Renormalized dual basis for scalable simulations of (2+1)D compact quantum electrodynamics

Marc Miranda-Riaza, Pierpaolo Fontana, Alessio Celi

Trento, 4th September 2025

Lagrangian formulation

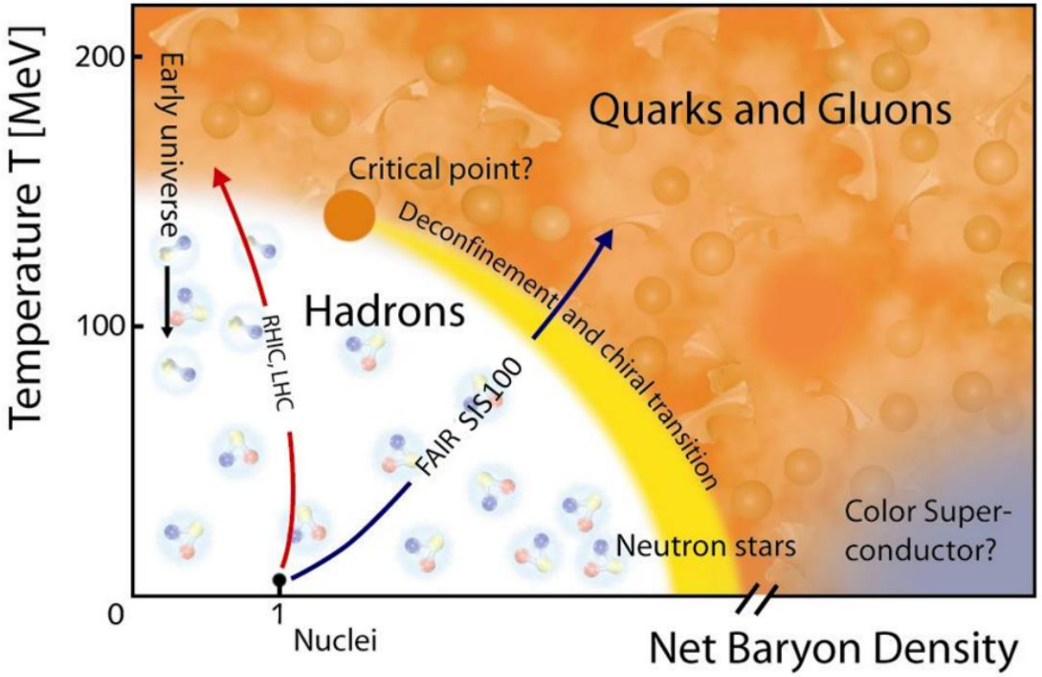
Studying LGTs with Monte Carlo:

➢ Finite baryon density

➢ Real-time evolution

Sign problem!

[Creutz, *Quarks, Gluons and Lattices* (1983)]
[Rothe, *Lattice Gauge Theories: An Introduction* (1992)]



[Durante et al. Phys. Scr. **94** (2019)]

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Hamiltonian formulation (U(1))

$$[U_\mu(\boldsymbol{n}), E_\nu(\boldsymbol{m})] = -\delta_{\mu\nu}\delta_{\boldsymbol{mn}} U_\mu(\boldsymbol{n})$$

- Electric field Hamiltonian:

$$H_E = \frac{g^2}{2}\sum_{\boldsymbol{n},\hat{\mu}} E_\mu^2(\boldsymbol{n})$$

- Magnetic field Hamiltonian

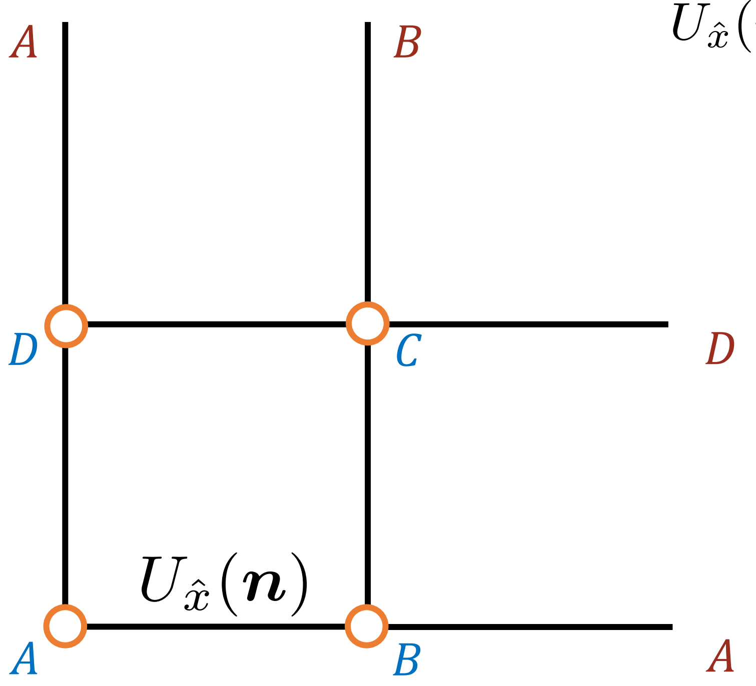
$$H_B = -\frac{1}{4g^2}\sum_P (U_P^\dagger + U_P)$$

$$U_P \equiv U_{\hat{x}}(\boldsymbol{n})U_{\hat{y}}(\boldsymbol{n}+\hat{x})U_{\hat{x}}^\dagger(\boldsymbol{n}+\hat{y})U_{\hat{y}}^\dagger(\boldsymbol{n})$$

g – Coupling strength

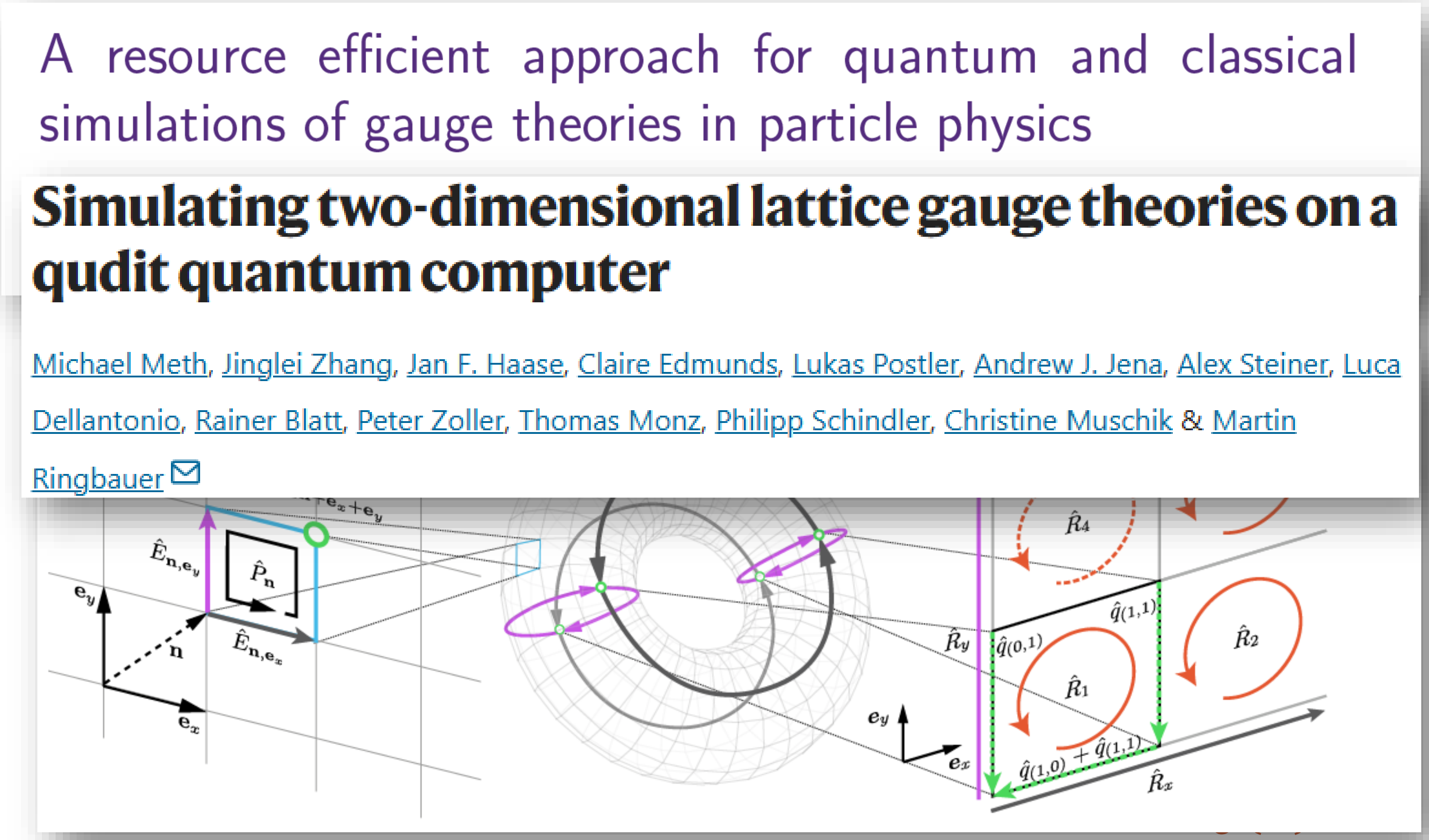
Possible gauge groups G: U(1), U(N), SU(N), Sp(N),...

$U_{\hat{x}}(\boldsymbol{n}) \in G$



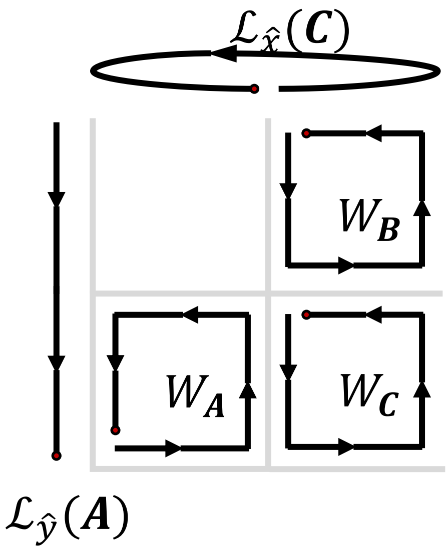
[Kogut & Susskind, Phys. Rev. D **11** (1975)]

Reformulation


A resource efficient approach for quantum and classical simulations of gauge theories in particle physics

Simulating two-dimensional lattice gauge theories on a qudit quantum computer

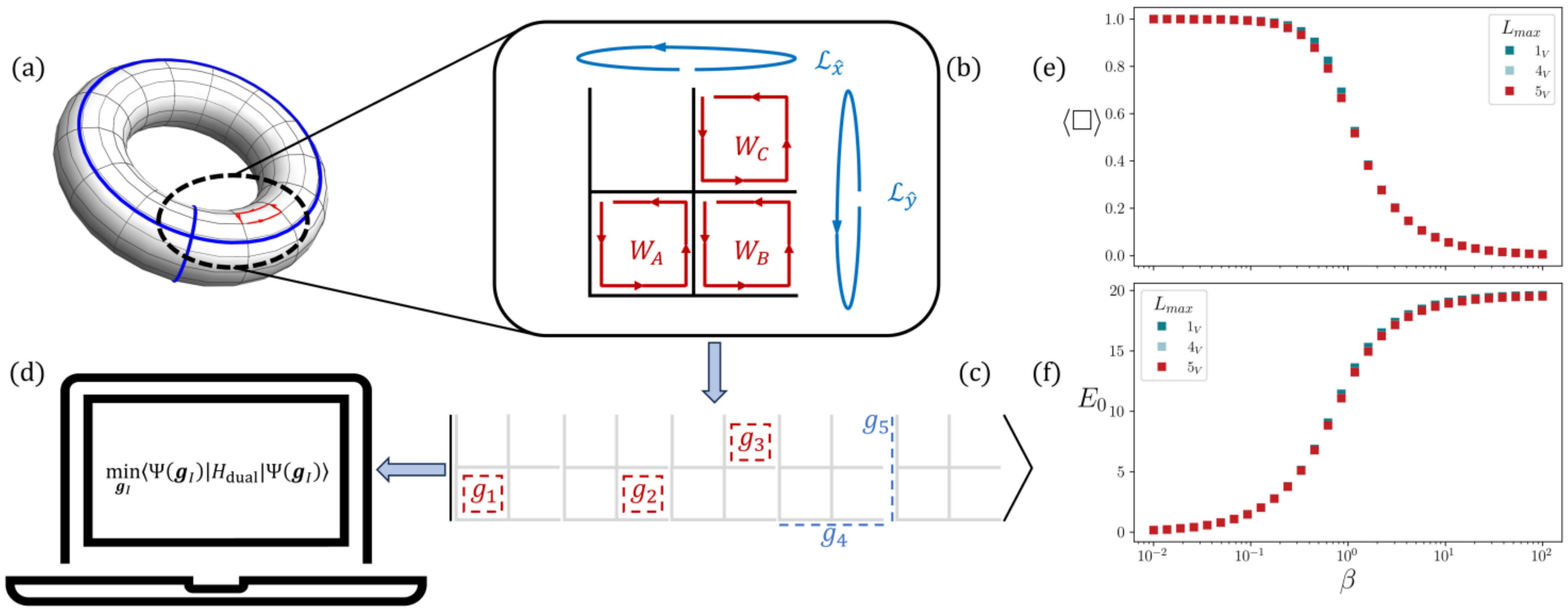
Michael Meth, Jinglei Zhang, Jan F. Haase, Claire Edmunds, Lukas Postler, Andrew J. Jena, Alex Steiner, Luca Dellantonio, Rainer Blatt, Peter Zoller, Thomas Monz, Philipp Schindler, Christine Muschik & Martin Ringbauer



