Quantum Computing and Applications to Lattice Gauge Theory

Christian Bauer
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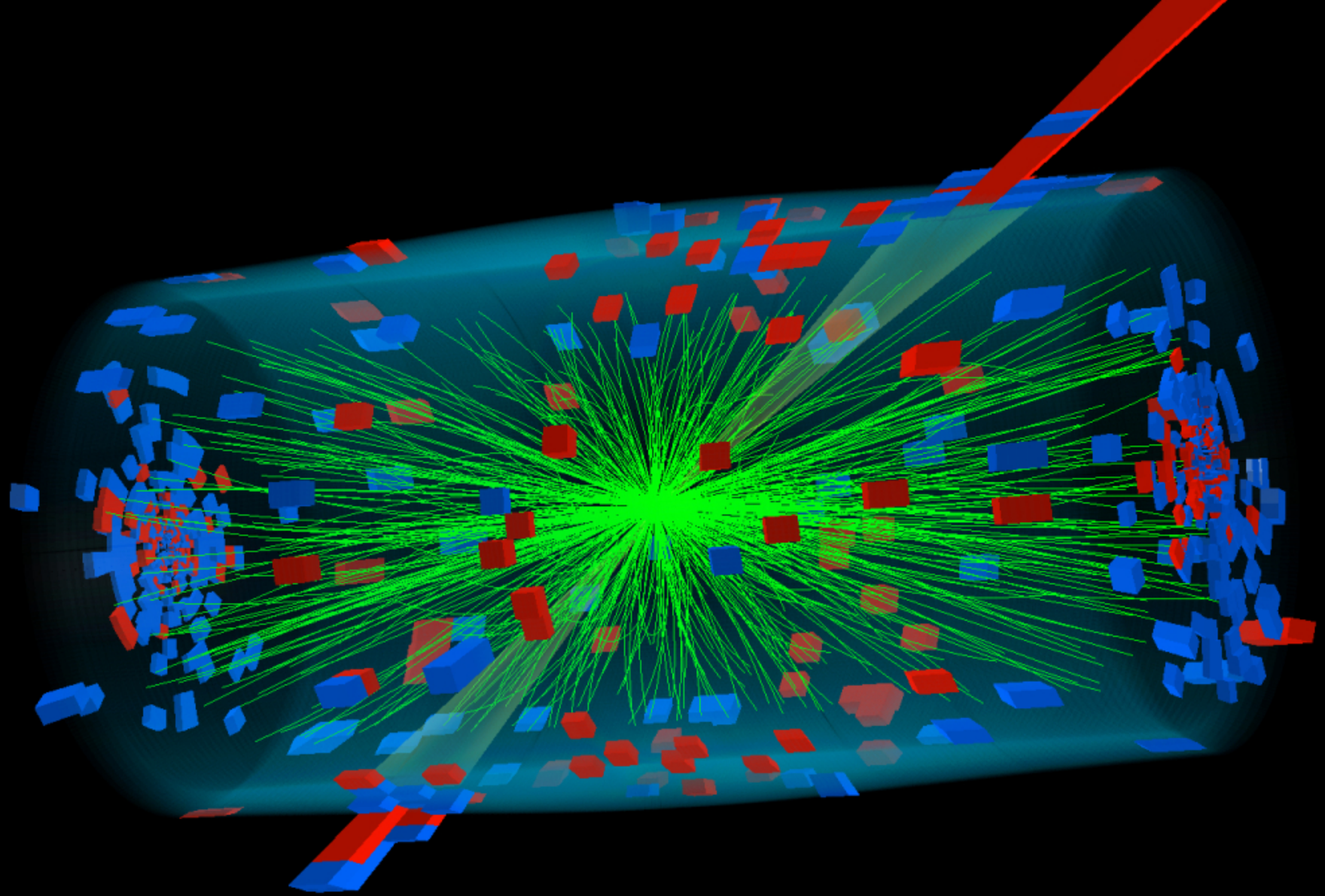
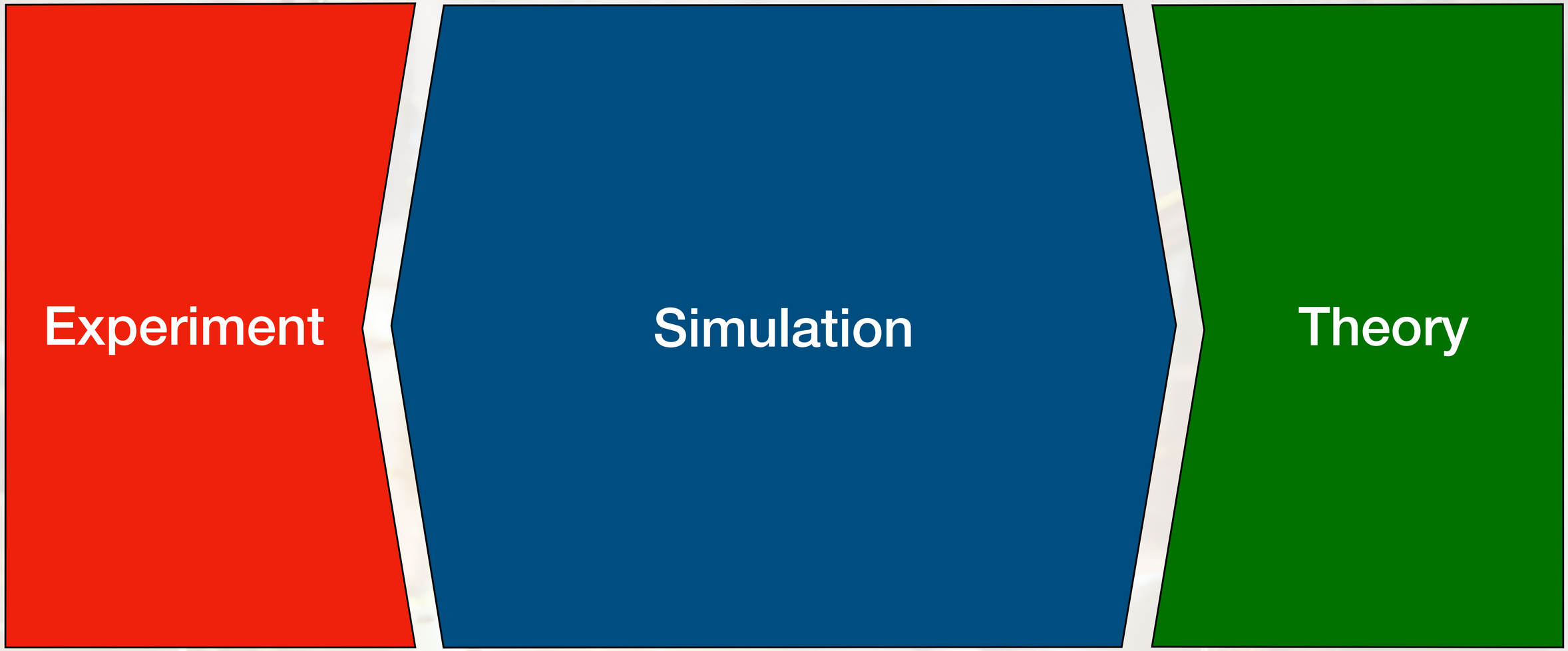
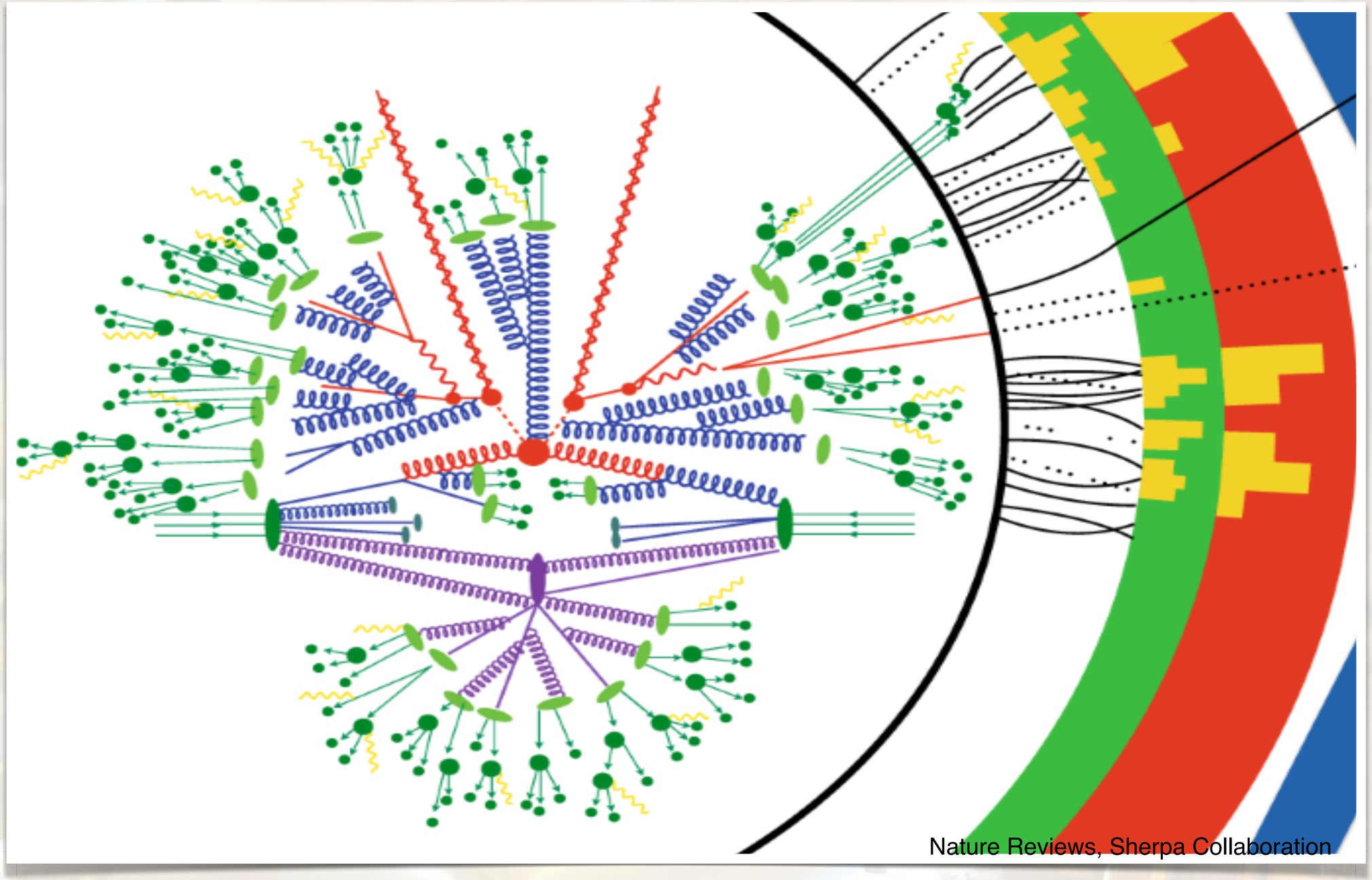


