LARGE N TRUNCATIONS OF LGT

Anthony Ciavarella

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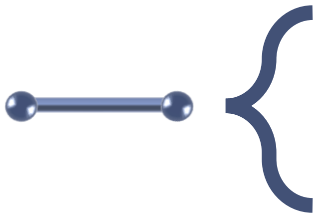
HAMILTONIAN FORMULATION OF LATTICE GAUGE THEORY

Kogut and Susskind

Phys. Rev. D 11 (1975) 395



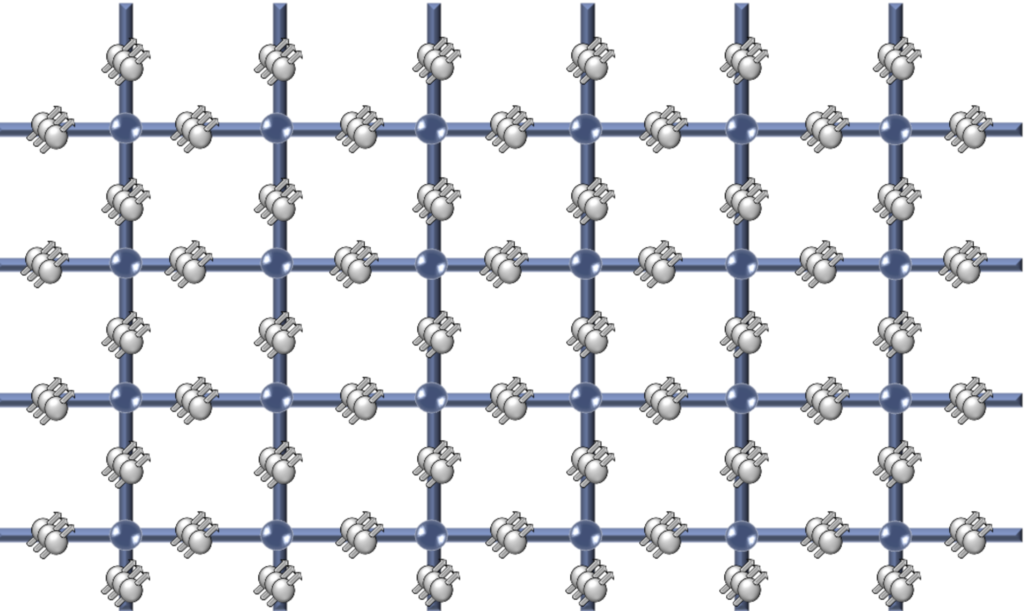
Magnetic Basis $|U\rangle$

SU(2) SU(3)

Electric Basis $|j\ m_L\rangle \otimes |j\ m_R\rangle$ $|p, q, T_L, T_L^z, Y_L, T_R, T_R^z, Y_R\rangle$

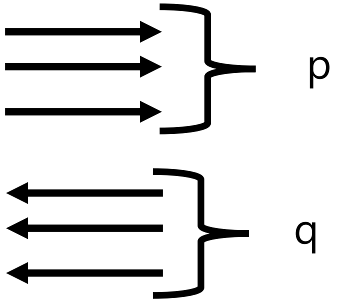
$$U^R_{s_L\ s_R}|j\ m_L\rangle\otimes|j\ m_R\rangle = \sum_{j'm'_Lm'_R}\sqrt{\frac{\dim(j)}{\dim(j')}}C^{j'm'_L}_{j\ m_L;R\ s_L}C^{j'm'_R}_{j\ m_R;R\ s_R}|j'\ m'_L\rangle\otimes|j'\ m'_R\rangle$$

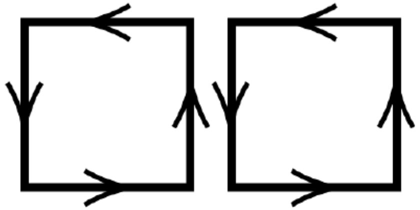
$$\hat{H} = \frac{g^2}{2a^{d-2}}\sum_{b,\text{links}}|\hat{\mathbf{E}}^{(b)}|^2 + \frac{1}{2a^{4-d}g^2}\sum_{\text{plaquettes}}\left[6 - \hat{\Box}(\mathbf{x}) - \hat{\Box}^\dagger(\mathbf{x})\right]$$



Gauge invariant electric basis states can be represented graphically by loops

$$T^{a_1,a_2,\cdots a_p}_{b_1,b_2,\cdots b_q}$$





ELECTRIC BASIS TRUNCATION

- Quantum computers have finite memory ⟶ Place a truncation on the electric energy / link
- Works well in the strong coupling limit, but the continuum is in the weak coupling limit
- Instead of just truncating the electric energy, one can generate basis states through a local Krylov subspace construction

$$\left|\{P_p, \bar{P}_p\}\right\rangle \equiv \prod_p \hat{\Box}_p^{P_p} \hat{\Box}_p^{\dagger \bar{P}_p} |0\rangle$$