5 independent physical variables

[Haase et al., Quantum **5** (2021)] [Meth et al., Nat. Phys. **21** (2025)]

Reformulation for SU(N)

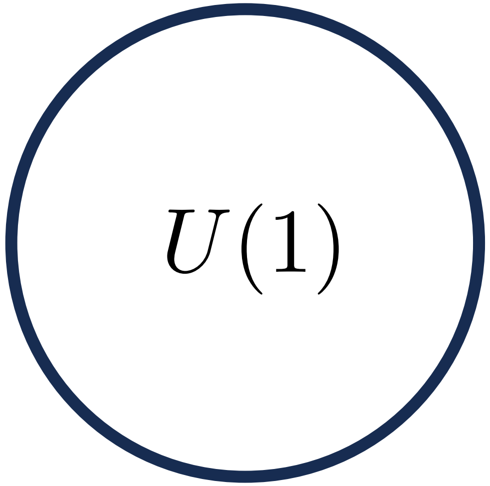


[Fontana, **MMR** & Celi, Phys. Rev. X (accepted) (2025)]

[Mathur & Rathor, Phys. Rev. D **107** (2023)]

The renormalized dual basis (RDB)

$$H_E = \frac{g^2}{2} \sum_{\boldsymbol{n},\hat{\mu}} E_\mu^2(\boldsymbol{n}) \qquad H_B = -\frac{1}{4g^2} \sum_P \left(U_P^\dagger + U_P\right)$$

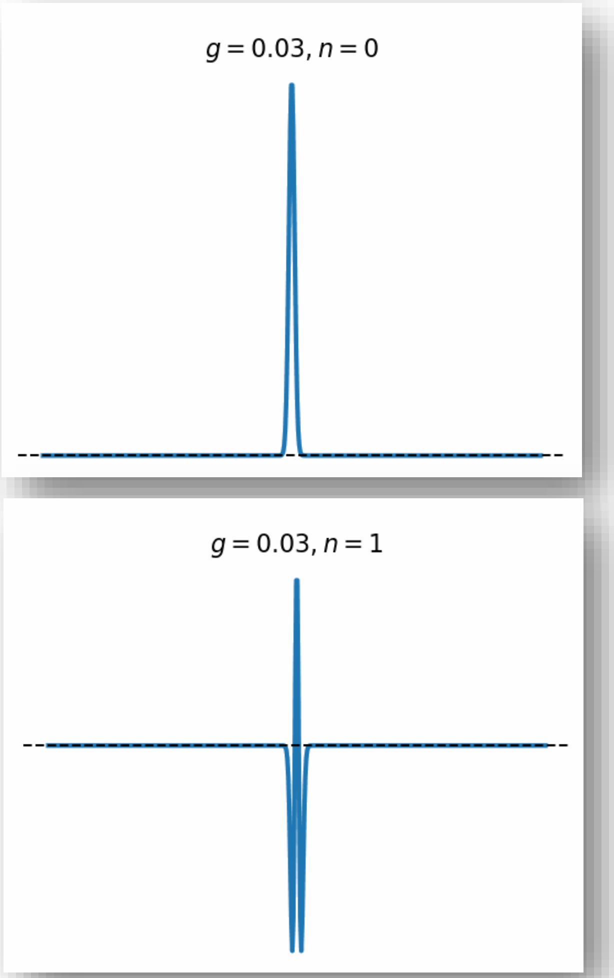


- Electric basis $|k\rangle$
- Group basis $|e^{i\theta}\rangle$

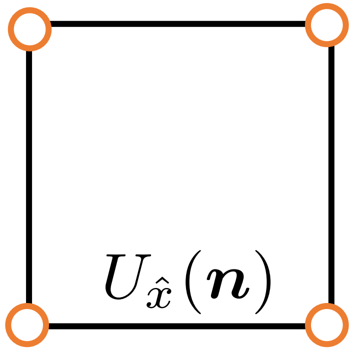
Peter-Weyl theorem $\langle k|e^{i\theta}\rangle = e^{ik\theta}$

- *Truncation* of degrees of freedom
- Span *weak & strong couplings* with the same precision

The renormalized dual basis (RDB)



$$H_E = \frac{g^2}{2} \sum_{\boldsymbol{n},\hat{\mu}} E_\mu^2(\boldsymbol{n}) \qquad H_B = -\frac{1}{4g^2} \sum_P (U_P^\dagger + U_P)$$



Minimal setup for the competition between non-commuting electric and magnetic fields

$L^2(G)$

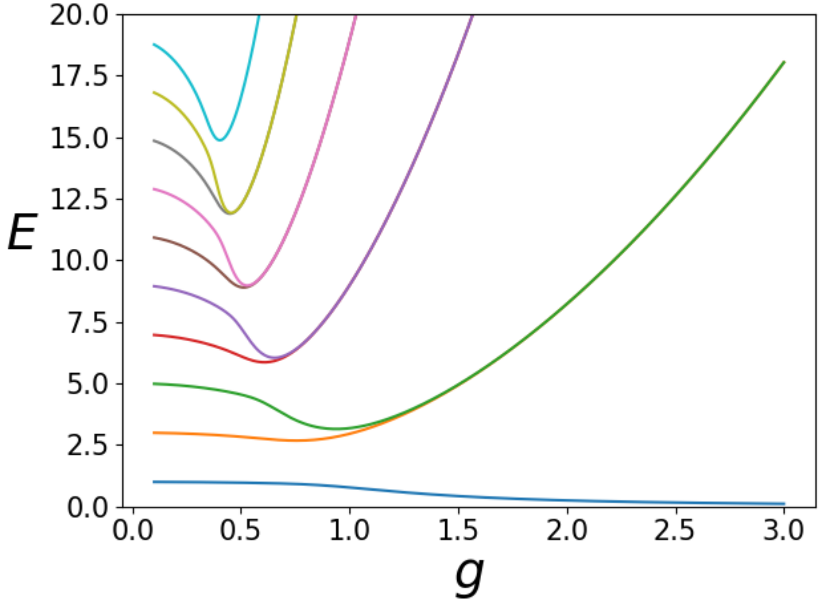
In coordinate representation

$$|\Psi\rangle = \int_0^{2\pi} \frac{d\theta}{2\pi} \Psi(\theta) |e^{i\theta}\rangle$$

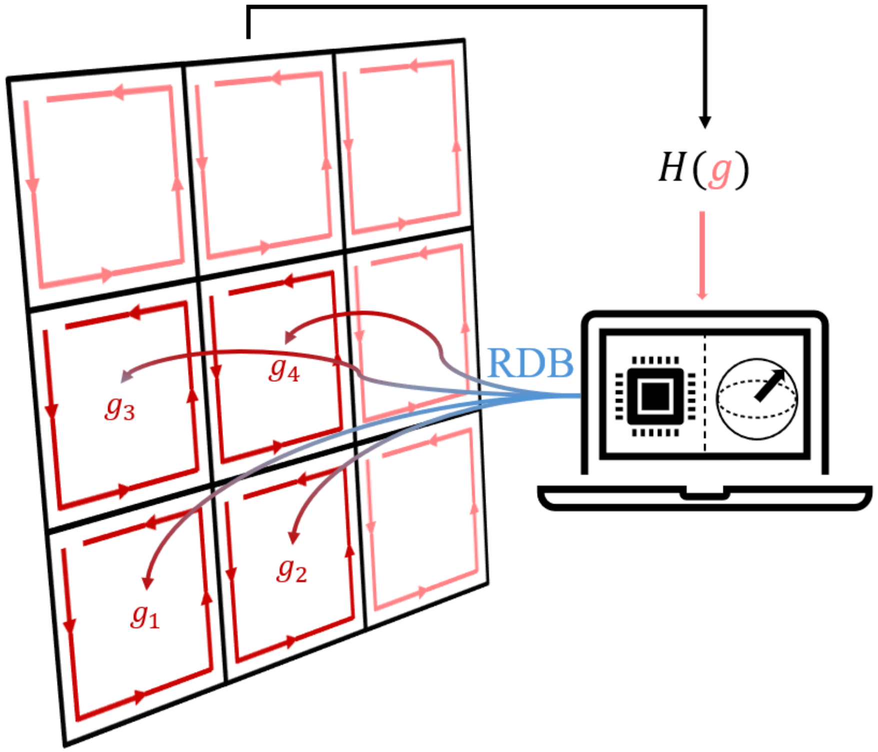
$$H(g) = -2g^2 \partial_\theta + \frac{1 - \cos\theta}{g^2}$$

[Fontana, **MMR** & Celi, Phys. Rev. X (accepted) (2025)]

The renormalized dual basis (RDB)

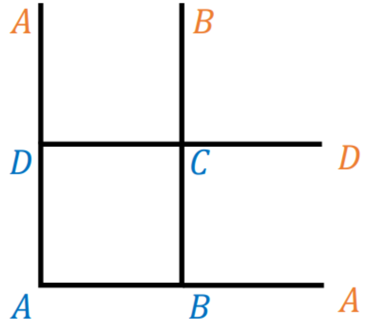


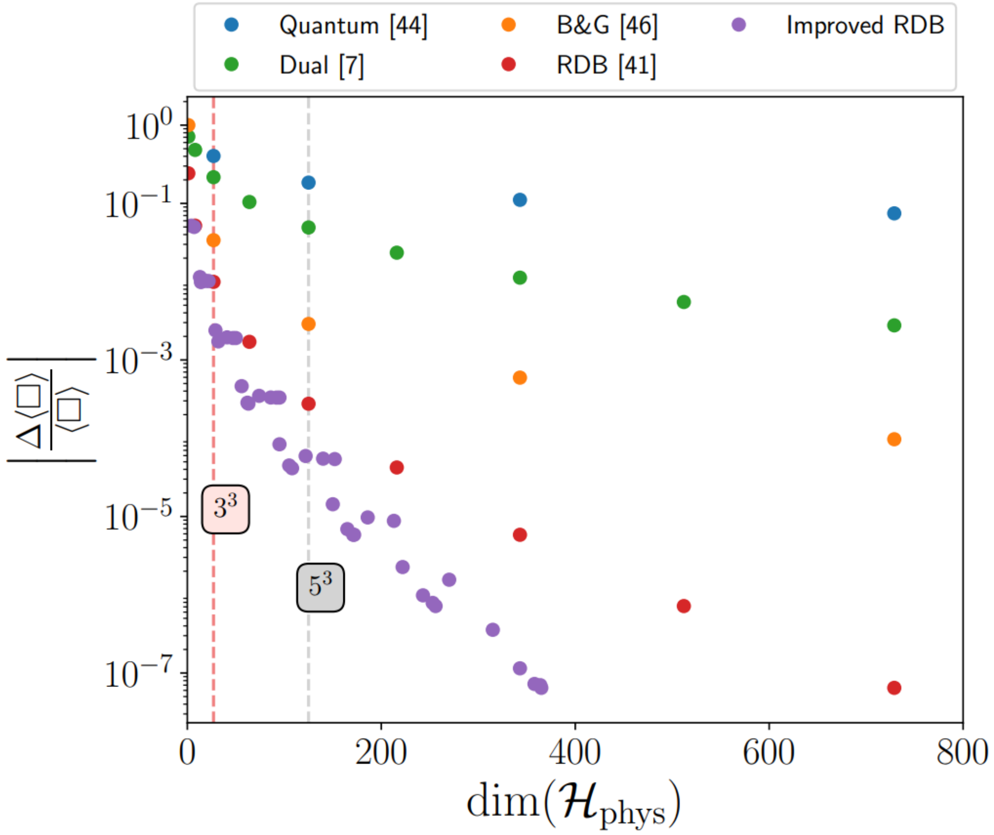
Truncation $L_{\max}$

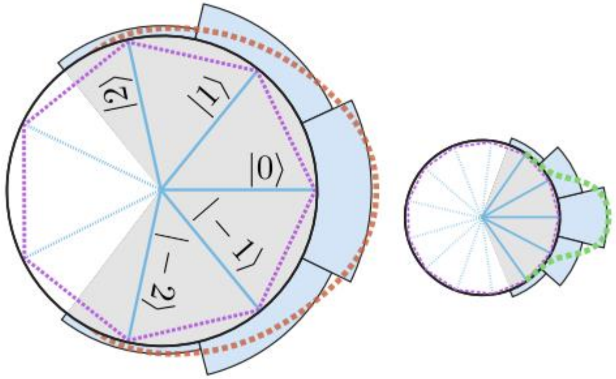


[**MMR**, Fontana & Celi (in preparation)]

