Tensor Networks for Lattice Field Theory

Selected Topics

Stefan Kühn

Hamiltonian Lattice Gauge Theories: Status, Novel Developments and Applications, ECT*, 02.09.2025

Theoretical Particle Physics

- Gauge field theories
- Analytical access often hard

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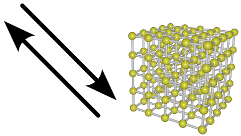

Classical simulation

- Monte Carlo methods in euclidean space-time

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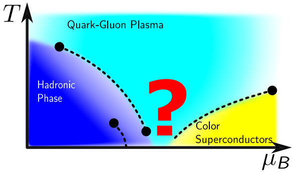

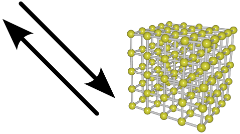

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- Monte Carlo methods in euclidean space-time
  - No real-time dynamics
  - Sign problem in certain regimes

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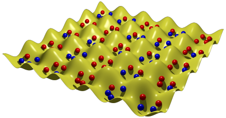

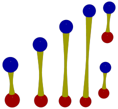

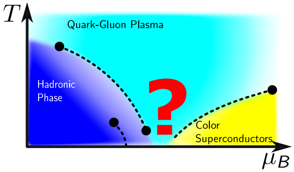

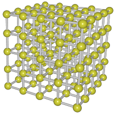

Quantum computing

- Free from purely numerical limitations
- First proof-of-principle experiments

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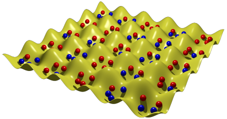

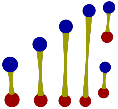

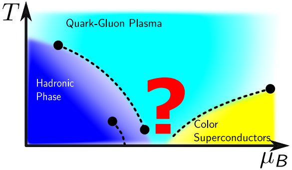

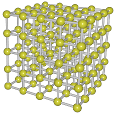

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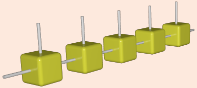
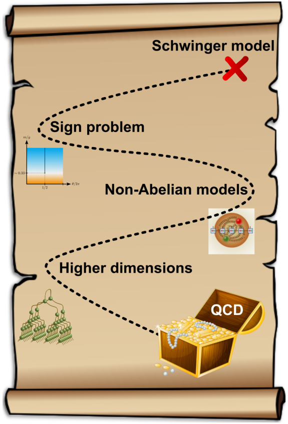


Motivation

Roadmap

> Full QCD in 3+1 dimensions is too complicated
> Start with simpler models that share some of the relevant features of QCD
> Incrementally increase complexity to develop new techniques



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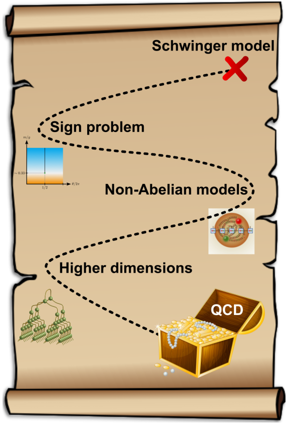
Challenges

- Techniques for the Hamiltonian formulation?
- How to discretize and deal with fermions?
- Resource efficient bases?



Outline

Motivation

Tensor Network States

Application to lattice field theory

Synergies with Quantum Computing

Summary & Outlook

2.

Motivation

Tensor Network States

Application to lattice field theory

Synergies with Quantum Computing

Summary & Outlook

Tensor Network States (TNS)

Basic idea of tensor networks states

- Wave function for an interacting N-body lattice system

$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N=1}^{d} c_{i_1,i_2,\ldots,i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

$\Rightarrow$ Tensor $c_{i_1,\ldots,i_N}$ with d^N entries, **exponential** number of parameters

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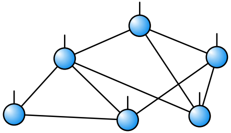

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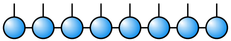

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Tensor Network States (TNS)

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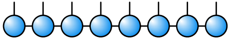

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- Maximum entanglement entropy is limited by log D

M. B. Hastings, J. Stat. Mech. **2007**, P08024 (2007)
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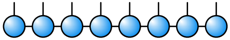

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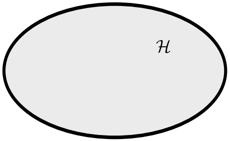
- Quantum information theory: $D =$ poly(N) for many physically interesting scenarios
- **Entanglement** in **many relevant situations** is **moderate**



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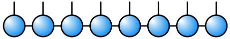

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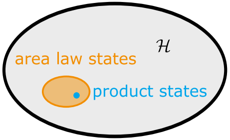

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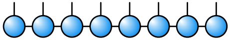

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- Maximum entanglement entropy is limited by log D
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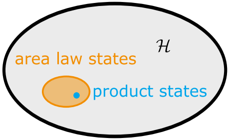
- **Entanglement** in **many relevant situations** is **moderate**
- **TNS** efficiently **parametrize** the subspace of **slightly entangled states**



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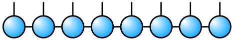
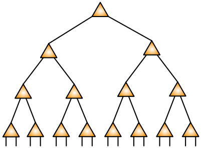
Tensor Network States (TNS)

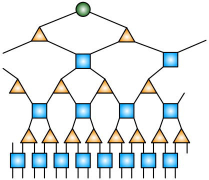
Tensor Network States in one dimension



Matrix product state

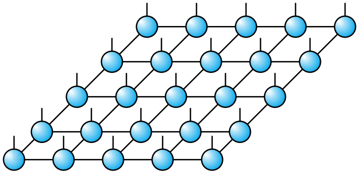


Tree Tensor Network

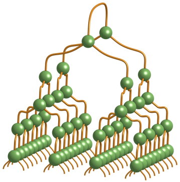


MERA

Tensor Network States in two dimensions



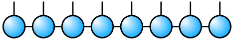
Projected Entangled Pair State



Tree Tensor Network

Introduction to Tensor Network States (TNS)

Tensor Network States in one dimension



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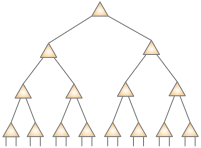

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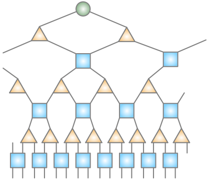

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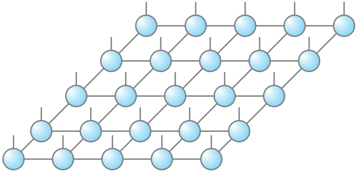

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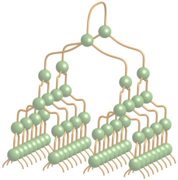


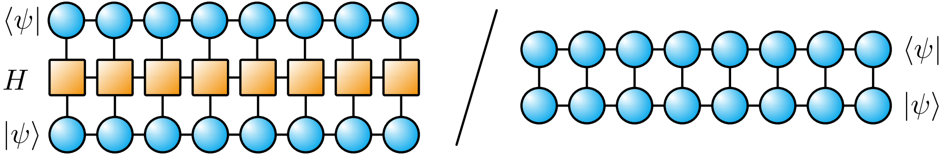
Tree Tensor Network

Tensor Networks as numerical tool

Finding ground states of Hamiltonians

- Computing ground states: variationally minimize the energy

$$E_0 = \min_{|\psi\rangle} \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$$



