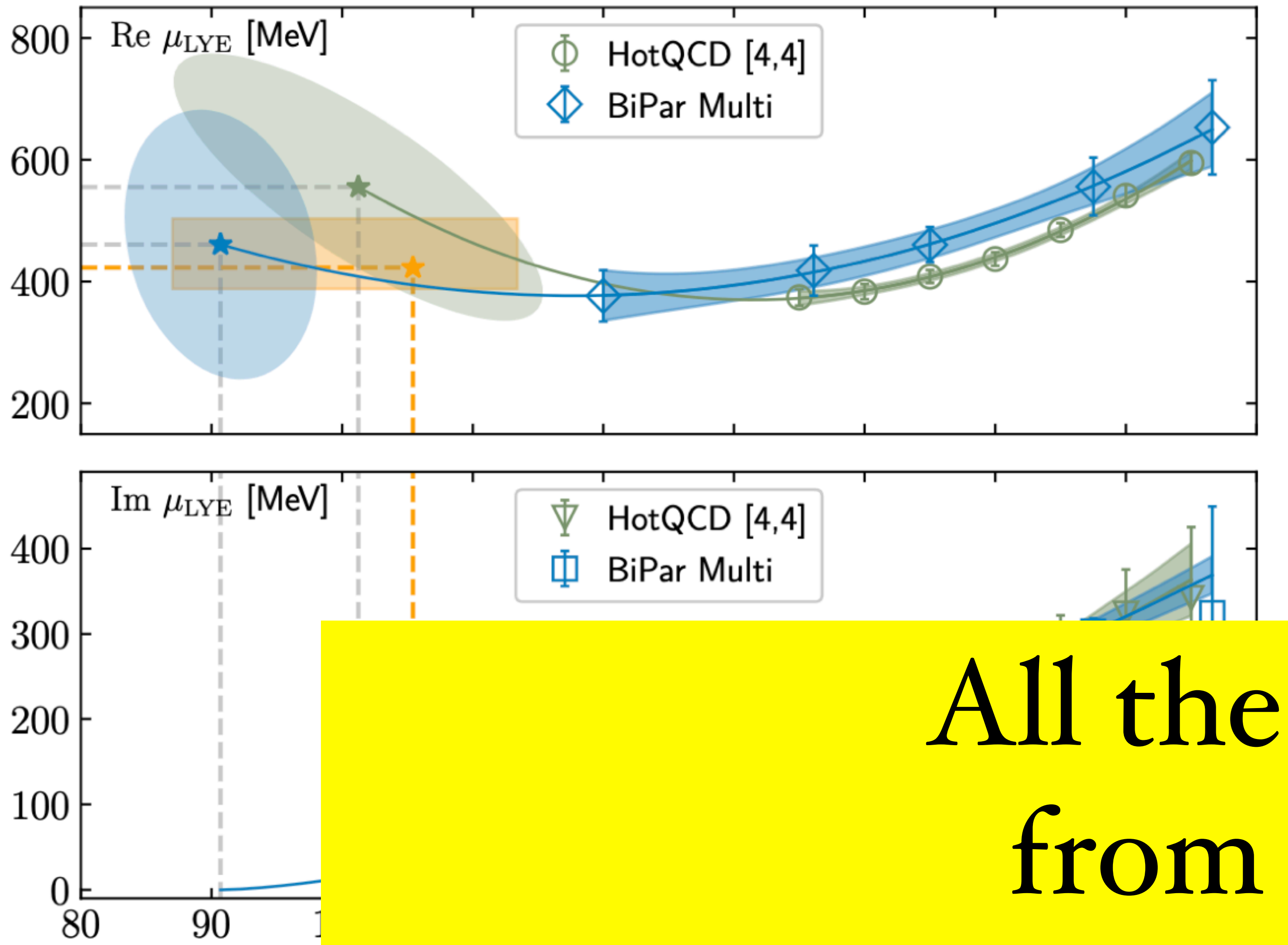

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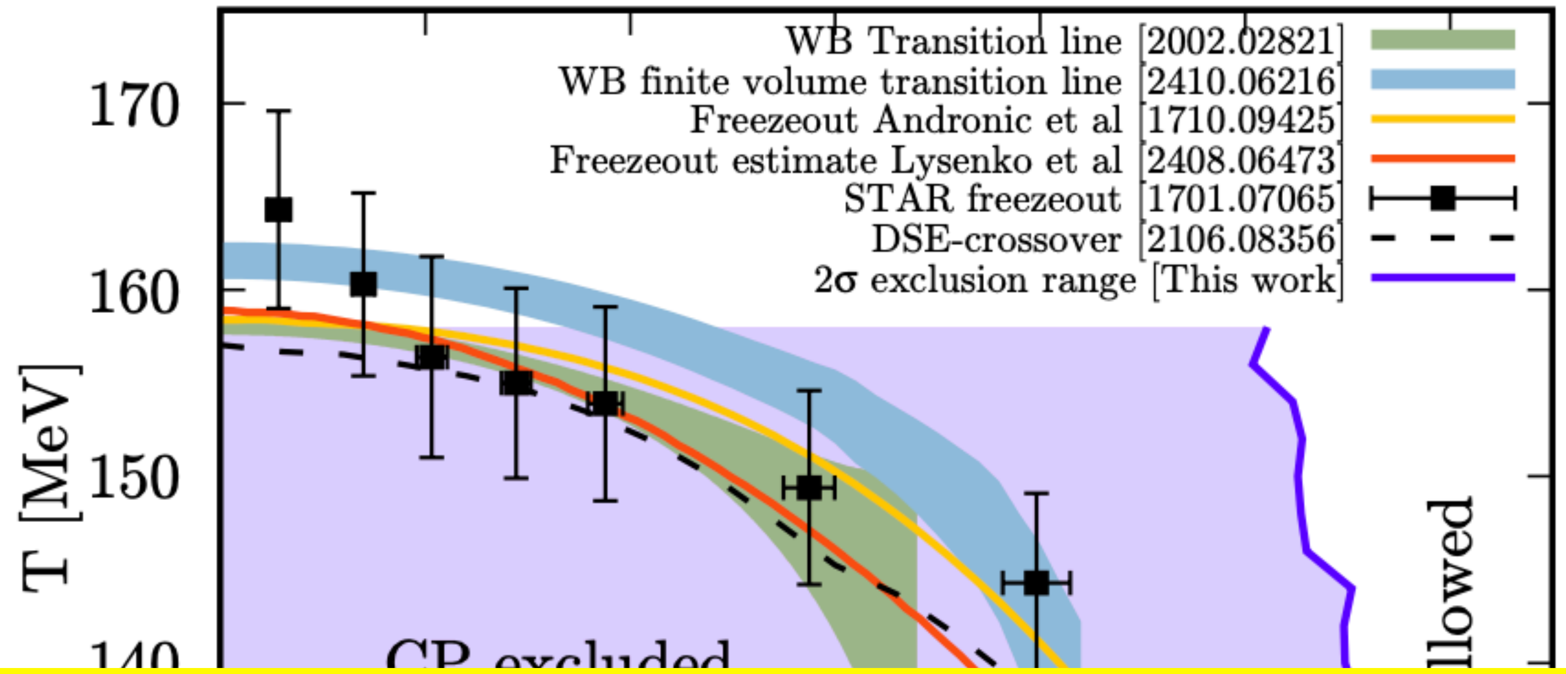

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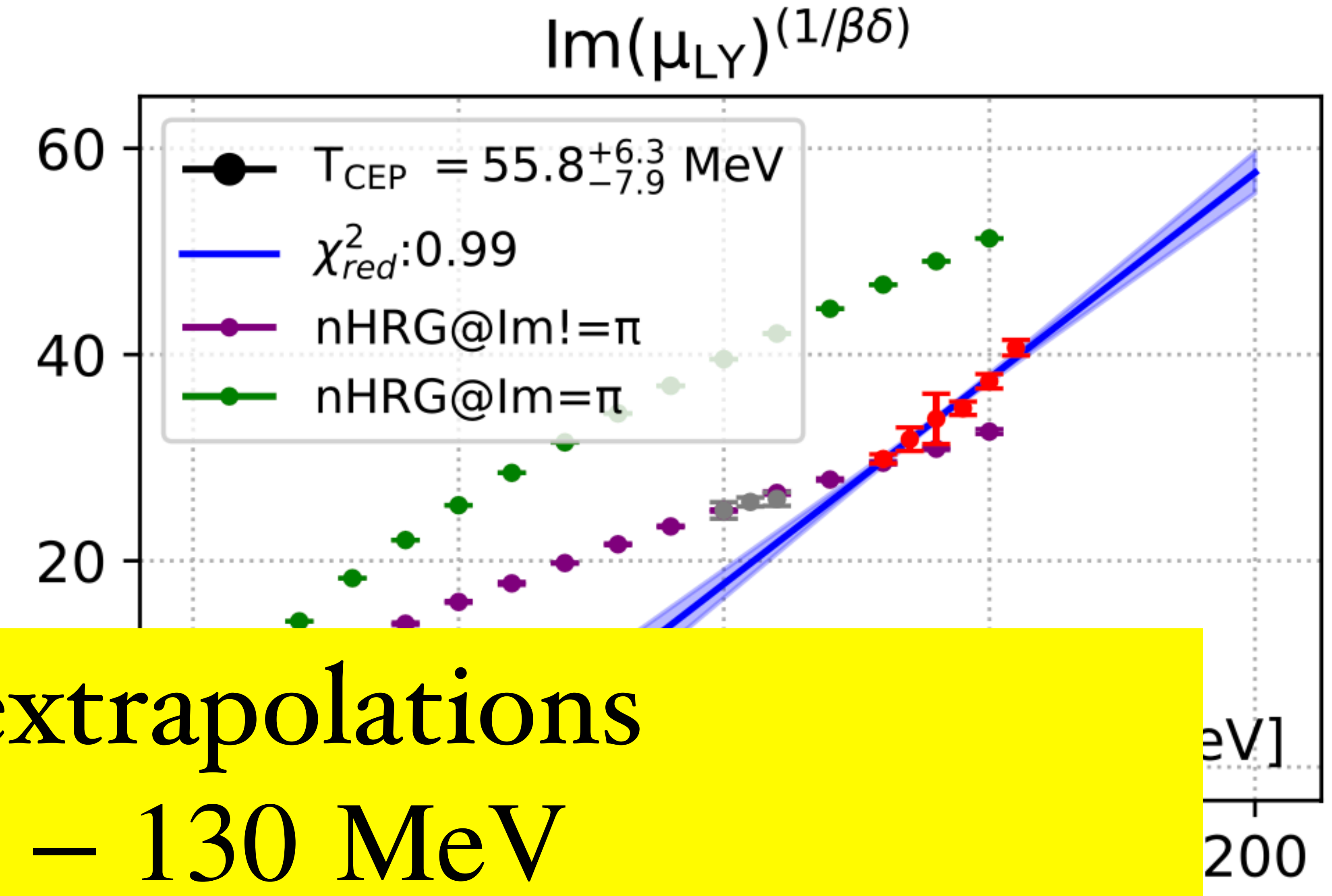

All these analyses are done based on extrapolations
from lattice QCD results at $T \gtrsim 120 - 130$ MeV
A direct simulation at T=108 MeV in lattice QCD

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$\mu_B^{\rm CEP} = 422^{+80}_{-35}$ MeV

Bielefeld-Parma, arXiv:2405.10196

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Shah et al., arXiv:2410.16206

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Lattice setup at T=108 MeV

- N_f=2+1 QCD on 20^3x10 lattices with at T=107.7 MeV using HISQ fermions at the physical point with $\mu_u = \mu_d = \mu_s = \mu_B/3$
- 24 values of $i\mu_B \in [0, i\pi]$ each for χ^B_{1-4}
- 400k configurations for zero chemical potential, 200k for others
- Additional 32^3x10 lattices generated to check the volume dependence

