Thermodynamics of the hadronic phase and the phase diagram

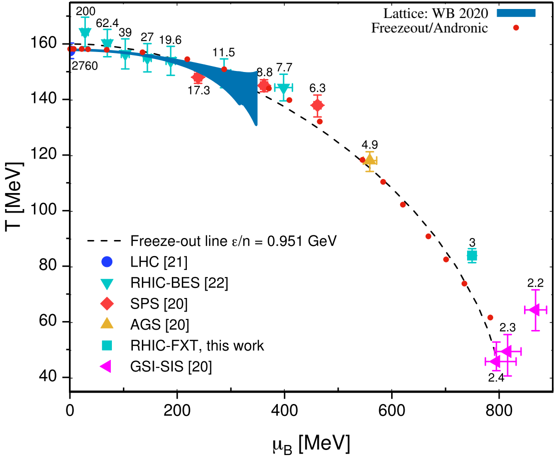
Szabolcs Borsányi

Wuppertal-Budapest collaboration.
A. Adam, Z. Fodor, J. N. Günther, P. Parotto (Torino),
A. Pásztor (Budapest), L. Pirelli K. K. Szabó, C. H. Wong

Bergische Universität Wuppertal

Analytic structure of QCD and Yang-Lee edge singularity
Trento, 8 Sep 2025

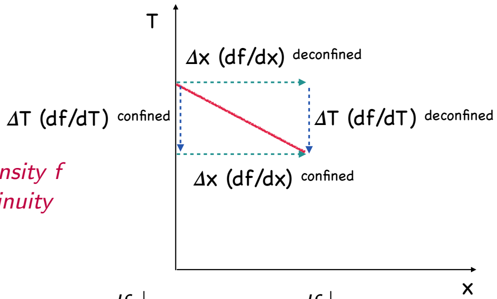
QCD phase diagram



Compiled by [Vovchenko et al 2408.06473]

Lattice result on the chiral transition line:[Wuppertal-Budapest PRL 125 (2020) 052001]

What can Lattice QCD add to this phase diagram?



Free energy density f has no discontinuity

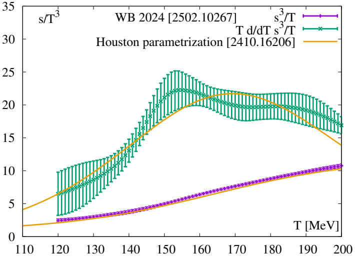
$$\Delta x \left.\frac{df}{dx}\right|_{\text{confined}} + \Delta T \left.\frac{df}{dT}\right|_{\text{confined}} = \Delta x \left.\frac{df}{dx}\right|_{\text{deconfined}} + \Delta T \left.\frac{df}{dT}\right|_{\text{deconfined}}$$

$$-\Delta s = \left.\frac{df}{dT}\right|_{\text{deconfined}} - \left.\frac{df}{dT}\right|_{\text{confined}} \quad ; \quad -\frac{1}{2}\Delta\chi = \left.\frac{df}{dx}\right|_{\text{deconfined}} - \left.\frac{df}{dx}\right|_{\text{confined}}$$

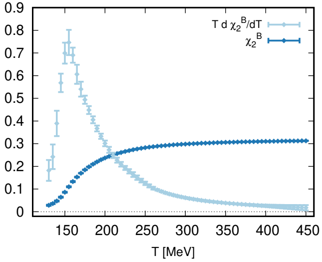
$$\Delta T \Delta s = -\Delta x \frac{1}{2}\Delta\chi$$

$$\kappa := \frac{1}{T}\cdot\frac{\Delta T}{\Delta x} = -\frac{\Delta\chi/T^4}{2\Delta s/T^3}$$

$$\frac{s}{T^3} \qquad T\frac{d}{dT}\frac{s}{T^3} \qquad \chi_2^B \qquad T\frac{d}{dT}\chi_2^B$$



[Wuppertal Budapest [2502.10267]]



[Wuppertal Budapest [2102.06660]]

$$\kappa = -\frac{1}{2}\cdot\frac{\Delta\text{susceptibility}}{\Delta\text{entropy}} \to -\frac{1}{2}\cdot\frac{\frac{d\text{susceptibility}}{dT}}{\frac{d\text{entropy}}{dT}} \approx -\frac{1}{2}\cdot\frac{0.75}{22.3} \approx -0.017$$

- *Taylor coefficients* of the pressure

$$\frac{p(\mu_B)}{T^4} = \frac{p(0)}{T^4} + \frac{1}{2!}\frac{\mu_B^2}{T^2}\chi_2^B(T) + \frac{1}{4!}\frac{\mu_B^4}{T^4}\chi_4^B(T) + \frac{1}{6!}\frac{\mu_B^6}{T^6}\chi_6^B(T) + \ldots$$

- These Taylor coefficients are equal to the Grand Canonical fluctuations

$$\chi_2^B(T) = \left\langle B^2 \right\rangle - \left\langle B \right\rangle^2 = \frac{1}{VT}\frac{\partial^2 \log Z(V, T, \mu_B, \mu_Q, \mu_S)}{\partial {\mu_B}^2}$$

- Higher fluctuations are the Taylor coefficients of lower fluctuations

$$\chi_2^B(\mu_B) = \chi_2^B(\mu_B = 0) + \frac{1}{2!}\frac{\mu_B^2}{T^2}\chi_4^B(\mu_B = 0) + \frac{1}{4!}\frac{\mu_B^4}{T^4}\chi_6^B(\mu_B = 0) + \ldots$$

- Taylor coefficients can be used to reveal analytic structure of the thermodynamic potential
  - Repulsive interactions beyond ideal HRG
  - Searching the critical end point
- Hints for chiral O(4) universality

How to calculate the χ coefficients?

$$\chi_{200}^{uds} = +\langle B_u \rangle + \langle A_u A_u \rangle$$

$$\begin{aligned}\chi_{400}^{uds} = {} & +\langle D_u \rangle + 3\langle B_u B_u \rangle - 3\langle B_u \rangle\langle B_u \rangle + 4\langle A_u C_u \rangle \\ & +\langle A_u A_u A_u A_u \rangle - 3\langle A_u A_u \rangle\langle A_u A_u \rangle + 6\langle A_u A_u B_u \rangle - 6\langle B_u \rangle\langle A_u A_u \rangle\end{aligned}$$

$$\begin{aligned}\chi_{600}^{uds} = {} & +\langle F_u \rangle + 10\langle C_u C_u \rangle + 15\langle B_u D_u \rangle + 15\langle B_u B_u B_u \rangle + 6\langle A_u E_u \rangle + 60\langle A_u B_u \\ & +45\langle A_u A_u B_u B_u \rangle + 20\langle A_u A_u A_u C_u \rangle + 15\langle A_u A_u A_u A_u B_u \rangle + \langle A_u A_u A_u A \\ & -15\langle D_u \rangle\langle A_u A_u \rangle - 45\langle B_u \rangle\langle B_u B_u \rangle - 60\langle B_u \rangle\langle A_u C_u \rangle - 90\langle B_u \rangle\langle A_u A_u B_u \\ & -15\langle B_u \rangle\langle A_u A_u A_u A_u \rangle - 45\langle B_u B_u \rangle\langle A_u A_u \rangle - 60\langle A_u C_u \rangle\langle A_u A_u \rangle - 90\langle A \\ & -15\langle A_u A_u \rangle\langle A_u A_u A_u A_u \rangle + 30\langle B_u \rangle\langle B_u \rangle\langle B_u \rangle + 90\langle B_u \rangle\langle B_u \rangle\langle A_u A_u \rangle \\ & +90\langle B_u \rangle\langle A_u A_u \rangle\langle A_u A_u \rangle + 30\langle A_u A_u \rangle\langle A_u A_u \rangle\langle A_u A_u \rangle\end{aligned}$$