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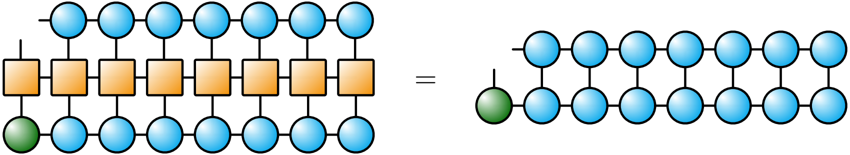
$$H_{\text{eff}}\vec{A}_k^{i_k} = N\vec{A}_k^{i_k}$$



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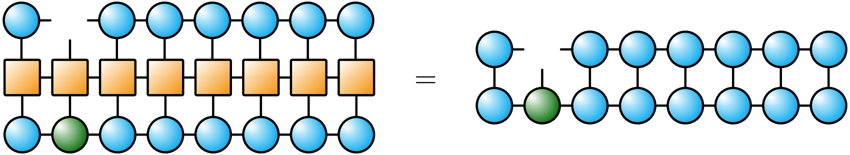

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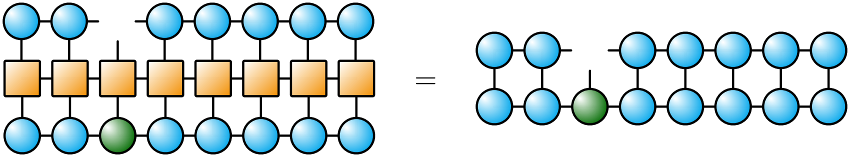

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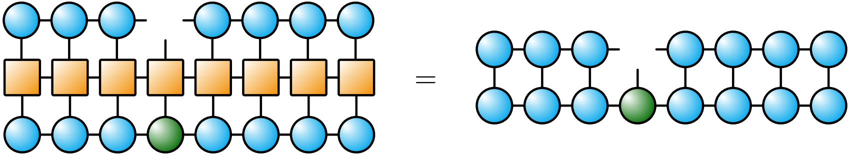
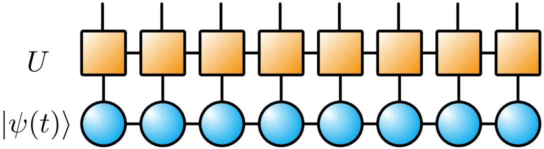


Tensor Networks as numerical tool

Computing time evolution of a quantum state

- Computing time evolution

$$|\psi(t+\Delta t)\rangle = \exp(-iH\Delta t)|\psi(t)\rangle$$



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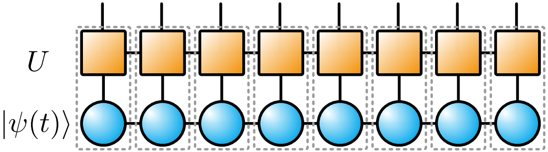

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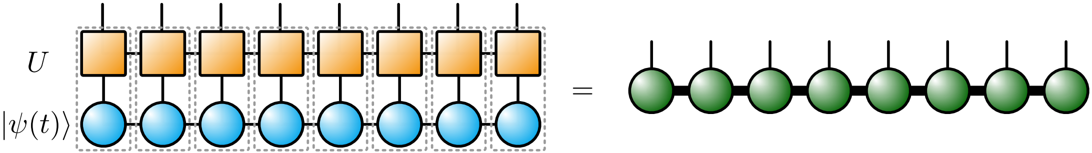

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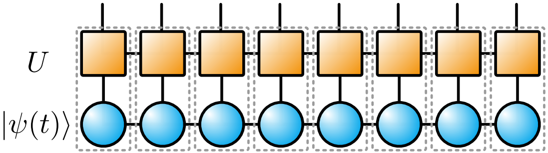

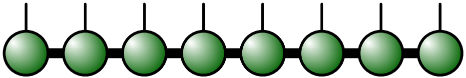

- Dimension of bonds grows exponentially in the number of time steps
- Approximate $|\psi(t+\Delta t)\rangle$ with a MPS of $|\phi\rangle$ with smaller bond dimension D

$$\|\,|\psi(t+\Delta t)\rangle - |\phi\rangle\|_2 \rightarrow \text{min}$$

- As long as the entanglement grows moderately one can follow the time evolution

F. Verstraete, D. Porras, J. I. Cirac, Phys. Rev. Lett. **93**, 227205 (2004)

3.

Motivation

Tensor Network States

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Synergies with Quantum Computing

Summary & Outlook

Quantum electrodynamics in 1+1 dimensions: the Schwinger Model

The Schwinger model

Example: Schwinger model

- Lagrangian of the theory: QED in 1+1 dimensions

$$\mathcal{L} = \int \mathrm{d}t\, \mathrm{d}x \left(i\overline{\Psi}\partial\!\!\!/\Psi - g\overline{\Psi}A\!\!\!/\Psi - m\overline{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right)$$

Kinetic term

Coupling to gauge field

Mass

Electromagnetic term

- Matter fields: $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

J. Schwinger, Phys. Rev. 128, 2425 (1962)

The Schwinger model

Example: Schwinger model

- Lagrangian of the theory: QED in 1+1 dimensions

$$\mathcal{L} = \int \mathrm{d}t\,\mathrm{d}x \left(\underbrace{i\overline{\Psi}\partial\!\!\!/\Psi}_{\text{Kinetic term}} - \underbrace{g\overline{\Psi}A\!\!\!/\Psi}_{\text{Coupling to gauge field}} - \underbrace{m\overline{\Psi}\Psi}_{\text{Mass}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Electromagnetic term}} \right)$$

- Matter fields: $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- Hamiltonian formulation in temporal gauge ($A^0 = 0$)

$$H = \int \mathrm{d}x \left(\underbrace{-i\bar{\Psi}\gamma^1\partial_1\Psi}_{\text{Kinetic term}} + \underbrace{\bar{\Psi}\gamma^1 g A_1\Psi}_{\text{Coupling to gauge field}} + \underbrace{m\bar{\Psi}\Psi}_{\text{Mass}} + \underbrace{\frac{1}{2}E^2}_{\text{Electric energy}} \right)$$

- Additional constraint on the physical states: Gauss law

$$\partial_1 E = g\bar{\Psi}\gamma^0\Psi$$

J. Schwinger, Phys. Rev. 128, 2425 (1962)

The Schwinger model

Lattice Hamiltonian formulation

- Kogut-Susskind staggered fermions in temporal gauge $A^0 = 0$

$$H = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\phi_n^\dagger U_n \phi_{n+1} - \text{h.c.}\right) + \sum_{n=1}^{N} (-1)^n m\, \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}\left(L_n + \frac{\theta}{2\pi}\right)^2$$

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kinetic part + coupling to gauge field

staggered mass term

electric energy + topological term

ϕ_n: single-component fermionic field, $U_n = e^{i\alpha_n}$: link operator, $[L_n, \alpha_{n'}] = i\delta_{nn'}$

The Schwinger model

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- **Toy model** for **QCD** sharing many relevant feature
  - Confinement
  - Nontrivial topological vacuum structure
  - Chiral symmetry breaking

