Digitised Hamiltonian SU(2) Lattice Gauge Theories at Weak Couplings

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[5]University Bern

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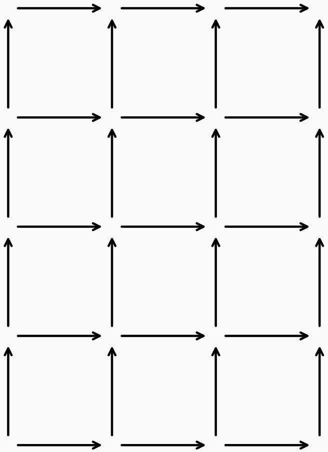
- efficient simulations near the continuum limit

The Hamiltonian

The Hilbert Space



Positions of links labelled by $\mathbf{x}$, directions by k

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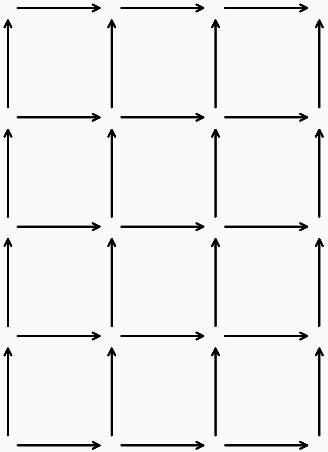

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Hamiltonian acts on wave functions:

$$\psi\,(\ldots, U_{\mathbf{x},k}, \ldots) \ : \ \mathrm{SU}(2)^{N_{\text{links}}} \ \to \ \mathbb{C}\,,$$

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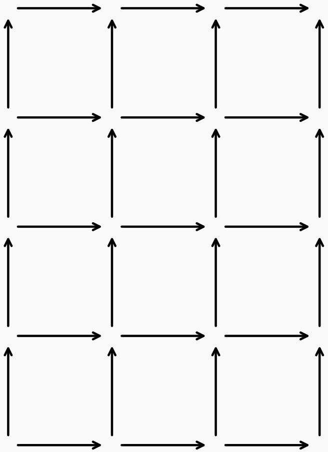

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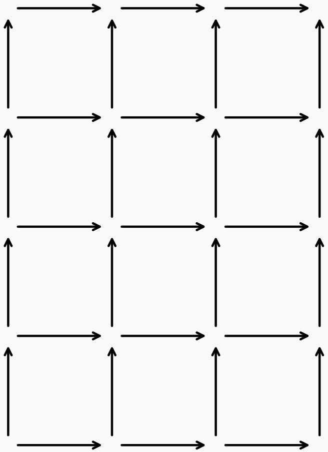

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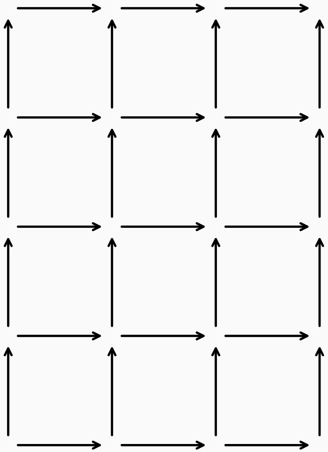
Basis for entire space:

$$|\ldots, n_{\mathbf{x},\mathbf{k}}, \ldots\rangle = \prod_{\mathbf{x},k} \hat{\phi}_{n_{\mathbf{x},\mathbf{k}}}(U_{\mathbf{x},k})$$

Kogut Susskind Hamiltonian[1]



$$\hat{H}_{\mathrm{KS}} = \frac{g^2}{2} \sum_{\mathbf{x},k,c} (\hat{L}^c_{\mathbf{x},k})^2 + \frac{2}{g^2} \sum_{\mathbf{x},j<k} \mathrm{Tr}\left[\mathbb{1} - \mathrm{Re}\left(\hat{P}_{\mathbf{x},jk}\right)\right]$$

[1] John Kogut and Leonard Susskind. **"Hamiltonian formulation of Wilson's lattice gauge theories".** In: *Phys. Rev. D* 11 (2 Jan. 1975), pp. 395–408. DOI: 10.1103/PhysRevD.11.395.