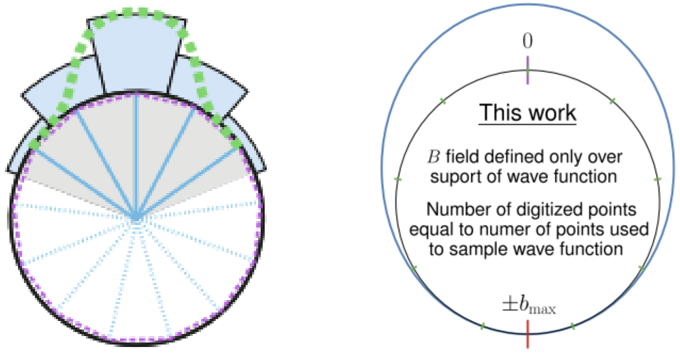
Comparison for U(1)







[44][Adapted from Haase et al., Quantum **5** (2021)]



$$\beta = (2g^2)^{-1} = 5 \qquad \langle\square\rangle \equiv \frac{g^2}{2N_{\mathrm{plaq}}}\langle\psi_0|H_B|\psi_0\rangle$$

[46][Adapted from Bauer & Grabowska, Phys. Rev. D **107** (2023)]

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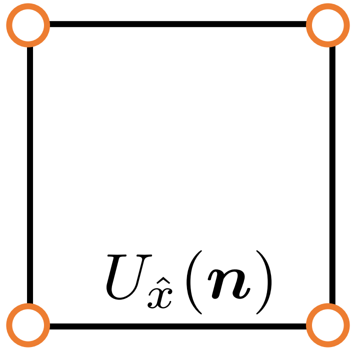
[7][Ligterink et al., Ann. Phys. **284** (2000)]

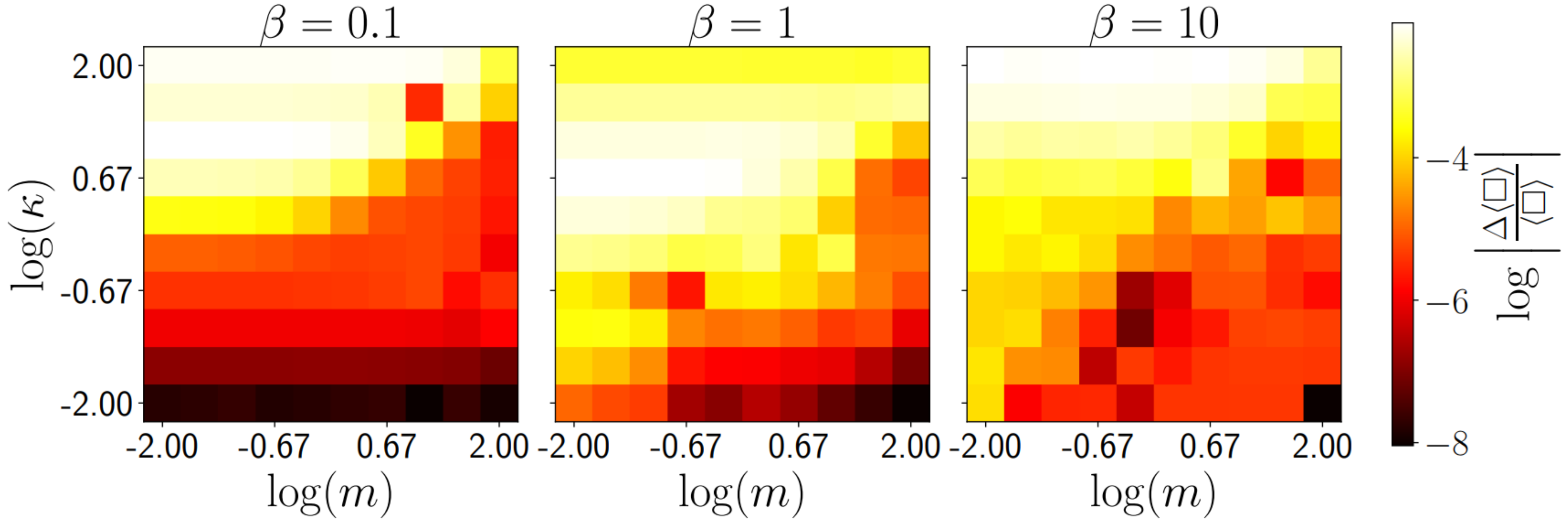
[41][Fontana, **MMR** & Celi, Phys. Rev. X (accepted) (2025)]

[**MMR**, Fontana & Celi (in preparation)]



Adding matter [U(1)]

$$H_M = H_m + H_k$$
$$= m \sum_{\mathbf{n}} (-1)^{n_x+n_y} \psi_{\mathbf{n}}^\dagger \psi_{\mathbf{n}}$$
$$+ \kappa \sum_{\mu,\mathbf{n}} \psi_{\mathbf{n}}^\dagger U_\mu(\mathbf{n}) \psi_{\mathbf{n}+\mu} + \text{H.c.}$$



$L_{\max} = 2$

$\beta \equiv (2g^2)^{-1}$

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[Haase et al., Quantum **5** (2021)]

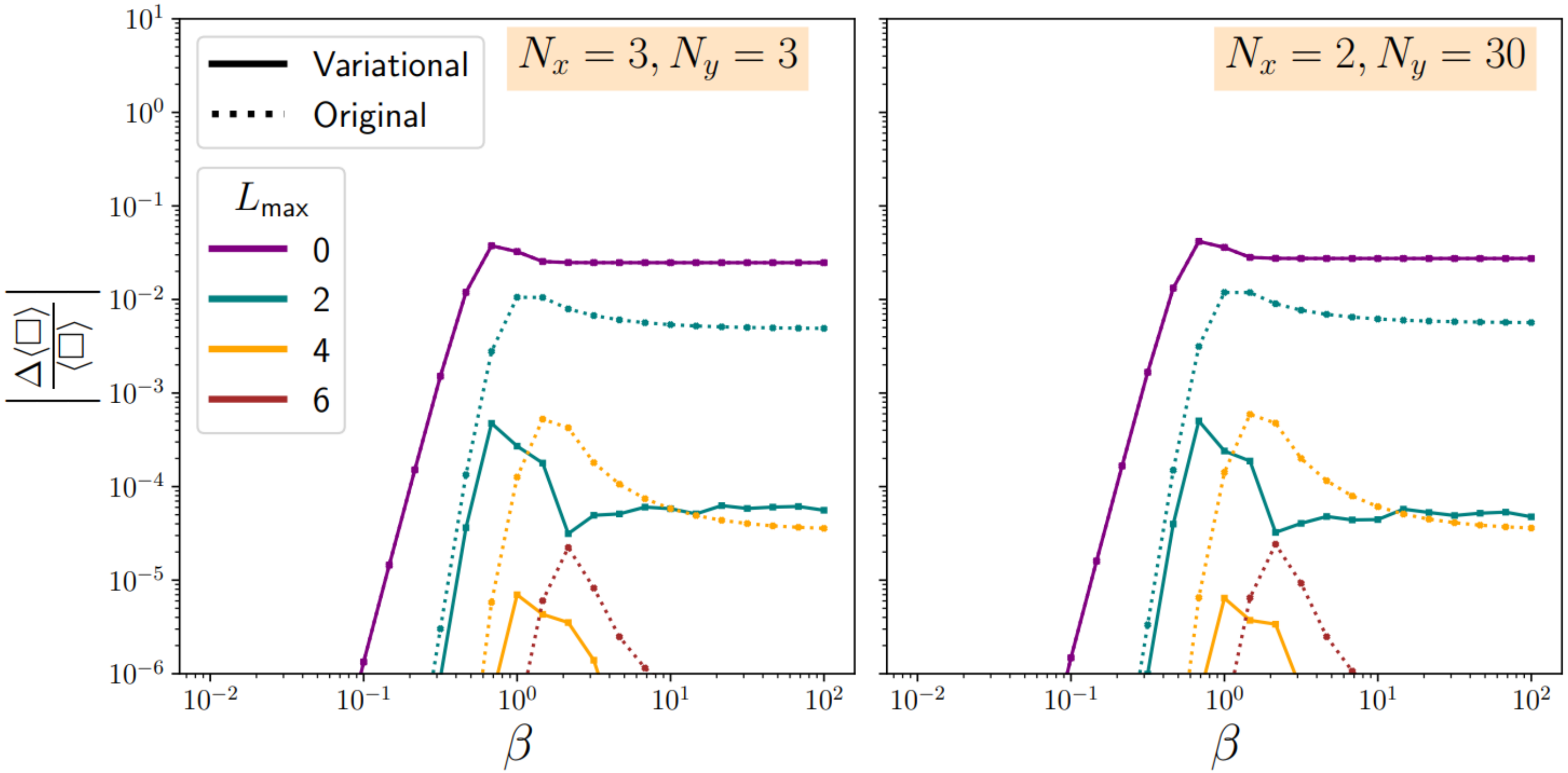
[**MMR**, Fontana & Celi (in preparation)]

Scaling to larger lattices [U(1)]

- MPS with DMRG
- Square/ladder latices in OBC
- Single variational parameter g

$$\langle\square\rangle \equiv \frac{g^2}{2N_{\text{plaq}}}\langle\psi_0|H_B|\psi_0\rangle$$

$$\beta \equiv (2g^2)^{-1}$$



[Bender, Zohar, Phys. Rev. D **102** (2020)]

[**MMR**, Fontana & Celi (in preparation)]

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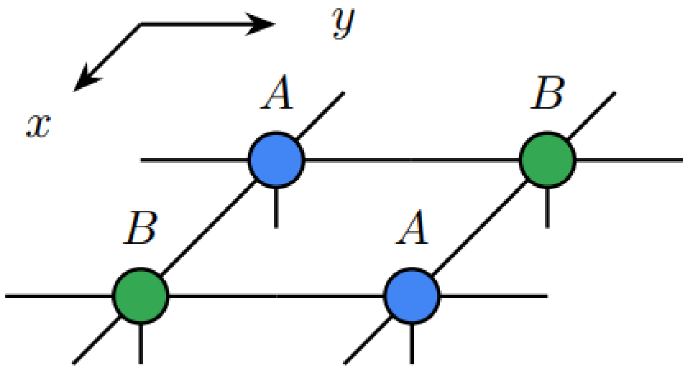
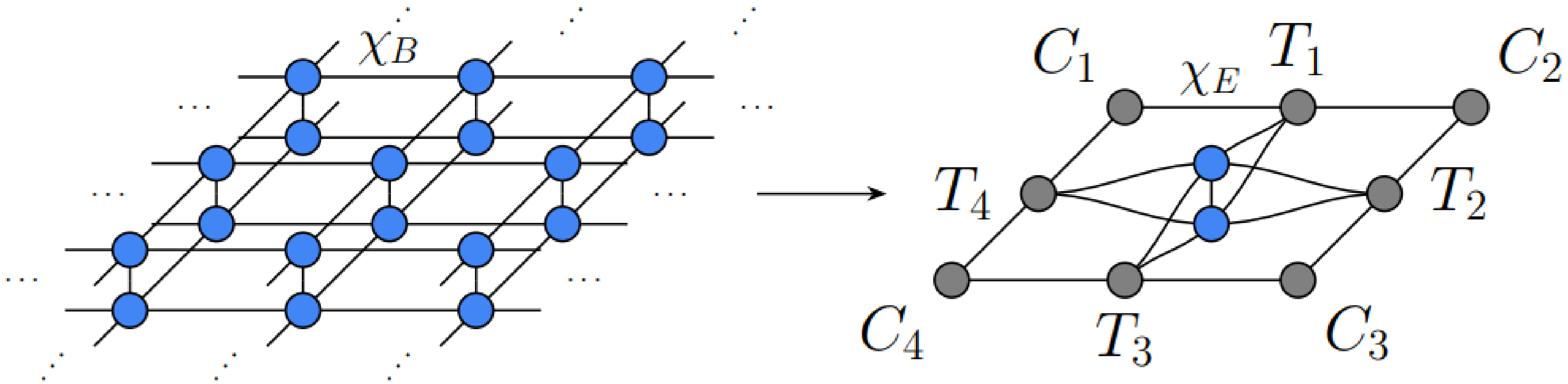
Even larger lattices [U(1)]



(infinite) Projected entangled pair states (iPEPS)

