Non-abelian $(1+1)$d gauge theories using infinite matrix product states

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Princeton University

Based on work with Ross Dempsey, Anna-Maria Glück, Igor Klebanov and Silviu Pufu
[2508.16363], [2409.19164], [2311.09334]

Trento, September 3rd, 2025

Introduction

- Many works on LGT using MPS: How to deal with the **gauge field**?
  - Integrate out: non-local charge-charge interaction ⇒ mostly finite lattices
    [Byrnes, Sriganesh, Bursill, Hamer 2002; Bañuls, Cichy, Jansen, Cirac 2013; Godfrey, McCulloch 2025; ...]
  - Same footing as matter: non-standard symmetric MPS ⇒ local, but mostly U(1)
    [Buyens, Haegeman, Van Acoleyen, Verschelde, Verstraete 2013; Fujii, Fujikura, Kikukawa, Okuda, Pedersen 2024; ...]

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  - Allows for **infinite lattices** and **non-abelian** gauge groups
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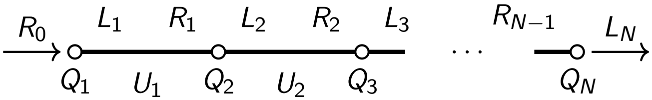
Hamiltonian lattice gauge theories

- Gauge group G: Parallel propagators $U_n \in G$, electric fields $L_n, R_n \in \mathfrak{g}$ on links and matter fields/charges ψ_n/Q_n on sites



- Dynamics + constraint:

Hamiltonian : $H = \frac{ag^2}{2} \sum_{n=1}^{N-1} L_n^2 + H_{\text{matter}}$, Gauss law: $L_n - R_{n-1} = Q_n$,

BCs for electric field set by irreps $\boldsymbol{r}_0/\boldsymbol{r}_N$ acted on by R_0/L_N

- Hilbert spaces:

$$\mathcal{H}_{\text{site},n} = \boldsymbol{R}_n , \qquad \mathcal{H}_{\text{gauge},n} = L^2(G) = \bigoplus_{\boldsymbol{r} \in \text{irreps}} \boldsymbol{r}_L \otimes \overline{\boldsymbol{r}_R} \; (= \text{span}\{|q\rangle\} \quad \text{for U(1)}) .$$

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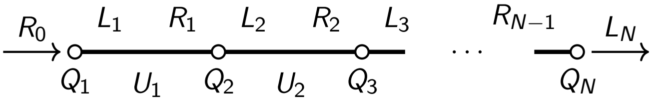

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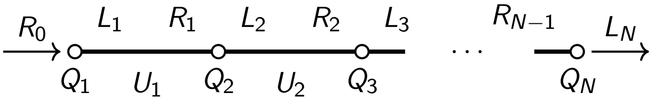

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  $\Rightarrow$ Solve using **symmetric MPS** $\Rightarrow$ restores **locality**

$$|A_1, \ldots, A_N\rangle = \sum_{\{i_n\}} A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N} |i_1\rangle |i_2\rangle \ldots |i_N\rangle =$$

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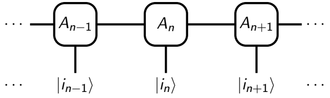

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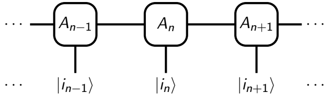

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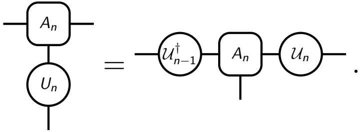
Symmetric matrix product states

- Global **on-site** symmetry $U = \prod_n U_n \Rightarrow$ local constraints on MPS tensors
  [Sanz, Wolf, Perez-Garcia, Cirac 2009]



- For infinitesimal transformation $\Rightarrow$ Gauss law for MPS tensors

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- Keep track of **virtual irreps** e.g. for U(1) symmetry

$= 0 \qquad \text{if} \qquad q(\alpha_{n-1}) + q(i_n) \neq q(\alpha_n)\,.$

In general, A_n can be non-zero when $\boldsymbol{r}_n \subset \boldsymbol{R}_n \otimes \boldsymbol{r}_{n-1}$

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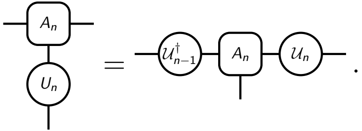