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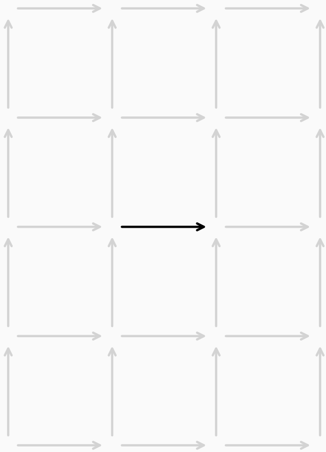

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Canonical Momentum Operators:

$$\hat{L}^c_{\mathbf{x},k}\,\psi = -\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}\beta}\,\psi\left(\dots, e^{-\mathrm{i}\,\beta\tau_c}\,U_{\mathbf{x},k},\dots\right)|_{\beta=0}$$

and

$$\hat{R}^c_{\mathbf{x},k}\,\psi = -\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}\beta}\,\psi\left(\dots, U_{\mathbf{x},k}\,e^{\mathrm{i}\,\beta\tau_c},\dots\right)|_{\beta=0}\,,$$

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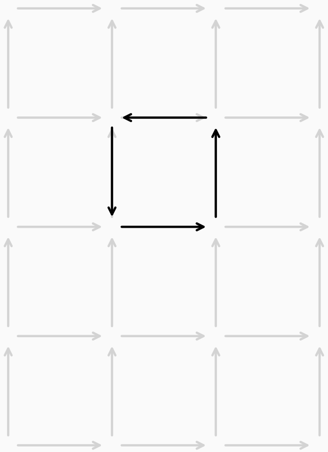

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Plaquette Operator:

$$\hat{P}_{\mathbf{x},ij} = \hat{U}_{\mathbf{x},i}\, \hat{U}_{\mathbf{x}+a\hat{\mathbf{i}},j}\, \hat{U}^\dagger_{\mathbf{x}+a\hat{\mathbf{j}},i}\, \hat{U}^\dagger_{\mathbf{x},j}$$

in terms of link operators

$$\hat{U}_{\mathbf{x},k}\, \psi = U_{\mathbf{x},k} \psi\left(\ldots, U_{\mathbf{x},k}, \ldots\right)$$

and

$$\hat{U}^\dagger_{\mathbf{x},k}\, \psi = U^\dagger_{\mathbf{x},k} \psi\left(\ldots, U_{\mathbf{x},k}, \ldots\right) .$$

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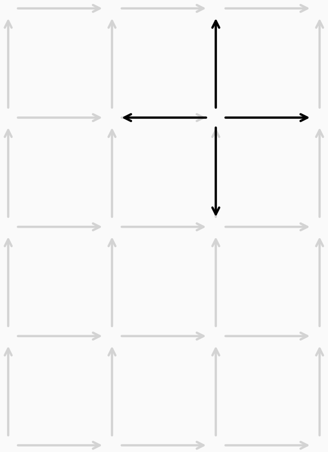

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Gauss Law for physical states:

$$\hat{G}^c_{\mathbf{x}} \left|\psi\right\rangle = \sum_k \left(\hat{L}^c_{\mathbf{x},k} + \hat{R}^c_{\mathbf{x}-a\hat{\mathbf{k}},k}\right) \left|\psi\right\rangle = 0$$

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Electric Basis Functions

Character / Clebsch-Gordon expansion:

- Eigenstates $|J, m_L, m_R\rangle$ of $\sum_c (\hat{L}_c)^2$, $\hat{L}_3$ and $\hat{R}_3$ known

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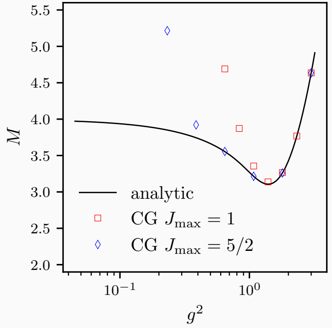