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Multi-point Padé fits

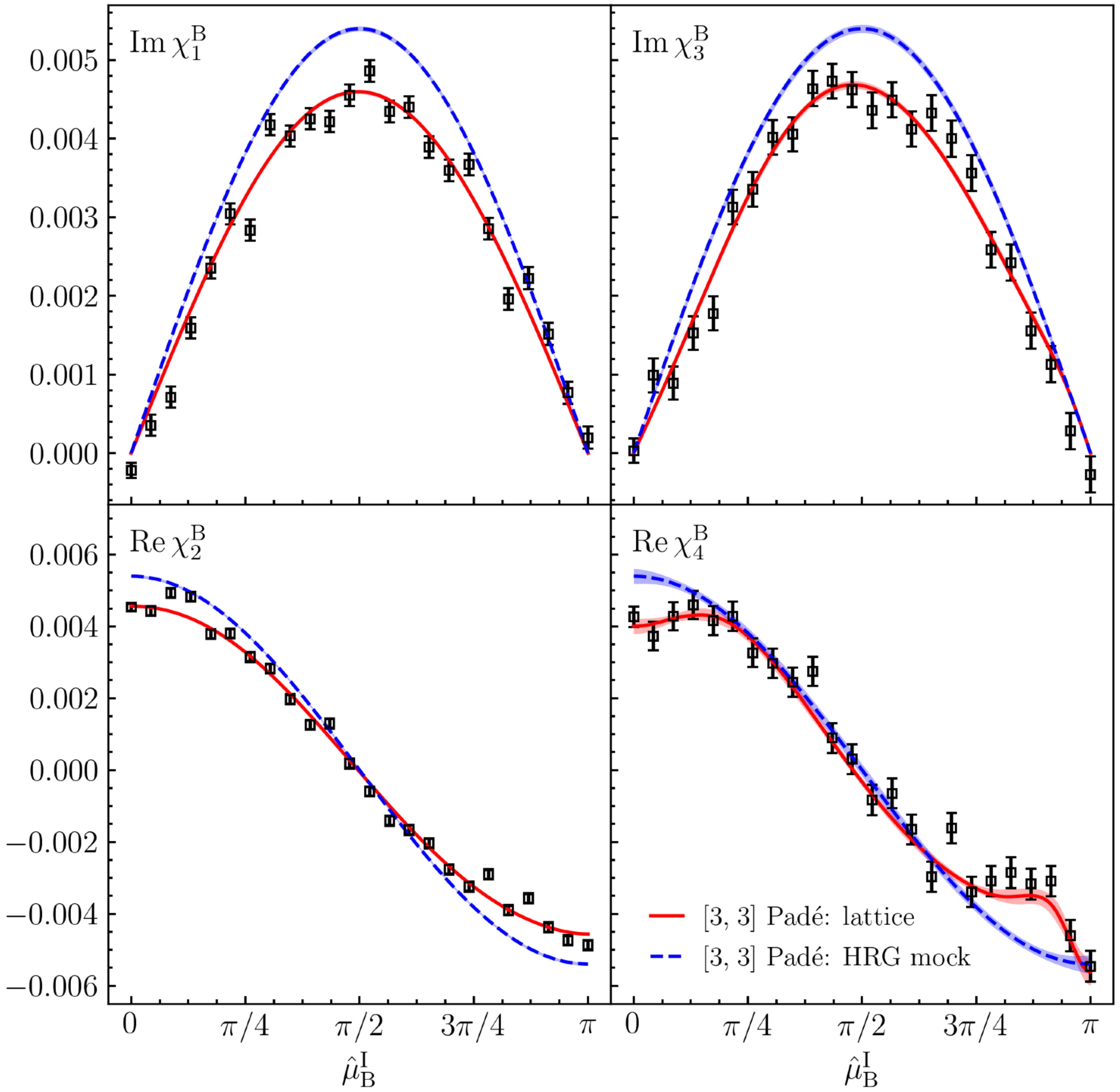
$$R_k^m(x) \equiv \frac{P_m(x)}{1+Q_k(x)} = \frac{\sum_{i=0}^m a_i x^i}{1+\sum_{j=1}^k b_j x^j}, x = \cosh(\mu_B) - 1$$

- Generate N_{boot} bootstrap samples by resampling $N_{conf,i}$ configurations allowing replacement for each one of 24 discrete values of $\mu_{B,i}$, and compute mean value of χ^B_{1-4} at $\mu_{B,i}$ within each bootstrap sample

- Joint fit to χ^B_n *with* $\partial^n R_k^m(x)/\partial\mu_B^n$ *with n=1-4 in each bootstrap sample with covariance matrix obtained from the original data sets*

- Final uncertainties from 68% confidence interval over bootstrap fits

[3,3]



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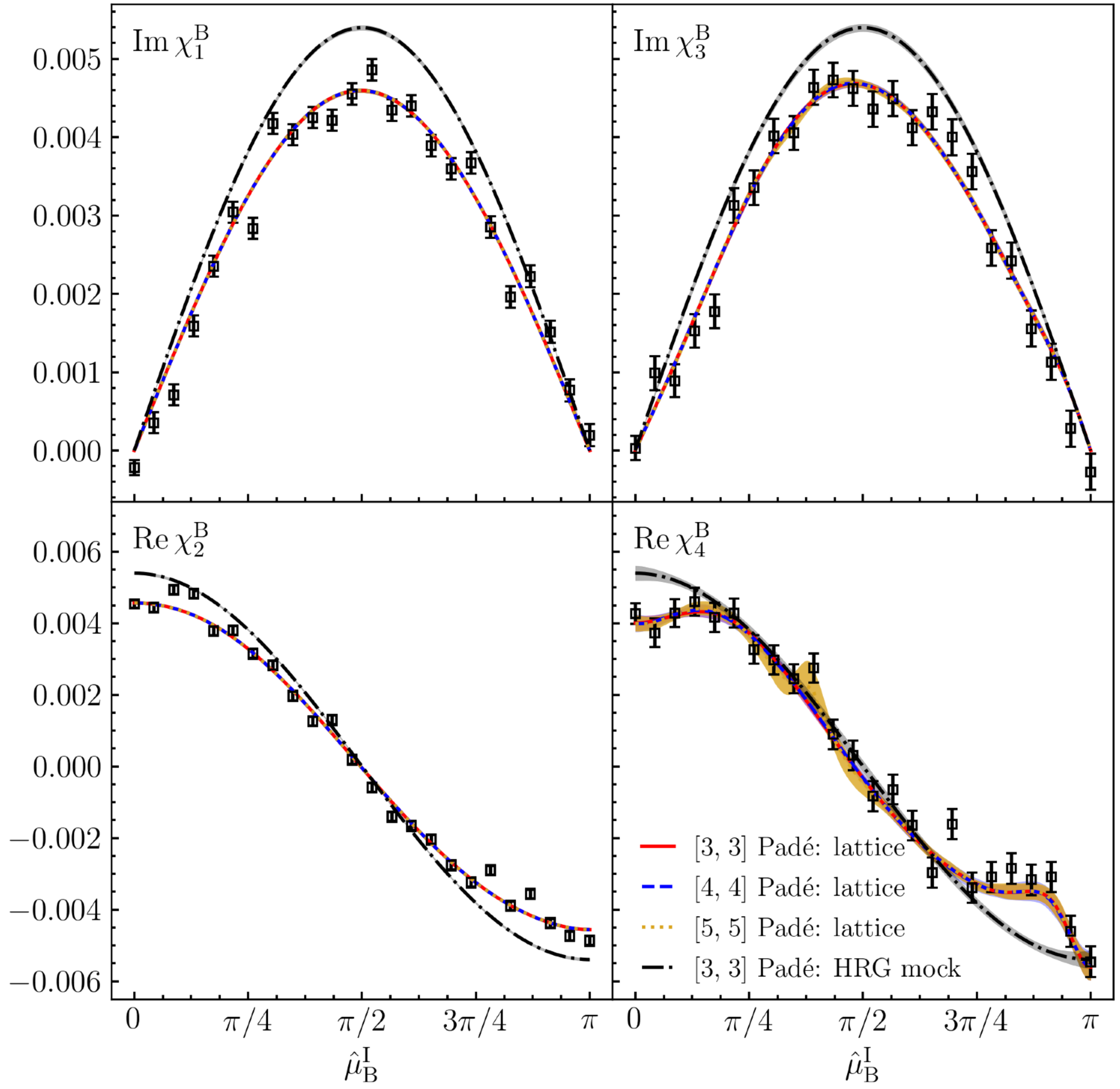
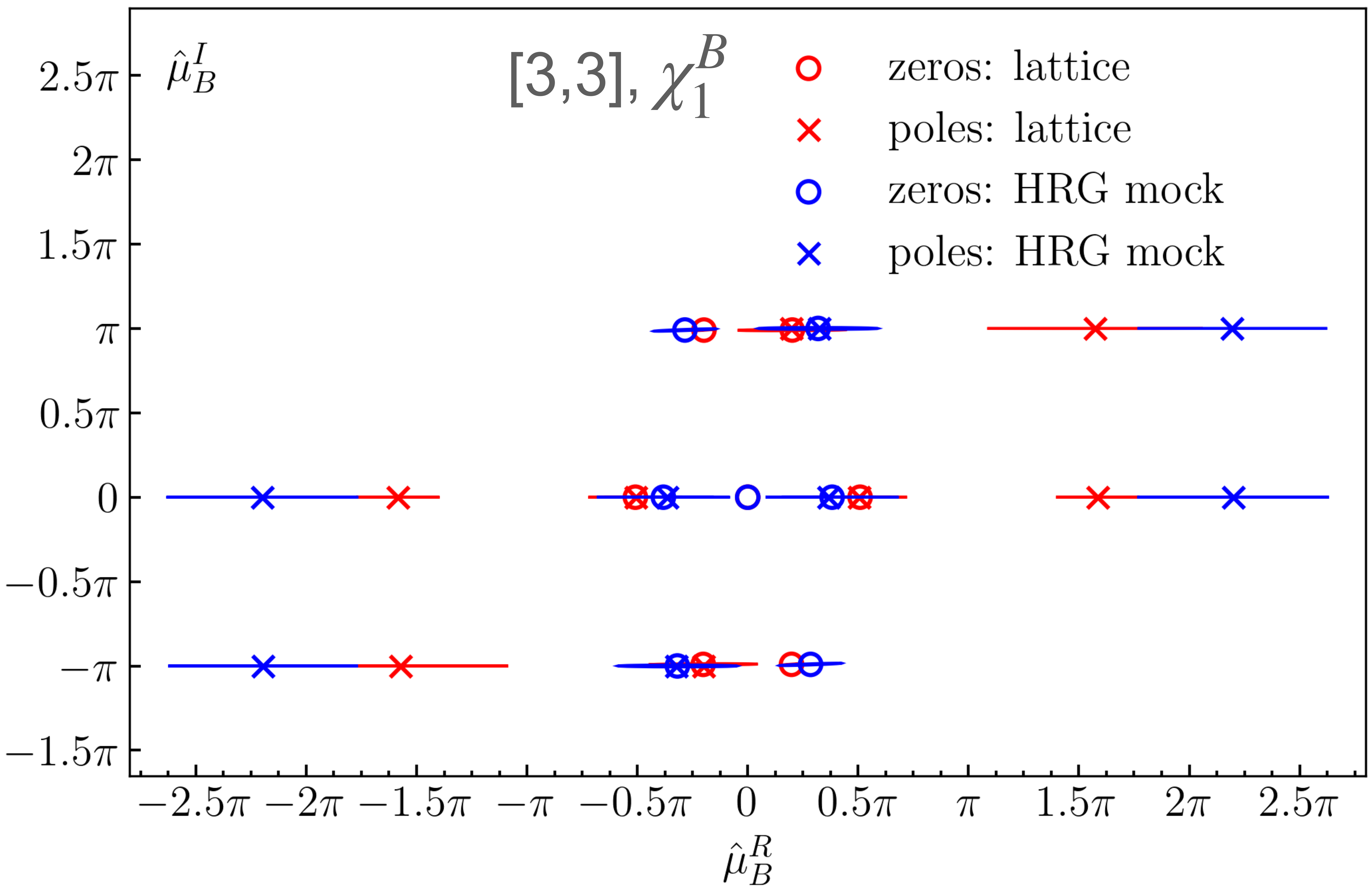


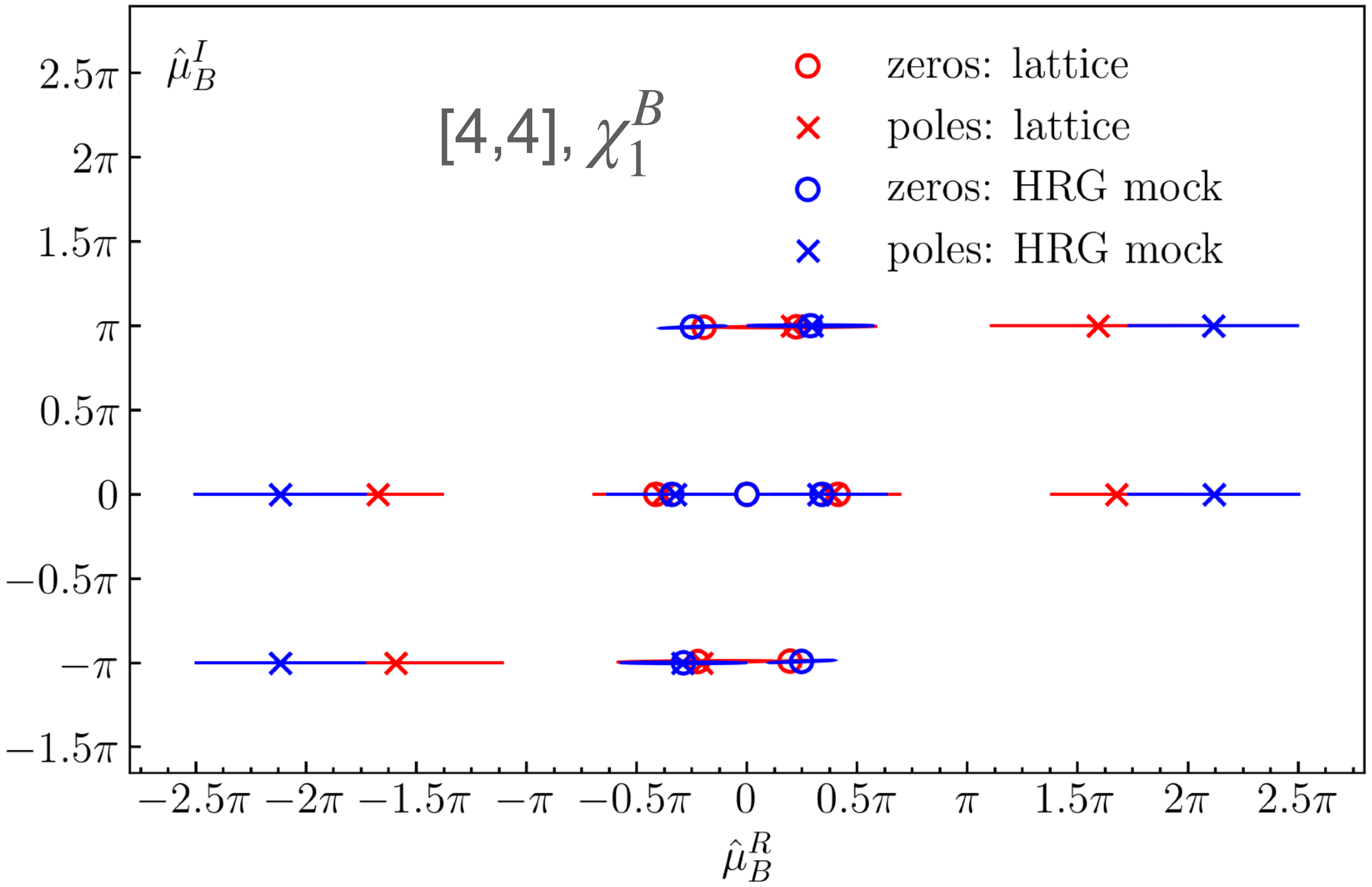
		Padé [3,3]		Padé [4,4]		Padé [5,5]	
		All 24 points	Random 12 points	All 24 points	Random 12 points	All 24 points	Random 12 points
χ^2/d.o.f.	lattice	4.00(55)	3.39(75)	4.06(57)	4.45(90)	4.03(57)	3.99(85)
	HRG mock	0.97(21)	0.95(30)	0.97(21)	0.94(31)	0.96(21)	0.93(31)
AICc	lattice	373(50)	156(31)	375(50)	198(36)	370(49)	178(32)
	HRG mock	101(19)	54(13)	103(18)	57(12)	105(18)	61(12)

