Simulating gauge-invariant SU(2) Yang-Mills Theory with near-term quantum computers

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Motivation

- Ultimate goal: Nonabelian LGT in higher dimensions (incl. matter)
- Crucial ingredient for any interacting QFT: n-point functions
  - Example: 2-point function, i.e. propagator
    $$\langle \boldsymbol{\Omega} | \widehat{\mathbf{W}}(\boldsymbol{x_2}, \boldsymbol{t_2}) \widehat{\mathbf{W}}(\boldsymbol{x_1}, \boldsymbol{t_1})^{+} | \boldsymbol{\Omega} \rangle$$
  - Spatial correlator ⇔ time-independent variant ($\boldsymbol{t_1} = \boldsymbol{t_2} = \mathbf{0}$)
- Acquire vacuum vector $\boldsymbol{\Omega}$ via VQE (variational quantum eigensolver)

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3 quantum numbers on each edge

- What basis to choose?
    - Large # of quantum d.o.f. on lattice
    - Bosonic nature of Hilbert space
    - Retain gauge-invariance
    - VQE scalability (barren plateau problem)

Gauge-invariant Hilbert space

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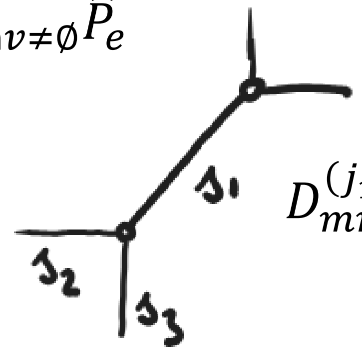
SU(2) Gauss constraint:

$$G_{SU(2)}(v) = \Sigma_{e \cap v \neq \emptyset} \hat{P}_e$$



$$D_{mn}^{(j_1)}(h_1)$$

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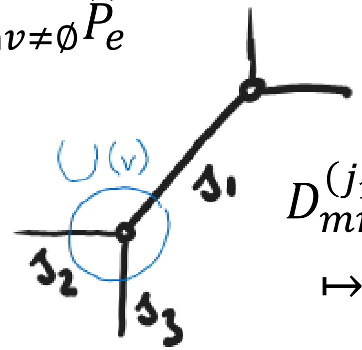

$$D^{(j_1)}_{mn}(h_1)$$

$$\mapsto \Sigma_{m'} [U(v)]_m^{\ m'} D^{(j_1)}_{m'n}(h_1)$$

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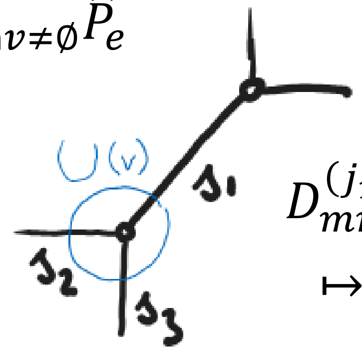

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Spin-network basis of $\mathcal{H}_G$:

[Penrose ´71]

$$|j_1, j_2, j_3, \ldots\rangle := \Sigma_{m_1 m_2 m_3} D^{(j_1)}_{m_1 \ldots}(h_1)\, D^{(j_2)}_{m_2 \ldots}(h_2)\, D^{(j_3)}_{m_3 \ldots}(h_3) \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \ldots$$

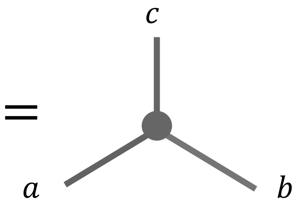
- Implement Gauss constraint (fully)
- Reduce quantum numbers

- 3j-symbols enforce symmetry conditions: *[Wigner ´93]*
  1. $j_1 + j_2 + j_3 \in 2\mathbb{N}$
  2. $|j_1 - j_2| \leq j_3 \leq j_1 + j_2$

Gauge transformations

Gauge transformation on holonomies: $h(e) \mapsto U(e_o)h(e)U(e_1)^+$

$$\begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \end{pmatrix} = \begin{pmatrix} a & b & c \\ \alpha' & \beta' & \gamma' \end{pmatrix} D^{(a)}_{\alpha'\alpha}(U) D^{(b)}_{\beta'\beta}(U) D^{(c)}_{\gamma'\gamma}(U) =$$



Spin network function form a basis of intertwiner space:

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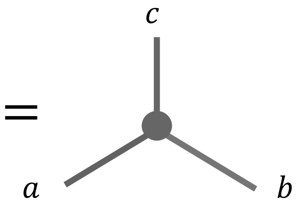

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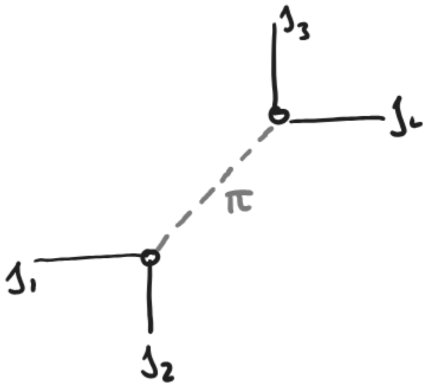
$|j_1, j_2, j_3, j_4, \pi\rangle :=$



$$= \Sigma_{m_1 m_2 m_3}\, D^{(j_1)}_{m_1 \ldots}(h_1)\, D^{(j_2)}_{m_2 \ldots}(h_2) D^{(j_3)}_{m_3 \ldots}(h_3) D^{(j_4)}_{m_4 \ldots}(h_4)$$

$$\begin{pmatrix} j_1 & j_2 & \pi \\ m_1 & m_2 & p \end{pmatrix} \begin{pmatrix} j_3 & j_4 & \pi \\ m_3 & m_4 & -q \end{pmatrix} (-1)^{\pi - p} D^{(\pi)}_{pq}(1)$$

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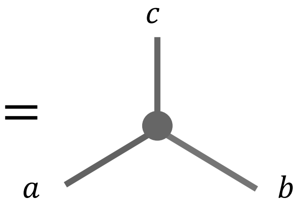
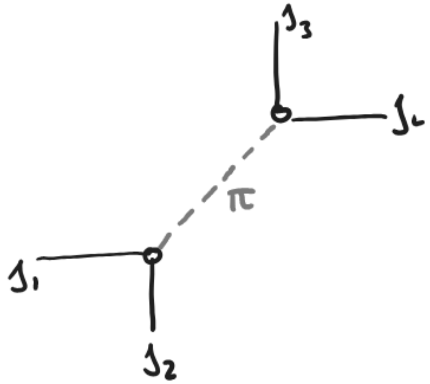


Spin network function form a basis of intertwiner space:

$$|j_1, j_2, j_3, \dots\rangle \coloneqq \Sigma_{m_1 m_2 m_3} D^{(j_1)}_{m_1 \dots}(h_1)\, D^{(j_2)}_{m_2 \dots}(h_2) D^{(j_3)}_{m_3 \dots}(h_3) \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \dots$$

$|j_1, j_2, j_3, j_4, \pi\rangle \coloneqq$



$$= \Sigma_{m_1 m_2 m_3}\, D^{(j_1)}_{m_1 \dots}(h_1)\, D^{(j_2)}_{m_2 \dots}(h_2) D^{(j_3)}_{m_3 \dots}(h_3) D^{(j_4)}_{m_4 \dots}(h_4)$$

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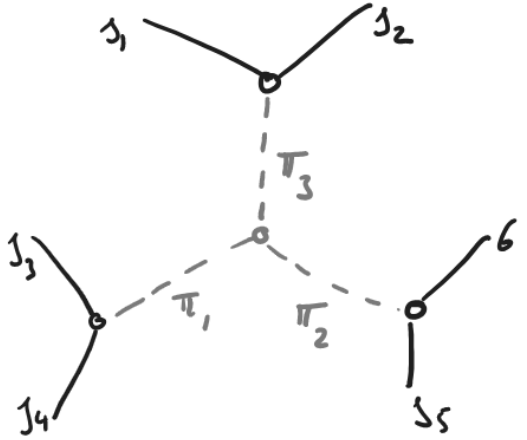
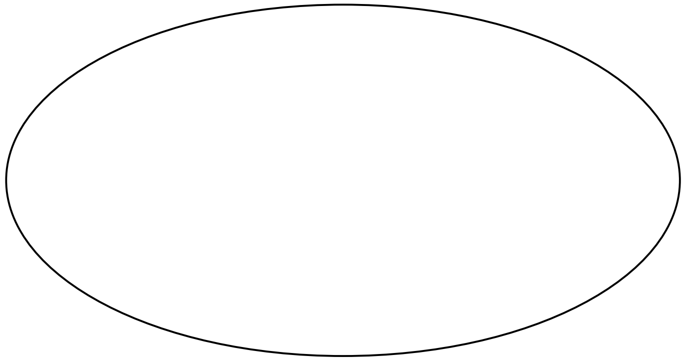
6-valent vertex decomposition:



LGT Overview

- Long range entangelement due to gauge constraint G_v

$$\mathcal{H} = \bigotimes_e \mathrm{L}_2(\mathrm{SU}(2))$$



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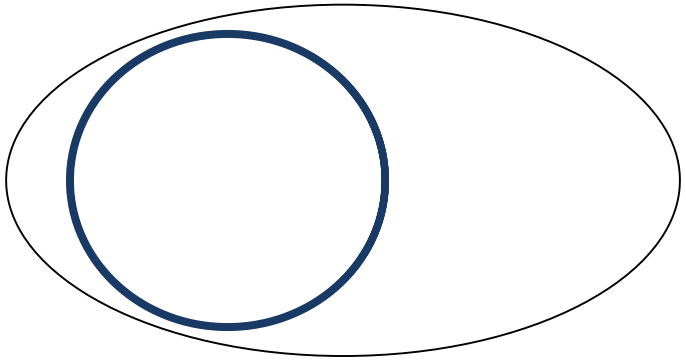

⇒ Restriction to

gauge-invariant Hilbert space

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- Long range entangelement due to gauge constraint G_v

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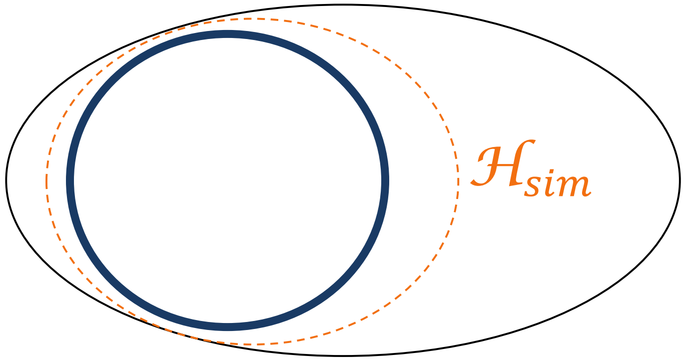

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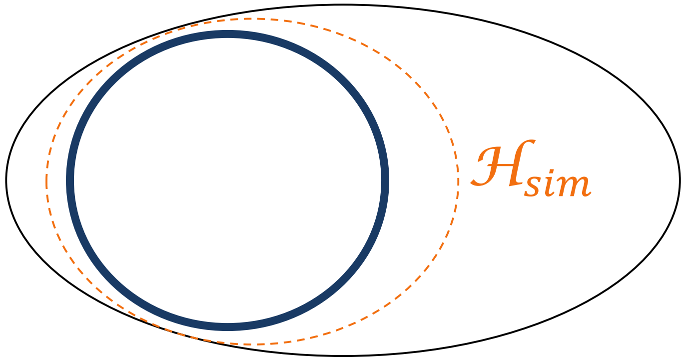
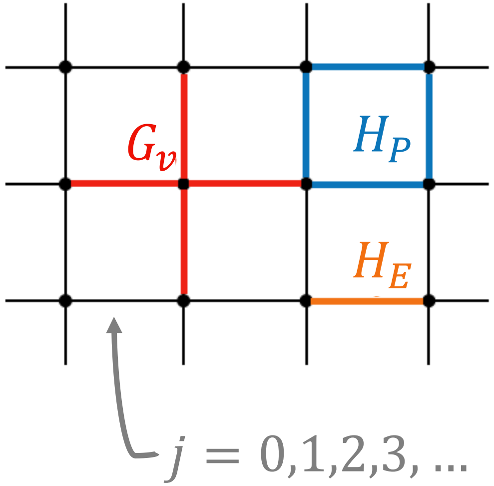
⇒ Restriction to (almost only) **gauge-invariant Hilbert space**