- Truncations are specified by 3 integers and a reference state (electric vacuum in this work)

n_P: # of times a plaquette operator can be applied to the reference state

n_l: # of times an individual link operator can be applied to the reference state

k: max # of lines of electric flux / link

arXiv:2503.11888

LARGE N EXPANSION

SU(3) → SU(N), Expand in 1/N

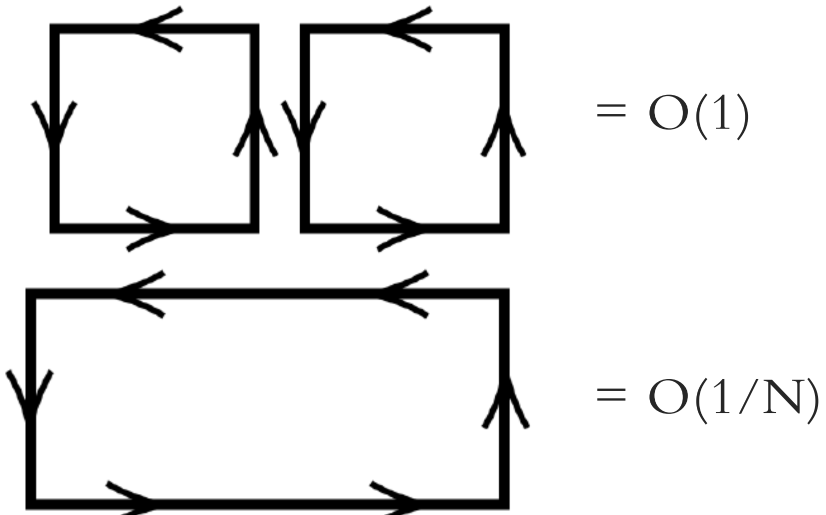
- Qualitatively reproduces many aspects of QCD
- Provides a starting point for describing interactions between mesons
- Used in event generators that simulate collider physics

- Expand plaquette operator matrix elements in powers of 1/N

- Truncate matrix elements of the plaquette operator based on the large N scaling and use the truncated plaquette operator in the local Krylov subspace construction

- The large N scaling of a state is determined by the maximum overlap of the state with

$$|\{P_p, \bar{P}_p\}\rangle \equiv \prod_p \hat{\square}_p^{P_p} \hat{\square}_p^{\dagger \bar{P}_p} |0\rangle$$



Phys. Rev. Lett. **133**, 111901

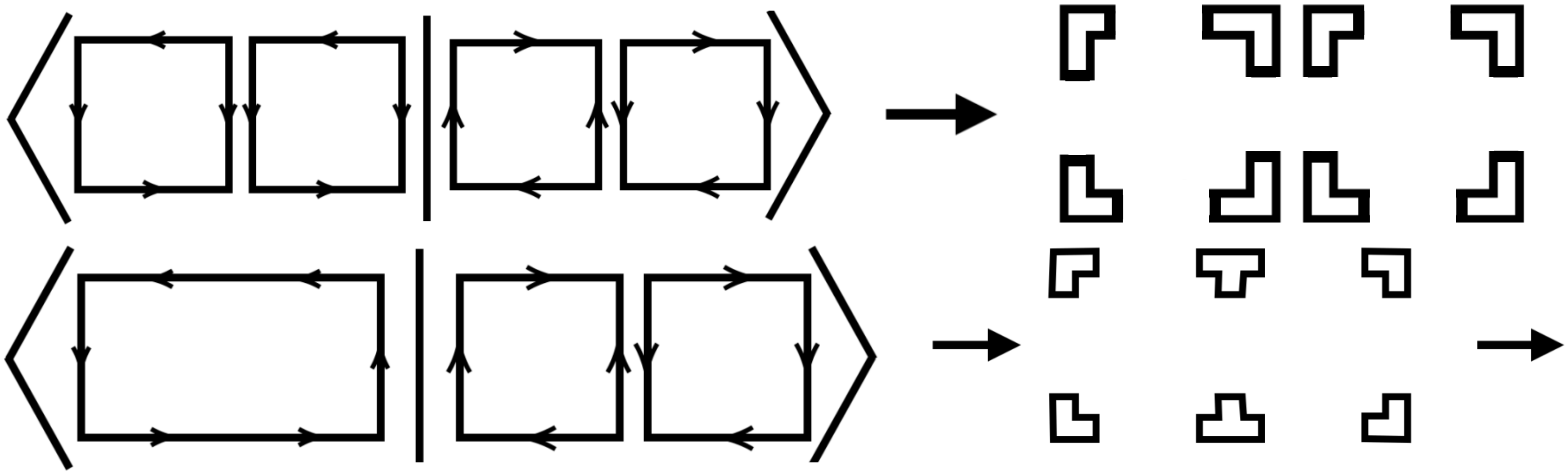
LARGE N SCALING OF STATES

- The Large N scaling is determined by $\left\langle \{L_i\} \middle| \{P_p, \bar{P}_p\} \right\rangle$ $\quad |\{P_p, \bar{P}_p\}\rangle \equiv \prod_p \hat{\square}_p^{P_p} \hat{\square}_p^{\dagger \bar{P}_p} |0\rangle$
- This can be evaluated in the magnetic basis, $\mathbf{1} = \prod_{\text{links } l} \int dU_l \, |U_l\rangle \langle U|_l$

  using the identity

$$\int dU \prod_{n=1}^{q} U_{i_n j_n} U^*_{i'_n j'_n} =$$

$$\frac{1}{N_c^q} \sum_{\text{permutations k}} \prod_{n=1}^{q} \delta_{i_n i'_{k_n}} \delta_{j_n j'_{k_n}} + \mathcal{O}\left(\frac{1}{N_c^{q+1}}\right)$$



