SUMMARY & DISCUSSION: Hamiltonian Lattice Gauge Theories: Status, Novel Developments and Applications

K. Jansen, R. Lewis, G. Magnifico, E. Rico Ortega, S. Romiti, C. Urbach

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General impression

Positive impression

- Very interesting workshop, many topics touched in the talks, useful discussions during the breaks
- Overview on the state of the art, limitations, new ideas
- New ideas both at the formulation level as well as the algorithmic one

Facing reality

- Clearer statements about the limitations about phenomenological applications
- Most applications limited to U(1).
- New ideas for SU(N): large N truncation, tailored basis of functions (g-dependence), AI, QLM
- Exponential scaling of Hilbert space: not solved, but mitigating techniques proposed

Opening questions

What are the methods and algorithms available for quantum and tensor network gauge field simulations in the Hamiltonian formulation?

Methods

Mostly KS Hamiltonian:

- Loop-string-hadron, gauge invariant bases
- Orbifold + ΔH converging to KS in $m \to \infty$ limit

Quantum Link Models

Algorithms

- ED (good for checking simple systems)
- TN (massive reduction of Hilbert space size, low entanglement)
- AI (variational eigensolver, learning overlap with ground state)

The Orbifold Approach

- exponential speedup due to Cartesian coordinates?
- MC test in SU(2) and SU(3) w/o discretisation looks very promising [Bergner, Hanada (2025)]
- what about gauge invariance breaking w/ necessary discretisation
- overhead of unneeded states
  structurally very similar to proposal Romiti, Urbach (2024)
- are there $\mathcal{O}(a^2m^2)$ artefacts in observables with vacuum quantum numbers?

General comments

Technology (Quantinuum)

- Possibility to write programs, open source library
- Fault-tolerant machine by 2029
- Not clear how they encode them (industrial secret)
- Possible overhead from gates applications

Orbifold formulation

Controversial, should be discussed further

Opening questions

what are the advantages and disadvantages of quantum and tensor network methods, i.e. in which situation is their application preferable?

Low dimensional systems

- Efficient applications seem limited to 1+1 systems, especially for time evolution and non-abelian groups

Mitigating scaling of effective Hilbert space size

- Quantitative arguments on entaglement justify modest bond dimensions
- Room for improvement by using different fermion discretizations. TM fermions also reducing lattice artifacts

Opening questions

is there a multi-grid approach possible in the Hamiltonian formalism?
Does not seem to be the case...

which phenomenologically relevant applications are in reach for current quantum devices and tensor network algorithms?
None for the moment, restricted to very small systems

can the results from Hamiltonian simulations and large scale Monte Carlo simulations be combined to obtain physically relevant results for experiments?
Maybe, one should be careful in finding the Matching point: Hamiltonian limit $a_t \to 0$ is delicate

Opening questions

how to perform non-perturbative renormalisation in the Hamiltonian formulation?
No true answer from the talks

can non-perturbative improvement be implemented?
Tadpole improvement

Interesting insights (biased!)

- Important to have a quantitative estimate of energy scale of interest
- Gauge invariance checked frequently in time evolution confines the state in the gauge-invariant sector (Zeno effect)
- Quantitative statement about the truncation needed
- Diagonalization by using spatially separated operators: 1- and 2- point functions

DJT and Orbifold

Probably similar, they share:

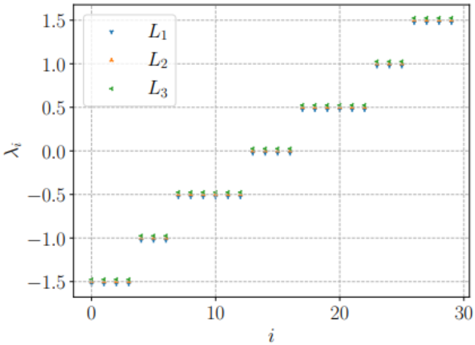
- Large number of points, but most discarded in the bulk
- Necessarily evenly distributed points
- Well-defined procedure to project out the "garbage" states (mathematically exact)

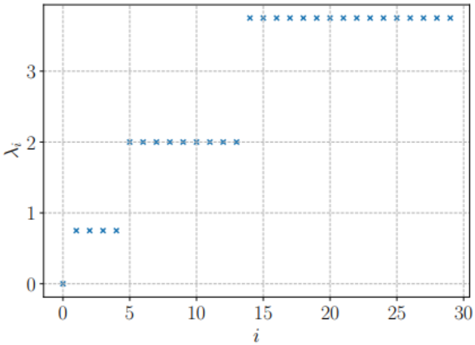
$$N_\alpha = \begin{cases} (q+1)\cdot(4q+1)\cdot(4q+1)\,, & \text{if } q\in\mathbb{N}\,, \\ (q+1/2)\cdot(4q+1)\cdot(4q+1)\,, & \text{otherwise}\,, \end{cases}$$

DJT and Orbifold

$$\theta_a\,,\; a = 1, \ldots, N_\theta$$
$$\phi_b = \frac{4\pi}{N_\phi} b\,,\; b = 1, \ldots, N_\phi\,,$$
$$\psi_c = \frac{4\pi}{N_\psi} c\,,\; c = 1, \ldots, N_\psi\,.$$

$$L_a \to L_a + P_g \left[\mathbb{1}_{N_q \times N_q} \oplus (\tau_a \otimes \mathbb{1}_{\frac{N_r}{2} \times \frac{N_r}{2}}) \right] P_g.$$





DJT and Orbifold

$$
\begin{aligned}
\left(\sum_a R_a^2\right)|j,m_L,m_R\rangle = \left(\sum_a L_a^2\right)|j,m_L,m_R\rangle = j(j+1)|j,m_L,m_R\rangle\,,\\
L_3|j,m_L,m_R\rangle = m_L|j,m_L,m_R\rangle\,,\\
R_3|j,m_L,m_R\rangle = -m_R|j,m_L,m_R\rangle\,,\\
(L_1 \pm iL_2)|j,m_L,m_R\rangle = \sqrt{j(j+1)-m_L(m_L\pm 1)}|j,m_L\pm 1,m_R\rangle\,,\\
(R_1 \mp iR_2)|j,m_L,m_R\rangle = -\sqrt{j(j+1)-m_R(m_R\pm 1)}|j,m_L,m_R\pm 1\rangle\,.
\end{aligned}
$$