→ variPEPS library

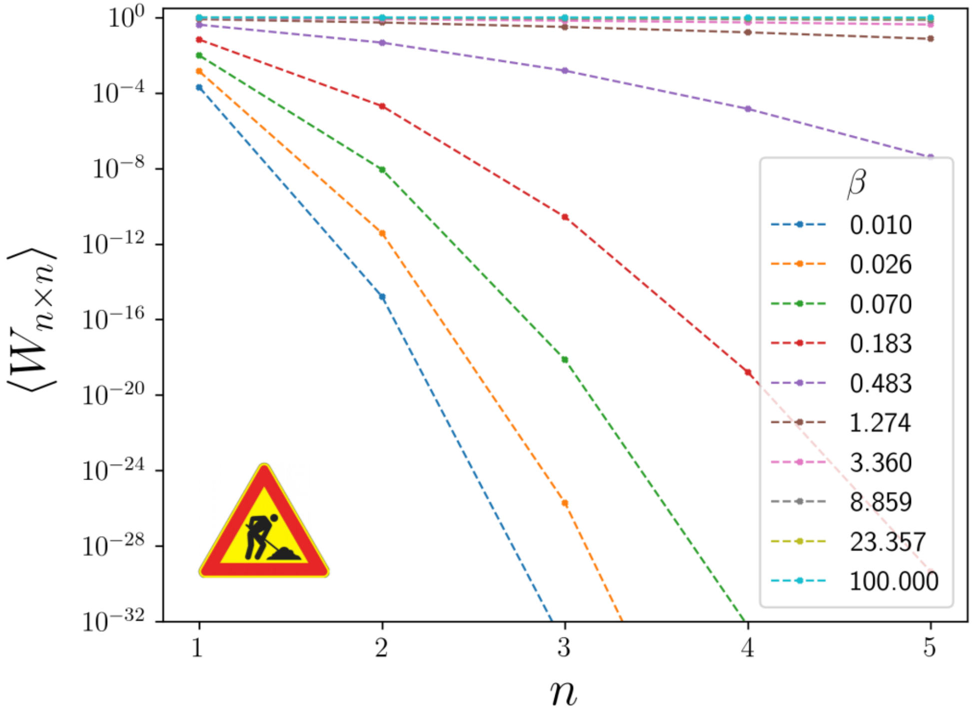


[Naumann, Weerda et al., SciPost Phys.Lect.Notes **86** (2024)]

Even larger lattices [U(1)]

- iPEPS for infinite 2D (**variPEPS**)

$$\beta \equiv (2g^2)^{-1}$$



[**MMR**, Fontana & Celi (part 2: in preparation)]

$$\langle W_{i\times j}\rangle \propto e^{-\kappa_A (i\times j) - \kappa_P (i+h)}$$

String tension: κ_A

Probe of confinement

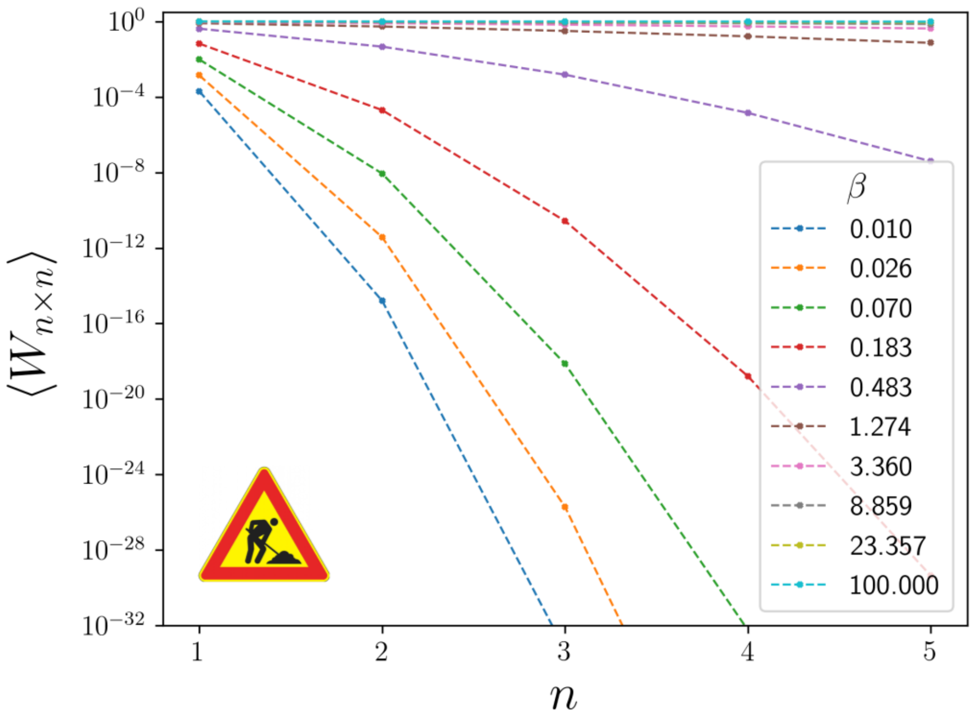
Creutz ratio: $\chi_{i\times j} = -\ln \frac{W_{i\times j} W_{i-1\times j-1}}{W_{i\times j-1} W_{i-1\times j}}$

[Wilson, Phys. Rev. D **10** (1974)]
[Creutz, Phys. Rev. Lett. **45** (1980)]
[Zohar, Phys. Rev. Research **3** (2021)]
[Bender, Zohar, Phys. Rev. D **102** (2020)]

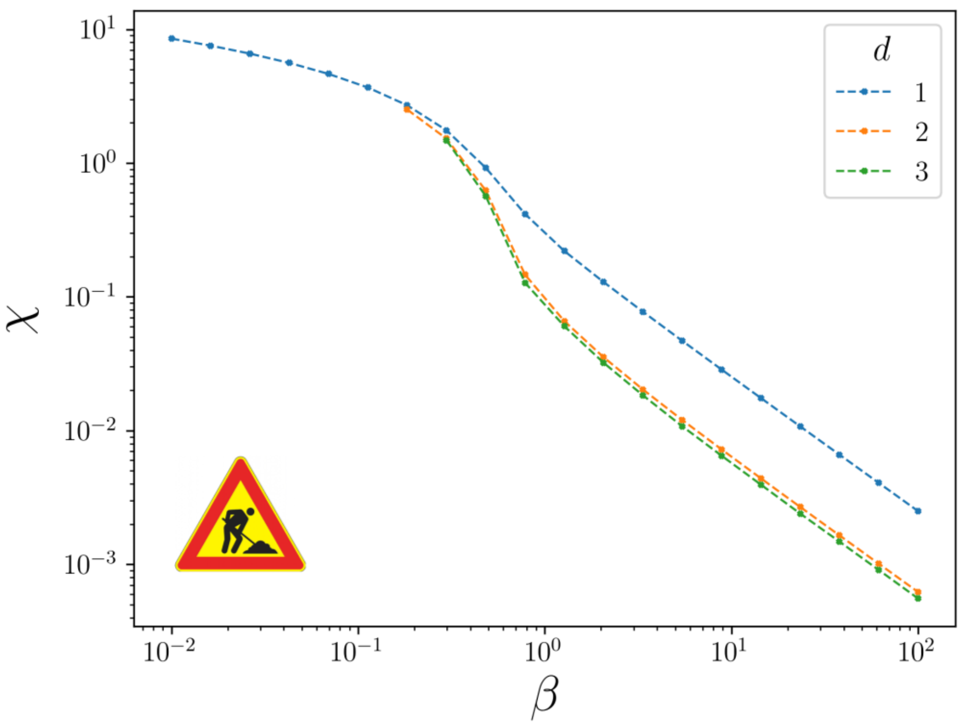
Even larger lattices [U(1)]

➢iPEPS for infinite 2D (**variPEPS**)

$\beta \equiv (2g^2)^{-1}$



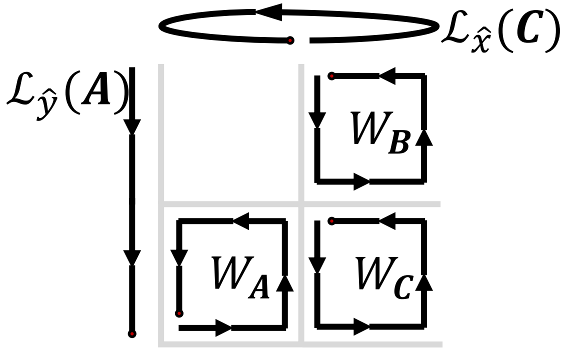
[**MMR**, Fontana & Celi (part 2: in preparation)]

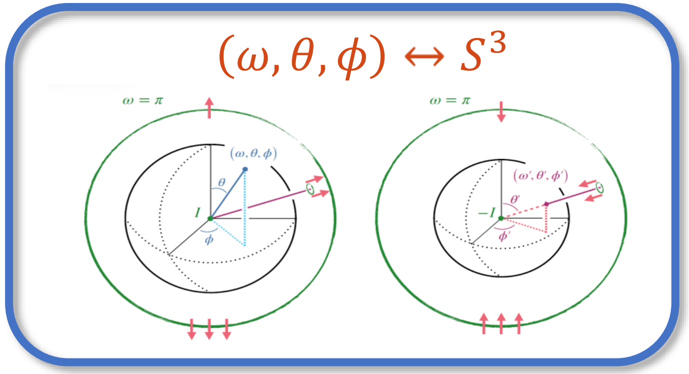


Creutz ratio: $\chi_{i\times j} = -\ln \frac{W_{i\times j} W_{i-1\times j-1}}{W_{i\times j-1} W_{i-1\times j}}$

[Wilson, Phys. Rev. D **10** (1974)]
[Creutz, Phys. Rev. Lett. **45** (1980)]
[Zohar, Phys. Rev. Research **3** (2021)]
[Bender, Zohar, Phys. Rev. D **102** (2020)]

Renormalized dual basis for SU(2)





[Adapted from D'Andrea et al., Phys. Rev. D **109** (2024)]

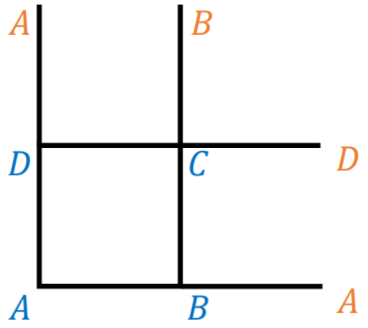
Single plaquette Hamiltonian for each loop:

$$H_{0,\boldsymbol{n}}(g_i) \equiv \frac{1}{2g_i^2}\mathrm{Tr}[2 - (W_{\boldsymbol{n}} + W_{\boldsymbol{n}}^\dagger)] + 2g_i^2\mathcal{E}^2(\boldsymbol{n})$$

$$\mathrm{SU}(2) \cong S^3 \qquad \Psi(\omega, \theta, \phi) = \sum_{\ell, m} c_m^\ell \frac{u_m^\ell(\omega)}{2\sin\frac{\omega}{2}} Y_m^\ell(\theta, \phi).$$

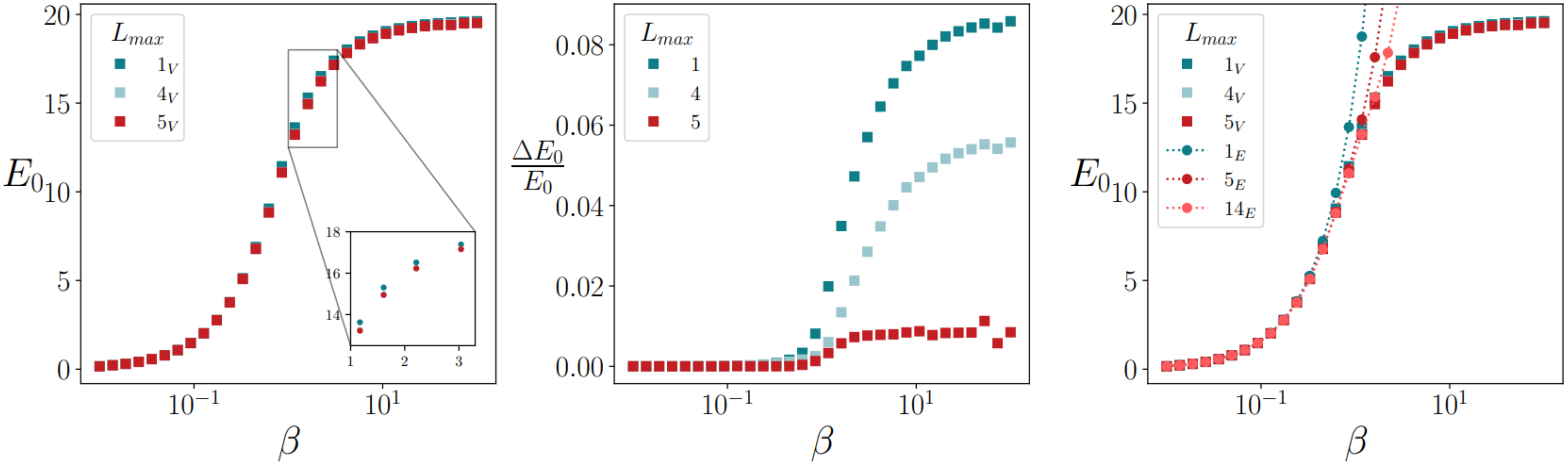
[D'Andrea et al., Phys. Rev. D **109** (2024)]

Renormalized dual basis for SU(2)



$$\beta \equiv (2g^2)^{-1}$$

$$\frac{\Delta E_0}{E_0} \equiv \frac{E_0(\boldsymbol{g_0}) - E_0(\boldsymbol{g_V})}{E_0(\boldsymbol{g_V})}$$