The Schwinger Model

Dealing with the fermionic degrees of freedom

- **Tensor Network** algorithms can deal with **fermionic degrees of freedom**
- For one 1d problems it is often more practical to translate them to spins using a **Jordan-Wigner transformation** $\phi_n = \prod_{l<n} (iZ_l)\, \sigma_n^-$

P. Corboz, R. Orús, B. Bauer, G. Vidal, Phys. Rev. B **81**, 165104 (2010)
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- For one 1d problems it is often more practical to translate them to spins using a **Jordan-Wigner transformation** $\phi_n = \prod_{l<n} (iZ_l)\, \sigma_n^-$
- Resulting Hamiltonian in spin language

$$H = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\sigma_n^+ U_n \sigma_{n+1}^- - \text{h.c.}\right) + \sum_{n=1}^{N}(-1)^n m \frac{1}{2}\left(\mathbb{1} + Z_n\right) + \frac{ag^2}{2}\sum_{n=1}^{N-1}\left(L_n + \frac{\theta}{2\pi}\right)^2$$

- Gauss law: $L_n - L_{n-1} = Q_n = \frac{1}{2}\Big(Z_n - (-1)^n\Big)$

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The Schwinger Model

Dealing with the gauge degrees of freedom

- Electric field on the links is unbounded $\Rightarrow$ infinite dimensional Hilbert spaces
- These cannot be directly treated in numerical simulations

Truncation

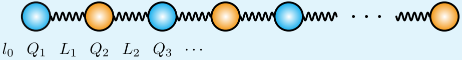
- Truncate the links to a finite number of degrees of freedom
- **Quantum link models**

$$U_n \to S_n^+, \quad L_n \to S_n^z$$

- Unitarity of link operators is lost

Removing the gauge fields

- **Only** possible for **1+1d** and OBC
- Use Gauss law to express the electric field



- **Long-range interactions**

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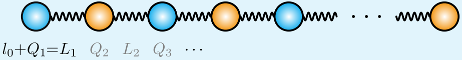

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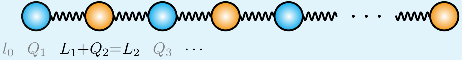

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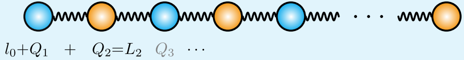
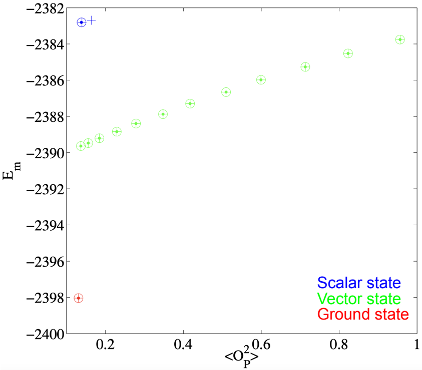


- **Long-range interactions**

The Schwinger Model

Particle spectrum of the Schwinger model with MPS

- Both approaches have been used in the context of Tensor Networks
- Low-lying particle spectrum of the Schwinger model after full extrapolation



Vector binding energy

m/g	exact	DMRG[1]	MPS[3]	MPS[4]
0	0.5641895	0.5641859	0.56414(26)	0.56418(2)
0.125	-	0.53950(7)	0.53946(20)	0.539491(8)
0.25	-	0.51918(5)	0.51915(14)	0.51917(2)
0.5	-	0.48747(2)	0.48748(6)	0.487473(7)

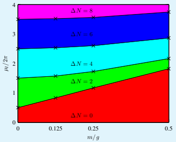
Scalar binding energy

m/g	exact	SCE[2]	MPS[3]	MPS[4]
0	1.12838	1.128379	1.1283(10)	-
0.125	-	1.22(2)	1.221(2)	1.222(4)
0.25	-	1.24(3)	1.239(6)	1.2282(4)
0.5	-	1.20(3)	1.231(5)	1.2004(1)

[1] C. J. Hamer, Z. Weihong, J. Oitmaa, Phys. Rev. D **56**, 55 (1997)
[2] T. M. R. Byrnes, P. Sriganesh, R. J. Bursill, C. J. Hamer, Phys. Rev. D **66**, 013002 (2002)
[3] M.C. Bañuls, K. Cichy, K. Jansen, J. I. Cirac, JHEP **2013**, 158 (2013)
[4] B. Buyens, J. Haegeman, K. Van Acoleyen, H. Verschelde, F. Verstraete, Phys. Rev. Lett. **113**, 091601 (2014)

Tensor Networks for lattice field theories: 1+1d problems

Chemical potential



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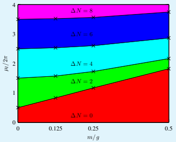

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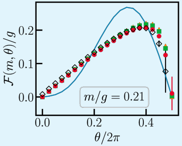
Topological term



Funcke et al., Phys. Rev. D **101**, 054507 (2020)

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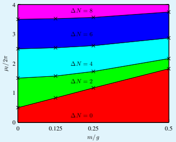

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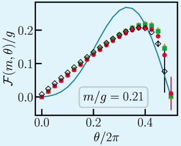

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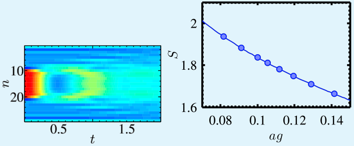
Non-Abelian theories



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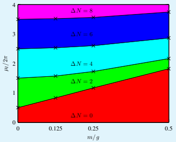

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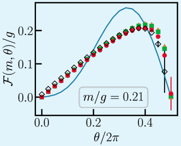

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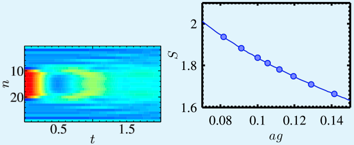

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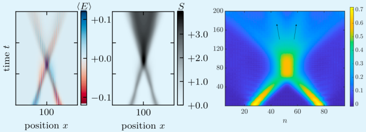
Scattering



Rigobello et al., PRD **104**, 114501 (2021)
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Chai et al., arXiv:2505.21240

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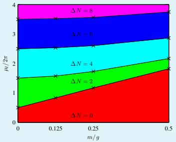

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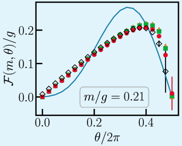

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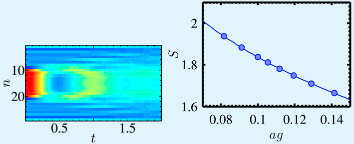

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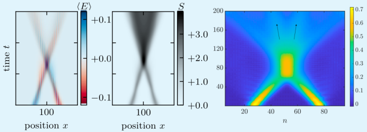

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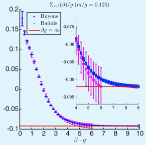
Finite temperature



Buyens et al., PRD **94**, 085018 (2016)
Banuls et al., PRD **92**, 034519 (2015)

Tensor Networks for lattice field theories: 1+1d problems

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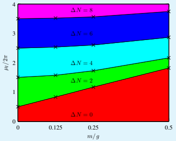

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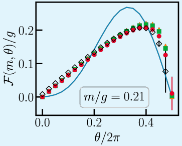