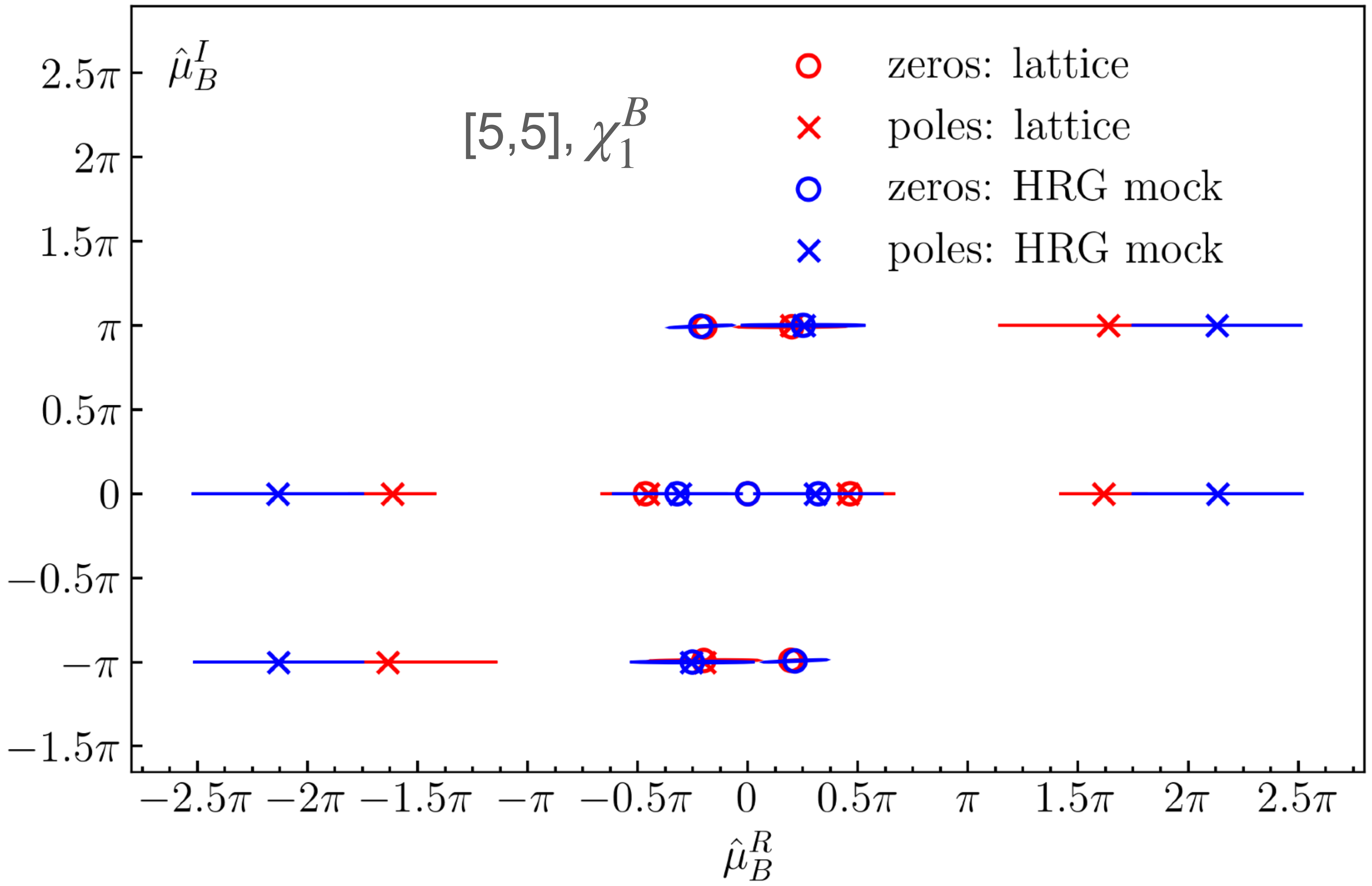
Zero/poles extracted from Padé [3,3] analyses



- No uncanceled poles
- Similar pattern between Lattice and HRG mock

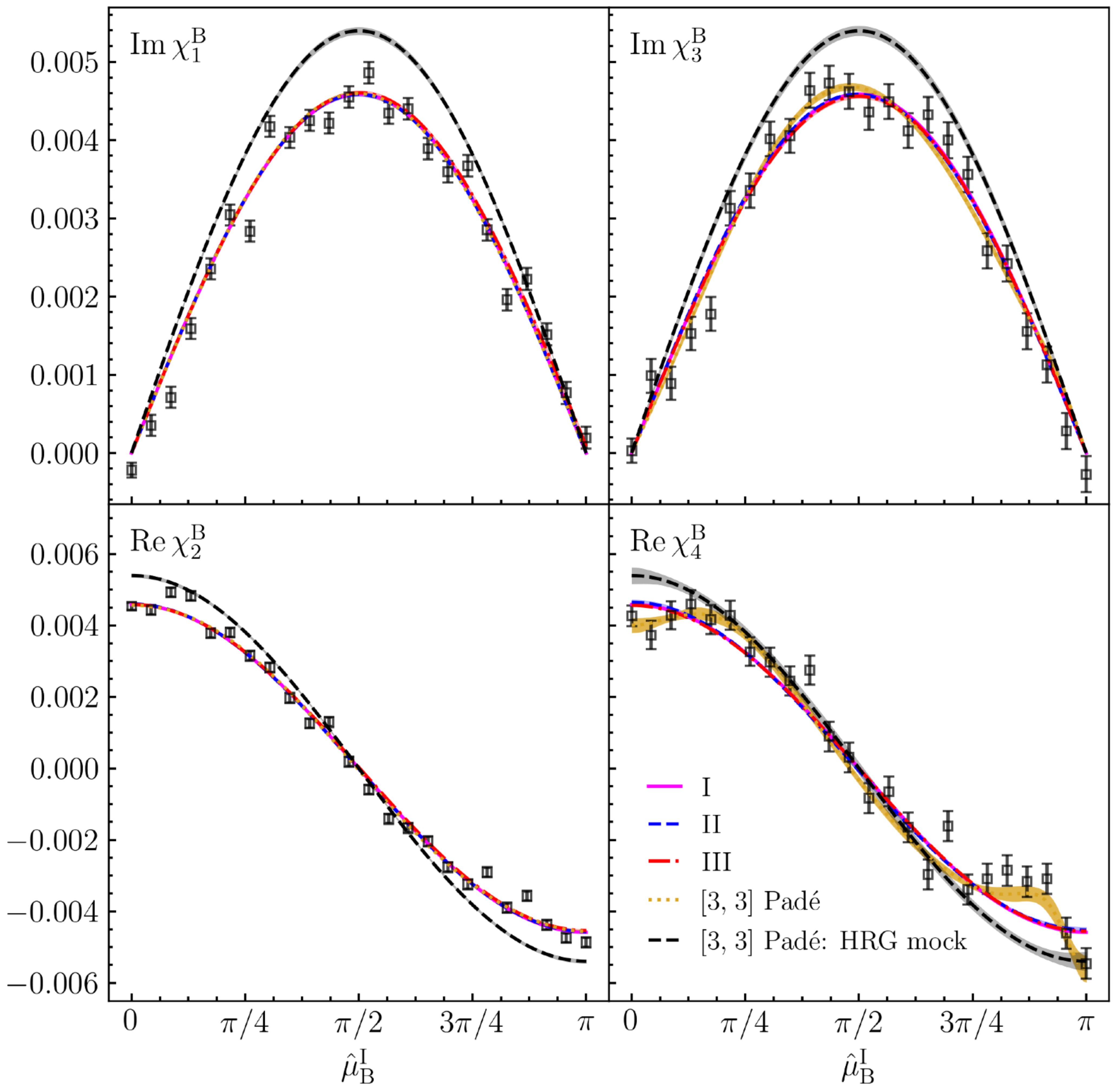
Zero/poles extracted from higher order Padé analyses





Similar observation as in the case of Padé [3,3]

Fits with physical-motivated non-critical ansatz



Non-critical ansatz for pressure:

Ansatz I: $f(\mu_B) = A\cosh(\mu_B)$

Ansatz II: $f(\mu_B) = A\cosh(\mu_B) + B\cosh(2\mu_B)$

	Lattice QCD Data			
	All 24 points		Random 12 points	
	Ansatz I	Ansatz II	Ansatz I	Ansatz II
Parameters				
A ($\times 10^{-3}$)	4.584(35)	4.582(34)	4.528(48)	4.662(50)
B ($\times 10^{-5}$)	–	0.38(64)	–	0.001(898)
Statistics				
χ^2/d.o.f.	4.20(55)	4.13(55)	4.45(82)	4.23(81)
AICc	401(52)	401(52)	211(38)	199(37)

Singularities in the real plane?

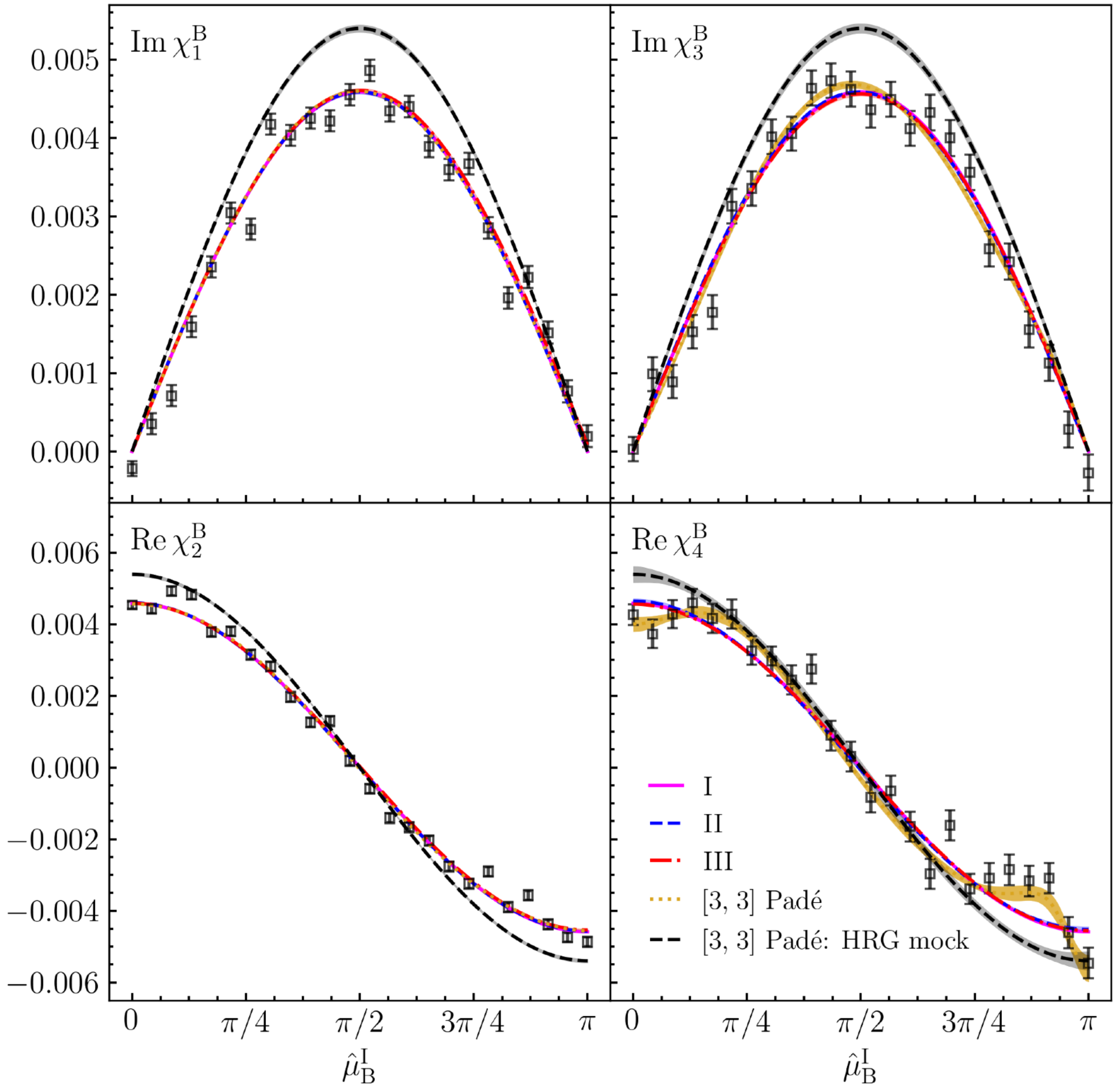
Critical ansatz III: $f(\mu_B) = A\cosh(\mu_B) + C(|\mu_B - \mu_{B,c}|^\alpha + |\mu_B + \mu_{B,c}|^\alpha)$