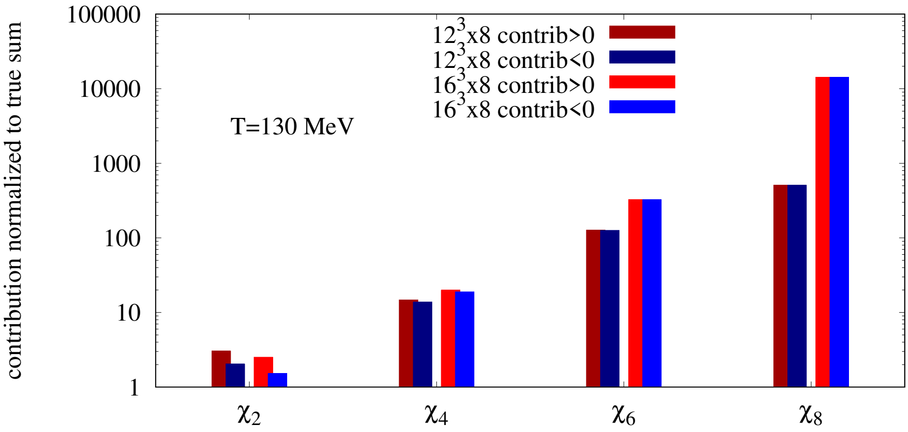
$$\chi_{800}^{uds} = 79 \text{ terms} \ldots$$

$A, B, C, \ldots$ are defined as

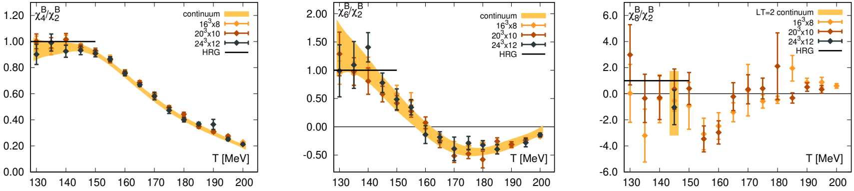
$$[\det M(\mu_u)]^{1/4} = [\det M(0)]^{1/4} \exp\left(1 + A_u \mu_u + \frac{B_u}{2!}\mu_u^2 + \frac{C_u}{3!}\mu_u^3 + \ldots\right)$$

Data analysis uses computer generated code.

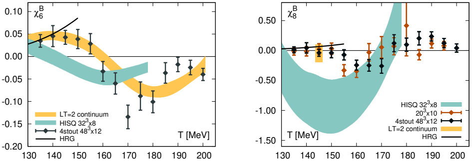
Sign problem in the Taylor coefficients



4Hex continuum result [Phys.Rev.D 110 (2024) 1, L011501]



Comparison with literature



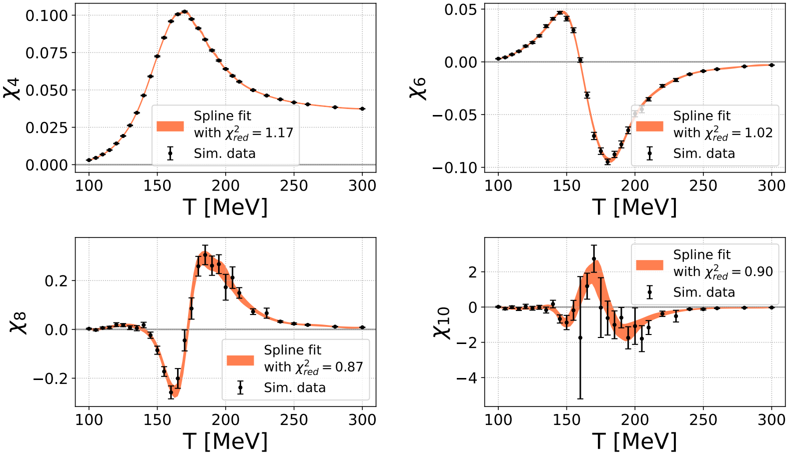
4stout data (imaginary μ_B, $48^3 \times 12$ lattice) : [Wuppertal-Budapest, 1805.04445]

HISQ data: ($\mu_B = 0$, $32^3 \times 8$ lattice, inexact charge conservation) [BNL-Bielefeld, 2202.09184,2212.09043]

$16^3 \times 8$ lattice, 4HEX action, full μ-dependence is recorded.

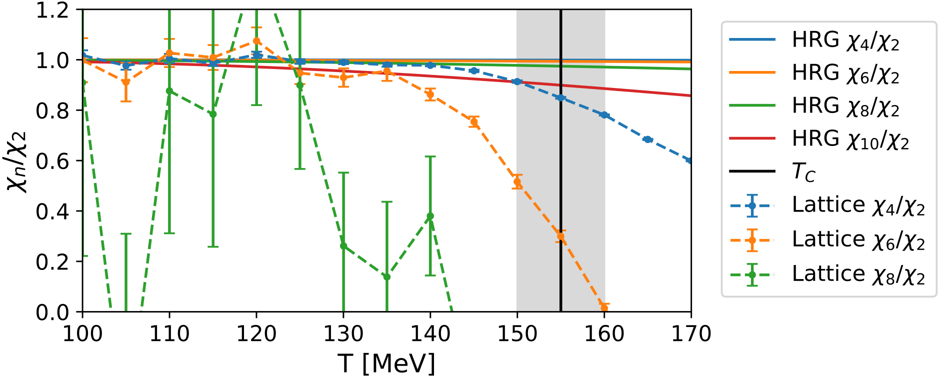
T [MeV]	# configurations at $\mu_B = i\pi(j/8)$								
	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$
100	0.9M	-	-	-	-	-	-	-	-
105	0.9M	-	-	-	-	-	-	-	-
110	0.8M	-	-	-	-	-	-	-	-
115	1.1M	-	-	-	-	-	-	-	-
120	1.1M	-	-	-	-	-	-	-	-
125	1.7M	-	-	-	-	-	-	-	-
130	2.1M	-	96k	-	96k	-	96k	-	96k
135	1.5M	111k	105k	104k	120k	103k	104k	103k	103k
140	4.0M	100k	105k	105k	359k	104k	99k	103k	114k
145	2.6M	107k	106k	105k	122k	104k	103k	102k	115k
150	1.7M	114k	114k	112k	117k	110k	98k	96k	109k
155	1.6M	109k	109k	107k	126k	104k	103k	101k	116k
160	2.2M	112k	111k	109k	128k	104k	96k	101k	109k
165	1.8M	104k	102k	90k	121k	96k	93k	87k	105k
170	1.2M				-				

Baryon cumulant ratios



[Wuppertal-Budapest 2507.13254]

Do we see any differences to the HRG model?

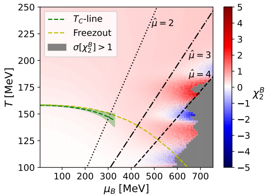


We see a clear signal from baryon interactions for $T \leq 130$ MeV.

