Generalizing the loop-string-hadron formulation to SU(3) Yang-Mills theory

Jesse Stryker

in collaboration with Aahiri Naskar, Saurabh Kadam, & Indrakshi Raychowdhury

PRD 111, 074516 (2025) & work in prep.

ECT* 2025-09-01

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Motivation

- Hamiltonian lattice gauge theory: Framework for quantum simulation and tensor network calculations
- Gauge symmetry → redundancy in description → multiple possible formulations possible/being considered for calculations
- For lattice QCD, a formulation must be adapted to *SU(3) gauge fields and 3+1 D*
- Gauge-invariant formulations offer some advantages (but are not the only possibility)

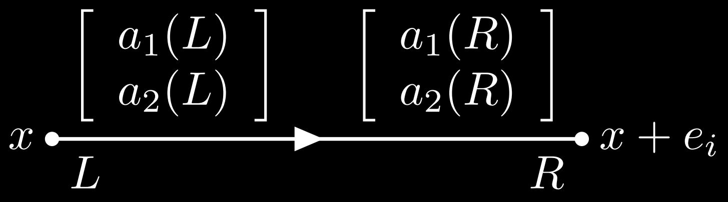
Davoudi, Raychowdhury, & Shaw, PRD (2021)

Path to quantum simulation

- Choose a formulation
- Choose an orthonormal basis
- Associate qudits with field d.o.f.
  - (truncation)
- Evaluate matrix elements of fields/Hamiltonian w.r.t. basis
- Map Hamiltonian evolution to hardware operations

This talk: Loop-string-hadron formulation

Schwinger-boson parent formulation



SU(2) gauge link

- Defining features
  - Construct local lattice fields from multiplets of gauge-covariant harmonic oscillators
  - Gauge fields and electric fields: specially crafted bilinears of harmonic oscillators
  - Simple, discrete basis: Occupation numbers
  - Clebsch-Gordon coefficients follow from SHO factors
  - Non-Abelian Gauss's law
  - More d.o.f. than usual → extra "Abelian Gauss's law" constraints

SU(3) irreducible Schwinger bosons

- SU(2): Arbitrary irrep *j* constructible by tensor-producting enough spin-1/2's → One doublet to construct all |*j*,*m*> states

$$a^\dagger = \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix}$$

- In SU(3): Arbitrary irrep (P,Q) constructible by tensor products of one 3 and one 3*

$$A(L) = \begin{pmatrix} A^1(L) \\ A^2(L) \\ A^3(L) \end{pmatrix} \qquad A(R) = \begin{pmatrix} A^1(R) \\ A^2(R) \\ A^3(R) \end{pmatrix}$$

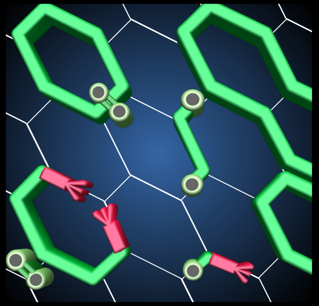
$$B(L) = \begin{pmatrix} B_1(L) \\ B_2(L) \\ B_3(L) \end{pmatrix} \qquad B(R) = \begin{pmatrix} B_1(R) \\ B_2(R) \\ B_3(R) \end{pmatrix}$$

- Ex: 3 $\quad |(1,0)\rangle_\alpha = A^\dagger_\alpha |\Omega\rangle$
- Ex: 3* $\quad |(0,1)\rangle^\beta = B^{\dagger\beta} |\Omega\rangle$

Anishetty, Mathur, & Raychowdhury (2009)

Loop-string-hadron formulation: SU(2)

- Defining features
  - Derived from Schwinger bosons, but resulting framework stands independently
  - Elementary fields are SU(2)-neutral and local
  - Non-Abelian constraints are removed
  - Remnant constraints are Abelian
  - Developed for D=1+1, 2+1, 3+1, with or without staggered fermions



Raychowdhury, & JRS
Phys. Rev. D (2020)

