A Quantum Link Model with Heavy Hex Topology

Uwe-Jens Wiese

Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics, Bern University

Hamiltonian Lattice Gauge Theories:
Status, Novel Developments
and Applications
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Outline

From Wilson's Lattice Gauge Theory to Quantum Link Models

$U(1)$ Quantum Link Model on a Triangular Lattice

Quantum Computation on IBM's Heavy Hex Architecture

Conclusions

Additional Material:
$SU(N)$ Quantum Links and D-Theory Formulation of QCD

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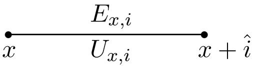
Hamiltonian formulation of Wilson's $U(1)$ lattice gauge theory



$$U = \exp(i\varphi), \; U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

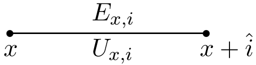
$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{e^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2e^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link

Quantum link formulation of $U(1)$ lattice gauge theory



$$U = S^+, \; U^\dagger = S^-$$

Electric field operator E

$$E = S^3, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

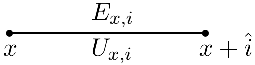
$U(1)$ gauge invariant Hamiltonian

$$H = \frac{e^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2e^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in a finite-dimensional Hilbert space per link

Outline

$U(1)$ quantum links from spins $\frac{1}{2}$



$$U = S^1 + iS^2 = S^+, \; U^\dagger = S^1 - iS^2 = S^-$$

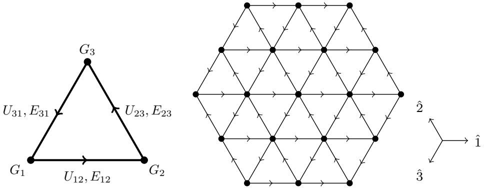
Electric flux operator E

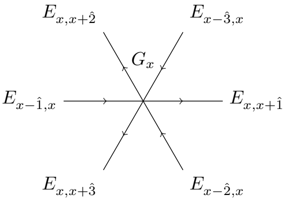
$$E = S^3, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 2E$$

Ring-exchange plaquette Hamiltonian

$$H = -\frac{1}{2e^2} \sum_{\triangle} (U_\triangle + U_\triangle^\dagger) \; , \quad U_\triangle = U_{12} U_{23} U_{31}$$

Triangular lattice and Gauss law



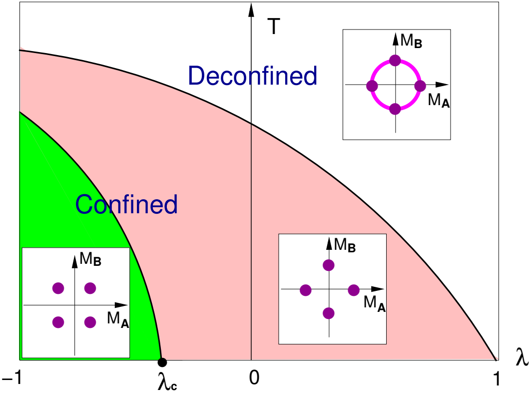


D. Horn, Phys. Lett. B100 (1981) 149
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Hamiltonian with Rokhsar-Kivelson term

$$H = -\frac{1}{2e^2}\left[\sum_{\triangle}(U_{\triangle} + U_{\triangle}^{\dagger}) - \lambda \sum_{\triangle}(U_{\triangle} + U_{\triangle}^{\dagger})^2\right]$$

Phase diagram



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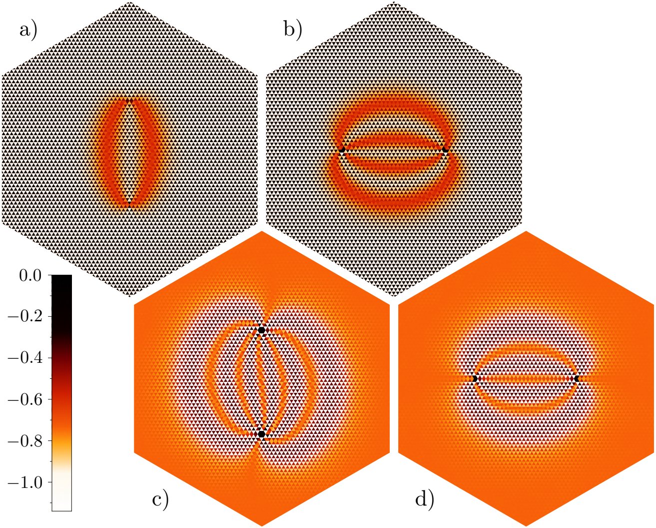
Energy distribution for the strings connecting external charges

