Composite particle spectroscopy in the Hamiltonian approach

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科研費 KAKENHI
SQAI サスティナブル量子AI研究拠点
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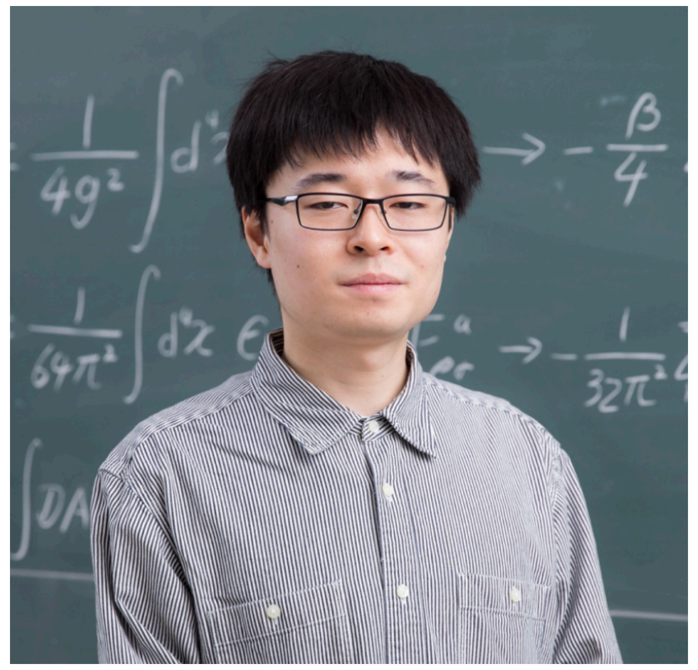


Based on
E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 11 (2023) 231
E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 09 (2024) 155

Hamiltonian Lattice Gauge Theories: Status, Novel Developments and Applications, ECT*, Trento, Italy, 2025/09/01

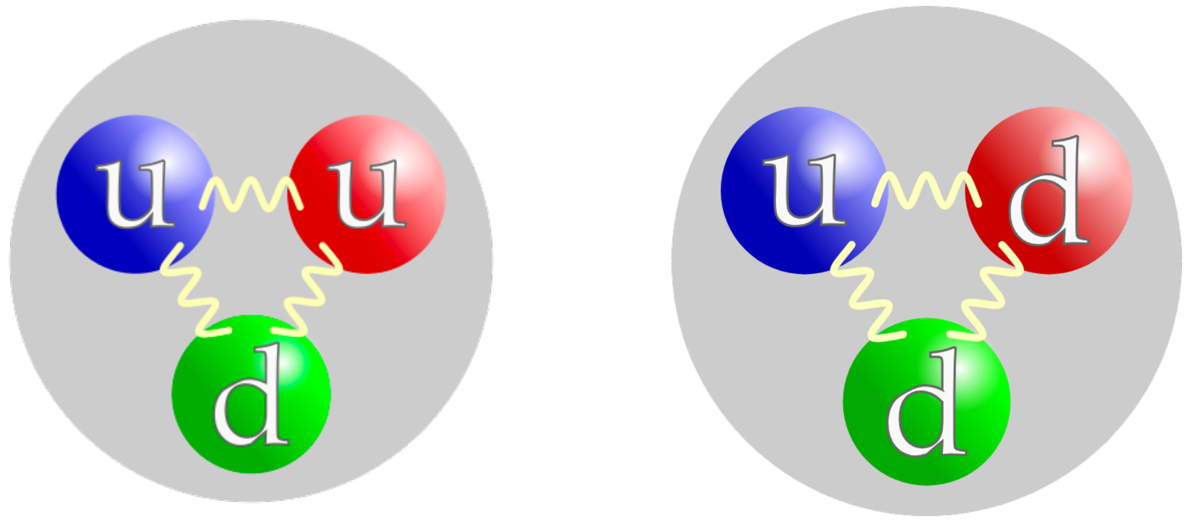
1.Introduction

One of nontrivial phenomena of QCD

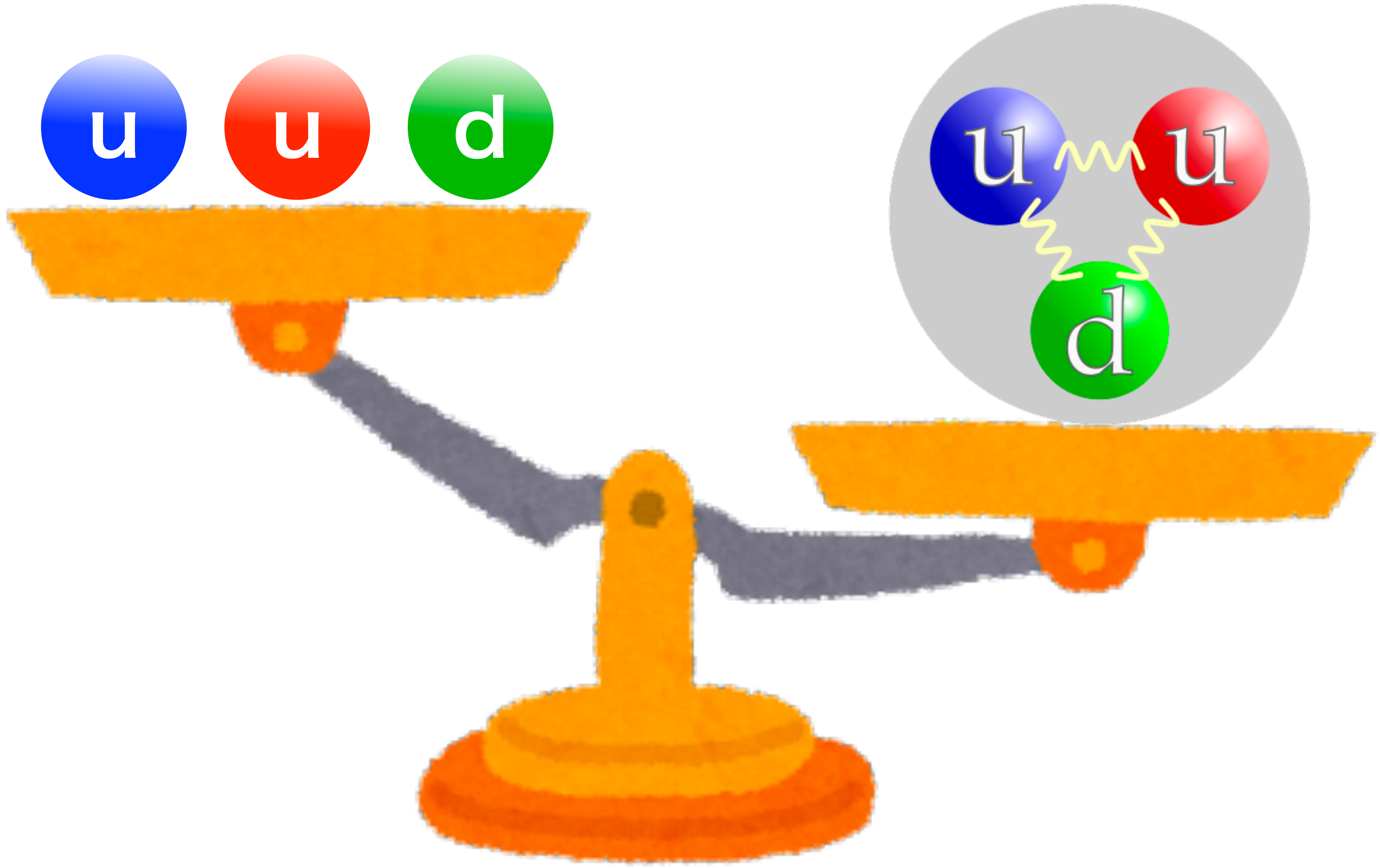
- Low-lying states are given by composite particles (Hadrons) because of quark confinement

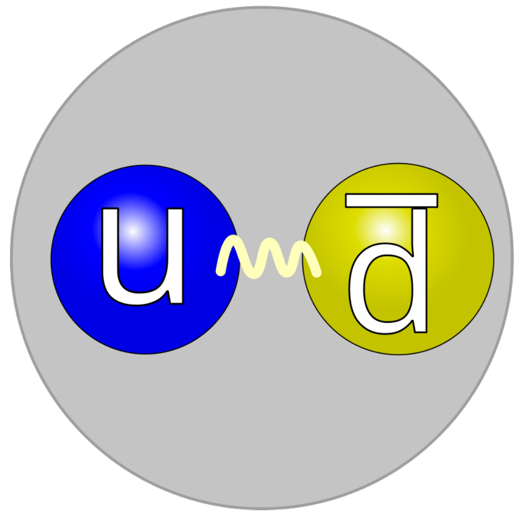
u,d quark mass ~ 2-5MeV

proton mass ~ 938MeV



composed of 3 quarks



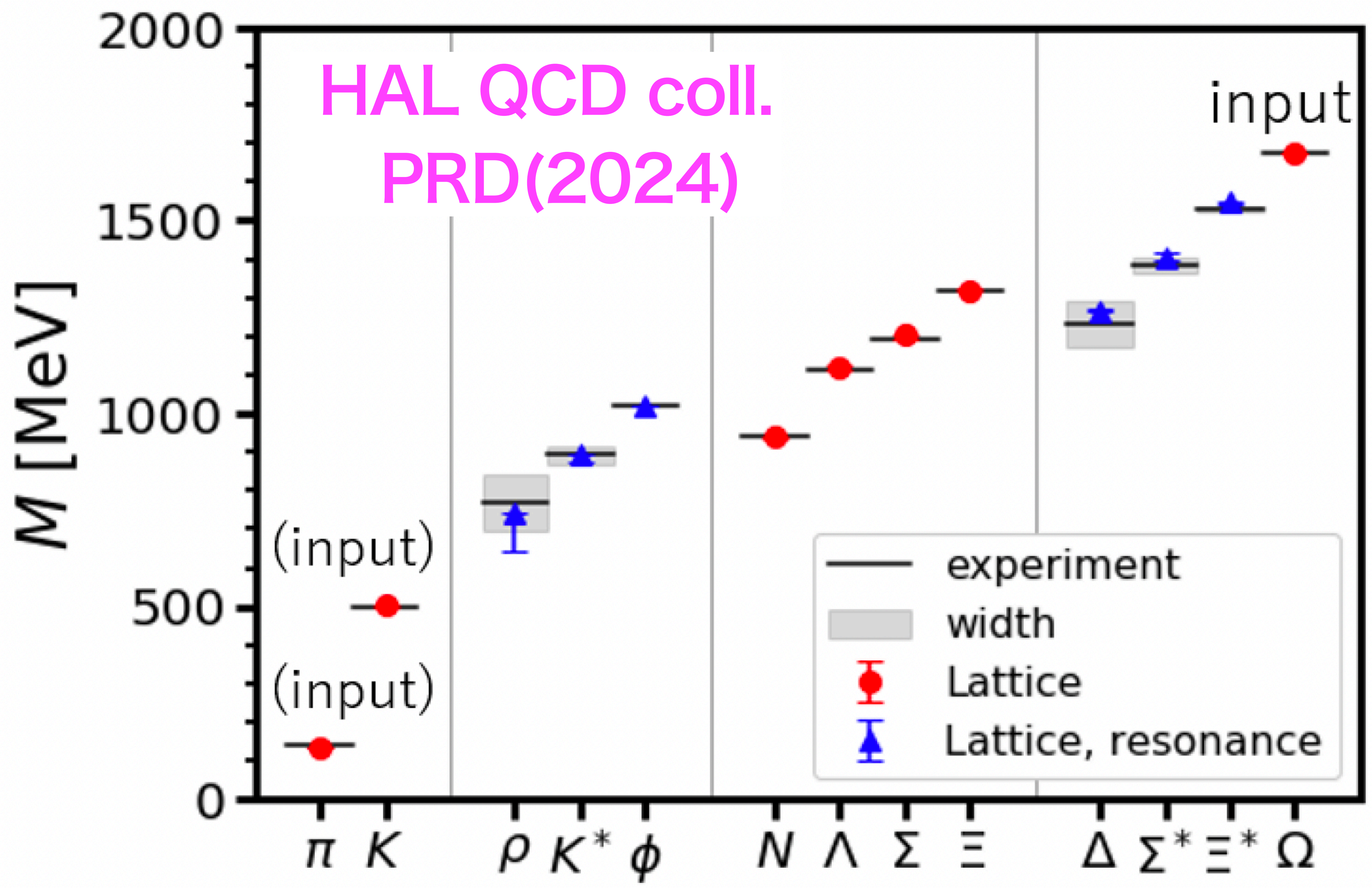


composed of quark and anti-quark

- The masses of hadrons are much heavier than the sum of quarks

Numerical results by Lattice MC QCD

Agreement between QCD predictions and experiments



- Input parameters are in QCD action
  lattice bare coupling g_0 ($\leftrightarrow a$)

  bare quark masses

  (Left panel: $m_{u,d}^0, m_s^0$)

- Only 3 inputs give more than 10 hadron masses, which are consistent with experimental data within a few % errors

- This is quantitative evidence that hadron micro-theory is QCD

Today's talk

- New calculation methods to obtain mass spectra of hadrons, which works for the gauge theories in Hamiltonian formalism

- Demonstrate the calculation for the Schwinger model using DMRG
  Find ground state (Matrix Product State, MPS)
  using variational algorithm: cost fn. is ${}_{try}\langle \Psi | H | \Psi \rangle_{try}$

ITensor
High-Performance Tensor Software
Inspired By Tensor Diagrams

- Introduce the topological θ term
  sign problem emerges in the conventional method

- We investigate near CFT (level-1, SU(2) WZW theory)
  the DMRG works well

Outline

1. Introduction

2. 2-flavor Schwinger model

3. Our proposal for calculating "Hadron" spectra ($\theta = 0$)

   Correlation-function scheme

   One-point function scheme

   Dispersion-relation scheme

   E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 11 (2023) 231

4. "Hadron" spectra ($\theta \neq 0$)

   θ dependence of hadron spectra

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5. Summary

2. 2-flavor Schwinger model

Schwinger model + θ term (Nf=1)

Kogut and Susskind (1975)

Shaw et al. Quantum 4, 306 (2020)

- Lagrangian w/ non-zero θ_0: Sign problem in conventional method

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

photon propagation   Kinetic/mass terms of electron

- Hamiltonian written by spin variables fits to Quantum Computation (QC) and Tensor Network(TN)

To minimize the Hilbert space…

Legendre transformation

Gauge fixing

Gauss' law

Open BC

Jourdan-Winger trans.

- Hamiltonian by spin variables (X, Y, Z: Pauli matrix)

$$H = J\sum_{n=0}^{N-2}\left[\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\theta}{2\pi}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1} + Y_nY_{n+1}\right] + \frac{m_{\text{lat.}}}{2}\sum_{n=0}^{N-1}(-1)^n Z_n$$

kinetic term of electric field   kinetic/mass terms of electron

- θ is constant shift of electric field

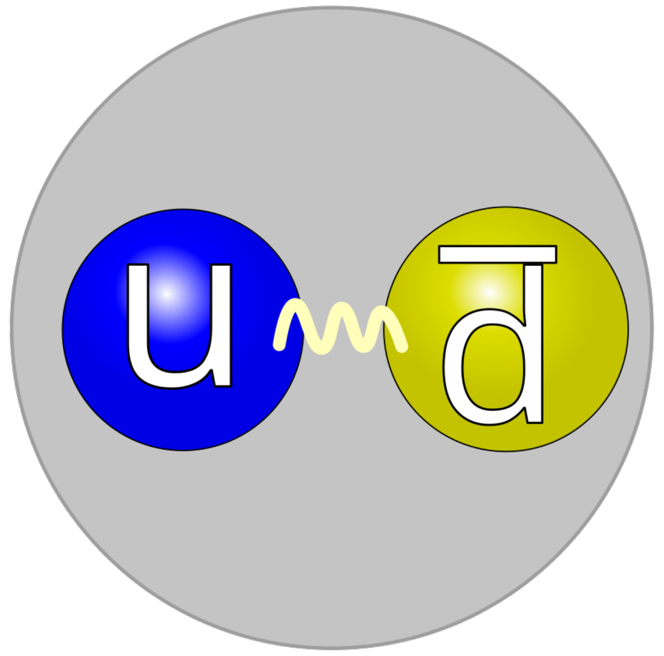
- Gaped system (even in massless fermion for Nf=1)

Property of Nf=2 Schwinger model

- Confinement of fermions occurs

  Low-lying states are given by composite particle (=boson)

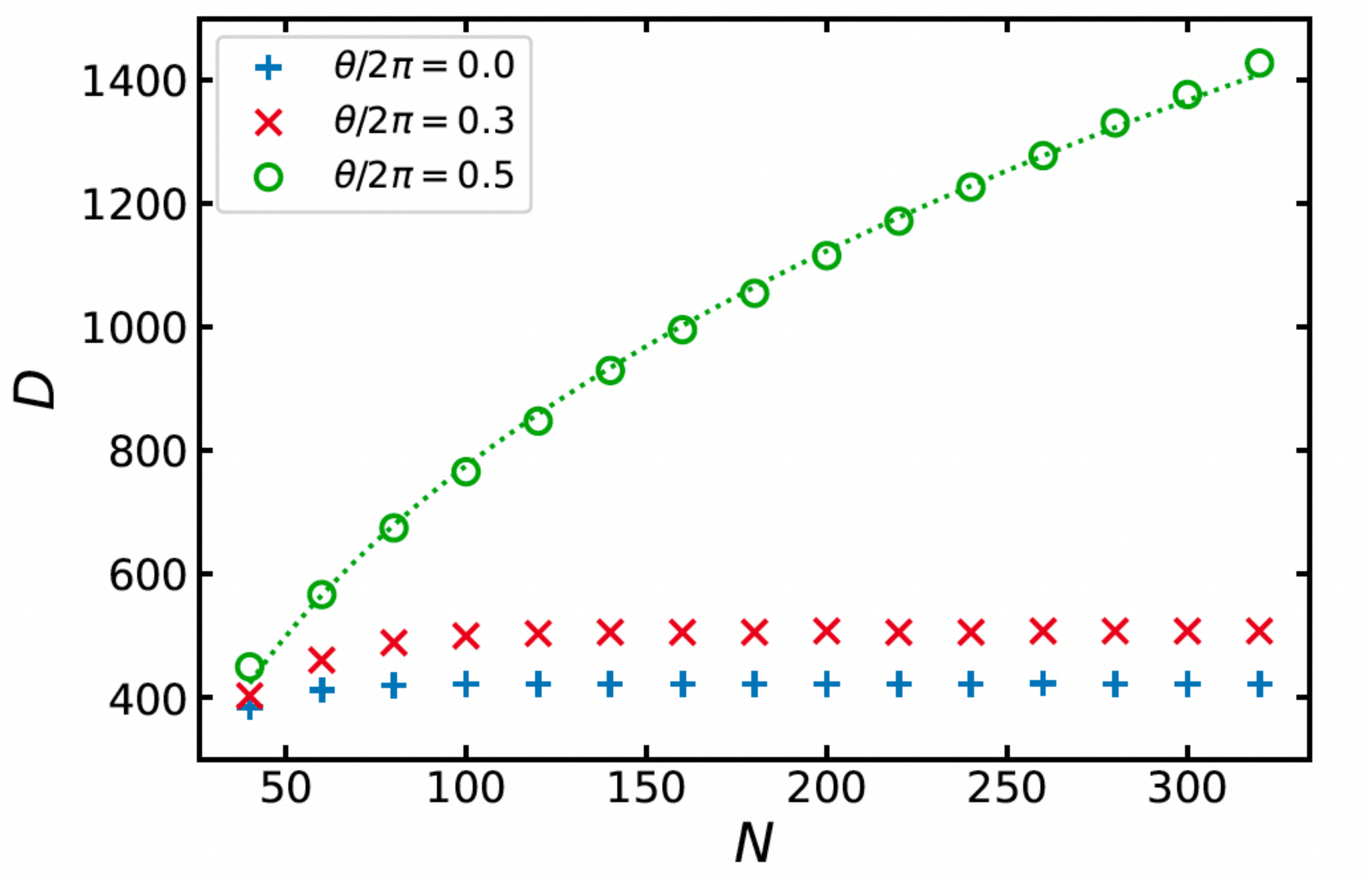
  = meson (like a pion) in QCD



- At $\theta = \pi$, it is expected the model becomes (nearly) conformal field theory

  Can we do DMRG?

bond dim.