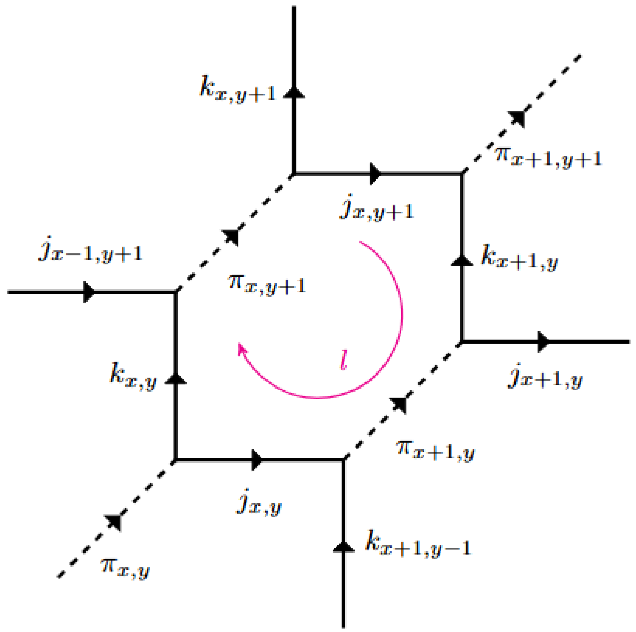
- **Yang-Mills Hamiltonian** *[Kogut&Susskind ´75]*

$$H_{YM} = H_E + g^2 H_P$$

- SU(2): Bosonic nature of $\mathcal{H}$ requires qubit-registers / qudits and finite cut-off j_{max} (limiting accuracy)

- **Yang-Mills Hamiltonian** *[Kogut&Susskind ´75]*:

$$H_{YM} = \Sigma_e \widehat{P}_e \widehat{P}_e + g^2 \Sigma_\square \mathrm{Re}(\widehat{h}(\square))$$



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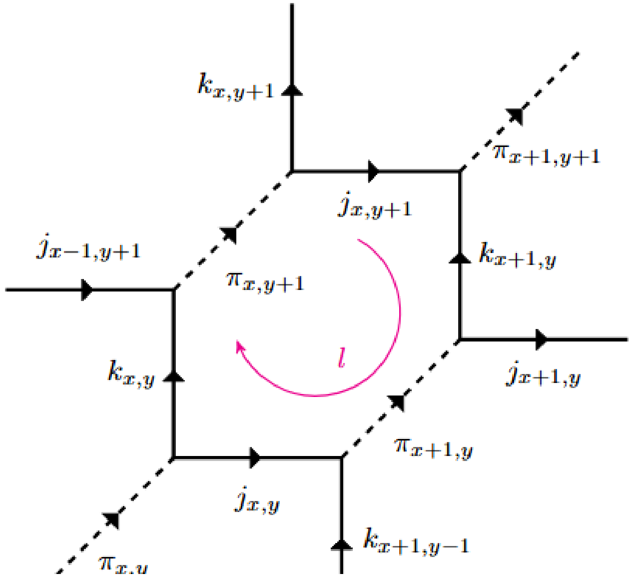

$$\langle \{\pi'_{x,y}, j'_{x,y}, k'_{x,y}\}_{x,y\in\mathbb{Z}} | \, \Sigma_e \hat{P}_e \hat{P}_e \, | \{\pi_{x,y}, j_{x,y}, k_{x,y}\}_{x,y\in\mathbb{Z}} \rangle =$$
$$= (j_{x,y}(j_{x,y}+1) + k_{x,y}(k_{x,y}+1))\delta_{j_{1,1},j'_{1,1}} \dots \delta_{k_{N,N},k'_{N,N}}$$

$$\langle \{\pi'_{x,y}, j'_{x,y}, k'_{x,y}\}_{x,y\in\mathbb{Z}} | \, \hat{h}(\square) | \{\pi_{x,y}, j_{x,y}, k_{x,y}\}_{x,y\in\mathbb{Z}} \rangle = \delta_{j_{1,1} j'_{1,1}} \dots \not\delta_{j_{x,y} j'_{x,y}} \dots \not\delta_{j_{x+1,y+1} j'_{x+1,y+1}} \dots \delta_{j_{N,N} j'_{N,N}}$$
$$= \sqrt{d_{j_{x,y}} d_{k_{x,y}} d_{j_{x,y+1}} d_{\pi_{x,y+1}} d_{k_{x+1,y}} d_{\pi_{x+1,y}} d_{j'_{x,y}} d_{k'_{x,y}} d_{j'_{x,y+1}} d_{\pi'_{x,y+1}} d_{k'_{x+1,y}} d_{\pi'_{x+1,y}}}$$
$$(-1)^{\pi'_{x,y} + \pi'_{x,y+1} + \pi_{x,y+1} + \pi'_{x+1,y} + \pi_{x+1,y} + \pi'_{x+1,y+1} + j'_{x,y} + j_{x,y} + j'_{x,y+1} + j_{x,y+1} + j'_{x+1,y} + j'_{x-1,y+1} + k'_{x,y} + k_{x,y} + k'_{x,y+1} + k'_{x+1,y} + k_{x+1,y} + k'_{x-1,y+1}}$$

$$\begin{Bmatrix} 1/2 & k'_{x,y} & k_{x,y} \\ \pi'_{x,y} & j_{x,y} & j'_{x,y} \end{Bmatrix} \begin{Bmatrix} \pi_{x,y+1} & j'_{x-1,y+1} & k_{x,y} \\ k'_{x,y} & 1/2 & \pi'_{x,y+1} \end{Bmatrix} \begin{Bmatrix} j_{x,y+1} & k'_{x,y+1} & \pi_{x,y+1} \\ \pi'_{x,y+1} & 1/2 & j'_{x,y+1} \end{Bmatrix} \times$$
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VQE for LGT

- **Yang-Mills Hamiltonian** *[Kogut&Susskind ´75]* restricted to $\mathcal{H}_G$:

$$H_{YM} = \Sigma_e \hat{P}_e \hat{P}_e + g^2 \Sigma_\square \mathrm{Re}(\widehat{h}(\square))$$

$$= \Sigma_e \omega_e \hat{a}_e^+ \hat{a}_e + \alpha_e \hat{a}_e^+ \hat{a}_e^+ \hat{a}_e \hat{a}_e + g^2 \Sigma_{\square=(e_1,e_2,e_3,e_4)} [\, f_{3j}(\vec{e})\, \hat{a}_{e1} \hat{a}_{e2} \hat{a}_{e3} \hat{a}_{e4} +$$

$$+ f'_{3j}(\vec{e}) \hat{a}^+_{e1} \hat{a}_{e2} \hat{a}_{e3} \hat{a}_{e4} + \cdots + f''_{3j}(\vec{e}) \hat{a}^+_{e1} \hat{a}^+_{e2} \hat{a}^+_{e3} \hat{a}^+_{e4}\,]$$

diagonal part

creates excitations obeying 3j-symbol conditions

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- **VQE** *[Peruzzo et al. 2014, …]* to solve $H_{YM}\mathbf{\Omega} = \mathrm{E_0}\mathbf{\Omega}$
  - Algorithm $A(\vec{\theta})$ to prepare $\psi_{\vec{\theta}} \coloneqq A(\vec{\theta})\, |0\rangle$
  - $\min_\theta \langle \psi_{\vec{\theta}}, H_{YM} \psi_{\vec{\theta}} \rangle$ equivalent to $\psi_{\vec{\theta}} \equiv \mathbf{\Omega}$

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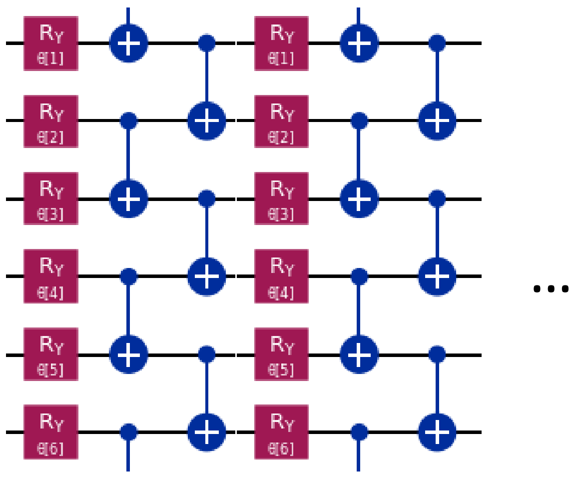
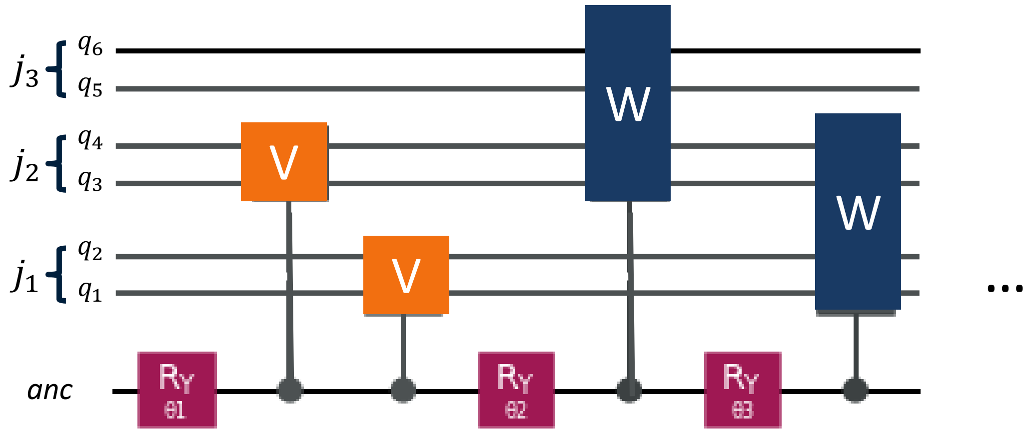
⇒ Choice of $A(\vec{\theta})$ motivated by Hamiltonian !

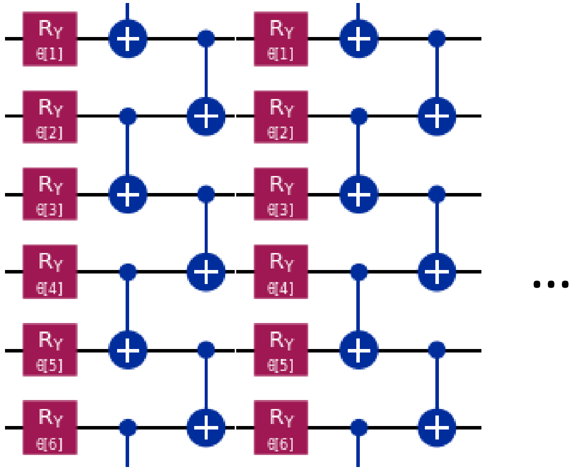
Gauge-invariant ansatz $A(\theta)$



Replaced by



Gauge-invariant ansatz A(θ)



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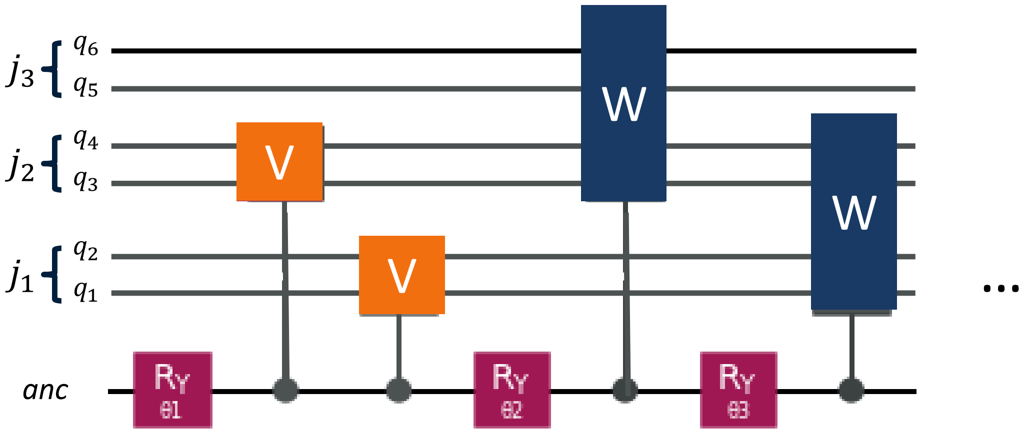
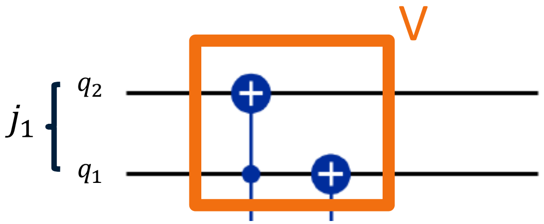
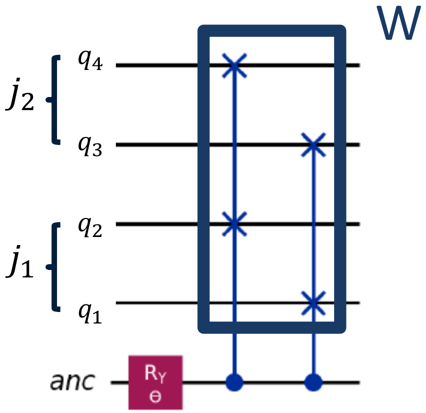
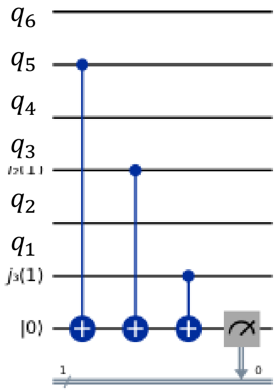


- Cut-Off, e.g. $j_{max} = 3 \Rightarrow \{0,1,2,3\} \equiv |q_1, q_2\rangle$
- Creation of gauge-invariant excitations
- In-bulk symmetry verification by testing symmetry conditions