Charges ± 1 (a) and ± 2 (b) for $\lambda > \lambda_c$



Charges ± 3 (c) and ± 2 (d) for $\lambda < \lambda_c$

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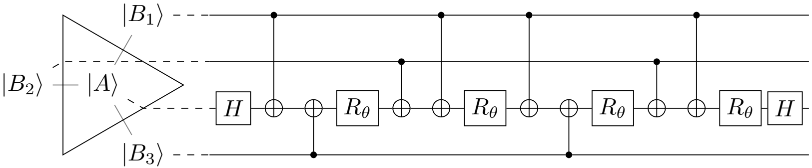
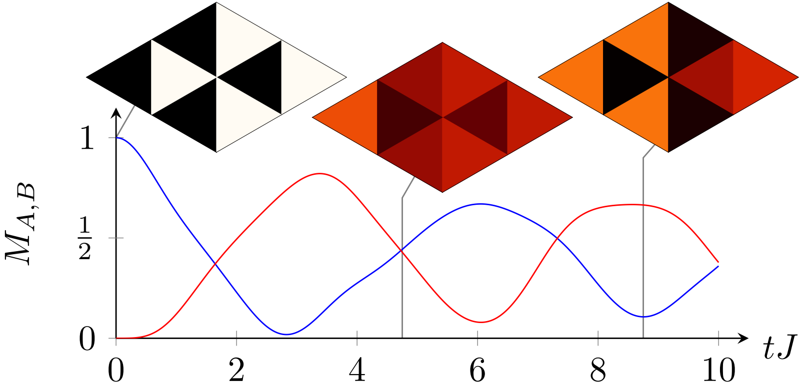
Circuit decomposition of the time-evolution operator



Time-evolution of the order parameters



D. Banerjee, S. Caspar, F.-J. Jiang, J.-H. Peng, UJW, Phys. Rev. Research 4 (2022) 023176

Outline

Collaboration: Debasish Banerjee, Anthony Gandon, Emilie Huffman, Gurtej Kanwar, Alessandro Mariani, Francesco Tacchino, Ivano Tavernelli, UJW

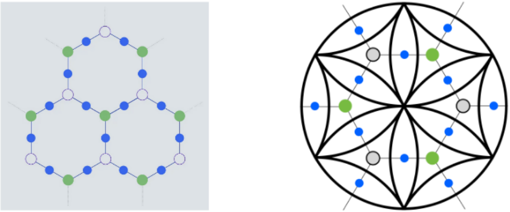
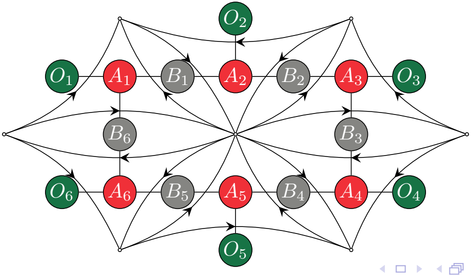


FIGURE 1: Left: Heavy hex topology. Right: Hexafoil lattice, the dual of the heavy hex topology.



Dynamics in the dual height formulation

Trotter decomposition into triangle and petal and steps:

$$\exp(i(H_\triangle + H_\diamond)t) = \lim_{n\to\infty} \left(\exp(iH_\triangle t/n)\exp(iH_\diamond t/n)\right)^n$$

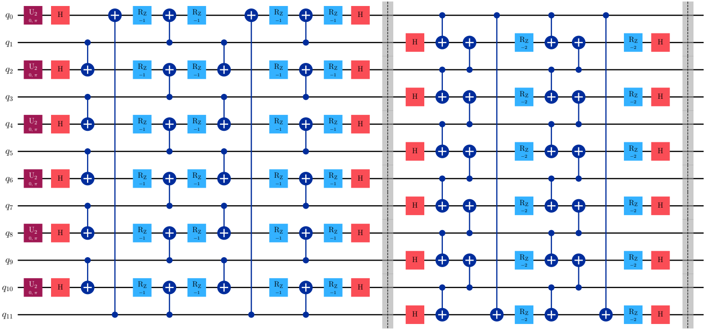


Figure: Quantum circuit for one Trotter step.