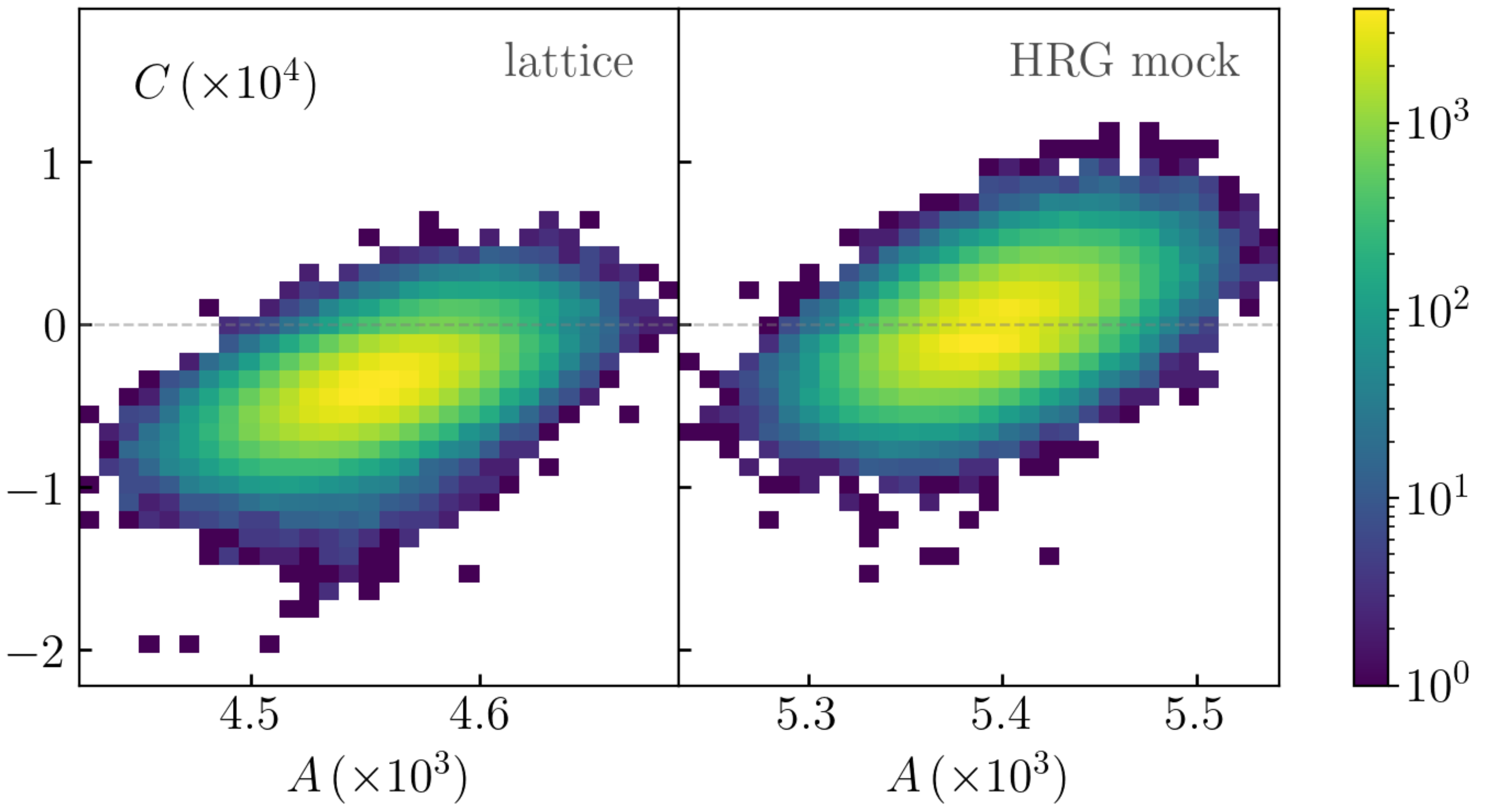
Impose lower bound of $\mu_{B,c}$: 350, 360,380, 400 MeV

$\alpha = 1 + \delta_{ISNG}$ = 1.208

Hasenbusch , PRB 82(2010)174433

Karch, Schmidt & Singh, PRD109(2024)014508





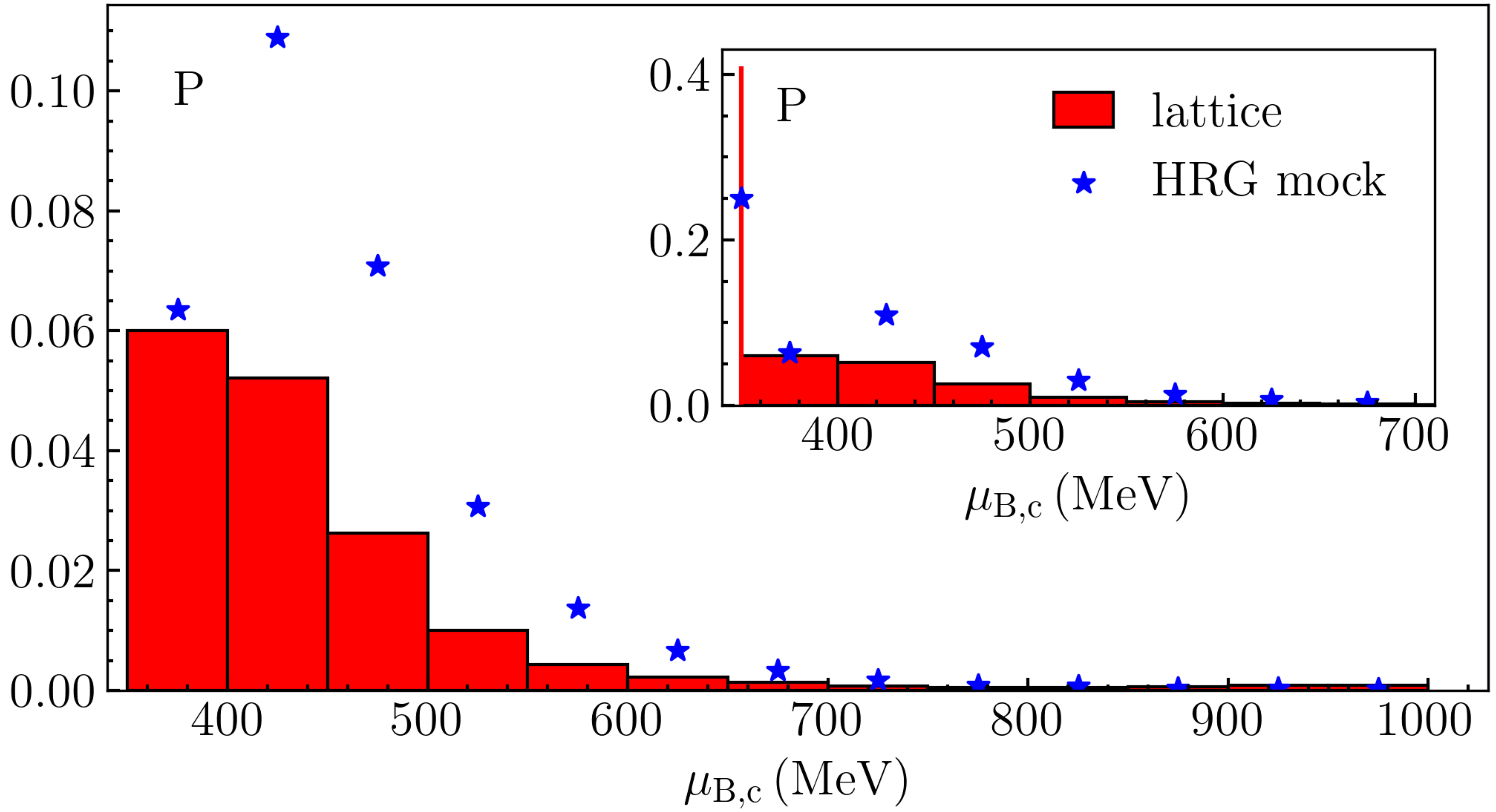
Probability of $\mu_{B,c}$ in a certain region $\mathscr{R}$

Define a probability P for $\mu_{B,c}$ in a region $\mathscr{R}$: $$P = \frac{N_{\mathscr{R}}}{N_{total}}$$

$N_{\mathscr{R}}$: # of bootstrap samples yield $\mu_{B,c}$ in the region $\mathscr{R}$

N_{total}: total # of bootstrap samples used in the fit

Lower bound of $\mu_{B,c}$=350 MeV



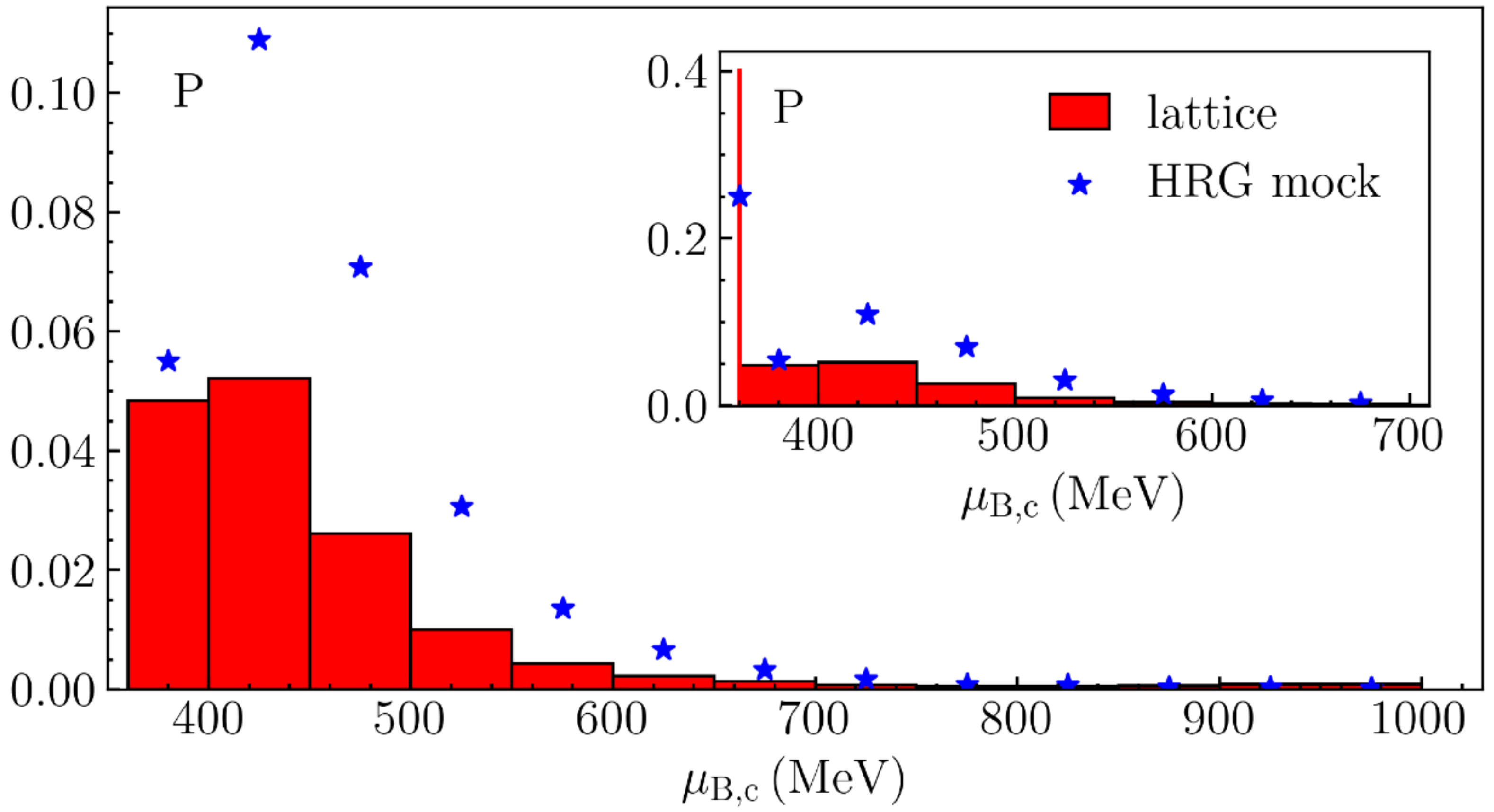
- Nonzero false-positive $P_{\text{HRG}} > P_{\text{LAT}}$!