$$\rightarrow \frac{1}{N_c^8} \times N_c^8 = 1$$

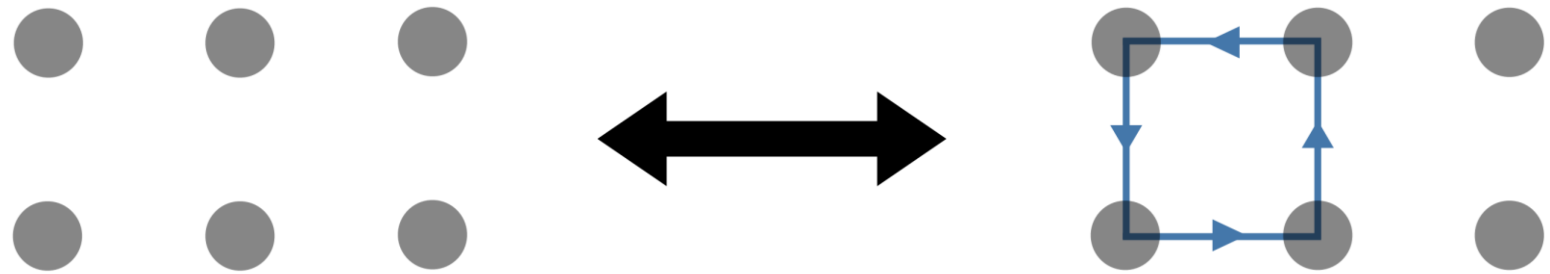
$$\rightarrow \frac{1}{N_c^7} \times N_c^6 = \frac{1}{N_c}$$

Simple Scaling Rule $\quad \left\langle \{L_i, a_\ell\} \middle| \{P_p, \bar{P}_p\} \right\rangle \propto \prod_i N_c^{1-m_i}$

m_i =# Plaquettes enclosed by loop i

$(n_P = 1,\ n_l = 1,\ K = 1)$ TRUNCATION

- Each plaquette operator can be applied once
- Neighboring plaquette operators cannot both be applied



- One qutrit per plaquette, specifies if the plaquette is excited and the orientation of electric flux

$$\hat{H} = \sum_p g^2 \left(N_c - \frac{1}{N_c}\right)\left(|\square\rangle_p \langle\square|_p + |\blacksquare\rangle_p \langle\blacksquare|_p\right) - \frac{1}{2g^2}\left(\prod_{\hat{n}} |\mathbf{1}\rangle_{p+\hat{n}} \langle\mathbf{1}|_{p+\hat{n}}\right)\left(|\square\rangle_p \langle\mathbf{1}|_p + |\blacksquare\rangle_p \langle\mathbf{1}|_p + \text{h.c.}\right)$$

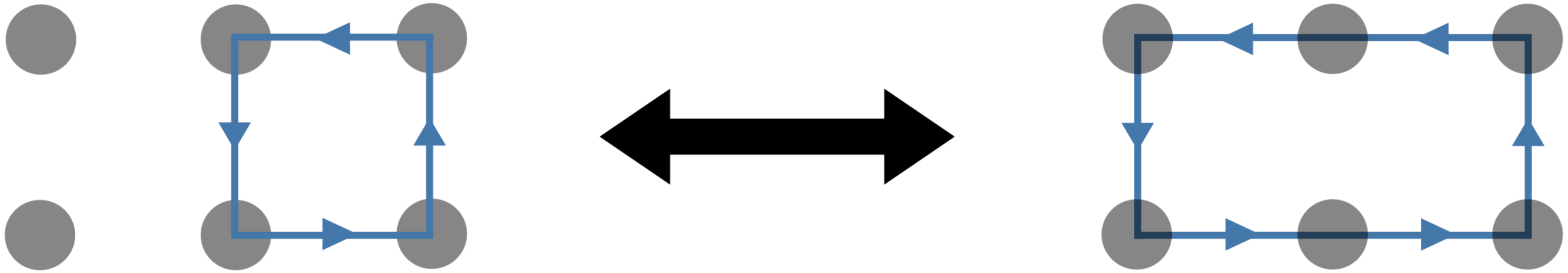
- In the even C sector, only get equal superpositions of each orientation of flux, so only qubits are needed

$$|0\rangle_p = |\mathbf{1}\rangle_p$$
$$|1\rangle_p = \frac{1}{\sqrt{2}}\left(|\square\rangle_p + |\blacksquare\rangle_p\right)$$

$$\hat{H} = \sum_p g^2 \left(N_c - \frac{1}{N_c}\right)|1\rangle_p \langle 1|_p - \frac{1}{\sqrt{2}g^2}\left(\prod_{\hat{n}} |0\rangle_{p+\hat{n}} \langle 0|_{p+\hat{n}}\right)\hat{X}_p$$

$(n_P = 1,\ n_l = 2,\ K = 1)$ TRUNCATION

- Each individual plaquette operator can be applied once, but neighboring plaquette operators can also be applied
- Only keep states where links are in the trivial or fundamental irrep
- At large N, identical to the previous truncation, but has a 1/N correction