Mass gap M for a single plaquette as a function of g^2 using Clebsch-Gordon Operators

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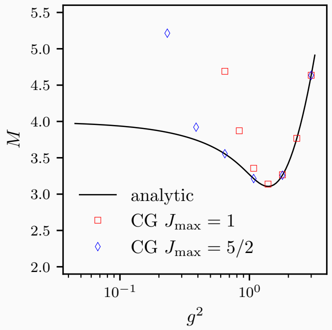
For $g^2 \to 0$ (Continuum limit):

$$\psi \to \prod_{\mathbf{x},jk} \delta\left(\mathbb{1} - P_{\mathbf{x},jk}\right)$$

$$\Rightarrow J_{\max} \to \infty$$



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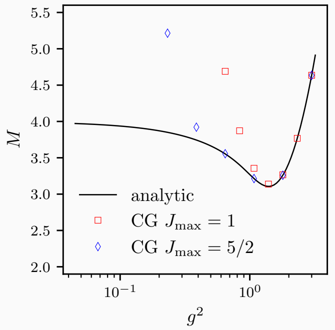
How to solve this:

1. Reformulate the KS-Hamiltonian s.t. the magnetic contributions become local
2. Choose a set of appropriate basis functions



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Magnetic Hamiltonians

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- non-local electric terms
  $\rightarrow$ dealbreaker for TN, much more demanding on QC (Swap Gates etc.)

Magnetic Hamiltonian via Plaquette Separation?

Manu Mathur, Atul Rathor (2023) arxiv:2109.0099

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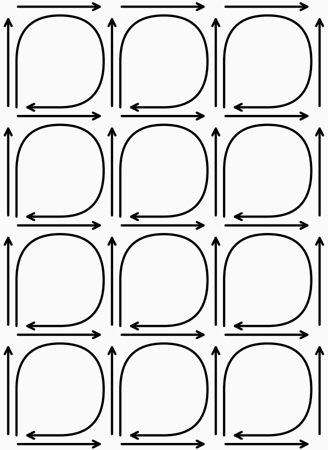

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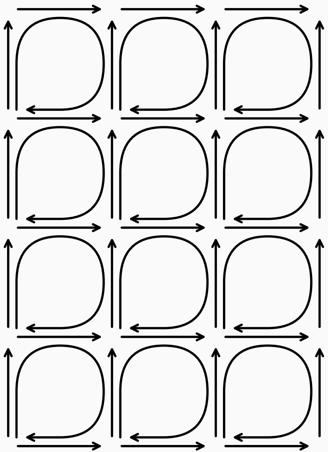

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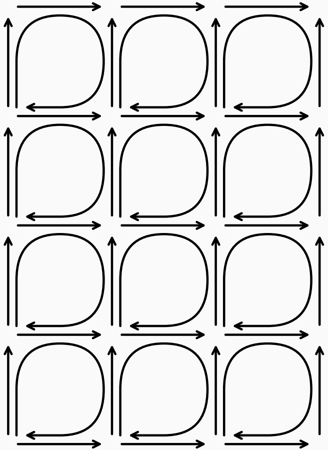

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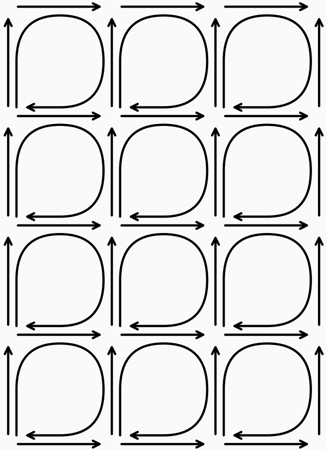