There are no long-range interactions. All CNOTs are local.

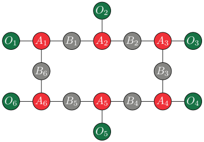
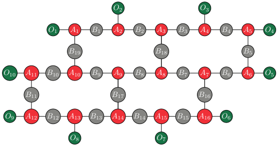
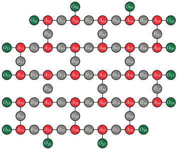
Lattice	Qubits	CNOTs/Trotter
	12	48
	35	164
	63	308

Table: Depths for a single Trotter step and different system sizes.

4-hexagon results: picking the best qubit loops

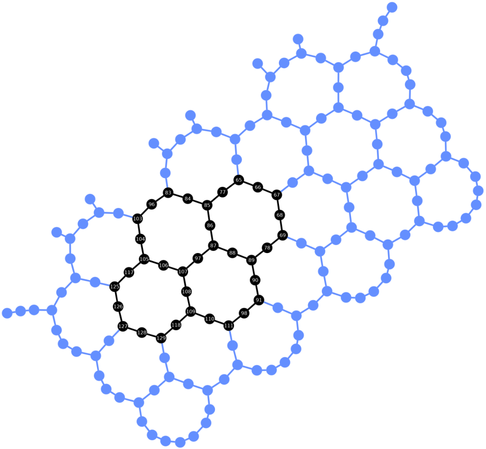
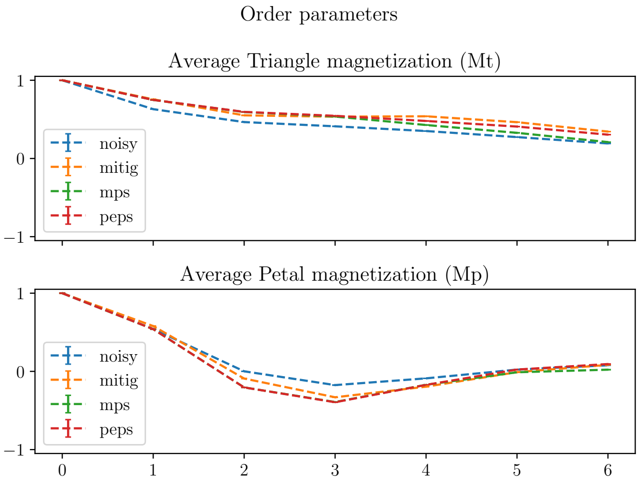


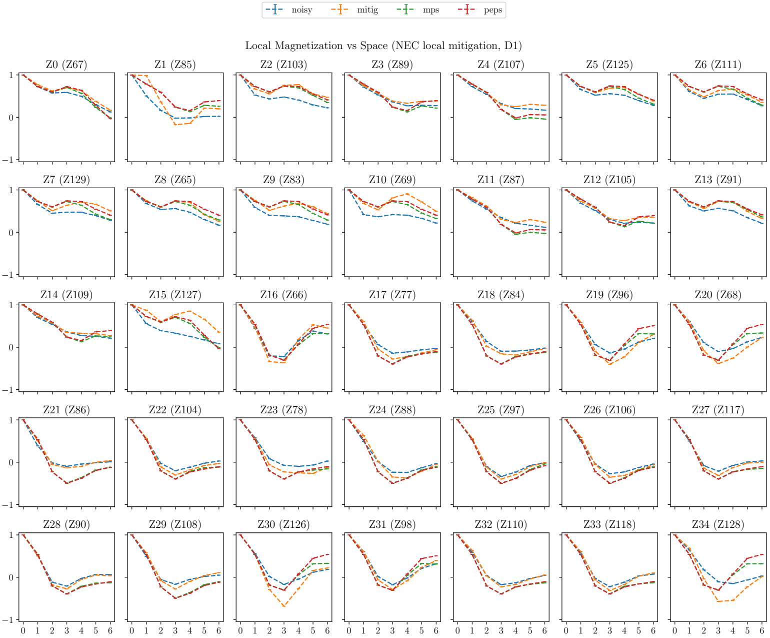
Figure: Best loop: $\prod_{2q} \mathcal{F}_{2q} = 77.2\%$, $\min(\mathcal{F}_{readout}) = 87.4\%$, entangling gate fidelity (averaged over 114 qubits): $\mathcal{F}_{2q} = 99.5\%$.