Region $\mathcal{R}$ (MeV)	$P(\mathcal{R})$ lattice	$P(\mathcal{R})$ HRG mock
387–502 (Predicted Region I)	9.1%	20.4%
420–750 (Experimental Search Region I)	7.3%	19.4%
420–632 (Experimental Search Region II)	7.1%	18.7%
540.0–664.2 (Predicted Region II)	0.86%	2.6%

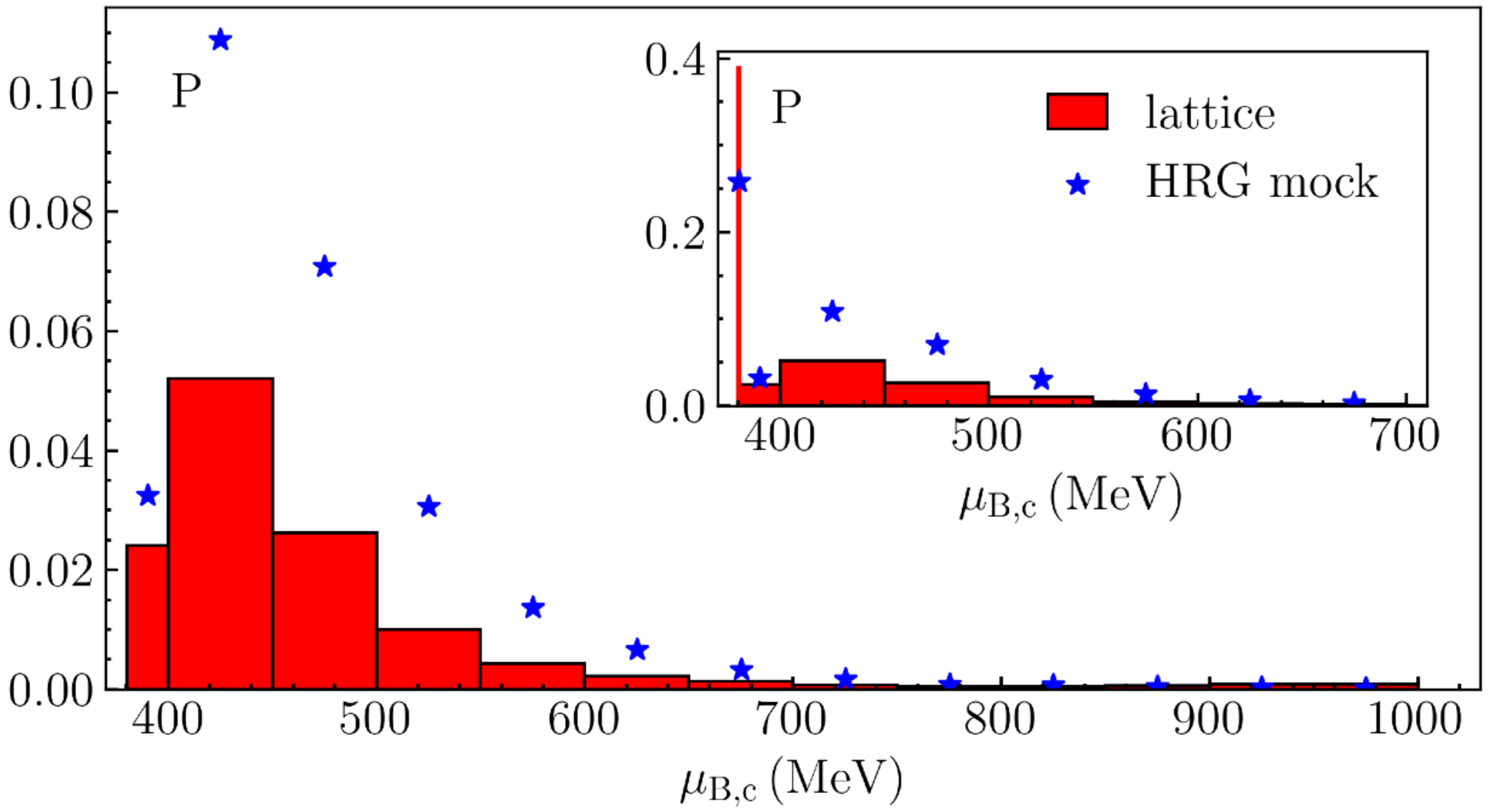
- Accumulation of large samples at lower bound

Dependence on the lower bound & # of bootstrap samples

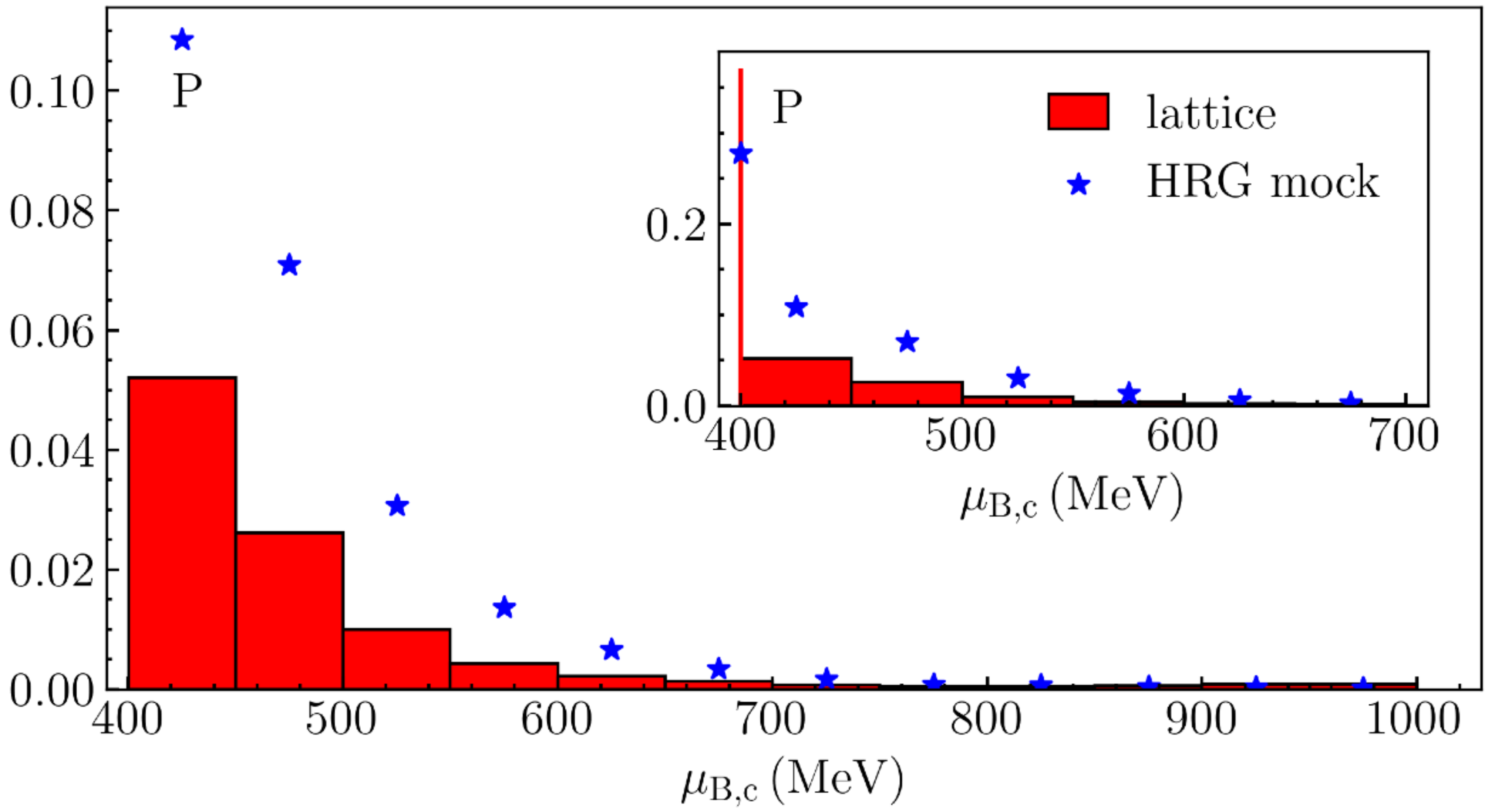
Lower bound 360 MeV



Lower bound 380 MeV

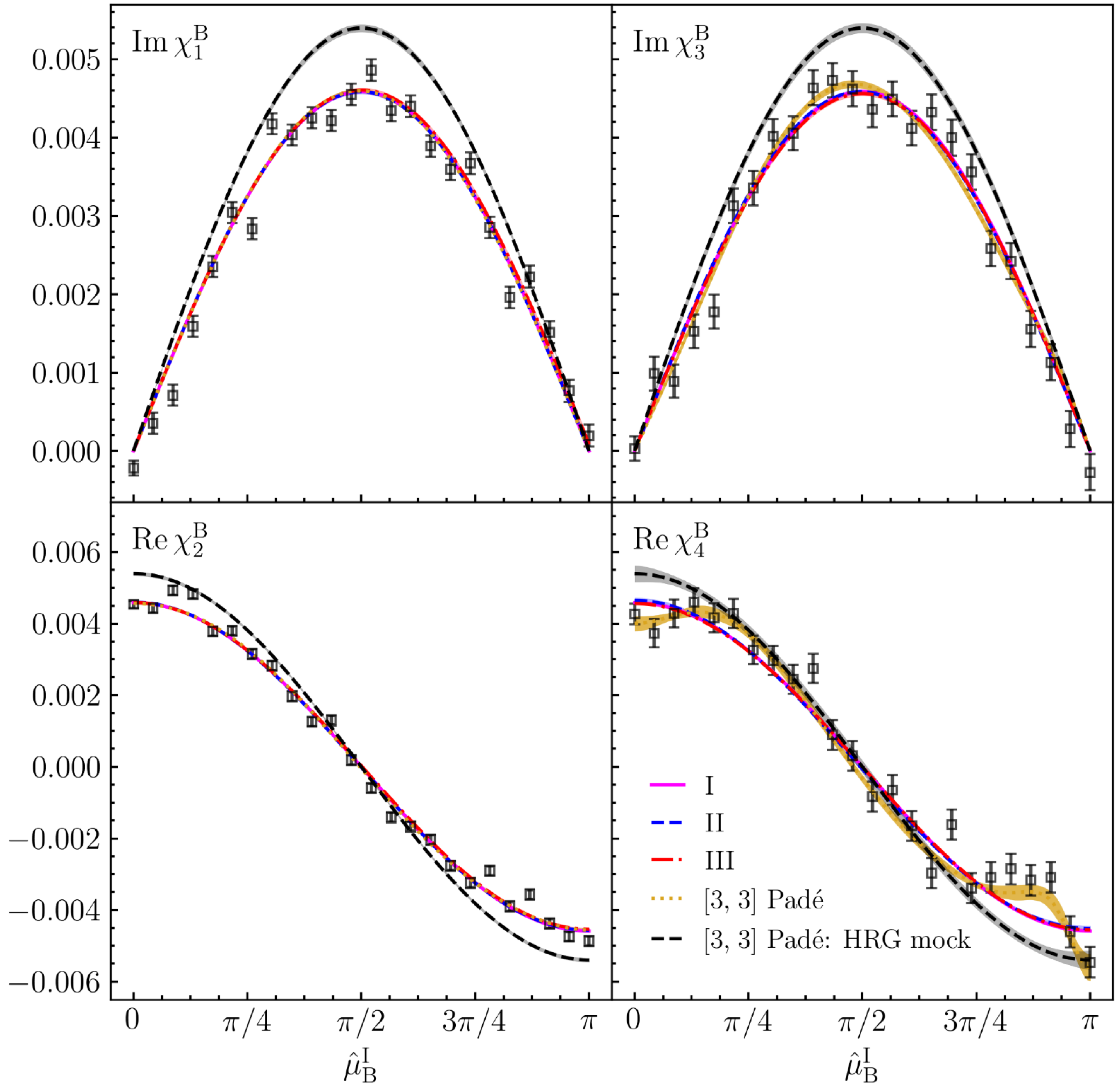


Lower bound 400 MeV



	$P(\mathcal{R})$											
	100K Samples				200K Samples				300K Samples			
Region $\mathcal{R}$ (MeV)	350	360	380	400	350	360	380	400	350	360	380	400
387–502	0.091	0.091	0.091	-	0.092	0.091	0.091	-	0.092	0.092	0.092	-
420–750	0.074	0.074	0.074	0.074	0.073	0.073	0.073	0.073	0.073	0.073	0.073	0.073
420–632	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071
540.0–664.2	0.0086	0.0086	0.0087	0.0087	0.0087	0.0087	0.0086	0.0086	0.0085	0.0085	0.0084	0.0085

Dependence on the reduced/full data sets



Lattice

	$P(\mathcal{R})$							
	All 24 points				Random 12 points			
Region $\mathcal{R}$ (MeV)	350	360	380	400	350	360	380	400
387–502	0.092	0.092	0.092	-	0.124	0.124	0.124	-
420–750	0.073	0.073	0.073	0.073	0.133	0.133	0.132	0.133
420–632	0.071	0.071	0.071	0.071	0.126	0.126	0.126	0.126
540.0–664.2	0.0085	0.0085	0.0085	0.0085	0.023	0.023	0.022	0.023