$\Leftrightarrow \quad j_1 + j_2 + j_3 \in 2\mathbb{N}$

VQE for LGT: expected advantages

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Scaling advantages for VQE in LGT:

- Local lattice interactions in Hamiltonian H_{YM}
- Highly symmetric Hamiltonian & vacuum state
- Gauge-invariant Hilbert space and gauge-invariant ansatz

VQE for LGT: expected advantages

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Scaling advantages for VQE in LGT:

- Local lattice interactions in Hamiltonian H_{YM}

  → Mild scaling of # Pauli strings for H_{YM} decomposition & parallelizable readout

  $\mathcal{O}(N^4 \log_2(j_{max})) \to \mathcal{O}(N j_{max}^4)$

- Highly symmetric Hamiltonian & vacuum state

- Gauge-invariant Hilbert space and gauge-invariant ansatz

VQE for LGT: expected advantages

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Scaling advantages for VQE in LGT:

- Local lattice interactions in Hamiltonian H_{YM}

  → Mild scaling of # Pauli strings for H_{YM} decomposition & parallelizable readout

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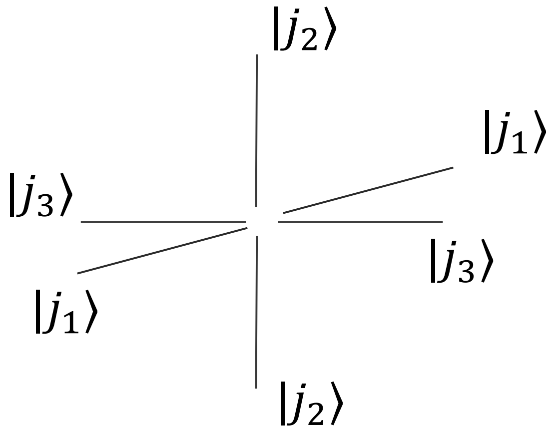
- Gauge-invariant Hilbert space and gauge-invariant ansatz

  → Reduced # qubits & # optimization parameters for state preparation

  $\mathcal{O}(3dN \log_2(2j_{max} + 1) + 2Nd \log_2 2)$
  $\to \mathcal{O}\big((3d - 3)N \log_2(2j_{max} + 1) + (2d - 3)N \log_2 2\big)$

Toy model

- **Toy model:**
  1 vertex with 6 edges & periodic boundary conditions



- Physical motivation: symmetry reduction before quantization

Gauge-invariant Hilbert space

- $\mathbf{\Omega} = \mathbf{\Omega}(j_1, j_2, j_3)$ characterized by 3 quantum numbers
  $\Rightarrow$ **6 data qubits + 1 ancilla** (instead of 18 qubits naively)

$$\mathcal{H}_G \Big|_{j_{max}=3} =$$

$$= \text{span}(|j_1, j_2, j_3\rangle \mid j_k \in \{0,1,2,3\})$$

- Completely emulatable with classical numerics

Systematic state preparation (SSP)

$$H_{YM} = (1 - g^2)\, H_E + g^2\, H_P$$

coupling parameter

- Strength of coupling $g \in [0,1]$ ⇔ phase transitions
- $g = 0$: free theory
- $g \to 1$: tuning coupling strength
  ⇔ tuning correlations
  ⇔ $\boldsymbol{\Omega}$ containing more excitations n_{tot}
  ⇔ increasing algorithm depth

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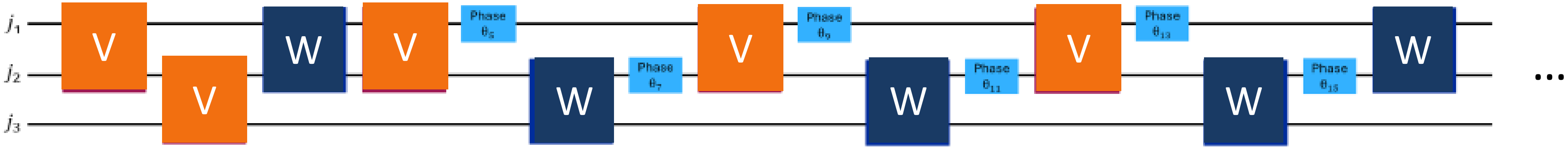

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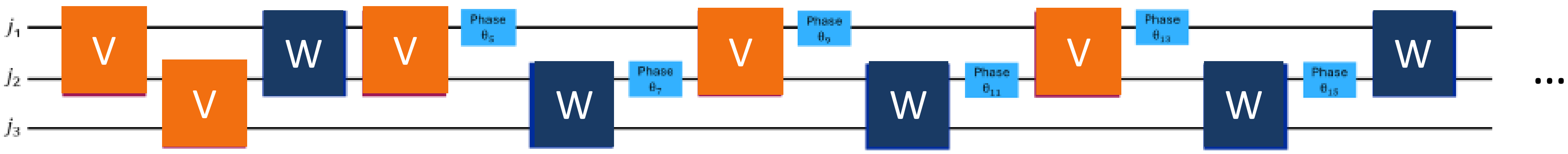

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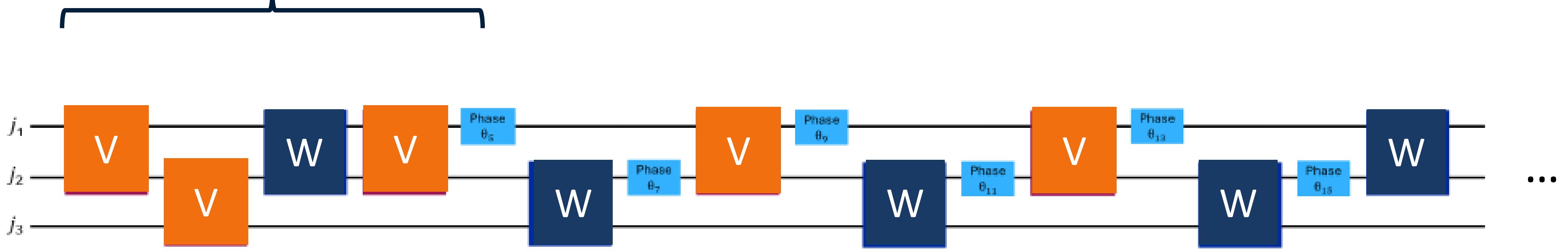

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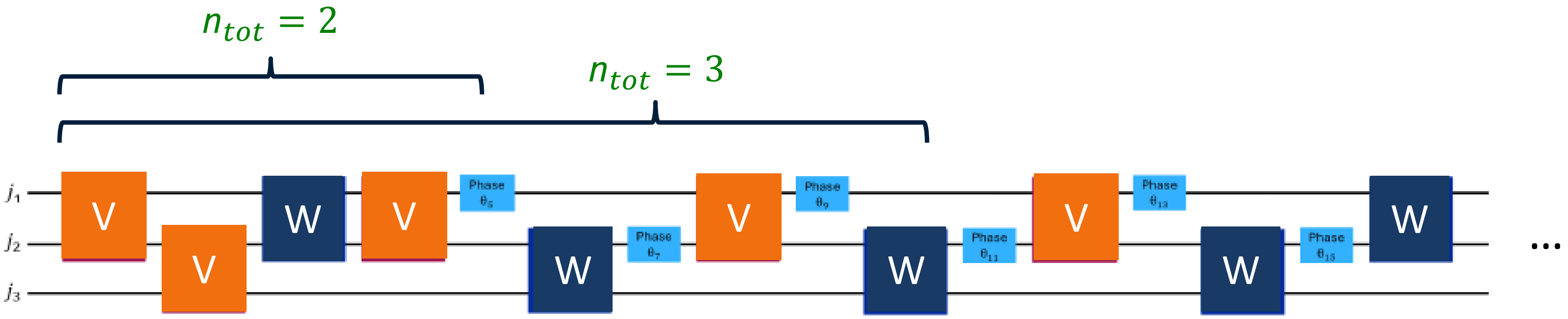

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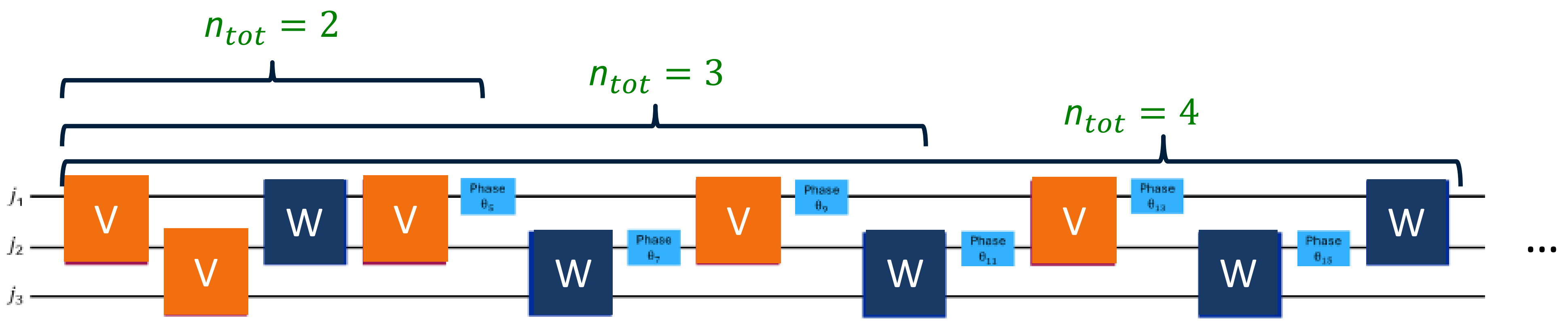

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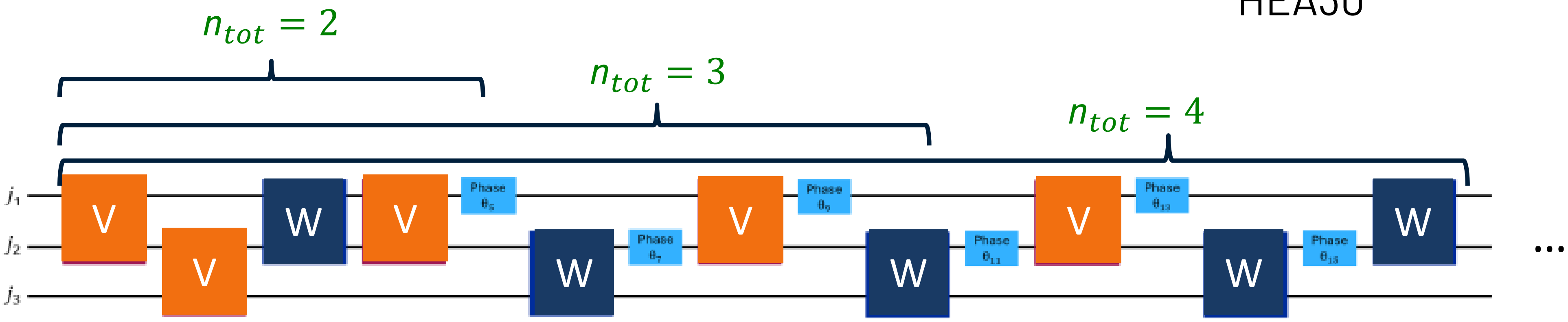

Ideal VQE simulations

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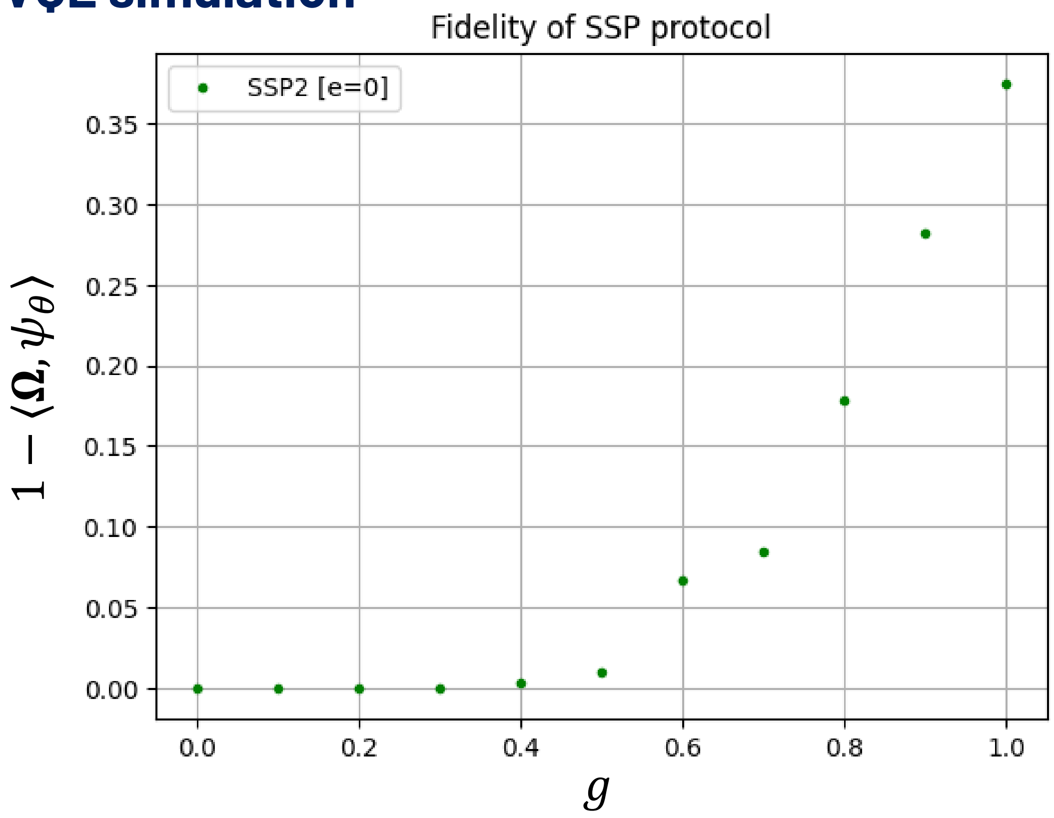