CMS Run/Event 262548/458269

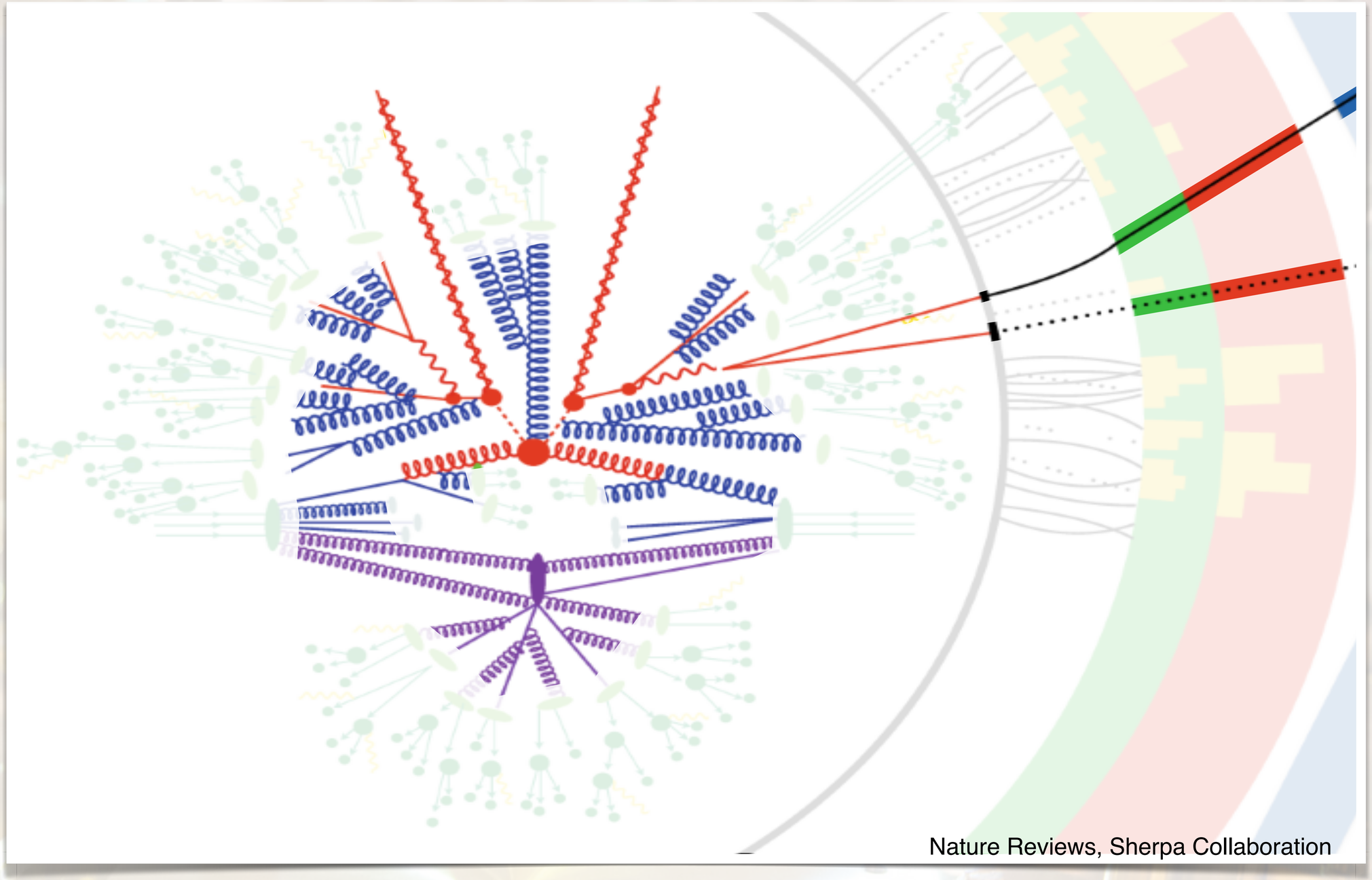
Experiment

Theory


Experiment
Simulation
Theory



Nature Reviews, Sherpa Collaboration



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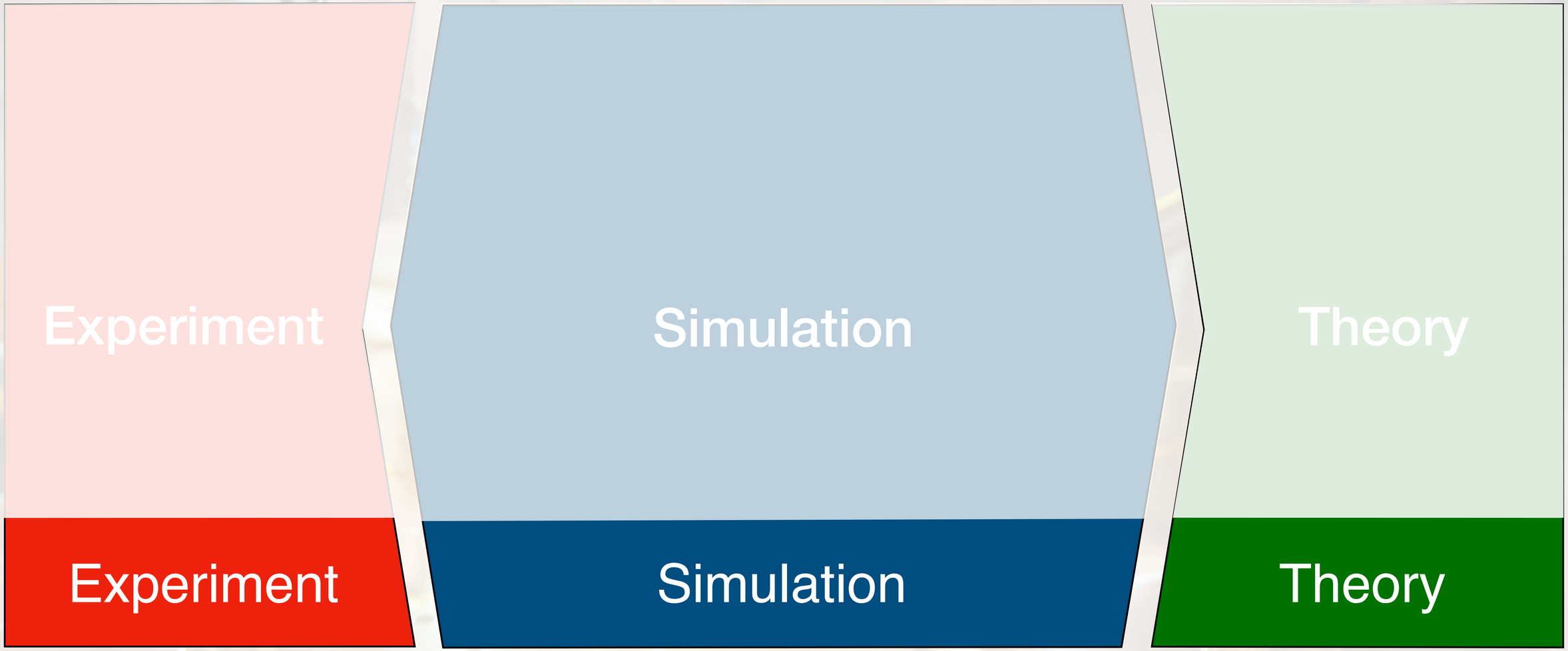
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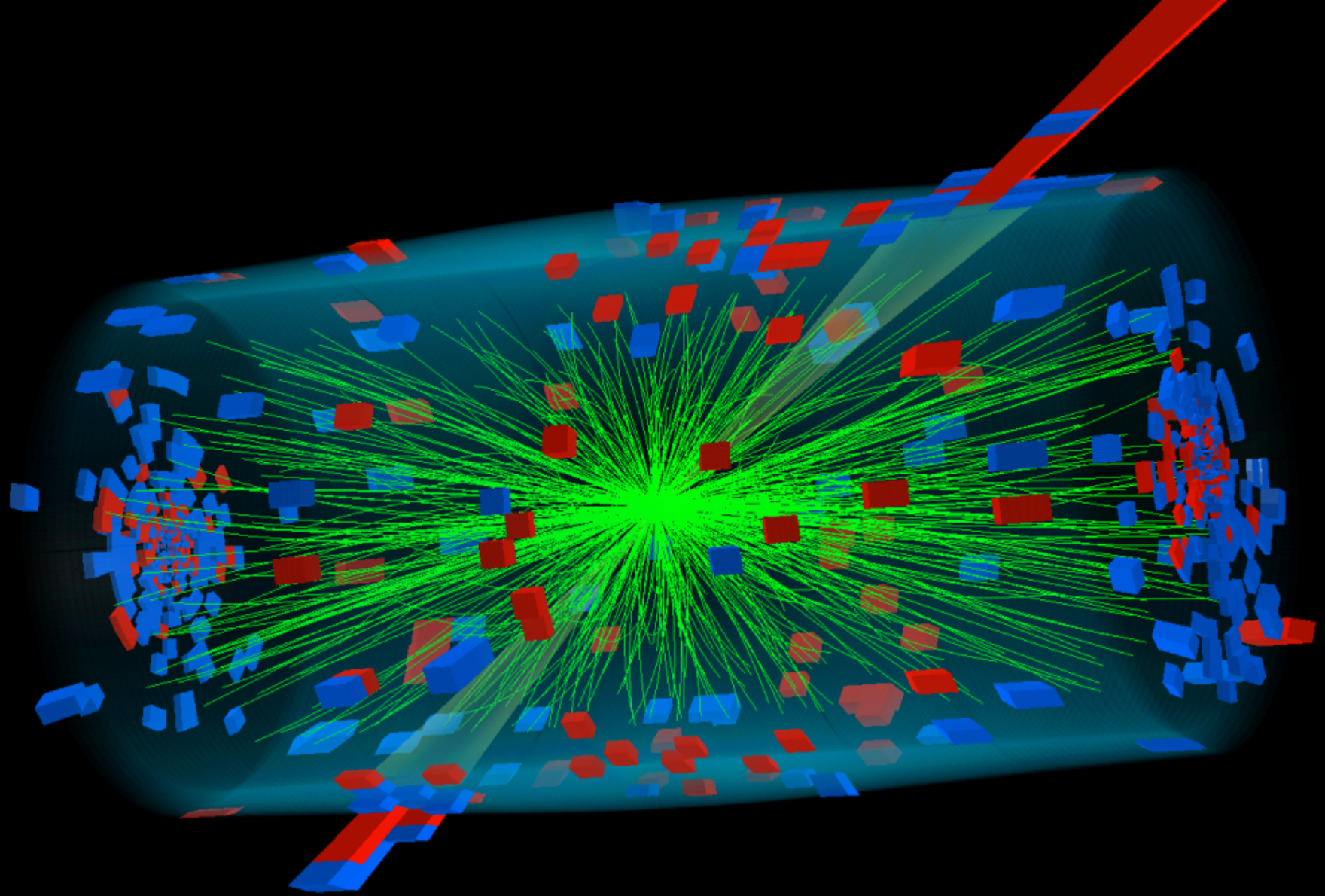

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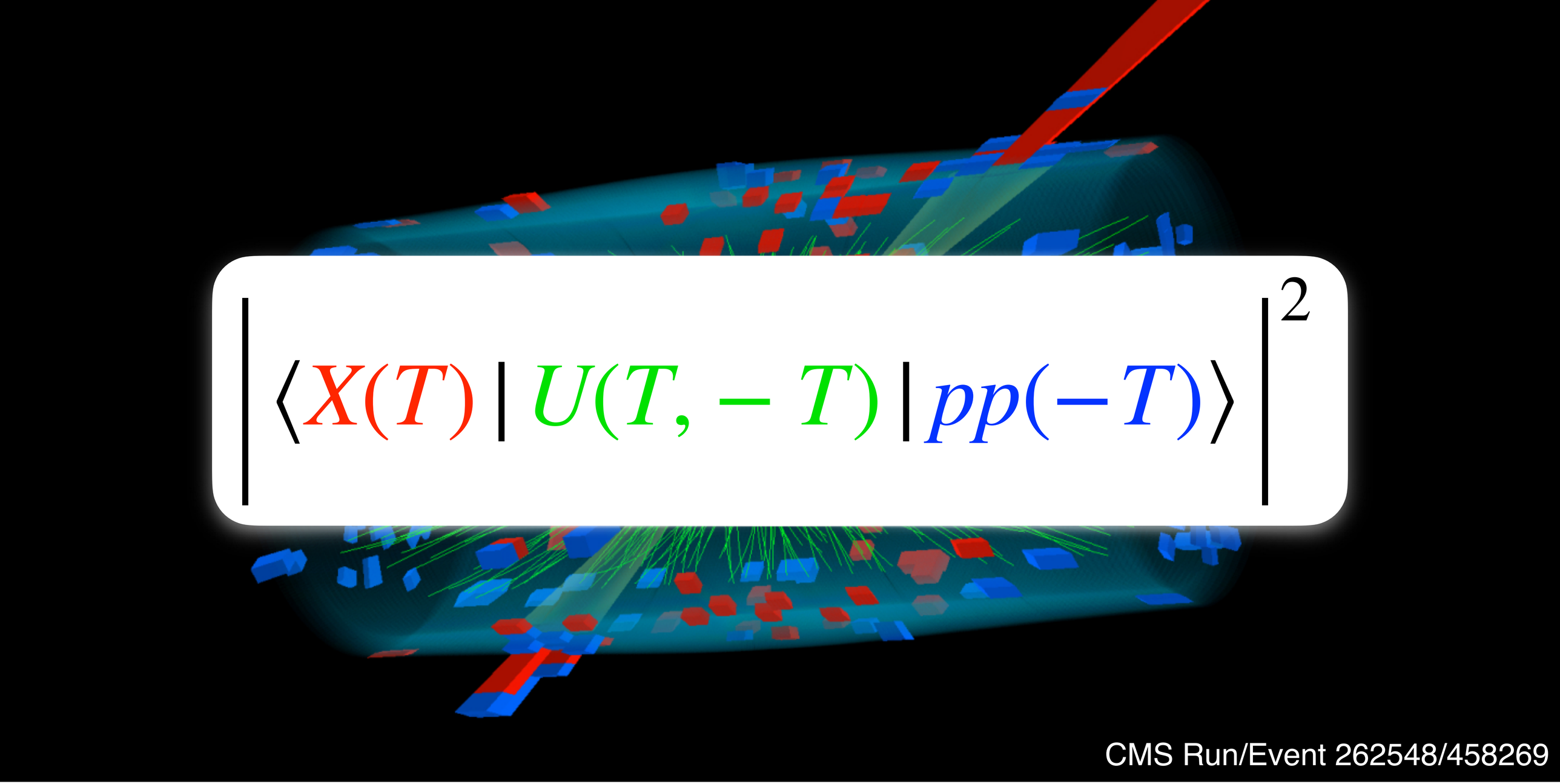

$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$
CMS Run/Event 262548/458269

Lattice Gauge Theory relies on Feynman's path integral formulation, which sums over all possible "paths" or configurations weighted by the action

Quantum mechanics

$$\langle q_f, t_f | q_i, t_i \rangle = \int \mathcal{D}q \, e^{iS[q]}$$

Lattice Gauge Theory relies on Feynman's path integral formulation, which sums over all possible "paths" or configurations weighted by the action

Quantum Field Theory

$$\left\langle \phi_f, t_f \middle| \phi_i, t_i \right\rangle = \int \mathcal{D}\phi \, e^{iS[\phi]}$$

Lattice Gauge Theory relies on Feynman's path integral formulation, which integrates over all configurations

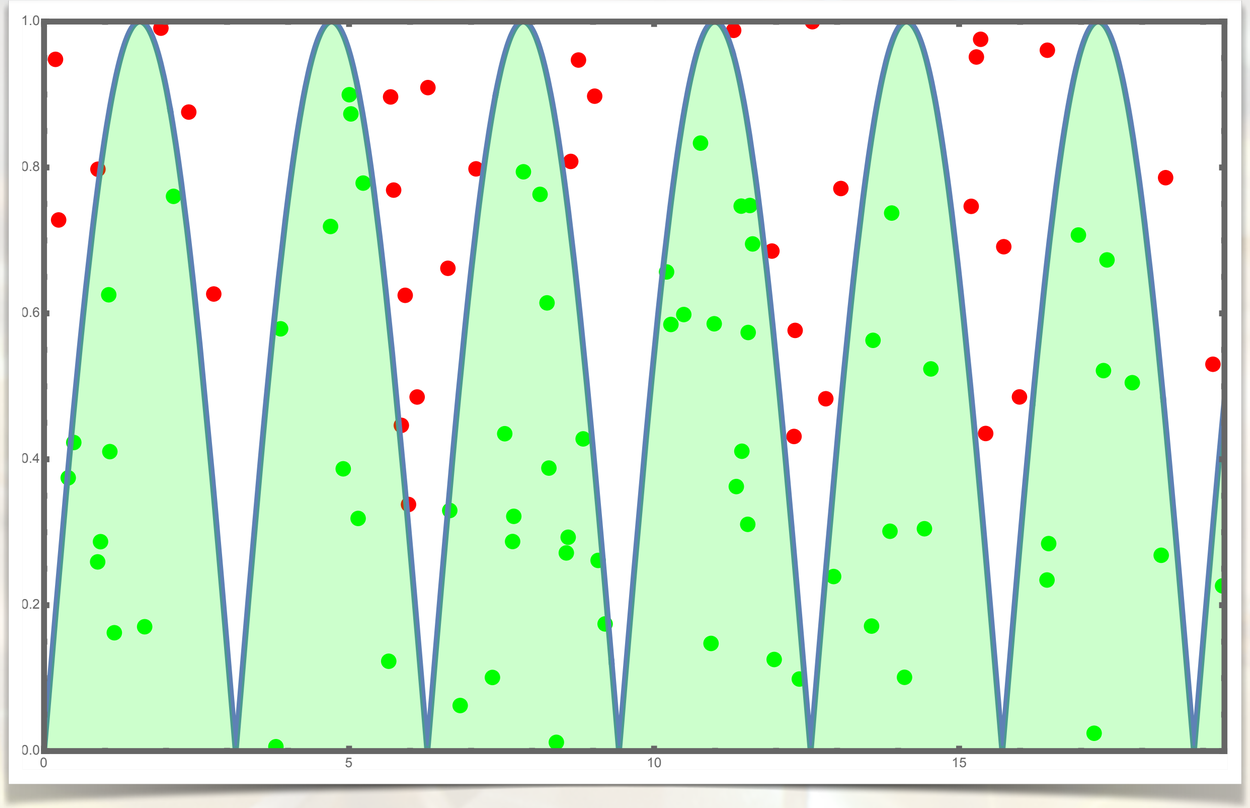
$$\left\langle \phi_f, t_f \middle| q_i, t_i \right\rangle = \frac{1}{V} \sum \phi_j(x_i)\, \Delta\phi_j(x_j)\, e^{iS[\phi_j(x_i))]}$$

Number of terms in integral:

$$n = n_j^V$$

Well known technique to integrate high dimensional functions with resources that are independent of n (Monte-Carlo integration)

$$\int_0^1 \mathrm{d}x_1 \ldots \mathrm{d}x_n \; f(x_1, \ldots x_n) = V \frac{N_\bullet}{N_{\mathrm{tot}}}$$



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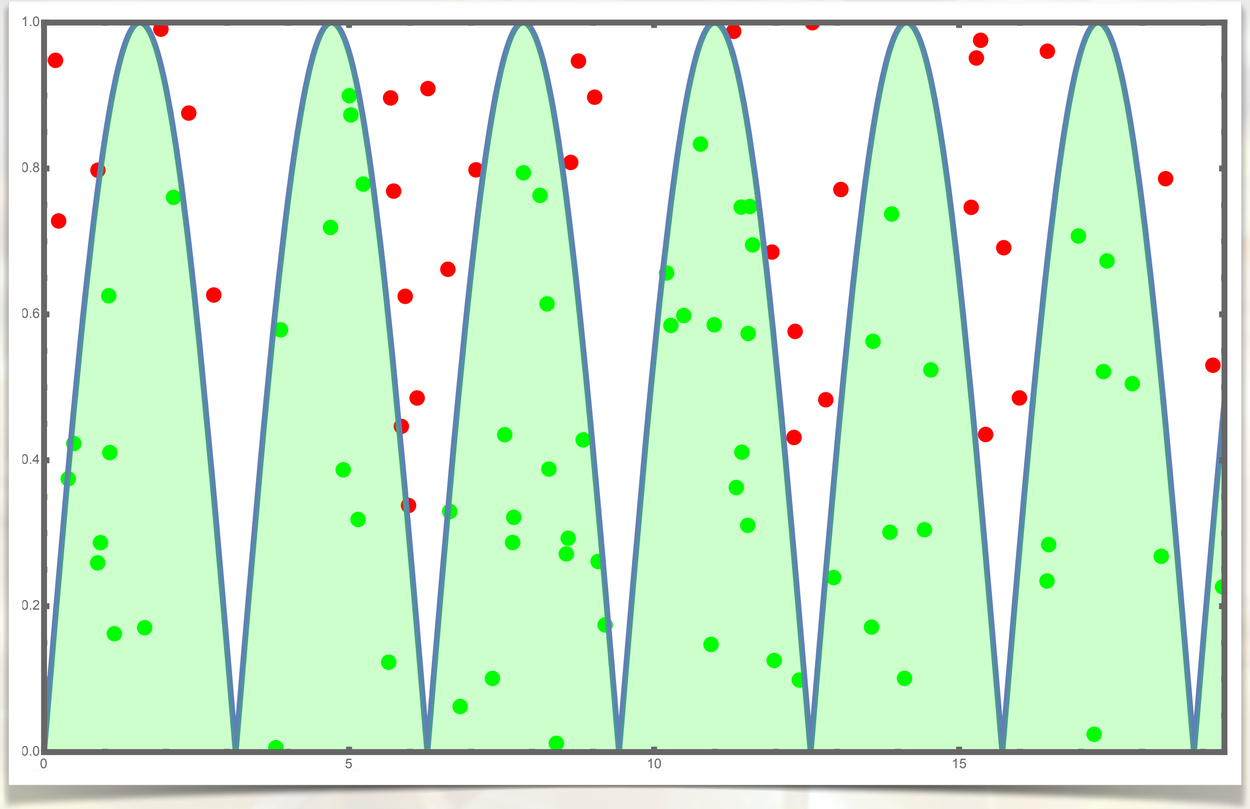