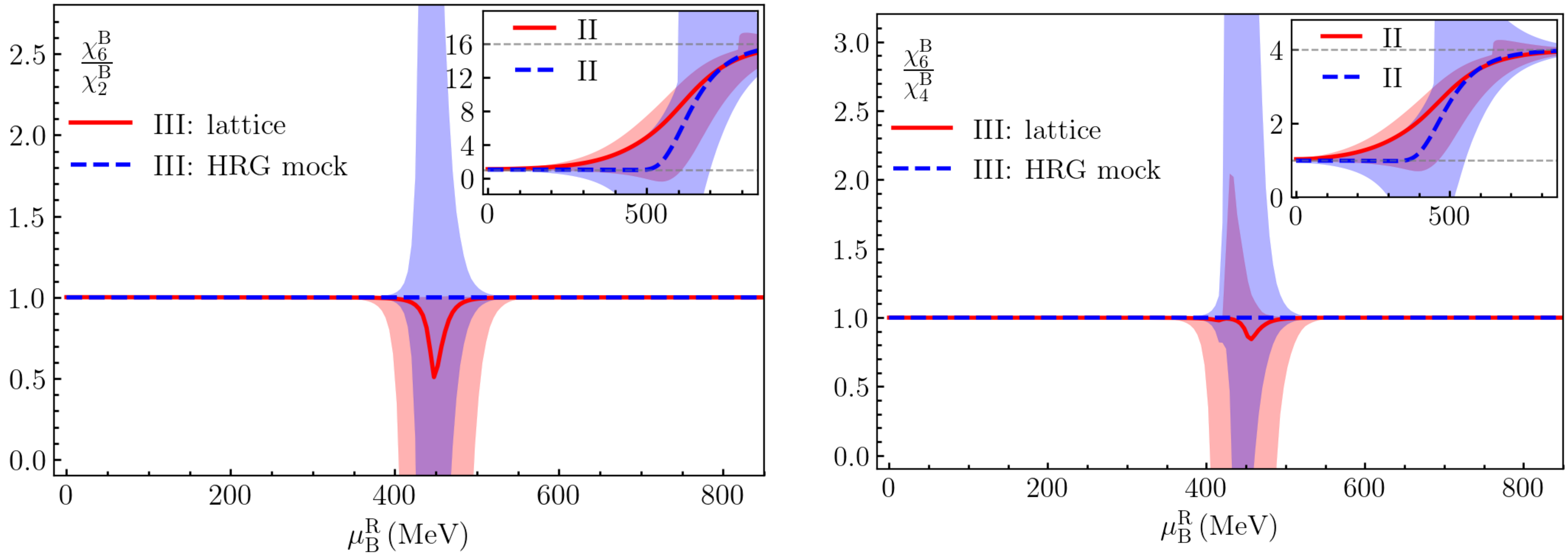
HRG

	$P(\mathcal{R})$							
	All 24 points				Random 12 points			
Region $\mathcal{R}$ (MeV)	350	360	380	400	350	360	380	400
387–502	0.204	0.204	0.204	-	0.263	0.225	0.197	-
420–750	0.194	0.194	0.194	0.194	0.301	0.218	0.175	0.184
420–632	0.187	0.187	0.187	0.187	0.293	0.211	0.168	0.177
540.0–664.2	0.0258	0.0259	0.0258	0.0258	0.0327	0.0266	0.0233	0.0247

Analytic continuation

- Baryon number density $\hat{n}_B$ and $\partial\hat{n}_B/\partial\hat{\mu}_B$ shall be positive, which restricts $\mu_B^R \lesssim 800$MeV
- Results from ansatz I (non-interacting) and III (critical) are very much similar
- Results from ansatz II (interacting) differ from others at $\mu_B^R \gtrsim 500$MeV

Manifestation in the high order susceptibilities



Although about 7% probability allowed to have $\mu_{B,c} \in (420{,}750)$ MeV
No significant sign change, deviation from HRG etc are observed

No signal in Exp

Analyses using critical ansatz in the complex plane

Critical ansatz: Ansatz III: $f(\mu_B) = A\cosh(\mu_B) + C(|\mu_B - \mu_{B,c}|^\alpha + |\mu_B + \mu_{B,c}|^\alpha)$

$\alpha = 1 + \sigma_{\mathrm{LYE}} = 1.085$

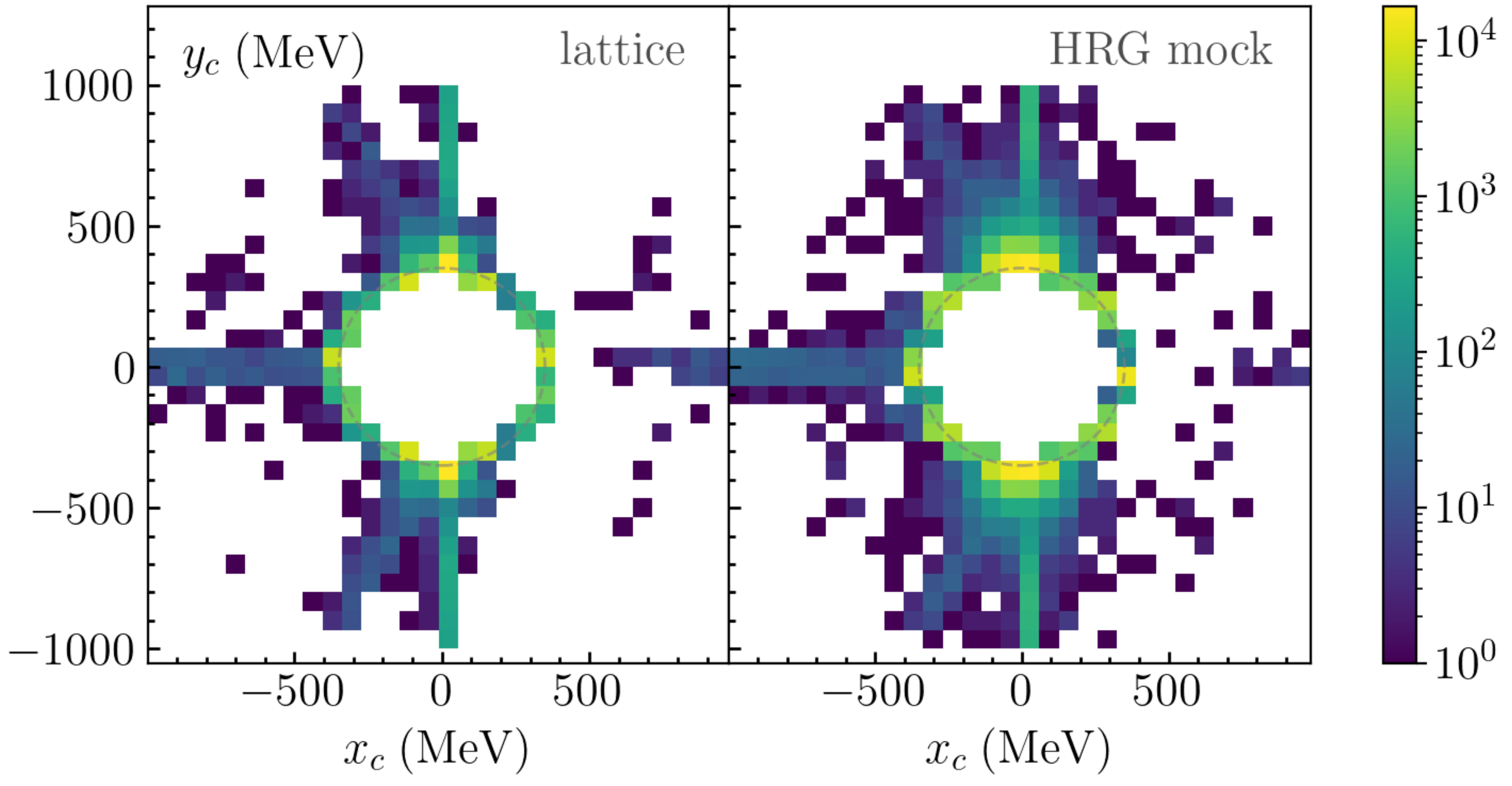
Gliozzi and Rago, JHEP 10 (2014) 042 [1403.6003]

In the complex plane to fit the LQCD data:

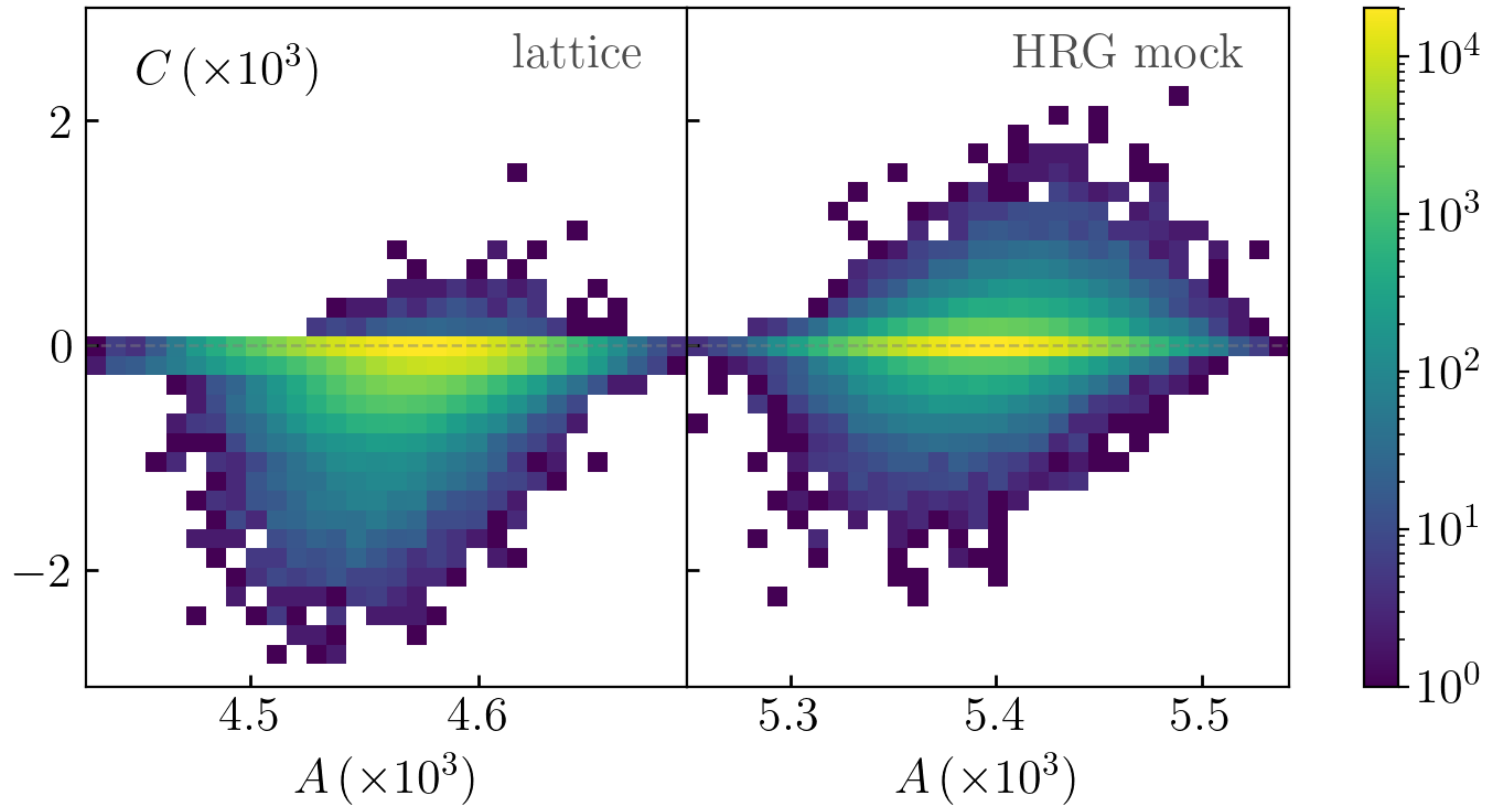
$$f(i\theta_B) = A(T)\ \cos(\theta_B) + C\left[\left(x_c^2 + (\theta_B - y_c)^2\right)^{\alpha/2} + \left(x_c^2 + (\theta_B + y_c)^2\right)^{\alpha/2}\right]$$

- Lee-Yang edge singularity (x_c, y_c)
- Radius of convergence $|\mu_{B,c}|_{min} = \left(\sqrt{x_c^2 + y_c^2}\right)_{min}$

Fit with lower bound of $|\mu_{B,c}|$=350 MeV in the complex plane

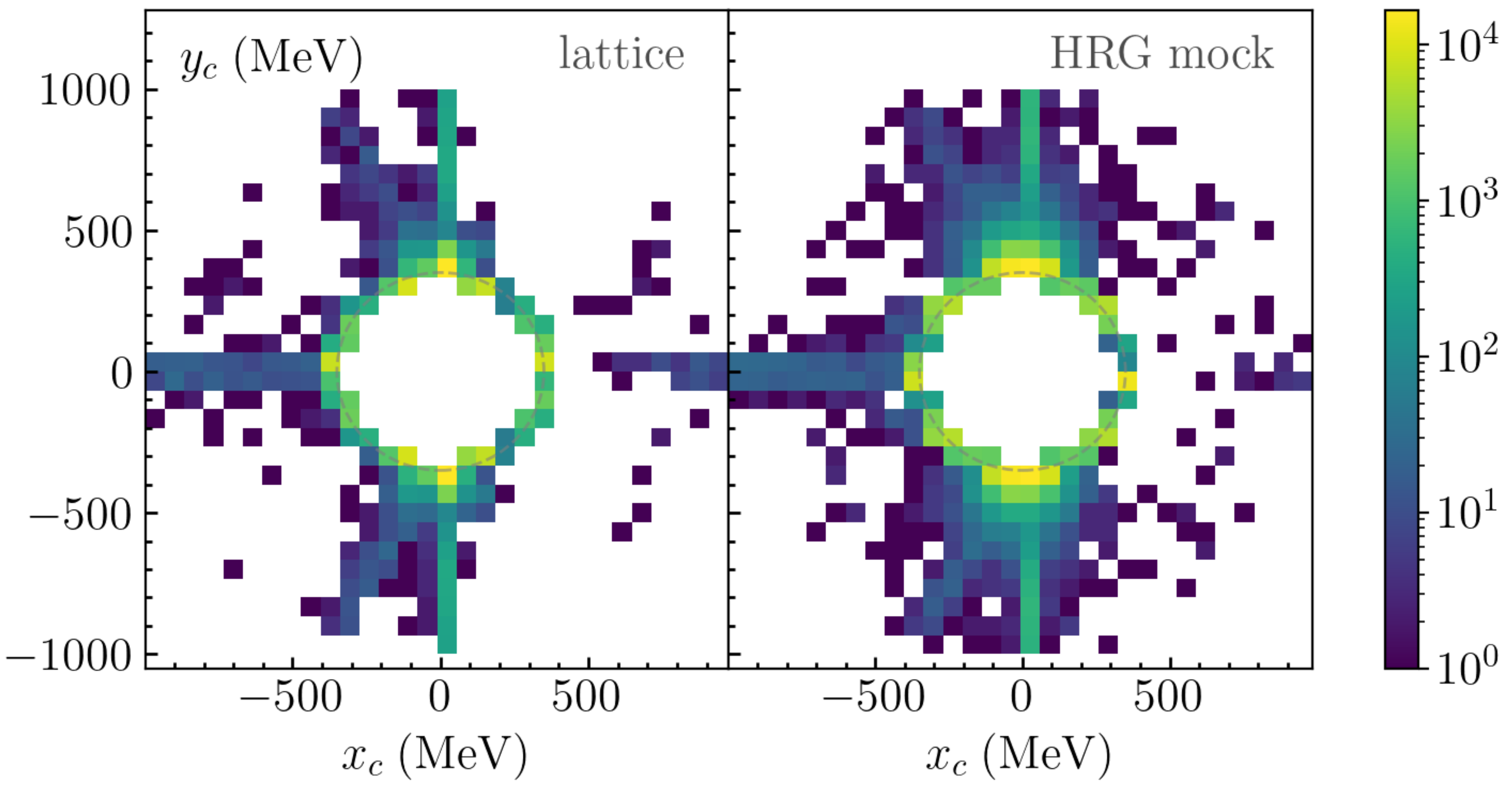


Heat map of (x_c, y_c)