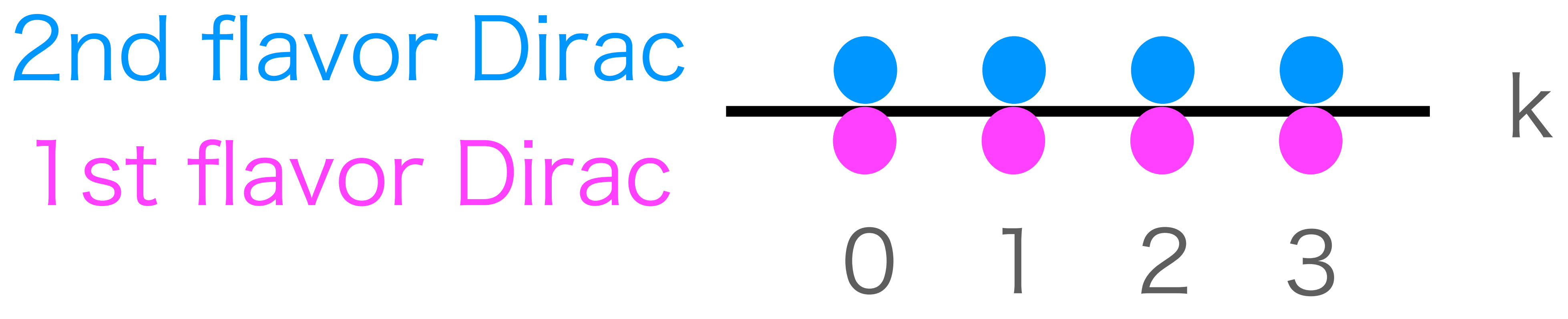
fitting fn.: $c_1 N^{1/3} + c_2$

Entanglement entropy (theoretical)

$S_{EE} \sim (c/3) \log N$ w/ c=1

Multi-flavor Schwinger model: ordering

- Dirac fermion -> lattice fermion (staggered fermion)



M.C. Banuls et al, PRL 118, 071601 (2017)
R.Dempsey et al., arXiv:2305.00437
E.I., Matsumoto, Tanizaki, JHEP 11 (2023) 231
M.Rigobello et al., arXiv:2308.04488

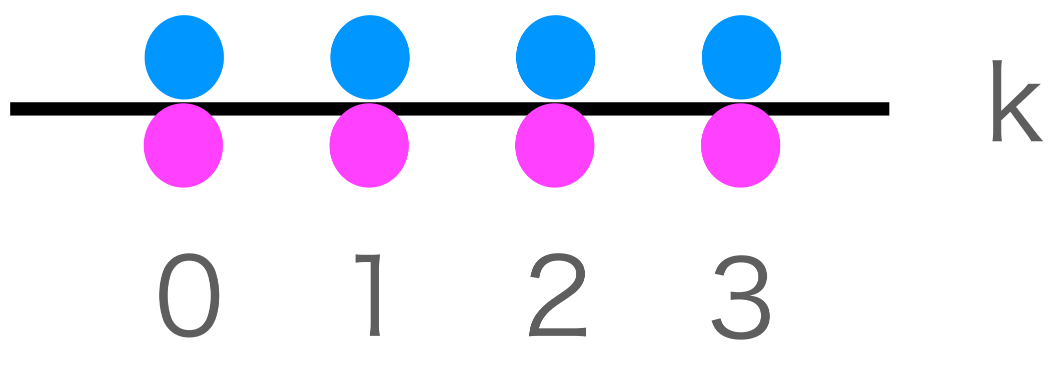
- lattice fermion -> spin variable (Jordan-Wigner trans.)

Multi-flavor Schwinger model: ordering

- Dirac fermion -> lattice fermion (staggered fermion)

2nd flavor Dirac

1st flavor Dirac



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Flavor ordering (n=k+N(f-1))

Staggered ordering (n=2k+(f-1))

- lattice fermion -> spin variable (Jordan-Wigner trans.)

Multi-flavor Schwinger model: ordering

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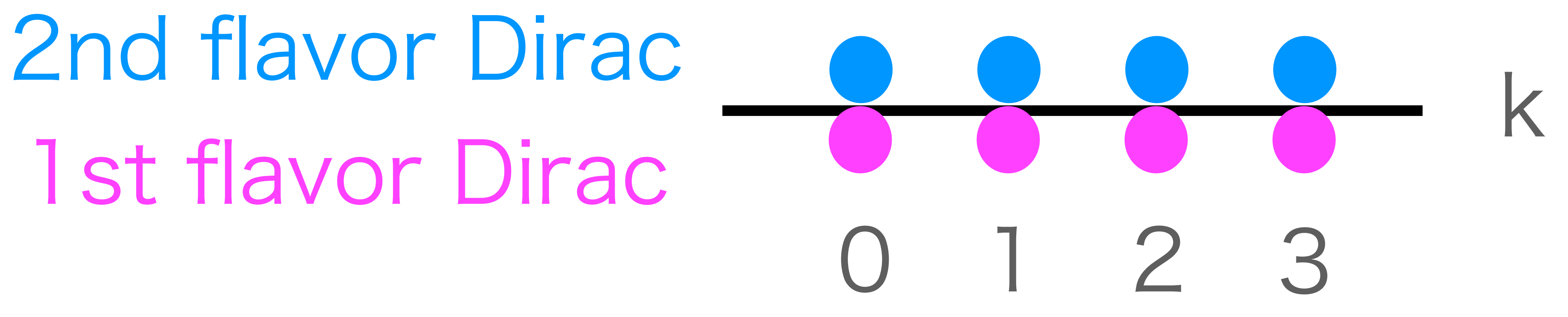

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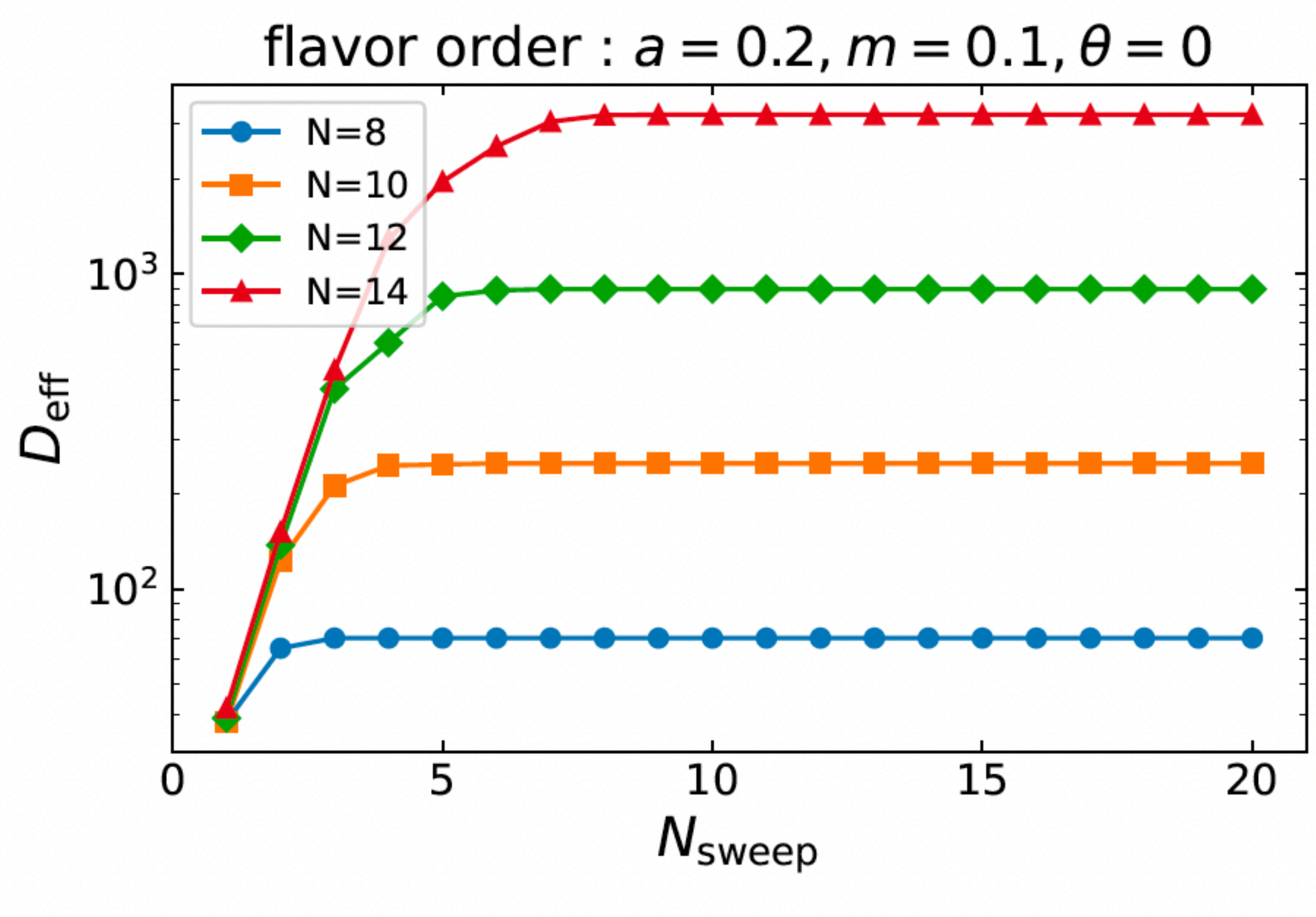

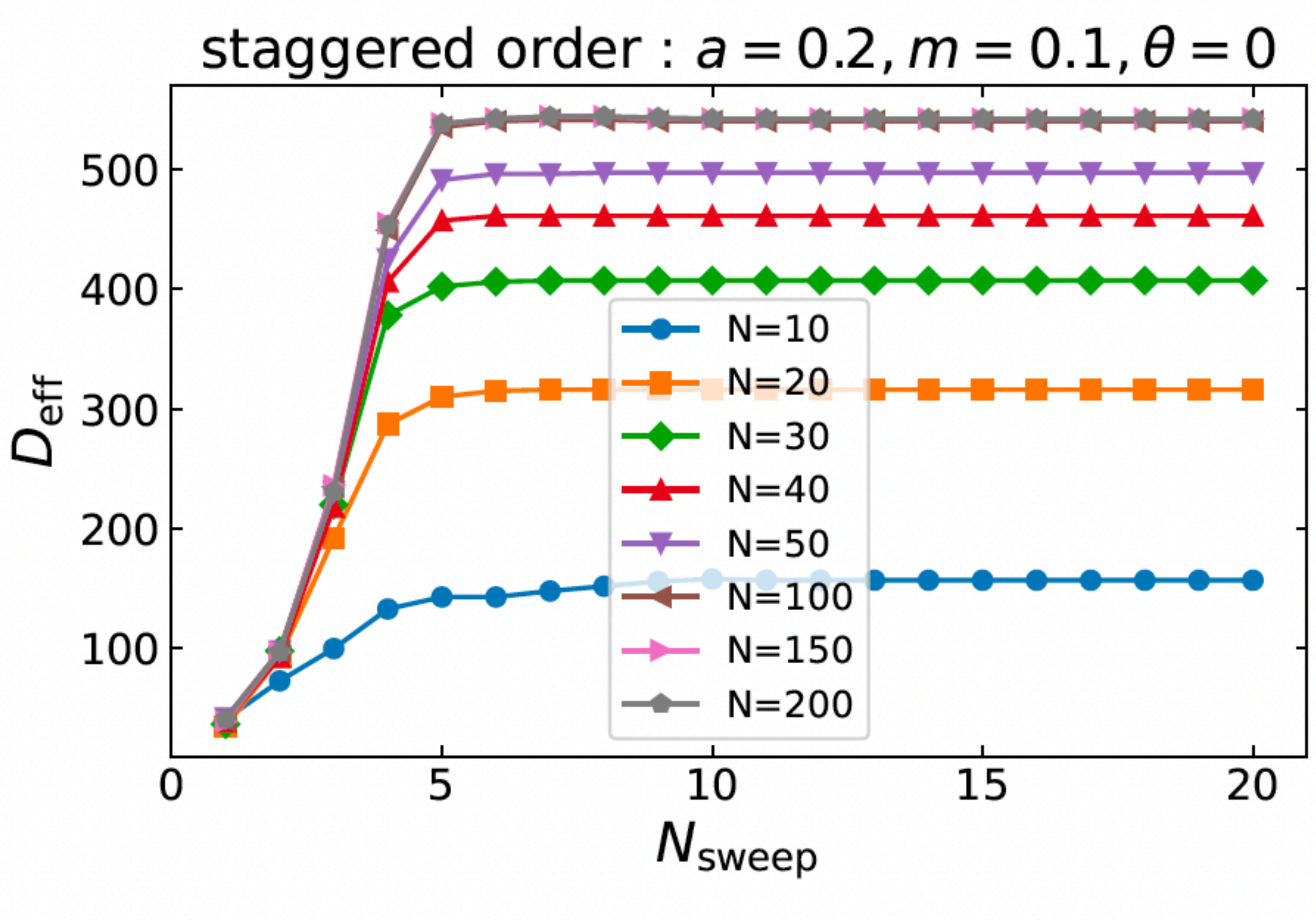

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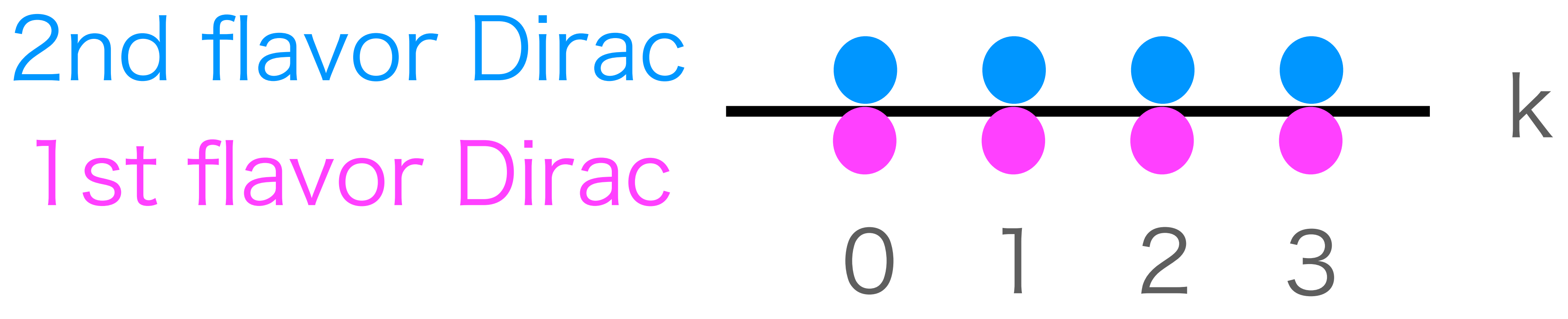
- Dirac fermion -> lattice fermion (staggered fermion)



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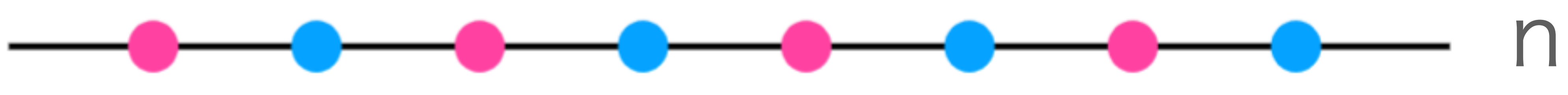
Staggered ordering (n=2k+(f-1))



- lattice fermion -> spin variable (Jordan-Wigner trans.)

Conditions for Nf -fermion

$$\{\chi^\dagger_{f,n}, \chi_{\tilde{f},m}\} = \delta_{f,\tilde{f}}\delta_{n,m}$$
$$\{\chi_{f,n}, \chi_{\tilde{f},m}\} = \{\chi^\dagger_{f,n}, \chi^\dagger_{\tilde{f},m}\} = 0$$

our choice

$$\chi_{1,n} = \frac{\sigma^x_{1,n} - \sigma^y_{1,n}}{2} \prod_{j=0}^{n-1} (-\sigma^z_{2,j}\sigma^z_{1,j})$$
$$\chi_{2,n} = \frac{\sigma^x_{2,n} - \sigma^y_{2,n}}{2} (-i\sigma^z_{1,n}) \prod_{j=0}^{n-1} (-\sigma^z_{2,j}\sigma^z_{1,j})$$

local op. (isospin and so on)
becomes only a few # of Pauli matrices

"Hadron" state in Nf=2 Schwinger model

- Prediction by analytical study (Coleman, 1976) at $\theta = 0$

(1)pion (Iso-triplet pseudo-scalar meson)

$$\pi = -i\left(\bar{\psi}_1\gamma^5\psi_1 - \bar{\psi}_2\gamma^5\psi_2\right)$$

$$J^{PG} = 1^{-+}\ (J_z = -1,0,1)$$

(2)sigma(Iso-singlet scalar meson)

$$\sigma = \bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2,\ J^{PG} = 0^{++}$$

(3)eta(Iso-singlet pseudo-scalar meson)

$$\eta = -i\left(\bar{\psi}_1\gamma^5\psi_1 + \bar{\psi}_2\gamma^5\psi_2\right),\ J^{PG} = 0^{--}$$

Quantum numbers:

$\mathbf{J}^2, J_z$ Isospin

associate with SU(2) flavor sym.

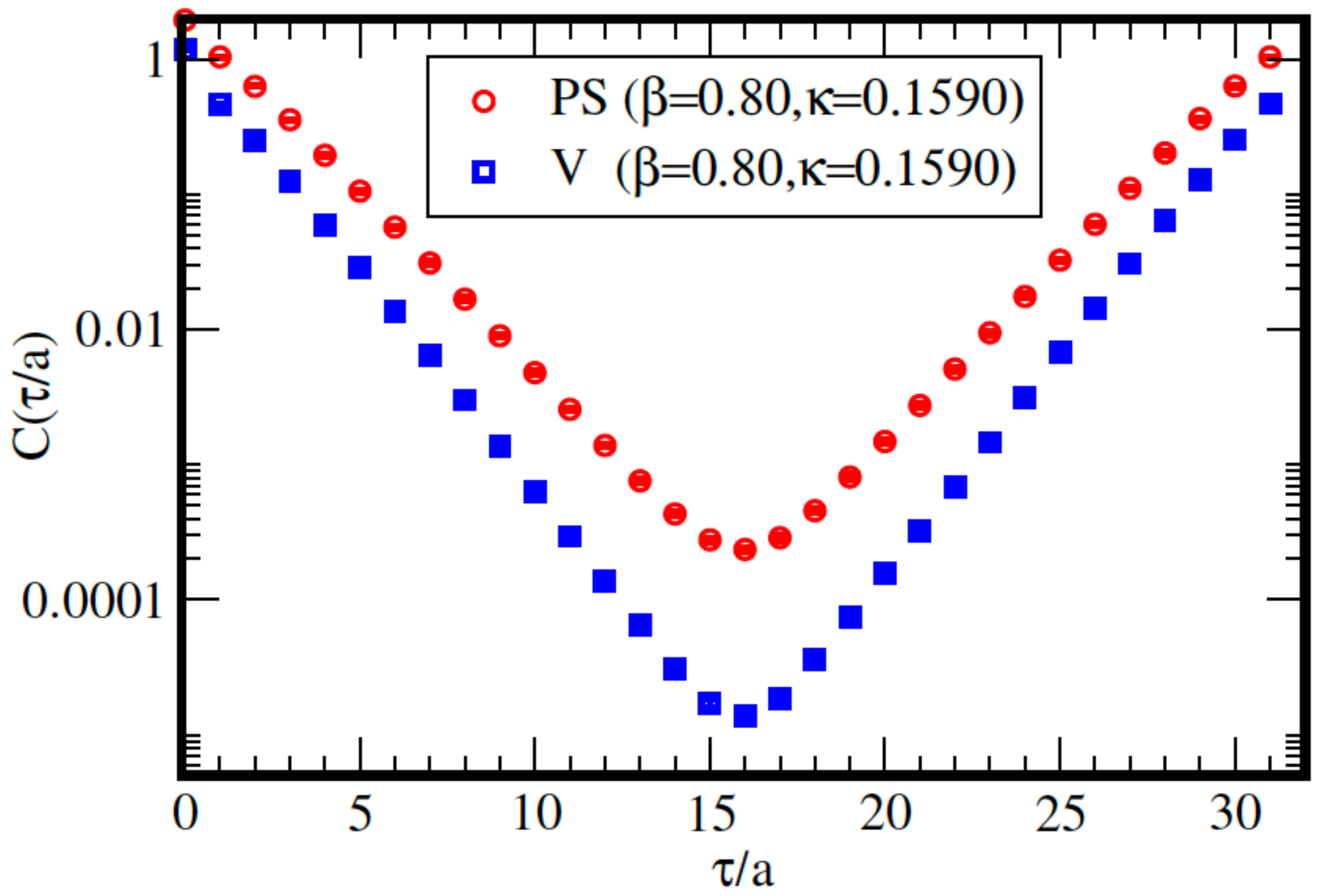
$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \to \mathcal{U}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

P: Parity

G-parity (generalized C.C.)

How can we calculate the mass spectra of hadrons?