[Data from Wupperta-Budapest 2507.13254]

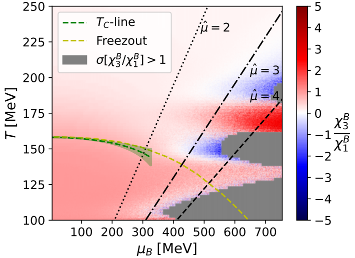
Baryon 2nd cumulant $\chi_2^B(\mu_B)$ to $\mathcal{O}(\mu_B^8)$ order:

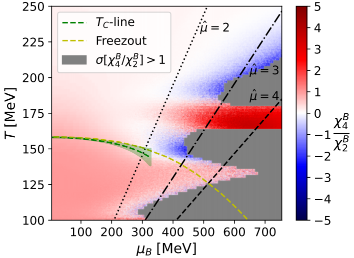
[Wuppertal-Budapest 2507.13254]



Using the baryon Taylor coefficients we express fluctuation ratios, e.g.

$$\frac{\chi_4^B(\mu_B)}{\chi_2^B(\mu_B)} = \frac{\chi_4^B + \frac{\hat{\mu}_B^2}{2!}\chi_6^B + \frac{\hat{\mu}_B^4}{4!}\chi_8^B + \frac{\hat{\mu}_B^6}{6!}\chi_{10}^B}{\chi_2^B + \frac{\hat{\mu}_B^2}{2!}\chi_4^B + \frac{\hat{\mu}_B^4}{4!}\chi_6^B + \frac{\hat{\mu}_B^6}{6!}\chi_8^B + \frac{\hat{\mu}_B^8}{8!}\chi_{10}^B}$$





Stephanov [0809.3450,1104.1627] :

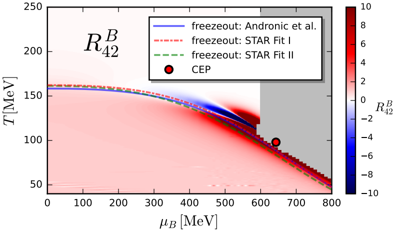
Non-monotonic high order fluctuations near a critical end point.

Detailed predictions for χ_4/χ_2 are given in various setups.

FRG-Lattice assisted LEFT

Fu et al. [2308.15508]

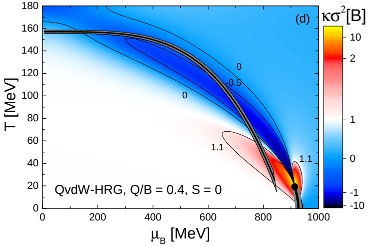
CEP = (98 MeV, 643 MeV)



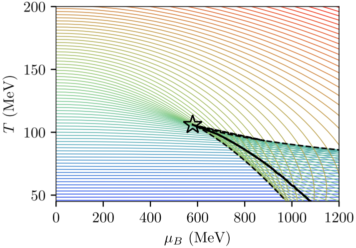
QVdW gas

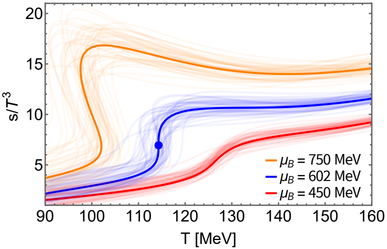
Vovchenko et al [1906.01954]

Liquid-Gas CEP = (19.7 MeV, 922 MeV)



Idea: look at contours of constant entorpy/T^3, find spinodal regions.





[Black-Hole-Engineering model Hippert et al[2309.00579]]

[QCD: Shah et al [2410.16206]]

Leading order: Entropy contours are exact parabolas: $T' = A + B\mu_B^2$

Optimistic assumption on error propagation.

Critical endpoint estimate: $T_c = 114.3 \pm 6.9$ MeV, $\mu_{B,c} = 602.1 \pm 62.1$ MeV

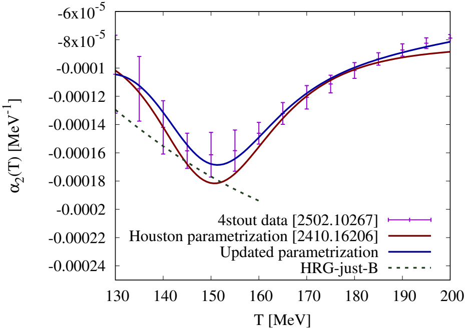
Is this a first principles result on a CEP?

No, an expansion is defined, where
each order can be computed from first principles,
this expansion does not automatically break down near the CEP.

New imlicit T'-expansion for the entropy [Houston]

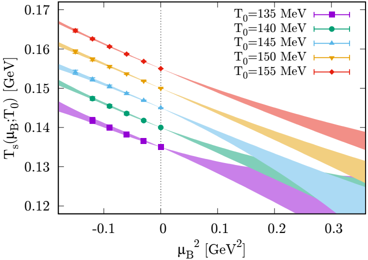
The Houston group introduced a sysematic scheme to expand the entropy [2410.16206]

$$s[T'(\mu_B;T),\mu_B] = s[T,\mu_B=0]$$
$$T'(\mu_B;T) = T+\alpha_2(T)\frac{1}{2}\mu_B^2+\dots$$
$$\mu_{B,c} = \sqrt{-2/\alpha_2'(T_*)}, \qquad \text{where } \alpha_2''(T_*)=0$$



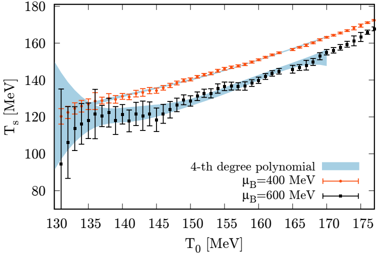
1. Calculate entropy at imaginary μ_B

2. Extrapolate constant s contours to $\mu_B > 0$



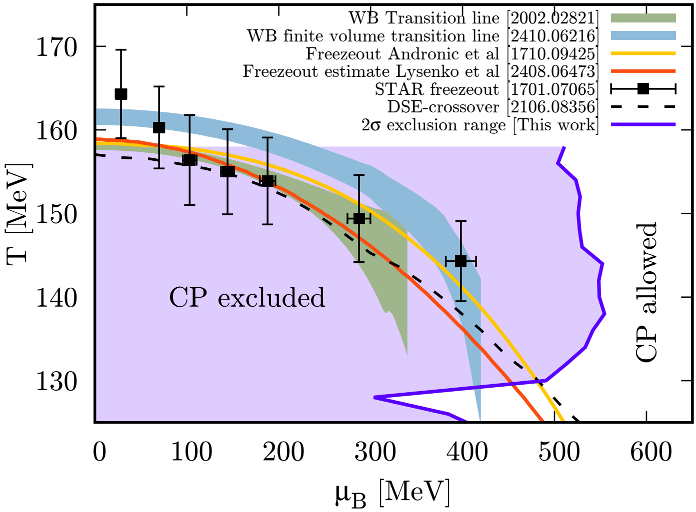
3. Pick a μ_B : $s(T_s, \mu_B) = s(T_0, 0)$

4. $\mu_B \geq \mu_{B,c}$ means multivalued s, that is $T_s(T_1) = T_s(T_2)$.



For how big μ_B can we show that $T_s(T_0)$ is strictly monotonic?
That will be the exclusion range.

This is valid for the strangeness neutral case:



[Wuppertal-Budapest 2502.10267]

Fugacity series

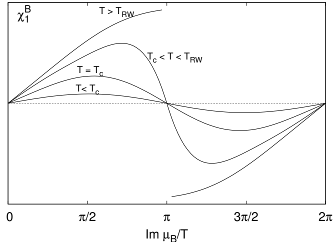
Exact symmetries of the QCD thermodynamic potential $\log Z \approx pV$:

$$p(T,\mu_B) = p(T,\mu_B + i2\pi T) \qquad \textit{Center symmetry}$$
$$p(T,\mu_B) = p(T,-\mu_B) \qquad \textit{Charge conjugation symmetry}$$

With $\hat{p} = p/T^4$ and $\hat{\mu}_B = \mu_B/T$ we can define the fugacity series

$$\hat{p}(T,\hat{\mu}_B) = p_0 + p_1 \cosh(\hat{\mu}_B) + p_2 \cosh(2\hat{\mu}_B) + p_3 \cosh(3\hat{\mu}_B) + \dots$$

which is the Fourier series of the pressure in imaginary μ_B.