To get integral with uncertainty ϵ need

$$N_{\mathrm{tot}} \sim \frac{1}{\epsilon^2}$$

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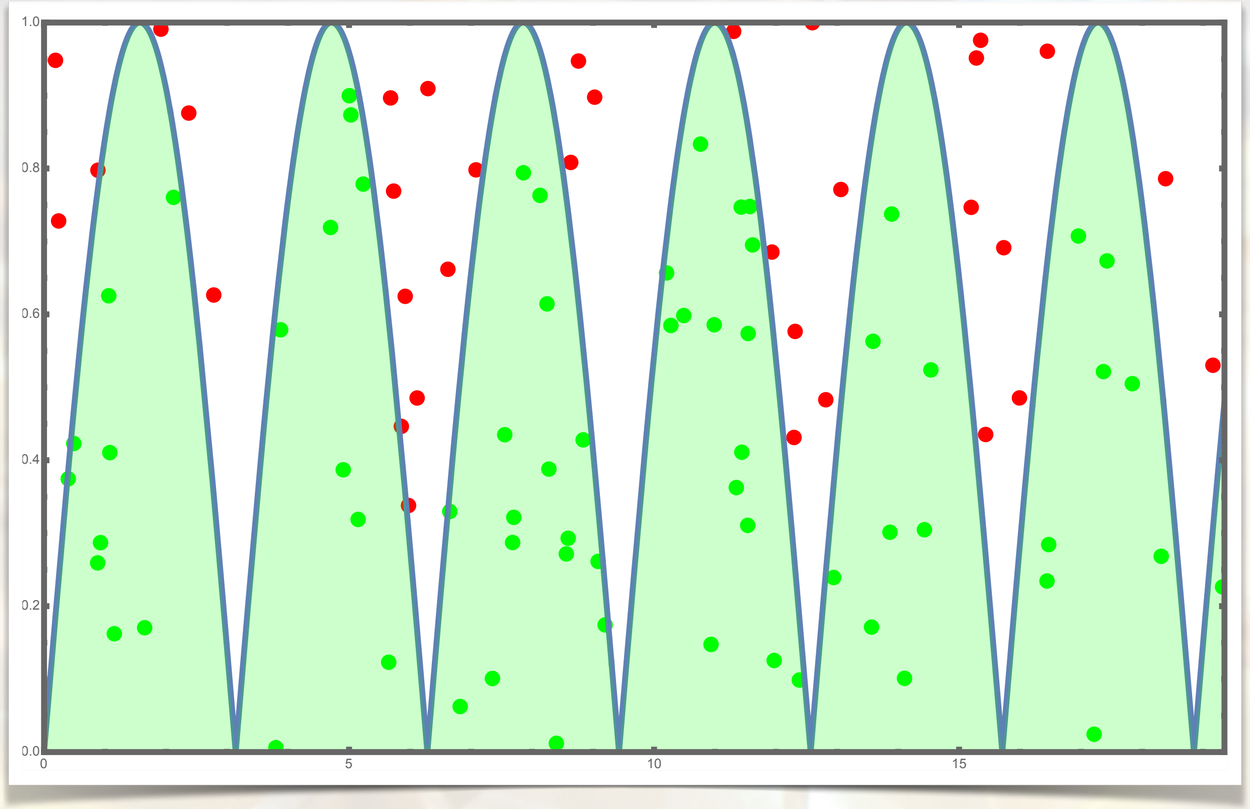

To get integral with uncertainty ϵ need

$$N_{\mathrm{tot}} \sim \frac{1}{\epsilon^2}$$

Requires function to be positive definite

Lattice Gauge Theory relies on Feynman's path integral formulation, which integrates over all configurations

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For imaginary time

$$e^{iS[\phi_j(x_i))]} \rightarrow e^{-S[\phi_j(x_i))]}$$

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For imaginary time

$$e^{iS[\phi_j(x_i))]} \longrightarrow e^{-S[\phi_j(x_i))]}$$

Can answer many static questions, but calculating dynamics requires real time, not imaginary time

Instead of doing Monte-Carlo simulation of path integral, can try to do time evolution using Schrödinger equation

Go back to the S matrix elements mentioned before

$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

All elements in this expression in terms of fields $\phi(x)$

Both position x and field $\phi(x)$ are continuous

Discretizing position x and digitizing field value $\phi(x)$ turn continuous (QFT) problem into discrete (QM) problem

Basic idea is to map the infinite Hilbert space of QFT on a finite dimensional HS making this a QM problem

$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

3 basic steps:

1. Create an initial state vector at time (-T) of two proton wave packets
2. Evolve this state forward in time from to time T using the Hamiltonian of the full interacting field theory
3. Perform a measurement of the state

Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan,[1]* Keith S. M. Lee,[2] John Preskill[3]

Quantum field theory reconciles quantum mechanics and special relativity, and plays a central role in many areas of physics. We developed a quantum algorithm to compute relativistic scattering probabilities in a massive quantum field theory with quartic self-interactions (ϕ^4 theory) in spacetime of four and fewer dimensions. Its run time is polynomial in the number of particles, their energy, and the desired precision, and applies at both weak and strong coupling. In the strong-coupling and high-precision regimes, our quantum algorithm achieves exponential speedup over the fastest known classical algorithm.

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Since quantum computers can only work in a finite dimensional Hilbert space, need to formulate problem in terms of truncated Hamiltonian lattice gauge theory

1. Formulate a lattice theory that reproduces SU(3) in the limit of vanishing lattice spacing
2. Choose basis for the Hilbert space
3. Truncate the theory (how to choose a discrete set of field values)

Goal is Hamiltonian Lattice theory that reproduces QCD in continuum limit and can be simulated efficiently on a quantum computer

We have to remember that truncated lattice formulations are always an approximation to the real world

Truncated Hamiltonian Lattice Gauge Theories …

1. …remove physics below some length scale (lattice spacing)
2. …remove physics above some length scale (lattice volume)
3. …only approximately represent bosonic degrees of freedom

How do we estimate the uncertainties made by the above approximations?

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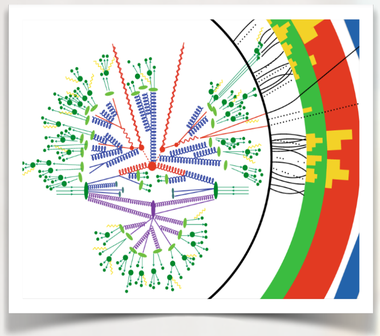
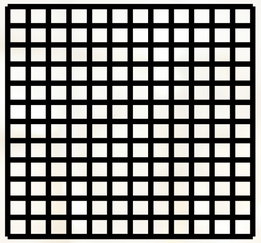
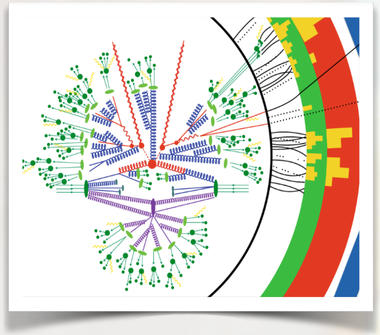


What observables should we calculate on a quantum computer?



How do we take the continuum limit in of quantum lattice simulations?

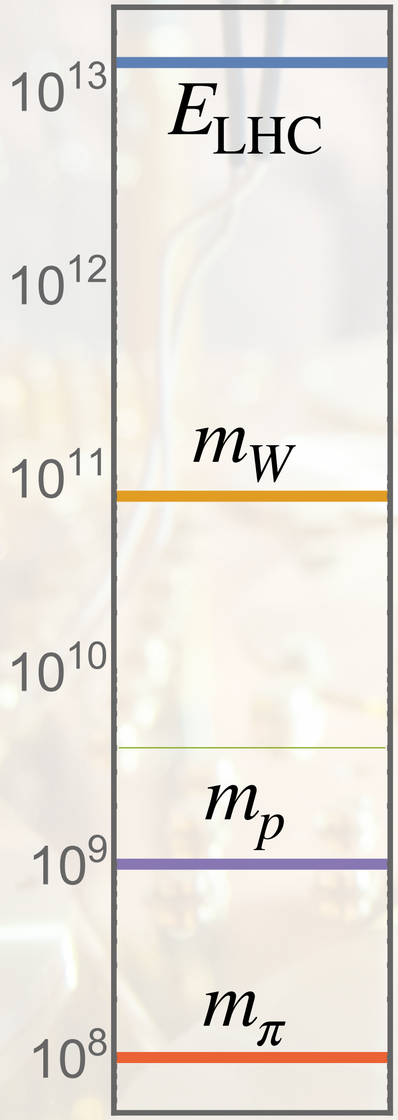
How do we determine what level of truncation is needed for the physics we are after?



What observables should we calculate on a quantum computer?

There are many energy scales that are present in LHC events, and all need to be accounted for in an adequate description

CWB, 2503.16602



Energy of colliding protons

Scale of electroweak gauge bosons

Mass of the proton

Mass of the pion, the lightest hadron

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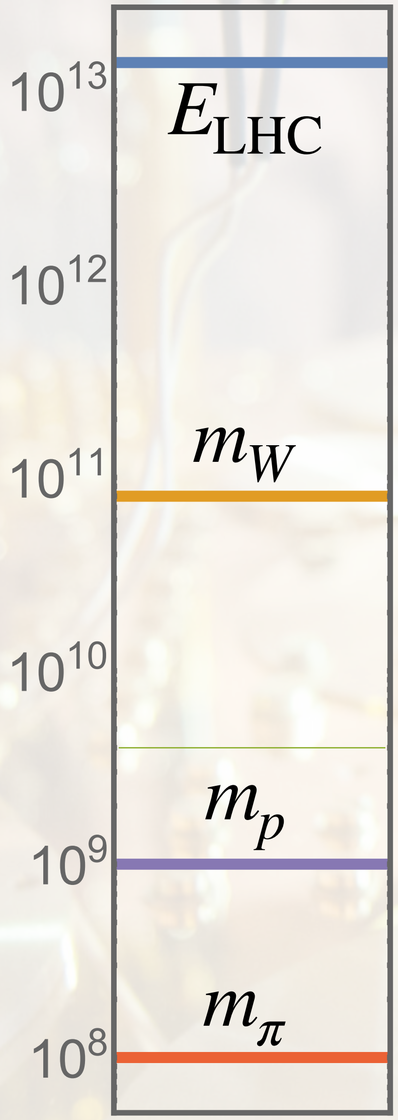

Field configurations corresonding to given energy have wavelength

$$l \sim 1/E$$

The largest and smallest energy scales set the maximum and minimum wavelength of field configurations that need to be considered

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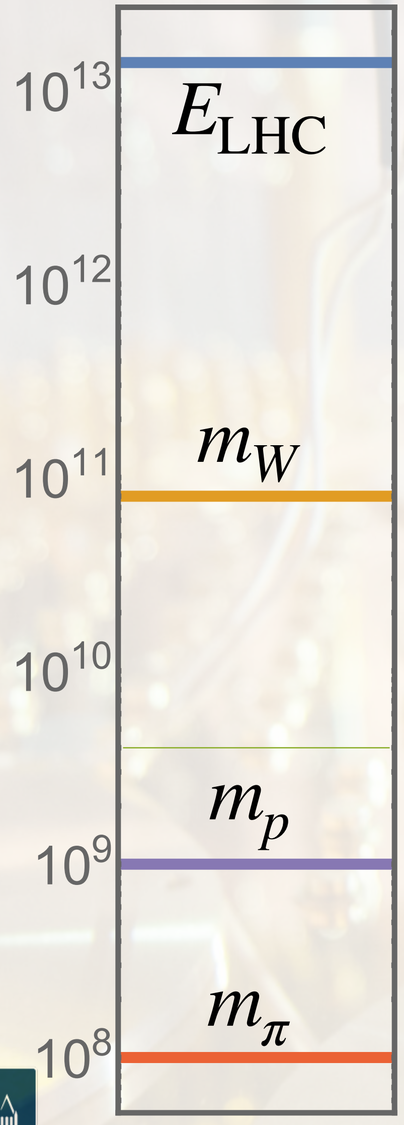

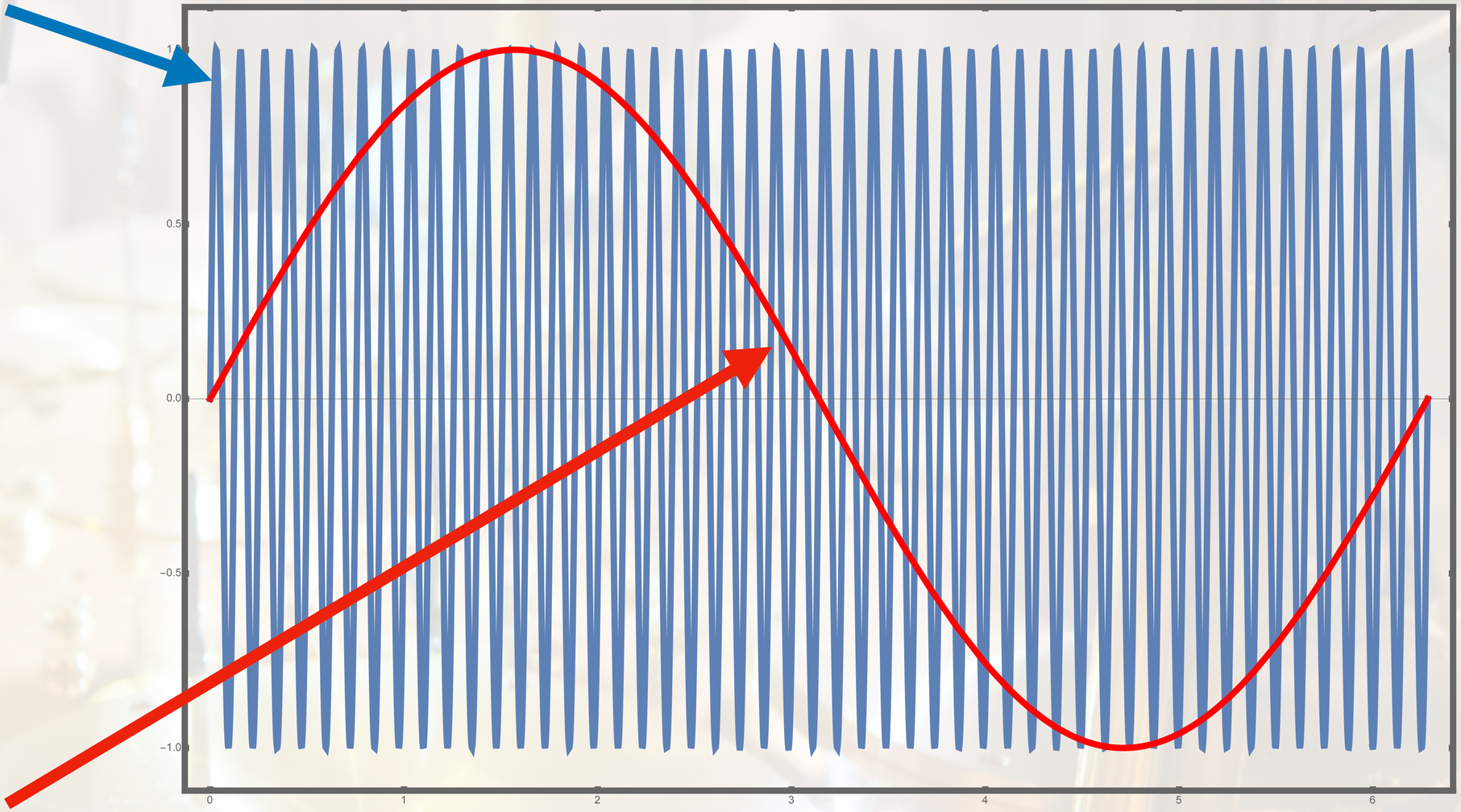

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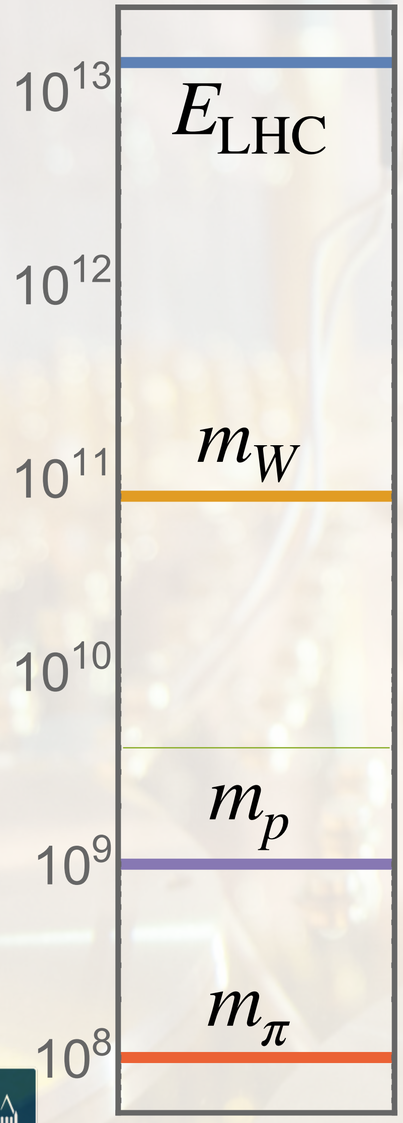