- Conventional lattice MC: two-point correlation function

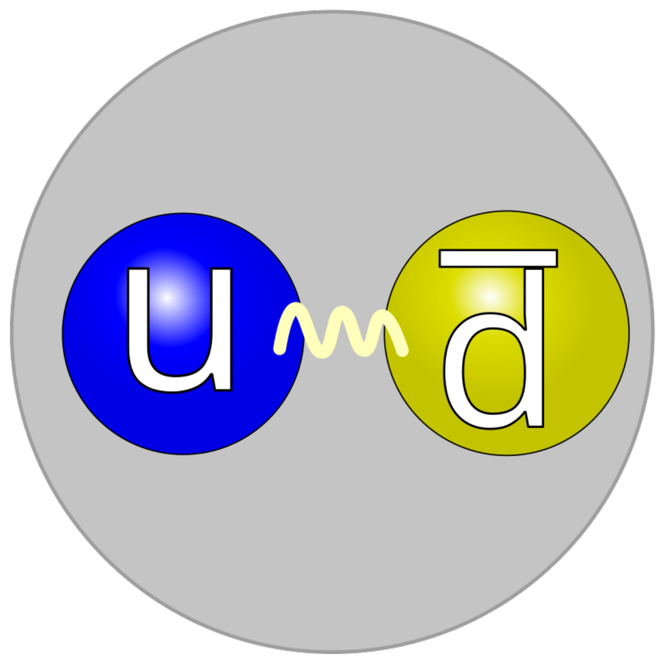


$$C(\tau) = \langle O(\tau) O(0) \rangle$$

$$\lim_{\tau\to\infty} C(\tau) \sim e^{-m\tau}$$

pion: $O = \bar{\psi}\gamma_5\psi$

rho meson: $O = \bar{\psi}\gamma_1\psi$



How we obtain them In Hamiltonian formalism

- At $\theta = \pi$, it is expected the model becomes (nearly) conformal field theory

- Shape of correlation function?

- How can we see the (almost) massless state?

3. Mass spectra in the Hamiltonian formalism

E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 11 (2023) 231

Three calculation methods (at $\theta = 0$)

E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 11 (2023) 231

(1) (Spatial) correlation-function scheme (conventional method)

$$\langle \mathcal{O}(r)\mathcal{O}(0)\rangle \propto K_0(Mr) \sim \frac{1}{\sqrt{r}}e^{-Mr}$$

ex.) $\pi = -i\left(\bar{\psi}_1\gamma^5\psi_1 - \bar{\psi}_2\gamma^5\psi_2\right)$

(2) One-point-function scheme

Calculate $\langle \mathcal{O}(x)\rangle = \langle \text{Vac.}\,|\,\mathcal{O}(x)\,|\,\text{Bdry}\rangle \sim e^{-Mx}$

one-point fn. = correlation fn. with edge state

By tuning the b.c. and value of θ , we obtain the desired meson state

(3) Dispersion-relation scheme

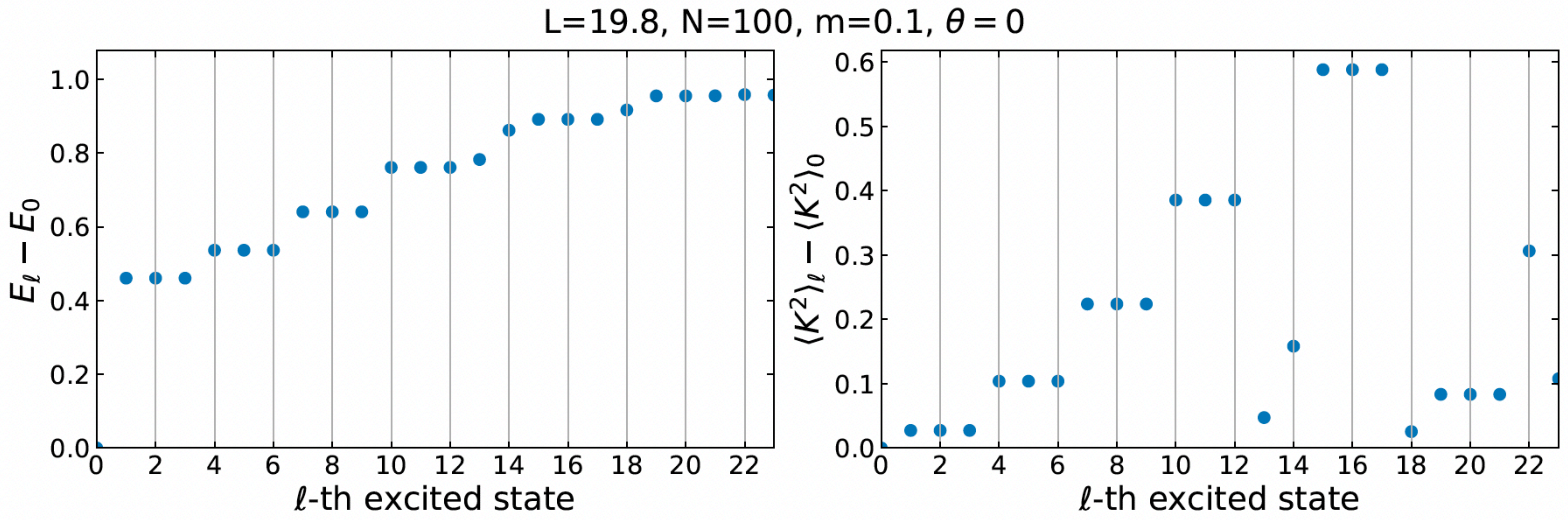
Construct excited states and measure energy, momentum and quantum numbers

excited state calc.: M.C. Banuls, K. Cichy, J.I. Cirac and K. Jansen (2013)

(3) Dispersion-relation scheme

MPS for ℓ-th excited state is given by the modified cost fn.: $H_{eff} = H + \lambda \sum_{k=0}^{\ell-1} |\psi_k\rangle\langle\psi_k|$

Upto 23rd excited state



Measure the quantum number (Iso-spin, G-parity, Parity) of generated MPS to identify each meson

(3)Momentum op. and Quantum number op.

- Momentum op.(flavor-dependent, $[\hat{k}_f, H] \neq 0$)

1st flavor
$$\hat{k}_{1,n} = \frac{i}{4a}(S^-_{1,n-1}Z_{2,n-1}Z_{1,n}Z_{2,n}S^+_{1,n+1} - S^+_{1,n-1}Z_{2,n-1}Z_{1,n}Z_{2,n}S^-_{1,n+1}),$$

2nd flavor
$$\hat{k}_{2,n} = \frac{i}{4a}(S^-_{2,n-1}Z_{1,n}Z_{2,n}Z_{1,n+1}S^+_{2,n+1} - S^+_{2,n-1}Z_{1,n}Z_{2,n}Z_{1,n+1}S^-_{2,n+1}).$$

- Isospin operator (flavor SU(2) sym.), $\mathbf{J}^2, J_z$

$$[H,\, J_z] = 0$$

$$J_z = \sum_{n=0}^{N-1} j_z(n) = \frac{1}{2}\sum_{n=0}^{N-1}\left(\chi^\dagger_{1,n}\chi_{1,n} - \chi^\dagger_{2,n}\chi_{2,n}\right) = \frac{1}{4}\sum_{n=0}^{N-1}(Z_{1,n} - Z_{2,n})$$

$$[H,\, \mathbf{J}^2] = \left[H,\, \left(\frac{1}{2}J_+J_- + \frac{1}{2}J_+J_- + J_z^2\right)\right] = 0$$

(3)Quantum number op.

- Charge conjugation (broken due to OBC and finite lattice spacing)

$$C := \prod_{f=1}^{N_f} \left(\prod_{n=0}^{N-1} \sigma^x_{f,n} \right) \left(\prod_{n=0}^{N-2} (\mathrm{SWAP})_{f;N-2-n,N-1-n} \right)$$

- Parity (broken due to OBC, N=even)

$$P := \prod_{f=1}^{N_f} \left(\prod_{j=0}^{N/2-1} \sigma^z_{f,2j+1} \right) \times \left(\prod_{n=0}^{N-2} (\mathrm{SWAP})_{f;N-2-n,N-1-n} \right) \left(\prod_{n=0}^{N/2-1} (\mathrm{SWAP})_{f;n,N-1-n} \right)$$

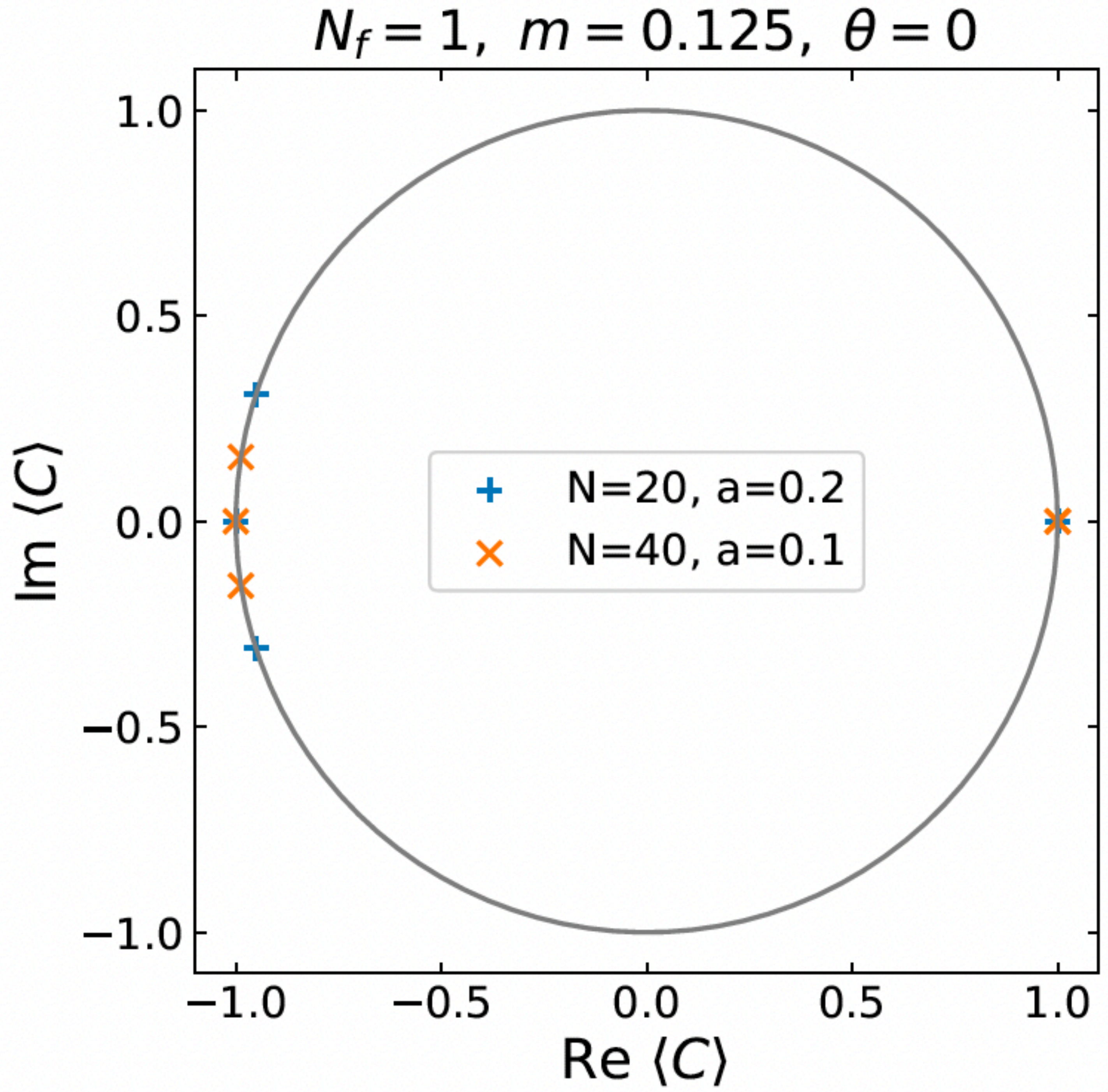
1 site translation

x <->L-x

p <-> ap flip

- G-Parity (commute with iso-spin)

$$G := Ce^{i\pi J_y},$$



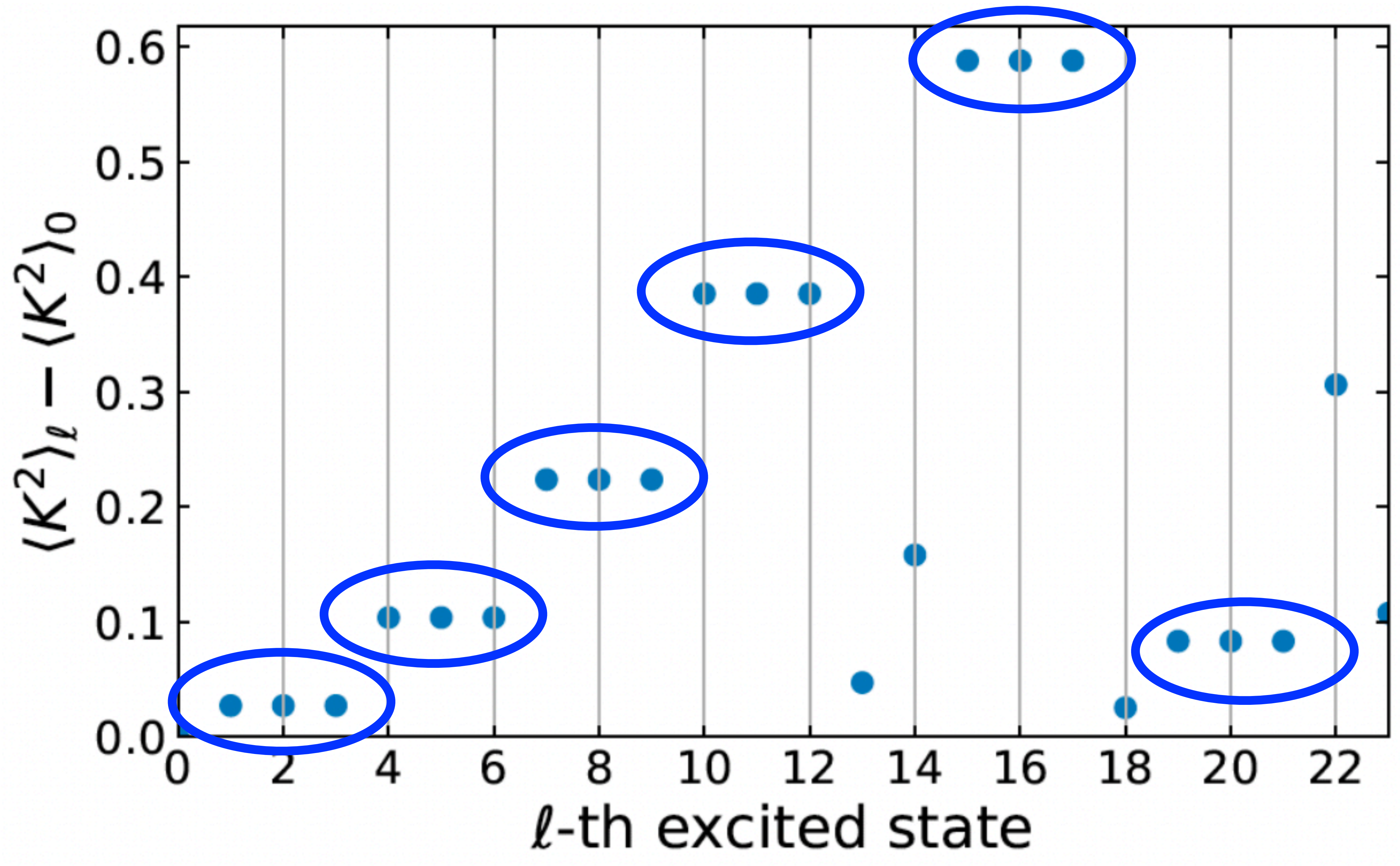
Free theory w/ PBC

In cont. lim., $\langle C \rangle = \pm 1$

the sign of Re$\langle C \rangle$ is

a remnant of exact C

(3)Results: iso-triplet channel



ℓ	$\boldsymbol{J}^2$	J_z	G	P
1	2.00000004	0.99999997	0.27872443	-6.819×10^{-8}
2	2.00000012	-0.00000000	0.27872416	-6.819×10^{-8}
3	2.00000004	-0.99999996	0.27872443	-6.819×10^{-8}
4	2.00000007	0.99999999	0.27736066	7.850×10^{-8}
5	2.00000006	0.00000000	0.27736104	7.850×10^{-8}
6	2.00000009	-0.99999998	0.27736066	7.850×10^{-8}
7	2.00000010	1.00000000	0.27536687	-8.838×10^{-8}
8	2.00000002	0.00000000	0.27536702	-8.837×10^{-8}
9	2.00000007	-0.99999998	0.27536687	-8.838×10^{-8}
10	2.00000007	0.99999998	0.27356274	9.856×10^{-8}
11	2.00000005	0.00000001	0.27356277	9.856×10^{-8}
12	2.00000007	-0.99999999	0.27356274	9.856×10^{-8}
15	1.99999942	0.99999966	0.27173470	-1.077×10^{-7}
16	2.00000052	0.00000000	0.27173482	-1.077×10^{-7}
17	2.00000015	-1.00000003	0.27173470	-1.077×10^{-7}
19	2.00009067	1.00004377	0.27717104	-3.022×10^{-8}
20	2.00002578	-0.00000004	0.27717020	-3.023×10^{-8}
21	2.00003465	-1.00001622	0.27717104	-3.023×10^{-8}

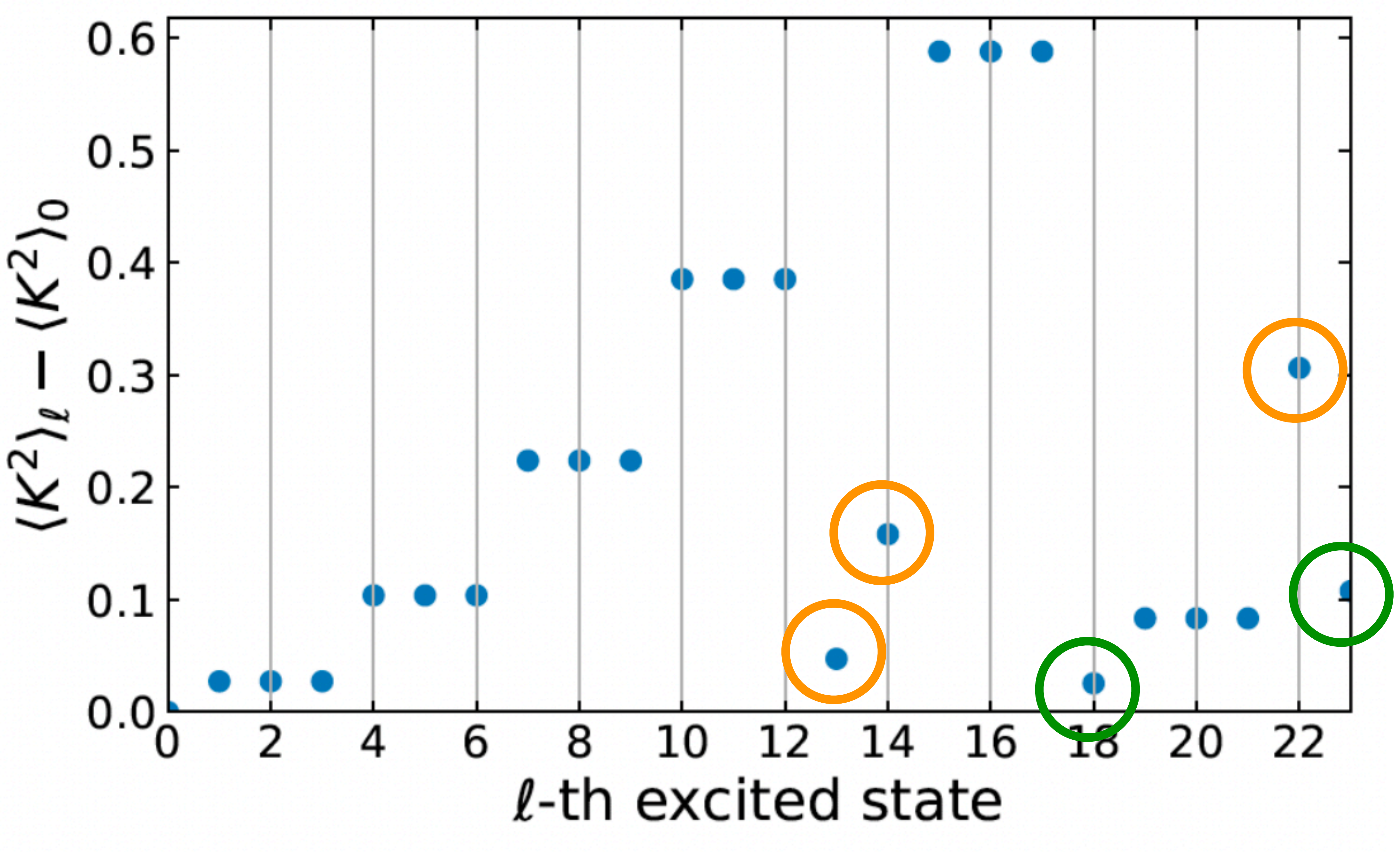
zero-mode

$P < 0$

J=1 $J_z = \pm 1$ $G > 0$ $P < 0$

pion : $J^{PG} = 1^{-+}$

(3)Results: iso-singlet channel



ℓ	$\boldsymbol{J}^2$	J_z	G	P
0	0.00000003	-0.00000000	0.27984227	3.896×10^{-7}
13	0.00000003	0.00000000	0.27865844	1.273×10^{-7}
14	0.00000003	0.00000000	0.27508176	-2.765×10^{-8}
18	0.00000028	0.00000006	-0.27390909	-6.372×10^{-7}
22	0.00001537	0.00000115	0.26678987	7.990×10^{-8}
23	0.00003607	-0.00000482	-0.27664779	5.715×10^{-7}