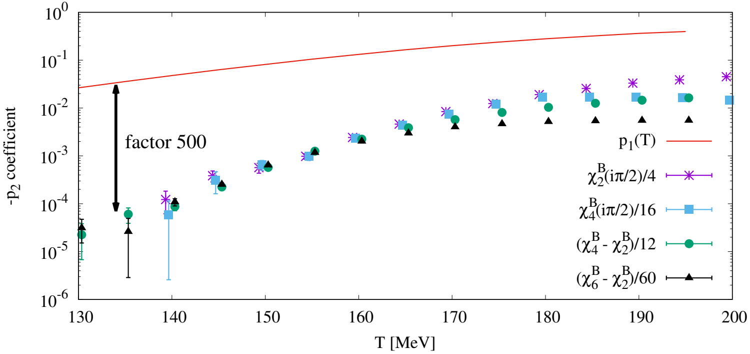
Is this an alternative to Taylor?
No straightforward truncation.

At low T only B states contribute:
$$\chi_1^B \sim \sin(\mu_B/T)$$

At high T fractional charges are required to make a first order transition at $\mu_B = \pi T$:
$$\chi_1^B \sim \sin(\mu_B/T/3)$$

The sub-leading coefficients are small. (In HRG: almost zero)

$$\begin{aligned} \chi_n^B(\hat\mu_B=0) &= p_1 + 2^n p_2 + 3^n p_3 + \ldots \\ \chi_2^B - \chi_4^B &= p_1(1^2-1^4) + p_2(2^2-2^4) \approx -12 p_2 \\ \chi_2^B - \chi_6^B &= p_1(1^2-1^6) + p_2(2^2-2^6) \approx -60 p_2 \\ \chi_2^B(\hat\mu_B = i\pi/2) &= p_1 \cos(\pi/2) + 2^2 p_2 \cos(\pi) + \cdots \approx 4 p_2 \end{aligned}$$

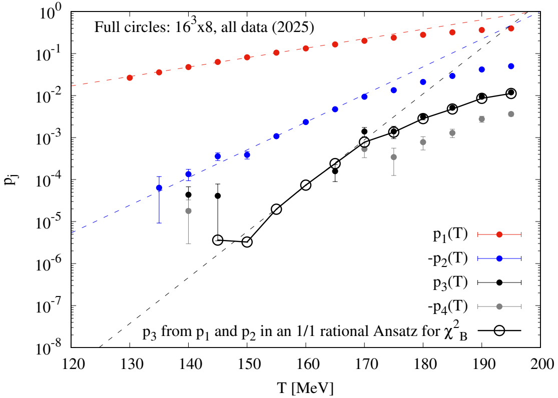


Factor 500 means that we need 6% relative error on χ_6 for 50% error on p_2.

(too) simple rational model:

$$\chi_2^B(\hat\mu_B) \sim \frac{\cosh(\hat\mu_B)}{1+\epsilon(T)\cosh(\hat\mu_B)}\,, \qquad \text{implies} \quad \frac{p_3}{p_2} \approx 1.78\frac{p_2}{p_1}\,, \quad \frac{p_2}{p_1} = -\frac{\epsilon(T)}{4}$$

Other models (e.g. cluster expansion [Vovchenko 1711.01261]) differ in the temperature independent coefficient.



[this result: Wuppertal-Budapest preliminary, see also Vevchenko et al 1708.02852]

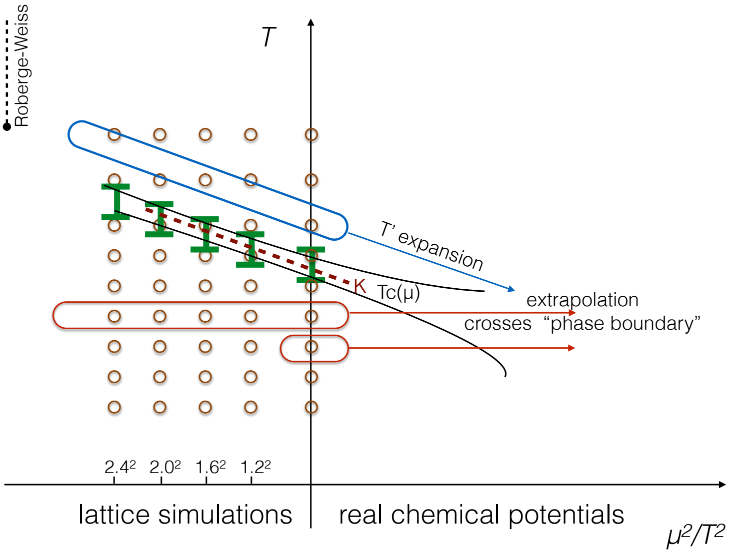
$p_1(T)$: baryon abundance,

$p_2(T)/p_1(T)$: baryon interactions,

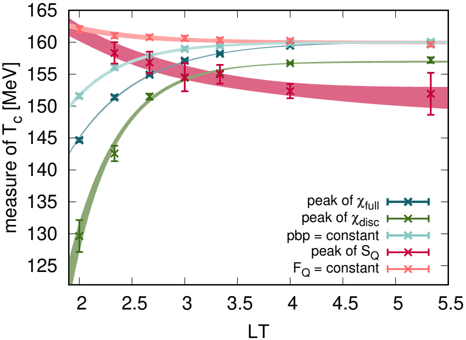
$p_3(T)/p_1(T)$: two-body + three-body effects

Goal: calculate $p_3(T)$ - model prediction.

Extrapolation schemes on the phase diagram



How is physics distorted in a smaller volume?

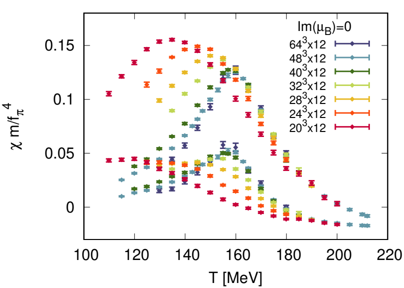


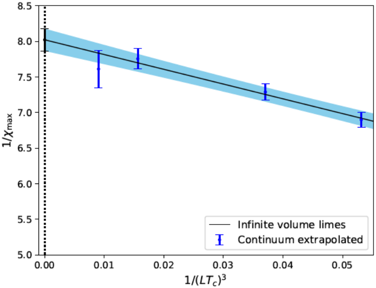
Chiral observables suffer from the reduced volume, but heavy quark observables (e.g. Polyakov loop) are much less affected. *[Wuppertal-Budapest 2405.1232]*

Transition temperature from chiral observables

Upper curves: *Full chiral susceptibility*
Lower curves: *Disconnected part of chiral susceptibility*

$$\chi_{\bar\psi\psi} \sim \frac{\partial^2 \log Z}{\partial m^2}$$





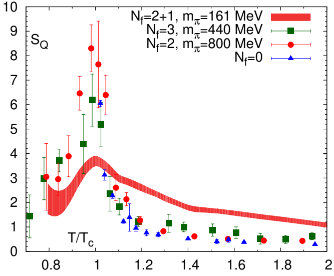
[Wuppertal-Budapest 2405.12320]

$$P = \frac{1}{V}\left\langle \mathrm{Tr} \prod_i U_i \right\rangle$$

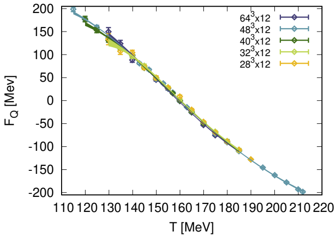
$$F_Q = -T \log P$$

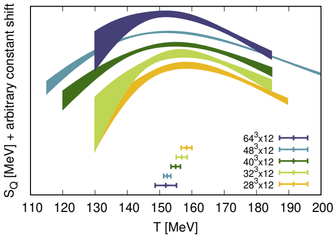
$$S_Q = -\frac{\partial F_Q(T,\mu_B)}{\partial T}$$

T_c : *peak of S_Q in T*



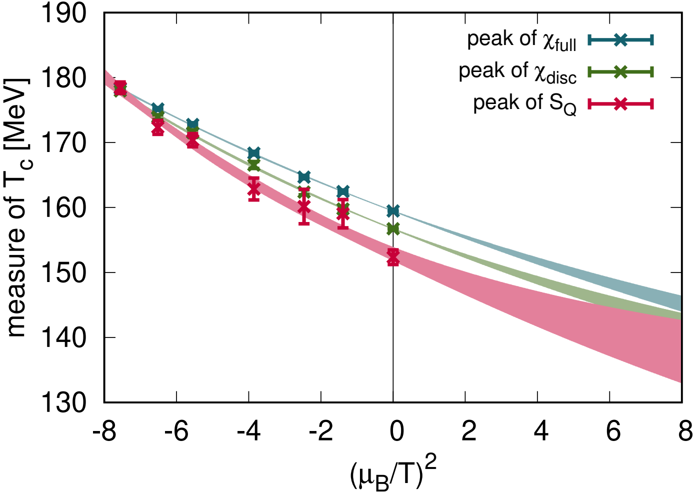
[Left: TUM QCD, Bazavov et al 1603.06637]





[Right: Wuppertal-Budapest 2405.12320]

We repeat the T_c calculation at many imaginary chemical potentials.