[Fontana, **MMR** & Celi, Phys. Rev. X (accepted) (2025)]

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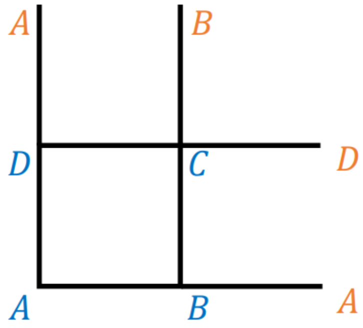
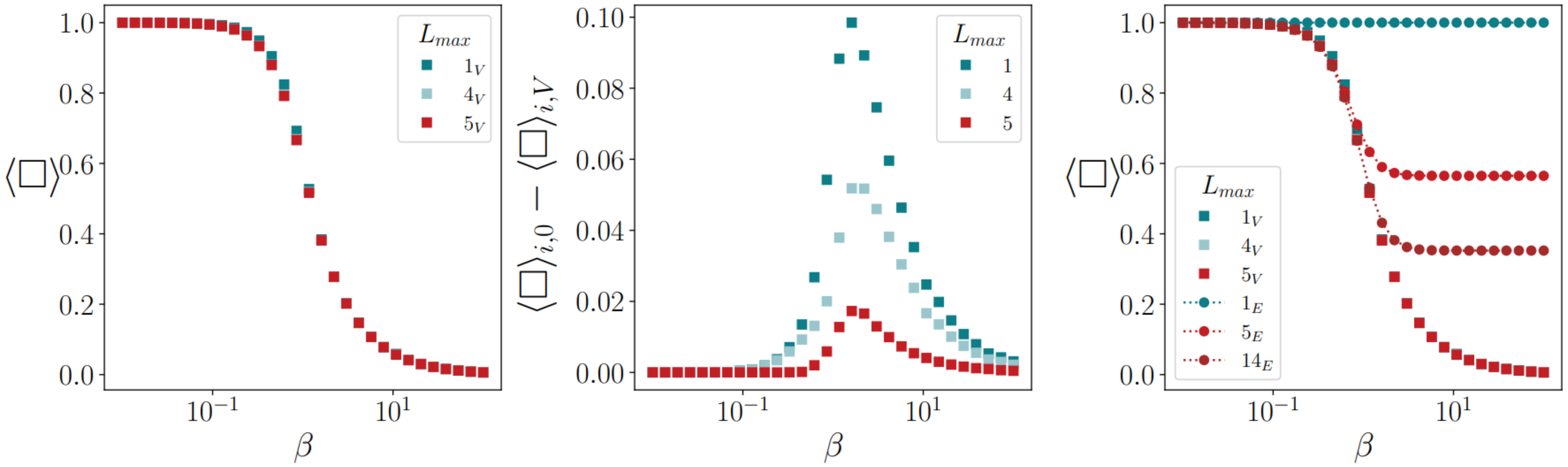
Renormalized dual basis for SU(2)



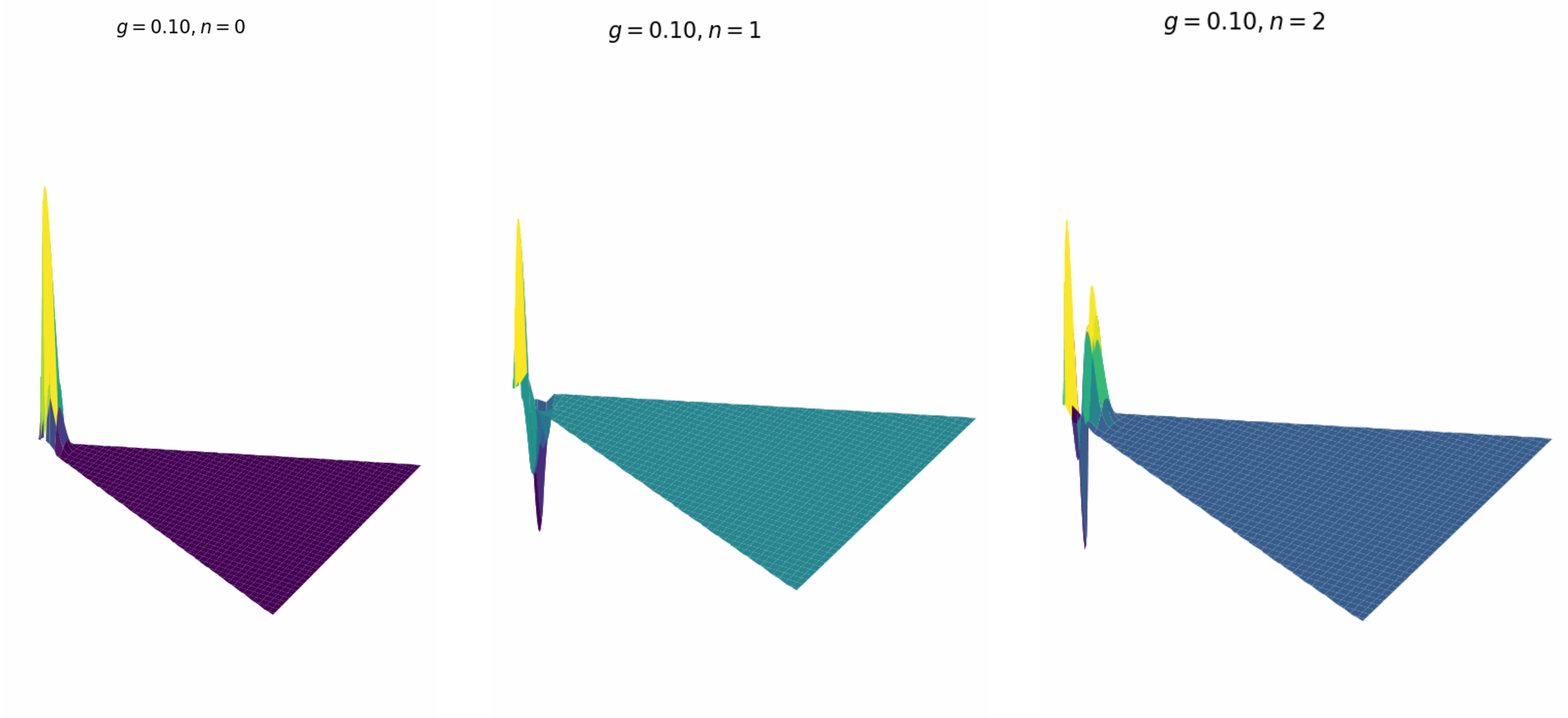
$$\beta \equiv (2g^2)^{-1}$$

$$\langle \square \rangle \equiv \frac{g^2}{2N_{\text{plaq}}} \langle \psi_0 | H_B | \psi_0 \rangle$$



[Fontana, **MMR** & Celi, Phys. Rev. X (accepted) (2025)]

Renormalized dual basis for SU(3)



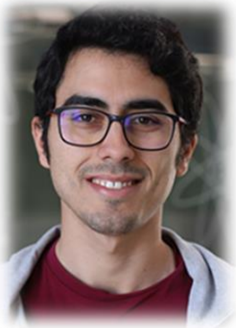
Outlook

Resource-efficiency of the renormalized dual basis (RDB)

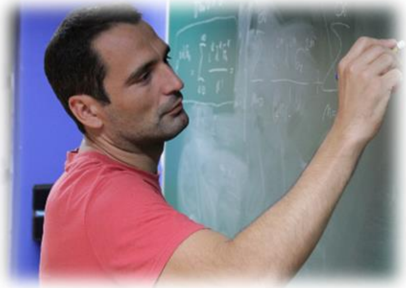
- For U(1) it enabled large-scale simulations at all values of the coupling
- Proof-of-concept demonstration for SU(2)

Outlook

- RDB for SU(3) and possibly other compact Lie groups
- Inclusion of matter and large-scale non-Abelian LGTs
- VQE implementation



Pierpaolo Fontana

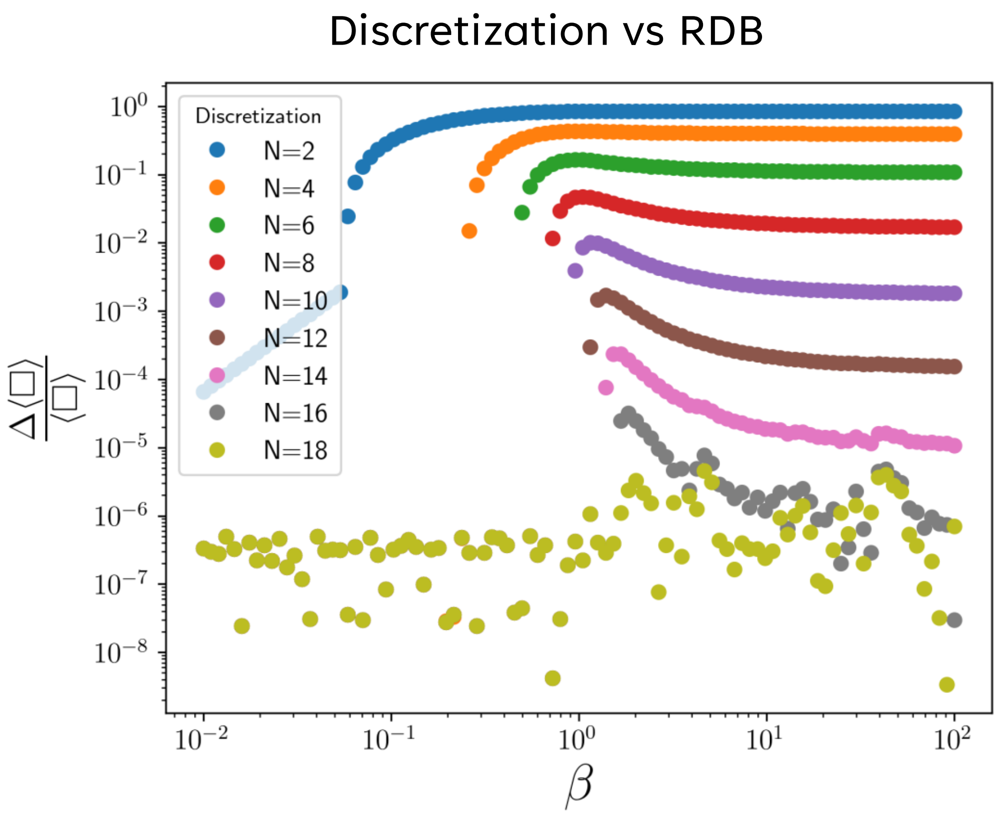


Alessio Celi

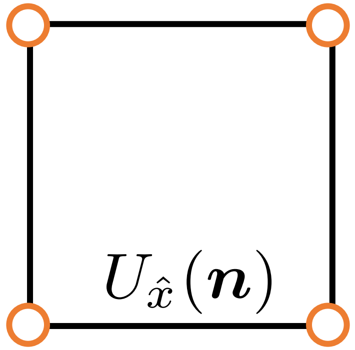
Universitat Autònoma de Barcelona

Back-up slides

Comparison in SU(2)

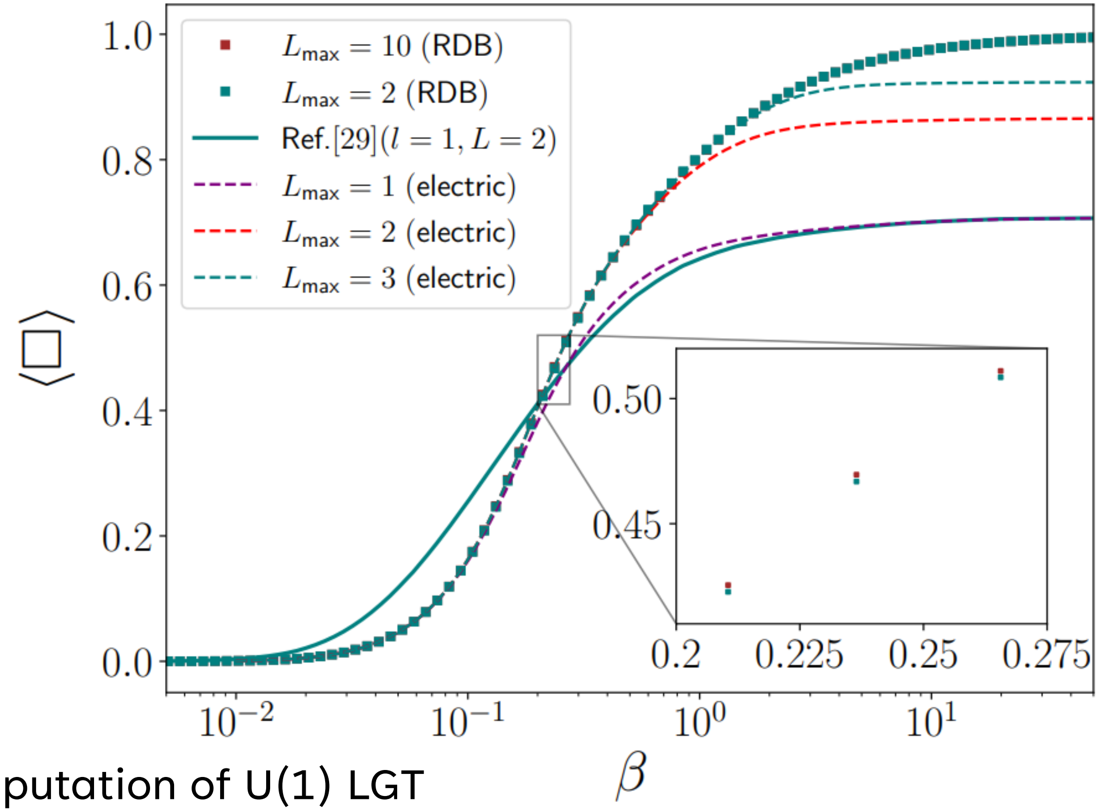


$$\beta \equiv (2g^2)^{-1}$$



Adding matter [U(1)]

$$
\begin{aligned}
H_M &= H_m + H_k \\
&= m \sum_{\mathbf{n}} (-1)^{n_x + n_y} \psi_{\mathbf{n}}^{\dagger} \psi_{\mathbf{n}} \\
&+ \kappa \sum_{\mu, \mathbf{n}} \psi_{\mathbf{n}}^{\dagger} U_{\mu}(\mathbf{n}) \psi_{\mathbf{n}+\mu} + \text{H.c.}
\end{aligned}
$$



$$\beta \equiv (2g^2)^{-1}$$

State of the art quantum computation of U(1) LGT
[Meth et al., Nat. Phys. **21** (2025)]