zero-mode

$P > 0$

zero-mode

$P < 0$

J=0 $J_z = 0$ $G > 0$ $P > 0$

sigma meson : $J^{PG} = 0^{++}$

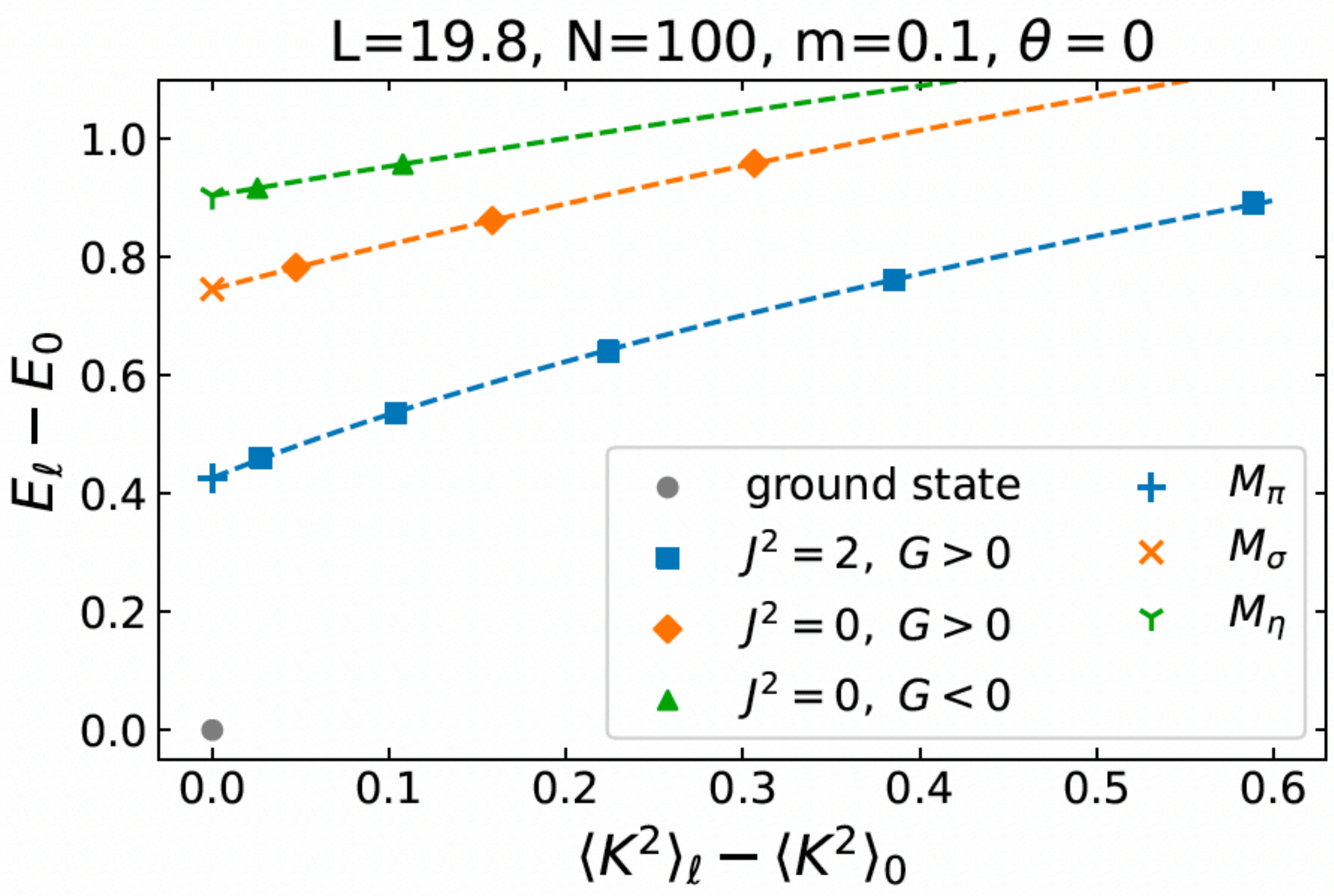
J=0 $J_z = 0$ $G < 0$ $P < 0$

eta meson : $J^{PG} = 0^{--}$

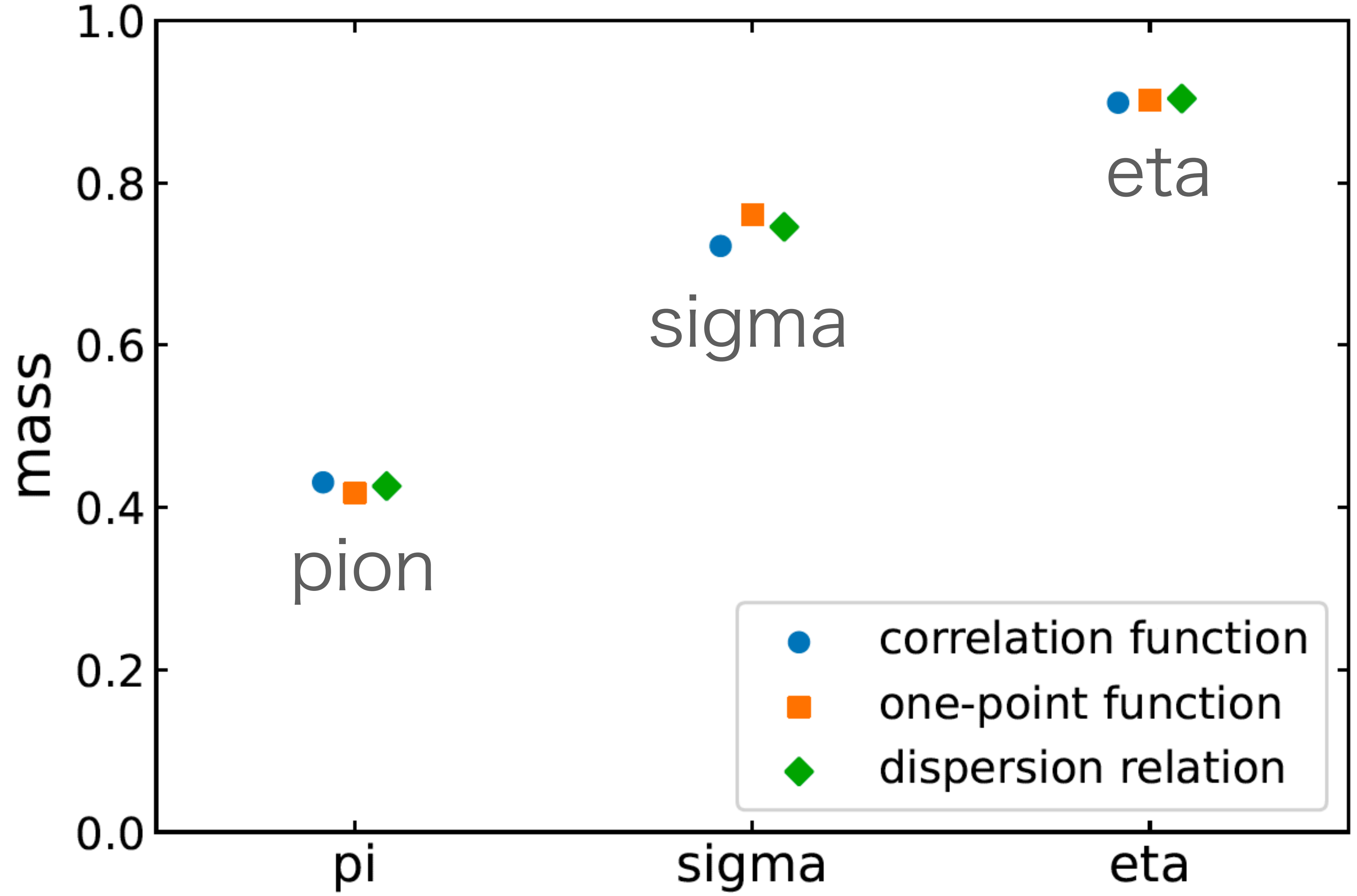
(3)Results of dispersion-relation scheme

Plot ΔE_ℓ against ΔK_ℓ^2 for each meson

Fit the data using $\Delta E = \sqrt{M^2 + b^2 \Delta K^2}$



Three meson masses obtained by three methods



Theoretical predictions

Coleman(1976), Dashen et al. (1975)

✓ $M_\pi < M_\sigma < M_\eta$: U(1) problem

✓ $M_\eta = \mu + O(m)$ $(\mu = g\sqrt{N_f/\pi} \sim 0.8, \quad m = 0.1)$

✓ $M_\sigma / M_\pi = \sqrt{3}$ (within 5% deviation)

Data obtained by directly measure the 2-pt. fn. or bulk 1-pt. of composite state.
Data of dispersion-relation scheme, naturally obtain these composite states as a low-lying state.

4. $\theta \neq 0$

E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 09 (2024) 155

At $\theta \neq 0$ (theoretical things)

- Sign problem appears in Lattice Monte Carlo

- operator mixing between Scalar and Pseudo-Scalar ops. occurs,
  $\mathcal{O} = C_S S + C_{PS} PS$

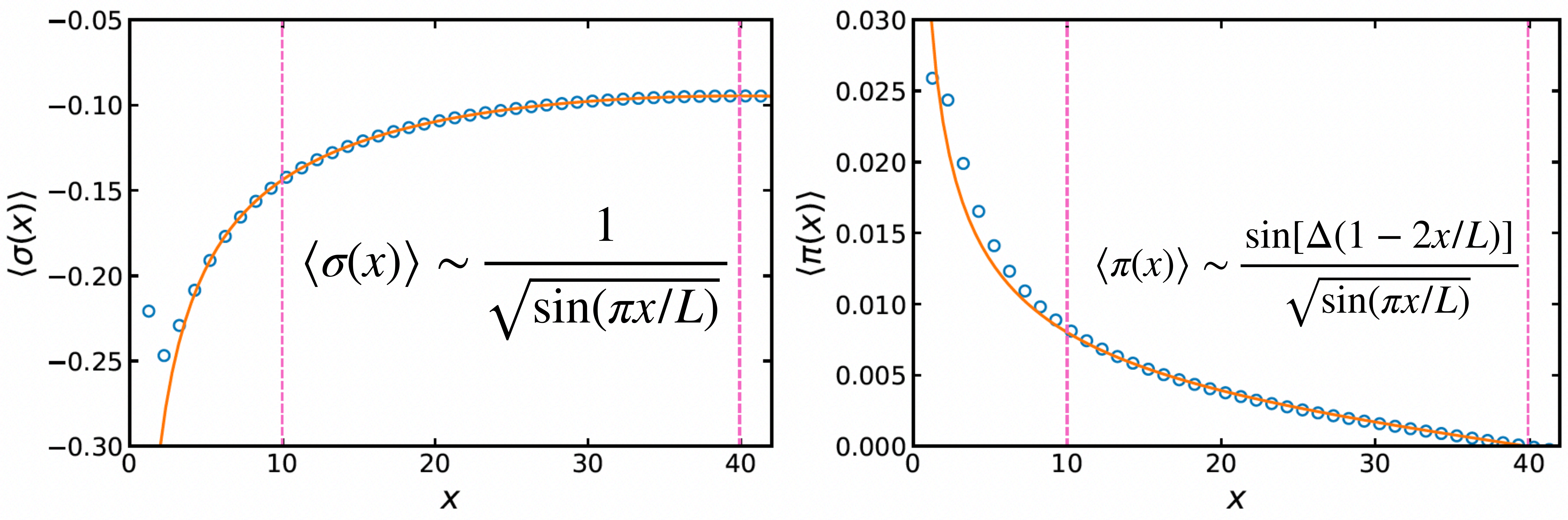
- loss of quantum numbers (G-parity is broken, η-decay is no longer prohibited)

- decay mode: η meson -> 2 pions
  η meson is not a stable particle

- (almost) conformal theory at $\theta = \pi$ (level-1, SU(2) WZW theory)
  DMRG is hard, shape of correlation fn. is changed

One-point fn. scheme at $\theta = \pi$ (near CFT)

- Analytic form of one-point fn. with Dirichlet b.c.



cf.) 2-flavor Schwinger model at $\theta = \pi$

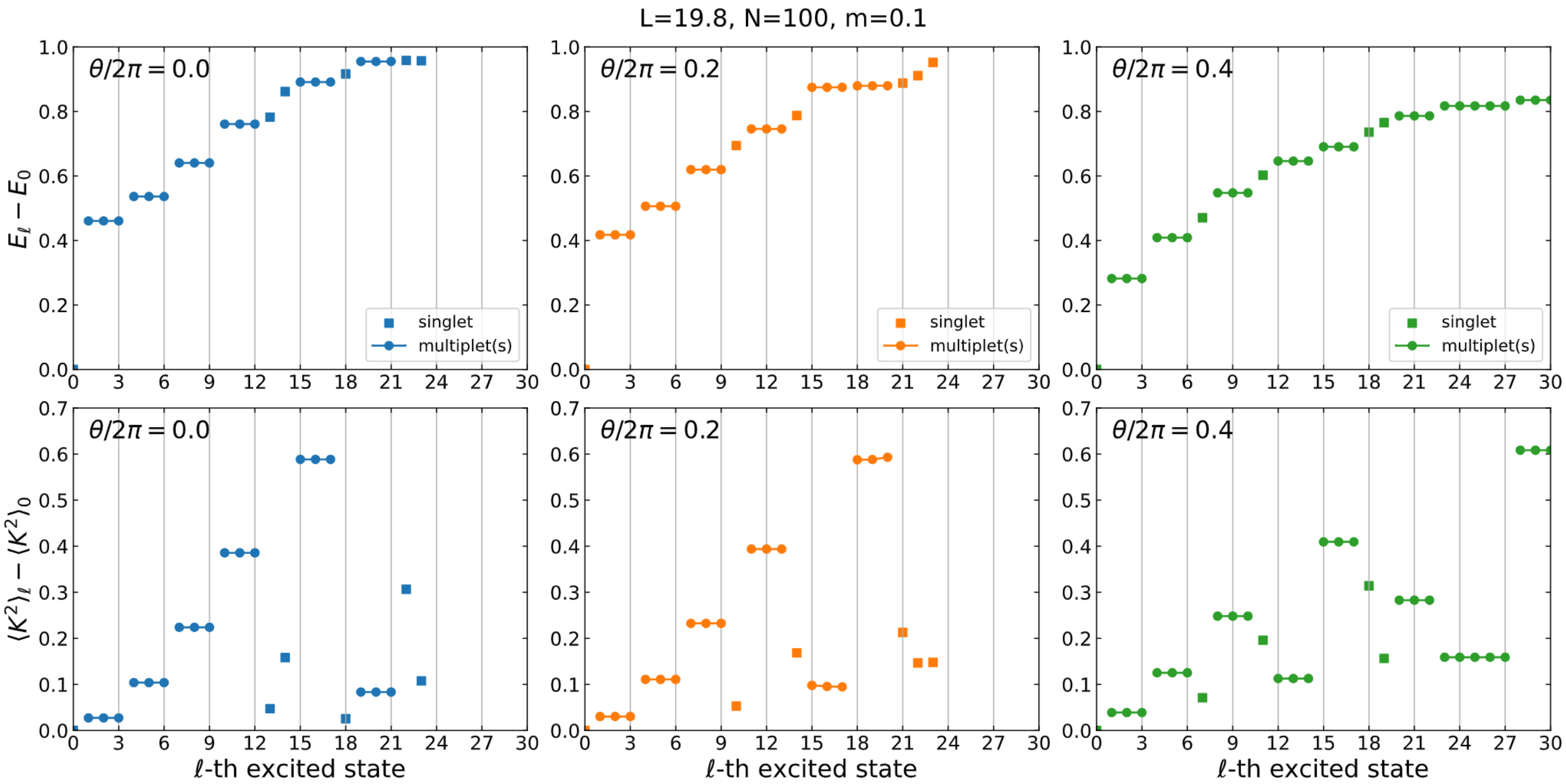
a small mass gap $\sim e^{-Ag^2/m^2}$ remains (Not exact CFT if $m \neq 0$)