This plot: $48^3 \times 12$ *lattice, 4stout*

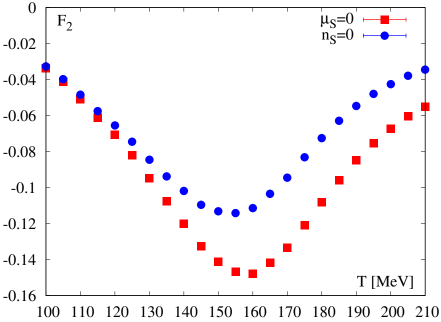
[Wuppertal-Budapest 2405.12320]

The Taylor coefficients for $F_Q(T)$

$$F = -\frac{T}{2}\log|\langle P\rangle|^2 = T\sum_{n=0,2,\dots}\frac{F_n}{n!}\left(\frac{\mu_B}{T}\right)^n$$

$$F_0 = -\frac{1}{2}\log(|P|^2)$$

$$F_2 = -\frac{1}{2|P|^2}\frac{\partial^2}{(\partial\mu_B/T)^2}|P|^2$$



[This result: Wuppertal-Budapest 2410.06216, first computed: D'Elia et al 1907.09461]

Expansion of the Polyakov loop

$$Z = \int \mathcal{D}U e^{-S_g(U)} \cdot \det M_u(U, m_l, \hat{\mu}_u)^{1/4} \cdot \det M_d(U, m_l, \hat{\mu}_d)^{1/4} \cdot \det M_s(U, m_s, \hat{\mu}_s)^{1/4}$$

The Polyakov loop depends on U only, μ_B appears in the determinant only.

$$\begin{aligned} \log\det M_j^{1/4}(U, m_j, \mu_j) &= \log\det M_j^{1/4}(U, m_j, 0) + \overbrace{A_j(U)}^{\text{Imaginary density}} \mu_j \\ &\quad + \frac{1}{2!} B_j(U)\mu_j^2 + \frac{1}{3!} C_j(U)\mu_j^3 + \ldots \end{aligned}$$

To expand a variable X in the chemical potential μ_j we write $\partial_j = \frac{\partial}{\partial \mu_j / T}$

$$\begin{aligned} \partial_j \langle X \rangle &= \langle X A_j \rangle - \langle X \rangle \langle A_j \rangle + \cancel{\langle \partial_j X \rangle} \\ \partial_k \partial_j \langle X \rangle &= \frac{1}{2} \langle X A_j A_k \rangle - \frac{1}{2} \langle X \rangle \langle A_j A_k \rangle - \langle X A_j \rangle \langle A_k \rangle \\ &\quad + \langle X \rangle \langle A_k \rangle \langle A_j \rangle + \cancel{\langle (\partial_k X) A_j \rangle} - \cancel{\langle \partial_k X \rangle} \langle A_k \rangle \\ &\quad + \frac{1}{2} \cancel{\langle \partial_k \partial_j X \rangle} - \frac{1}{2} \langle X \rangle \langle \partial_k A_j \rangle + (j \leftrightarrow k) \end{aligned}$$

$B_j = \partial_j A_j$ and $C_j = \partial_j B_j$ and so on ...

Expansion of the Polyakov loop in imaginary μ

$$P = P_R + P_I$$

Magenta color: real and C-even.
Blue color: real and C-odd. Imagine ∂_j as imaginary-μ derivative!
A straightforward application of these rules yields up to second order

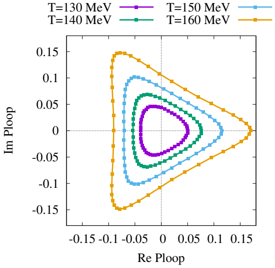
$$\begin{aligned}
\partial_j \langle P_R \rangle|_{\mu\equiv 0} &= 0\,, \\
\partial_j \langle P_I \rangle|_{\mu\equiv 0} &= \langle A_j P_I \rangle\,, \\
\partial_j \partial_k \langle P_R \rangle|_{\mu\equiv 0} &= \delta_{jk} \left[\langle B_j P_R \rangle - \langle B_j \rangle \langle P_R \rangle \right] \\
&\quad \langle A_j A_k P_R \rangle - \langle A_j A_k \rangle \langle P_R \rangle\,, \\
\partial_j \partial_k \langle P_I \rangle|_{\mu\equiv 0} &= 0\,.
\end{aligned}$$

These can be combined into the following relation *(all terms are real, even)*:

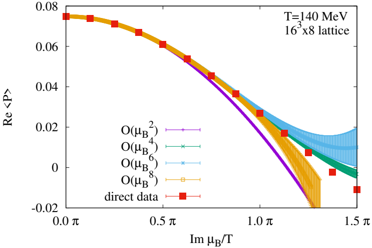
$$\begin{aligned}
\frac{1}{2} \partial_j \partial_k |\langle P \rangle|^2 &= +\delta_{jk} \left[\langle P_R \rangle \langle B_j P_R \rangle - \langle P_R \rangle^2 \langle B_j \rangle \right] \\
&\quad + \langle P_R \rangle \langle A_j A_k P_R \rangle - \langle A_j P_I \rangle \langle A_j P_I \rangle \\
&\quad - \langle A_j A_k \rangle \langle P_R \rangle^2\,,
\end{aligned}$$

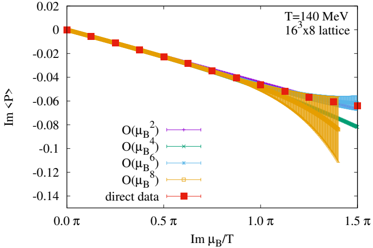
Expansion of the Polyakov loop in imaginary μ

$$P = P_R + P_I$$



$$
\begin{aligned}
\partial_j \langle P_R \rangle \big|_{\mu\equiv 0} &= 0\,, \\
\partial_j \langle P_I \rangle \big|_{\mu\equiv 0} &= \langle A_j P_I \rangle\,, \\
\partial_j \partial_k \langle P_R \rangle \big|_{\mu\equiv 0} &= \delta_{jk}\left[\langle B_j P_R \rangle - \langle B_j \rangle \langle P_R \rangle\right] \\
& \quad \langle A_j A_k P_R \rangle - \langle A_j A_k \rangle \langle P_R \rangle\,, \\
\partial_j \partial_k \langle P_I \rangle \big|_{\mu\equiv 0} &= 0\,.
\end{aligned}
$$





Varying the imaginary μ_B, P_R stays real, P_I stays imaginary.