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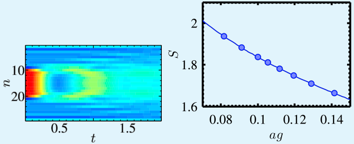

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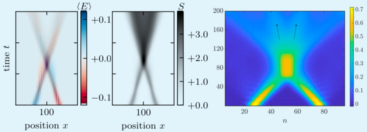

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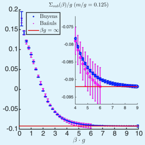

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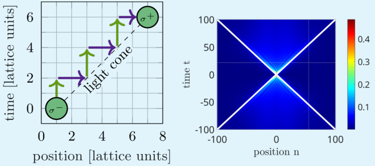
PDFs



Kang et al., arXiv:2501.09738
Bañuls et al. arXiv:2504.0750

Higher dimensions

Quantum Electrodynamics higher dimensions

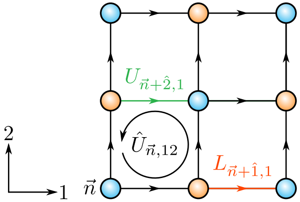
Hamiltonian formulation

- Kogut-Susskind staggered fermions in temporal gauge $A^0 = 0$

$$H = -t\sum_{\mathbf{n},\mu}\left(\phi^\dagger_{\mathbf{n}} U_{\mathbf{n},\mu}\phi_{\mathbf{n}+\mu} + \text{h.c.}\right) + m\sum_{\mathbf{n}}(-1)^{\sum_i n_i}\phi^\dagger_{\mathbf{n}}\phi_{\mathbf{n}} + \frac{g_e^2}{2}\sum_{\mathbf{n},\mu} L^2_{\mathbf{n},\mu}$$
$$-\frac{g_m^2}{2}\sum_{\text{plaquettes } p}\left(\Box_p + \text{h.c.}\right)$$

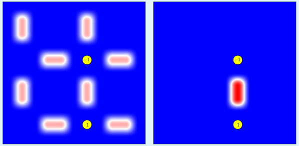
- $\Box_p$: plaquette operator representing magnetic interaction
- Gauss law: $\forall n\ G_{\mathbf{n}}\,|\psi\rangle = 0$, $G_{\mathbf{n}} = \sum_\mu L_{\mathbf{n},\mu} - Q_{\mathbf{n}}$
- Fermions can no longer be integrated out

$\Rightarrow$ Gauge degrees of freedom have to be truncated



Tensor Networks for lattice field theories: beyond 1+1d

$\mathbb{Z}_3$ LGT in (2+1)d



Robaina et al., Phy. Rev. Lett. **126**, 050401 (2021)

G. Magnifico, G. Cataldi, M. Rigobello, P. Majcen, D. Jaschke, P. Silvi, S. Montangero, Commun. Phys. **8**, 322 (2025)

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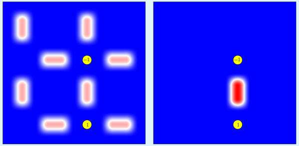

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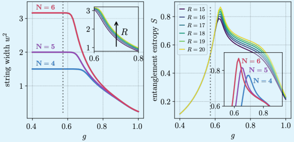

Di Marcantonio et al., arXiv:2505.23853
Xu et al., arXiv:2507.01950

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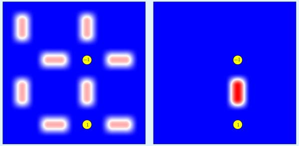

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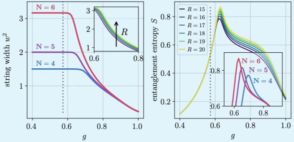

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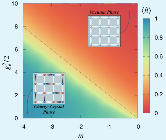
U(1) Quantum Link Model in (2+1)d



T. Felser et al., Phys. Rev. X **10**, 041040 (2020)

G. Magnifico, G. Cataldi, M. Rigobello, P. Majcen, D. Jaschke, P. Silvi, S. Montangero, Commun. Phys. **8**, 322 (2025)

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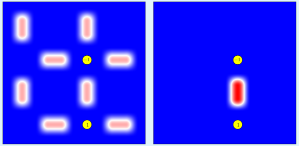
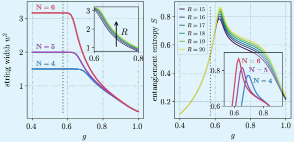


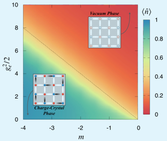
Robaina et al., Phy. Rev. Lett. **126**, 050401 (2021)

$\mathbb{Z}_2$ LGT in 2+1d



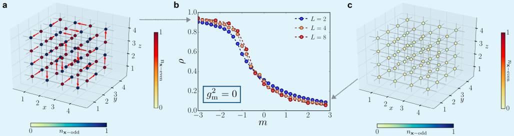
Di Marcantonio et al., arXiv:2505.23853
Xu et al., arXiv:2507.01950

U(1) Quantum Link Model in (2+1)d



T. Felser et al., Phys. Rev. X **10**, 041040 (2020)

U(1) Quantum Link Model in (3+1)d



G. Magnifico et al., Nat. Commun. **12**, 3600 (2021)

G. Magnifico, G. Cataldi, M. Rigobello, P. Majcen, D. Jaschke, P. Silvi, S. Montangero, Commun. Phys. **8**, 322 (2025)

Different fermion discretizations

The Schwinger model

Wilson fermions

- Staggered fermions fail to fully remove the doublers beyond one spatial dimension
- Wilson fermions remove doublers by adding a second derivative $r\frac{a^2}{2}\sum_n \bar{\phi}_n \partial_1^2 \phi_n$
- Wilson formulation fully removes all doublers in any dimension

T. V. Zache, F. Hebenstreit, F. Jendrzejewski, M. K. Oberthaler, J. Berges, P.. Hauke, Quantum Sci. Technol. **3** 034010 (2018)
T. Angelides, L. Funcke, K. Jansen, SK, PRD **108**, 014516 (2023)

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- Hamiltonian formulation with Wilson fermions with $r = 1$

$$H = \sum_{n=1}^{N-1} \left(\bar{\phi}_n \left(\frac{1 + i\gamma_1}{2a} \right) U_n \, \phi_{n+1} + \text{h.c.} \right) + \sum_{n=1}^{N} \left(m_{\text{lat}} + \frac{1}{a} \right) \bar{\phi}_n \phi_n + \frac{ag^2}{2} \sum_{n=1}^{N-1} \left(L_n + \frac{\theta}{2\pi} \right)^2$$

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kinetic part + coupling to gauge field mass term electric energy + topological term

ϕ_n: Two-component Dirac spinor, $U_n = e^{i\alpha_n}$: link operator, $[L_n, \alpha_{n'}] = i\delta_{nn'}$

T. V. Zache, F. Hebenstreit, F. Jendrzejewski, M. K. Oberthaler, J. Berges, P.. Hauke, Quantum Sci. Technol. **3** 034010 (2018)
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kinetic part + coupling to gauge field

mass term

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T. Angelides, L. Funcke, K. Jansen, SK, PRD **108**, 014516 (2023)

The Schwinger model

Wilson fermions

- Additive mass renormalization

$$\frac{m_r}{g} = \frac{m_{\text{lat}}}{g} + \underbrace{\boxed{\text{MS}(N, ag, \theta, r)}}_{\text{mass shift}}$$