R. Dempsey et al., 2023

Dispersion-relation scheme in $\theta \neq 0$

E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 09 (2024) 155

- Can be applied to the $\theta \neq 0$ regions straightforwardly



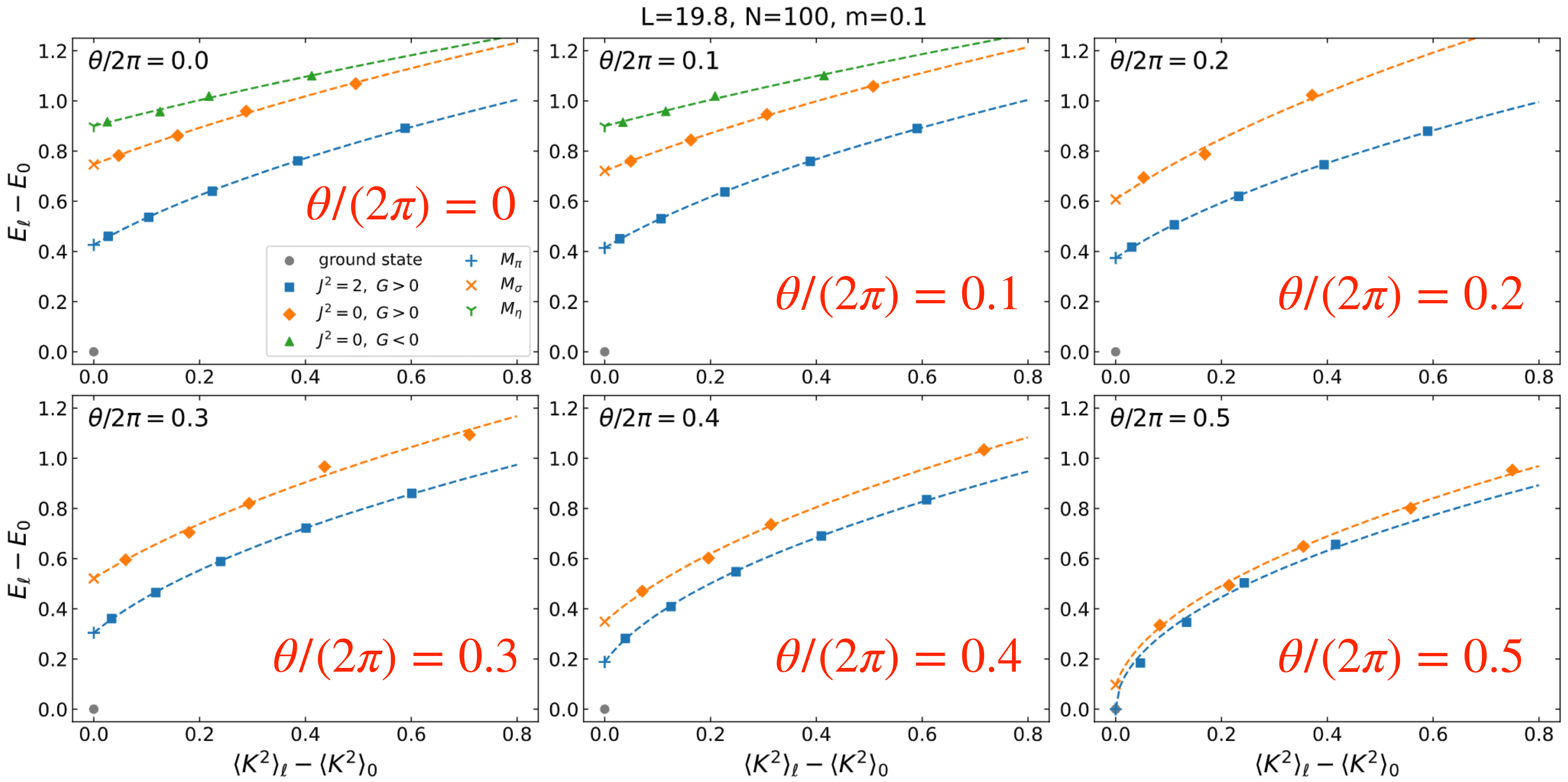
Iso-triplet must be pion

We cannot distinguish between eta and sigma

Dispersion-relation scheme in $\theta \neq 0$

E.I., Akira Matsumoto, Yuya Tanizaki, JHEP 09 (2024) 155

- Fit the data for each meson using $\Delta E = \sqrt{M^2 + b^2 \Delta K^2}$



eta meson

sigma meson

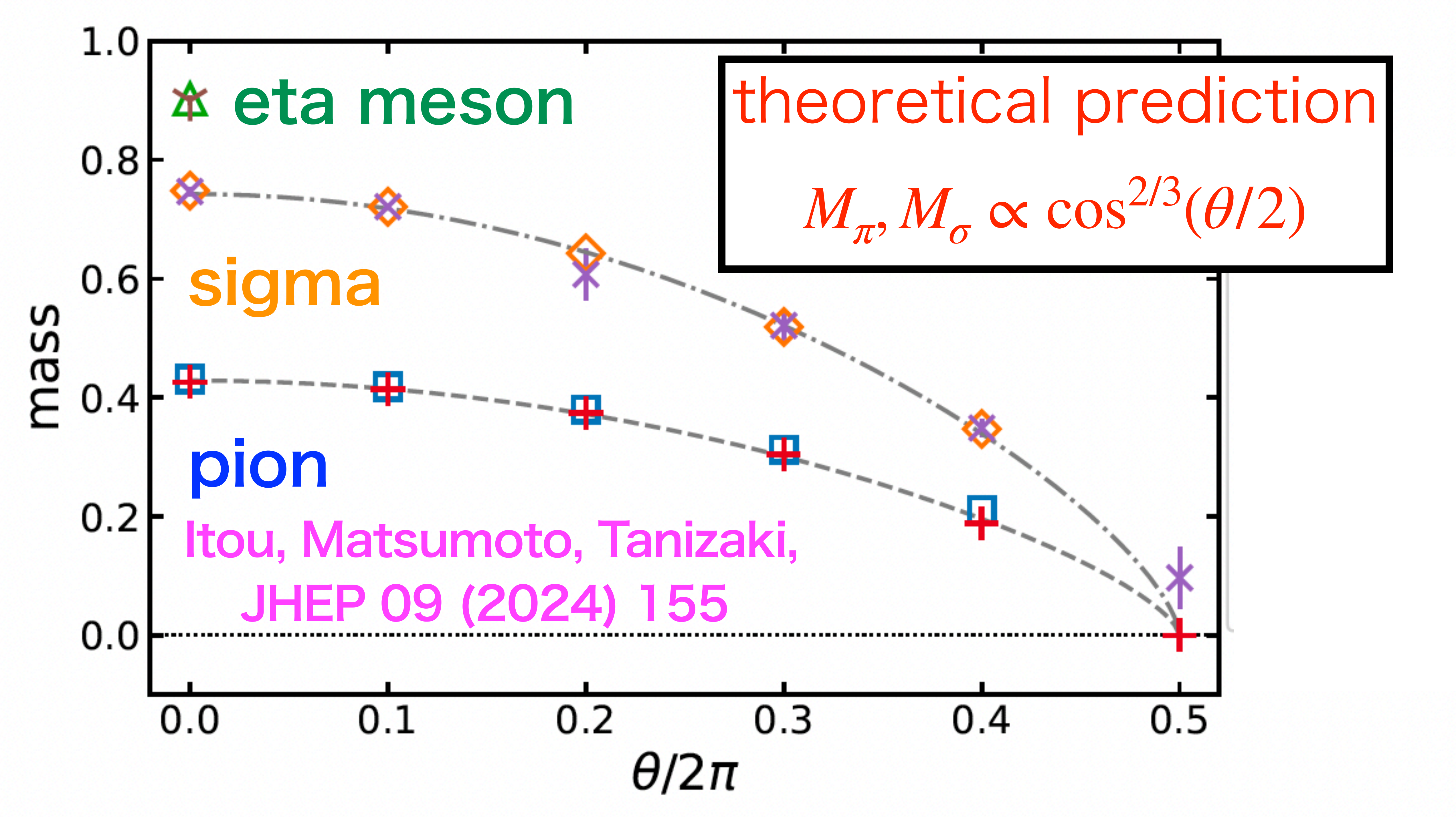
pion

η disappear $\theta/2\pi > 0.2$

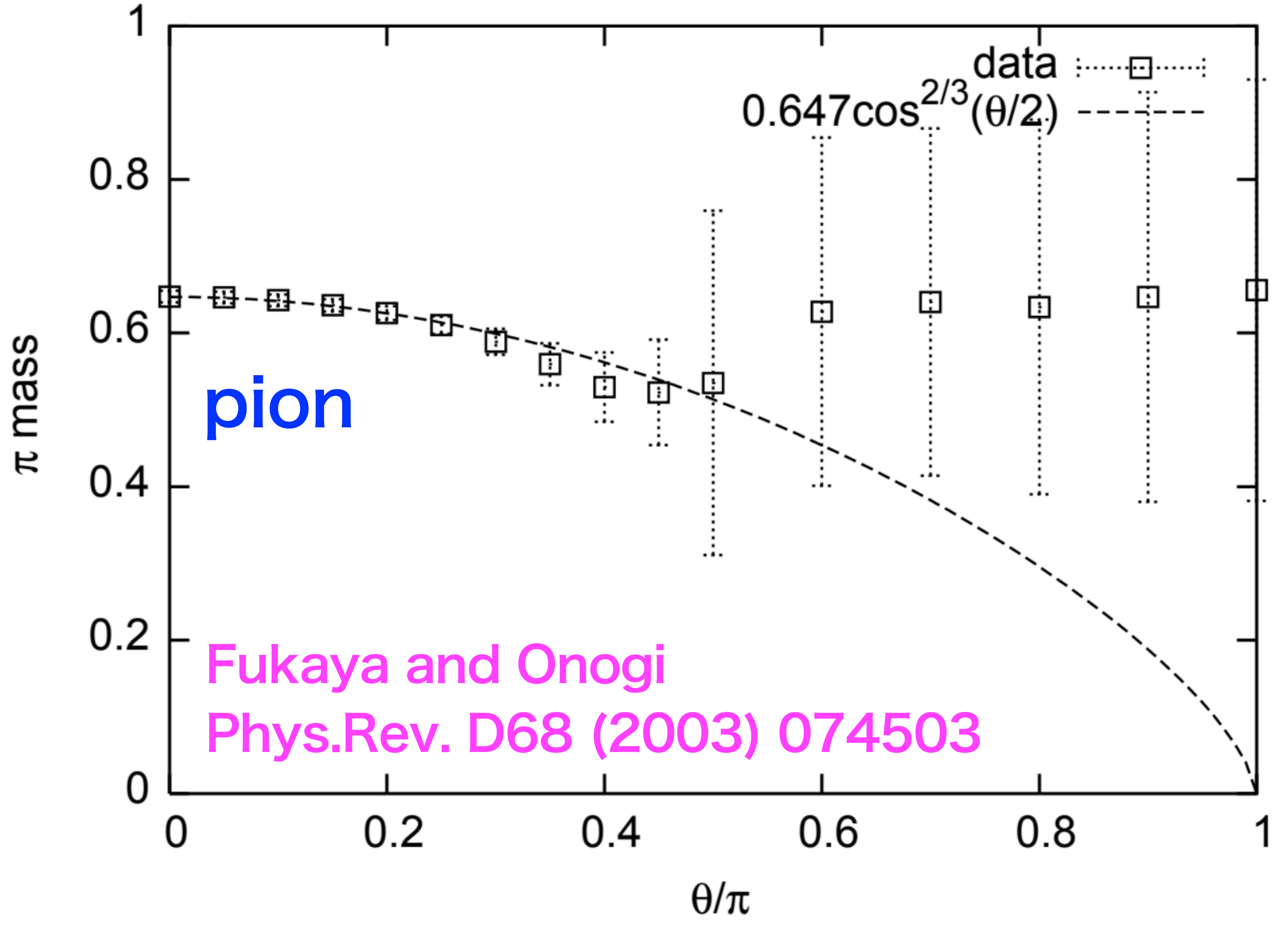
sigma (singlet) and pion (triplet) are degenerating at $\theta = \pi$

Comparison w/ previous work in Lattice MC

Tensor network based on Hamiltonian



Monte Carlo based on Lagrangian (w/ improved techniques)



- Straightforwardly apply to $\theta = \pi$ regime (even near CFT)
- Higher states are heuristically found
- Consistent with several theoretical predictions

- In large θ, the signal is very noisy because of the sign problem
- Difficult to find a heavy η-meson and σ-meson

Summary and future directions

Summary and future prospect

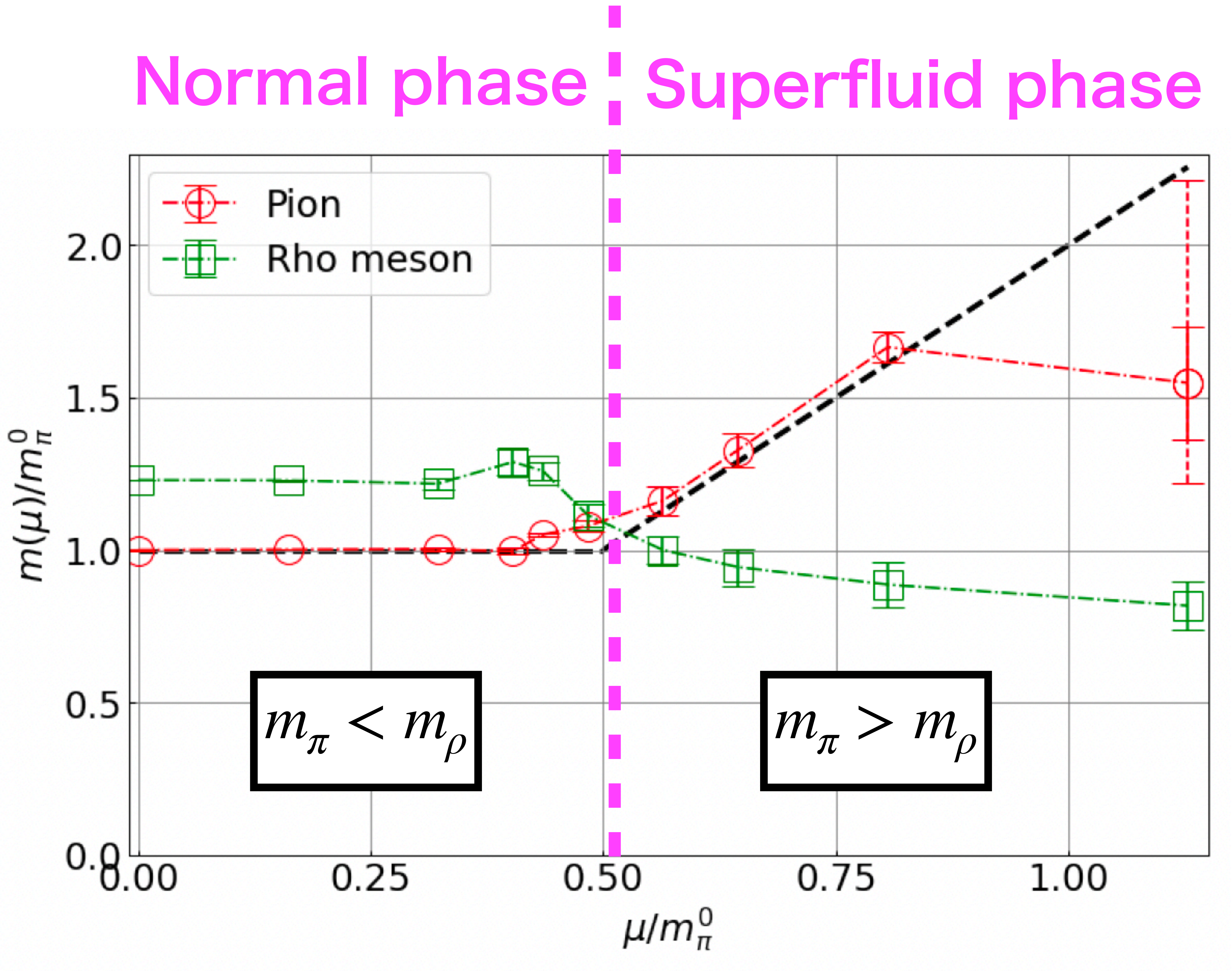
- We reformulate the particle theory (L to H) and search for new calculation methods to fit tensor network and/or quantum computation

- New strategy expands the range of theories that can be explored
  In particular, the theory with the sign problem in conventional MC
  ** Our methods for mass spectra can apply to the finite-density QCD

Neutron star
(high-density QCD)

Pions are not the lightest particles?

Lattice MC results for dense 2color QCD



K.Murakami, D.Suenaga, K.Iida, EI, PoS LATTICE2022 (2023) 154

- Lattice MC simulation of dense QCD is extremely difficult because of the sign problem
- Ab initio calculations in gauge theory that avoid the sign problem for toy model of QCD (2color QCD)
- It is shown that rho is lighter than pion in high-density regime

Analytical study: cf.) Hatsuda-Lee

Future directions

- Toward QCD = 3+1 dim. SU(3) gauge theory

  cf.) Schwinger model = 1+1 dim. U(1) gauge theory

- Implementation of QCD Hamiltonian still has some issues
  - How to deal with gauge dof. w/ infinite dimensional Hilbert space
  - Practical computation methods for TN and QC

	cond-mat model	<	Schwinger model	≈	quantum chemistry	≪ QCD
# of T-gate	$\sim 10^8$		$\sim 10^{12}$		$\sim 10^{12}$	$\sim ??$

K.Sakamoto, H.Morisaki, J.Haruna, EI, K.Fujii, K.Mitarai, Quantum 8 (2024) 1474

- Stay tuned for these progresses!

backup

Short summary of scheme

Three calculation methods for hadron spectra in Hamiltonian formalism

(1)correlation-function scheme

👍 applicability to broad class of theories

😥 sensitive to the bond dimension (DMRG) —> 😊 quantum computation

(2)one-point-function scheme

👍 need to increase neither the bond dimension nor the system size L

😥 need theoretical knowledge

only the lowest state with the same quantum number of boundary state

(3)dispersion-relation scheme

👍 obtain various states heuristically / directly see wave functions

😥 computational cost to generate excited states

Two calculation methods (at $\theta \neq 0$)

(1) 2-pt. correlation-function for mixed op. and find the mixing angle

$C(\tau) = \langle O(\tau) O(0) \rangle$, for $O = C_S S + C_{PS} PS$

+ (1') One-point-function scheme

one-point fn. = correlation fn. with source state

(SPT phase, at θ iso-singlet state / at $\theta + 2\pi$ iso-triplet state)

near $\theta = \pi$, a shape of corr. fn. change to CFT-like

(2) Dispersion-relation scheme

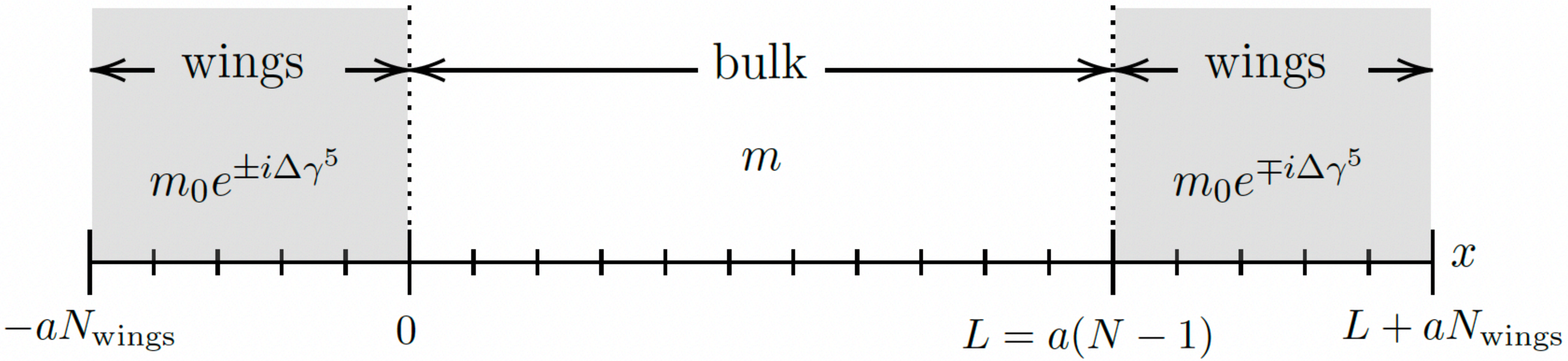
Construct excited states and measure energy, momentum and (approximate) quantum numbers

exact sym. is only isospin, e.g. iso-singlet and iso-triplet

(1) Improvement for 1pt fn. scheme

- Introduce a wings regime and putting flavor-dependent masses to extract desired state

ex.) Lattice setup to extract the pions (triplet states)



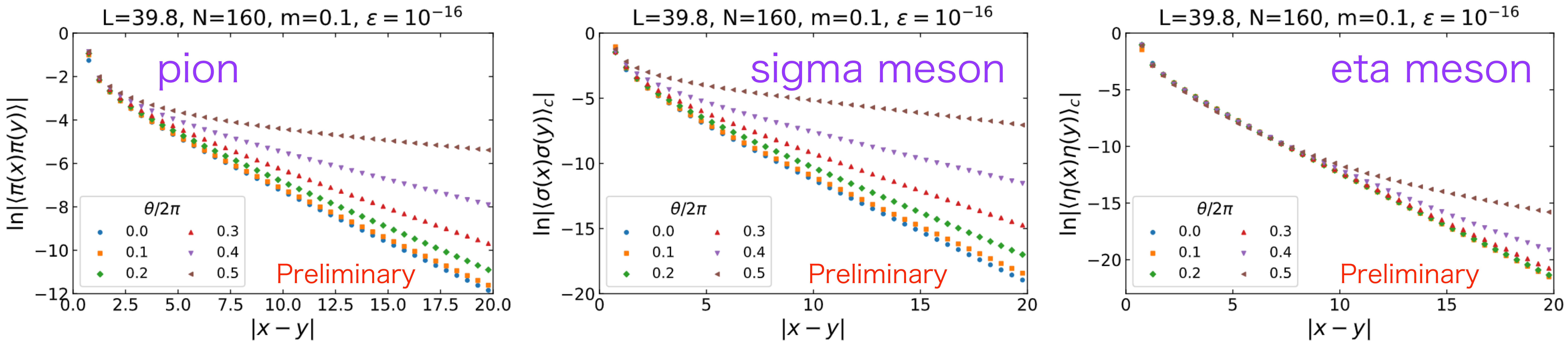
(1) correlation fn. scheme

Operator mixing between Scalar and Psuedo-Scalar ops. occurs, $\mathcal{O} = C_S S + C_{PS} PS$

Diagonalise 2pt. correlation matrix: $C_\pm(x,y) = \begin{pmatrix} \langle S_\pm(x) S_\pm(y)\rangle_c & \langle S_\pm(x) PS_\pm(y)\rangle_c \\ \langle PS_\pm(x) S_\pm(y)\rangle_c & \langle PS_\pm(x) PS_\pm(y)\rangle_c \end{pmatrix}$

-------> $C_+(x,y) = R_+^{\mathrm{T}} \begin{pmatrix} \langle \sigma(x)\sigma(y)\rangle_c & 0 \\ 0 & \langle \eta(x)\eta(y)\rangle_c \end{pmatrix} R_+$ for iso-singlet mesons

-------> $C_-(x,y) = R_-^{\mathrm{T}} \begin{pmatrix} ** & 0 \\ 0 & \langle \pi(x)\pi(y)\rangle_c \end{pmatrix} R_-$ for iso-triplet mesons

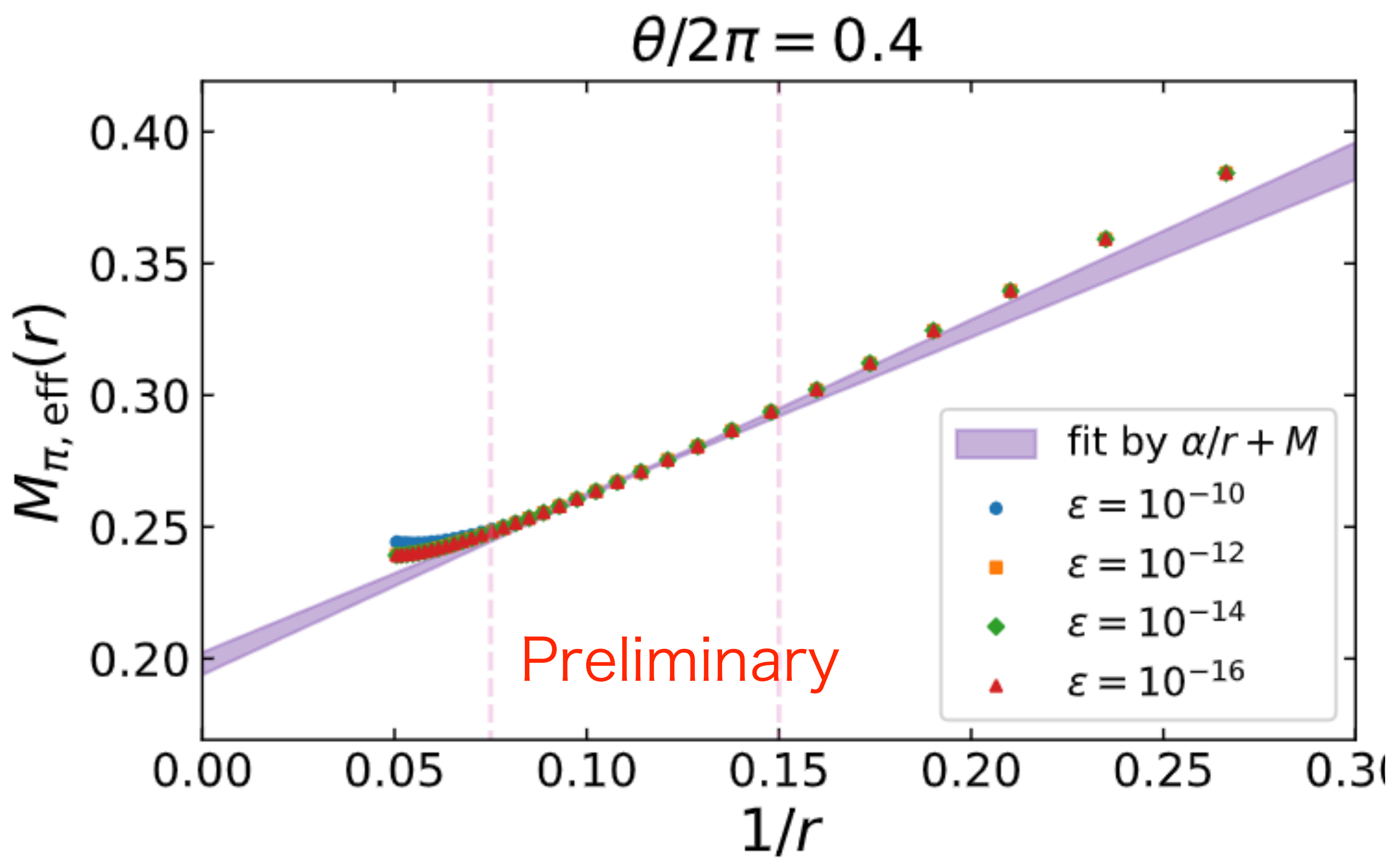


The slope is slower in the larger θ.