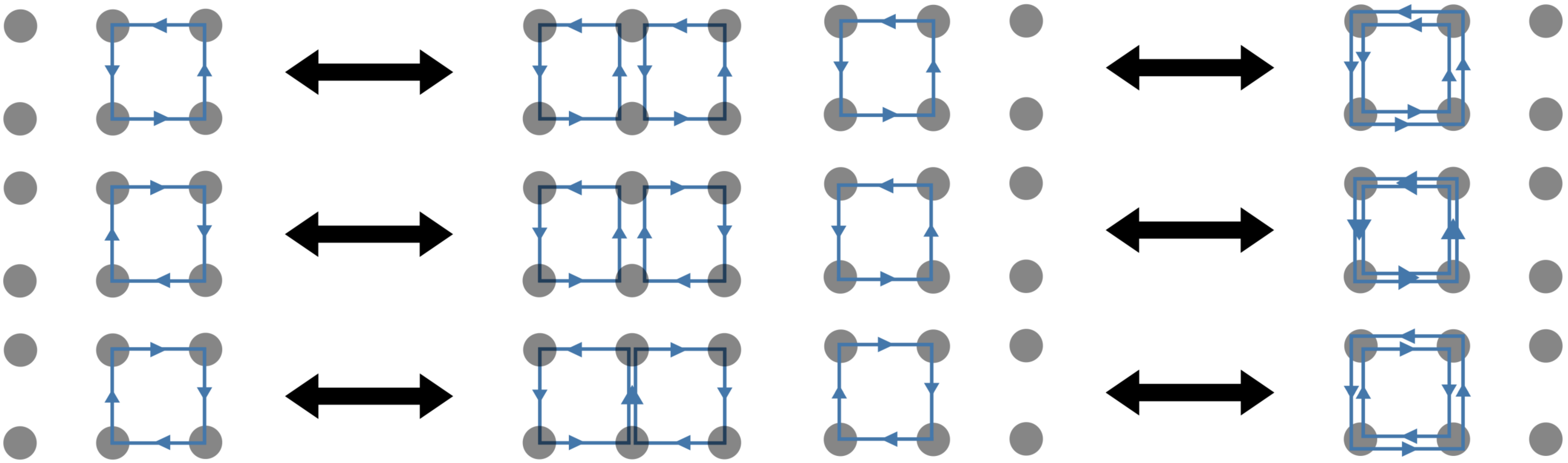


$$
\begin{aligned}
\hat{H}^{(1,2,1),1/N_c} =& \hat{H}_E^{(1,2,1),1/N_c} + \hat{H}_B^{(1,2,1),1/N_c} \\
\hat{H}_E^{(1,2,1),1/N_c} =& g^2 \left(N_c - \frac{1}{N_c}\right) \sum_p \Bigg[\left(|\square\rangle_p \langle\square|_p + |\blacksquare\rangle_p \langle\blacksquare|_p \right) \\
& - \frac{1}{4} \sum_{\hat{n}} \left(|\square\rangle_p \langle\square|_p |\square\rangle_{p+\hat{n}} \langle\square|_{p+\hat{n}} + |\blacksquare\rangle_p \langle\blacksquare|_p |\blacksquare\rangle_{p+\hat{n}} \langle\blacksquare|_{p+\hat{n}} \right) \Bigg] \\
\hat{H}_B^{(1,2,1),1/N_c} =& -\frac{1}{2g^2} \sum_p \left(\prod_{\hat{n}} |\mathbf{1}\rangle_{p+\hat{n}} \langle\mathbf{1}|_{p+\hat{n}} \right) \left(|\square\rangle_p \langle\mathbf{1}|_p + |\blacksquare\rangle_p \langle\mathbf{1}|_p + \text{h.c.} \right) \\
& - \frac{1}{2g^2 N_c} \sum_p \sum_{\hat{k}} |\square\rangle_{p+\hat{k}} \langle\square|_{p+\hat{k}} \prod_{\hat{n}\neq\hat{k}} \left(|\mathbf{1}\rangle_{p+\hat{n}} \langle\mathbf{1}|_{p+\hat{n}} \right) \left(|\square\rangle_p \langle\mathbf{1}|_p + \text{h.c.} \right) \\
& - \frac{1}{2g^2 N_c} \sum_p \sum_{\hat{k}} |\blacksquare\rangle_{p+\hat{k}} \langle\blacksquare|_{p+\hat{k}} \prod_{\hat{n}\neq\hat{k}} \left(|\mathbf{1}\rangle_{p+\hat{n}} \langle\mathbf{1}|_{p+\hat{n}} \right) \left(|\blacksquare\rangle_p \langle\mathbf{1}|_p + \text{h.c.} \right)
\end{aligned}
$$

Still only need a qutrit per plaquette, and can project the even C sector onto a single qubit per plaquette