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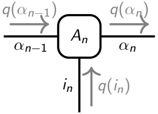

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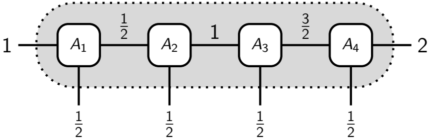
Symmetric MPS as gauge theory states

- SU(2) example of symmetric (matrix) product state:



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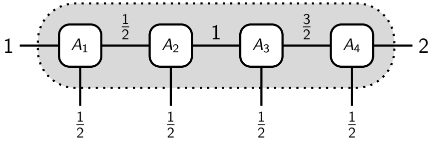

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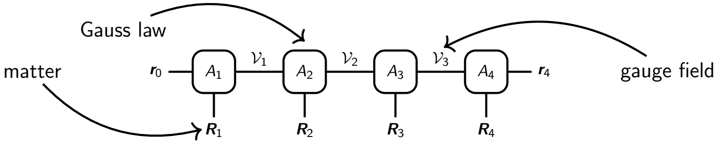

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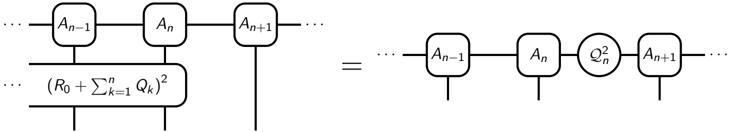
Link operators

- We can extract the state of the electric field by acting on **virtual** indices
- L_n^2 acts as new type of operators acting on links [Dempsey, Glück, Pufu, BTS 2025]
  $\Rightarrow$ restores **locality** and **translational invariance**

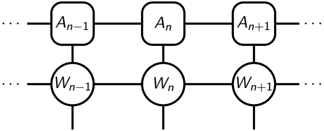
$L_n^2 \, |A_1, \ldots, A_N\rangle =$  ,

with $\mathcal{Q}_n^2$ block diagonal computing the Casimir of irreps

Link-enhanced matrix product operator

- Gauge-fixed matter Hamiltonian as an MPO [Verstraete, García-Ripoll, Cirac 2004; McCulloch 2007]

$$H_{\text{matter}} \left| A_1, \ldots, A_N \right\rangle =$$



- Combine ordinary and link MPO into a **LEMPO** [Dempsey, Glück, Pufu, BTS 2025]

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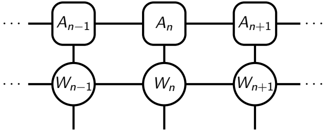

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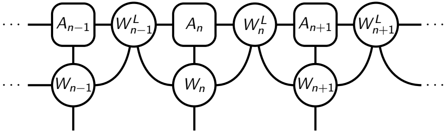

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LEMPO for the Schwinger model

- Bosonized lattice Schwinger model $Q_n = \frac{1}{2}(\sigma_z - \delta_{n,\text{odd}})$ [see Takuya Okuda's talk]

$$H = \sum_n \left[\frac{g^2 a}{2} \left(L_n + \frac{\theta}{2\pi} \right)^2 + \frac{1}{2a} \left(\sigma_n^+ U_n \sigma_{n+1}^- + \sigma_{n+1}^+ U_n^\dagger \sigma_n^- \right) + \frac{m_\text{lat}}{2} (-1)^n \sigma_n^z \right] .$$

- LEMPO representation:

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What can LEMPOs do for you?

- **Translational invariance**: Works directly in the thermodynamic limit
- **Abstracts the group theory**: Gauss law, fusion of irreps etc. are handled by well-known symmetric MPSs ⇒ Existing codes e.g. `TeNPy`, `ITensors.jl`, `QSpace`, or `MPSKit.jl`
- **Physical virtual bonds**: MPS ansatz for non-trivial flux tube sector
- **No fixed electric cut-off**: The virtual space can be dynamically adjust (e.g. DMRG)
- **Works with MPS algorithms**: DMRG, VUMPS, TDVP etc.