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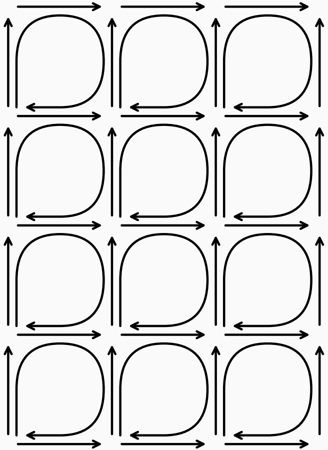

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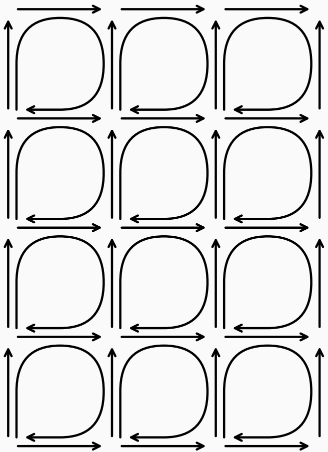

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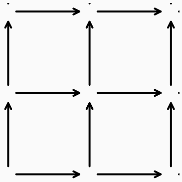
We need a solution to this, in order to simulate physics!

Results

3×3 System, Open Boundaries



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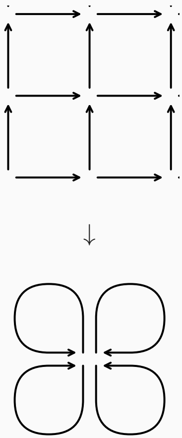

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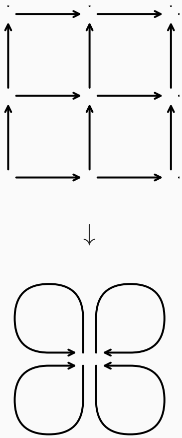

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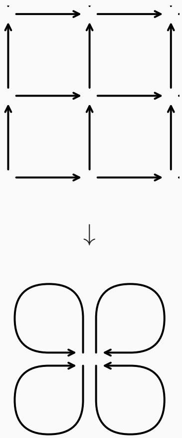

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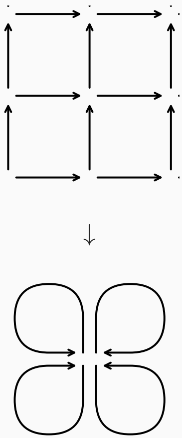

Plaquette State Basis

Single Plaquette System

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Parametrisation of $\mathrm{SU}(2)$**:**

$$U(\psi,\theta,\phi) = \cos(\psi)\mathbb{1} - \mathrm{i}\sin(\psi)\,\vec{n}(\theta,\phi)\cdot\vec{\sigma}$$

where $$\vec{n}(\theta,\phi) = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

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$$\phi_{n,l,m} \sim \frac{\mathrm{se}_{2n+2}(\psi/2;q)}{\sin(\psi)} Y_{l,m}(\theta,\phi)\ , \quad q = 16/g^4$$

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- for a finite basis we truncate by demanding $n \leq n_{\mathrm{max}}$ and $l \leq l_{\mathrm{max}}$

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$$U(\psi,\theta,\phi) = \cos(\psi)\mathbb{1} - \mathrm{i}\sin(\psi)\,\vec{n}(\theta,\phi)\cdot\vec{\sigma}$$

where $$\vec{n}(\theta,\phi) = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

Eigenstates(-ish)

$$\phi_{n,l,m} \sim \frac{\mathrm{se}_{2n+2}(\psi/2;q)}{\sin(\psi)} Y_{l,m}(\theta,\phi)\ , \quad q = 16/g^4$$