$(n_P = 2,\ n_l = 2,\ K = 2)$ TRUNCATION

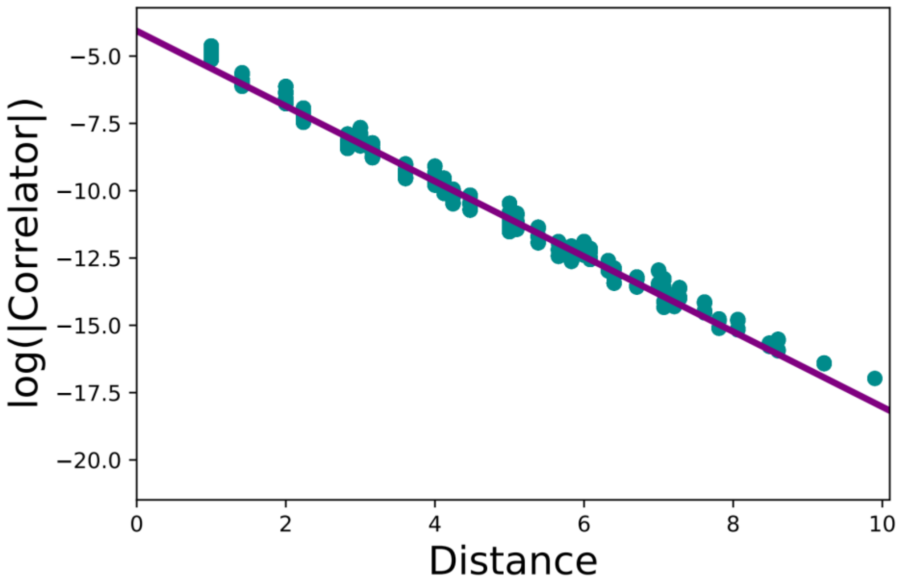
- Individual plaquette operators can now be applied twice



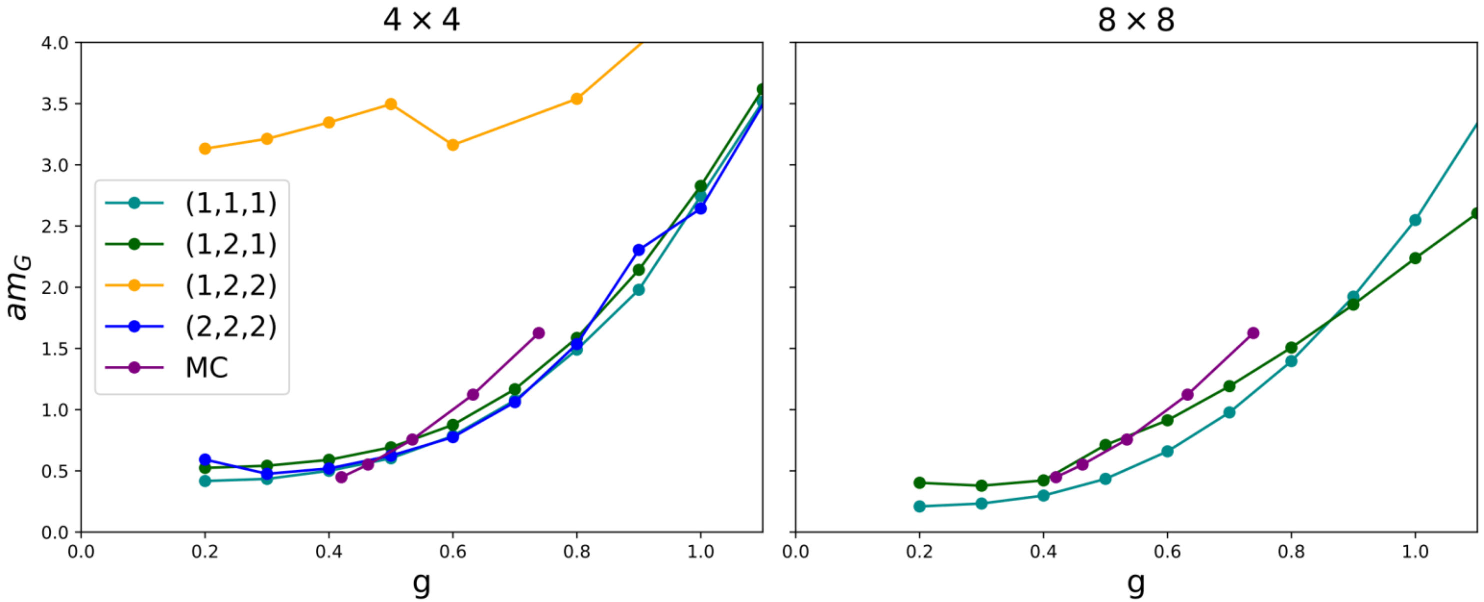
- Need a qu8 per plaquette to specify the irrep of flux flowing around the plaquette
- Also, need a qubit / link to resolve ambiguities

GLUEBALL MASS CALCULATION

- As a test of these truncations, the scalar glueball mass was computed for SU(3) LGT in 2+1D and compared to traditional MC results
- This was done by obtaining the vacuum state from DMRG and fitting correlation functions to an exponential