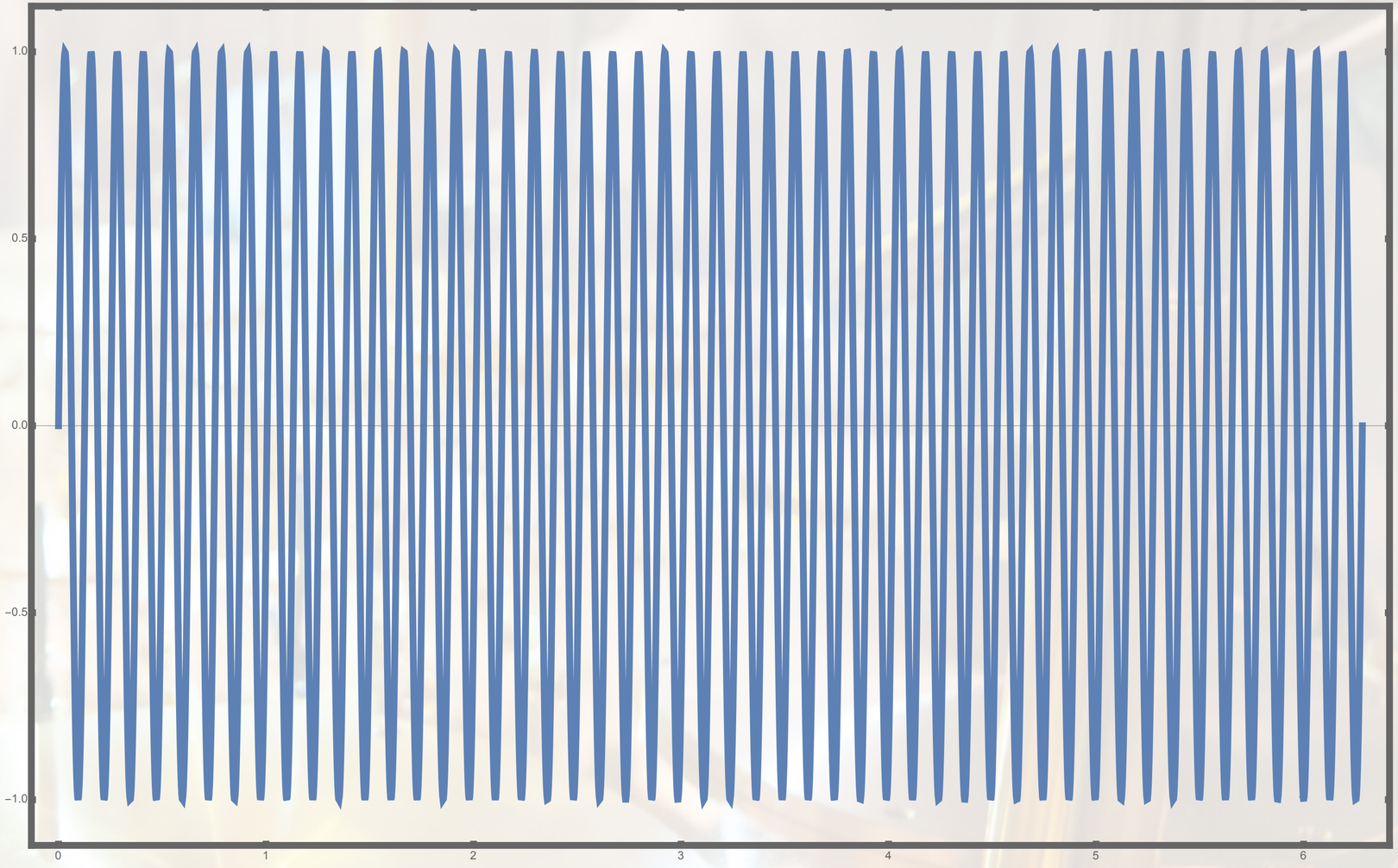

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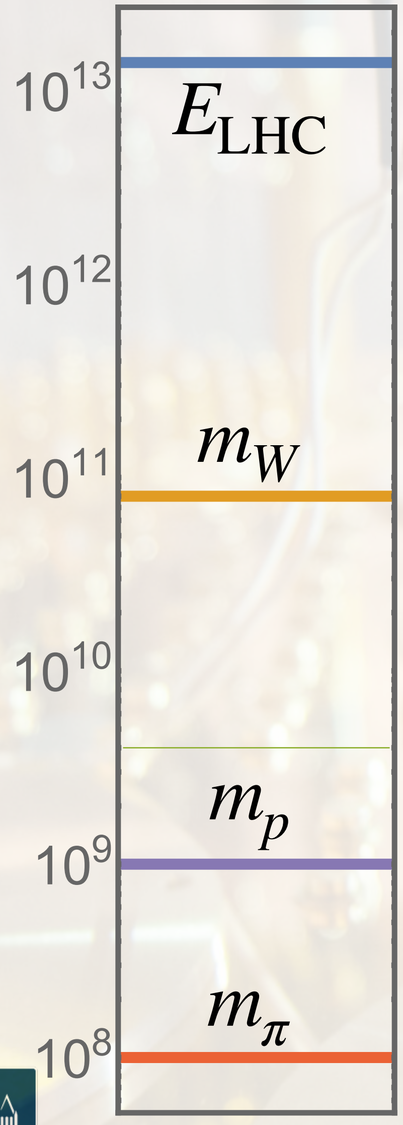

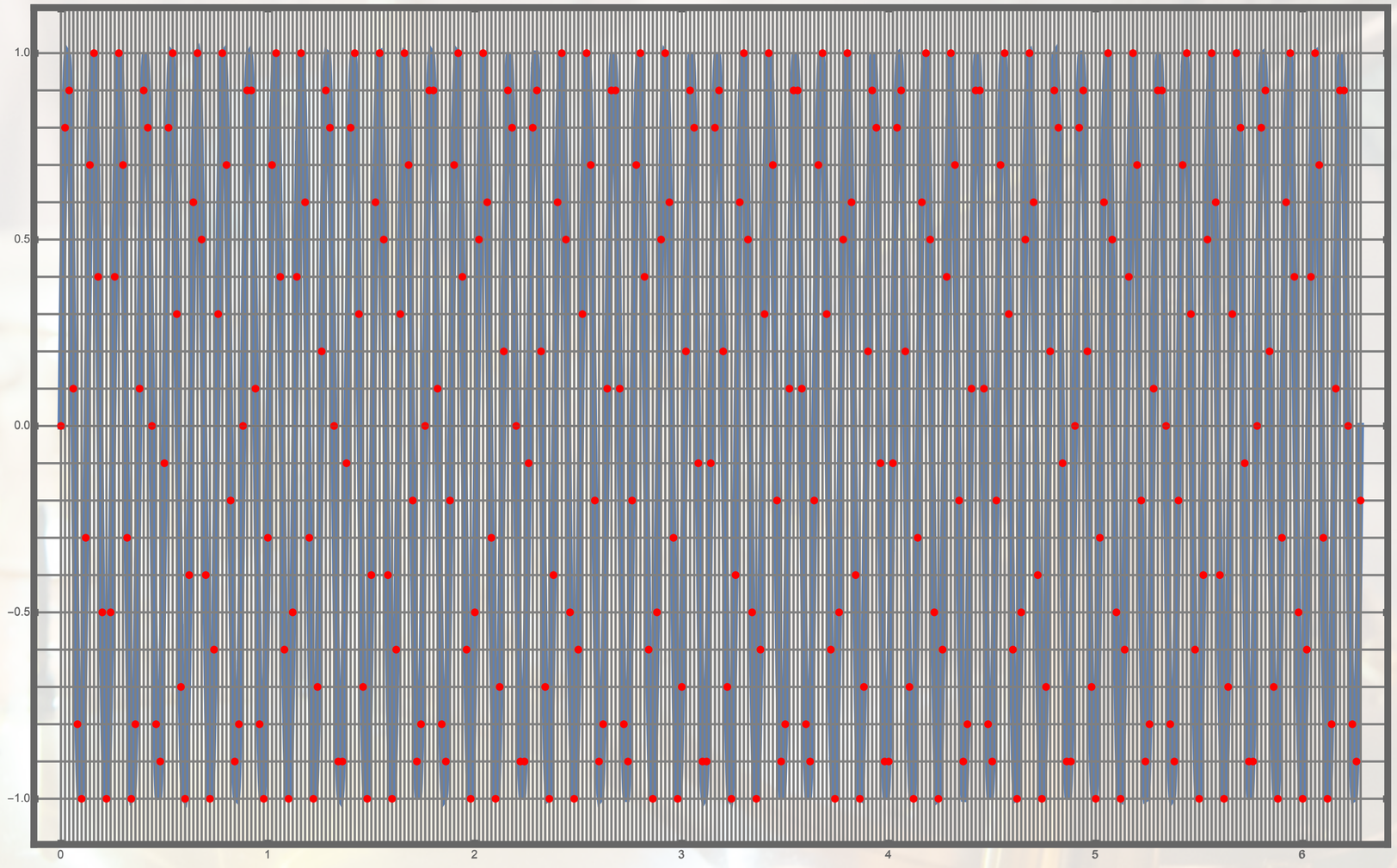

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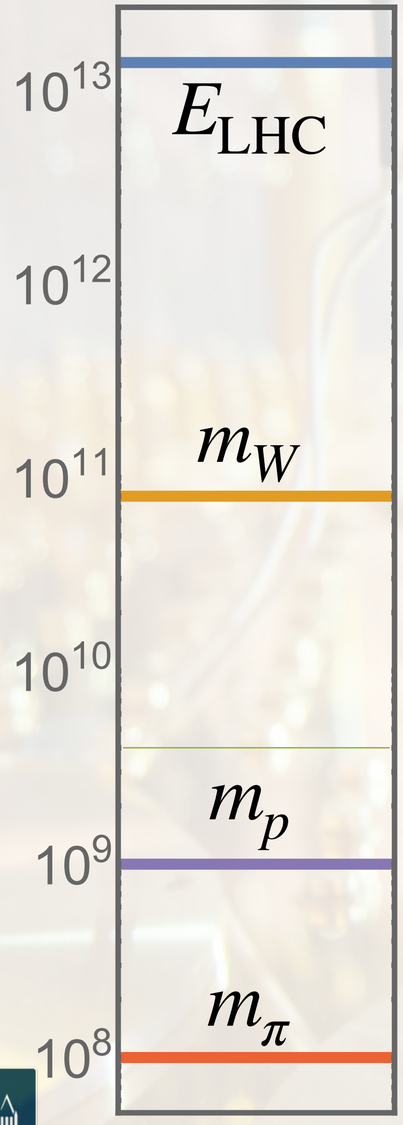
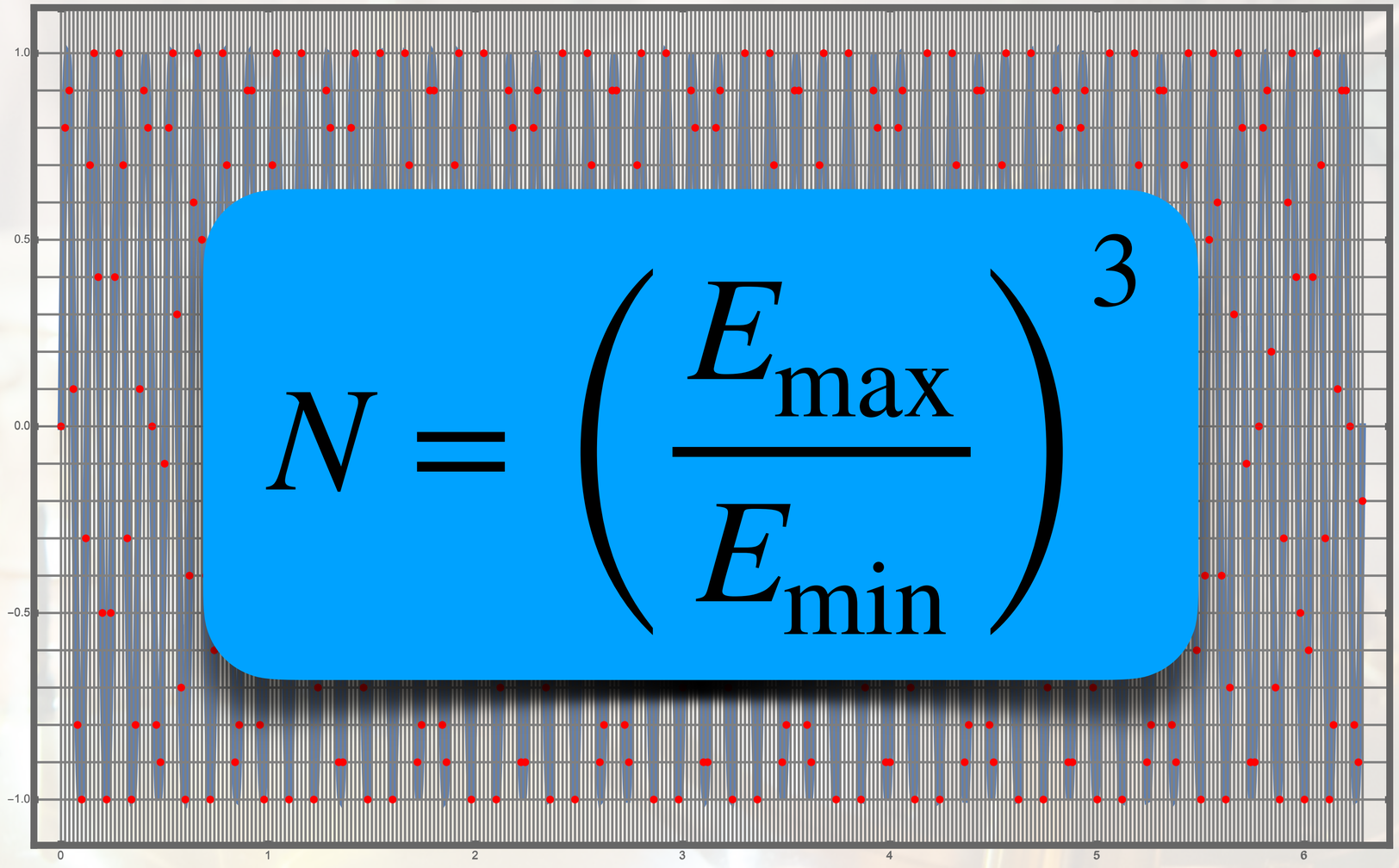
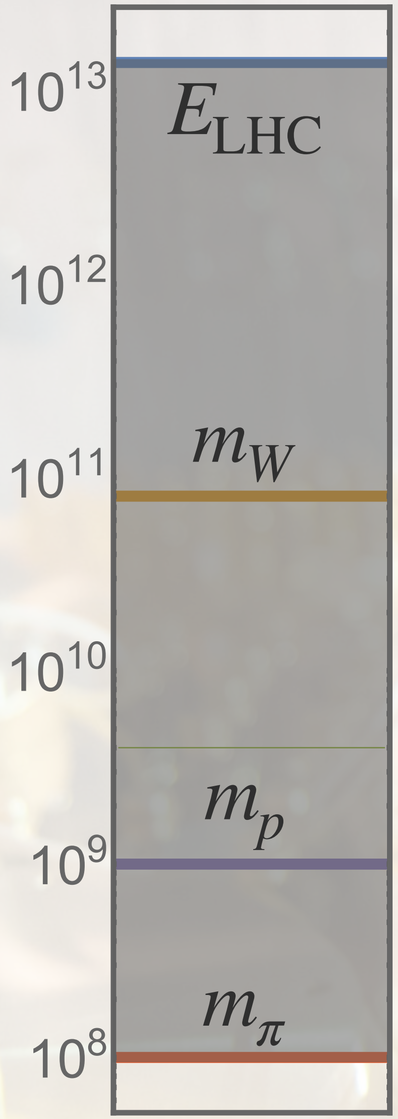
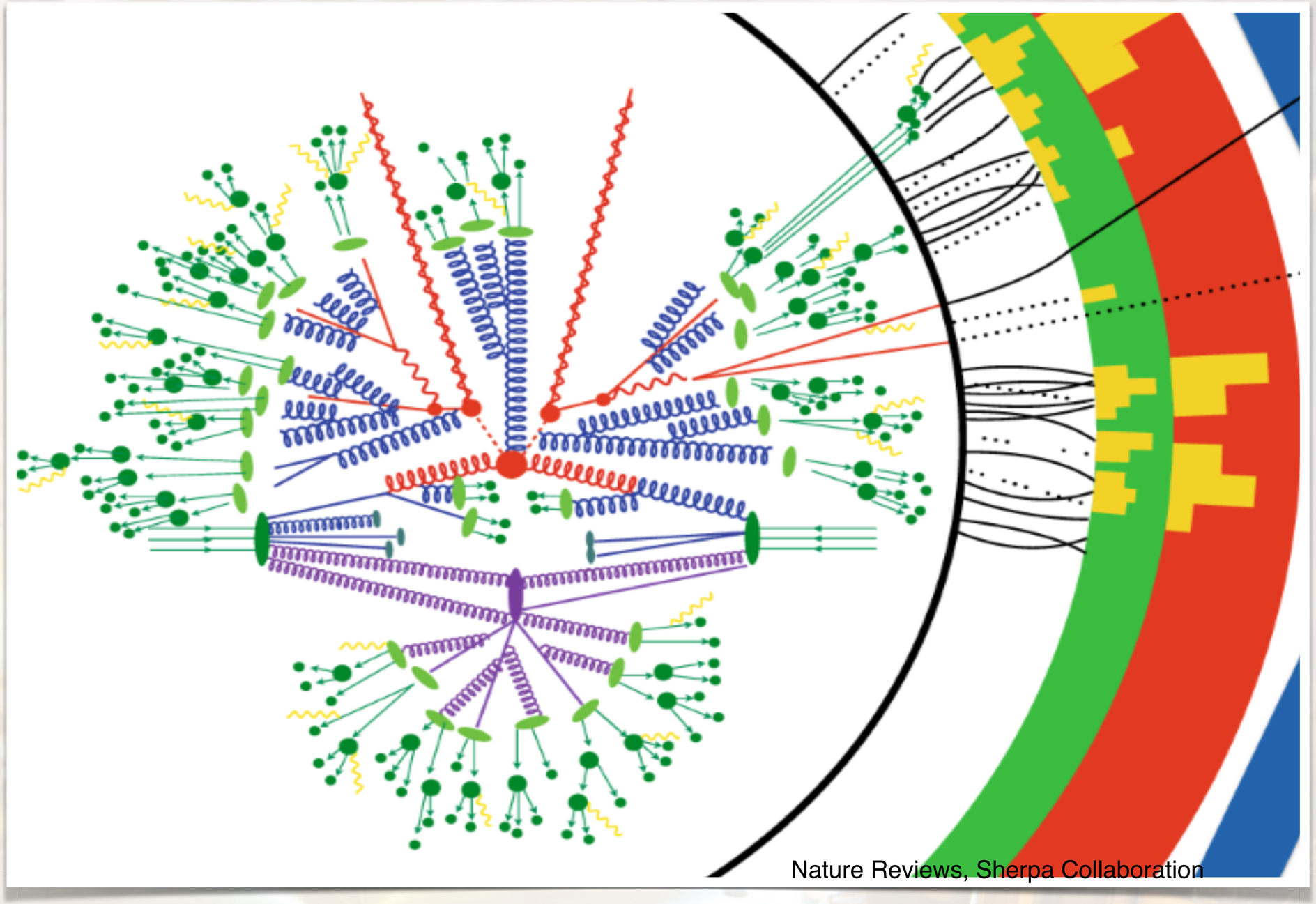




$$N = \left(\frac{E_{\text{max}}}{E_{\text{min}}}\right)^3$$





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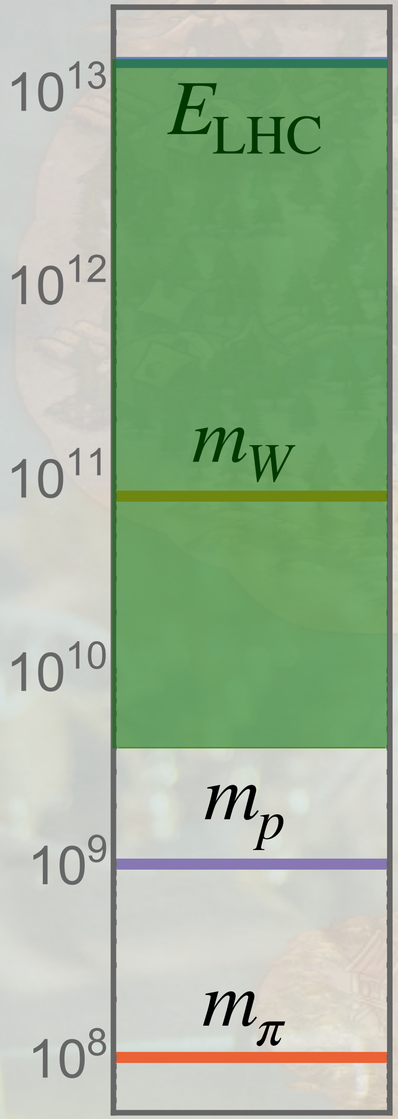