4-hexagon results

Error mitigation by twirling, Clifford perturbation theory in Δt.
Successful strategy: reduce bias at the cost of increased variance.



4-hexagon results: local magnetization per dual site



Flux strings connecting external charges

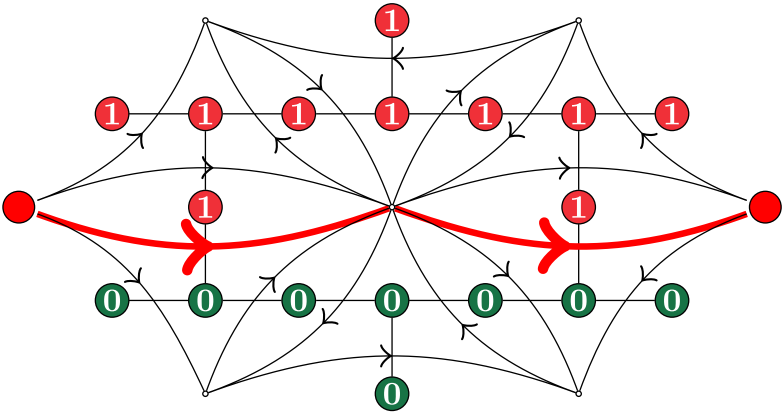
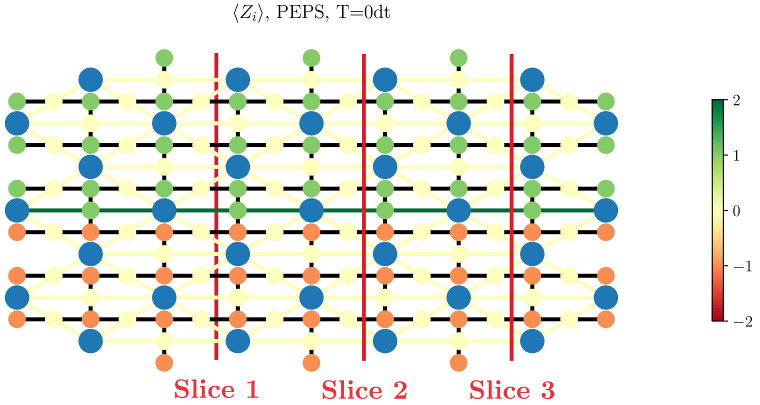
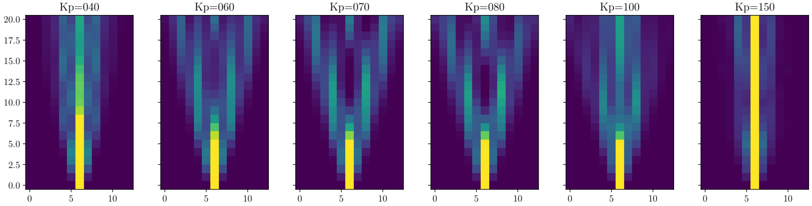