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Uniform MPS

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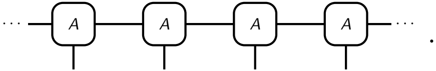

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- Quasiparticle ansatz: Bound states as local excitations of the ground state [Haegeman, Osborne, Verstraete 2013]

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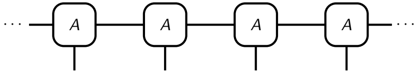
$|\psi\rangle =$ 

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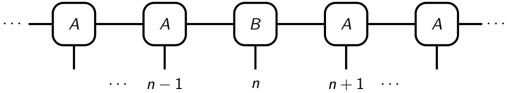
$|B, k\rangle = \sum_n e^{ikn}$ 

Adjoint QCD_2: Continuum theory

- Two dimensional $SU(N_c)$ gauge theory with adjoint Majorana fermion of mass m
  [Dalley, Klebanov 1993;. . .; Katz, Tavares, Xu 2014; Cherman, Jacobson, Tanizaki, Ünsal 2020;. . .]

$$\mathcal{L} = \mathrm{tr}\left(-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi - m\bar{\Psi}\Psi\right) .$$

- One parameter m/g, by $(\mathbb{Z}_2)_\chi : \psi \to -\psi$, we take $m \geq 0$
  - $\boxed{m = 0}$, mass gap + vanishing string tension [Gross, Klebanov, Matytsin, Smilga 1996]
  - Supersymmetry at $\boxed{m_{\mathrm{SUSY}} = g\sqrt{\frac{N_c}{2\pi}}}$ [Kutasov 1994; Popov 2022; Klebanov, Pufu, BTS, Witten 2025]
  - Generic m, mass gap + confinement
- Center symmetry: N_c different flux tube sectors $\Leftarrow$ probe of confinement

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Adjoint QCD$_2$: Lattice Hamiltonian

- Staggered real fermion w/ $\{\chi_n^a, \chi_m^b\} = \delta^{ab}\delta_{nm}$, $\{a, b, \ldots\}$ adjoint indices
  [Dempsey, Klebanov, Pufu, BTS 2023, 2024]

$$H = \sum_n \left[\frac{ag^2}{2} L_n^a L_n^a - \frac{i}{2a} \chi_n^a U_n^{ab} \chi_{n+1}^b - \frac{im}{2} (-1)^n \chi_n^a U_n^{ab} \chi_{n+1}^b \right]$$

- Exact diagonalization: $N_c = 2$ up to 12 sites, $N_c = 3$ only up to 6 sites
- On-site charge $Q_n^a = -\frac{i}{2} f^{abc} \chi_n^b \chi_n^c$:

$$\boldsymbol{R}_n = \begin{cases} \sqrt{2} \times \mathbf{2}\,, & N_c = 2\,, \\ 2 \times \mathbf{8}\,, & N_c = 3\,. \end{cases}$$

- Flux tube sectors on the lattice, $N_c = 3$ e.g.

$p = 0$: $\qquad$ $p = 1$:

$\Rightarrow$ Construct MPS ansatz with virtual spaces in pth sector

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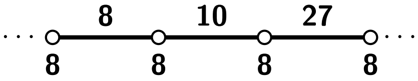

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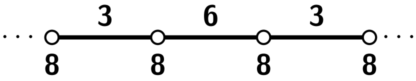


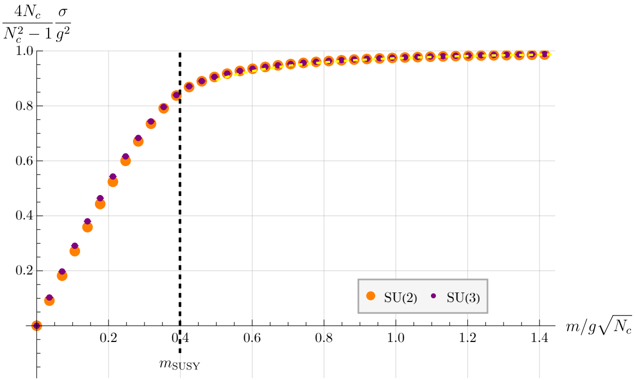
$\Rightarrow$ Construct MPS ansatz with virtual spaces in pth sector