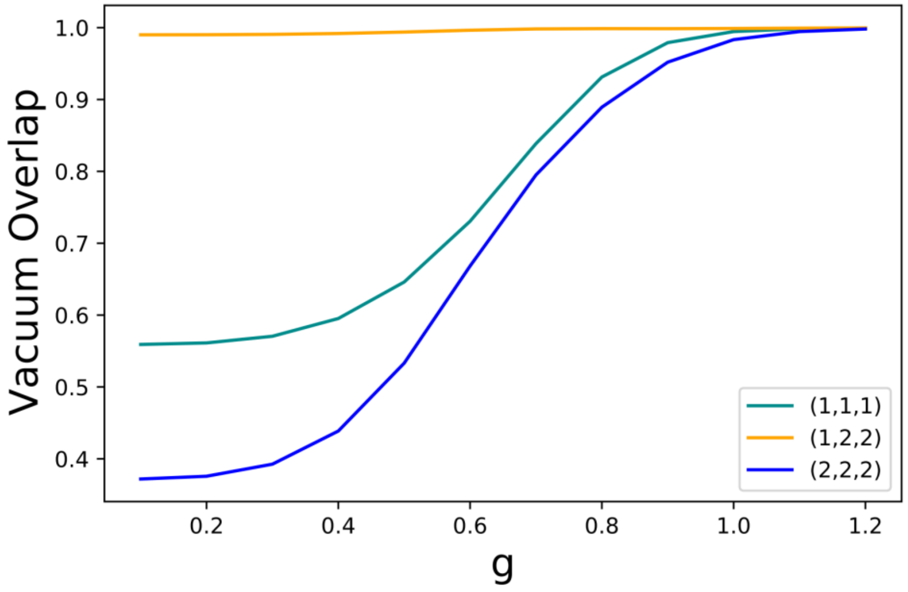


Correlation functions in the (1,1,1) truncation already show signs of the restoration of rotation symmetry at g=0.8 on an 8x8 lattice



WHY DOES (1,2,2) FAIL?

- All of the truncations except for (1,2,2) look roughly consistent with the untruncated theory.
- The (1,2,2) truncation lies close to a point in theory space where the correlation length vanishes for all values of g
- The correlation free limit comes from dropping all 1/N dependence in the electric energy and plaquette operator



RESOURCES NEEDED FOR QUANTUM SIMULATION

- These low lying truncations can reach non-trivial lattice spacings and can be used for near-term quantum simulations of lattice gauge theories
- Time evolution can be performed using Trotterization by breaking the plaquette operator into a sum over generators of gauge invariant basis rotations

	$(1,1,1)$ $2D$	$(1,1,1)$ $3D$	$(1,2,1)$ $2D$	$(1,2,1)$ $3D$	$(2,2,2)$ $2D$ at order $1/N_c$	$(2,2,2)$ $3D$ at order $1/N_c$
Qubits	1	1	1	1	5	4
CNOT	16	134	20	1,744	249,274	249,226
H	2	46	2	598	72,030	72,030
R_z	17	157	19	2,030	247,216	247,096

	$(1,1,1)$ 10×10	$(1,1,1)$ $10 \times 10 \times 10$	$(1,2,1)$ 10×10	$(1,2,1)$ $10 \times 10 \times 10$	$(2,2,2)$ 10×10 at order $1/N_c$	$(2,2,2)$ $10 \times 10 \times 10$ at order $1/N_c$
Qubits	100	3,000	100	3,000	500	12,000
T	64,600	1.79×10^7	72,700	2.3×10^8	9.4×10^8	2.8×10^{10}

REAL TIME EVOLUTION ON IBM'S QUANTUM COMPUTERS (1,1,1) TRUNCATION

Interaction Picture Trotterization

$$\hat{H}_{B,I}(t) = e^{i\hat{H}_E t}\hat{H}_B e^{-i\hat{H}_E t}$$

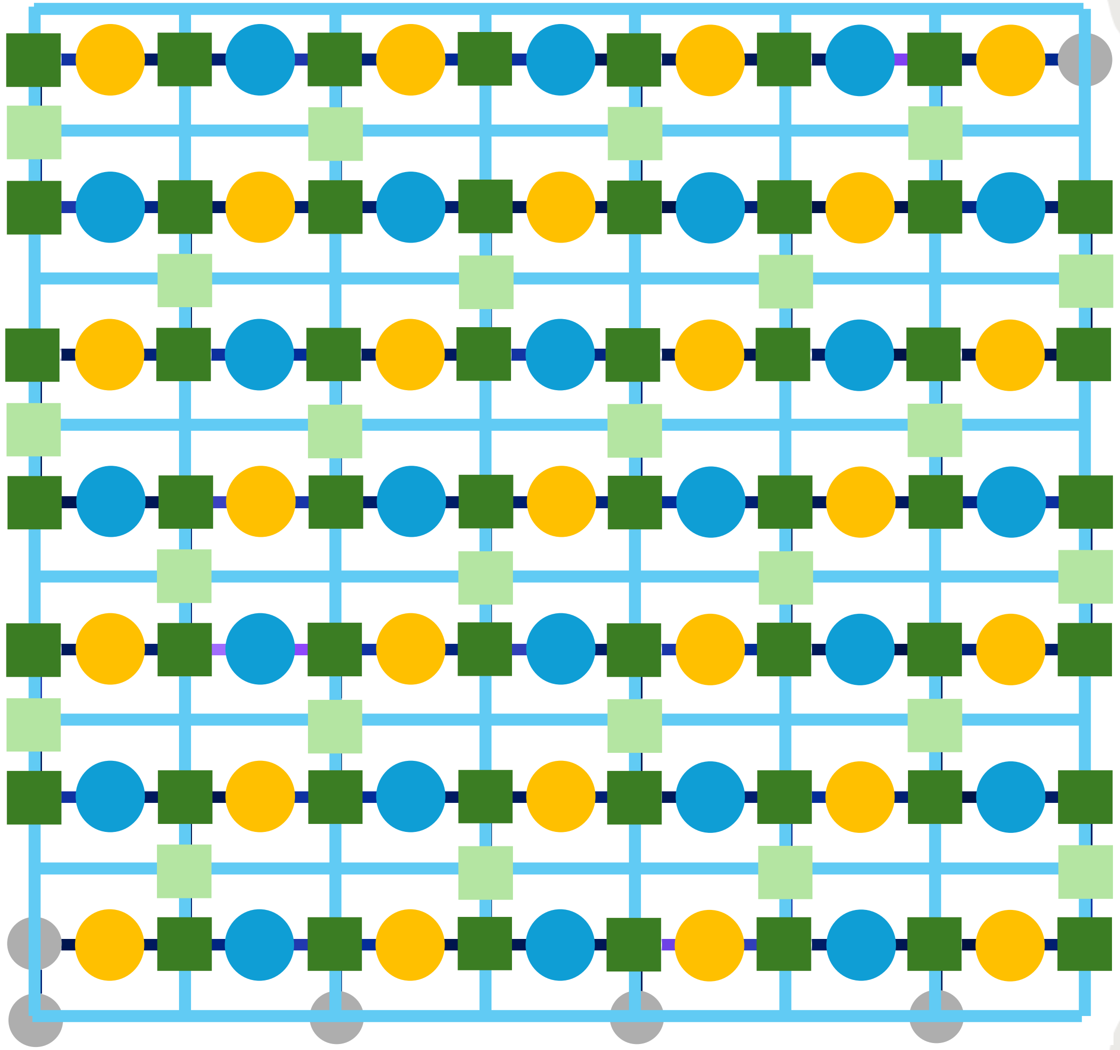
$$e^{-i\hat{H}t} = e^{-i\hat{H}_E t}\,\mathcal{T}e^{-i\int_0^t ds \hat{H}_{B,I}(s)}$$