Defining “success” for simulation candidate

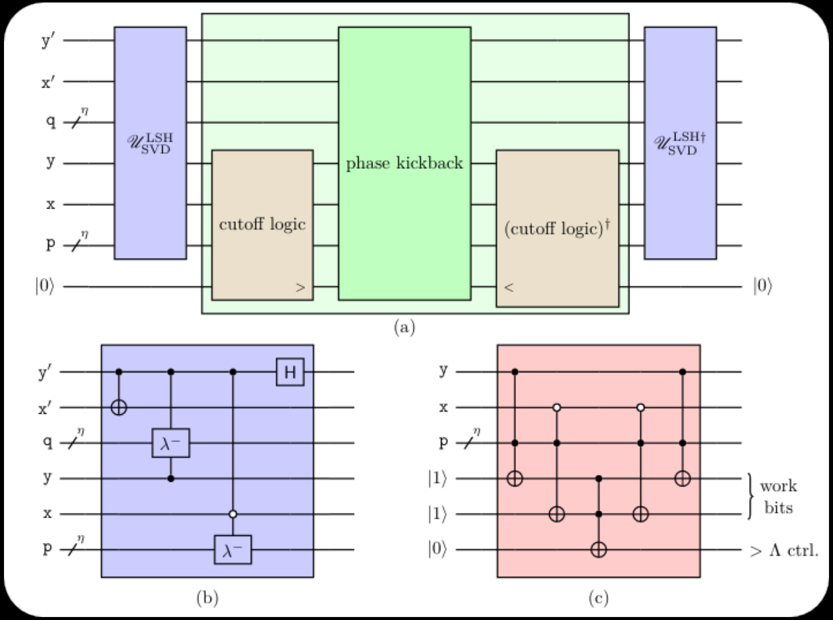
- Complete specification of discretized basis for lattice Hilbert space
- Constraints are known, including implementation
- Matrix elements for field operators in Hamiltonian are known
- Clear path to circuitizing the Hamiltonian operators

Loop-string-hadron formulation: SU(2)

- Complete specification of SU(2) LSH Hamiltonian was converted into time evolution circuits for 1+1D (Davoudi, Shaw, & JRS 2022)
- Modest gate reductions observed compared to gauge-covariant formulation
- Also have “physicality” circuits (Raychowdhury & JRS 2020)

General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory

Zohreh Davoudi[1,2,3,4], Alexander F. Shaw[1,3], and Jesse R. Stryker[1,2,5]



LSH SU(3) in 1+1

- Using ISBs, a direct generalization of 1+1 D follows
- Analytic understanding is on par with SU(2) theory

Kadam, Raychowdhury, & JRS
Phys. Rev. D (2023)

- Circuit-ready!

$$[A^\dagger(\underline{1}) \cdot B^\dagger(1)]^{n_P} \to$$

$$[B^\dagger(\underline{1}) \cdot A^\dagger(1)]^{n_Q} \to$$

$$|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 0, 0\rangle \to$$

$$\psi^\dagger \cdot B^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 0, 1\rangle \to$$

$$\psi^\dagger \cdot B^\dagger(\underline{1})\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 0, 0\rangle \to$$

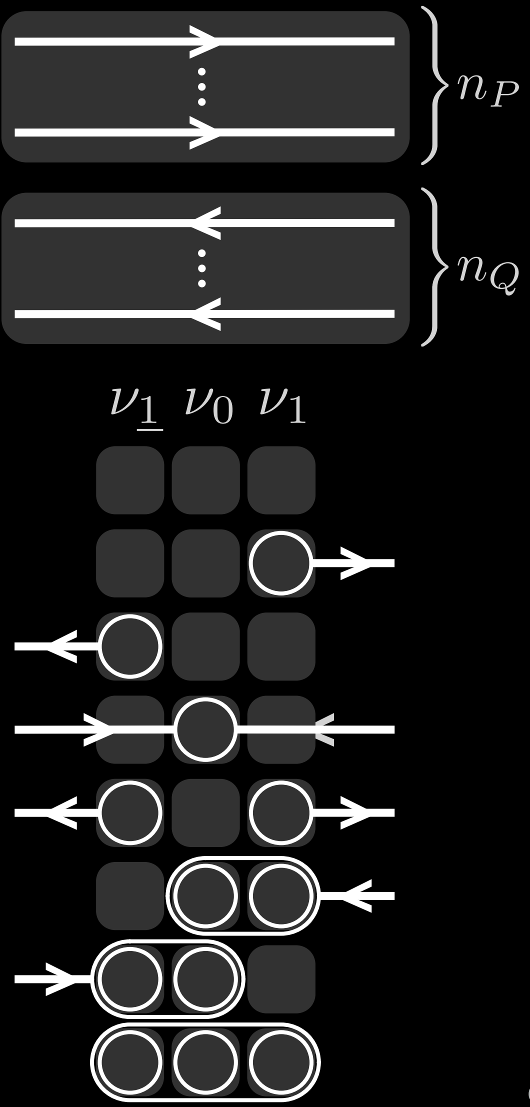
$$\psi^\dagger \cdot A^\dagger(\underline{1}) \wedge A^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 1, 0\rangle \to$$

$$\psi^\dagger \cdot B^\dagger(\underline{1})\psi^\dagger \cdot B^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 0, 1\rangle \to$$