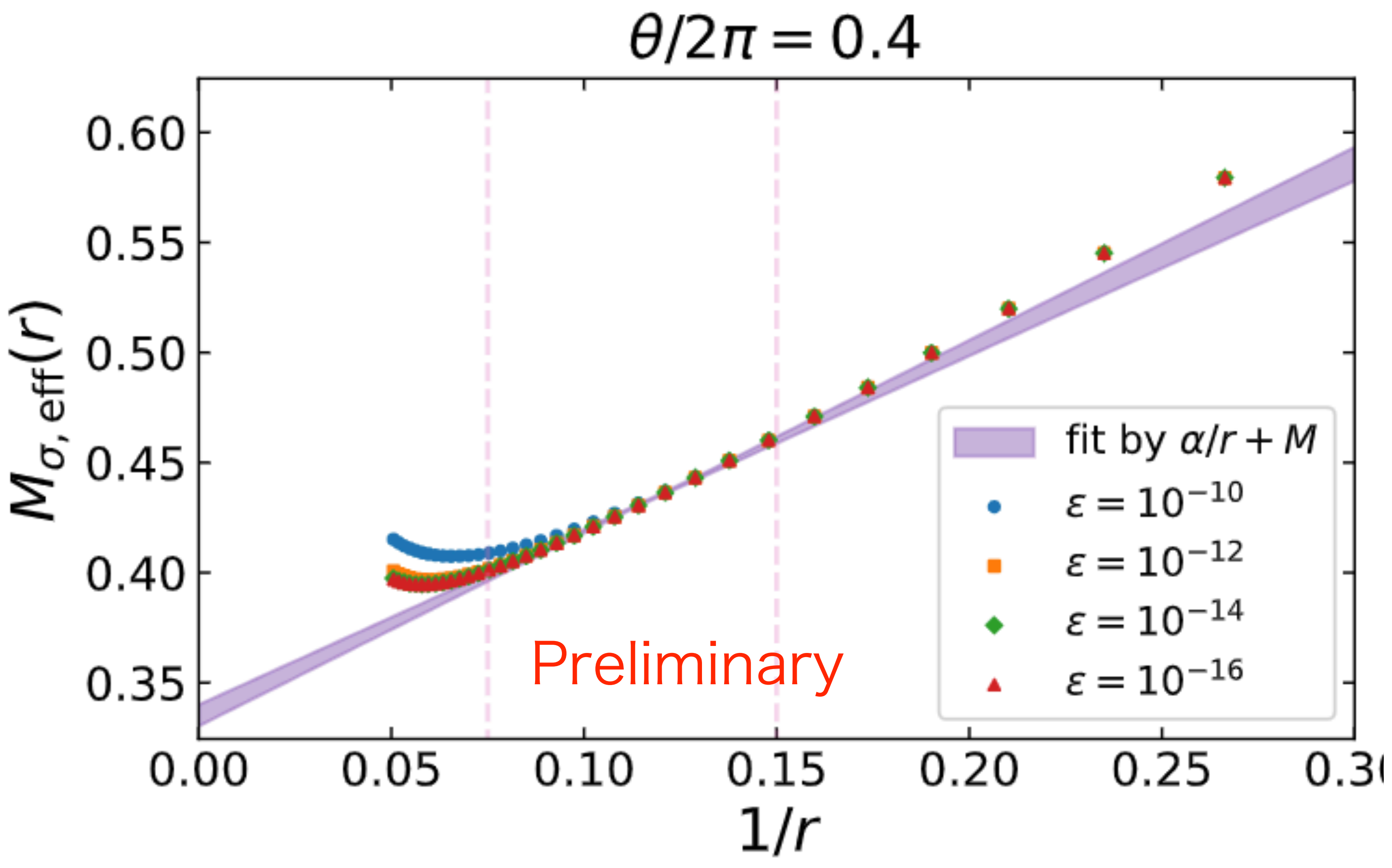
(1) correlation fn. scheme

- Effective mass as a function of 1/r at large θ (large mixing angle, near conformal)

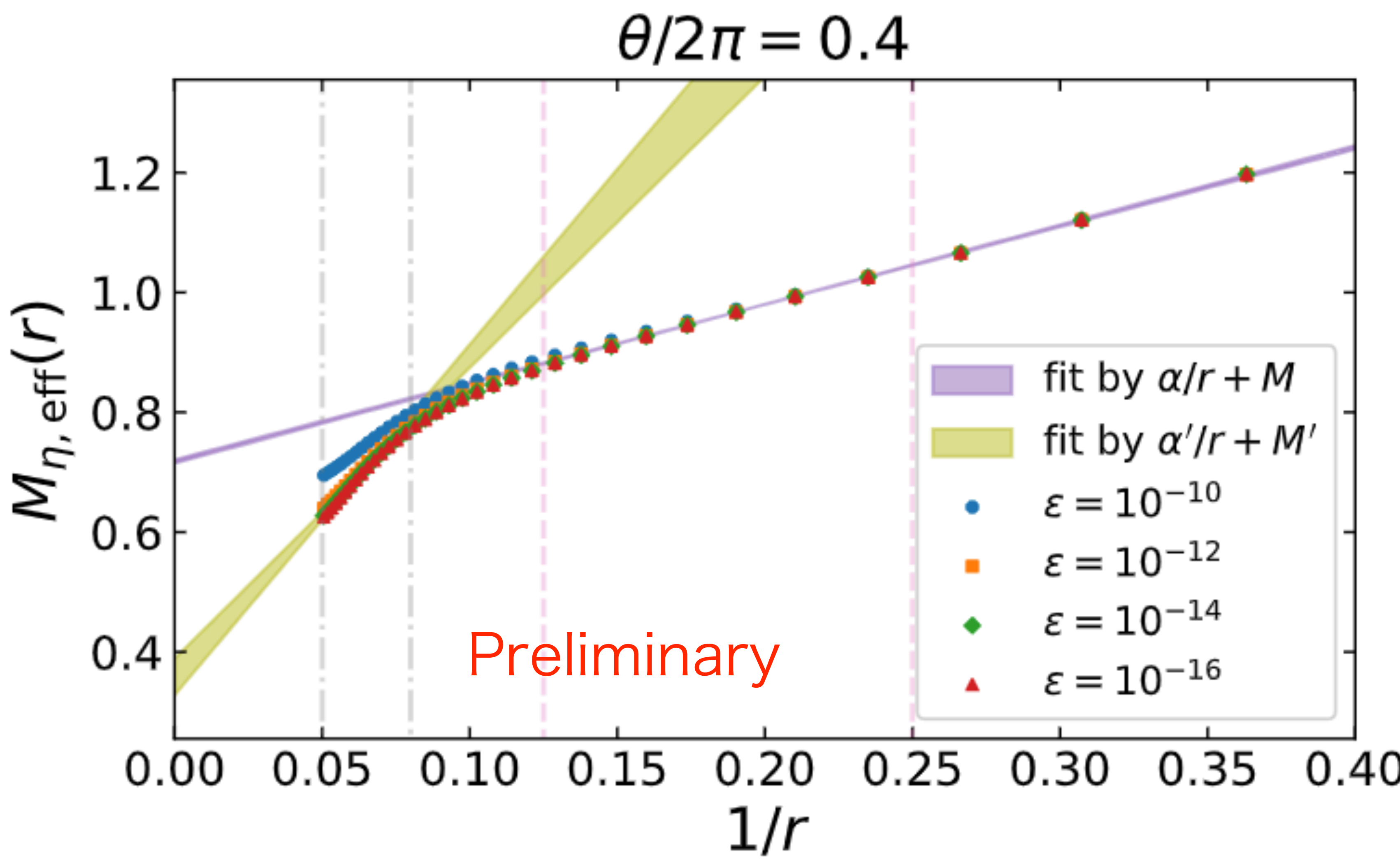
pion



sigma meson



eta meson



The mass becomes smaller (pion and sigma)

Eta meson decays into a lighter mode over long distances.

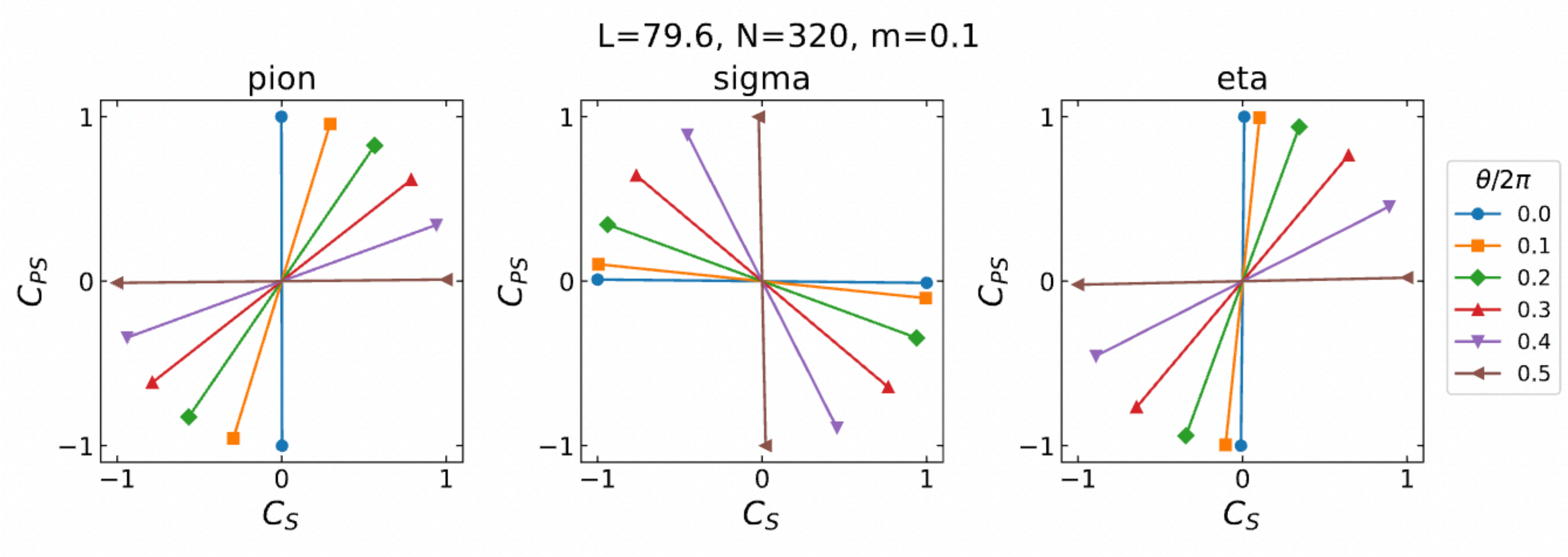
(1') one-point fn. scheme in $\theta < \pi$

- Need to increase neither the bond dimension nor the system size L

- To find the mixing of ops., $\mathcal{O} = C_S S + C_{PS} PS$, we use the rotation matrices by the 2-pt. fn. scheme : $\langle \mathcal{O}(x) \rangle \propto e^{-Mx}$ for $\theta < \pi$

$$\begin{pmatrix} * \\ \pi(x) \end{pmatrix} = R_- \begin{pmatrix} S_-(x) \\ PS_-(x) \end{pmatrix}$$

$$\begin{pmatrix} \sigma(x) \\ \eta(x) \end{pmatrix} = R_+ \begin{pmatrix} S_+(x) \\ PS_+(x) \end{pmatrix}$$

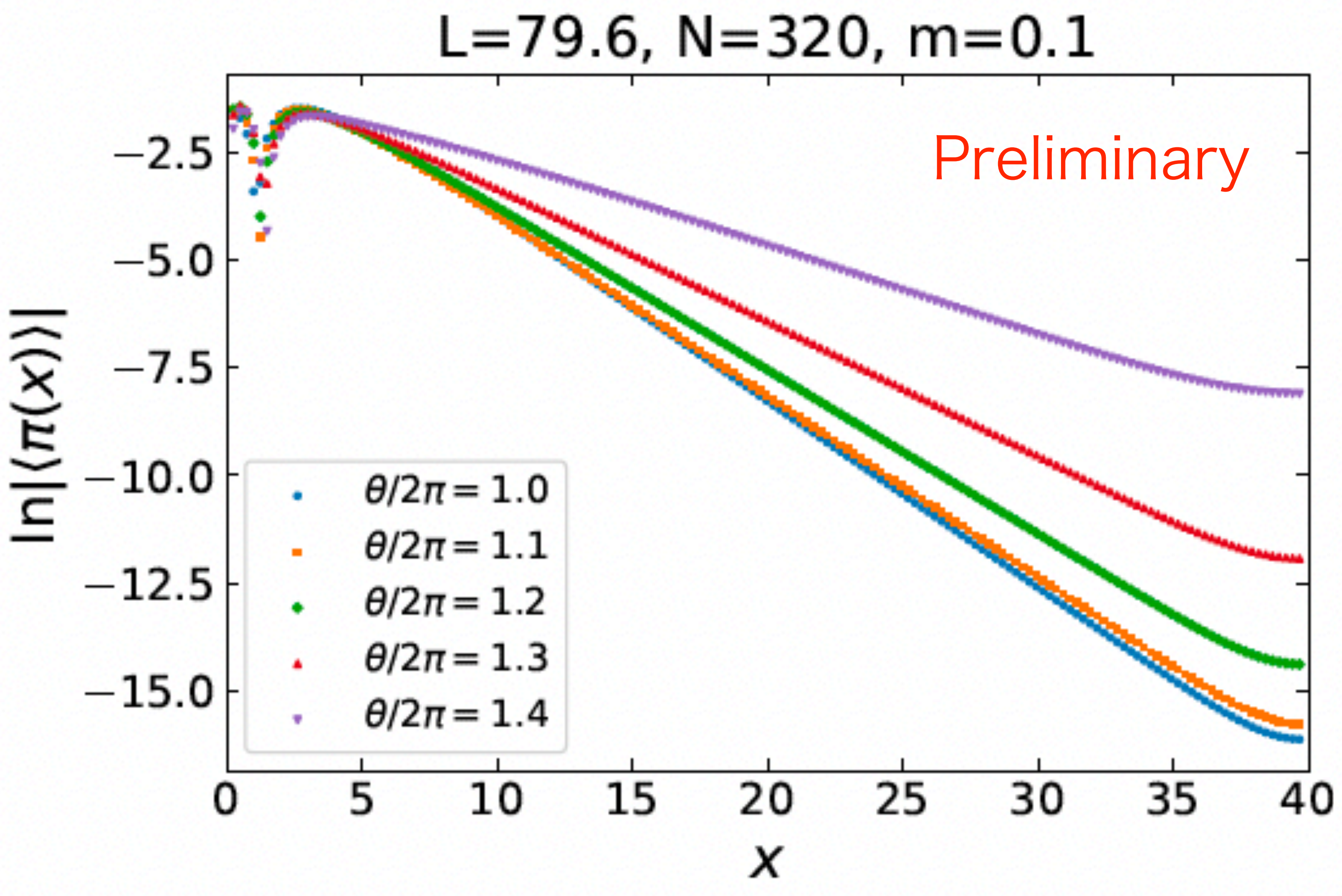


(1') one-point fn. scheme in $\theta < \pi$

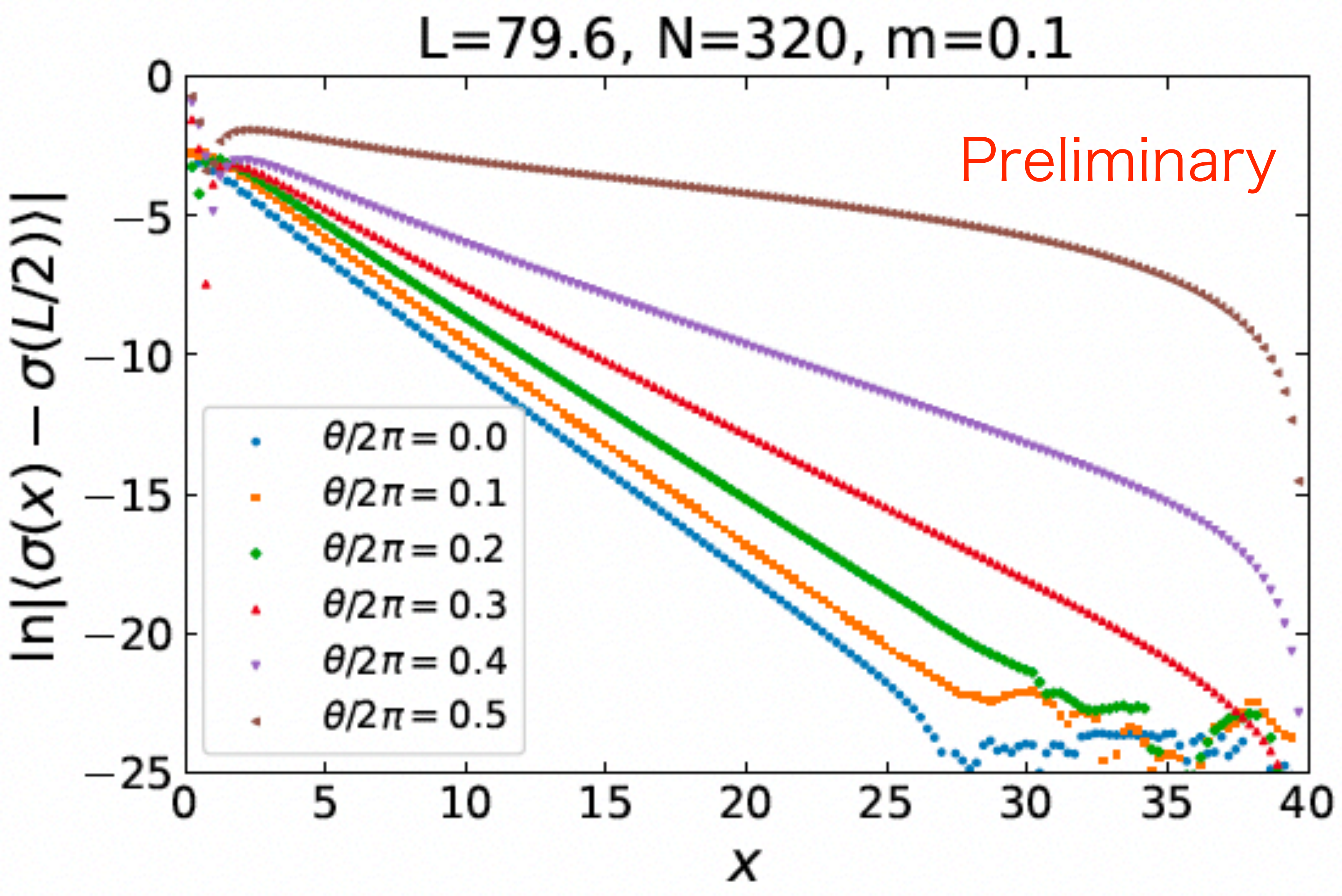
- Need to increase neither the bond dimension nor the system size L

- No longer an independent scheme
  To find the mixing of ops., we use the mixing matrix by the 2-pt. fn.
  scheme : $\langle \mathcal{O}(x) \rangle \propto e^{-Mx}$ for $\theta < \pi$