Figure: Initial configuration with external charges at the boundary

PEPS results for 42 triangles and 72 petals





IBM Heron results for 16 triangles and 29 petals

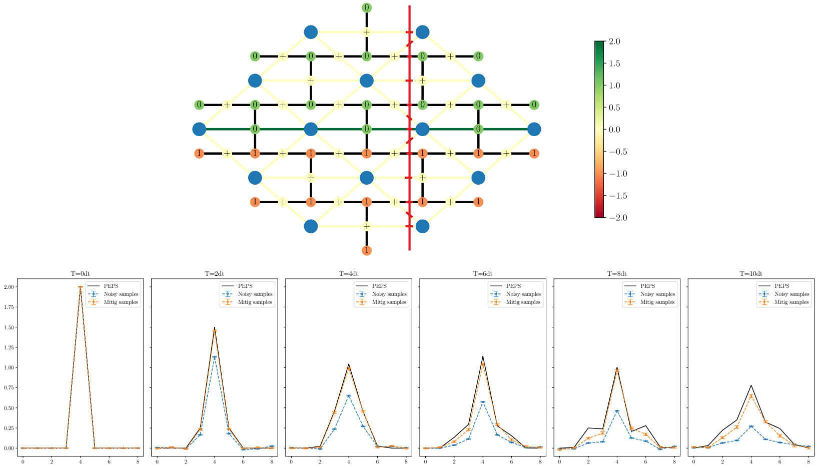


Figure: Lattice geometry of initial state and time evolution.

IBM Heron results for 42 triangles and 72 petals

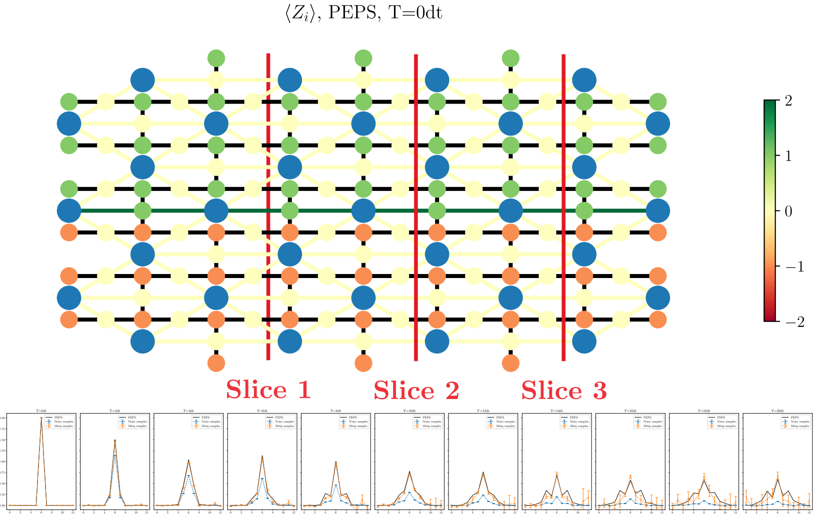


Figure: Lattice geometry of initial state and time evolution.

Outline

Conclusions

• Quantum link models provide a generalization of Wilson's lattice gauge theory with a finite-dimensional Hilbert space per link, which allows their resource-efficient realization in quantum computations.

• The hexafoil quantum link model is taylor-made for IBM's heavy hex qubit topology. It has a rich dynamics with quantum phase transitions separating distinct confined phases. The corresponding strings are interfaces separating different phases.

• This allows quantum computations of string dynamics, in particular, of the real-time evolution of string fluctuation and fractionalization.

• Properly error-mitigated quantum computations are consistent with PEPS results for moderate system sizes. Computations have been performed with up to 114 qubits over 20 Trotter steps, corresponding to a large number of CNOT gates.

• Quantum links can be used to regularize asymptotically free gauge theories including QCD. The continuum limit is then not pursued a la Wilson, but is taken efficiently via dimensional reduction.

• The path towards quantum computation of QCD will be a long one. However, with a lot of interesting physics along the way.