- $\mathrm{se}_n(\varphi;q)$ - sine elliptic functions, solution to the Mathieu differential equation
- $Y_{l,m}$ - spherical harmonics
- for a finite basis we truncate by demanding $n \leq n_{\max}$ and $l \leq l_{\max}$
- Operator Matrices of operator $\mathcal{O}$ obtained by (numerically) evaluating

$$\int \mathrm{d}V\, \phi_{n',l',m'}\mathcal{O}\phi_{n,l,m}$$

Plaquette State Basis

Single Plaquette System

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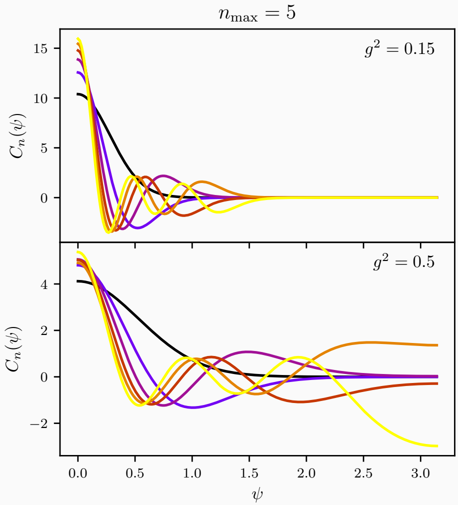
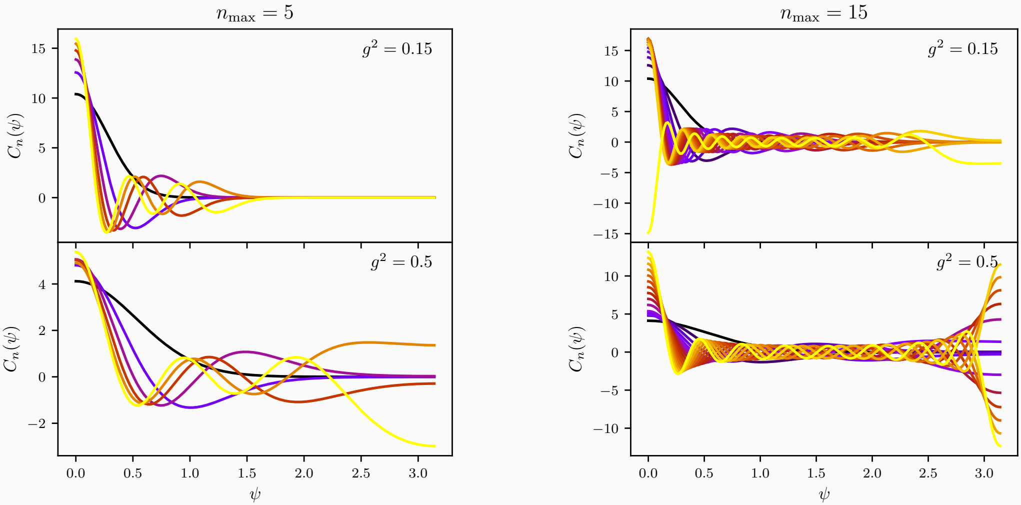
- Square operators $\hat{\vec{L}}^2$ and $\hat{\vec{L}}\cdot\hat{\vec{R}}$ integrated out separately

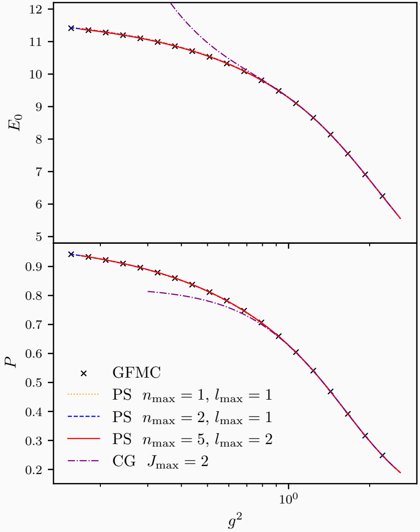
Plaquette Link Radial Basis Functions



Plaquette Link Radial Basis Functions



3×3 System, Open Boundaries



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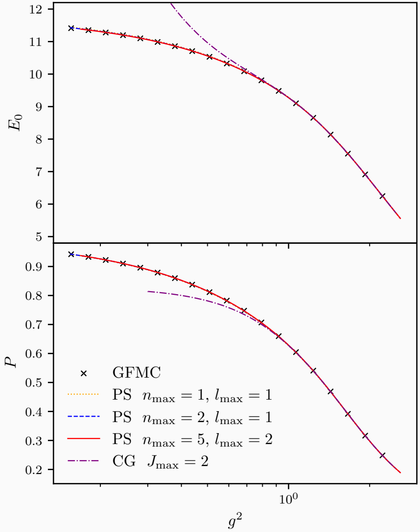