[29][Haase et al., Quantum **5** (2021)]

[**MMR**, Fontana & Celi (in preparation)]

Matrix elements (U(1))

Electric squared

```
array([[0.02703348, 0.        , 0.16254987],
       [0.        , 1.00483055, 0.        ],
       [0.16254987, 0.        , 0.97783219]])
```

Electric

```
array([[ 0.        , -0.16438674,  0.        ],
       [ 0.16438674,  0.        ,  0.98803202],
       [ 0.        , -0.98803202,  0.        ]])
```

Cosinus

```
array([[ 0.22976364,  0.        ,  0.67412683],
       [ 0.        ,  0.04010691,  0.        ],
       [ 0.67412683,  0.        , -0.18946245]])
```

Matrix elements (SU(2))

Electric squared

$$\begin{pmatrix} 0.0198131 & -0.121078 & 0 & 0 & 0 \\ -0.121078 & 0.742694 & 0 & 0 & 0 \\ 0 & 0 & 0.758256 & 0 & 0 \\ 0 & 0 & 0 & 0.758256 & 0 \\ 0 & 0 & 0 & 0 & 0.758256 \end{pmatrix}$$

Electric

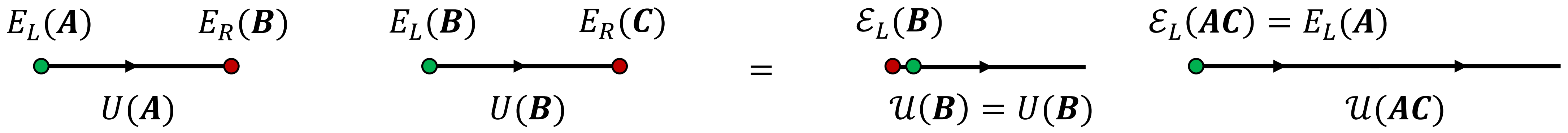
$$\begin{pmatrix} 0 & 0 & 0.+0.114915\,i & 0.+0.\,i & 0.-0.114915\,i \\ 0 & 0 & 0.-0.701258\,i & 0.+0.\,i & 0.+0.701258\,i \\ 0.-0.114915\,i & 0.+0.701258\,i & 0 & -1 & 0 \\ 0.+0.\,i & 0.+0.\,i & -1 & 0 & -1 \\ 0.+0.114915\,i & 0.-0.701258\,i & 0 & -1 & 0 \end{pmatrix}$$

Cosinus

$$\begin{pmatrix} 0.161206 & -0.484301 & 0 & 0 & 0 \\ -0.484301 & -0.0611284 & 0 & 0 & 0 \\ 0 & 0 & 0.0662457 & 0 & 0 \\ 0 & 0 & 0 & 0.0662457 & 0 \\ 0 & 0 & 0 & 0 & 0.0662457 \end{pmatrix}$$

«Fusion rules» for physical variables

Combine links & electric fields:



1. Add (left) electric fields & multiply adjacent paths:

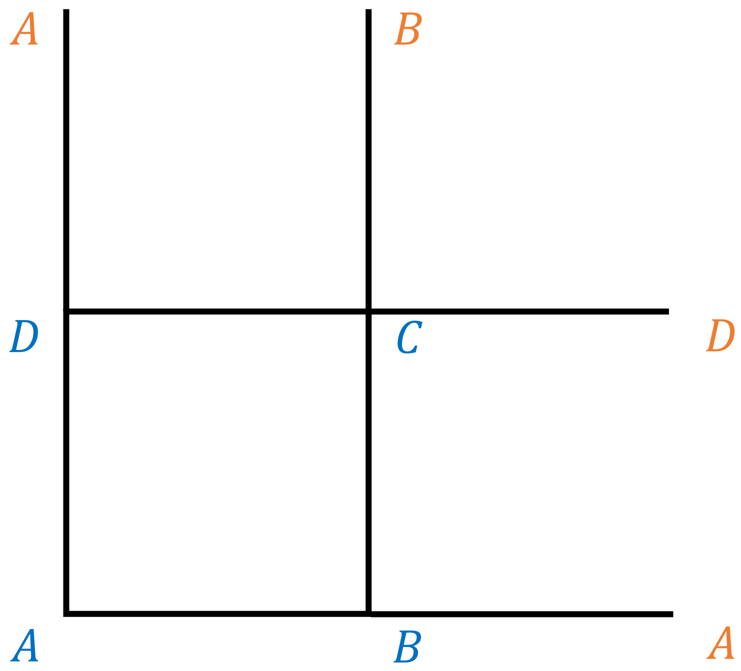
$$\mathcal{E}_L(\boldsymbol{B}) = E_R(\boldsymbol{B}) + E_L(\boldsymbol{B}) \qquad \mathcal{U}(\boldsymbol{AC}) = U(\boldsymbol{A})U(\boldsymbol{B})$$

2. Preserve canonical commutation relations & number of variables:

$$[\mathcal{E}_L, \mathcal{U}] = -T\mathcal{U}, \qquad [\mathcal{E}_R, \mathcal{U}] = \mathcal{U}T$$

[Mathur & Sreeraj (PRD 2015)]
[Mathur & Rathor (PRD 2023)]

Single periodic plaquette



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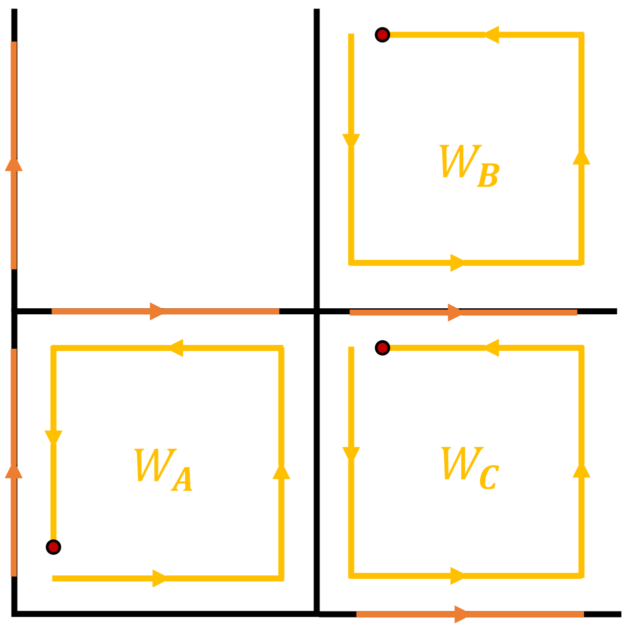
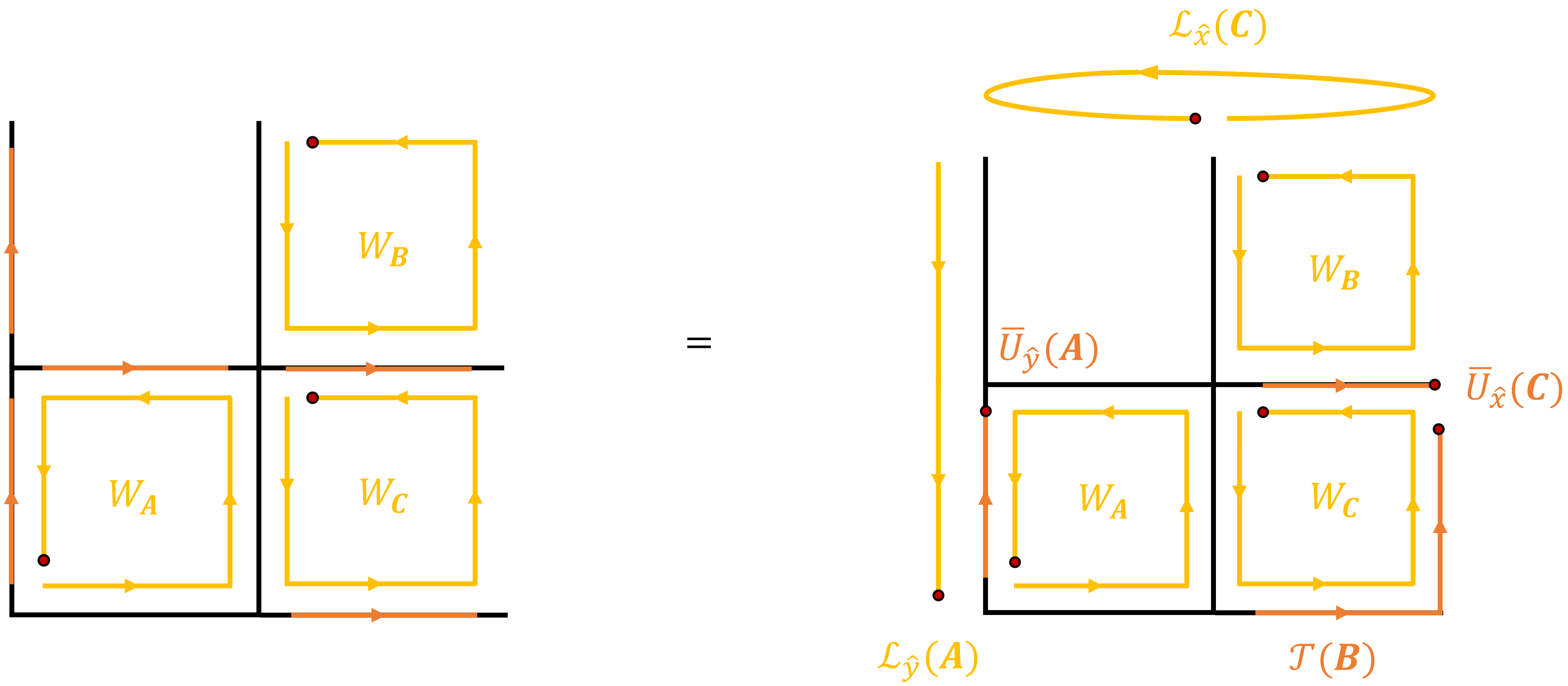
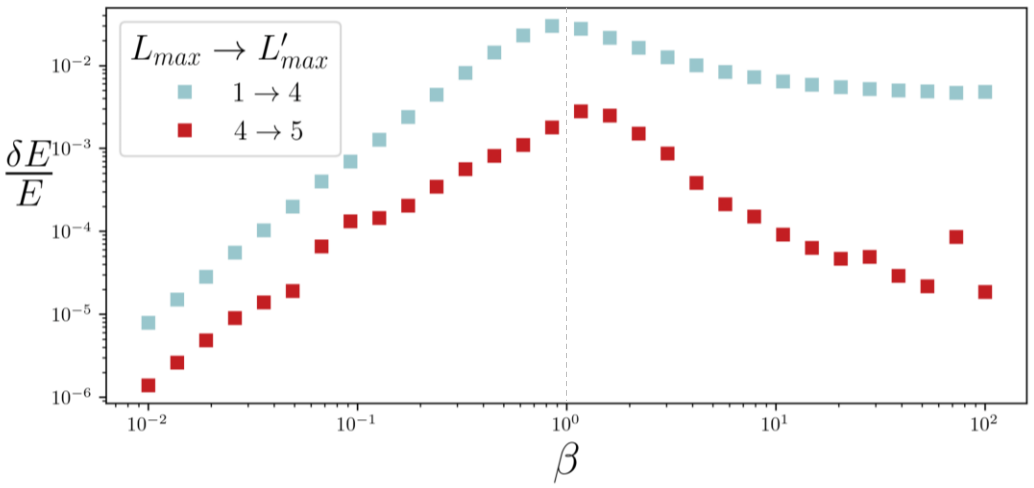
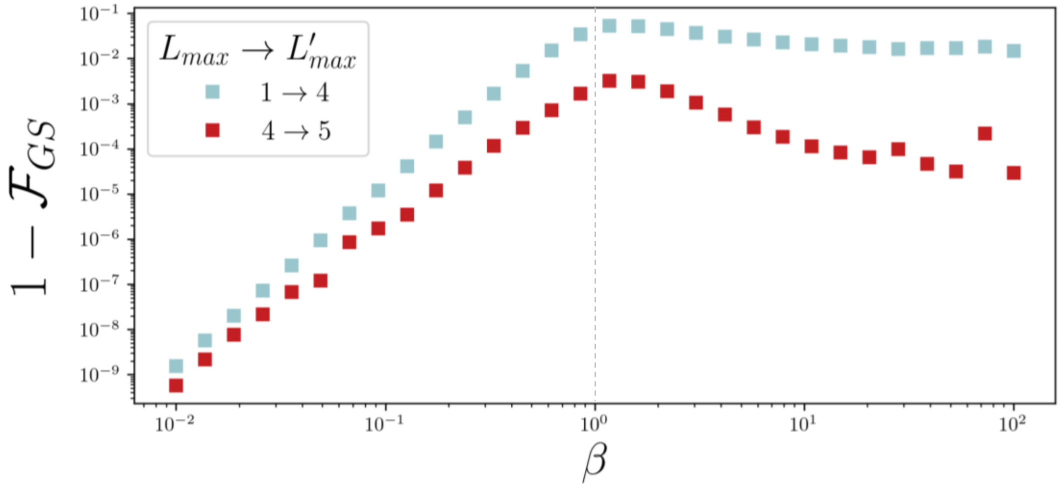