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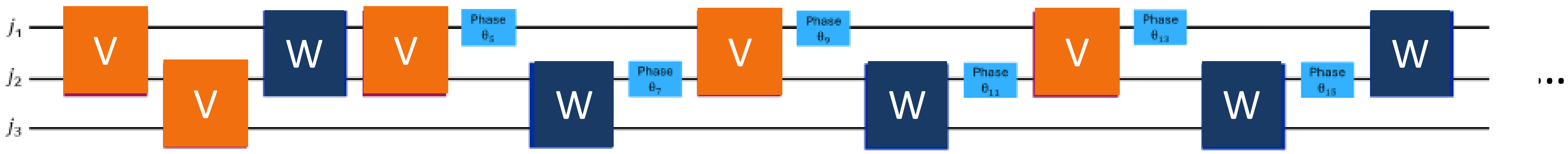

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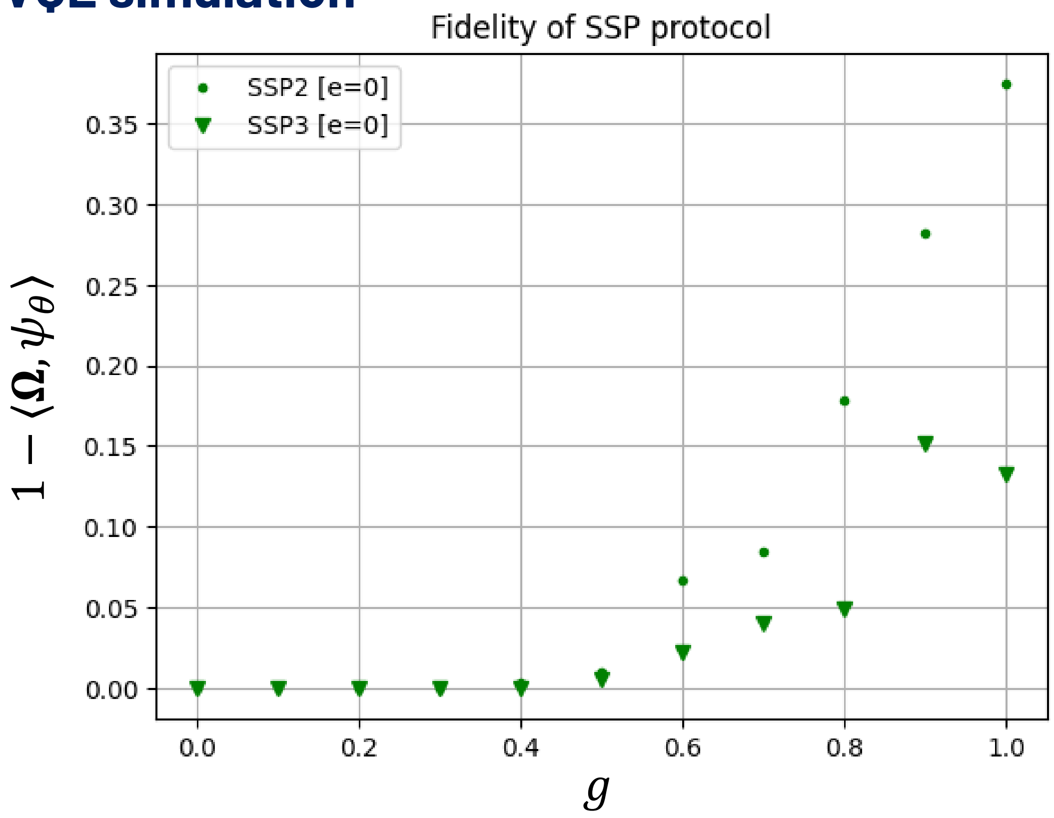

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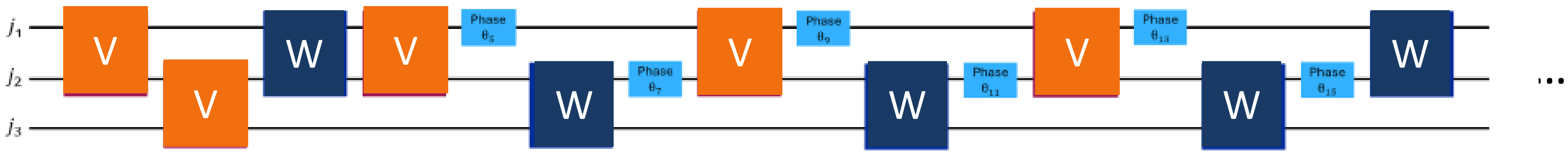

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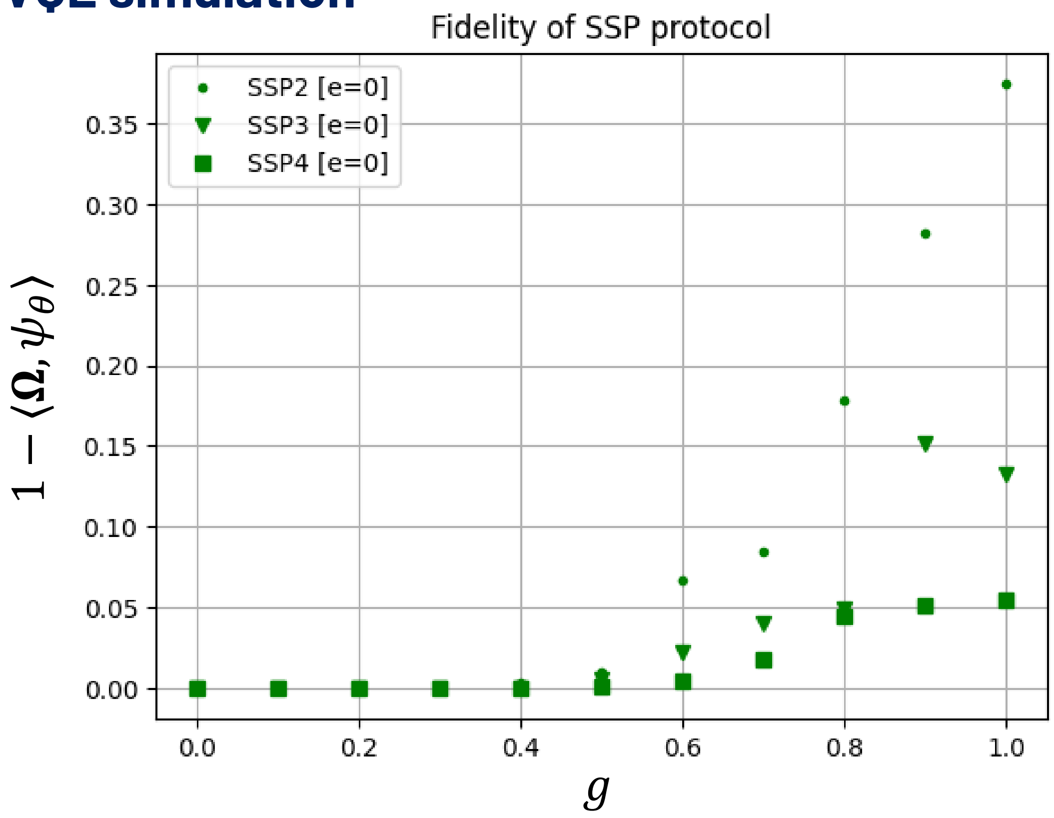

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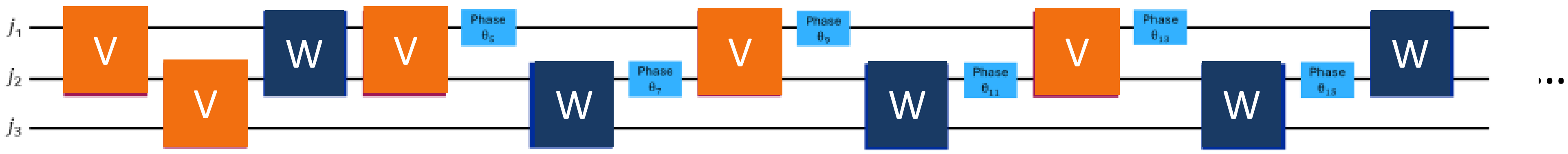

17-qubit chip at WMI

- 17-qubit-QPU in characterization at WMI
- Superconducting Transmons w/ tunable couplers
- Aiming at state-of-art KPIs:
    - $T_1 \sim 100\mu s$
    - $T_2 \sim 100\mu s$
    - $\mathrm{Infid}_{1QGate} \sim 0.005$
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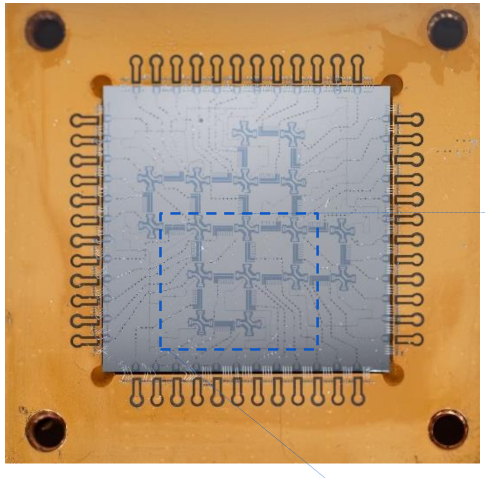
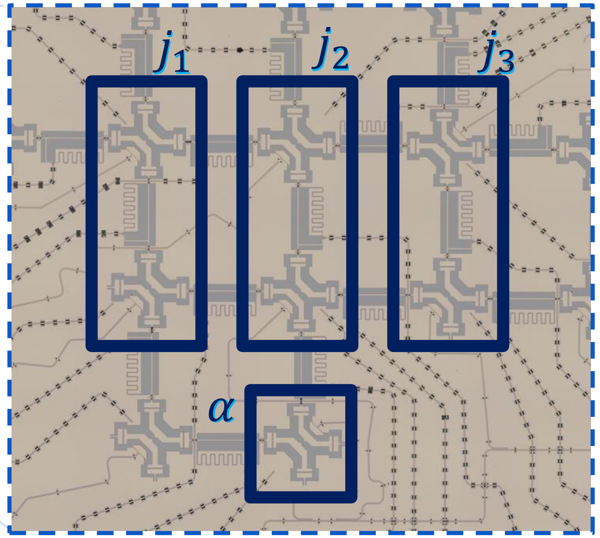


QPU with 17 superconducting (Transmon) qubits at WMI



Toy model with qubit registers transpiled on QPU

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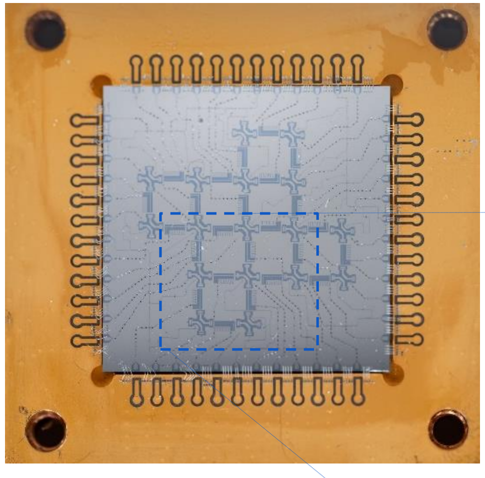
- Digitial twin emulator
  Kraus map for realistic markovian noise
  Parametric 2QGates *[Huber et al. 2024]*



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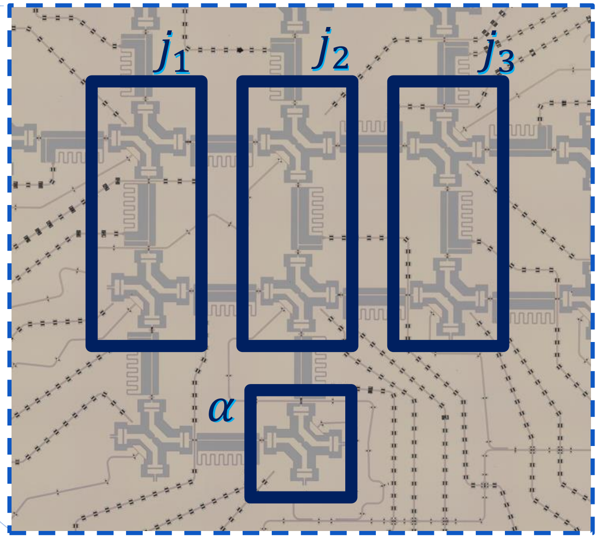