String tension $N_c = 2, 3$

- Fundamental string tension from vacuum energy density (extrapolated to $a = 0$)

$$\sigma = \lim_{a \to 0} [\varepsilon_{p=1}(a) - \varepsilon_{p=0}(a)] .$$

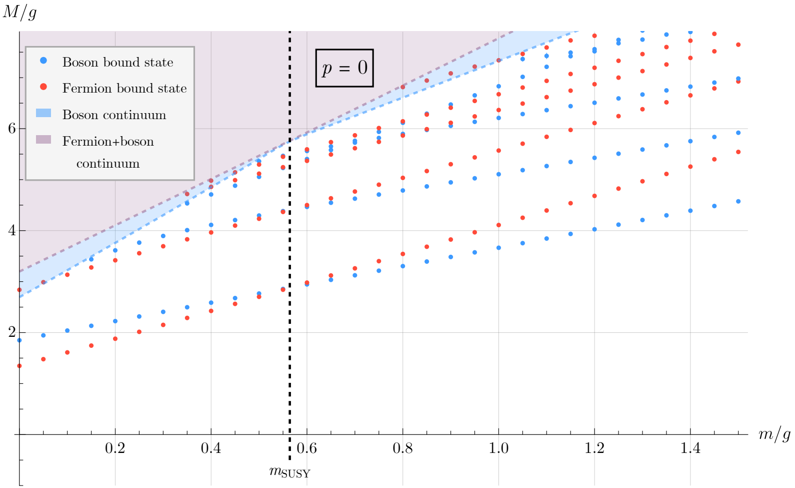
- Consistent with $m = 0$ prediction and 1-loop estimate for $m \gg g$



[Dempsey, Glück, Pufu, BTS 2025]

Bound states $N_c = 2$

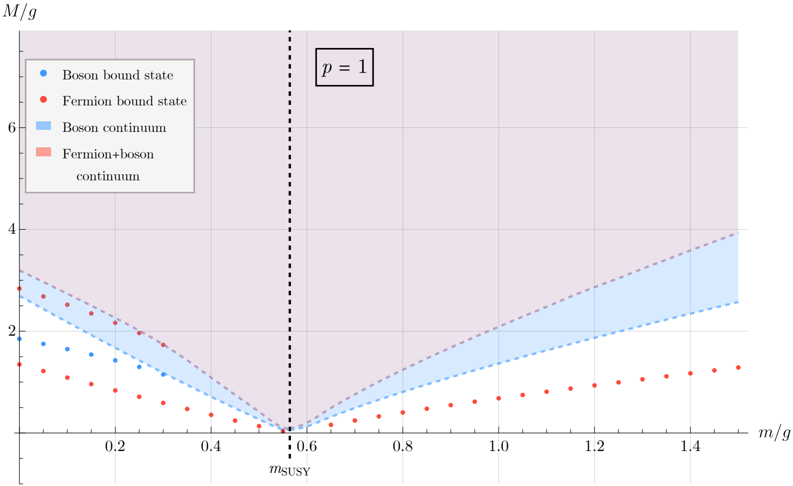
- Gapped @ $m = 0$, fermion/boson degeneracy @ $m = m_{\text{SUSY}}$
- Agrees with DLCQ at finite N_c [Dempsey, Klebanov, Lin, Pufu 2023]



[Dempsey, Glück, Pufu, BTS 2025]

Bound states $N_c = 2$

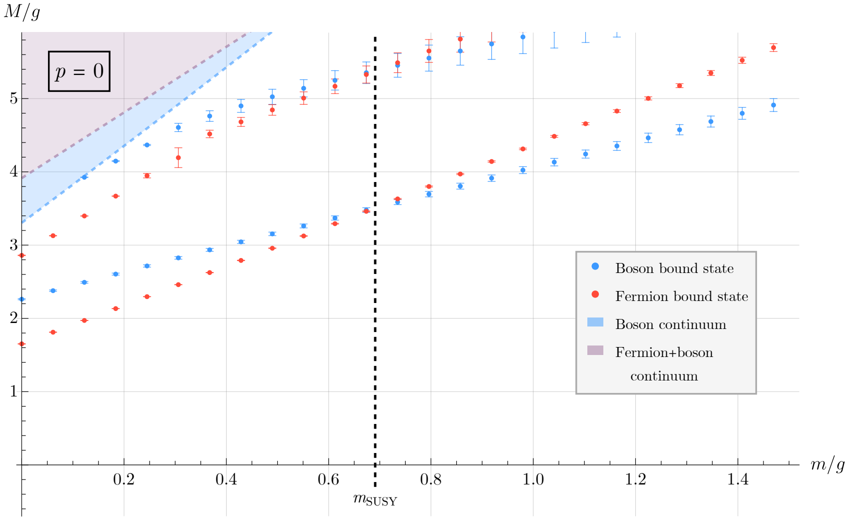
- m_{SUSY}: Non-zero string tension ⇒ spontaneous breaking of SUSY ⇒ goldstino
- Non-trivial flux tube sector not observable in DLCQ