$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0, 1, 1\rangle \to$$

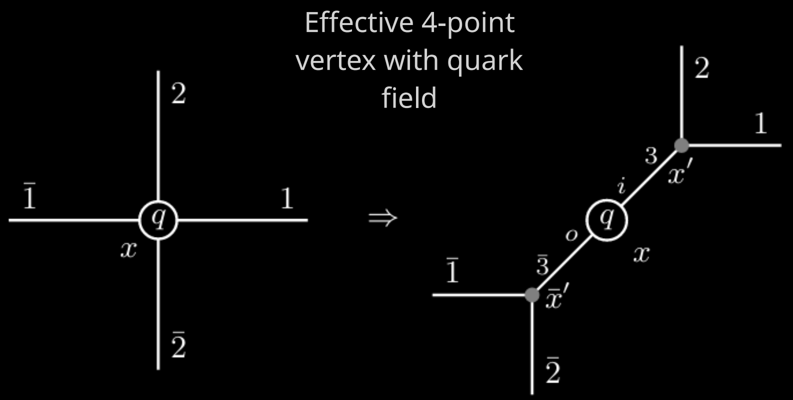
$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1})\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 1, 0\rangle \to$$

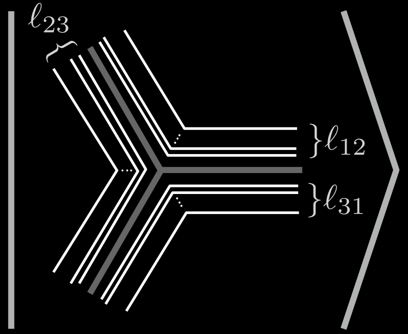
$$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1, 1, 1\rangle \to$$



The trivalent vertex

- Elementary building block of *multidimensional space* is trivalent vertex
  - Four- and six-leg vertices deconstructed via “point splitting”
- Trivalent vertex and point-splitting completely understood for LSH SU(2) (orthonormal basis, and operator matrix elements)





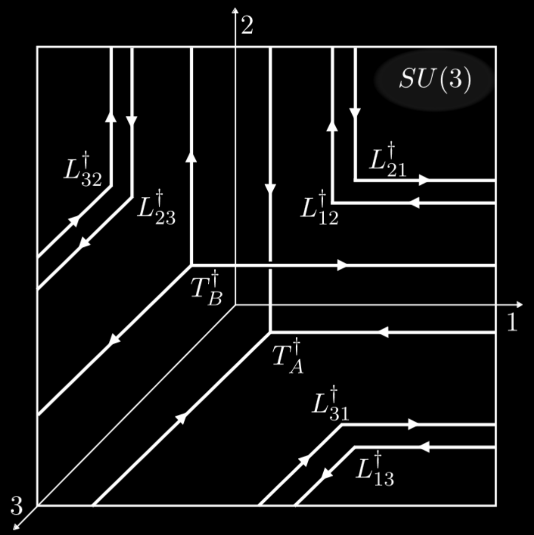
$$|\ell_{12}, \ell_{23}, \ell_{31}\rangle \equiv \frac{(\mathcal{L}_{12}^{++})^{\ell_{12}}(\mathcal{L}_{23}^{++})^{\ell_{23}}(\mathcal{L}_{31}^{++})^{\ell_{31}}}{\sqrt{\ell_{12}!\ell_{23}!\ell_{31}!(\ell_{12}+\ell_{23}+\ell_{31}+1)!}}|0\rangle$$

$$\mathcal{L}_{12}^{++}|\ell_{12}, \ell_{23}, \ell_{31}\rangle = \sqrt{(\ell_{12}+1)(\ell_{12}+\ell_{23}+\ell_{31}+2)}\,|\ell_{12}+1, \ell_{23}, \ell_{31}\rangle$$

$$\mathcal{L}_{12}^{+-}|\ell_{12}, \ell_{23}, \ell_{31}\rangle = -\sqrt{(\ell_{31}+1)\ell_{23}}\,|\ell_{12}, \ell_{23}-1, \ell_{31}+1\rangle$$