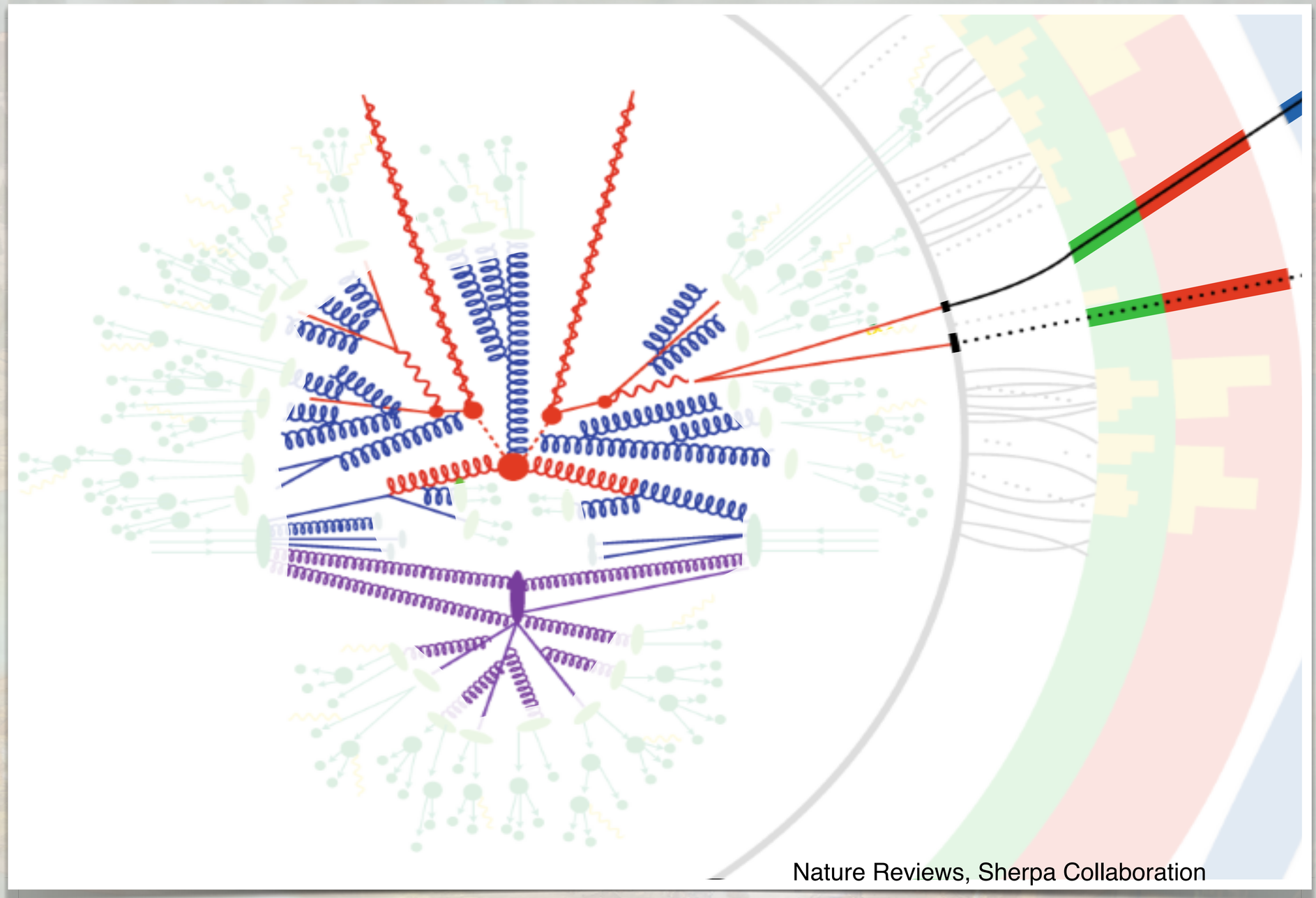

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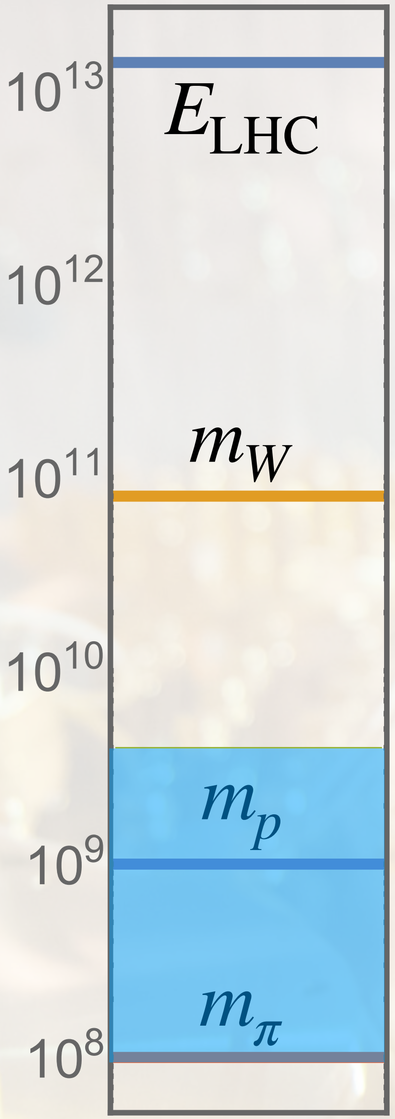