Expansion of the Polyakov loop in real μ

$$P = P_R + P_I$$

Magenta color: C-even (real or imaginary).
Blue color: C-odd (real or imaginary)

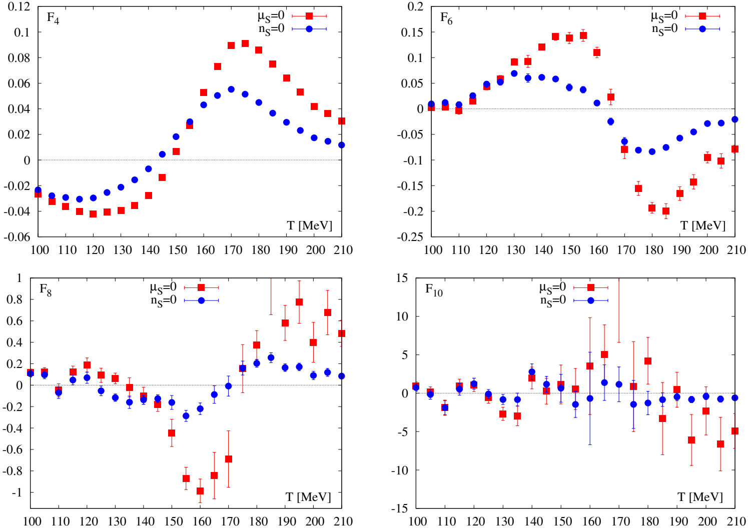
$$\begin{aligned}
\partial_j \langle P_R \rangle|_{\mu\equiv 0} &= 0\,, \\
\partial_j \langle P_I \rangle|_{\mu\equiv 0} &= \langle A_j P_I \rangle\,, \\
\partial_j \partial_k \langle P_R \rangle|_{\mu\equiv 0} &= \delta_{jk}\left[\langle B_j P_R \rangle - \langle B_j \rangle \langle P_R \rangle\right] \\
&\quad \langle A_j A_k P_R \rangle - \langle A_j A_k \rangle \langle P_R \rangle\,, \\
\partial_j \partial_k \langle P_I \rangle|_{\mu\equiv 0} &= 0\,.
\end{aligned}$$

If ∂_j is a **real chemical potential** derivative:

$$\langle P_I(\mu_B) \rangle = \underbrace{\langle A_j P_I(\mu_B = 0) \rangle}_{\text{real}} \mu_B + \ldots\,,$$

$\langle P_I \rangle$ was meant to be the imaginary part, but in fact,
it has a **real expectation value***, while being C-odd.*

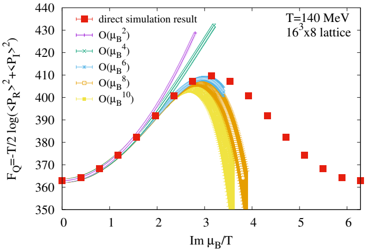
Higher orders:

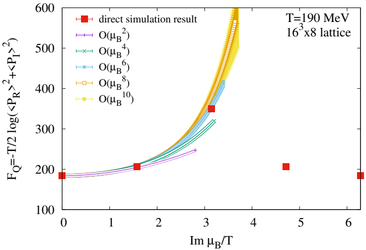


[Wuppertal-Budapest 2410.06216]

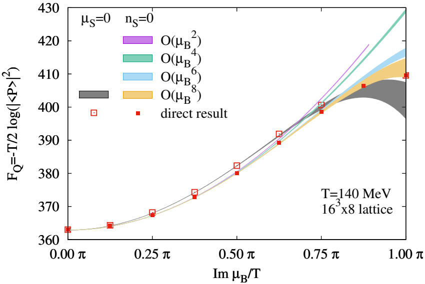
Testing the expansion at imaginary μ_B

Two temperatures in the $\mu_S = 0$:

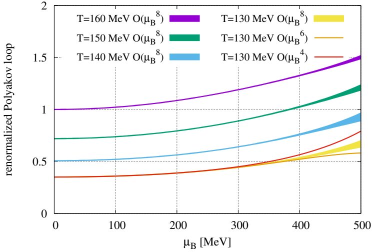


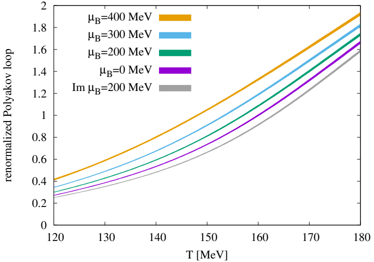


Testing the strangeness neutral extrapolation:



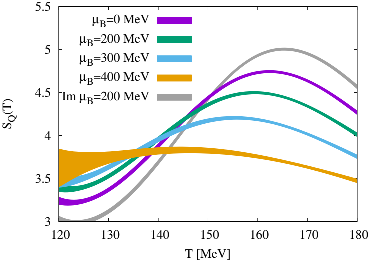
The renoramalized Polyakov loop:





$$F_Q = -\frac{T}{2}\log(|P|^2)$$

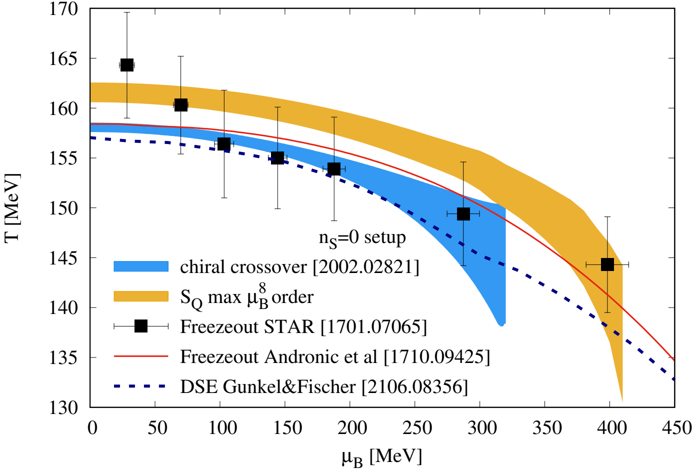
$$S_Q = -\frac{\partial F_Q(T)}{\partial T}$$



[Wuppertal-Budapest 2410.06216]

T_c extrapolated to eighth order

S_Q itself can be extrapolated *($16^3 \times 8$ lattices)*.
The temperature of the S_Q-peak is calculated for each μ_B.



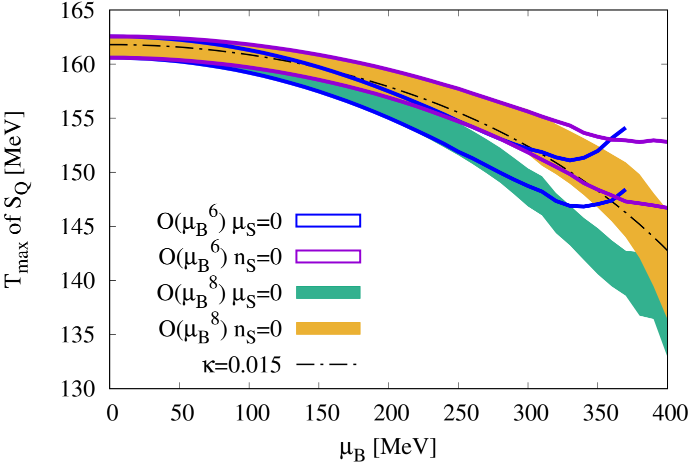
[Wuppertal-Budapest 2410.06216]

T_c extrapolated to eighth order

F_Q is extrapolated *($16^3 \times 8$ lattices)*.

Strangeness neutral ($n_S = 0$) and normal ($\mu_S = 0$) extrapolations differ.