- Determining the mass shift
  - At $m_r = 0$ physics is independent from θ
  - Electric field vanishes as it is $\propto m_r/g$
- Condition can be used to determine the mass shift

T. Angelides, L. Funcke, K. Jansen, SK, PRD **108**, 014516 (2023)

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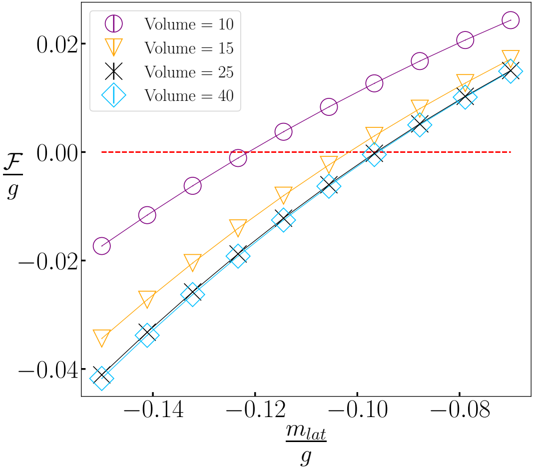

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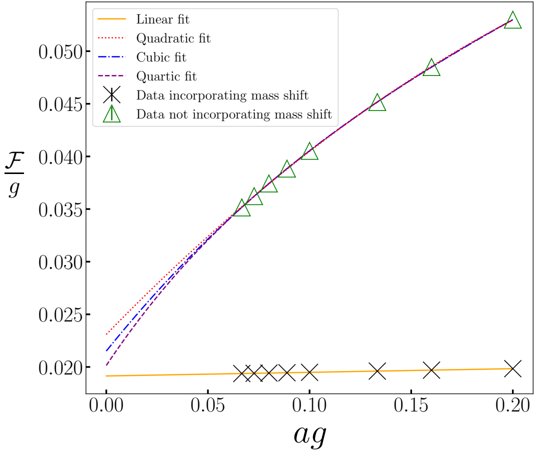
> Including MS improves convergence to the continuum



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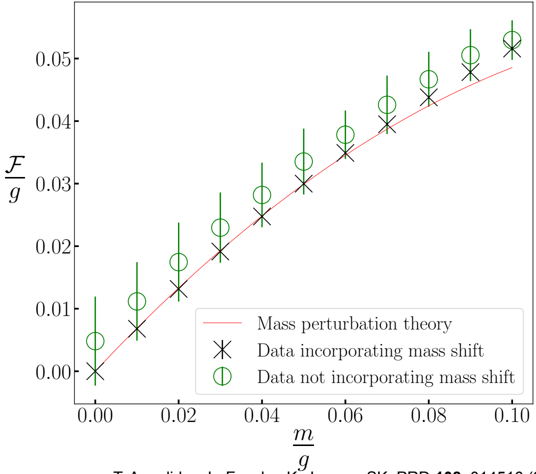

T. Angelides, L. Funcke, K. Jansen, SK, PRD **108**, 014516 (2023)

The Schwinger model

Twisted mass fermions in the continuum

- Wilson fermions generally seem to require larger volumes to converge
- Twisted mass fermions

$$\mathcal{L}_F^{\mathrm{TM}} = \sum_{f,f'=0}^{1} \bar{\chi}_f \left(i\gamma^\mu D_\mu \delta_{ff'} - (m\delta_{ff'} + i\mu\gamma^5\tau^3_{ff'})\right)\chi_{f'} = \bar{\chi}\left(i\gamma^\mu D_\mu - \boxed{M}\, e^{i\boxed{\varphi}\gamma^5\tau^3}\right)\chi$$

mass $M = \sqrt{m^2+\mu^2}$

twist angle

- Applying an axial rotation $\psi = e^{i\omega\gamma^5\tau^3/2}\chi, \quad \bar{\psi} = \bar{\chi}e^{i\omega\gamma^5\tau^3/2}$
- For $\omega = \varphi$ the Lagrangian reduces to

$$\mathcal{L}_F^{\mathrm{TM}} = \bar{\psi}(i\gamma^\mu D_\mu - M)\psi$$

$\Rightarrow$ Standard Lagrangian, twisted mass corresponds to a basis rotation

R. Frezzotti, G. C. Rossi, J. High Energy Phys. **2004** (08), 007
A. Shindler, Phys. Reports **461**, 37 (2008)

The Schwinger model

Twisted mass fermions on the lattice

- At **maximal twist** $\varphi = \pi/2$ the Lagrangian lattice formulation is $\mathcal{O}(a)$ **improved**
- Since $\tan\varphi = \mu/m$ this corresponds $m = 0$

$\Rightarrow$ One needs to tune to vanishing renormalized mass (up to $\mathcal{O}(a)$)

The Schwinger model

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$\Rightarrow$ One needs to tune to vanishing renormalized mass (up to $\mathcal{O}(a)$)

- Hamiltonian lattice formulation in the twisted basis

$$H^{\mathrm{TM}} = \boxed{H} - \mu \sum_{n=0}^{N-1} \sum_{f=0}^{F-1} (-1)^f \left(\chi^\dagger_{n,f,1}\chi_{n,f,1} - \chi^\dagger_{n,f,2}\chi_{n,f,2}\right)$$

Hamiltonian for Wilson fermions

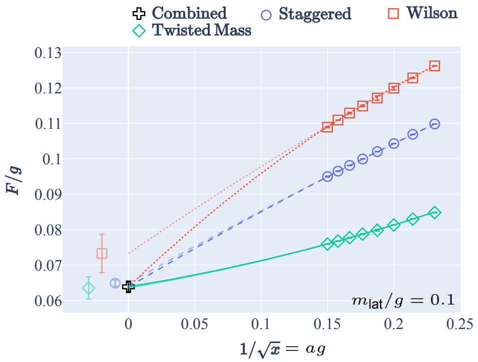
- Gauss law: $L_n - L_{n-1} = Q_n = \sum_{f=0}^{1} \left(\chi^\dagger_{n,f,1}\chi_{n,f,1} + \chi^\dagger_{n,f,2}\chi_{n,f,2}\right) - F$
- Care has to be taken with fermionic observables, need to be rotated in twisted basis

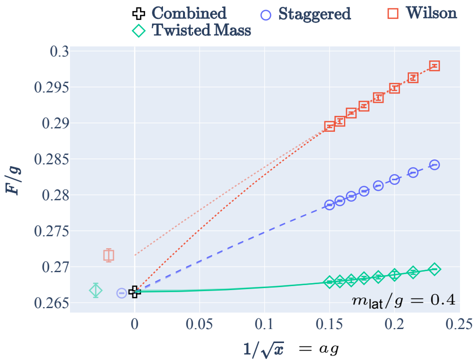
T. Schwägerl, K. Jansen, SK, arXiv:2509.02329

The Schwinger model

Twisted mass fermions on the lattice

> Electric field density of the Schwinger model for $F = 2$, mass shift incorporated





$\Rightarrow$ Dominant quadratic term for twisted mass

T. Schwägerl, K. Jansen, SK, arXiv:2509.02329

The Schwinger model

Twisted mass fermions on the lattice

> Electric field density of the Schwinger model for $F = 2$, no mass shift incorporated