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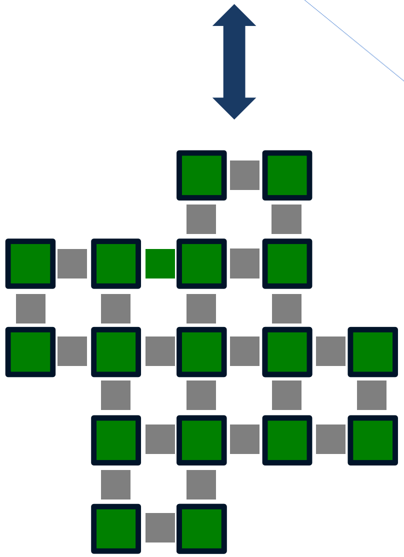
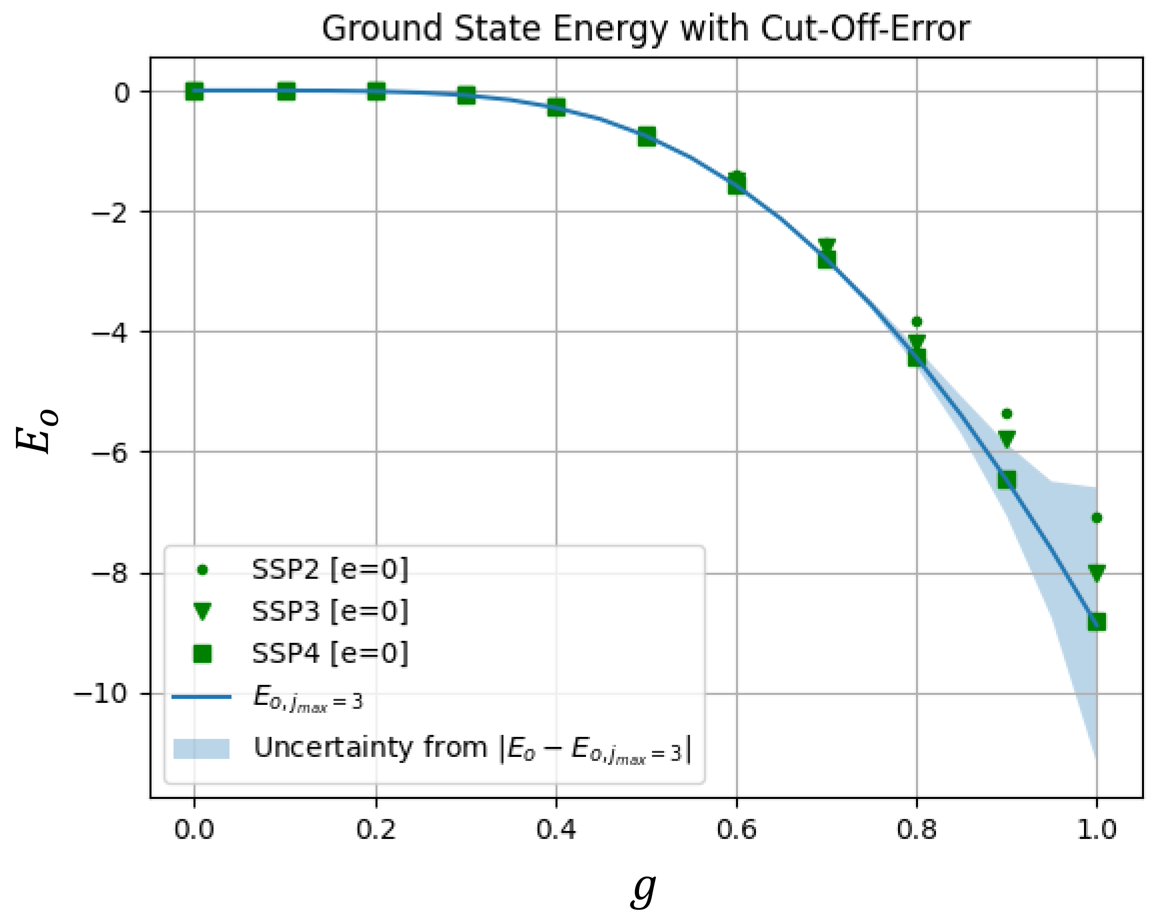


This talk: algorithm only tested on emulator

Ground state energy and error mitigation



- Noisy VQE simulations:
  - Antagonistic influence:
    2-Q gate error vs. circuit depth $N(n_{tot})$
  - Test on sc qubits **emulator with Markovian noise** with 0.5% error CZ gates

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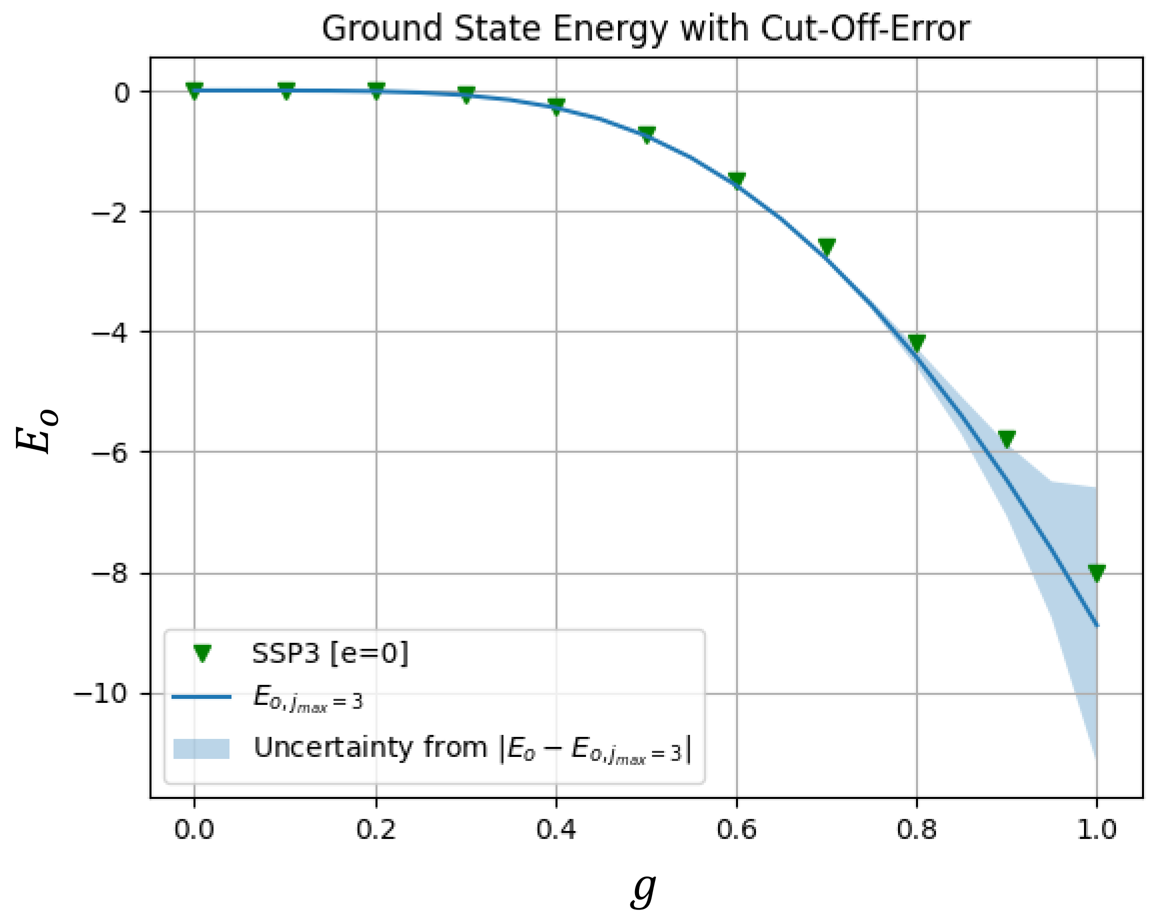

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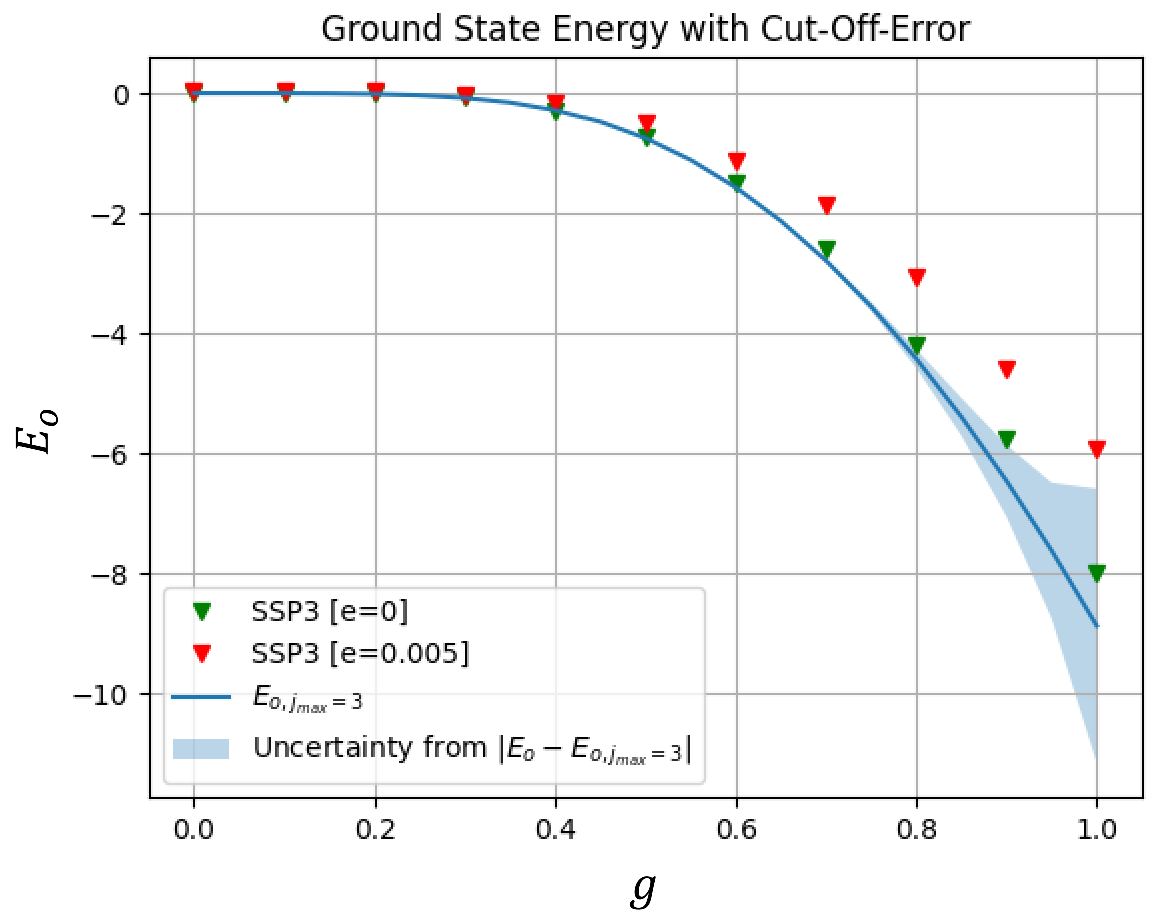