[Dempsey, Glück, Pufu, BTS 2025]

Bound states $N_c = 3$

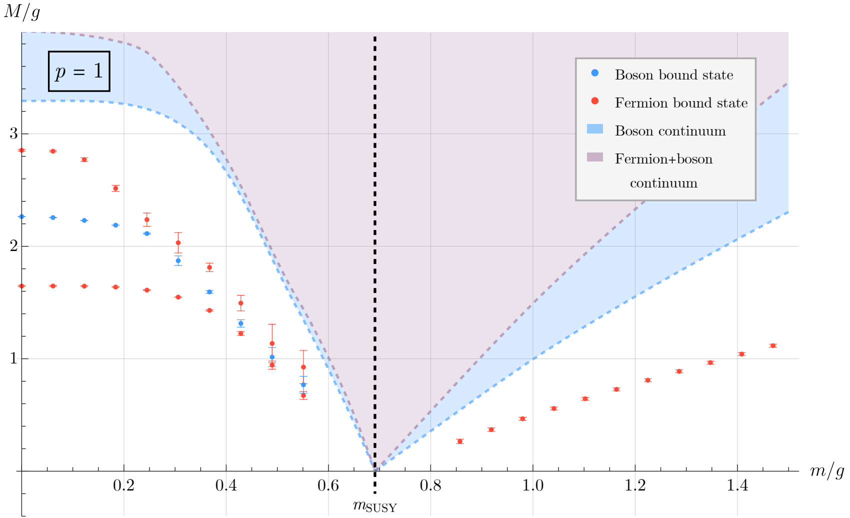
- Numerics are more challenging for $N_c = 3$, predicted behavior at $m = 0, m_{\text{SUSY}}$



[Dempsey, Glück, Pufu, BTS 2025]

Bound states $N_c = 3$

- Small mass gaps are computationally expensive: still trend towards SUSY mass



[Dempsey, Glück, Pufu, BTS 2025]

Summary

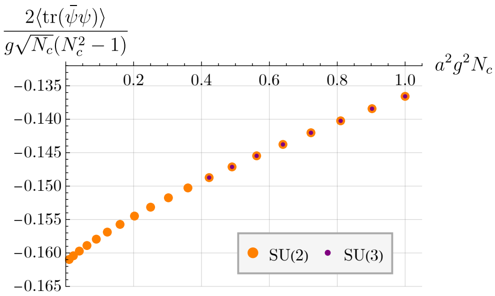
- Symmetric MPSs furnish a natural setting for Hamiltonian LGT states
- We introduced LEMPOs as a MPO formulation of general gauge theories compatible with standard MPS algorithms
- LEMPOs allow for infinite system size methods, which give high precision results for ground state properties and bounds state masses
- Analyzed lattice realization of $N_c = 2, 3$ adjoint QCD_2 and found agreement with theoretical predictions

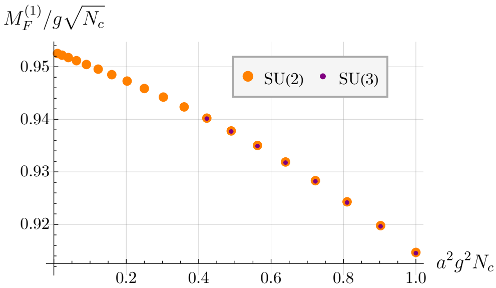
Future directions:

- Non-invertible symmetries from the lattice
- Real-time dynamics, 2-paricle S-matrix
- $(2+1)$d methods