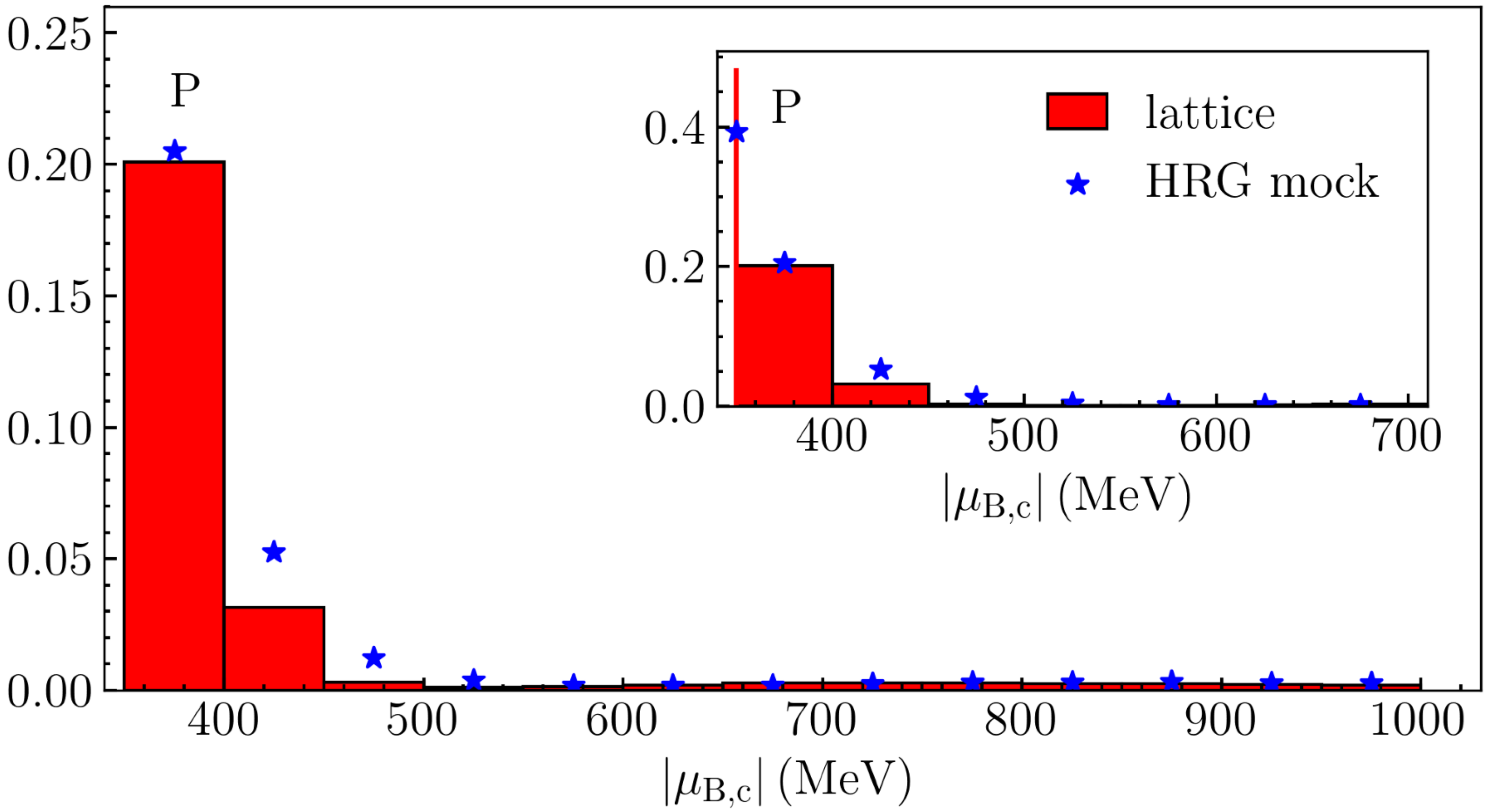


Heat map of (C, A)

Fit with lower bound of $|\mu_{B,c}|=350$ MeV



Heat map of (x_c, y_c)



Distribution P from Lat and HRG

Probabilities in certain rings in the complex plane

Region $\mathcal{R}$ (MeV)	$P(\mathcal{R})$ lattice	$P(\mathcal{R})$ HRG mock
387–502	8.8%	9.7%
420–750	1.9%	4.6%
420–632	1.3%	4.1%
540.0–664.2	0.41%	0.49%

With yc about zero, $|y_c| \leq 0.1$ MeV: $P_{\text{LAT}} = 0$

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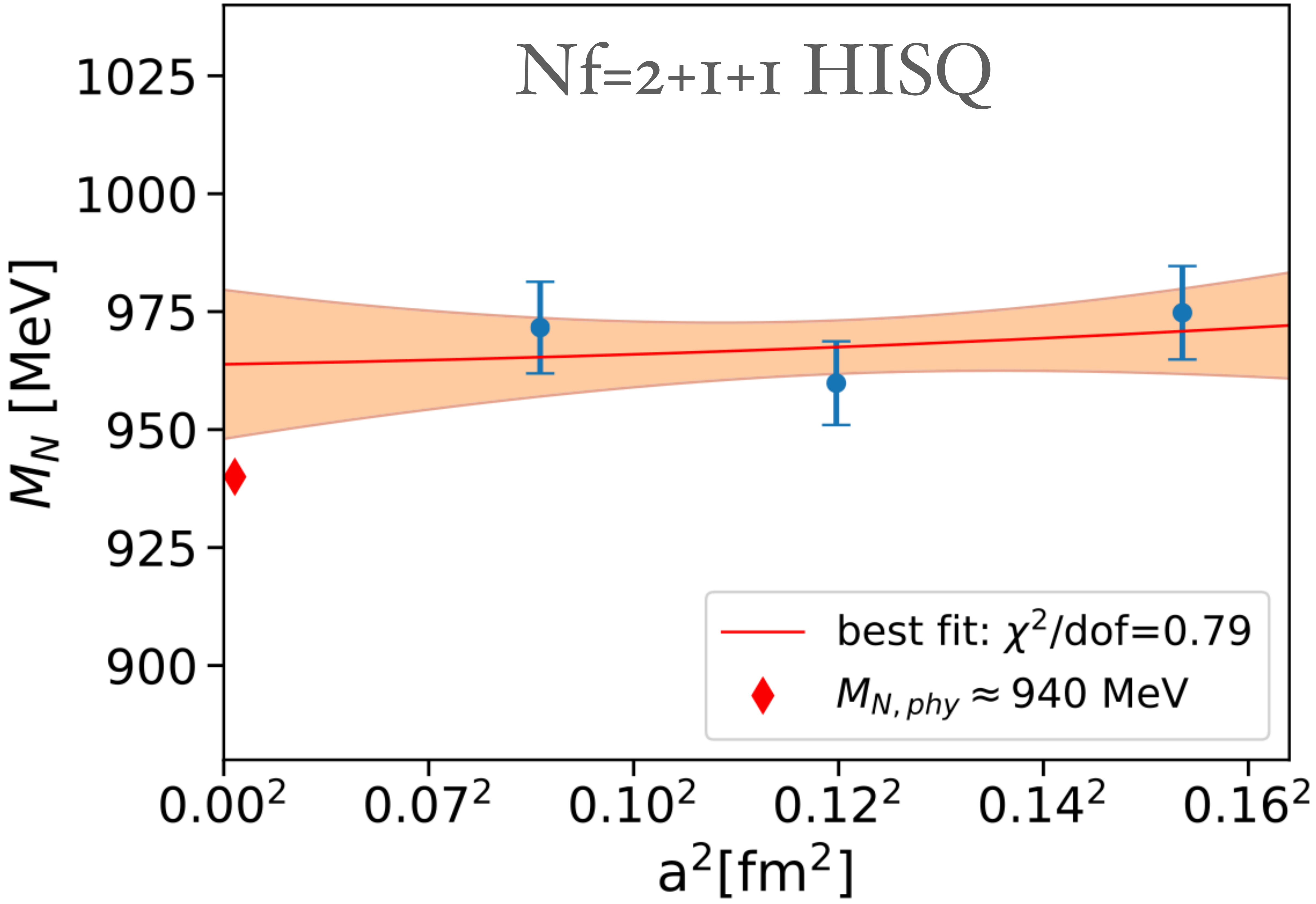
Our analyses provide direct evidence for smooth, analytic thermodynamics at T=108 MeV

Lattice cutoff effects



Yin Lin et al., *Phys.Rev.D* 103 (2021) 3, 034501

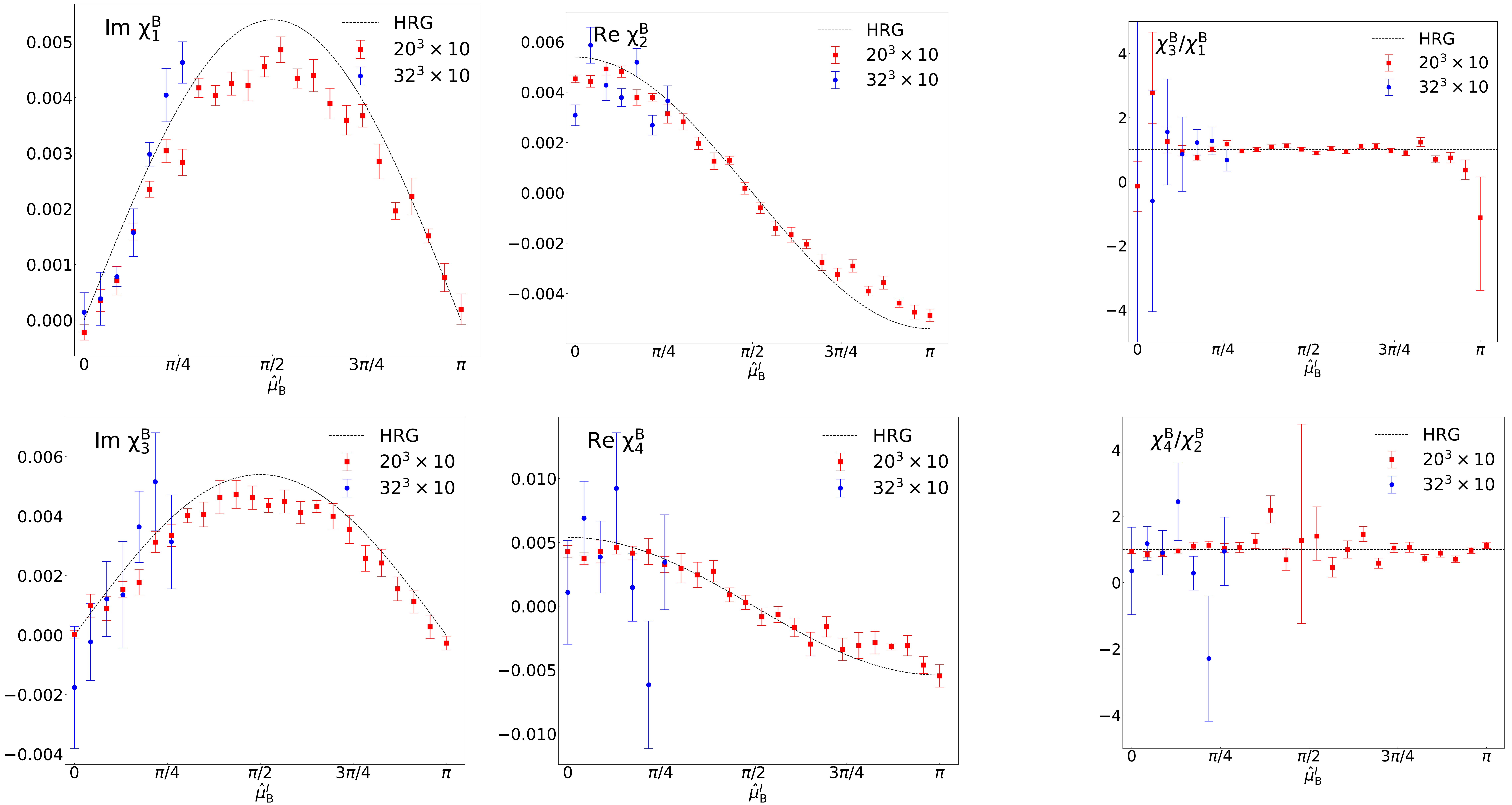
- Nt=10 at T=108 MeV⇒$a \approx 0.18$ *fm*