Mass	Fermion Type	A (x^2)	B (x)	C (1)	Linear Contribution	Quadratic Contribution
0.1	Staggered	$-0.099^{+0.017}_{-0.024}$	$0.22^{+0.01}_{-0.01}$	$0.0639^{+0.0007}_{-0.0009}$	$0.92^{+0.01}_{-0.02}$	$0.08^{+0.02}_{-0.01}$
	Wilson	$-0.383^{+0.021}_{-0.027}$	$0.36^{+0.01}_{-0.01}$		$0.835^{+0.002}_{-0.004}$	$0.165^{+0.004}_{-0.002}$
	Twisted Mass	$0.134^{+0.015}_{-0.028}$	$0.06^{+0.02}_{-0.01}$		$0.71^{+0.08}_{-0.06}$	$0.29^{+0.06}_{-0.08}$
0.4	Staggered	$-0.0505^{+0.0015}_{-0.0028}$	$0.088^{+0.002}_{-0.001}$	$0.2665^{+0.0001}_{-0.0002}$	$0.904^{+0.003}_{-0.006}$	$0.096^{+0.006}_{-0.003}$
	Wilson	$-0.21^{+0.01}_{-0.02}$	$0.184^{+0.004}_{-0.001}$		$0.827^{+0.004}_{-0.006}$	$0.173^{+0.006}_{-0.004}$
	Twisted Mass	$0.0581^{+0.0013}_{-0.0022}$	$0.000^{+0.002}_{-0.001}$		$0.02^{+0.09}_{-0.02}$	$0.98^{+0.02}_{-0.09}$

$\Rightarrow$ Dominant quadratic term for twisted mass

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The Schwinger model

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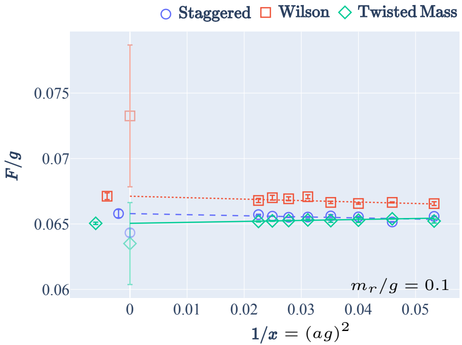
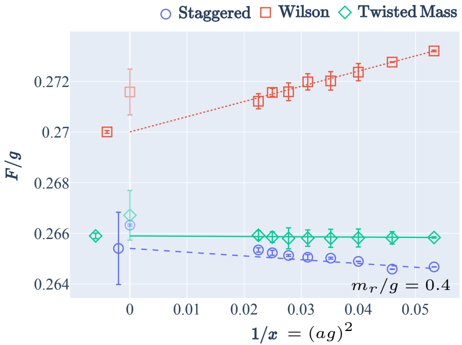




T. Schwägerl, K. Jansen, SK, arXiv:2509.02329

The Schwinger model

Twisted mass fermions on the lattice

- Low-lying spectrum of the Schwinger model for $F = 2$

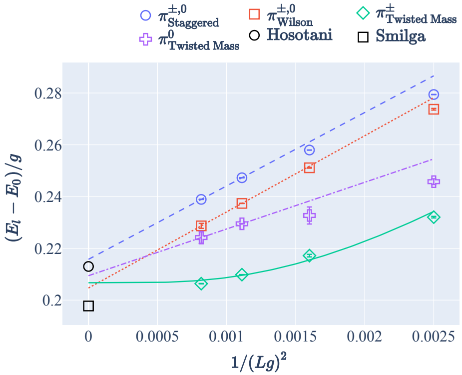
- Isospin breaking at finite lattice spacing similar to Lagrangian lattice QCD

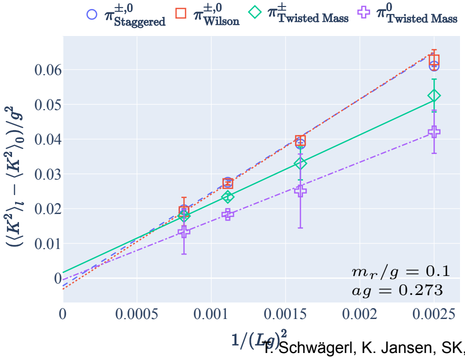
T. Schwägerl, K. Jansen, SK, arXiv:2509.02329

The Schwinger model

Twisted mass fermions on the lattice

- Infinite volume extrapolation of m_π
- Expected volume dependence $\Delta E = m_\pi + \frac{A}{(Lg)^2} + \mathcal{O}\left(\frac{1}{(Lg)^3}\right)$



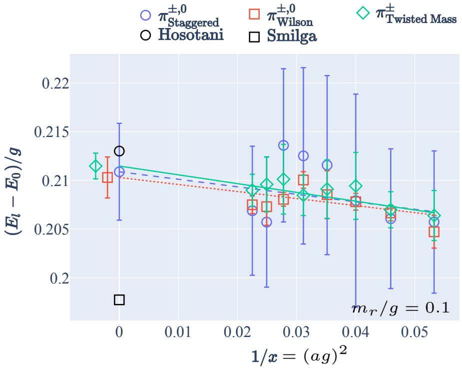


T. Schwägerl, K. Jansen, SK, arXiv:2509.02329

The Schwinger model

Twisted mass fermions on the lattice

> Continuum extrapolation of m_π



T. Schwägerl, K. Jansen, SK, arXiv:22509.02329

The Schwinger model

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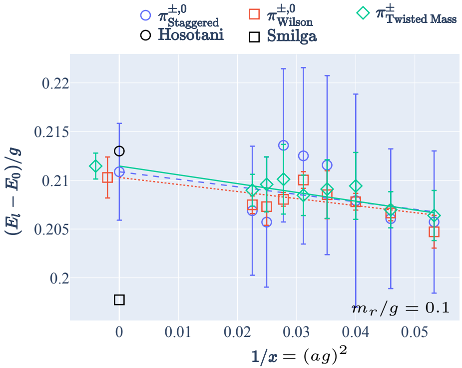

- Compatible with Y. Hosotani et al., J. Phys. A **31**, 9925 (1998)

T. Schwägerl, K. Jansen, SK, arXiv:22509.02329

The Schwinger model

Twisted mass fermions for the two-flavor Schwinger moel

- Results are compatible with expected $\mathcal{O}(a)$ improvement
- Twisted mass fermions with mass shift have smallest lattice artifacts
- Tuning the renormalized mass to zero is expensive, are there better ways?