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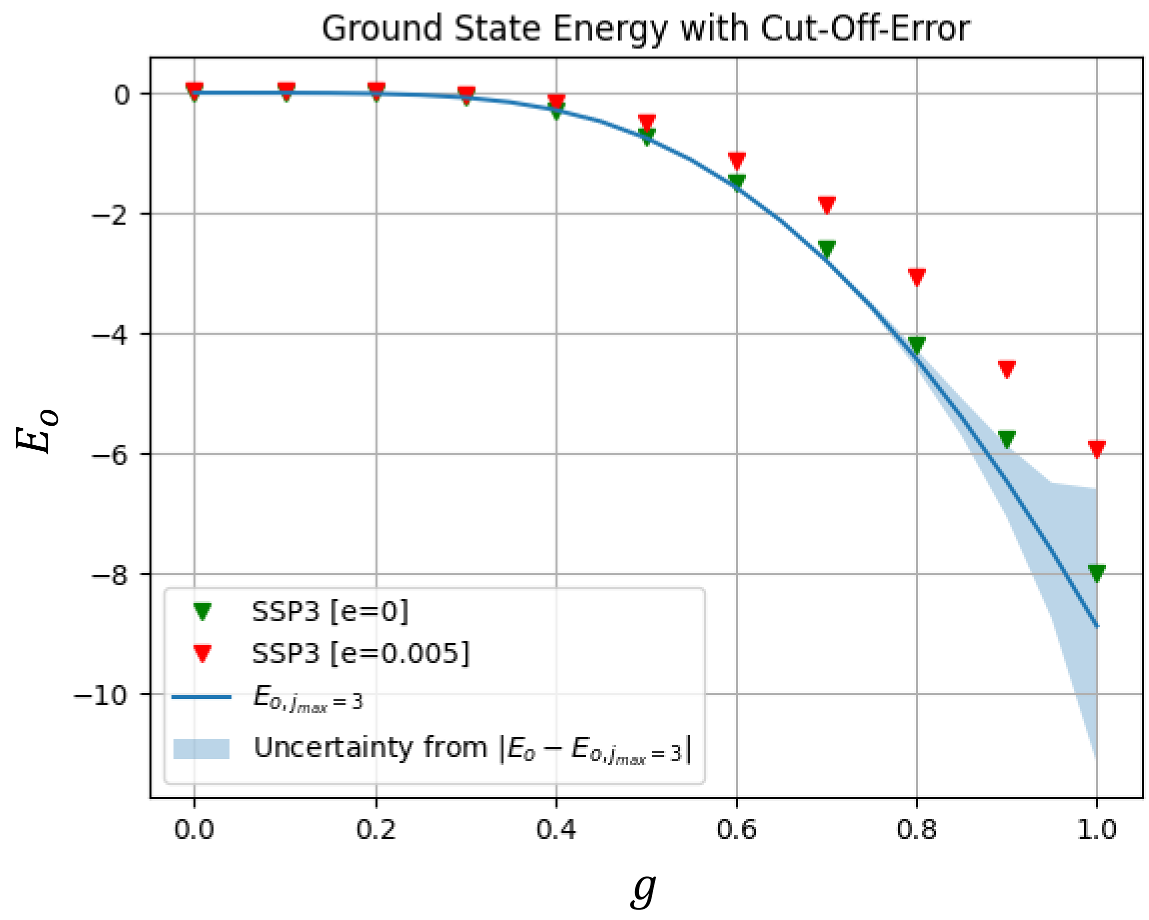

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  - Projection to gauge-invariant states
    - $j_1 + j_2 + j_3 \in \mathbb{N}$
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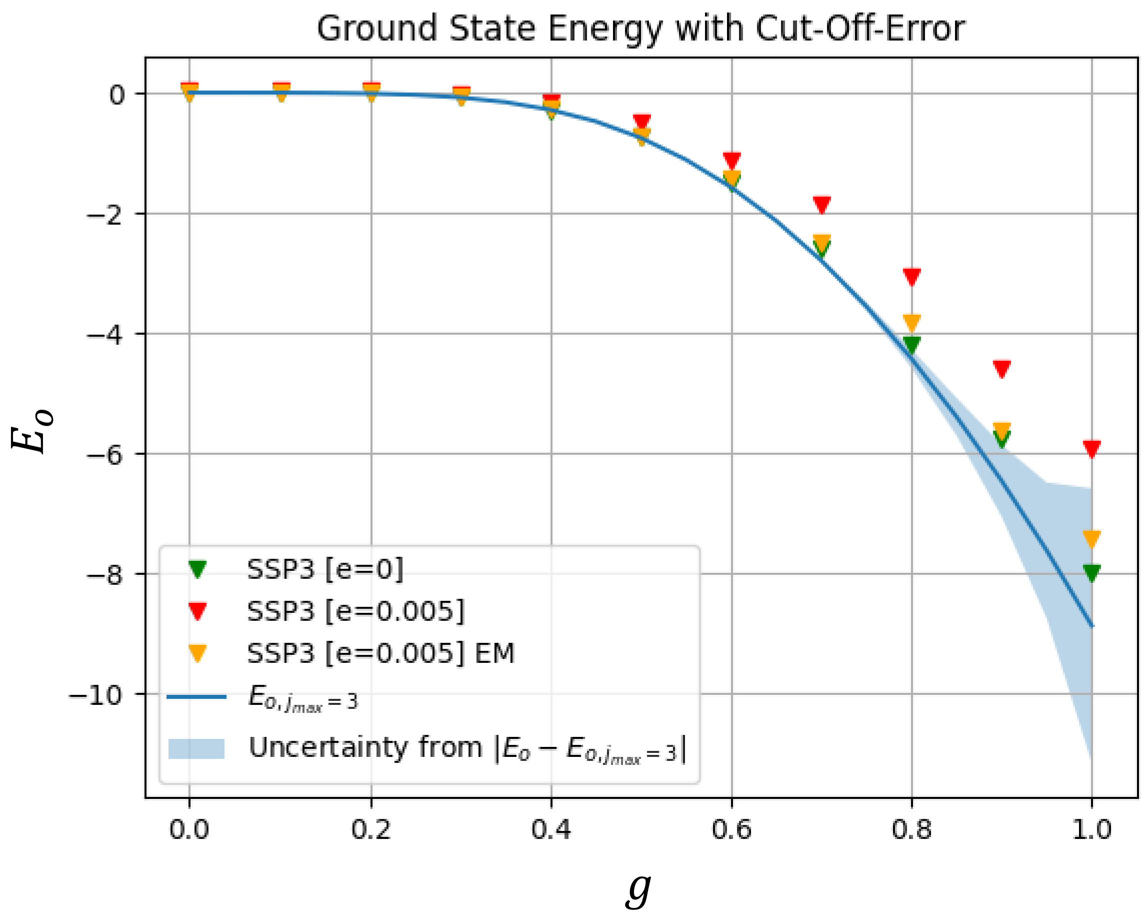

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Correlator on noisy emulator

2-point correlator

- Use corresponding circuit
  to prepare $\mathbf{\Omega}$ on QPU/**emulator**

- Evaluate correlations with prepared $\mathbf{\Omega}$

$$\langle \mathbf{\Omega} | \widehat{W}^{I}_{e_a} \left(\widehat{W}^{J}_{e_b} \right)^{+} | \mathbf{\Omega} \rangle$$
$$= \delta^{IJ} \left(\delta_{ab} 1 + \delta_{a \neq b} \, f_P(g) \right)$$

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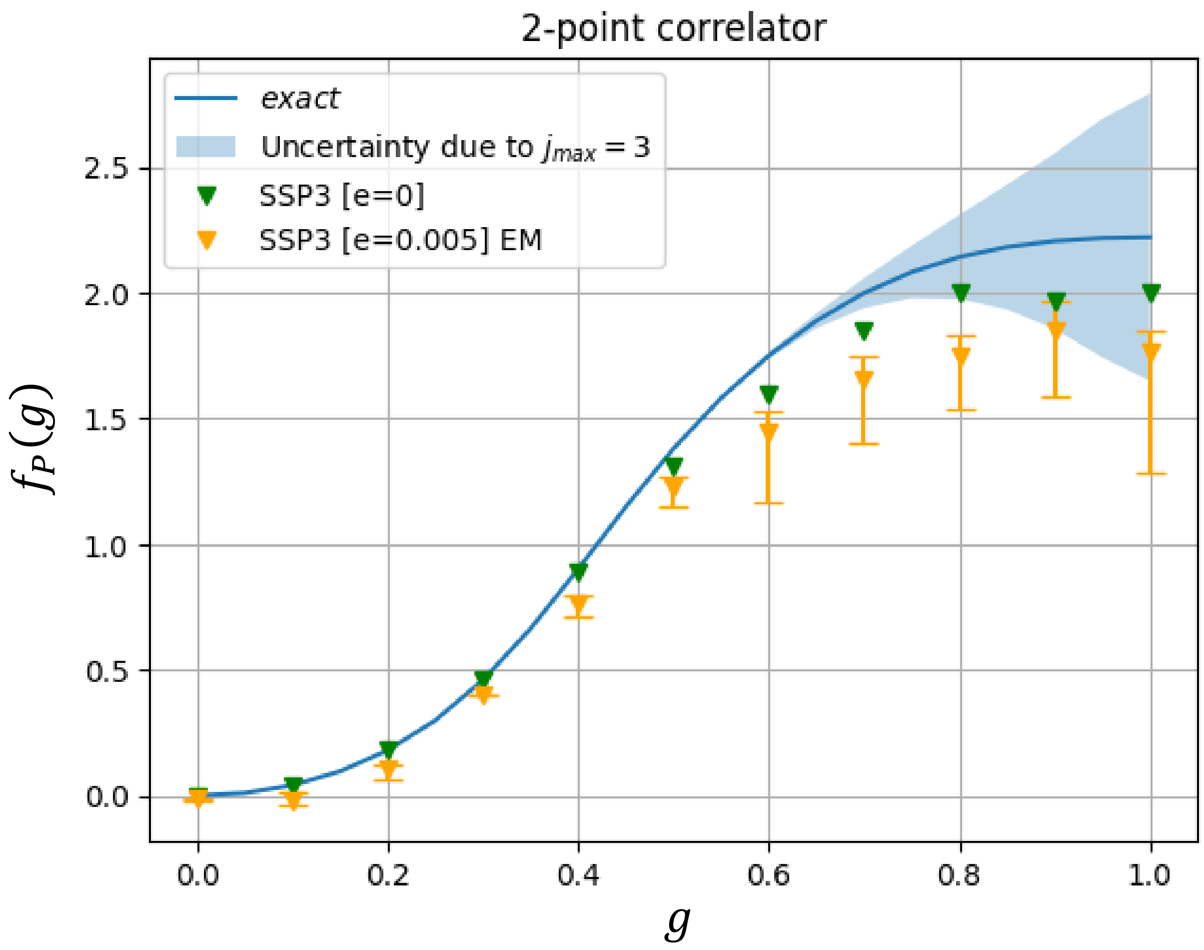
Emulating 0.5% 2-Q gate error:



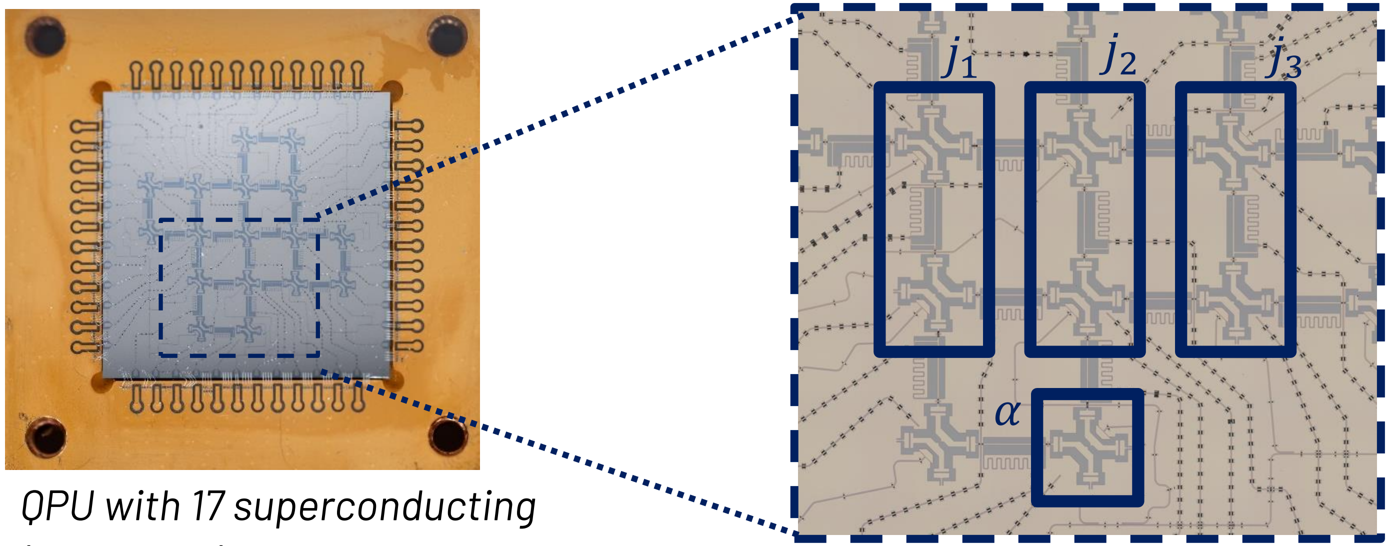
Summary and outlook

Summary:

- **Implemented Gauss constraint** making near term VQE accessible for non-abelian LGT
- **1-vertex toy model** approximating pure SU(2) Yang-Mills vacuum
- **Test on noisy emulator** simulating phase transitions with suitable accuracy

Outlook:

- **Implementation on real device**
  - Advanced error mitigation schemes (ZNE, ODR,...)
  - Sophisticated parameter optimizer (Adiabatic VQE, ...)
- **Extension of lattice size** and inclusion of matter