Naive LSH basis for SU(3) vertex

- Creation operators are constructed analogously
- SU(3) admits bilinear & *trilinear* excitations



$$
\begin{aligned}
L^\dagger_{12} &= A^\dagger_\alpha(1) B^{\dagger\alpha}(2) & L^\dagger_{21} &= A^\dagger_\alpha(2) B^{\dagger\alpha}(1) \\
L^\dagger_{23} &= A^\dagger_\alpha(2) B^{\dagger\alpha}(3) & L^\dagger_{32} &= A^\dagger_\alpha(3) B^{\dagger\alpha}(2) \\
L^\dagger_{31} &= A^\dagger_\alpha(3) B^{\dagger\alpha}(1) & L^\dagger_{13} &= A^\dagger_\alpha(1) B^{\dagger\alpha}(3) \\
T^\dagger_A &= \epsilon^{\alpha\beta\gamma} A^\dagger_\alpha(1) A^\dagger_\beta(2) A^\dagger_\gamma(3) & T^\dagger_B &= \epsilon_{\alpha\beta\gamma} B^{\dagger\alpha}(1) B^{\dagger\beta}(2) B^{\dagger\gamma}(3)
\end{aligned}
$$

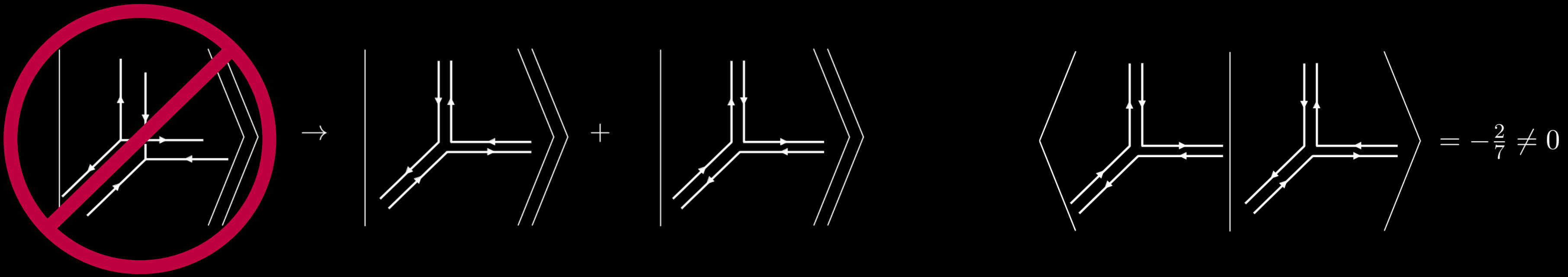
$$
|\ell_{12}\,\ell_{23}\,\ell_{31};\ell_{21}\,\ell_{32}\,\ell_{13};t\rangle\rangle \equiv L^{\dagger\,\ell_{12}}_{12} L^{\dagger\,\ell_{23}}_{23} L^{\dagger\,\ell_{31}}_{31} L^{\dagger\,\ell_{21}}_{21} L^{\dagger\,\ell_{32}}_{32} L^{\dagger\,\ell_{13}}_{13} \times \begin{cases} T^{\dagger\,t}_A\,|0\rangle\,, & t \geq 0 \\ T^{\dagger\,-t}_B\,|0\rangle\,, & t < 0 \end{cases}
$$

$$
\ell_{IJ} \in \{0,\ 1,\ 2,\ 3,\ \ldots\}, \quad t \in \{0,\ \pm1,\ \pm2,\ \pm3,\ \ldots\}
$$

arXiv:2407.19181

Naive LSH basis for SU(3) vertex

- Problem: { l_{12}, l_{23}, l_{31}, l_{21}, l_{32}, l_{13}, t } not always “good” quantum numbers
- Interesting things happen in sector $\overrightarrow{p\,q} = (p_1, q_1, p_2, q_2, p_3, q_3) = (1, 1, 1, 1, 1, 1)$



$$T_A^\dagger T_B^\dagger |0\rangle = |\ell_{12} = \ell_{23} = \ell_{31} = 1\rangle\rangle + |\ell_{21} = \ell_{32} = \ell_{13} = 1\rangle\rangle$$