Open systems

The Schwinger model

Setting of an open system

- Couple the Schwinger model to a φ^4 theory at high temperature in thermal equilibrium
- Study the thermalization of mesonic particles

$\Rightarrow$ **Toy model** for **quarkonium** moving in a **quark-gluon plasma**

K. Lee, J. Mulligan, F. Ringer, X. Yao, Phys. Rev. D **108**, 094518 (2023)
J. Lin, D. Luo, X. Yao, P. Shanahan, JHEP **2024**, 211 (2024)
T. Angelides, Y. Guo, Karl Jansen, SK, G. Magnifico, JHEP **2025**, 195 (2025)

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- Study the thermalization of mesonic particles

$\Rightarrow$ **Toy model** for **quarkonium** moving in a **quark-gluon plasma**

- In the Markovian limit the dynamics can be described by a master equation

$$\dot{\rho}(t) = e^{t\mathcal{L}}\rho_0$$

- Liouvillian superoperator

$$\mathcal{L} = -iH \otimes I + iI \otimes H + \sum_{n,k=0}^{N-1} D(n-k)\left(J_k \otimes J_n^\dagger - \frac{1}{2}J_n^\dagger J_k \otimes I - \frac{1}{2} I \otimes J_n^\dagger J_k\right)$$

- Evolution under $\mathcal{L}$ can be simulated with MPS using time-evolution techniques

K. Lee, J. Mulligan, F. Ringer, X. Yao, Phys. Rev. D **108**, 094518 (2023)
J. Lin, D. Luo, X. Yao, P. Shanahan, JHEP **2024**, 211 (2024)
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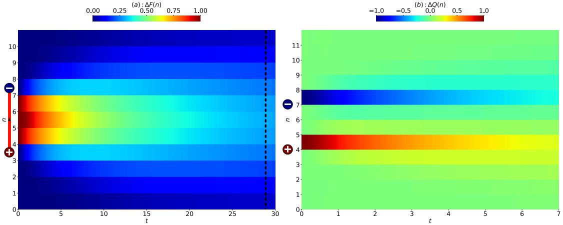
The Schwinger model

String thermalization in an open setting

- Create a noninteracting string
- Monitor the electric field and the charge over time to study thermalization



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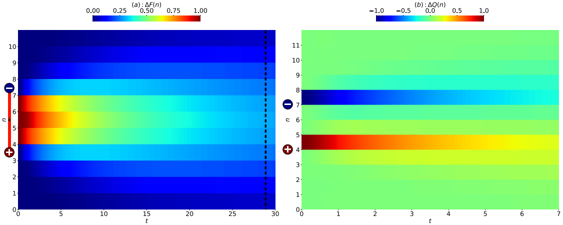

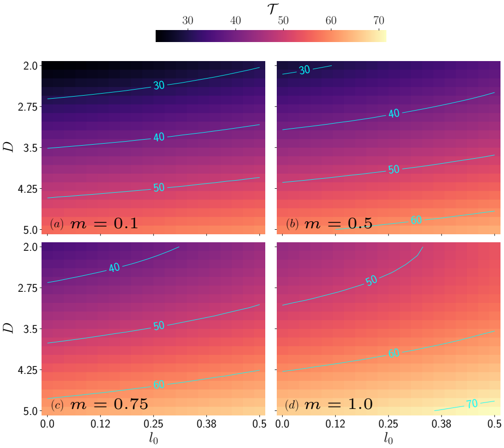

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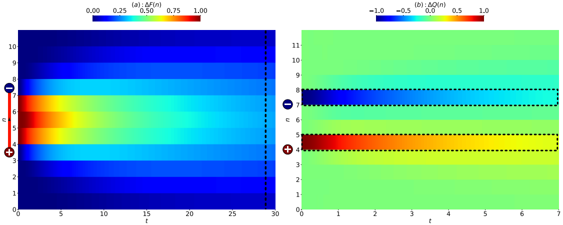
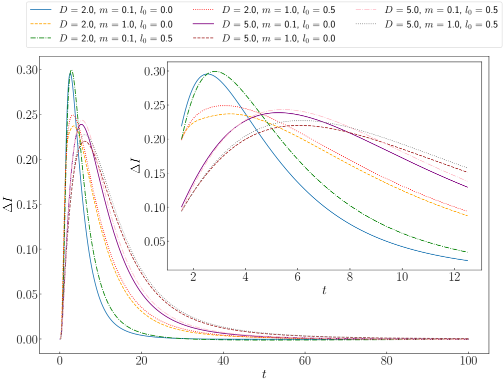
- We can access the quantum correlations in the state





T. Angelides, Y. Guo, Karl Jansen, SK, G. Magnifico, JHEP **2025**, 195 (2025)

4.

Motivation

Tensor Network States

Application to lattice field theory

Synergies with Quantum Computing

Summary & Outlook

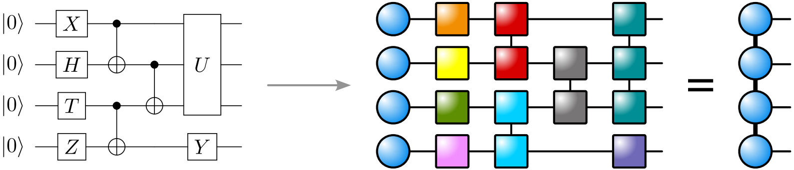
Synergies with Quantum Computing

Tensor Network States for simulating quantum computing

- > Tensor Networks States efficiently represent moderately entangled states

$\Rightarrow$ They can be used to simulate slightly entangled quantum computations

- > Unitary gate operations: evolution under some Hamiltonian for a time Δt



$\Rightarrow$ As long as the entanglement does not grow heavily with time this can be simulated efficiently with Tensor Network States

G. Vidal, Phys. Rev. Lett. **91**, 147902 (2003)
G. Vidal, Phys. Rev. Lett. **93**, 040502 (2004)

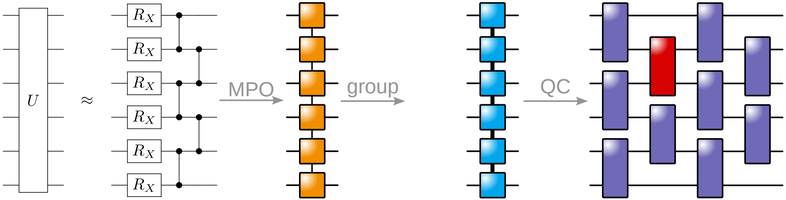
Synergies with Quantum Computing

Tensor Networks for circuit compilation and compression

- Implementing a trotterized version of e^{-iHt} typically leads to deep circuits
- Approximate the target unitary V_{targ} as an MPO

$\Rightarrow$ Can be multiple steps of a high-order trotter decomposition

- Find an circuit U_{QC} approximating V_{targ}



J. Gibbs, L. Cincio, Quantum **9**, 1789 (2025)

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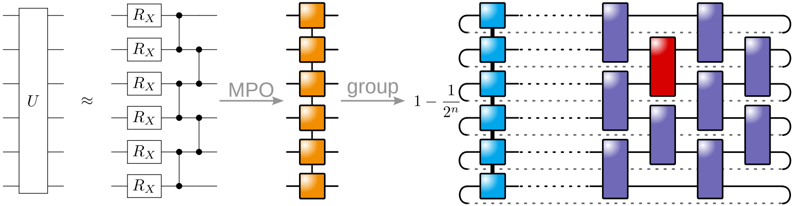
- Find an circuit U_{QC} approximating V_{targ} by minimizing $C = 1 - \frac{1}{2^n}\text{tr}(U_{\text{QC}}^\dagger V_{\text{targ}})^2$



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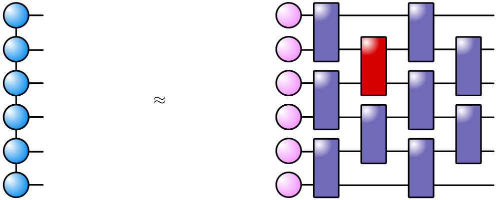