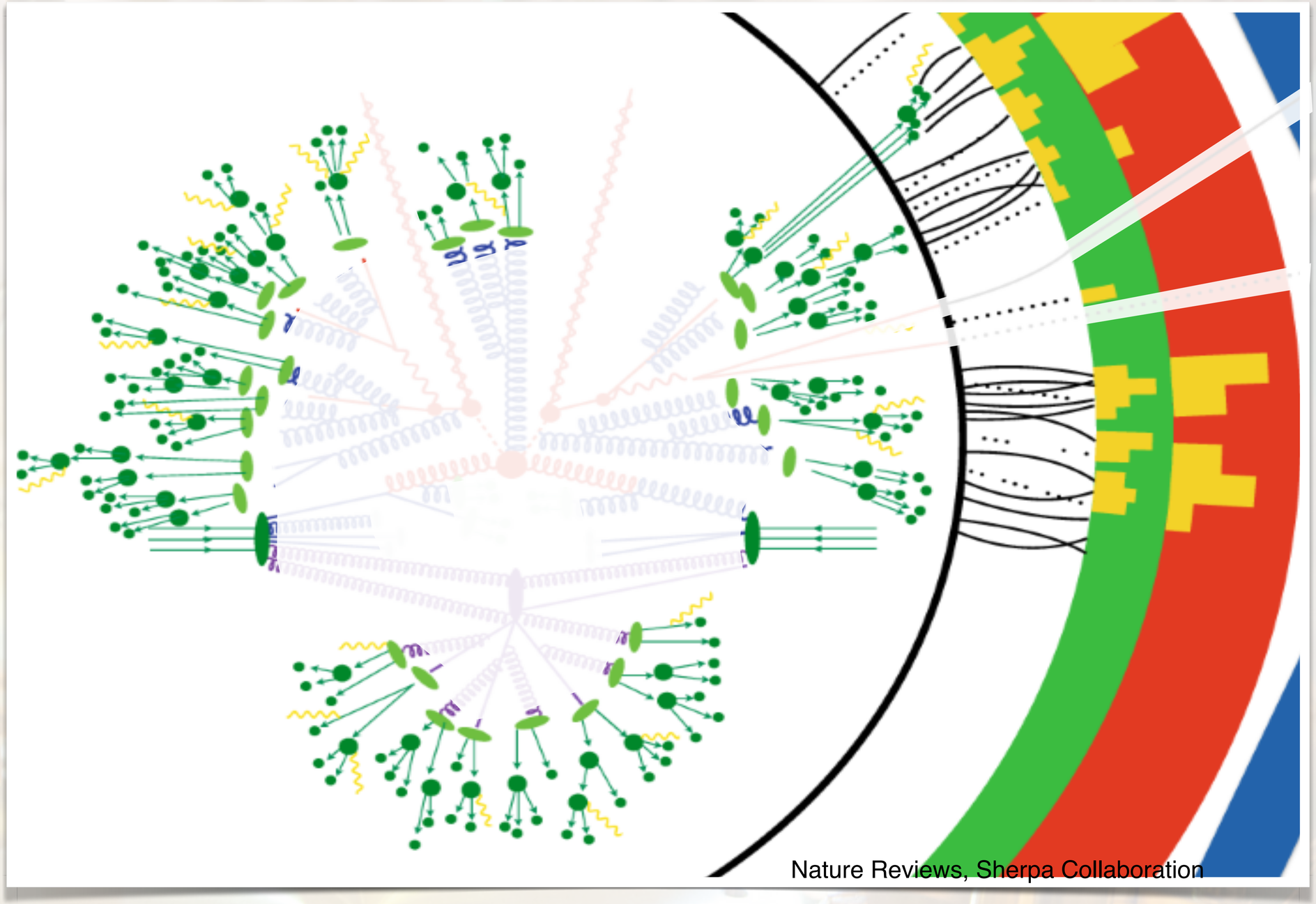

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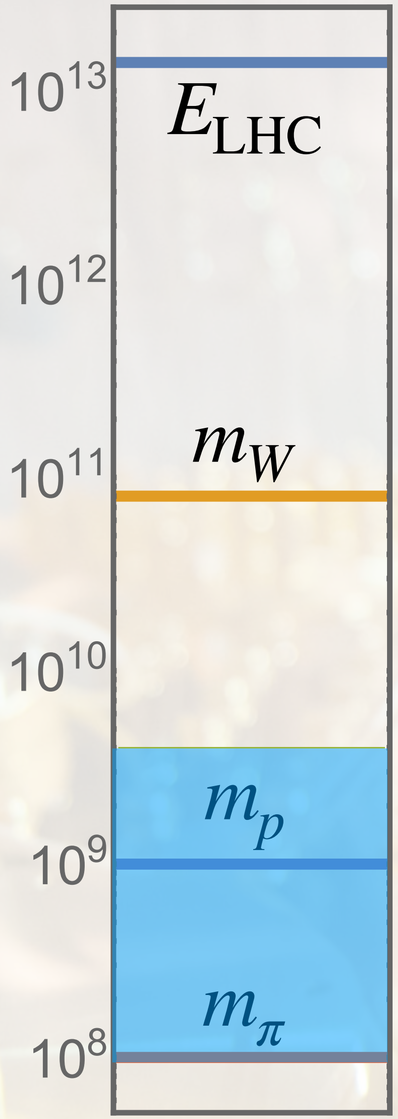
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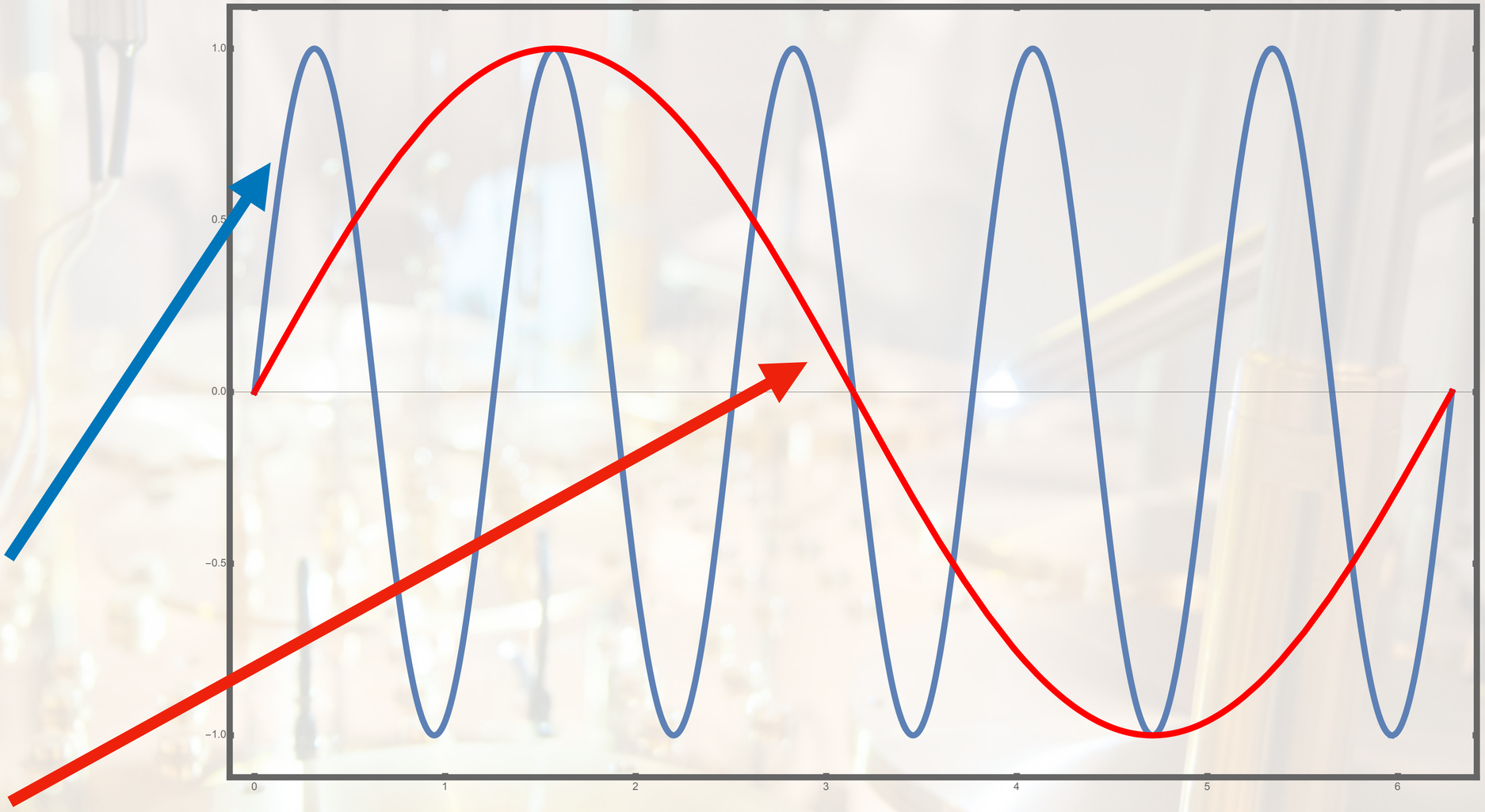
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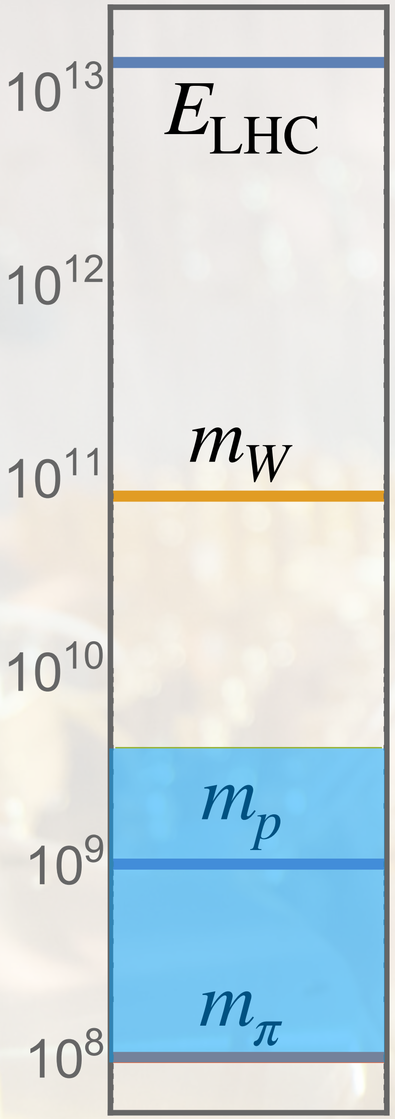
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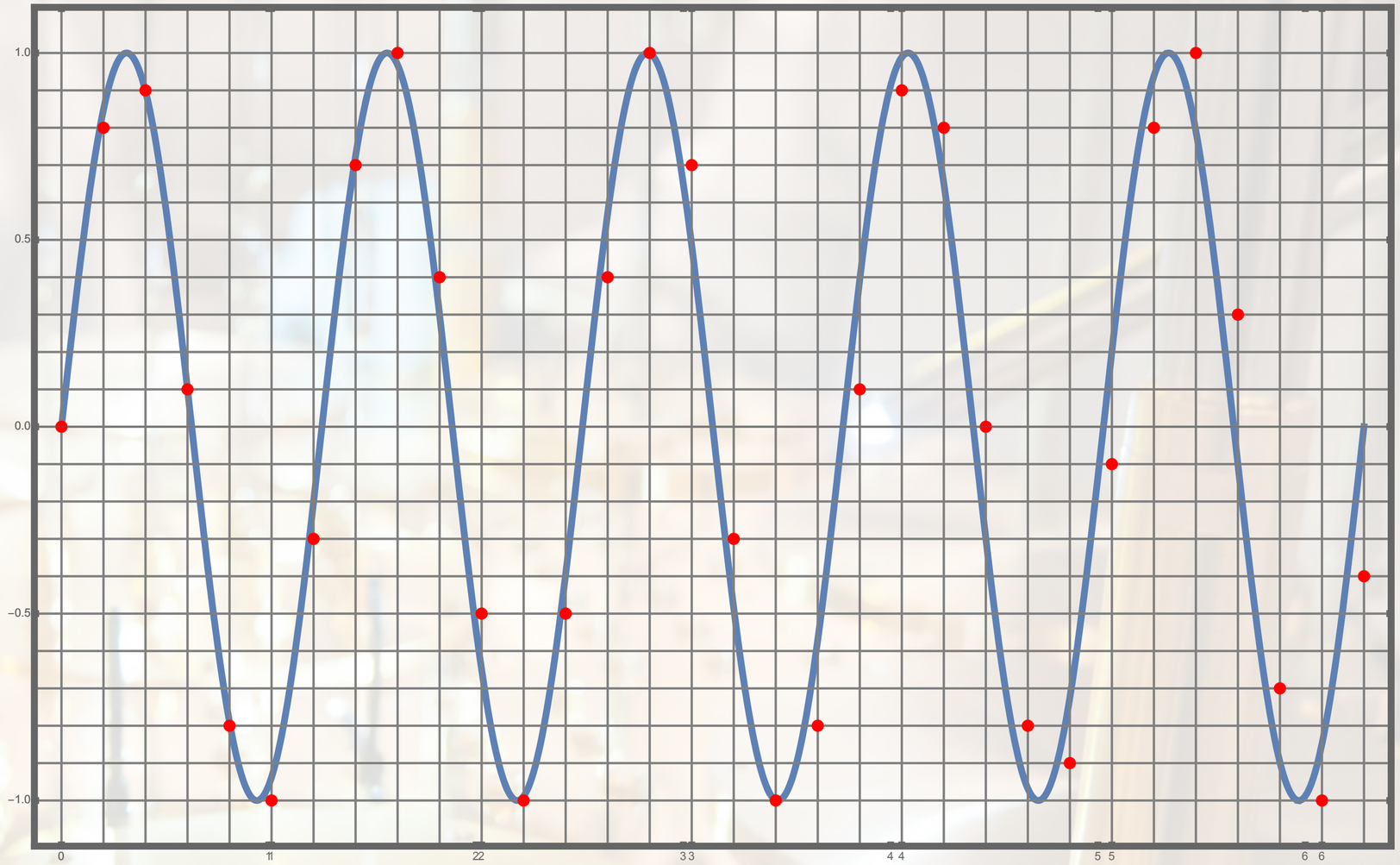
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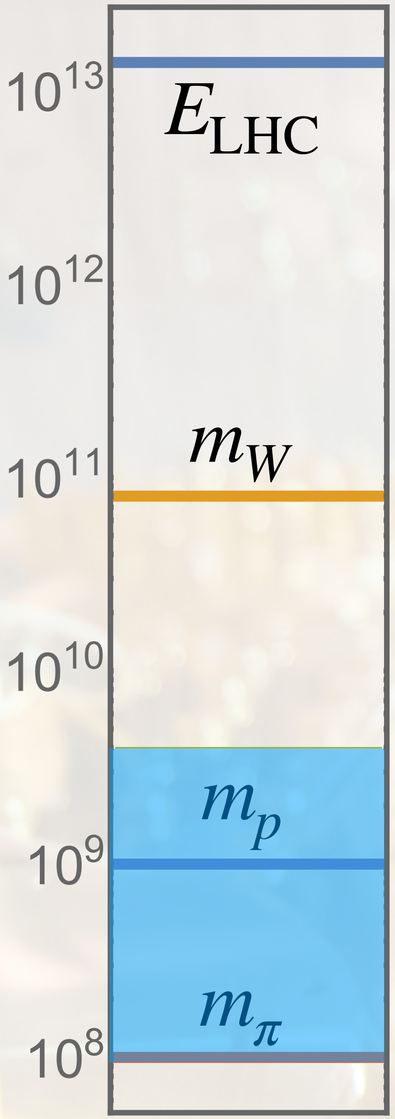
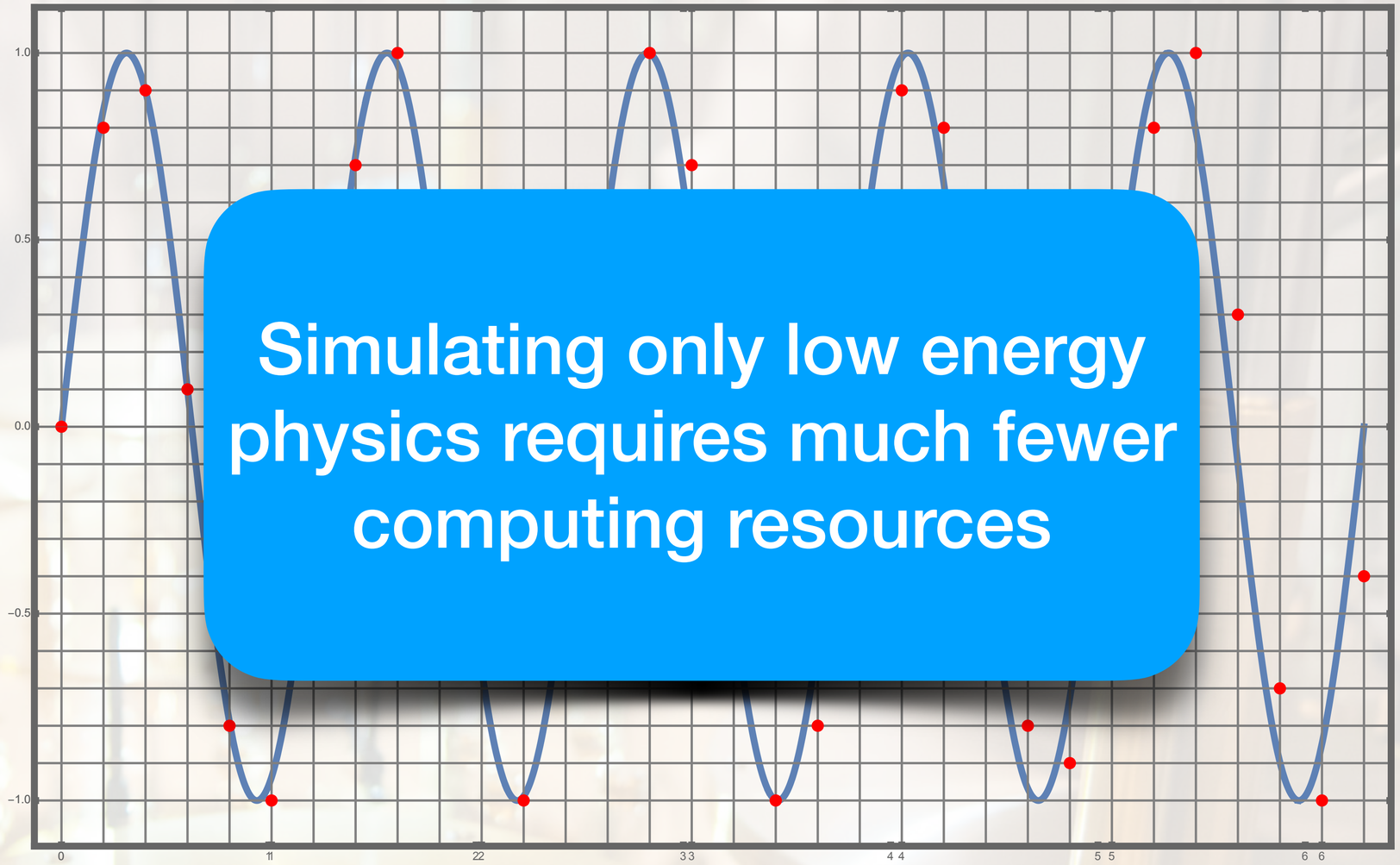
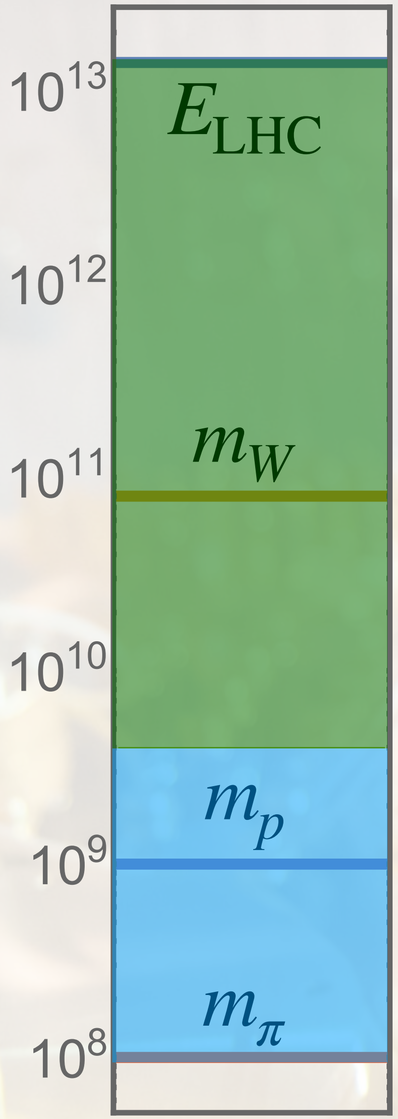




Simulating only low energy physics requires much fewer computing resources



Perturbation Theory

Quantum Simulaton

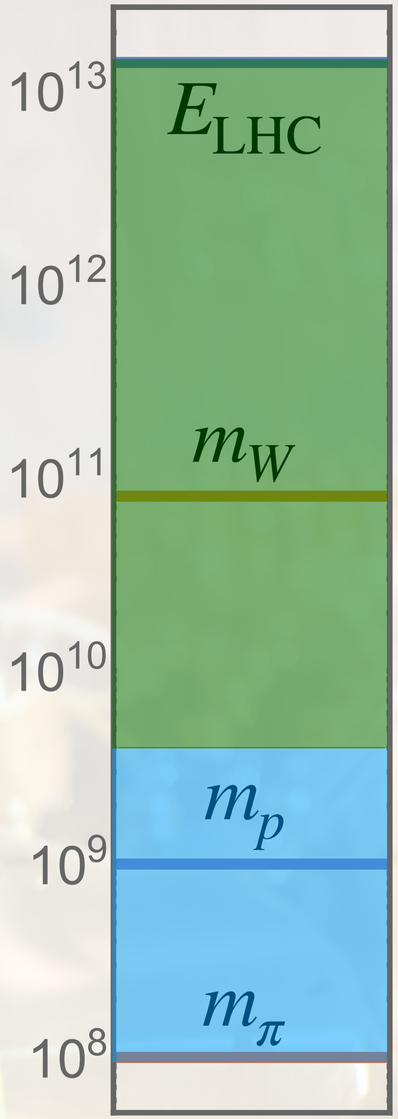
Can write almost all observables at colliders in the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}o} = \int \mathrm{d}\omega \frac{\mathrm{d}\sigma}{\mathrm{d}\omega} \delta(o - o(\omega))$$

Differential energy distribution can often be written in terms of perturbatively calculable pieces and non-perturbative pieces

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \Big[P(\omega_p) \;\; F(\omega_s) \Big](\omega)$$

Usual approach is to try to extract the non-perturbative pieces from data, which means that they need to be unversal



Perturbation Theory

Quantum Simulaton

Quantum computer allows to directly compute these non-perturbative functions from first principles

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \Big[P(\omega_p) \;\; F(\omega_s)\Big](\omega)$$

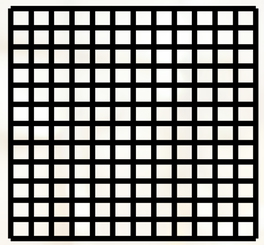
One of those function can be related to square of expectation value of relatively simple unitary operator

$$F(\omega_s) = \left| \left\langle X(\omega_s) \middle| T\left(Y_{n_1}^\dagger Y_{n_2}\right) \middle| \Omega \right\rangle \right|^2$$

As discussed, this function can be computed with much fewer resources than the whole process

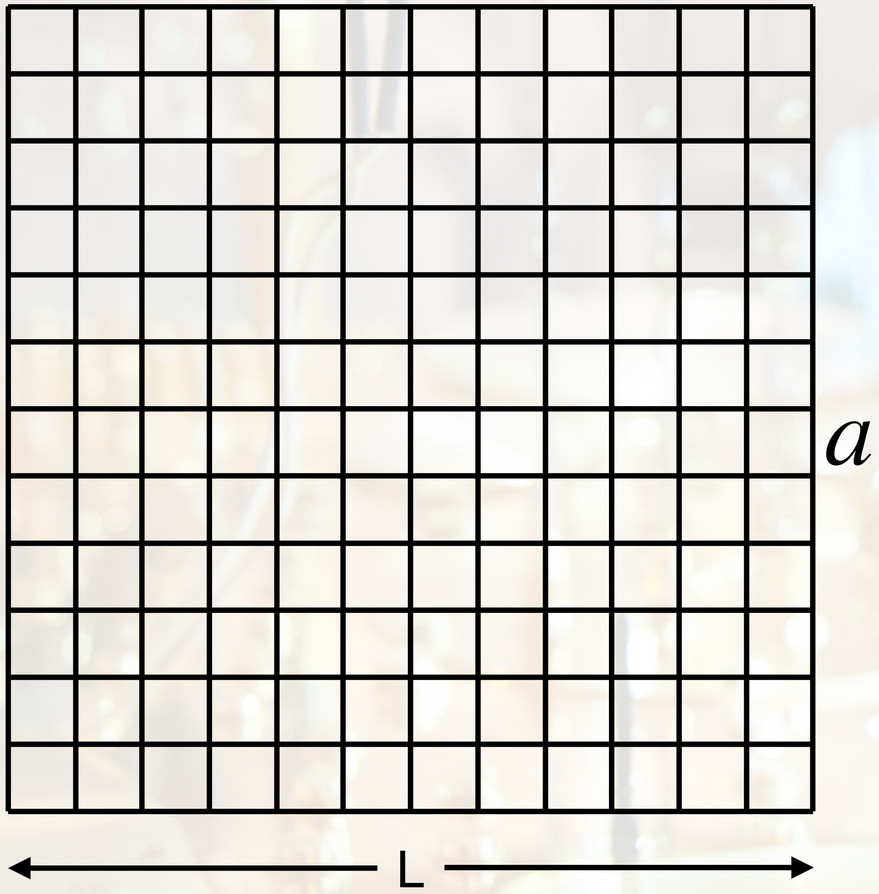
Summary of first part

- Simulating full scattering process from first principles too costly to be practical
- Using effective field theories, can isolate the perturbatively calculable short distance physics from non-perturbative physics
- Quantum computers will allow for the first time to calculate the relevant non-perturbative ingredients from first principles
- Such calculations likely the best way to use quantum computers for collider physics predictions



How do we take the continuum limit in of quantum lattice simulations?

The Kogut Susskind Hamiltonian many of us use as the starting point for quantum simulations depends on parameters that depend on lattice spacing



Several important questions:

- When we do a calculation, where does the lattice spacing enter the calculation?
- How do we perform calculations at different lattice spacings?
- How do we take the limit $a \to 0$?
- How do we estimate the uncertainty from working at finite a?