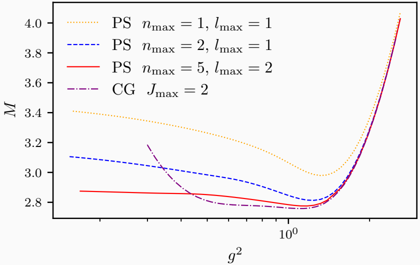

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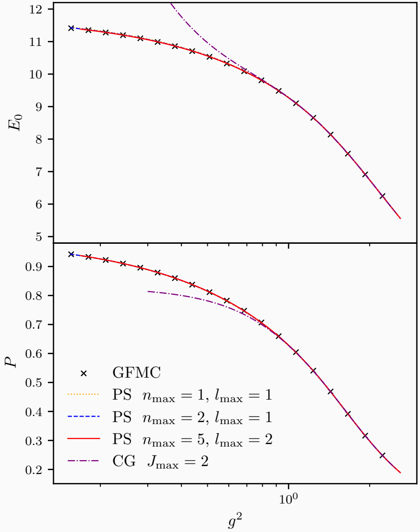

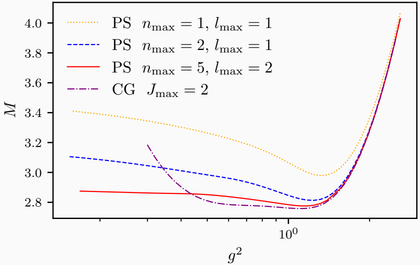

- Good matching with Greens function Monte-Carlo results, even at small couplings
- Mass gap still shows convergence behaviour, i.e. larger basis required

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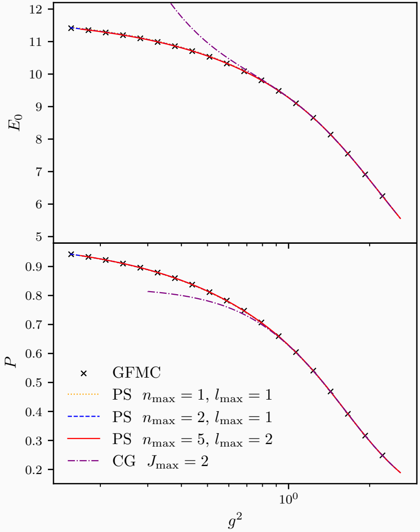

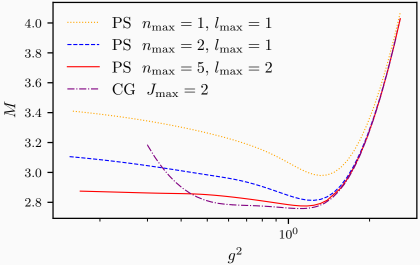

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$\rightarrow$ efficient operators are a solveable problem

Outlook

Outlook

Conclusion

- the digitisation problem has working solutions

$\Rightarrow$ This is essential to actually study QCD in quantum simulations!

Outlook

Conclusion

- the digitisation problem has working solutions

Homework

- figure a suitable dual formulation of the Kogut-Susskind Hamiltonian (non-abelian, (3+1) dimensions)
- figure out another efficiently to simulate Hamiltonian, that reproduces QCD in the continuum limit

$\Rightarrow$ This is essential to actually study QCD in quantum simulations!

The End

Thanks for listening