QPU with 17 superconducting (Transmon) qubits at WMI

WMI Quantum Computing Group

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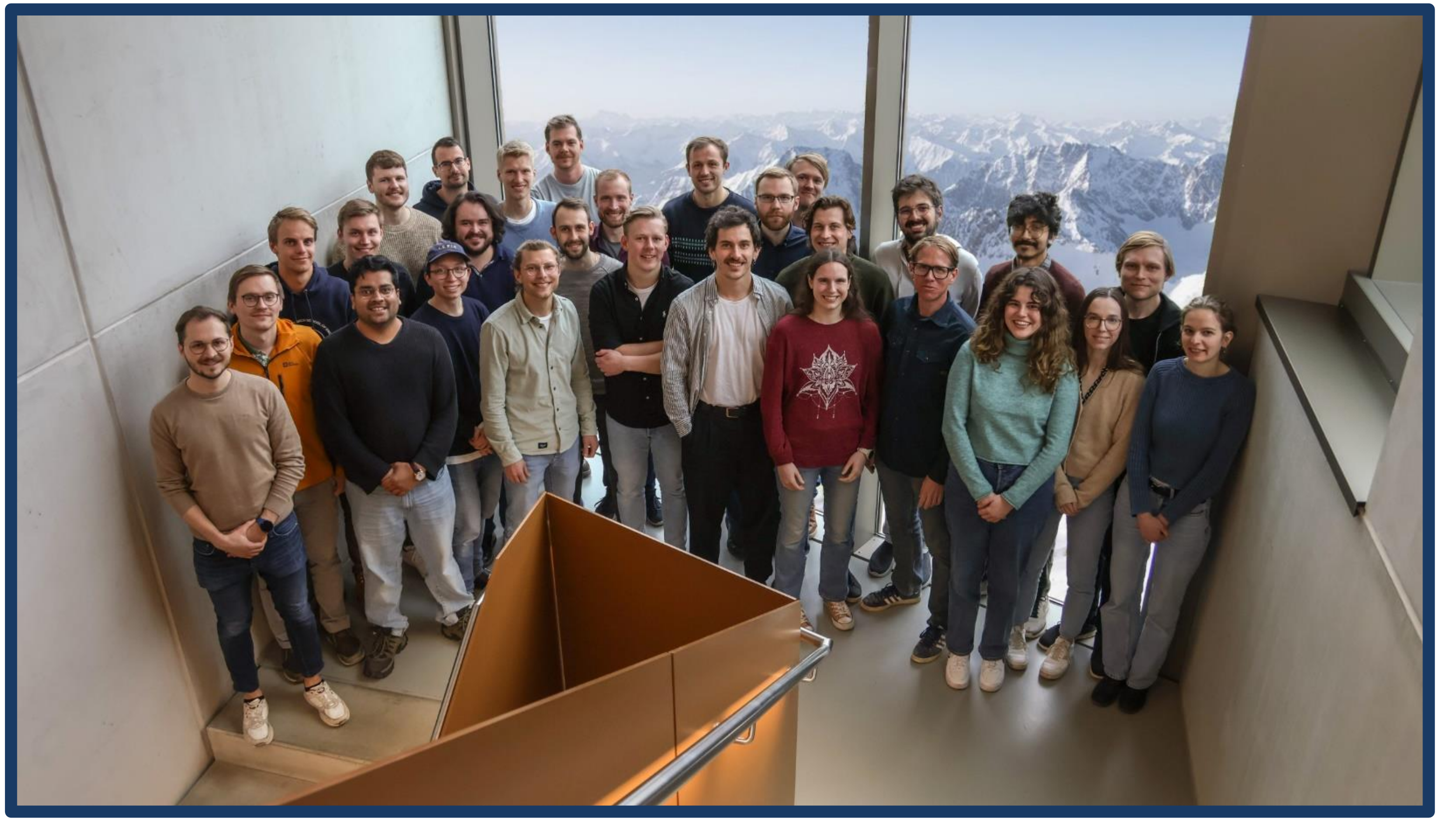

Aaron Anhalt
Nicolas Arlt
Vera Bader
Anirban Bhattacharjee
Daniil Bazulin
Niklas Bruckmoser
David Bunch
Julian Englhardt
Ege Erdem
Julius Feigl

Stefan Filipp
Johannes Friedrich
Niklas Glaser
Christian Gnandt
Karina Houska
Longxiang Huang
Gerhard Huber
Axel Karger
Kevin Kiener
Lisa Krüger

Martin Knudsen
Leon Koch
Vincent Koch
Gleb Krylov
Benedikt Lezius
Klaus Liegener
Benjamin Lienhard
Dominik Mattern
Frederik Pfeiffer
Lea Richard

Joao Romeiro
Federico Roy
Johannes Schirk
Christian Schneider
Tim Schneider
Saya Schöbe
Martin Schülein
Amanda Scoles
Malay Singh
Lasse Södergren

Ivan Tsitsilin
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Rui Wang
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Emily Wright
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Thank you for your attention !

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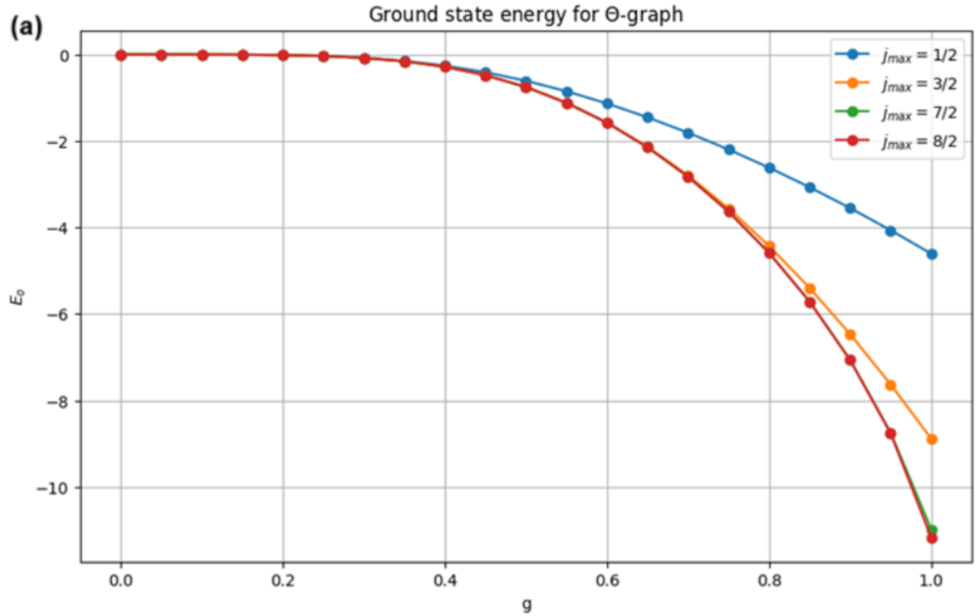
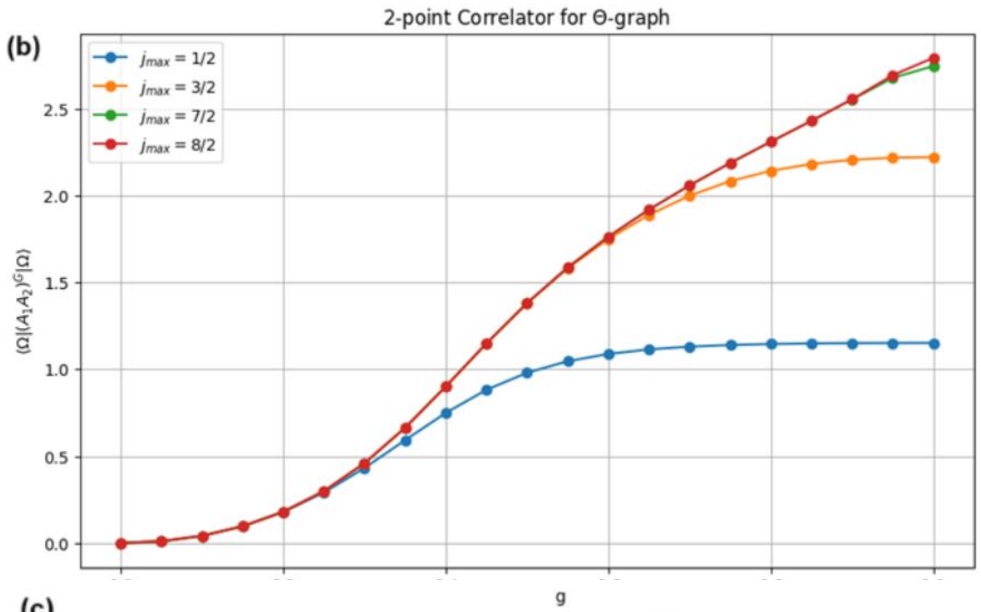
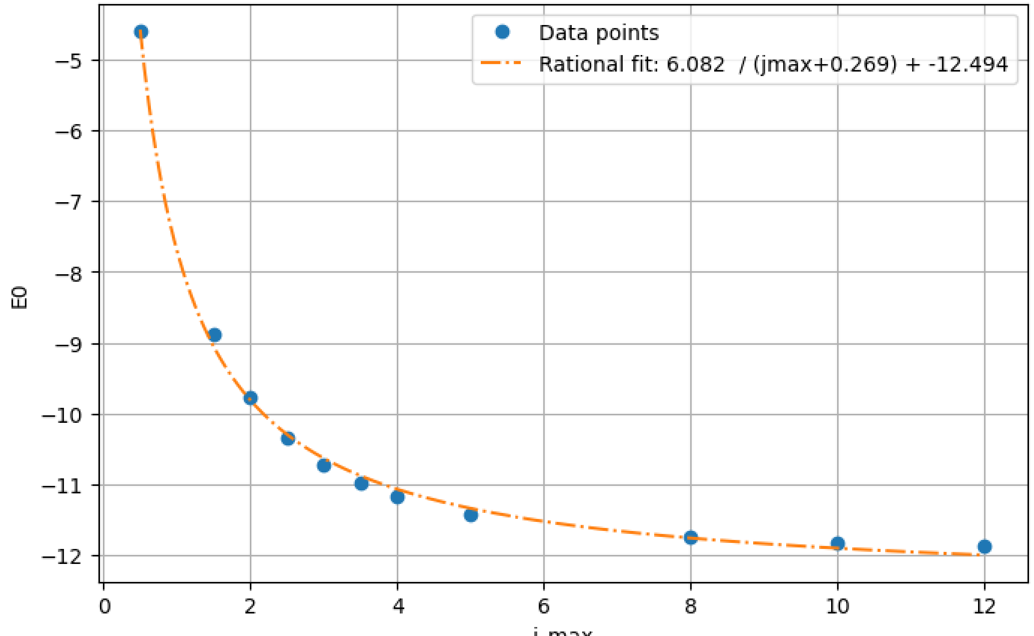
MUNIQC-SC Quantum Computing Demonstrator Superconducting Qubits
Munich Quantum Valley
MOQS Molecular Quantum Simulations
QuSCo
GeQCoS
Quromorphic
MCQST
QUSTEC