Single periodic plaquette



Ground state properties





$\delta E = |E_{0,L_{max}}(\beta, \boldsymbol{g_i}) - E_{0,L'_{max}}(\beta, \boldsymbol{g_i})|$

$1 - \mathcal{F}_{\text{GS}} = 1 - |\langle \psi_{0,L_{max}}(\beta, \boldsymbol{g_i}) | \psi_{0,L'_{max}}(\beta, \boldsymbol{g_i}) \rangle|^2$

Variational
energy precision

Max $\sim O(10^{-3})$ when $\beta \sim O(1)$
(magnetic to electric bases)

$SU(2)$ local basis choice

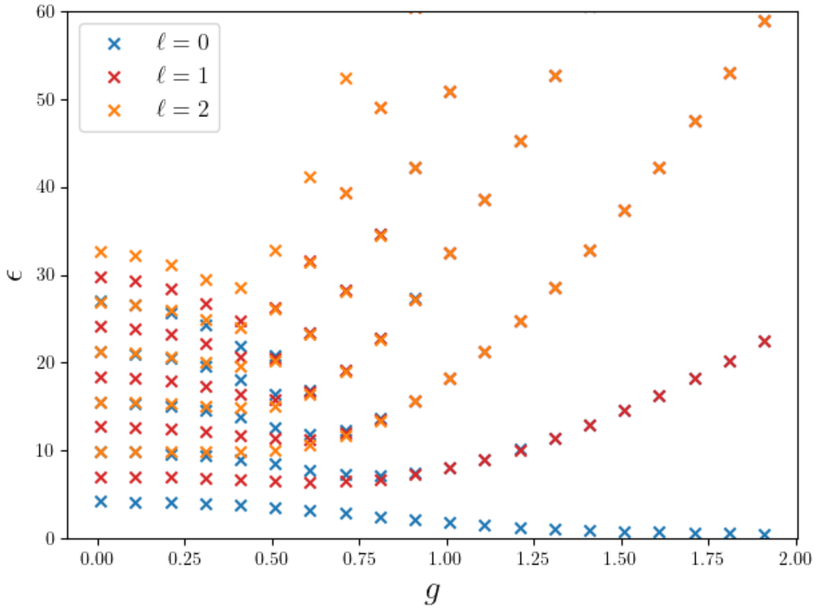
Single plaquette Hamiltonian:

$$H_0(g) = \frac{2}{g^2}\left(1 - \cos\frac{\omega}{2}\right) + 2g^2\hat{\mathcal{E}}^2$$

- «Mixed – angle» basis:

$$\mathbf{\Omega} = (\omega, \theta, \phi) \xrightarrow{Y_{lm}(\theta,\phi)} (\omega, l, m)$$

- L_{max} states for each g



$SU(2)$ local basis choice

- Work in the *group basis*:

Local Hamiltonian for $\mathcal{W}_i \in \mathcal{W}$:

$$H_{0,\mathcal{W}_i}(g_i) = V_B(\omega) + 2g_i^2 \hat{\mathcal{E}}^2$$

$$\hat{\mathcal{E}}^2 = \frac{\mathbf{L}^2}{\sin^2\frac{\omega}{2}} - 4\left[\partial_\omega^2 + \cot\frac{\omega}{2}\partial_\omega\right]$$

Central motion problem on S^3

$$V_B(\omega) = \frac{2}{g^2}\left(1 - \cos\frac{\omega}{2}\right)$$

L_{max} states for each g

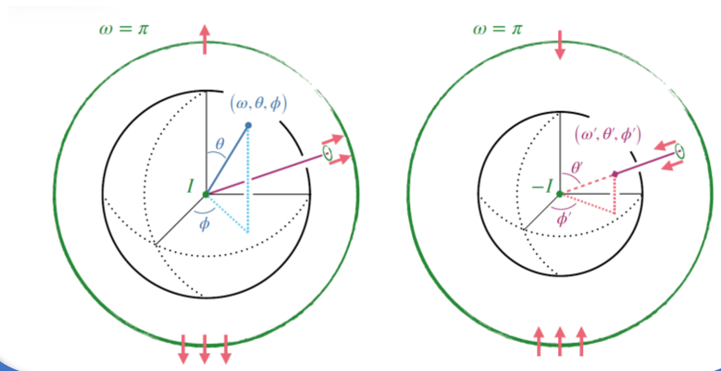
$$\mathcal{W} \equiv \{W_A, W_B, W_C, \mathcal{L}_{\hat{x}}, \mathcal{L}_{\hat{y}}\}$$

$\in SU(2)$

$$(\omega, \theta, \phi) \mapsto D^{1/2}(\omega, \theta, \phi) = e^{-i\omega\hat{n}\cdot\boldsymbol{\sigma}/2}$$

$$= \begin{pmatrix} \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta & -i\sin\frac{\omega}{2}\sin\theta e^{-i\phi} \\ -i\sin\frac{\omega}{2}\sin\theta e^{i\phi} & \cos\frac{\omega}{2} + i\sin\frac{\omega}{2}\cos\theta \end{pmatrix}$$

$(\omega, \theta, \phi) \leftrightarrow S^3$



[Adapted from Bauer & Grabowska (arXiv 2023)]

Dual Hamiltonian

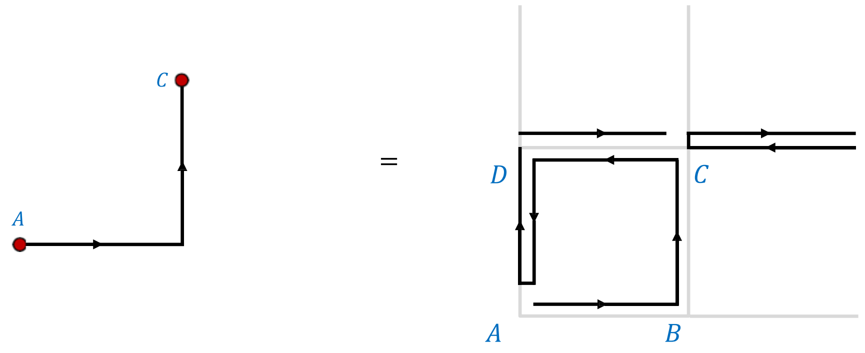
Main message 1: reformulating the theory in terms of physical quantities (gauge-invariant) we get rid of redundancies

- $H = H_E(\mathcal{E}, \mathcal{L}, \mathcal{W}) + H_B(\mathcal{W})$

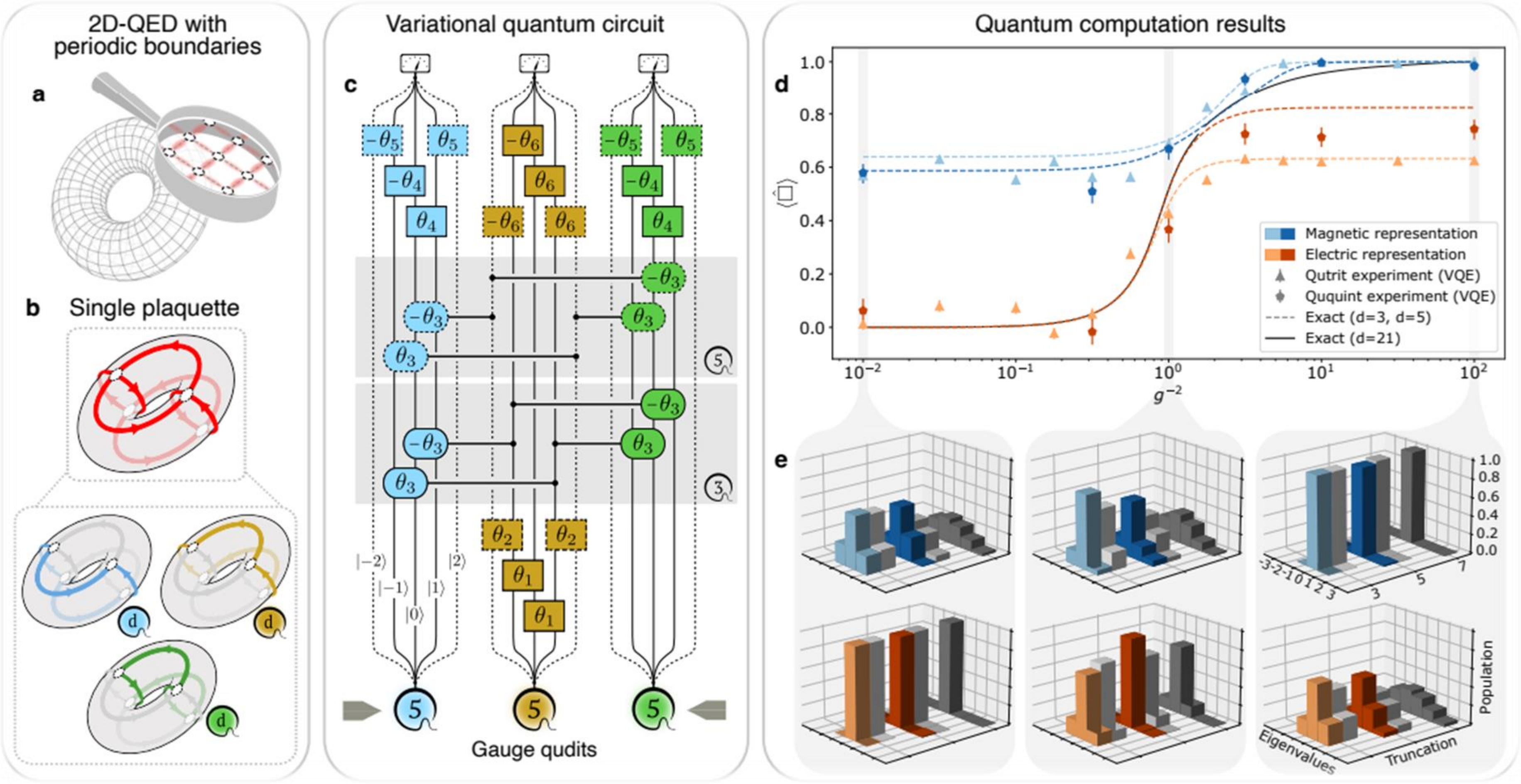
$$H_B(\mathcal{W}) = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \mathrm{Tr}\left[2 - \left(W_{\boldsymbol{n}}^{\dagger} + W_{\boldsymbol{n}}\right)\right]$$

$$H_E(\mathcal{E}, \mathcal{L}, \mathcal{W}) = H_{E,\text{local}} + H_{E,\text{non-local}}$$