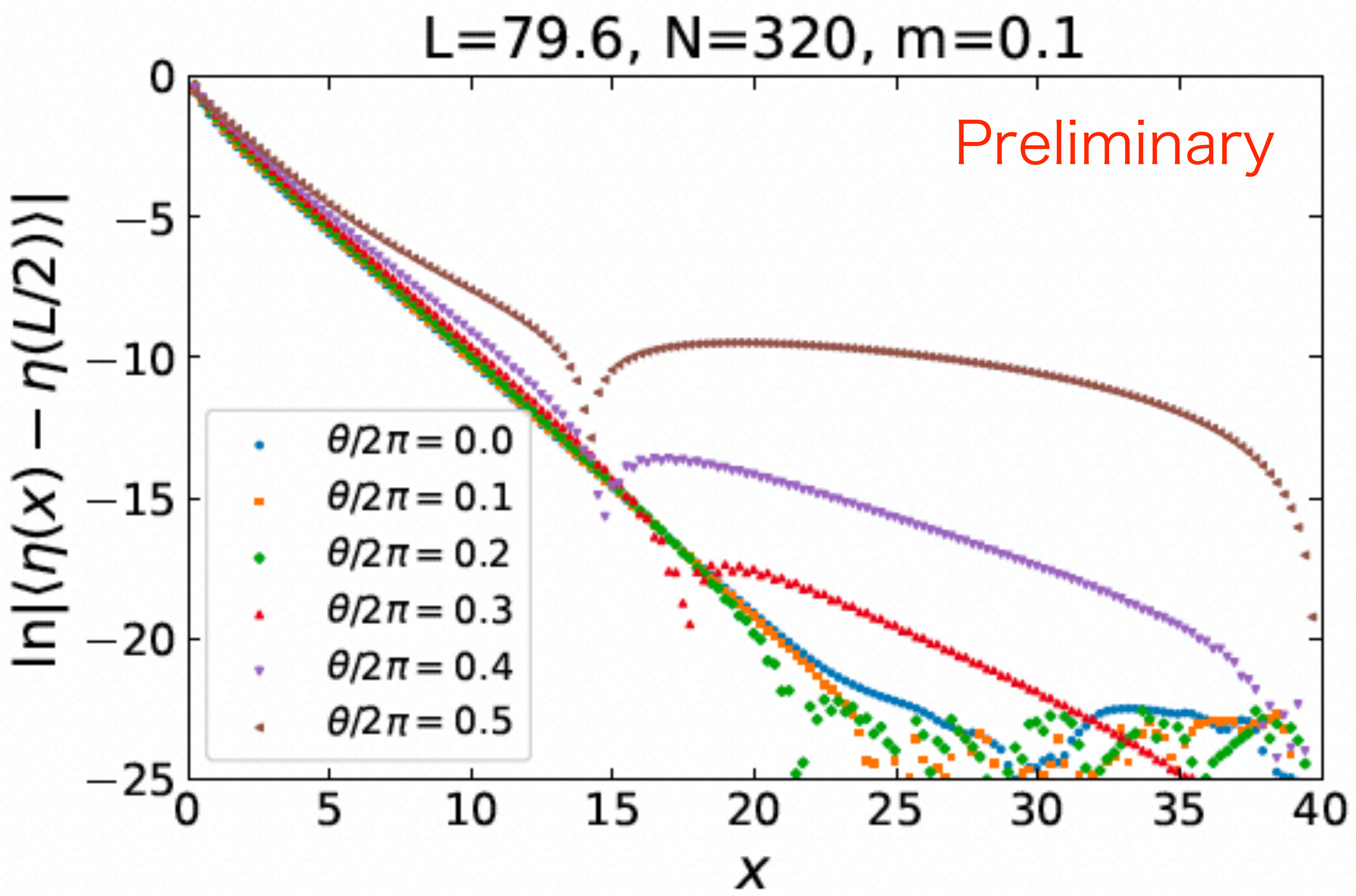
pion



sigma meson



eta meson



$\theta = \pi$ is difficult

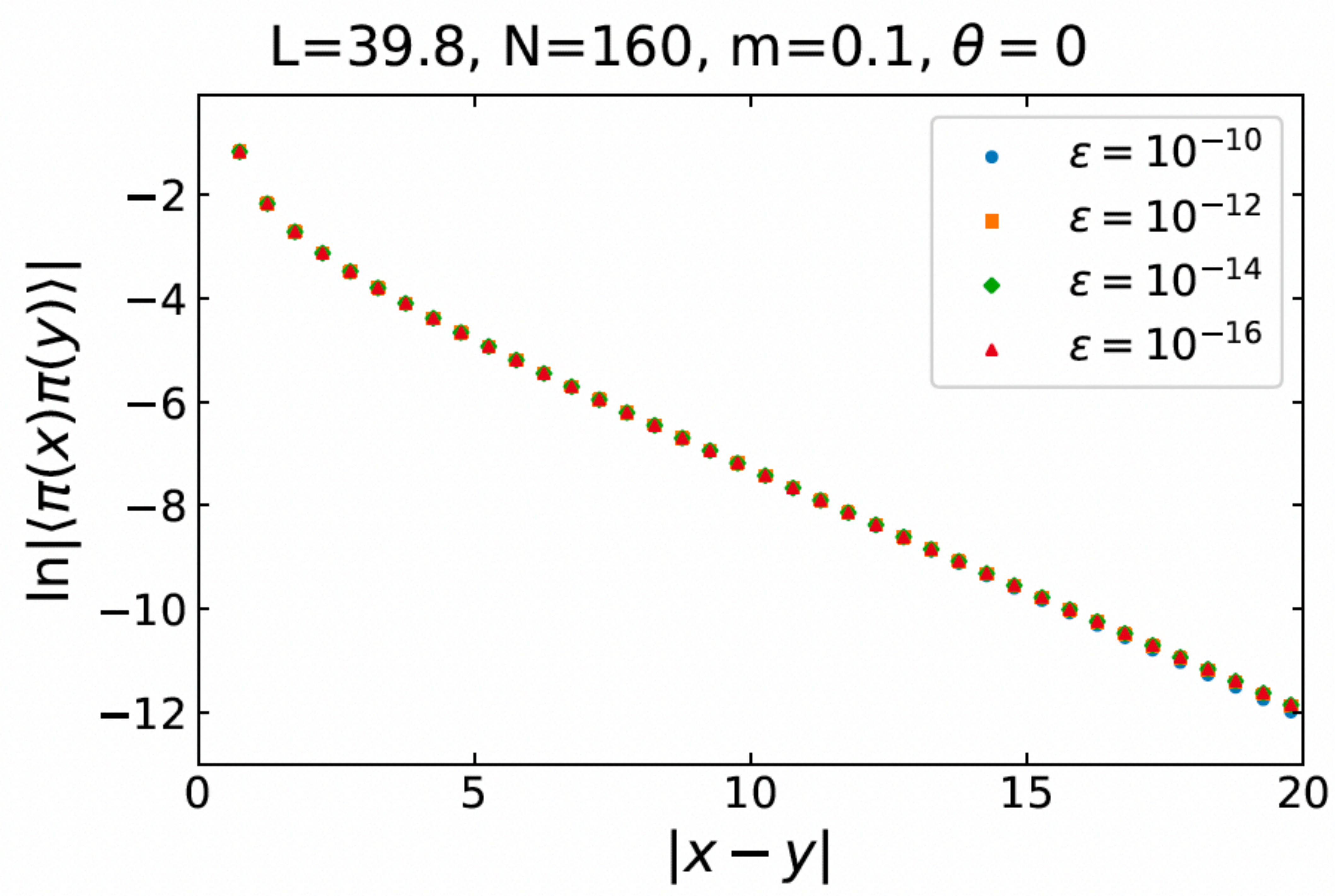
In middle x regime, there is a cusp

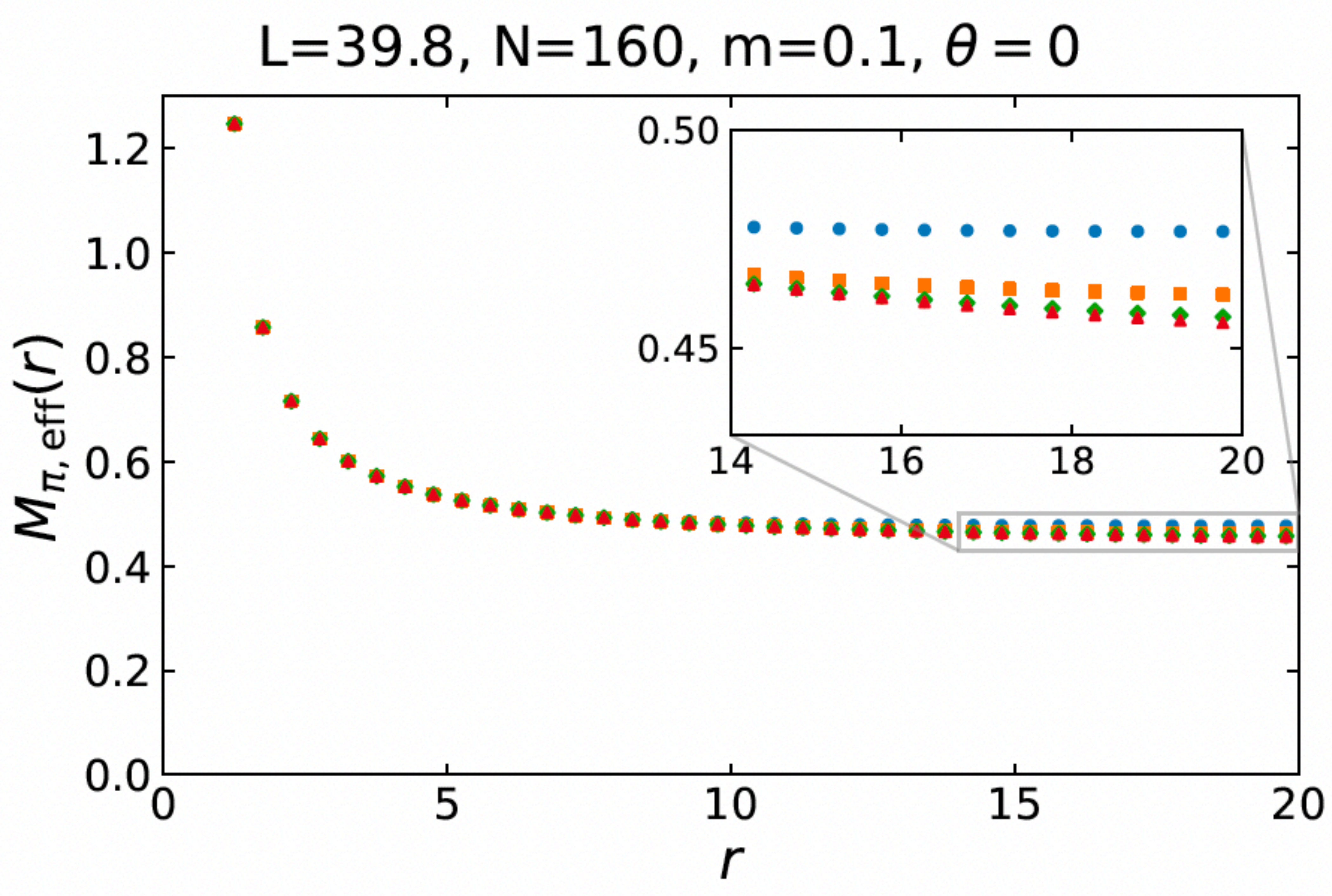
(1) (Spatial) correlation-function scheme

Effective mass

log plot of $C_\pi(r) = \langle \pi(r)\pi(0) \rangle \sim e^{-M_{\rm eff} r}$

$$\tilde{M}_{\pi,\rm eff}(r) = -\frac{1}{2a} \log \frac{C_\pi(r+2a)}{C_\pi(r)}$$





Plateau of effective mass = pion mass ??

High precision calculation shows a slope....

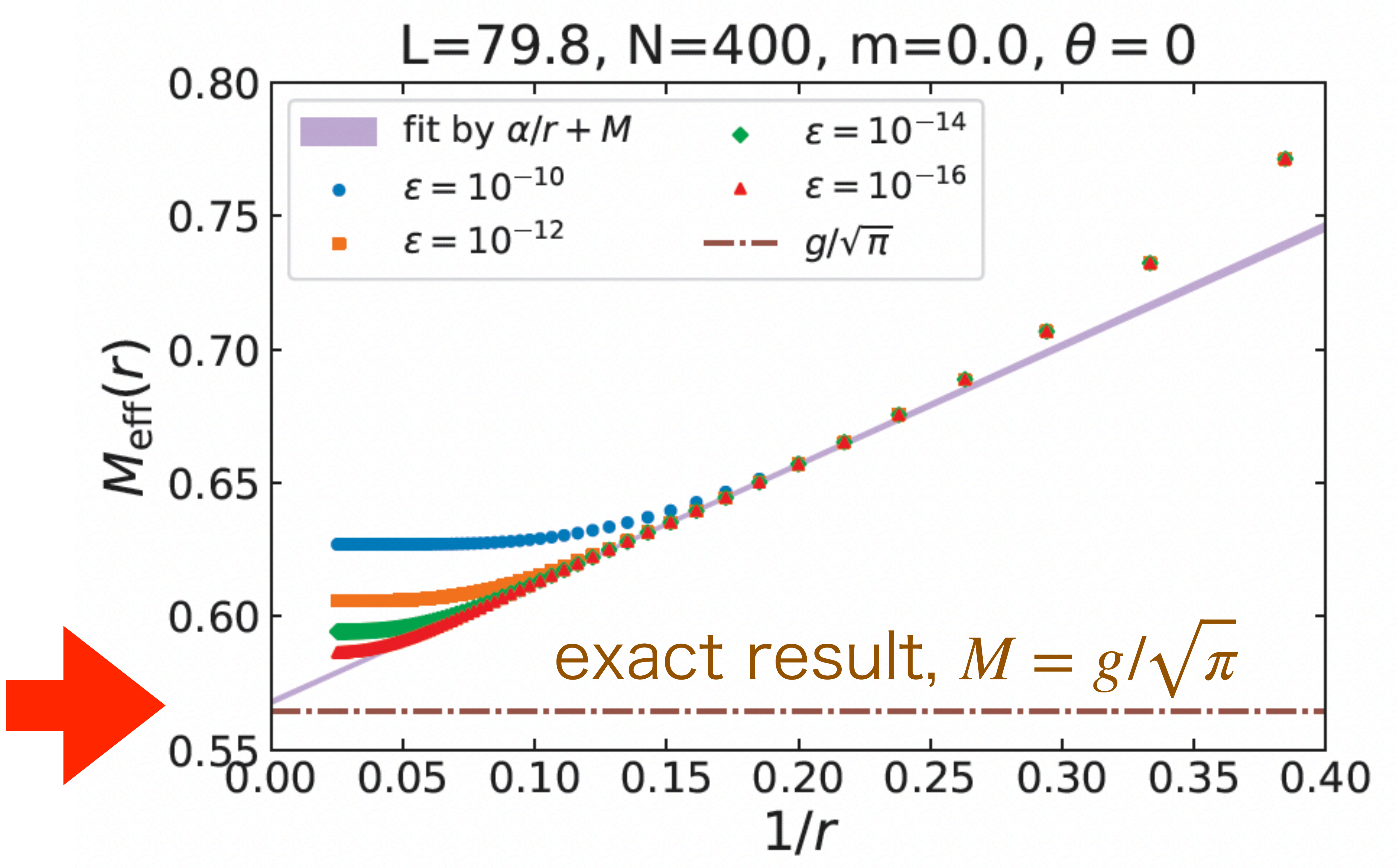
What's happen??

(1) 2-pt. correlation function scheme

(1+1)d. point-point correlation fn. has Yukawa-shape

$$\langle \pi(x,t)\pi(y,t)\rangle \propto K_0(Mr) \sim \frac{1}{\sqrt{Mr}} e^{-Mr}$$

here $\pi = -i\bar{\psi}\gamma^5\psi$ for Nf=1



Effective mass has power correction:

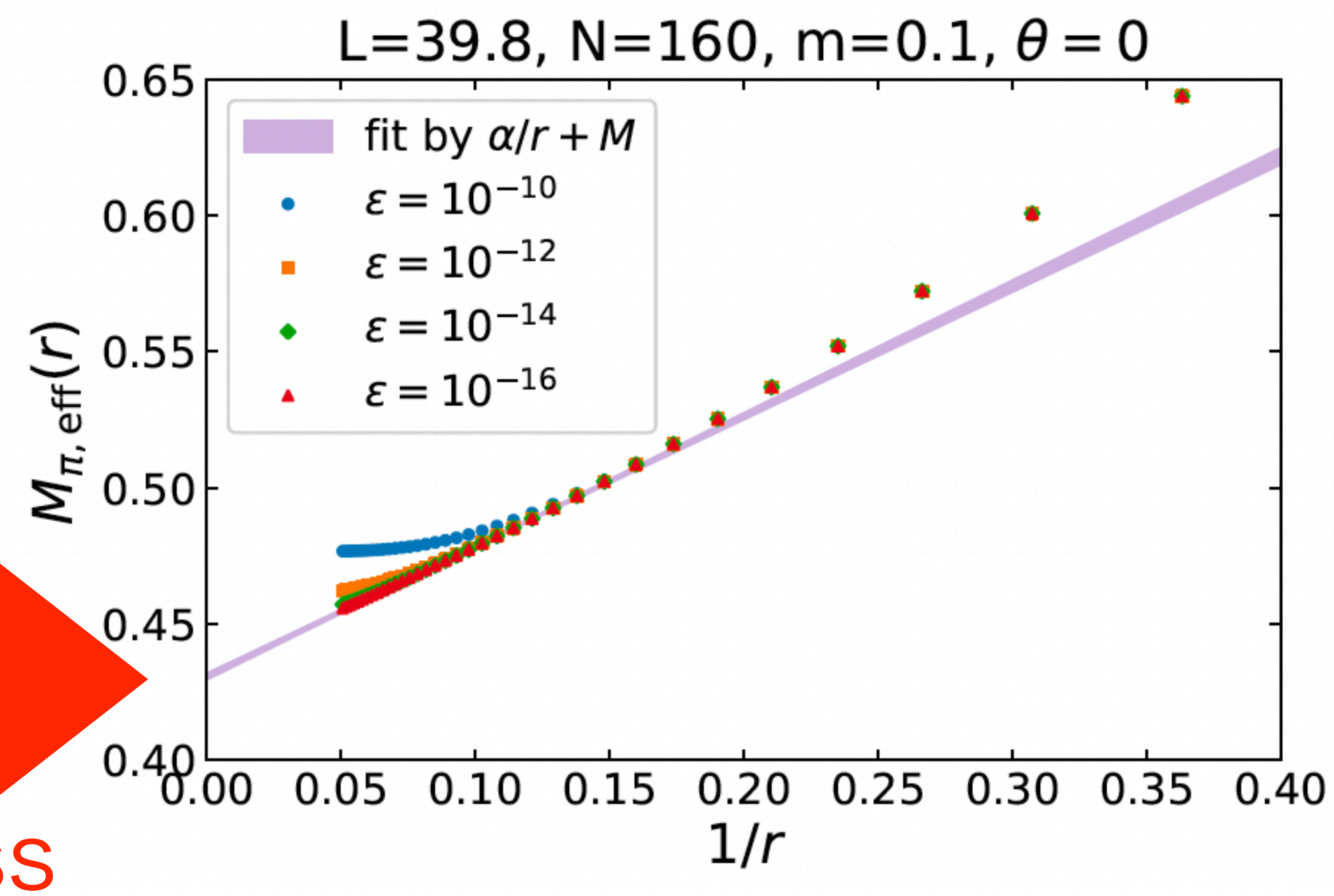
$$M_{\mathrm{eff}}(r) = -\frac{d}{dr}\log K_0(Mr) \sim \frac{1}{2r} + M$$

In $r \to \infty$ limit, obtained M is almost consistent with the exact result

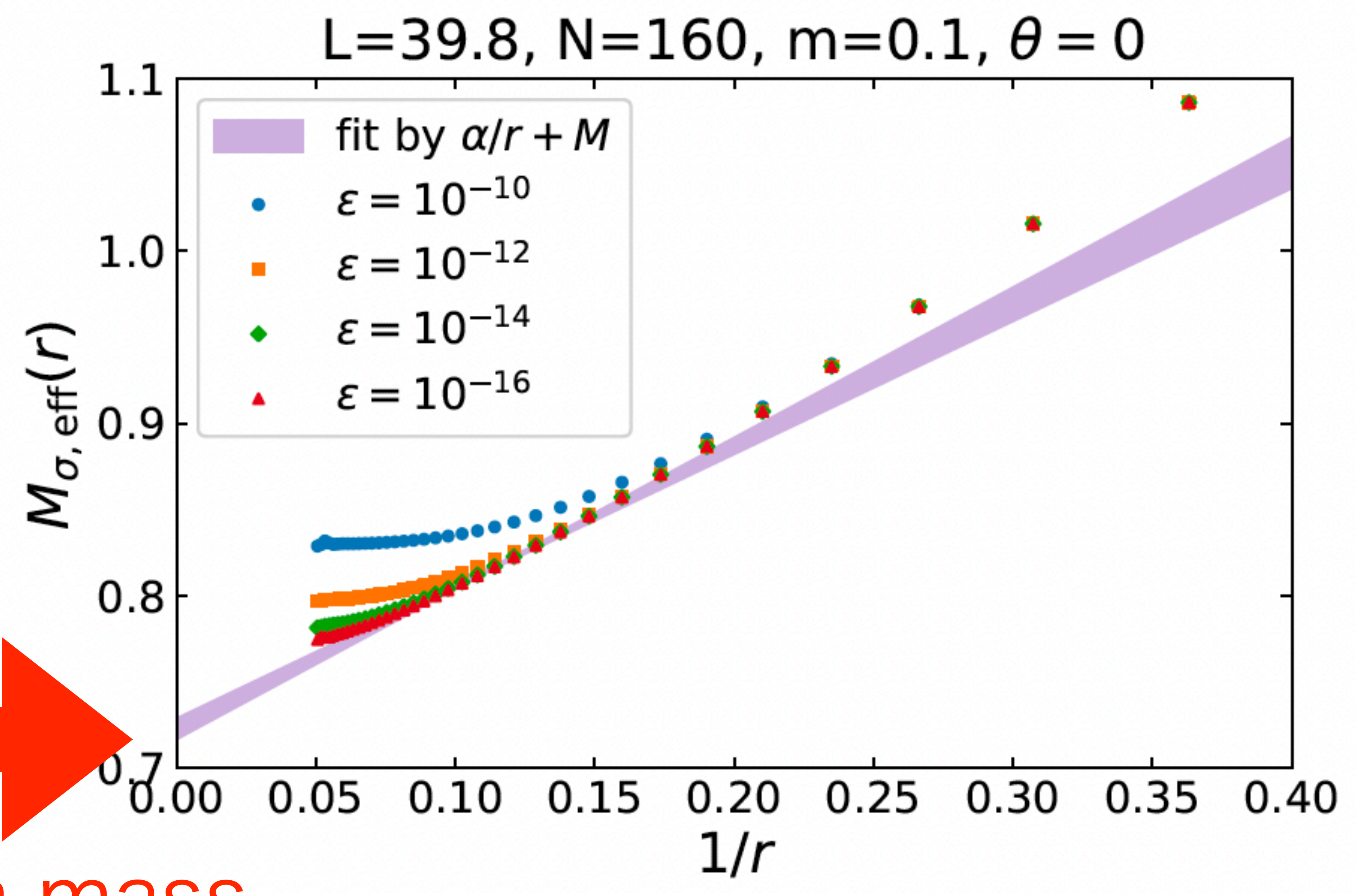
Why the convergence is slow?