$$e^{-i\int_0^{\Delta t} ds \hat{H}_{B,I}(s)} = e^{-i\int_0^{\Delta t} ds \hat{H}_{B,E}(s)} e^{-i\int_0^{\Delta t} ds \hat{H}_{B,O}(s)} + \mathcal{O}\left(\frac{\Delta t^2}{g^4}\right)$$

$$e^{-i\int_0^{\Delta t} \mathrm{ds}\, \hat{H}_{B,E}(s)} e^{-i\int_0^{\Delta t} \mathrm{ds}\, \hat{H}_{B,O}(s)} =$$

$$\left[e^{i\phi\sum_p \hat{Z}_p}\right]\left[e^{i\theta\sum_{p\in E}\hat{X}_p\prod_{q\in\partial p}\hat{P}_{0,q}}\right]\left[e^{i\theta\sum_{p\in O}\hat{X}_p\prod_{q\in\partial p}\hat{P}_{0,q}}\right]\left[e^{-i\phi\sum_p \hat{Z}_p}\right]$$

- Yellow and blue qubits are used to represent the state of the system
- Square qubits are used to enable communication between those used to represent the system
- One Trotter step = CNOT depth 45



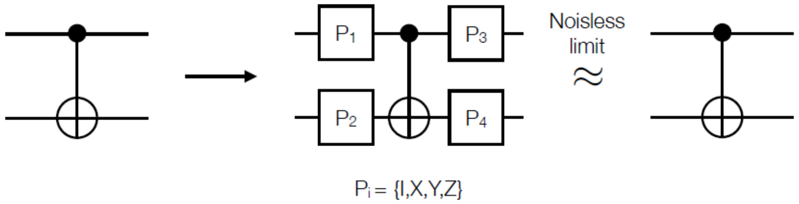
ERROR MITIGATION

- Quantum simulations have errors coming from inherent hardware errors
- Pauli twirling converts coherent errors into a Pauli error channel
- Decoherent Pauli noise renormalizes Pauli operators

$$\langle\psi|\hat{P}|\psi\rangle \to \eta_P \langle\psi|\hat{P}|\psi\rangle$$

- This can be mitigated by running a circuit with a known answer to determine η_P
- Other sources of hardware error can be mitigated by artificially introducing noise by applying more CNOT gates and extrapolating to zero noise.

Pauli Twirling (or randomized compiling)