- Irreps (six labels) are orthogonal, but insufficient
- LSH quantum numbers capture all d.o.f., but not always orthogonally

arXiv:2407.19181

Littlewood-Richardson coefficients

$$\lambda \otimes \mu = \bigoplus_{\nu} d^{\nu}_{\lambda,\mu}\, \nu$$

$$(1,1) \otimes (1,1) = (0,0) \oplus (1,1) \oplus (1,1) \oplus (3,0) \oplus (0,3) \oplus (2,2)$$

$$d^{(1,1)}_{(1,1),(1,1)} = 2$$

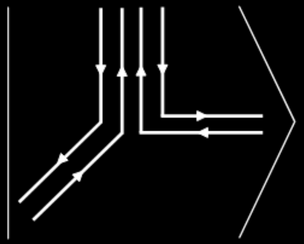
For SU(2), Littlewood-Richardson coefficients (LRCs) are either 0 or 1

For SU(3), LRCs can be any positive integer → Extra, seventh d.o.f.

arXiv:2407.19181

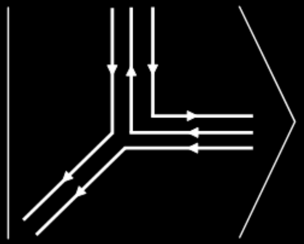
Nondegenerate states

- States corresponding to an LRC of one, have only LSH state
  → no orthogonality problem
- Can sort such states into two distinct classes



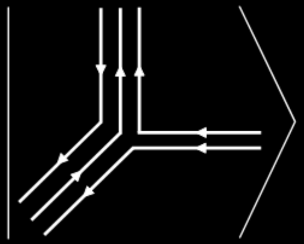
$= |\ell_{12} = \ell_{23} = \ell_{21} = \ell_{32} = 1\rangle$

Class I



$= |\ell_{12} = \ell_{23} = \ell_{21} = \ell_{13} = 1\rangle$

Class IIa



$= |\ell_{12} = \ell_{23} = \ell_{32} = \ell_{13} = 1\rangle$

Class IIb

arXiv:2407.19181

Nondegenerate states

- Without overlap problem, we normalized the states in closed form.

Class I:

$$\langle\langle \ell_{IJ}, \ell_{JK}, \ell_{JI}, \ell_{KJ}, t | \ell_{IJ}, \ell_{JK}, \ell_{JI}, \ell_{KJ}, t \rangle\rangle$$
$$= \tfrac{1}{2}(\ell_{IJ} + \ell_{JI} + \ell_{JK} + \ell_{KJ} + |t| + 2)\ell_{IJ}!\,\ell_{JK}!\,\ell_{JI}!\,\ell_{KJ}!\,|t|!\,(\ell_{IJ} + \ell_{KJ} + |t| + 1)!\,(\ell_{JK} + \ell_{JI} + |t| + 1)!\,.$$

Classes IIa:

$$\langle\langle \ell_{IJ}, \ell_{JK}, \ell_{JI}, \ell_{IK}, t | \ell_{IJ}, \ell_{JK}, \ell_{JI}, \ell_{IK}, t \rangle\rangle$$
$$= \frac{1}{2} \frac{(\ell_{IJ}+\ell_{JK}+\ell_{JI}+\ell_{IK}+|t|+2)\,\ell_{IJ}!\,\ell_{JK}!\,\ell_{JI}!\,\ell_{IK}!\,|t|!\,(\ell_{IJ}+\ell_{IK}+|t|+1)!\,(\ell_{JK}+\ell_{JI}+|t|+1)!\,\binom{\ell_{IJ}+\ell_{JK}+\ell_{JI}+\ell_{IK}+|t|+1}{\ell_{IK}}}{\binom{\ell_{IJ}+\ell_{JI}+\ell_{IK}+|t|+1}{\ell_{IK}}}$$

arXiv:2407.19181

Degenerate subspaces

- When LRC > 1, multiple LSH states exist in a sector, and fail to be orthogonal
- Counting LSH states provides a way to evaluate SU(3) LRCs
- Normalization becomes much harder (still maybe possible)
- Orthogonal basis is even less obvious