The Kogut Susskind Hamiltonian many of us use as the starting point for quantum simulations depends on parameters that depend on lattice spacing

$$H = H_E + H_B + H_M + H_K$$

$$H_E = \frac{g^2}{2} \sum_{\vec{x}} \sum_{j=1}^{d} E(\vec{x}, j)^2$$

$$H_B = -\frac{1}{2g^2} \sum_{p} (P_p + P_p^\dagger)$$

$$H_M = m \sum_{\vec{x}} (-1)^{\vec{x}} \psi^\dagger(\vec{x}) \psi(\vec{x})$$

$$H_K = \sum_{\vec{x}} \sum_{j=1}^{d} \eta(\vec{x}) \left(\psi^\dagger(\vec{x}) U(\vec{x}, j) \psi(\vec{x} + \hat{e}_j) + \text{h . c .} \right)$$

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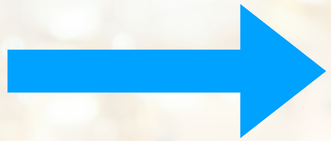
$$H = H_E + H_B + H_M + H_K$$

$$H_E = \frac{g^2}{2} \sum_{\vec{x}} \sum_{j=1}^{d} E(\vec{x}, j)^2$$

$$H_B = -\frac{1}{2g^2} \sum_{p} (P_p + P_p^\dagger)$$

$$H_M = m \sum_{\vec{x}} (-1)^{\vec{x}} \psi^\dagger(\vec{x}) \psi(\vec{x})$$

$$H_K = \sum_{\vec{x}} \sum_{j=1}^{d} \eta(\vec{x}) \left(\psi^\dagger(\vec{x}) U(\vec{x}, j) \psi(\vec{x} + \hat{e}_j) + \text{h.c.} \right)$$



$$H_E = \frac{\tilde{g}_t(a)^2}{2a_t} \sum_{\vec{x}} \sum_{j=1}^{d} E(\vec{x}, j)^2$$

$$H_B = -\frac{1}{2a_t \tilde{g}_s^2} \sum_{p} (P_p + P_p^\dagger)$$

$$H_M = \frac{\tilde{m}(a)}{a_t} \sum_{\vec{x}} (-1)^{\vec{x}} \psi^\dagger(\vec{x}) \psi(\vec{x})$$

$$H_K = \frac{\tilde{\kappa}(a)}{a_t} \sum_{\vec{x}} \sum_{j=1}^{d} \eta(\vec{x}) \left(\psi^\dagger(\vec{x}) U(\vec{x}, j) \psi(\vec{x} + \hat{e}_j) + \text{h.c.} \right)$$

$$c \equiv \frac{g_t}{g_s}$$

$$a_t \equiv \frac{a}{c}$$

These parameters need to be determined by demanding that Hamiltonian reproduces some known physics (measured observable)

4 dimensionless parameters ⇒ need dimensionless observables to fix them

$$\frac{\frac{a}{c}M_1(p_i)}{\frac{a}{c}M_2(p_i)} = \frac{M_1^{(\mathrm{phys})}}{M_2^{(\mathrm{phys})}}$$

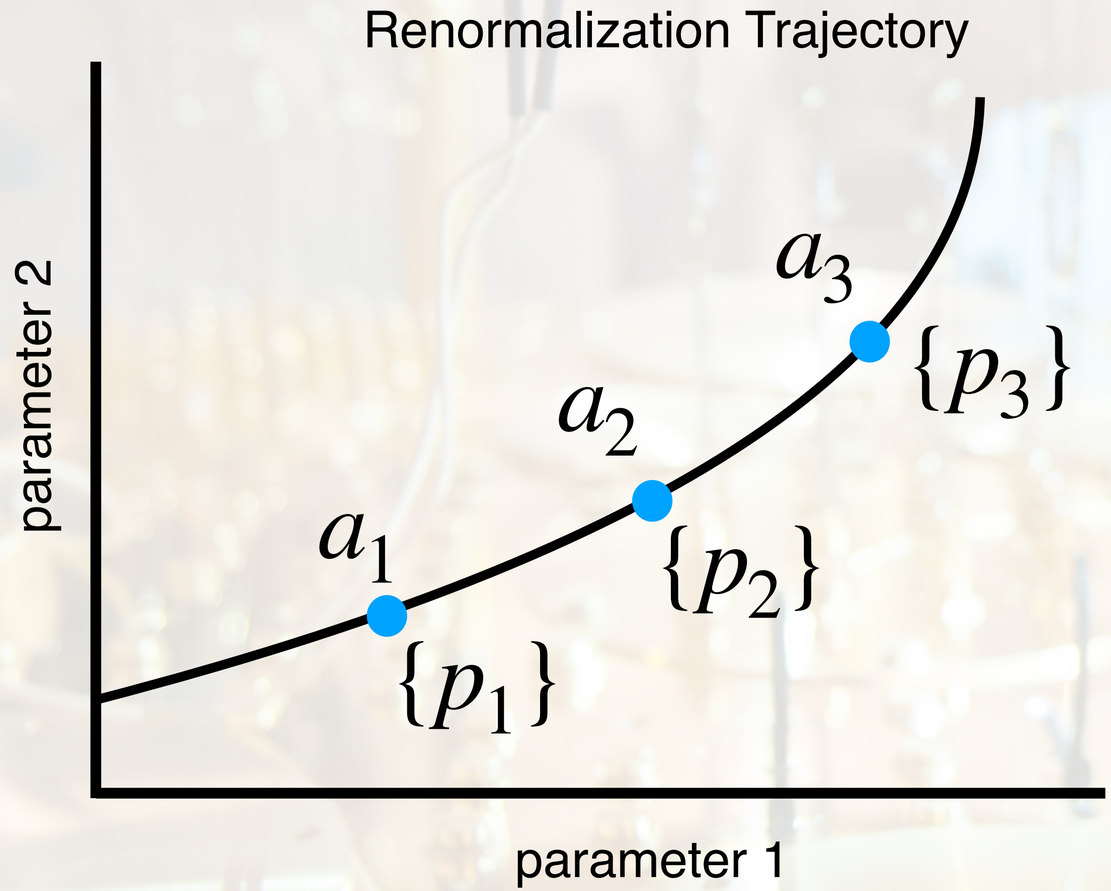
Find that there are many combinations of parameters that all reproduce the same 4 observable values

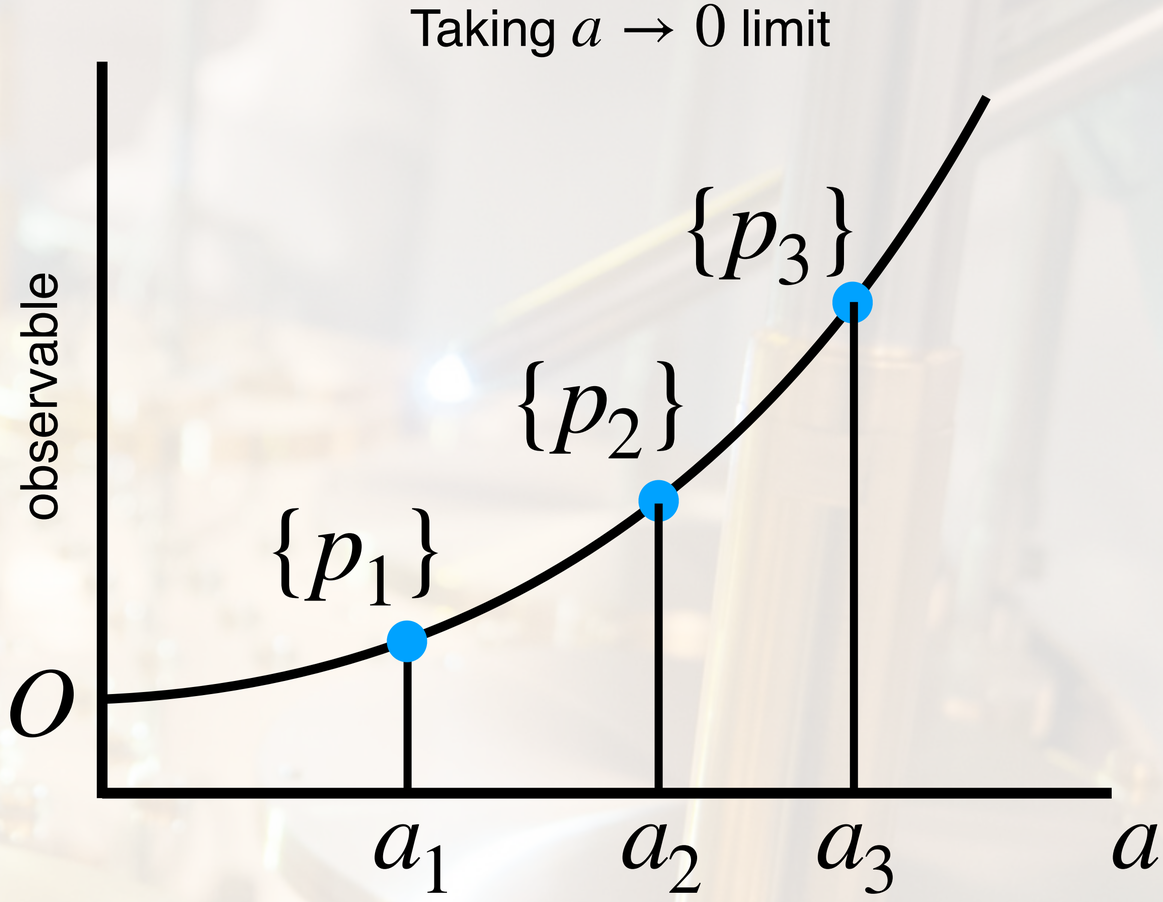
By using a one more observable (this time dimensionful) we can obtain the value of the lattice spacing.

$$a_t = \frac{\hat{M}(p_i)}{M^{(\mathrm{phys})}}$$

Different parameters sets give different values of lattice spacing

These parameters need to be determined by demanding that Hamiltonian reproduces some known physics (measured observable)





We can only implement that on quantum computers, and there are two different techniques to deal with this issue, which work for different approximations

Product formulas

$$H \to H(\delta_t) = H_1 + H_2 + i\frac{\delta_t}{2}\left[H_1, H_2\right] + \ldots$$

Various approaches to deal with parameter δ_t

One approach: treat δ_t as additional parameter, determined from data

C extract parameter dependence using classical simulations on Euclidean lattices

Carena, Lamm, Li, Liu, 2107.01166

We can only implement that on quantum computers, and there are two different techniques to deal with this issue

Approximations using Quantum Signal Processing

Difference of observable calculated in exact and approximate evolution by ϵ_δ

$$\left|\langle\hat{O}(\delta)\rangle - \langle\hat{O}\rangle\right| = \epsilon_\delta$$

Observables can only be calculated to given precision (noise, statistics, …)

$$\langle\hat{O}\rangle_{\text{calc}} = \langle\hat{O}\rangle \pm \sigma_{\hat{O}}$$

Choose approximate time evolution such that $\epsilon_\delta \ll \sigma_{\hat{O}}$ and proceed as for exactc time evolution possible for QSP, since resources logarithmic in ϵ_δ

Statistically bounded time evolution protocol

Kane, Hariprakash, Bauer, 2506.16559

Summary of second part

- Very important to remember that contrary to other fields, lattice gauge theories are only an approximation to the real world
- Lattice theory is related to continuum theory through renormalization procedure, and a sequence of lattice theories is required to calculate physical observables
- In presence of necessary approximation to time-evolution operator, different approaches suitable for product formulas and QSP-based techniques
- More work required to see which approach ultimately most efficient

How do we determine what level of truncation is needed for the physics we are after?

Bosonic degrees of freedom have an infinite dimensional Hilbert space, and a truncation is necessary to allow their simulation on quantum computers

Bosonic degrees of freedom have an infinite dimensional Hilbert space, and a truncation is necessary to allow their simulation on quantum computers

- What value of Lambda should we choose in a given simulation?
- How does the truncation affect the accuracy of a given simulation?
- For the rest of the talk, will work with electric basis states
- Will also work for simplicity with U(1) gauge theory

Bosonic degrees of freedom have an infinite dimensional Hilbert space, and a truncation is necessary to allow their simulation on quantum computers

Previous time independent bounds

Tong, Albert, McClean, Preskill, Su, 2110.06942

$$\langle 0|\, \hat{H}\, |0\rangle = \langle 0|\, e^{i\hat{H}t} \hat{H} e^{-i\hat{H}t}\, |0\rangle = \frac{1}{2g^2}$$

$$\langle 0|\, e^{i\hat{H}t} \hat{H} e^{-i\hat{H}t}\, |0\rangle \geq \langle 0|\, e^{i\hat{H}t} \hat{H}_E e^{-i\hat{H}t}\, |0\rangle = 2g^2 \langle 0|\, e^{i\hat{H}t} \hat{E}^2 e^{-i\hat{H}t}\, |0\rangle$$

$$\langle 0|\, e^{i\hat{H}t} \hat{E}^2 e^{-i\hat{H}t}\, |0\rangle \geq \Lambda^2 \langle 0|\, e^{i\hat{H}t} \hat{\Pi}_\Lambda e^{-i\hat{H}t}\, |0\rangle$$

$$\boxed{\langle 0|\, e^{i\hat{H}t} \hat{\Pi}_\Lambda e^{-i\hat{H}t}\, |0\rangle \leq \frac{1}{g^4 \Lambda^2}}$$

We can improve on this bound using the presence of Hilbert space fragmentation in Kogut-Susskind style gauge lattice theories

Ciavarella, CWB, Halimeh, 2502.03533

Why is there Hilbert fragmentation in Kogut-Susskind theories?

$$H = H_E + H_B = 2g^2 n^2 \, |n\rangle\langle n| + \frac{1}{2g^2} \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right)$$

Perform Schrieffer-Woff transformation, (find a matrix S that diagonalizes Hamiltonian to first order in H_B)

$$H_{\text{eff}} = e^S (H_E + H_B) e^{-S} = H_E + H_B + \left[S, H_E\right] + \ldots$$

Now find S such that $H_B + \left[S, H_E\right] = 0$. Working out next order one finds

$$H_{\text{eff}} = H_E + \frac{1}{2} \left[S, H_B\right] + \ldots$$

Can work to obtain expression of S and find condition under which this can be viewed as expansion

We can improve on this bound using the presence of Hilbert space fragmentation in Kogut-Susskind style gauge lattice theories

Ciavarella, CWB, Halimeh, 2502.03533

$$H_{\text{eff}} = H_E + \frac{1}{2}\left[S, H_B\right] + \ldots$$

Using the known form of H_B we can write

$$\langle n_f | \, [S, H_B] \, | n_i \rangle = \frac{1}{2g^2} [\langle n_f | \, S \, | n_i \pm 1 \rangle - \langle n_f \pm 1 \, | \, S \, | n_i \rangle]$$

Now using $H_B + \left[S, H_E\right] = 0$ can write

$$\langle n \, | \, S \, | m \rangle = \frac{1}{2g^2 n^2} \langle n \, | \, H_E \, S \, | m \rangle = \frac{1}{2g^2 n^2} \langle n \, | \, S \, H_E + H_B \, | m \rangle = \frac{m^2}{n^2} \langle n \, | \, S \, | m \rangle + \frac{1}{2g^2 n^2} \langle n \, | \, H_B \, | m \rangle$$

and obtain

$$\langle n \, | \, S \, | m \rangle = \frac{1}{2g^2 (n^2 - m^2)} \langle n \, | \, H_B \, | m \rangle$$

We can improve on this bound using the presence of Hilbert space fragmentation in Kogut-Susskind style gauge lattice theories

Ciavarella, CWB, Halimeh, 2502.03533

$$H_{\text{eff}} = H_E + \frac{1}{2}\left[S, H_B\right] + \ldots$$

Combining these results

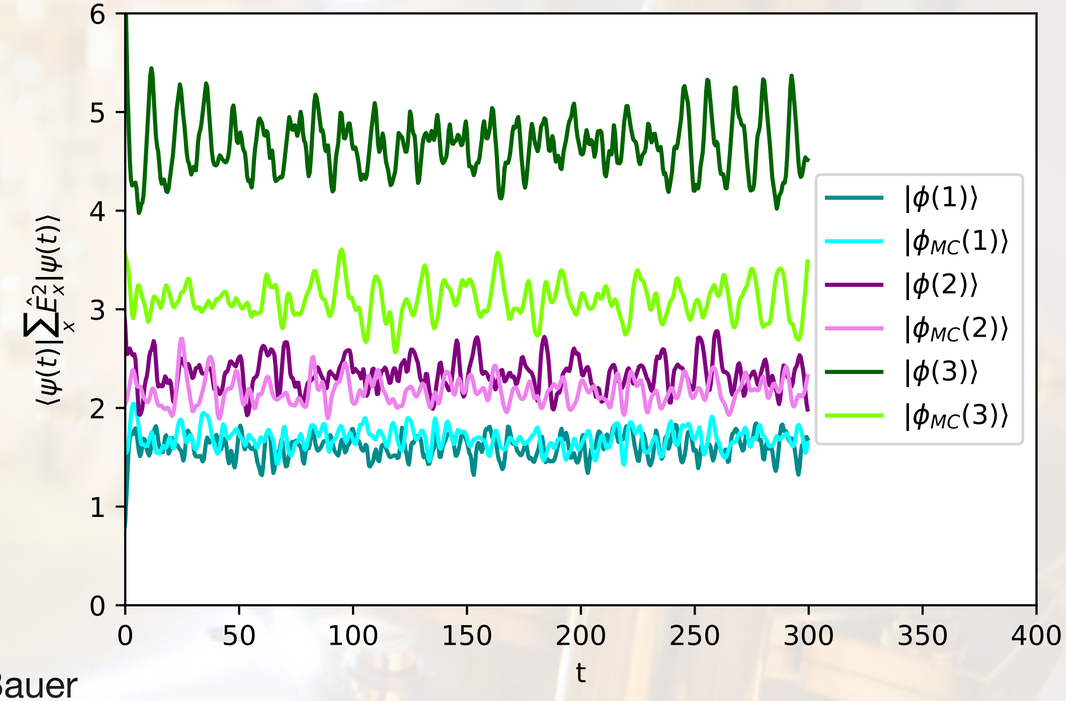
$$\langle n_f | [S, H_B] | n_i \rangle = \frac{1}{2g^2}[\langle n_f | S | n_i \pm 1\rangle - \langle n_f \pm 1 | S | n_i \rangle]$$

$$\langle n | S | m \rangle = \frac{1}{2g^2(n^2 - m^2)} \langle n | H_B | m \rangle$$

one finds that $n_f = n_i$ or $n_f = n_i \pm 2$ and

$$\langle n_f | [S, H_B] | n_i \rangle \sim \frac{1}{2g^6}\frac{1}{n}$$

This implies that for large enough n the off-diagonal interactions are suppressed, and Hilbert spaces with large n freeze out