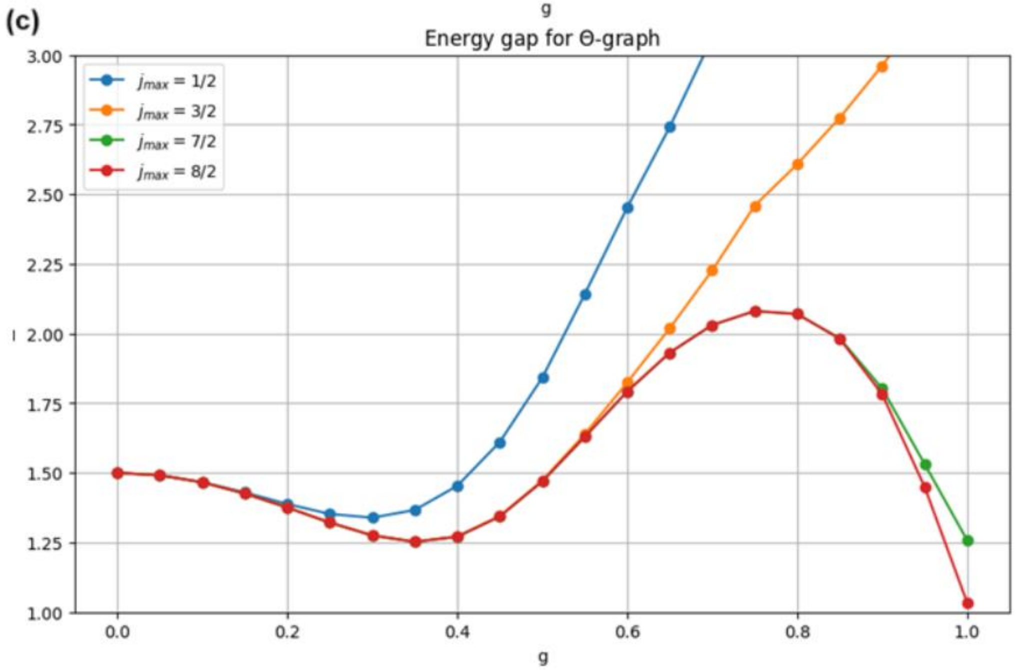
BackUp

Numerical comparison

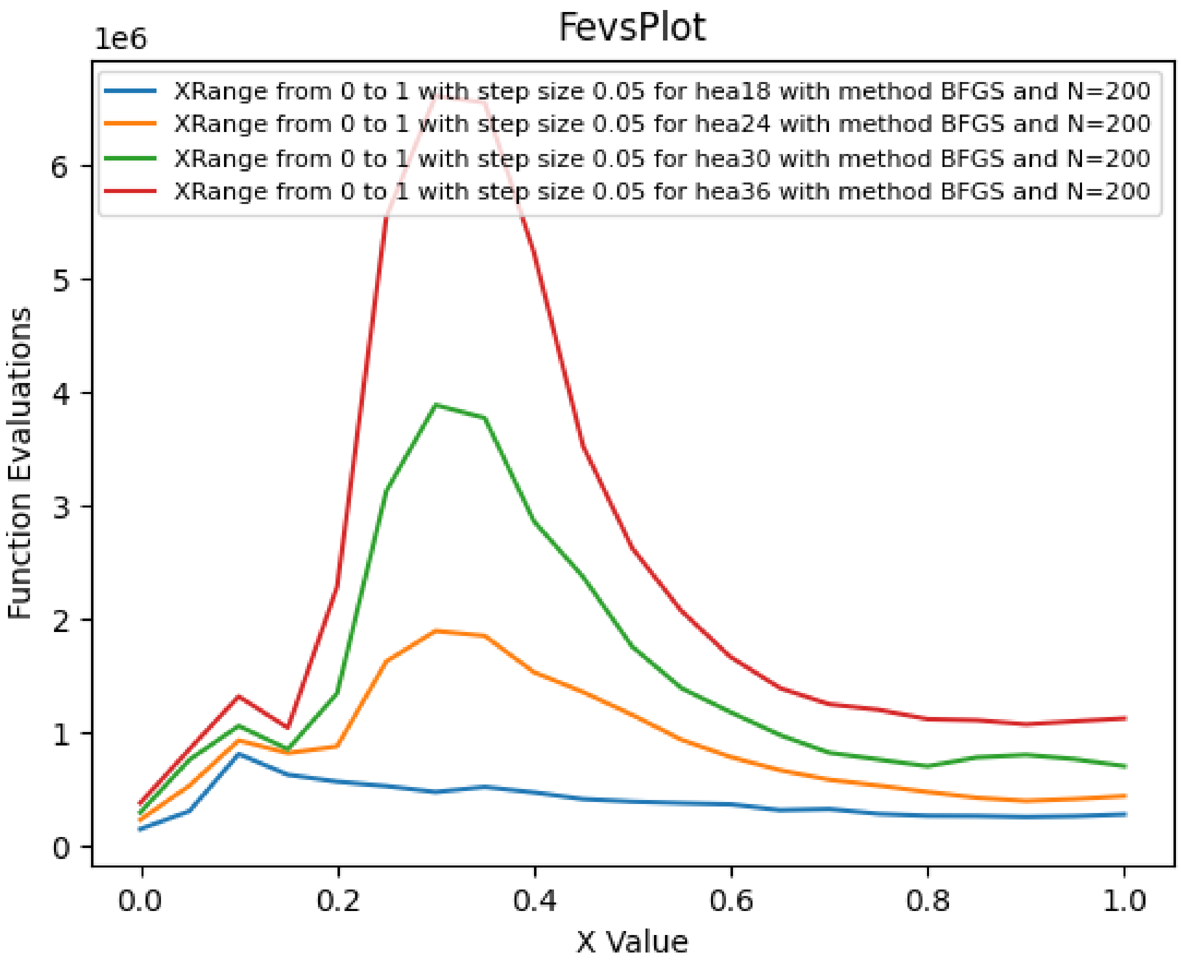


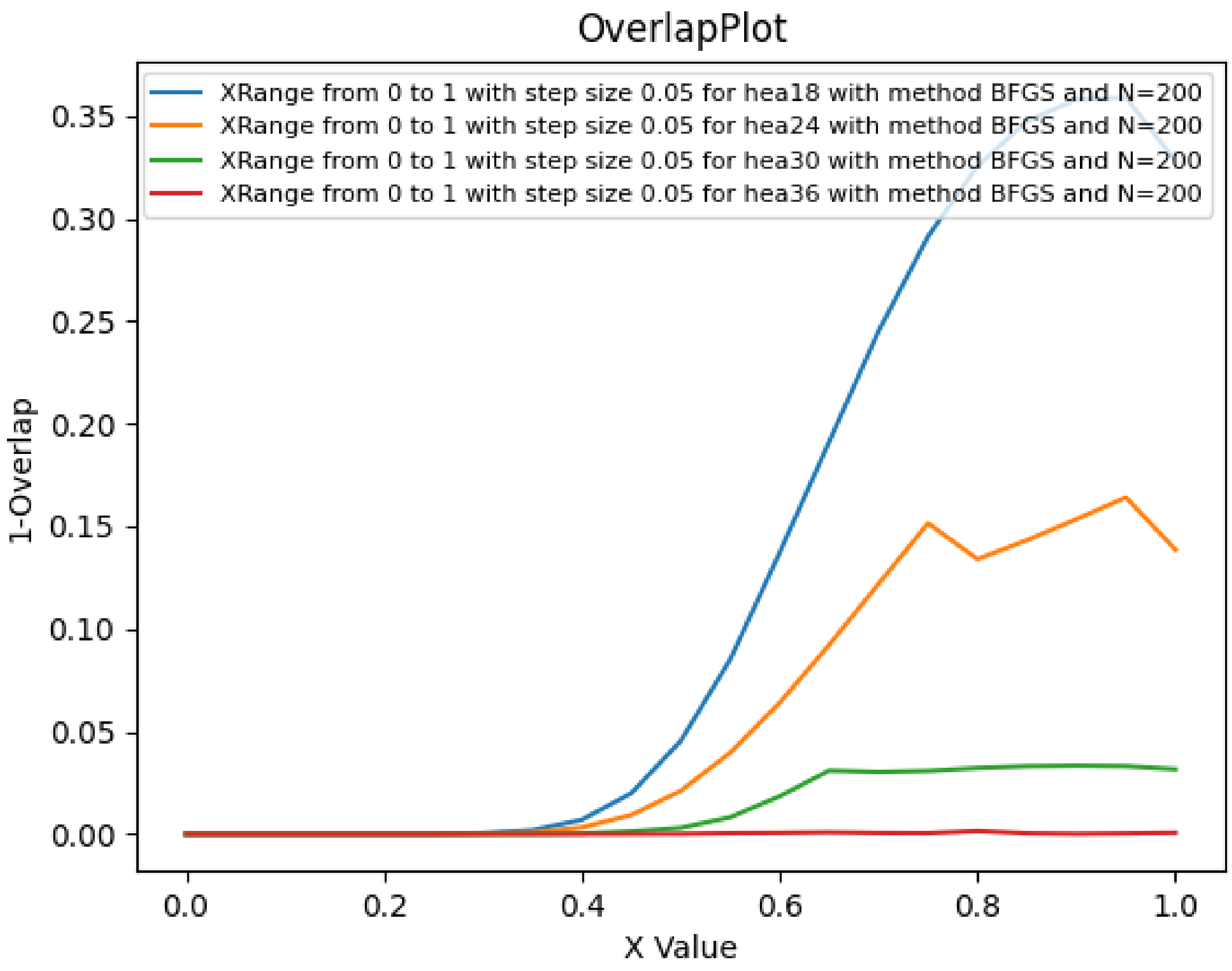






HEA comparison (ideal)



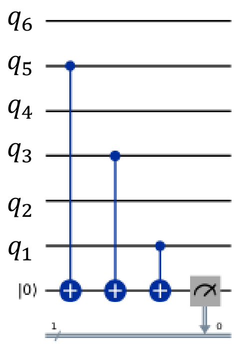


Error mitigation

- 3j-symbols enforce symmetry conditions: *[Wigner ´93]*

    1. $j_1 + j_2 + j_3 \in 2\mathbb{N}$
    2. $|j_1 - j_2| \leq j_3 \leq j_1 + j_2$

- In-bulk symmetry verification by testing symmetry conditions



$\Leftrightarrow \quad j_1 + j_2 + j_3 \in 2\mathbb{N}$

- Few remaining symmetry conditions via 3j-symbols

→ Implementation of in-bulk symmetry verfication for error mitigation

Rough estimator:

- Only total population for each excitation manifold
- Enforce gauge-inv.subspace
- Realize similarity to displaced harmonic oscillator

$$H = \omega(\hat{n})\hat{a}^{+}\hat{a} + x(\hat{n})(\hat{a} + \hat{a}^{+})$$

- Ground state given by Poisson distribution:

$$\langle \mathbf{\Omega}, n_{tot} \rangle \sim e^{-x^2/\omega^2} \frac{(x/\omega)^{2n_{tot}}}{n_{tot}!}$$

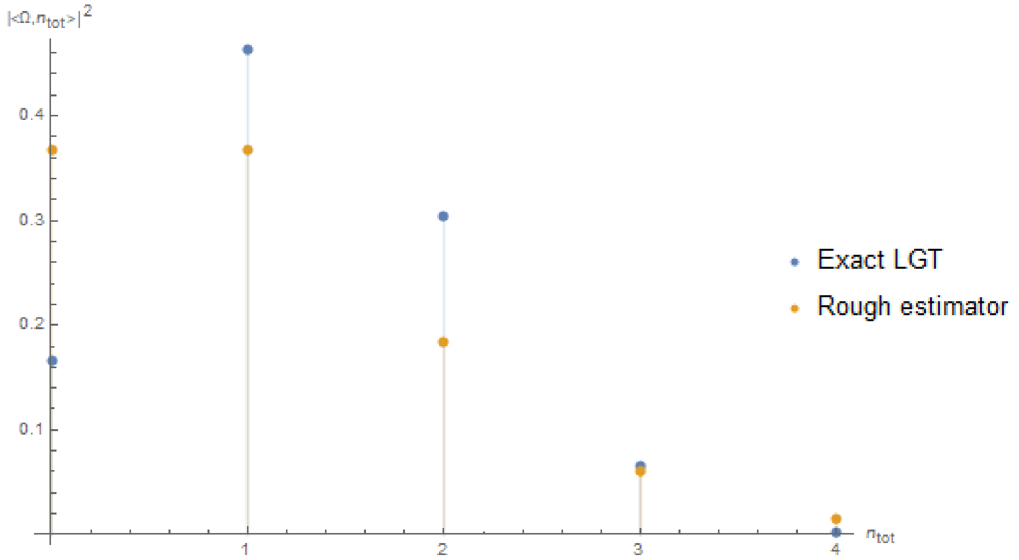
- Comparison with numerical exact results show rough similarity

$$H_{YM} = \Sigma_e \hat{P}_e \hat{P}_e + g^2 \Sigma_{\square} \mathrm{Re}(\widehat{h}(\square))$$

$$H_{gauge-inv} = \Sigma_e \omega_e \hat{a}_e^{+} \hat{a}_e + \alpha_e \hat{a}_e^{+\,2} \hat{a}_e^{2} + g^2 \Sigma_{\square=(e_1,e_2)} [\, f_{3j} \hat{a}_{e1} \hat{a}_{e2} + f'_{3j} \hat{a}_{e1}^{+} \hat{a}_{e2} + \text{h.c.} \,]$$

$$H_{approx} = (\omega - 2f'_{3j} g^2)\hat{a}^{+}\hat{a} + \alpha \hat{a}^{+\,2} \hat{a}^{2} + g^2 [\, f_{3j}\, \hat{a}^{2} + \text{h.c.} \,]$$

$$\begin{pmatrix} 0. & 0. & -1.41421 & 0. & 0. & \\ 0. & 1. & 0. & -2.44949 & 0. & \\ -1.41421 & 0. & 2.33333 & 0. & -3.4641 & \\ 0. & -2.44949 & 0. & 4. & 0. & \\ 0. & 0. & -3.4641 & 0. & 6. & \\ 0. & 0. & 0. & -4.47214 & 0. & \\ & & & & & \ddots \end{pmatrix} \Rightarrow \begin{pmatrix} 0. & 0 & -1.41421 & 0 & 0. & \\ 0 & 0 & 0 & 0 & 0 & \\ -1.41421 & 0 & 2.33333 & 0 & -3.4641 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0. & 0 & -3.4641 & 0 & 6. & \\ 0 & 0 & 0 & 0 & 0 & \\ & & & & & \ddots \end{pmatrix}$$



Aimed accuracy:

- Quantum simulations in LGT typically aim at verifying conceptual processes of physics instead of reaching a targetted accuracy
- Goal: Identify LGT analogon to chemical accuracy
- LGT models accuracy determined by discretization artefacts: $H_{cont} = H_{lattice} + \mathcal{O}(\epsilon)$
- Characteristic energy of the system is mass gap $\Delta := \frac{g^2 V^3}{2\epsilon} \min_g E_1(g) - E_0(g)$
- Identification of energy and length via photon-wave length λ and

$$\Delta = \hbar \frac{2\pi c}{\lambda_\Delta}$$

- Lattice spacing small compared to relevant energy scale $\frac{\epsilon}{\lambda_\Delta} \ll 1$ and gives the releative error rate
- Toy model: $\frac{\epsilon}{\lambda_\Delta} \approx 0.36\, \epsilon^3 \ll 1$
- Argumentation only valid in limit $\epsilon \to 0$, because not estimating the size of the prefactor of $\mathcal{O}(\epsilon)$