$\overrightarrow{p\,q} = (1,1,1,1,1,1)$:

$$\begin{pmatrix} \langle\langle 1\,1\,1; 0\,0\,0; 0| \\ \langle\langle 0\,0\,0; 1\,1\,1; 0| \end{pmatrix} \Big(|1\,1\,1; 0\,0\,0; 0\rangle\rangle\,,\ |0\,0\,0; 1\,1\,1; 0\rangle\rangle \Big) = \begin{pmatrix} \frac{56}{3} & \frac{-16}{3} \\ \frac{-16}{3} & \frac{56}{3} \end{pmatrix}$$

arXiv:2407.19181

Orthogonalization

arXiv:2407.19181

- Gram-Schmidt for a sector always possible, but...
  - no insight into seventh d.o.f.
  - not analytically solvable
- Alternate solution: Define a "seventh Casimir" operator
  - Should commute with (p_i,q_i)
  - Hermitian with nondegenerate spectrum → Eigenbasis will be orthogonal
  - One choice:

$$C_T \equiv (T_A T_B)^\dagger T_A T_B .$$

$$\mathrm{Spec}_{111111}(C_T) = \left\{0, \frac{80}{3}\right\},$$

$$\begin{pmatrix} |\phi_1\rangle\rangle_{111111} \\ |\phi_2\rangle\rangle_{111111} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} |1\,1\,1; 0\,0\,0; 0\rangle\rangle \\ |0\,0\,0; 1\,1\,1; 0\rangle\rangle \end{pmatrix}.$$

Orthonormalizing using Schwinger bosons gets costly

Mathematica notebook published on PRD & GitHub

Working with the nonorthogonal basis

- Electric Hamiltonian: function of number operators
- Magnetic Hamiltonian: number operators and various corner operators (contractions involving two link ends):

$$L^\dagger_{IJ}, \quad L_{IJ} = A(I) \cdot B(J)$$

$$N_{IJ} \equiv A(I)^\dagger_\alpha A(J)^\alpha$$

$$M_{IJ} \equiv A(I)^\dagger_\alpha A(J)^\alpha$$

$$\epsilon A(I)^\dagger A(J)^\dagger B(J) \equiv \epsilon^{\alpha\beta\gamma} A(I)^\dagger_\alpha A(J)^\dagger_\beta B(J)_\gamma$$

$$\epsilon B(I)^\dagger B(J)^\dagger A(J) \equiv \epsilon^{\alpha\beta\gamma} B(I)^\dagger_\alpha B(J)^\dagger_\beta A(J)_\gamma$$

- Other contractions exist, but understanding above is sufficient
- Normalizations involve $T_A, \quad T_B$

Working with the nonorthogonal basis

- Evaluating LSH operators on arbitrary LSH states is a LONG exercise
- 7-8 layers of proof-by-induction!
- Representative formulas

$$T_A^\dagger \left|\{\ell\};t\right\rangle\!\rangle = \begin{cases} t \geq 0: & \left|\{\ell\};t+1\right\rangle\!\rangle \\ t < 0: & \left|\ell_{ij}+1,\ell_{jk}+1,\ell_{ki}+1,\cdots;t+1\right\rangle\!\rangle + \left|\ell_{ji}+1,\ell_{kj}+1,\ell_{ik}+1,\cdots;t+1\right\rangle\!\rangle \end{cases}$$

$$N_{ij}\left|\{\ell\};t\right\rangle\!\rangle = \left(\frac{1}{\ell_{ij}+\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+1}\right)\Big[\ell_{jk}(\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+1)\left|\ell_{jk}-1,\ell_{ik}+1,\cdots;t\right\rangle\!\rangle - $$
$$- \ell_{ji}\ell_{kj}\left|\ell_{ij}+1,\ell_{ki}+1,\ell_{ji}-1,\ell_{kj}-1,\cdots;t\right\rangle\!\rangle\Big]$$

Symmetries give formulas for related operators.