- Principle can also be used to find a circuit loading a state on a quantum device

J. Gibbs, L. Cincio, Quantum **9**, 1789 (2025)

Synergies with Quantum Computing

Tensor Networks for circuit compression: scattering in the Thirring model

> Lattice formulation with staggered fermions

$$H = \frac{i}{2a}\sum_{n=0}^{N-2}\underbrace{\left(\xi_{n+1}^{\dagger}\xi_n - \xi_n^{\dagger}\xi_{n+1}\right)}_{\text{Kinetic energy}} + m\sum_{n=0}^{N-1}\underbrace{(-1)^n\xi_n^{\dagger}\xi_n}_{\text{Mass term}} + \frac{g}{a}\sum_{n=0}^{N-1}\underbrace{\xi_n^{\dagger}\xi_n\xi_{n+1}^{\dagger}\xi_{n+1}}_{\text{Four-body interaction}}$$

Y. Chai, A. Crippa, K. Jansen, SK, V. R. Pascuzzi, F. Tacchino, I. Tavernelli, Quantum 9, 1638 (2025)

Synergies with Quantum Computing

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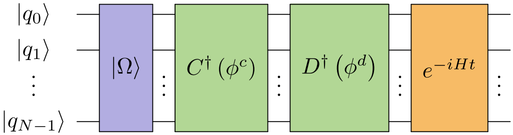
$$H = \frac{i}{2a}\sum_{n=0}^{N-2}\underbrace{\left(\xi^\dagger_{n+1}\xi_n - \xi^\dagger_n\xi_{n+1}\right)}_{\text{Kinetic energy}} + m\sum_{n=0}^{N-1}\underbrace{(-1)^n\xi^\dagger_n\xi_n}_{\text{Mass term}} + \frac{g}{a}\sum_{n=0}^{N-1}\underbrace{\xi^\dagger_n\xi_n\xi^\dagger_{n+1}\xi_{n+1}}_{\text{Four-body interaction}}$$

- Evolving particle wave packets: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle = e^{-iHt}\, D^\dagger(\phi^d)C^\dagger(\phi^c)\, |\Omega\rangle$

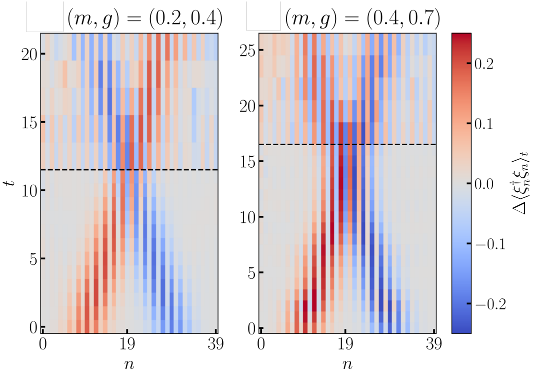


Y. Chai, A. Crippa, K. Jansen, SK, V. R. Pascuzzi, F. Tacchino, I. Tavernelli, Quantum 9, 1638 (2025)

Synergies with Quantum Computing

Tensor Networks for circuit compression: scattering in the Thirring model

- Simulation after compressing the initial part of the evolution and the time evolution operators



Y. Chai, J. Gibbs, V. R. Pascuzzi, Z. Holmes, SK, F. Tacchino, I. Tavernelli, arXiv:2507.17832

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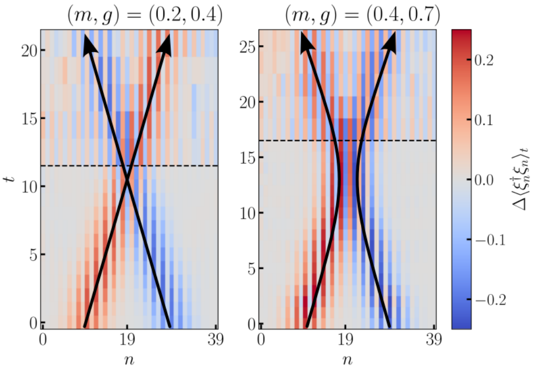

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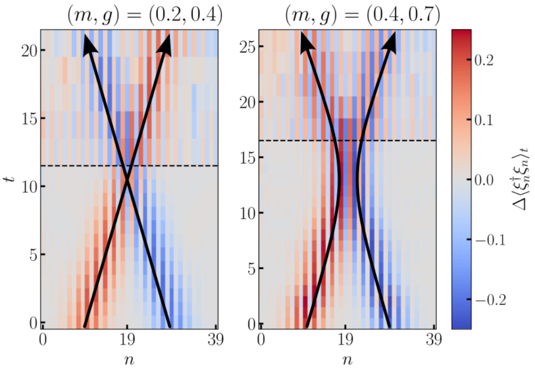

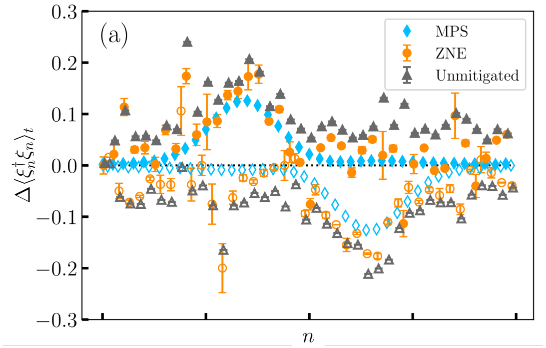

Y. Chai, J. Gibbs, V. R. Pascuzzi, Z. Holmes, SK, F. Tacchino, I. Tavernelli, arXiv:2507.17832

5.

Motivation

Tensor Network States

Application to lattice field theory

Synergies with Quantum Computing

Summary & Outlook

Summary

Tensor Network States

- Entanglement-based ansatz for the wave function of quantum many body state
- Numerical techniques for computing ground states and time-evolution
- These algorithms not suffer from the sign problem
- So far most successful for (1+1)d theories, but first simulations in (3+1)d exist

G. Magnifico, G. Cataldi, M. Rigobello, P. Majcen, D. Jaschke, P. Silvi, S. Montangero, Commun. Phys. **8**, 322 (2025)

Summary

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Theoretical foundations

- Hamiltonian formulation for problems typically given in the Lagrangian formulation
- Techniques suitable for the Hamiltonian formulation
- Efficient basis formulations

G. Magnifico, G. Cataldi, M. Rigobello, P. Majcen, D. Jaschke, P. Silvi, S. Montangero, Commun. Phys. **8**, 322 (2025)

Summary

Monte Carlo Methods

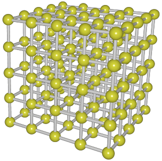
- ✓ Expectation values
- ✗ Access to the wave function
- ✗ Sign-problem free

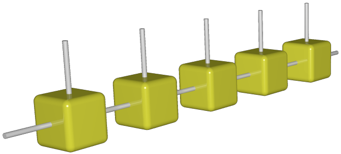
Tensor Network States

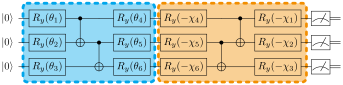
- ✓ Ground states / Low-lying excitations
- ○ Time evolution
- ✓ Sign-problem free

Quantum Computing

- ○ State preparation
- ○ Scaling of resources
- ✓ Free from purely numerical limitations







Thank you!

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