Extra Slides

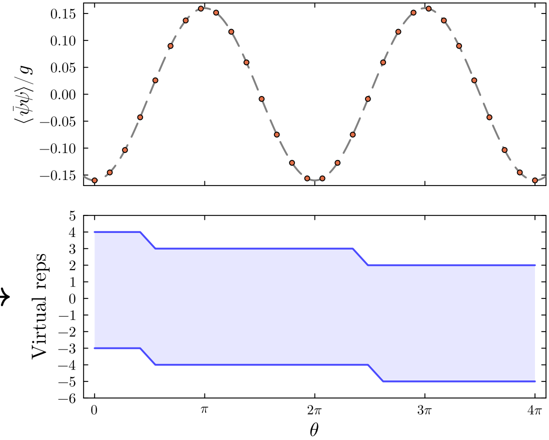
't Hooft scaling at finite a





Dynamical electric field cutoff

- For the effective electric field $L_{\text{eff}} = L + \frac{\theta}{2\pi}$ to be periodic in θ, the virtual reps have to compensate. This is done **dynamically** in VUMPS e.g. $\langle\bar{\psi}\psi\rangle$ is periodic



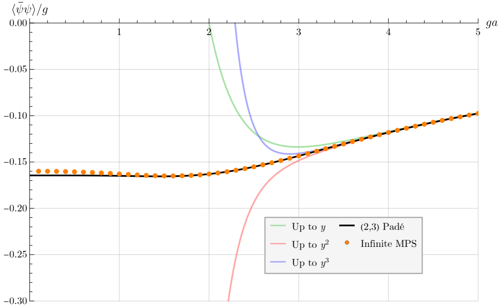
Range of U(1) irrep used to encode the electric field ⟶

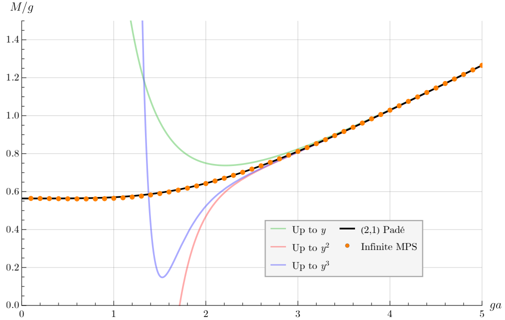
Exact result @ $m = 0$: $\langle\bar{\psi}\psi\rangle/g = -\frac{e^{\gamma}}{2\pi^{3/2}}\cos\theta$

Strong coupling expansion and Padé approximants

- Compute observables perturbatively in $y \equiv \frac{1}{(ag)^4} \ll 1$ and extrapolate to $a \to 0$ using Padé approximants [Banks, Susskind, Kogut 1976; Hamer, Weihong, Oitmaa 1997]
  $\Rightarrow$ Improve using mass shift [Dempsey, Klebanov, Pufu, Zan 2022]

$$M_S/g = \frac{ga}{4}\left(1 + 8y - 144y^2 + 4448y^3 + \ldots\right) \approx \frac{ga}{4}\left(\frac{3 + 190y + 2432y^2}{3 + 94y}\right)^{1/4} .$$





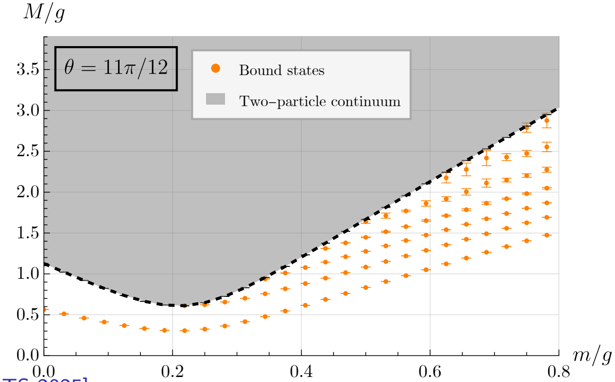
[Dempsey, Glück, Pufu, BTS 2025]

Precision study of observables and bound states

- Benchmark at $m = 0$ vs. exact solution

$$\langle\bar{\psi}\psi\rangle/g = -0.159926(3) \quad \text{(lattice)} \quad = -0.1599288\ldots \quad \text{(exact)},$$
$$M_S/g = 0.564191(3) \quad \text{(lattice)} \quad = 0.5641896\ldots \quad \text{(exact)}.$$

- Bound state spectrum



[Dempsey, Glück, Pufu, BTS 2025]