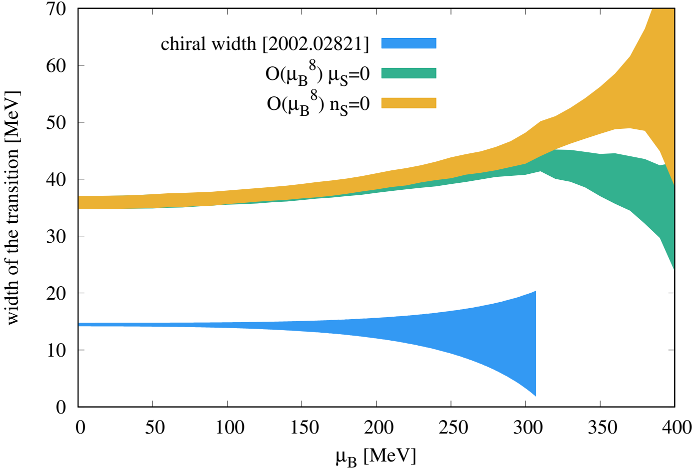
The temperature of the S_Q-peak is calculated for each μ_B for both cases.



[Wuppertal-Budapest 2410.06216]

Width of the transition extrapolated to eight order

S_Q itself can be extrapolated *($16^3 \times 8$ lattices)*.
The **width** of the S_Q-peak is calculated for each μ_B.



[Wuppertal-Budapest 2410.06216]

Besides light baryons hyperons are also generated with $\mu_B > 0$: $\Longrightarrow \langle S \rangle < 0$

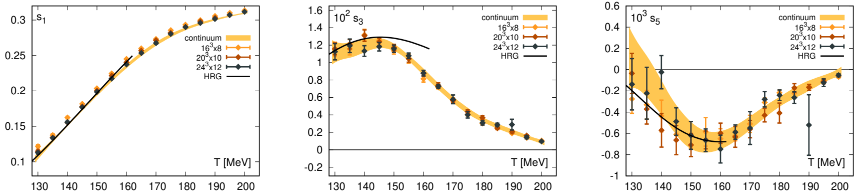
In experiment (at chemical freeze-out) $\langle S \rangle = 0$.
We achieve this by adding $\mu_S > 0$, this is T and μ_B dependent.

$$\mu_S^*(\mu_B, T) = s_1(T)\mu_B + s_3(T)\mu_B^3 + s_5(T)\mu_B^5 + \dots$$

One obtains $s_1(T)$, $s_3(T)$ and $s_5(T)$ from the standard Taylor coefficients

[HotQCD 1208.1220; 1701.04325]

Our recent continuum results [Wuppertal-Budapest 2312.07528]



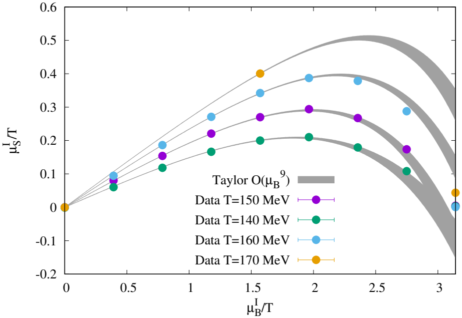
Strangeness neutrality in a crosscheck

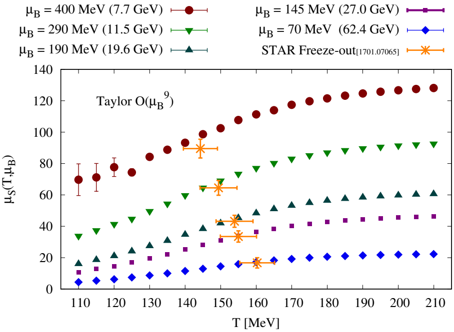
Besides ligth baryons hyperons are also generated with $\mu_B > 0$: $\Longrightarrow$ $\langle S \rangle < 0$

In experiment (at chemical freeze-out) $\langle S \rangle = 0$.

We achieve this by adding $\mu_S > 0$, this is T and μ_B dependent.

$$\mu_S^*(\mu_B, T) = s_1(T)\mu_B + s_3(T)\mu_B^3 + s_5(T)\mu_B^5 + s_7(T)\mu_B^7 + s_9(T)\mu_B^9 + \ldots$$





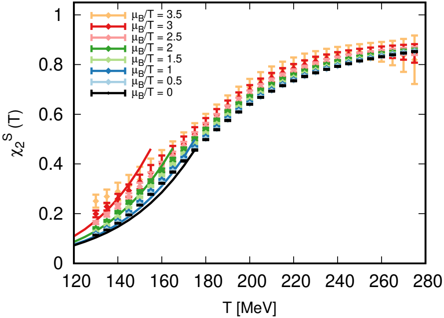
Wuppertal-Budapest preliminary data, $16^3 \times 8$ lattice.

A naive extrapolation of $\chi_2^S(\mu_B)$ is governed by the coefficients $\chi_{n,2}^{BS}(T)$.
With strangeness neutrality, however

$$\chi_2^S(\mu_B,\mu_S^*(\mu_B)) = \chi_2^S + \left(\chi_{22}^{BS} + 2\chi_{13}^{BS} s_1(T) + \chi_{04}^{BS}(s_1(T))^2\right)\hat{\mu}_B^2 + \dots$$

Alternatively, one can work in the T' expansion scheme

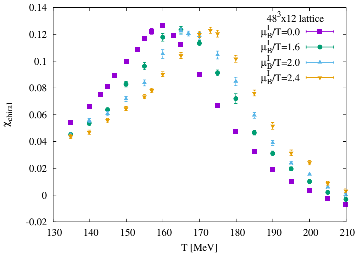
$$\chi_2^S(\mu_B,\mu_S^*(\mu_B)) = f\left(T(1+\lambda_{SS}\hat{\mu}_B^2 + \dots)\right)$$

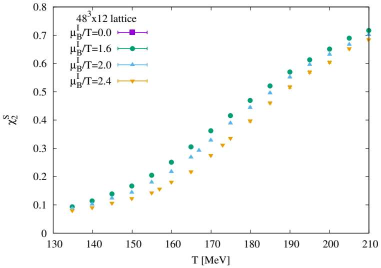


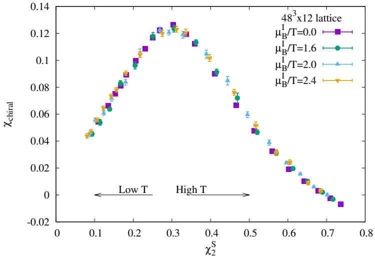
[Wuppertal-Budapest 2202.05574]

An observation about T_c and strangeness fluctuations

- Chiral susceptibility peaks at T_c
- The peak shifts with μ_B as expected
- Strange susceptibility $\chi_2^S(T)$ is monotonic in T
- *Collapse plot:* chiral susceptibility as a function of starngeness susceptibility is μ_B independent



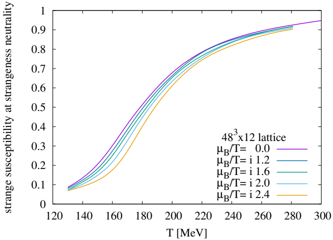


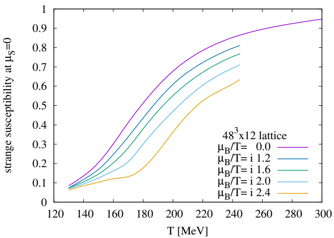


Lession from imaginary μ_B: $\chi_2^S \approx 0.3$ marks T_c for various μ_B

Strangeness susceptibility as a proxy for T_c

This behaviour is specific to the strangeness neutral case:

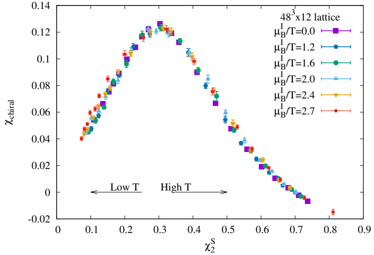


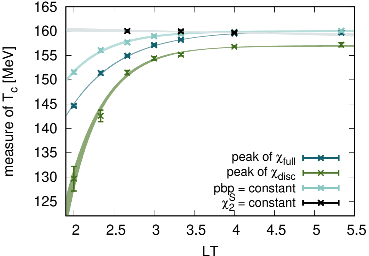


Observation at **zero and imaginary** μ_B:

The chiral susceptibility has a peak where $\chi_2^S(T_c) \approx 0.3$

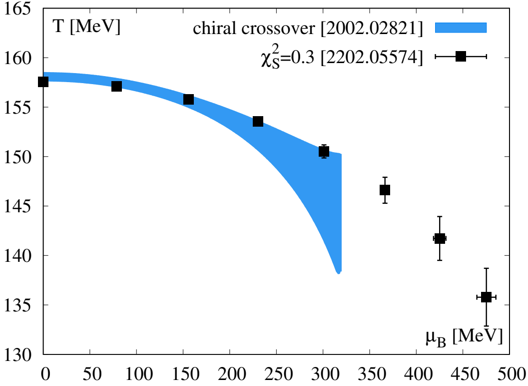
This proxy of T_c has very small volume dependence.