Working with the nonorthogonal basis

$$
\begin{aligned}
&L_{ij}\,|\{\ell\}\,;t\rangle\rangle \\
&= \left(\frac{1}{\ell_{ij}+\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+1}\right)\times \\
&\quad\times\Bigg(|\ell_{jk}+1,\ell_{ki}+1,\ell_{ji}-1,\ell_{kj}-1,\ell_{ik}-1,\cdots;t\rangle\rangle\times \\
&\qquad\times\ell_{ji}\ell_{kj}\ell_{ik}\left(\frac{\ell_{ij}+\ell_{jk}+\ell_{kj}+|t|+2}{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}\right)\left[\frac{\ell_{ki}}{\ell_{ij}+\ell_{ki}+\ell_{ji}+\ell_{ik}+|t|+1}-\left(\frac{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}{\ell_{ij}+\ell_{jk}+\ell_{kj}+|t|+2}+1\right)\right]+ \\
&\quad+|\ell_{ij}-1,\cdots;t\rangle\rangle\times \\
&\qquad\times\ell_{ij}\Bigg\{\frac{(-\ell_{ki})}{\ell_{ij}+\ell_{ki}+\ell_{ji}+\ell_{ik}+|t|+1}\left[(\ell_{kj}+1)(\ell_{ij}+\ell_{jk}+\ell_{kj}+|t|+1)+\frac{\ell_{ji}\ell_{ik}(\ell_{jk}+1)}{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}\right]+ \\
&\qquad\qquad+\left[\ell_{ik}(\ell_{ij}+\ell_{ji}+\ell_{kj}+|t|+1)+\left(\frac{\ell_{ik}\ell_{ji}(\ell_{jk}+1)}{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}\right)+\right. \\
&\qquad\qquad\qquad\left.+(\ell_{ij}+\ell_{kj}+|t|+1)(\ell_{ij}+\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+2)\right]\Bigg\}+ \\
&\quad+|\ell_{ij}-2,\ell_{jk}-1,\ell_{ki}-1,\ell_{ji}+1,\ell_{kj}+1,\ell_{ik}+1;t\rangle\rangle\left(\frac{\ell_{ij}(\ell_{ij}-1)\ell_{jk}\ell_{ki}}{\ell_{ij}+\ell_{ki}+\ell_{ji}+\ell_{ik}+|t|+1}\right)\Bigg)
\end{aligned}
$$

More annihilation operators
→ harder calculation

Working with the nonorthogonal basis

- All other operators have been evaluated explicitly, or can be derived from others. E.g.,

$$T_A = [L_{12}, \epsilon B(2)^\dagger A(2) A(3)]$$

- With matrix elements of all site-local operators available, classical calculations using purely LSH d.o.f. are being scripted and run quickly
  - Normalizations, orthogonalization, Hamiltonian matrix elements are all being tested
- Hope: Pure LSH coding will lead to orthogonal basis solution

Working with the nonorthogonal basis

localhost:8888/notebooks/trivalent-vertex_LSH.ipynb SU(3) 90%

File Edit View Insert Cell Kernel Widgets Help Not Trusted SageMath 9.5

SU(3) Yang-Mills vertex: LSH formulation

Jesse Stryker
Lawrence Berkeley National Laboratory

Script created: 2024-09-06

This SageMath notebook is a toolkit for explicit Hilbert-space calculations using SU(3) loop-string-hadron (LSH) operators and states in the naive nonorthogonal basis. The primary objects worked with are States (kets), and the operators are functions of those States. *This script is specially tailored to the use of the NONORTHOGONAL (but still linearly independent) basis.* The program was originally written in Mathematica and may differ slightly from the Mathematica version.

```
In [69]: testState=[[(2,1,1,1,1,1,2),1]]

In [70]: StateL(1,2)(testState)

[[(0, 0, 0, 2, 2, 2, 2), 1/32],
 [(1, 1, 1, 1, 1, 1, 2), 119/8],
 [(2, 2, 2, 0, 0, 0, 2), -1/4]]

In [71]: StateN(2,3)(testState)

[[(2, 1, 0, 2, 1, 1, 2), 6/7], [(3, 2, 1, 1, 0, 0, 2), -1/7]]

In [72]: StateTA(testState)

[[(3, 2, 2, 0, 0, 0, 1), 157/56],
 [(2, 1, 1, 1, 1, 1, 1), 5631/56],
 [(1, 0, 0, 2, 2, 2, 1), 585/112]]
```

```
In [73]: CTMatrixRep[(1,1,1,1,1,1)]

[40/3 40/3]
[40/3 40/3]

In [74]: print(CTMatrixRep[(1,1,1,1,1,1)].eigenvalues())

[80/3, 0]

In [75]: print(CTMatrixRep[(1,1,1,1,1,1)].eigenvectors_right())

[(80/3, [
(1, 1)
], 1), (0, [
(1, -1)
], 1)]
```