Operator Decoherence Renormalization

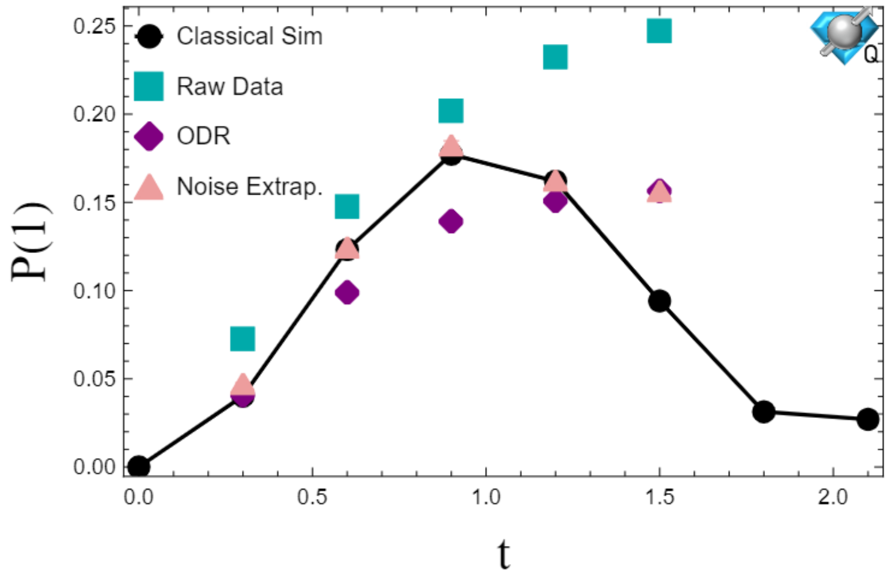
Phys. Rev. Lett. 127, 270502

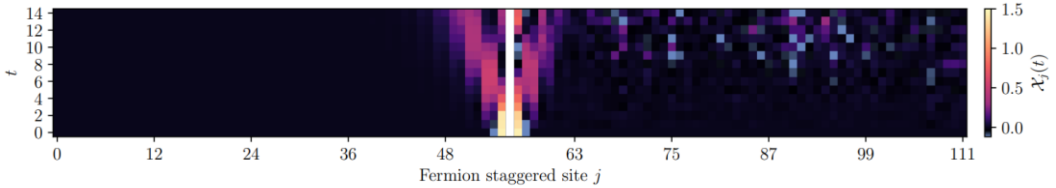
arXiv:2210.11606

PRX Quantum 5, 020315

Phys. Rev. D 109, 114510

5x5



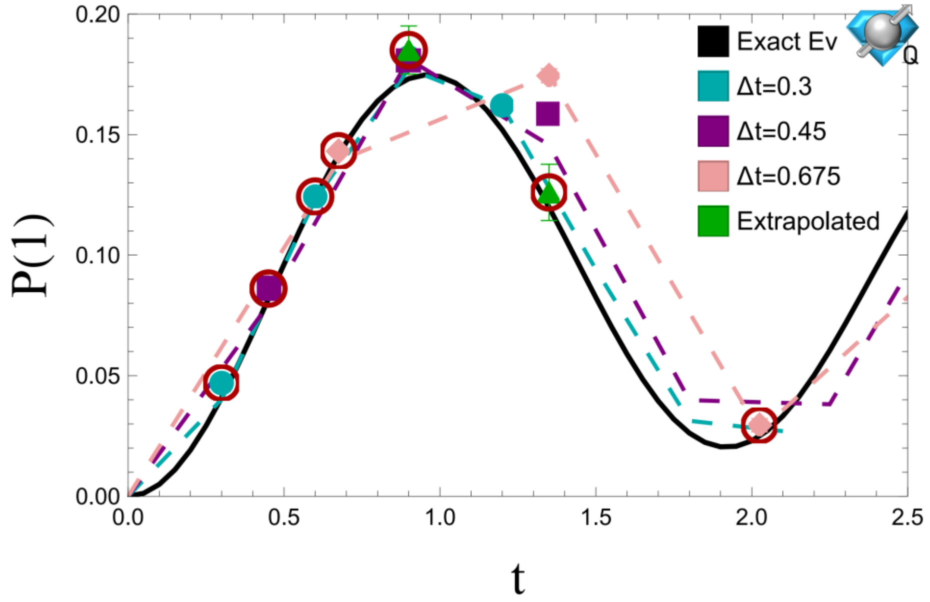


112 qubits, CNOT depth 370 (13,858 CNOTs)

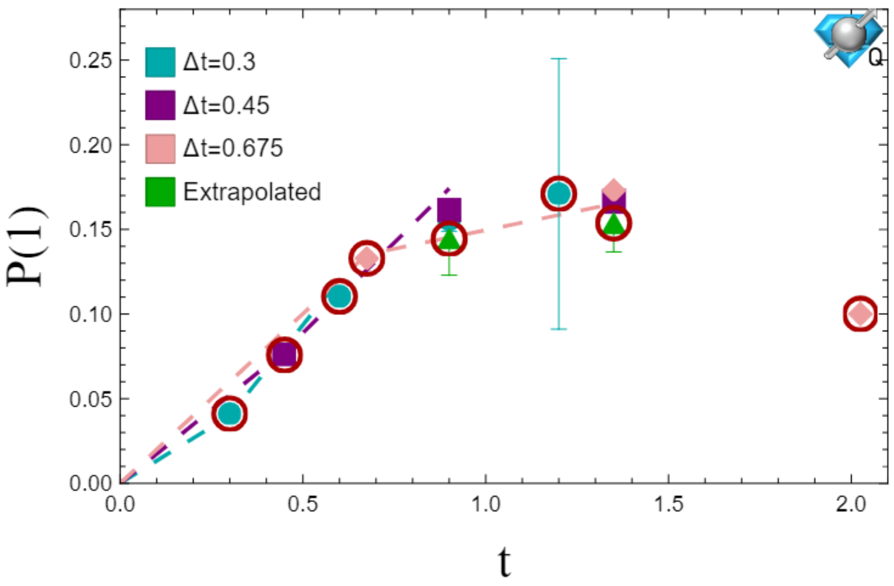
ALGORITHMIC ERRORS

- Errors also come from the Trotterization of the time evolution operator.
- This can be mitigated by performing the evolution with multiple step sizes that sample the same points in time and extrapolating.
- Noise in circuits scales with circuit depth not system size so small simulations can be used to validate the results of larger ones.
- CuQuantum was used to perform a classical simulation for a 8x8 lattice.

5x5



8x8



129 Qubits
CZ Depth 113
1500 CZ Gates
8733 1 Qubit Gates

SUMMARY & FUTURE GOALS

- Careful construction of electric bases can significantly reduce the gate count for quantum simulation
- The large N expansion can be used to reduce the resources needed for simulation further.
- Future work will look at including quarks and 1/N corrections.