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$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij},\ U^{ij\dagger} = S_1^{ij} - iS_2^{ij},\ i,j \in \{1,2,\ldots,N\},\ [U^{ij},(U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c,\ [R^a, R^b] = if_{abc}R^c,\ a,b,c \in \{1,2,\ldots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U,\ [R^a, U] = U\lambda^a,\ [E, U] = U$$

Algebraic structures of different quantum link models

$U(N) : U^{ij},\ L^a,\ R^a, E,\ 2N^2+2(N^2-1)+1 = 4N^2-1$ $SU(2N)$ generators

$SO(N) : O^{ij},\ L^a,\ R^a,\ N^2+2\frac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

$Sp(N) : U^{ij},\ L^a,\ R^a,\ 4N^2+2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

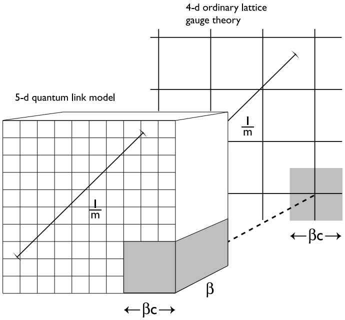
R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Low-energy effective action for a quantum link model in a $(4+1)$-d massless non-Abelian Coulomb phase

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \; \frac{1}{2e^2} \left(\mathrm{Tr}\; G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\; G_{\mu 5} G_{\mu 5} \right),$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

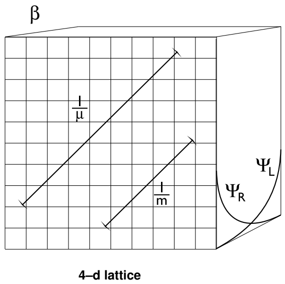
$$S[G_\mu] \to \int d^4x \; \frac{1}{2g^2} \mathrm{Tr}\; G_{\mu\nu} G_{\mu\nu}, \;\; \frac{1}{g^2} = \frac{\beta}{e^2}, \;\; \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$



R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Quarks as Domain Wall Fermions

$$
\begin{aligned}
H &= J \sum_{x,\mu\neq\nu} \mathrm{Tr}[U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}] + J' \sum_{x,\mu} [\det U_{x,\mu} + \det U^\dagger_{x,\mu}] \\
&+ \frac{1}{2} \sum_{x,\mu} [\Psi^\dagger_x \gamma_0 \gamma_\mu U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 \gamma_\mu U^\dagger_{x,\mu} \Psi_x] + M \sum_x \Psi^\dagger_x \gamma_0 \Psi_x \\
&+ \frac{r}{2} \sum_{x,\mu} [2\Psi^\dagger_x \gamma_0 \Psi_x - \Psi^\dagger_x \gamma_0 U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 U^\dagger_{x,\mu} \Psi_x].
\end{aligned}
$$

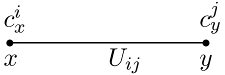


$$
\mu = 2M \exp(-M\beta), \; \frac{1}{m} \propto \exp\left(\frac{24\pi^2\beta}{(11N - 2N_f)e^2}\right), \; M > \frac{24\pi^2}{(11N - 2N_f)e^2}
$$

Fermionic rishons at the two ends of a link

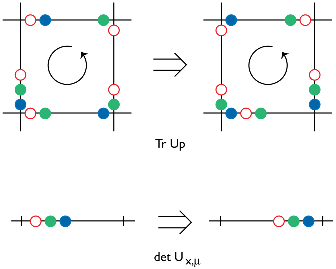
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra



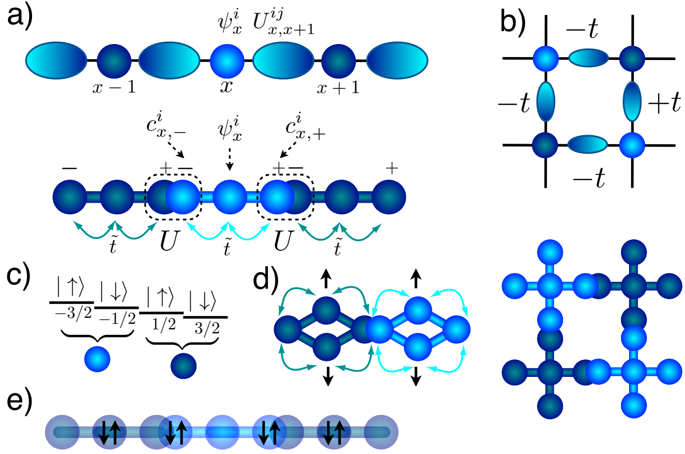
$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a "rishon abacus" implemented with ultra-cold atoms be used as a quantum simulator?



R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin



D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303