Constant $\chi_2^S(T, \mu_B)$ contour vs the crossover line

We can check the agreement between the $\chi_2^S = 0.3$ contour and the extrapolated chiral transition line with **published continuum data**.



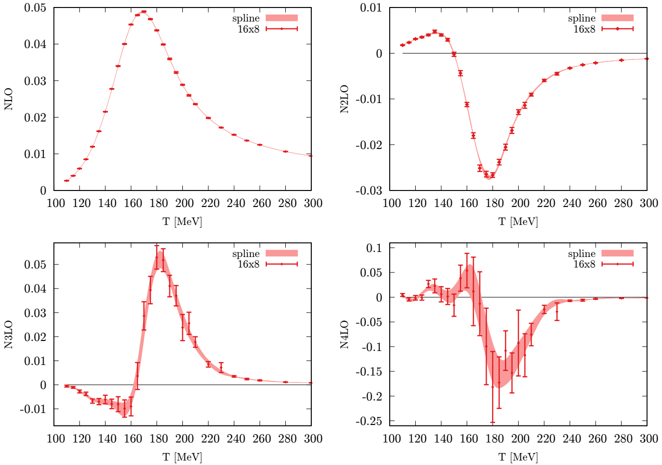
There is a small tension between the curvatures, but line is within 1 σ.

$\chi_2^S(T)$ is a result of NLO T' expansion

T_c (chiral) is the cubic polynomial extrapolation of the peak of $\chi_{\bar{\psi}\psi}$.

Taylor expansion coefficients with strangeness neutrality

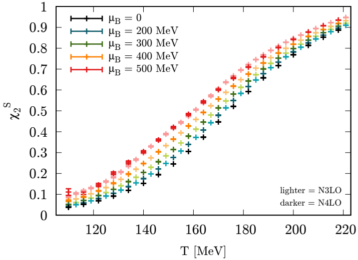
$$\chi_2^S(\mu_B, \mu_S^*(\mu_B)) = \chi_2^S + \underbrace{\left(\chi_{22}^{BS} + 2\chi_{13}^{BS} s_1(T) + \chi_{04}^{BS}(s_1(T))^2\right)}_{\chi^S_{2,NLO}} \hat{\mu}_B^2 + \ldots$$

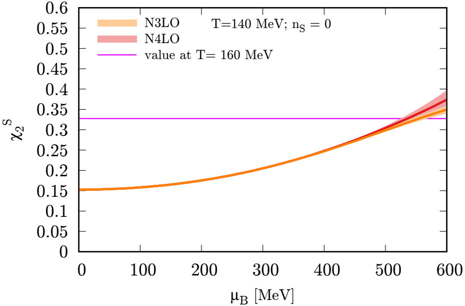


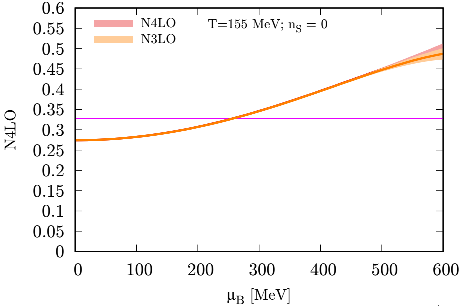
$16^3 \times 8$ lattice, 4HEX, (Wuppertal-Budapest)

Strangeness susceptibility extrapolated

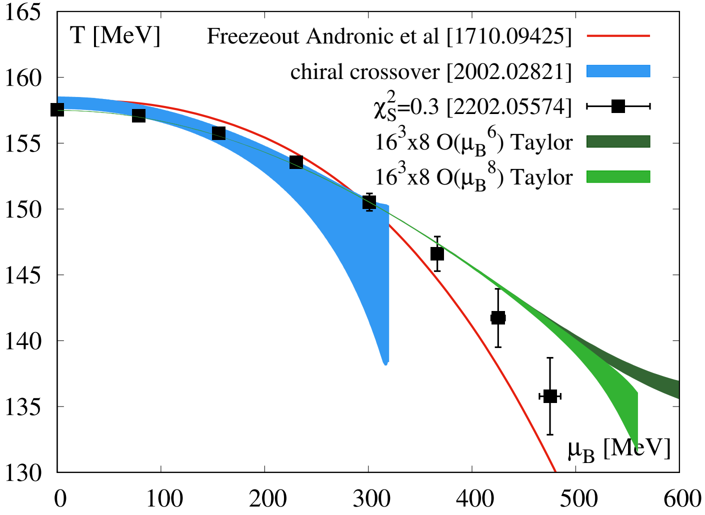
Taylor extrapolation from $\mu_B = 0$ of χ_2^S:





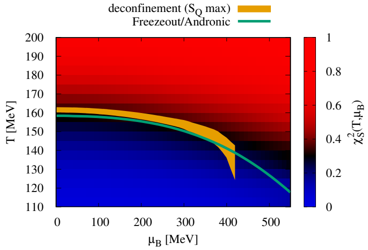


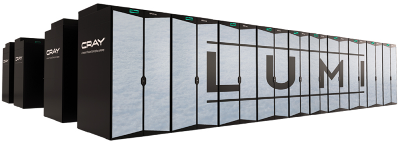
Strangeness contour with high statistics Taylor coefficients



It is still very expensive to access high density physics from the lattice.

- Continuum extrapolated high order coefficients
- Extreme statistics on $16^3 \times 8$ lattices
  ($\frac{1}{2}$ years in Jülich $+1$ year on LUMI)
  one can attempt to extrapolate to so far unattainable parts of the phase diagram.
  (today only on coarse lattices)



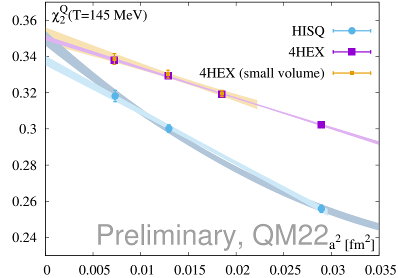


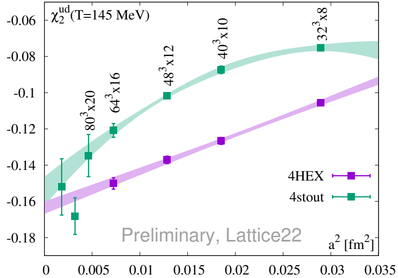
Testing continuum limits with the 4HEX action

4HEX staggered action with strongly reduced taste breaking.

Let's look at those fluctuations e.g. chanrge, that is sensitive to it.

Continuum extrapolation $T = 145$ MeV with large volume up to $N_t = 16$





4STOUT: Wuppertal-Budapest (2013–2023 [Wuppertal-Budapest [1507.04627]]

HISQ: BNL-Bielefeld (2011–. . .) [HotQCD [2107.10011]]

4HEX: Wuppertal-Budapest (2022–. . .) [Wuppertal-Budapest **QM2022**]