- proton mass is inflated by about 4-5%, ~980MeV at $a \approx 0.18$ *fm*, as estimated the from the difference in A fit parameter

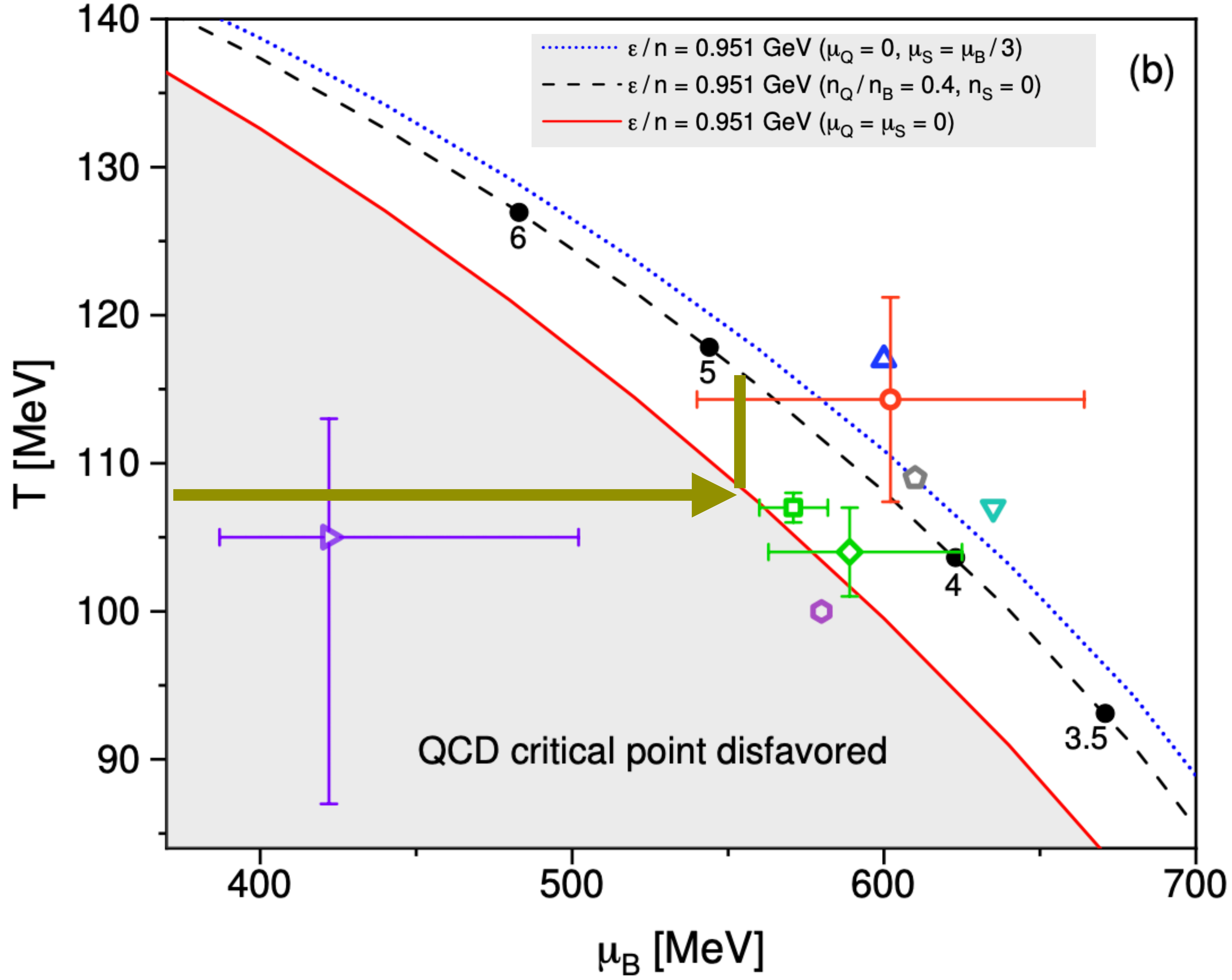
A_{LAT}=4.584(35)
A_{HRG}=5.397(42)

- Consistent with direct computation in Nf=2+1+1 QCD

Volume dependences



Connection to experiments

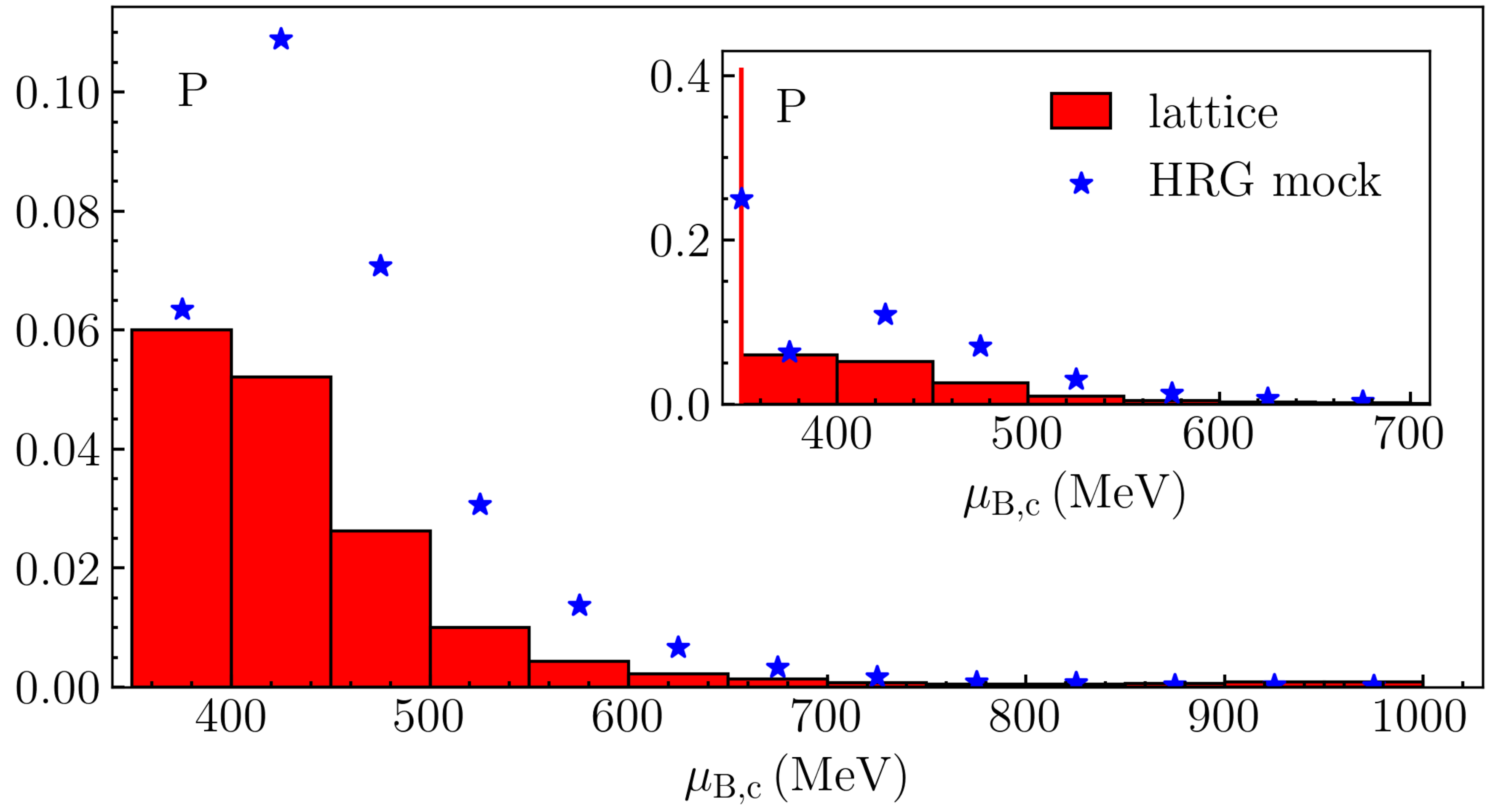


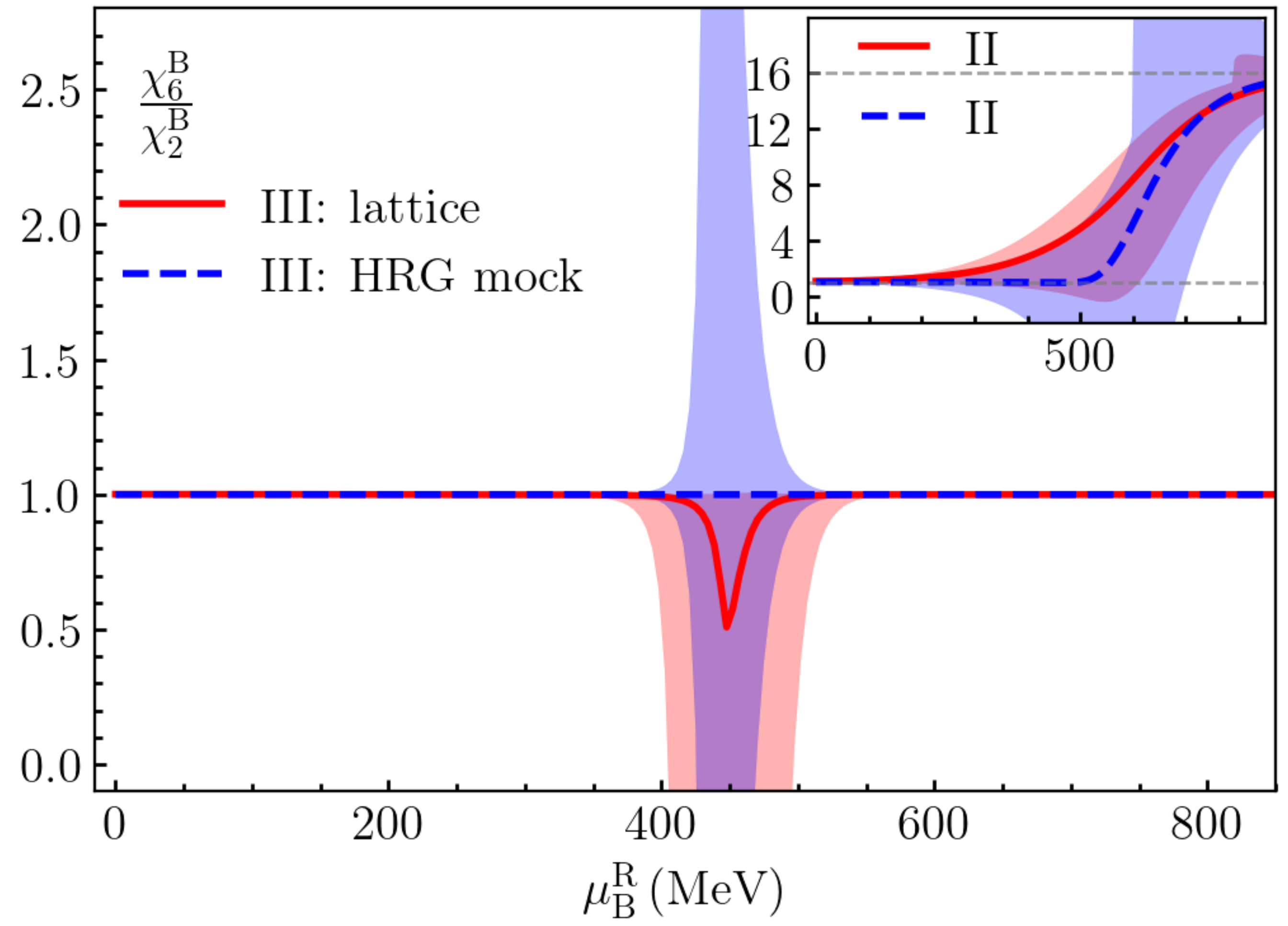
Lysenko, arXiv:2408.06473

- Possible upper bound: $T_c^{\text{CEP}} < 108\ \text{MeV}$
- Indication: if any CEP exists, it could be at $\sqrt{s_{\text{NN}}} \lesssim 5\ \text{GeV}$

Summary

- We have carried out the first LQCD simulations at nonzero imaginary chemical potential directly at a low temperature of 108 MeV and computed χ^B_{1-4}
- By performing fits using direct Padé approximants, physical motivated non-interacting, interacting and critical ansatz, lattice data at 108 MeV is compatible with smooth, analytic thermodynamics
- Although an small probability ~7% allows for a CEP in $\mu^{CEP}_{B,c} \in$ 420-750MeV at T=108 MeV, it does not manifest in the highest baryon susceptibilities that EXP can measure with reasonable precisions
- A possible upper bound of T=108 MeV for CEP, consequently indicating a CEP at $\sqrt{s_{NN}} \lesssim 5$ GeV if it exists





Backup

Multi-point Padé fits

$$R_n^m(x) \equiv \frac{P_m(x)}{1+Q_n(x)} = \frac{\sum_{i=0}^m a_i x^i}{1+\sum_{j=1}^n b_j x^j}, x = \cosh(\mu_B) - 1$$

[3,3] [4,4] [5,5]