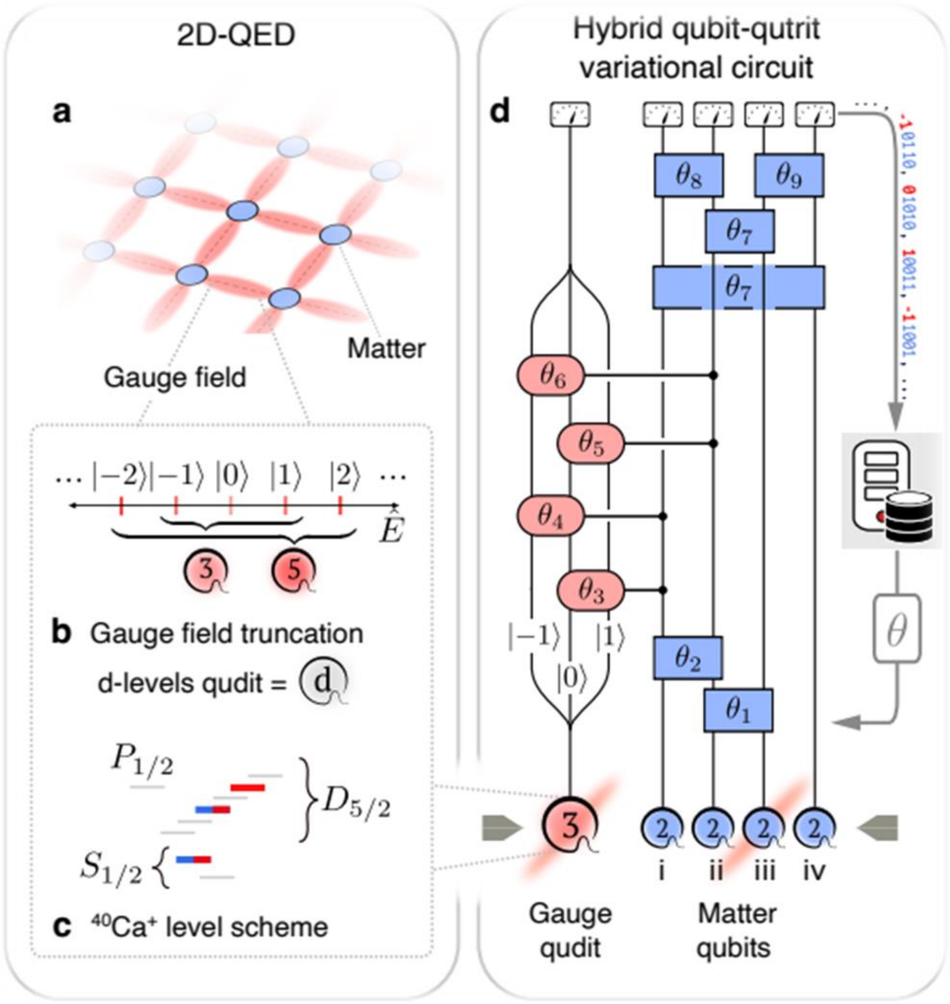
$$H_{E,\text{non-local}} \ni \mathcal{E}^a(\boldsymbol{n}) \mathcal{R}_{ab}(\mathcal{P}) \mathcal{E}^b(\boldsymbol{m})$$



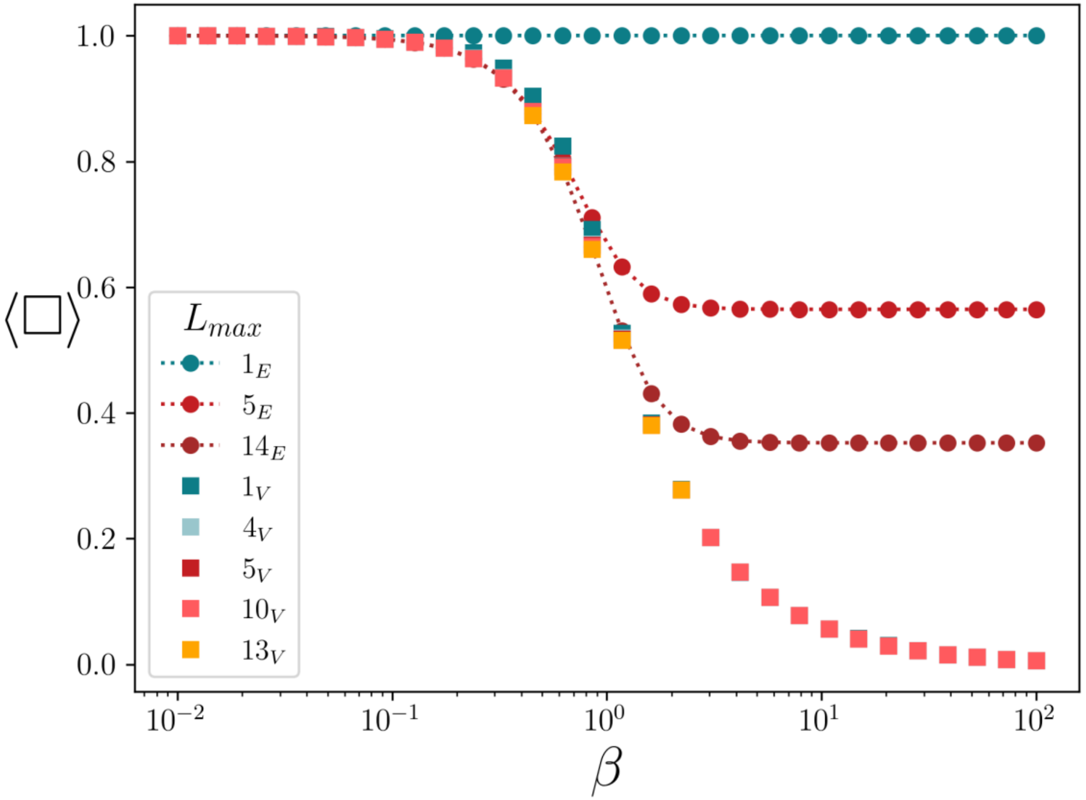
Trapped ions QPU: 2D QED



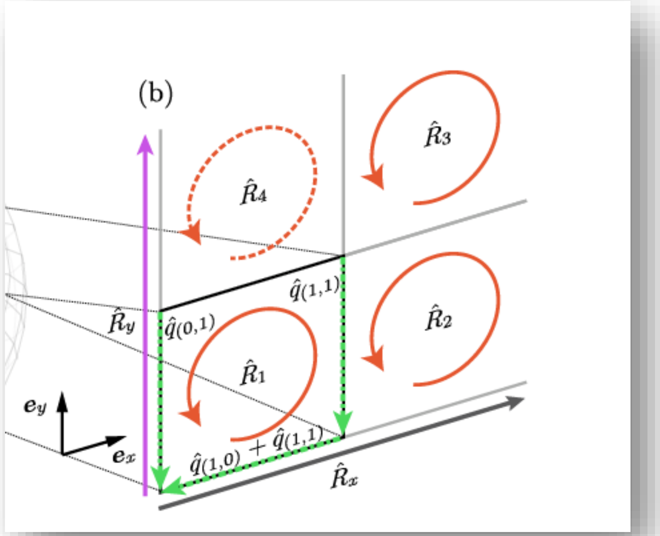
Trapped ions QPU: 2D QED



Plaquette in the electric basis



Another basis for U(1)



[Haase et al., Quantum **5**, 393 (2021)]

$$\hat{H}^{(\mathrm{e})} = \hat{H}_E^{(\mathrm{e})} + \hat{H}_B^{(\mathrm{e})},$$

$$\hat{H}_E^{(\mathrm{e})} = 2g^2 \left[\hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2 - \hat{R}_2(\hat{R}_1 + \hat{R}_3)\right],$$

$$\hat{H}_B^{(\mathrm{e})} = -\frac{1}{2g^2a^2}\left[\hat{P}_1 + \hat{P}_2 + \hat{P}_3 + \hat{P}_1\hat{P}_2\hat{P}_3 + \mathrm{H.c.}\right],$$

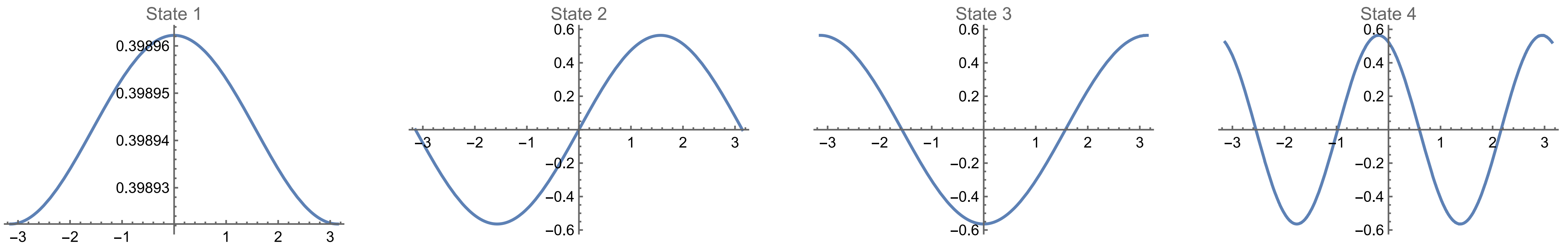
$$H_{0,i} \equiv 2g^2 R_i^2 + \frac{1}{2g^2}(2 - (P_i + P_i^\dagger))$$

$$H_{0,i}(\theta_i) = -2g^2\partial_{\theta_i}^2 + \frac{1}{g^2}(1 - \cos\theta_i)$$

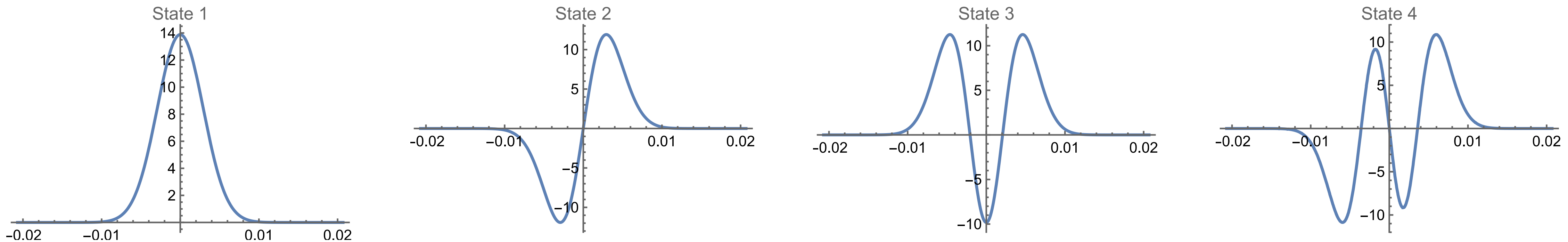
$$P_i = e^{i\theta_i} \qquad R_i = -i\partial_{\theta_i}$$

$$H_{0,i}(\theta_i) = -2g^2\partial^2_{\theta_i} + \frac{1}{g^2}(1-\cos\theta_i)$$

Strong coupling



Weak coupling



Another basis for U(1)

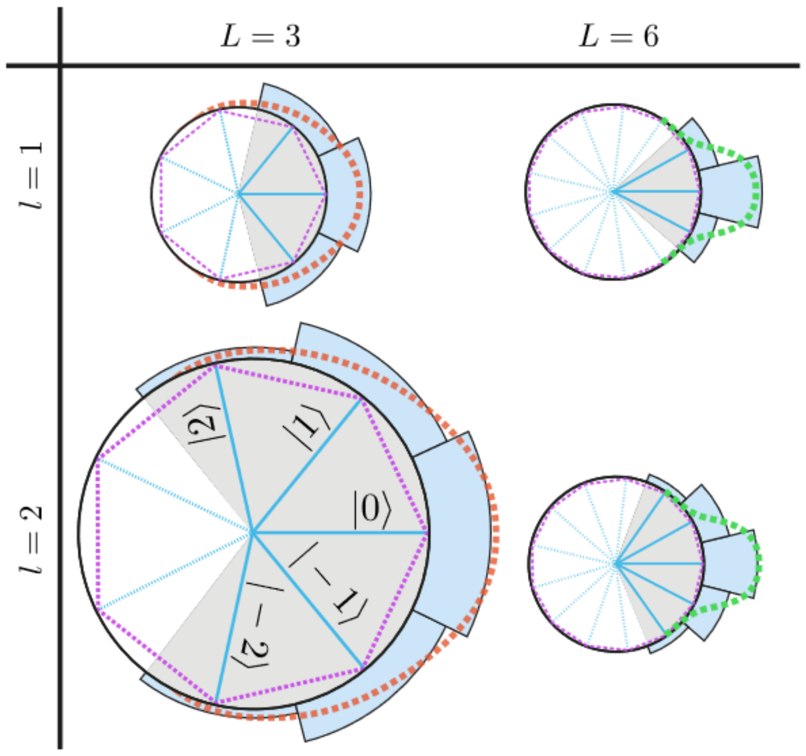


Figure 2: **Discrete approximation of a continuous distribution of states in the magnetic representation.** The ability

$$\hat{H}^{(\mathrm{e})} = \hat{H}_E^{(\mathrm{e})} + \hat{H}_B^{(\mathrm{e})},$$

$$\hat{H}_E^{(\mathrm{e})} = 2g^2 \left[\hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2 - \hat{R}_2(\hat{R}_1 + \hat{R}_3)\right],$$

$$\hat{H}_B^{(\mathrm{e})} = -\frac{1}{2g^2a^2}\left[\hat{P}_1 + \hat{P}_2 + \hat{P}_3 + \hat{P}_1\hat{P}_2\hat{P}_3 + \mathrm{H.c.}\right],$$

$$H_{0,i} \equiv 2g^2 R_i^2 + \frac{1}{2g^2}(2 - (P_i + P_i^\dagger))$$

$$H_{0,i}(\theta_i) = -2g^2\partial_{\theta_i}^2 + \frac{1}{g^2}(1 - \cos\theta_i)$$