=> DMRG can calculate exponential correlations and difficult to reproduce 1/r

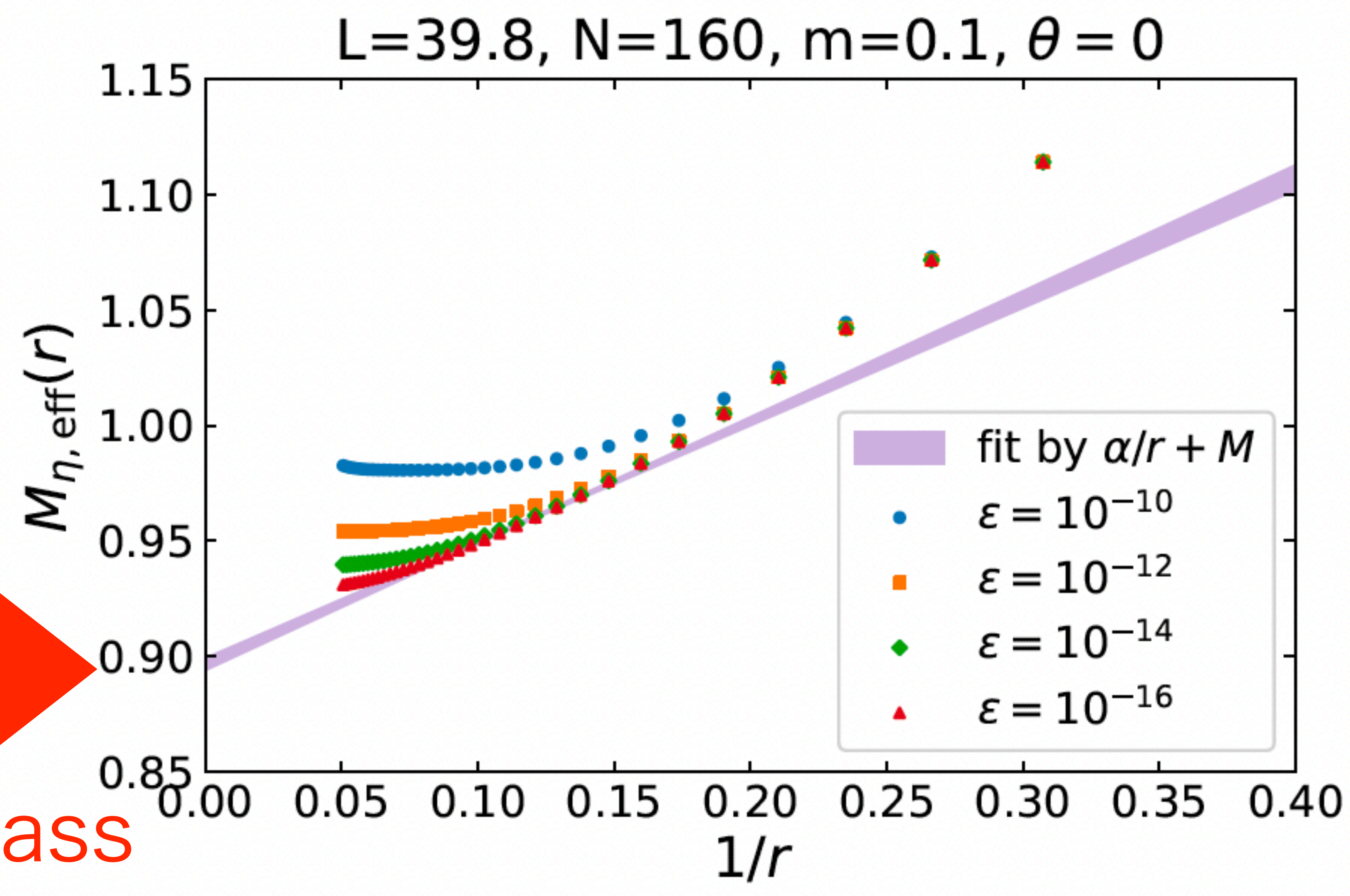
(1)Effective mass with a 1/r correction



pion mass



sigma mass



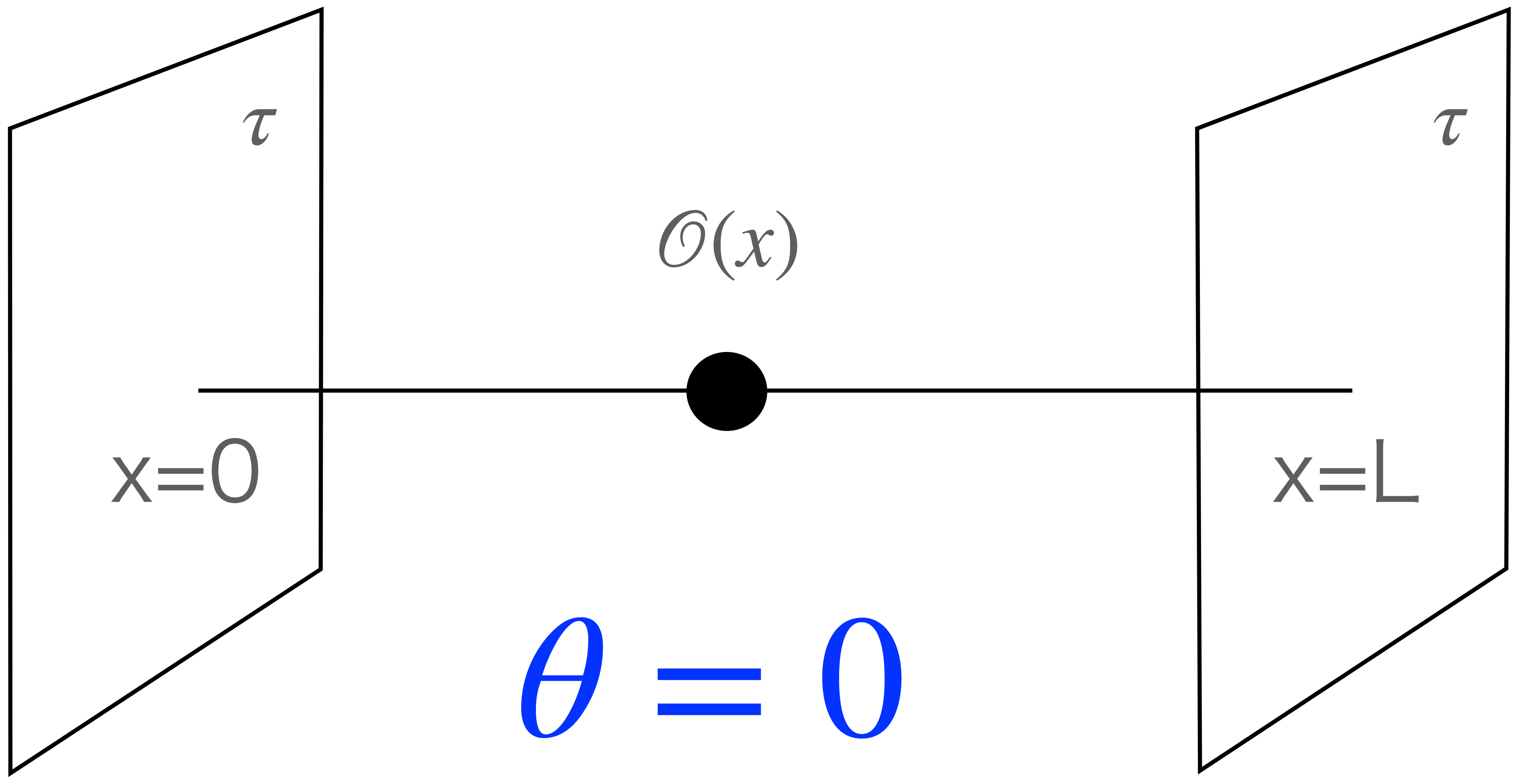
eta mass

(2)One-point-function scheme

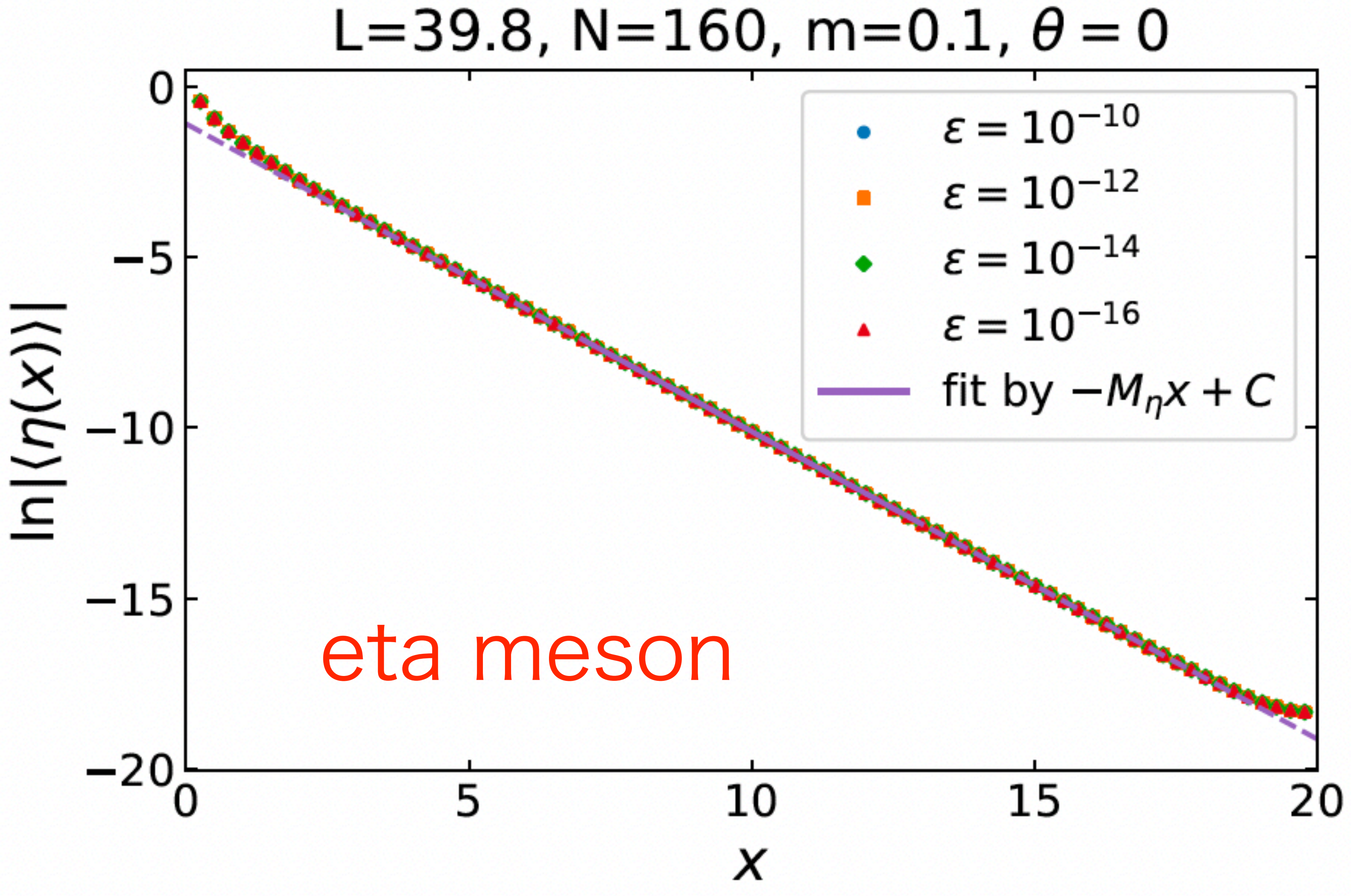
Calculate $\langle \mathcal{O}(x) \rangle$

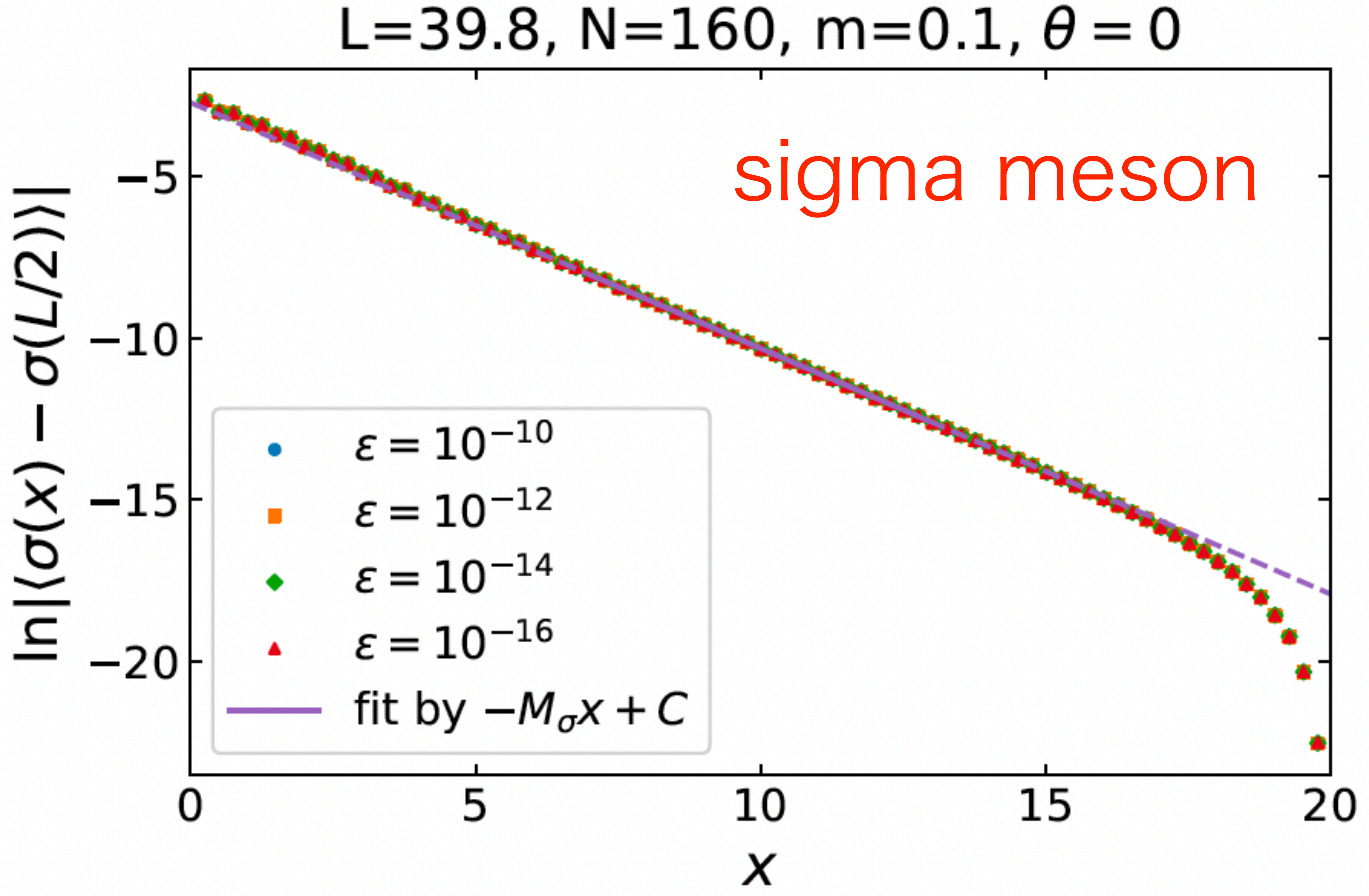
$$\sum_{\tau} \langle \mathcal{O}(x,\tau)\mathcal{O}_{wall}(x=0)\rangle \equiv \langle \text{Vac.} | \mathcal{O}(x) | \text{Bdry} \rangle \sim e^{-Mx}$$

Wall-point correlation function



$\theta = 0$

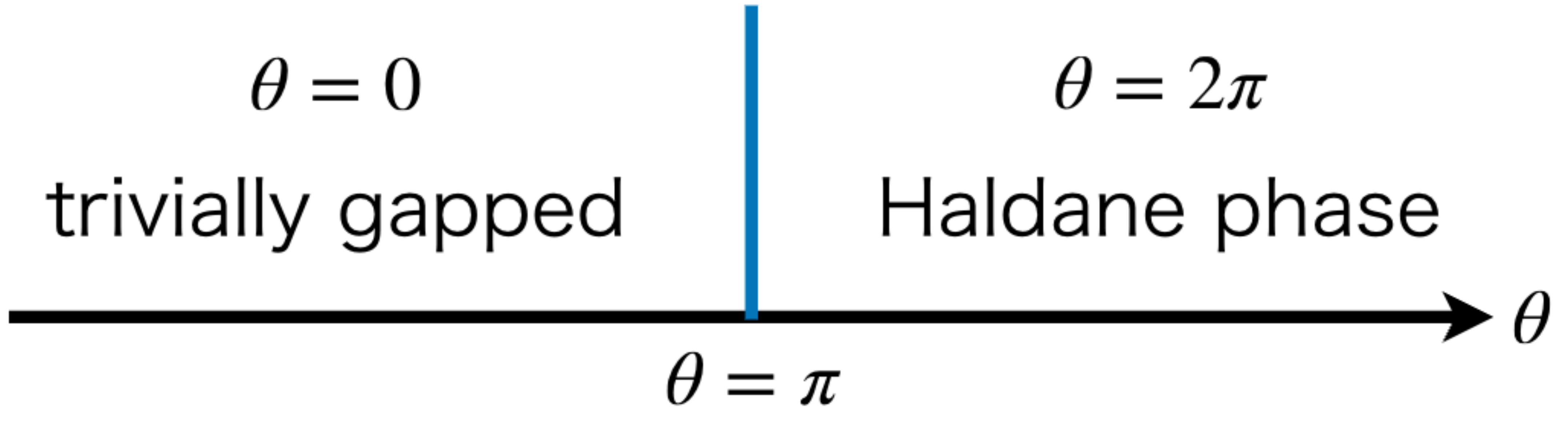




precision-dependence is not observed

(2)One-point-function scheme：pion

$\langle \pi(x) \rangle = 0$ everywhere, since the ground state is iso-singlet at $\theta = 0$



Haldane phase -> edge mode in OBC

isospin =1/2 at both edges = source of iso-triplet