Hilbert fragmentation can be used to obtain a much tighter bound on truncation uncertainties

Ciavarella, Hariprakash, CWB, Halimeh, 2508.00061

Split U(1) Hamiltonian into two pices

$$\begin{array}{c} \\ \Lambda \\ \Lambda \end{array} \begin{pmatrix} H_\Lambda & V_\Lambda \\ V_\Lambda & \end{pmatrix}$$

Λ is chosen large enought that Hilbert space fragmentation occurs at that value

State with support only on small n

$$\begin{aligned}
&(e^{-i\hat{H}t} - e^{-i\hat{H}_\Lambda t})\,|\phi(0)\rangle \\
&\quad = -ie^{-i\hat{H}_\Lambda t}\int_0^t ds\; e^{i\hat{H}_\Lambda s}\hat{V}_\Lambda e^{-i\hat{H}_\Lambda s}\,|\phi(0)\rangle \\
&\quad = -ie^{-i\hat{H}_\Lambda t}\,|\Lambda+1\rangle\int_0^t ds\; e^{i\langle\Lambda+1|\hat{H}_\Lambda|\Lambda+1\rangle s}\,\langle\Lambda+1|\,\hat{V}_\Lambda\,|\Lambda\rangle\,\langle\Lambda|\,e^{-i\hat{H}_\Lambda s}\,|\phi(0)\rangle \\
&\quad = -ie^{-i\hat{H}_\Lambda t}\,|\Lambda+1\rangle\int_0^t ds\; e^{i\langle\Lambda+1|\hat{H}_\Lambda|\Lambda+1\rangle s}\,\langle\Lambda+1|\,\hat{V}_\Lambda\,|\Lambda\rangle\;c\;e^{-i\langle\Lambda|\hat{H}_\Lambda|\Lambda\rangle s} \\
&\quad \leq \frac{2}{\langle\Lambda+1|\,\hat{H}_\Lambda\,|\Lambda+1\rangle - \langle\Lambda|\,\hat{H}_\Lambda\,|\Lambda\rangle}\left|\langle\Lambda+1|\,\hat{V}_\Lambda\,|\Lambda\rangle\right|
\end{aligned}$$

Hilbert fragmentation can be used to obtain a much tighter bound on truncation uncertainties

Ciavarella, Hariprakash, CWB, Halimeh, 2508.00061

Split U(1) Hamiltonian into two pices

$$\Lambda \begin{pmatrix} H_\Lambda & V_\Lambda \\ V_\Lambda & \end{pmatrix} \Lambda$$

Λ is chosen large enought that Hilbert space fragmentation occurs at that value

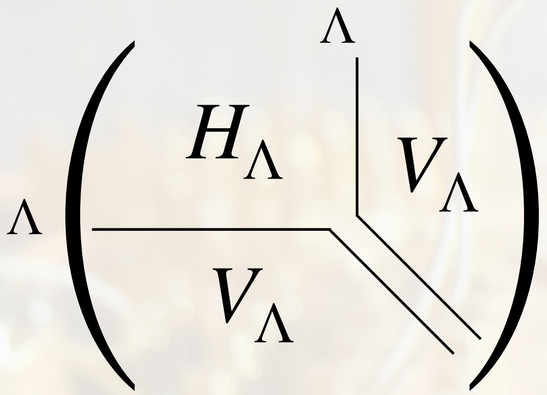
Going to higher Λ, above where HSF first occurs can apply same procedure multiple times

$$\begin{aligned}
&(e^{-i\hat{H}t} - e^{-i\hat{H}_\Lambda t})\,|\phi(0)\rangle \\
&\leq \; i \sum_{k=\Lambda_0}^{\Lambda} \left[\left(e^{-i\langle \Lambda+1|\hat{H}_\Lambda|\Lambda+1\rangle t} - e^{-i\langle k|H_\Lambda|k\rangle t} \right) \prod_{\substack{l=\Lambda_0 \\ l\neq k}}^{\Lambda+1} \frac{1}{\langle k|H_\Lambda|k\rangle - \langle l|H_\Lambda|l\rangle} \right] \\
&\quad \times \left[\prod_{k=\Lambda_0}^{\Lambda} \langle k+1|\hat{V}_\Lambda|k\rangle \right]
\end{aligned}$$

Hilbert fragmentation can be used to obtain a much tighter bound on truncation uncertainties

Ciavarella, Hariprakash, CWB, Halimeh, 2508.00061

Split U(1) Hamiltonian into two pices



Λ is chosen large enought that Hilbert space fragmentation occurs at that value

For U(1) gauge theory, we can plug the explicit matrix elements to find

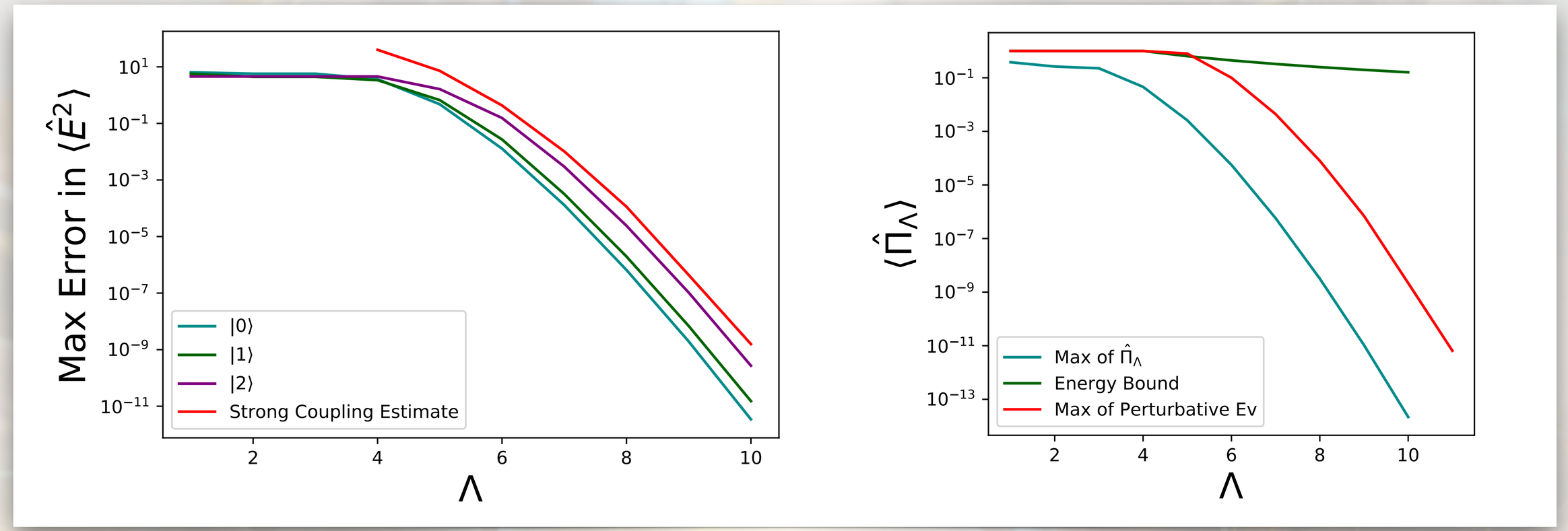
$$\left|\left(e^{-i\hat{H}t} - e^{-i\hat{H}_\Lambda t}\right)|\phi(0)\rangle\right| \leq 2\ \max_{t<T}\left|\sum_{k=\Lambda_0}^{\Lambda}\left[\left(e^{-i2g^2(\Lambda+1)^2 t} - e^{-i2g^2k^2t}\right)\prod_{\substack{l=\Lambda_0\\ l\neq k}}^{\Lambda+1}\frac{1}{k^2-l^2}\right]\right|\left(\frac{1}{2g^2}\right)^{\Lambda-\Lambda_0}$$

$$\leq 2\left(\frac{1}{g^2}\right)^{\Lambda-\Lambda_0}\frac{(2\Lambda_0-1)!!}{(2\Lambda-1)!!}\left(\frac{1}{2g^2}\right)^{\Lambda-\Lambda_0}$$

Hilbert fragmentation can be used to obtain a much tighter bound on truncation uncertainties

Ciavarella, Hariprakash, CWB, Halimeh, 2508.00061

Can check our bound against explicit calcuations, which also checks tightness of bound



Hilbert fragmentation can be used to obtain a much tighter bound on truncation uncertainties

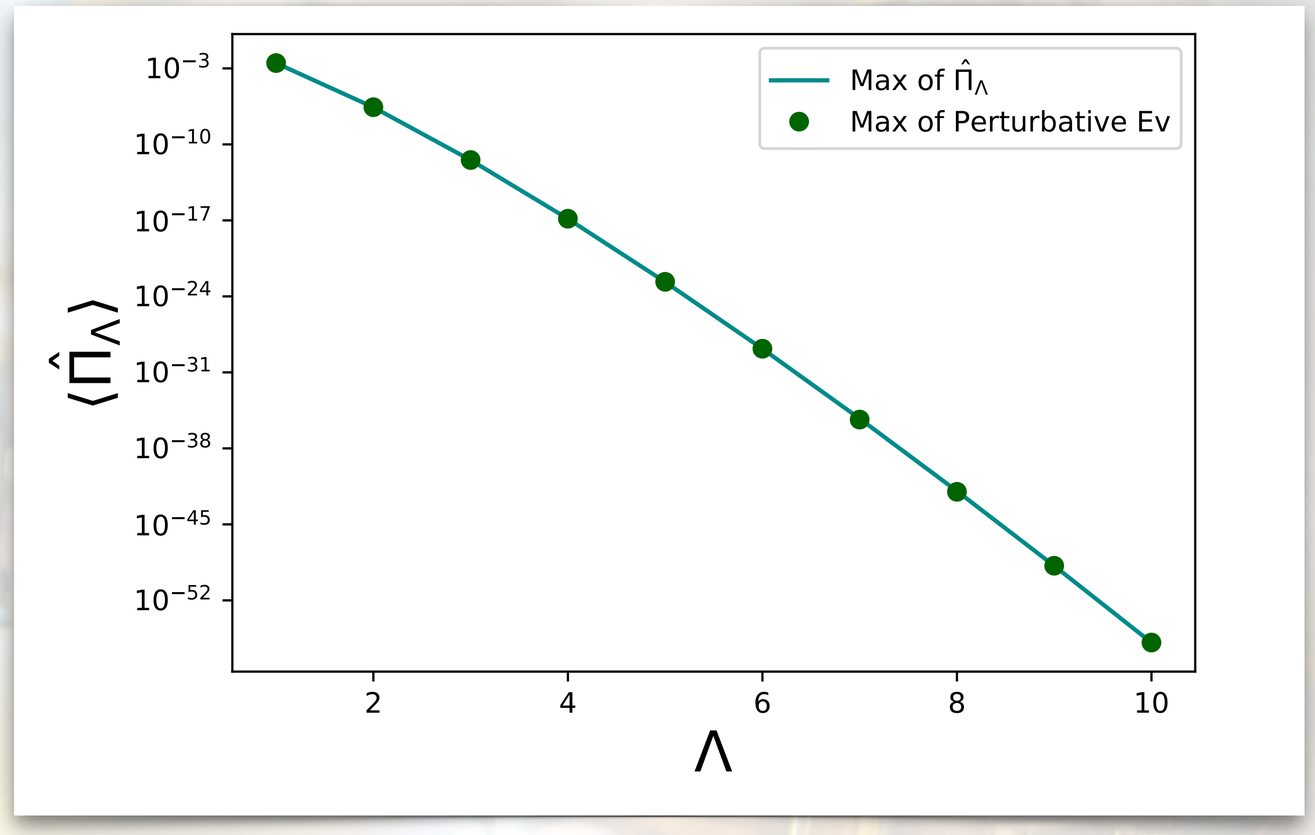
Ciavarella, Hariprakash, CWB, Halimeh, 2508.00061

Our results are dramatically tighter than the previous time independent bounds

Using time-independent bound results in statement that to simulate with $g = \sqrt{3}$ and to guarantee $\left|\langle k+\Lambda\,|\,e^{-iHt}\,|\,k\rangle\right| < 0.01$ with $t = 8$ requires $\Lambda > 100$

Results of HSF bounds give $\left|\langle k+100\,|\,e^{-iHt}\,|\,k\rangle\right| < 6\times 10^{-308}$

Actual simulation likely even much smaller



Summary of third part

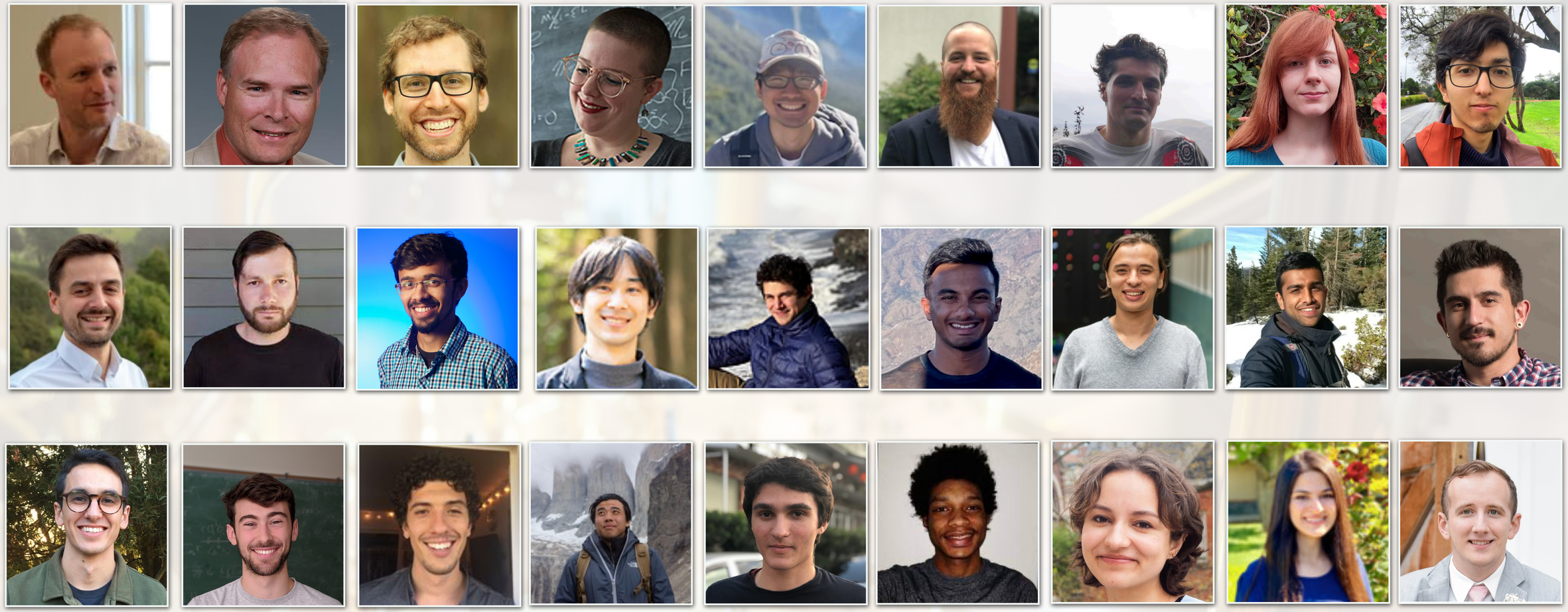
- Truncation the bosonic Hilbert space is a necessary ingredient in any quantum simulation
- It is important to understand what level of truncation is needed and how to bound the uncertainty
- Hilbert space fragmentation allows to derive much tighter bounds on truncation effects
- Will hopefully lead to full understanding of these effects in the future

Quantum computers open the door to perform currenty unattainable simulations

Using Effective Field Theories takes best advantage of quantum hardware

Need to take care of taking into account renormaliation and continuum limit

Obtained a new understanding of truncation effects that will hopefully lead to a much cleaner theoretical understanding




QUESTIONS?