- Jupyter notebook implements the nonorthogonal LSH basis formulas.
  - No Schwinger bosons, no Clebsch-Gordons, and fast.
- To be made public on Github

Status

- Much analytic control has been achieved for the naive basis
- Ideally: Find a seventh Casimir whose eigenstates can be constructed analytically
  - Ex: Some “ladder” operator applied to a reference state, similar to SU(2) irreps $|j,m\rangle$
- Point-splitting: We predict no significant departure from SU(2)
- Coupling to matter: Also believed to go like SU(2)
- Work in progress: lattice Hamiltonian in nonorthogonal basis

Summary

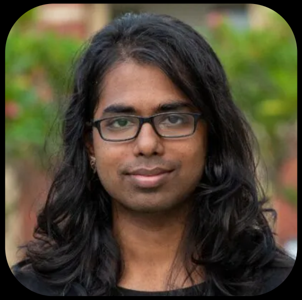
- SU(3) gauge invariant basis can be constructed by direct analogy with SU(2)
- For certain choices of irreps, the states are on par with SU(2) theory
- Subtleties arise for other choices of irreps
  - Basis is linearly independent, but not orthogonal
  - Analytic handle on these states is the main obstacle to putting SU(3) formulation *completely* on par with SU(2)

Acknowledgments

- Collaborators on this work

Indrakshi Raychowdhury (BITS-Pilani, Goa)

Saurabh Kadam (IQuS → Livermore)



Aahiri Naskar (BITS-Pilani, Goa)

- Insightful conversations with Anthony Ciavarella, Zohreh Davoudi, David B. Kaplan, John Lombard and Himadri Mukherjee

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Extra slides

Irreducible SU(3) Schwinger bosons

- SU(2): Arbitrary irrep *j* constructible by tensor-producting enough spin-1/2's → One doublet to construct all |*j*,*m*> states

$$a^\dagger = \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix}$$

- In SU(3): Arbitrary irrep (P,Q) constructible by tensor products of one 3 and one 3* →

$$a^\dagger = \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_3^\dagger \end{pmatrix}, \quad b^\dagger = \begin{pmatrix} b^{\dagger 1} \\ b^{\dagger 2} \\ b^{\dagger 3} \end{pmatrix}$$

- Ex: 3 $\quad |1,0\rangle_\alpha = a_\alpha^\dagger |\Omega\rangle$
- Ex: 3* $\quad |0,1\rangle^\beta = b^{\dagger\beta} |\Omega\rangle$

Irreducible SU(3) Schwinger bosons

- Ex: 8 $a^\dagger_\alpha b^{\dagger\beta}\left|\Omega\right\rangle \in (1,1)?$ *No!*... $3 \otimes 3^* = 8 \oplus 1$ $a^\dagger_\alpha b^{\dagger\alpha}\left|\Omega\right\rangle \in (0,0)$

  To be *irreducible*, the rep should be *traceless*

$$\left|1,1\right\rangle^\beta_\alpha \equiv a^\dagger_\alpha b^{\dagger\beta}\left|\Omega\right\rangle - \frac{1}{3}\delta^\beta_\alpha a^\dagger \cdot b^\dagger \left|\Omega\right\rangle, \qquad a^\dagger \cdot b^\dagger \equiv a^\dagger_\gamma b^{\dagger\gamma}$$

- One can generalize solution to all states/irreps, but hopeless to work with directly

- Solution: "irreducible Schwinger bosons"

Anishetty, Mathur, & Raychowdhury, J. Math. Phys. 50, 053503 (2009)

$$A^\dagger_\alpha \equiv a^\dagger_\alpha - \frac{1}{P+Q+1}(a^\dagger \cdot b^\dagger)b_\alpha,$$
$$B^{\dagger\alpha} \equiv b^{\dagger\alpha} - \frac{1}{P+Q+1}(a^\dagger \cdot b^\dagger)a^\alpha.$$

$$P \equiv a^\dagger \cdot a$$
$$Q \equiv b^\dagger \cdot b$$

With ISBs: $\left|1,1\right\rangle^\beta_\alpha \equiv A^\dagger_\alpha B^{\dagger\beta}\left|\Omega\right\rangle$

All irrep states have this 'monomial' form

Irreducible SU(3) Schwinger bosons