Symmetry verification for noisy quantum simulations of non-Abelian lattice gauge theories

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01/09/2025, Trento

PI: Philipp Hauke

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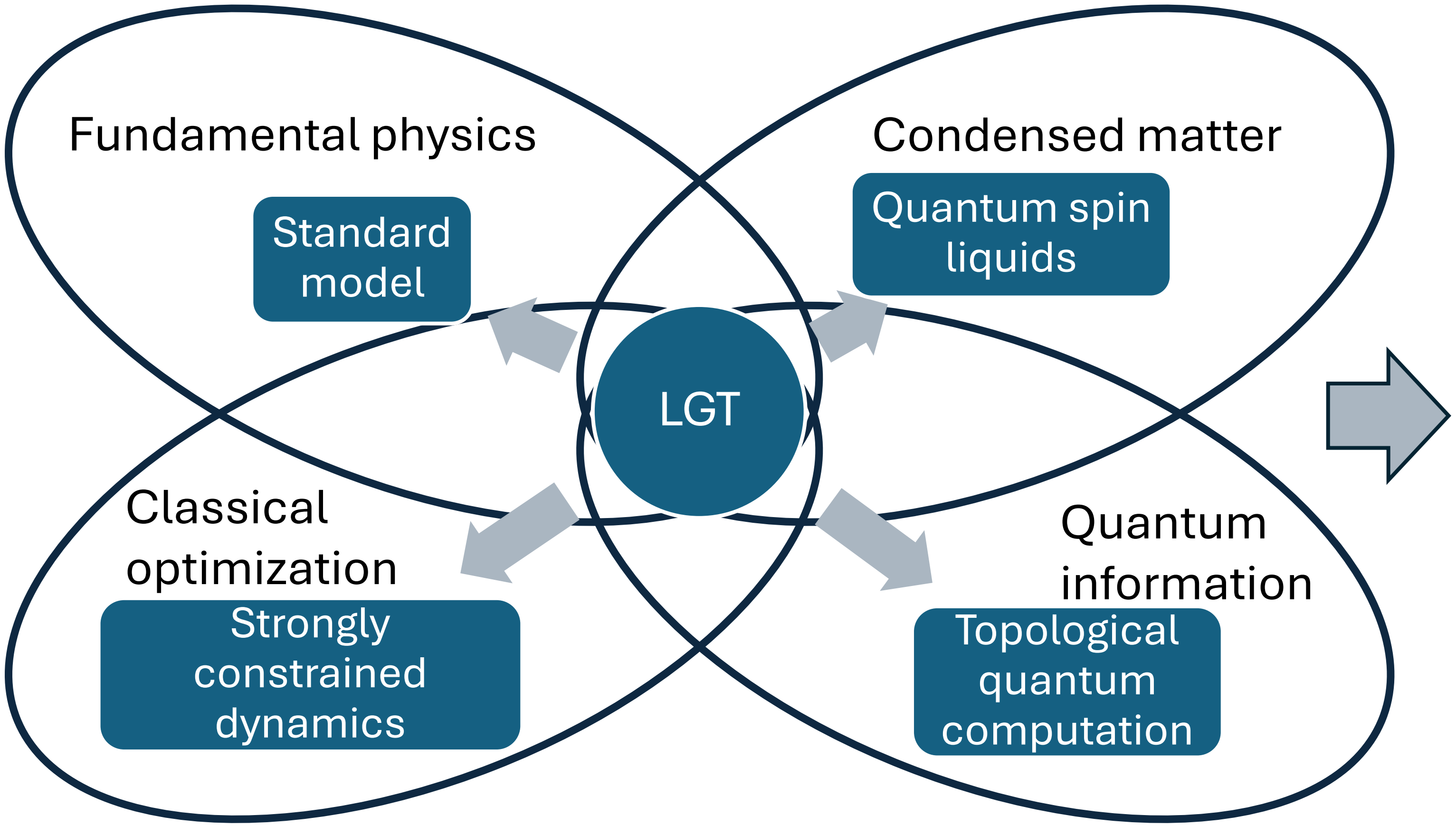
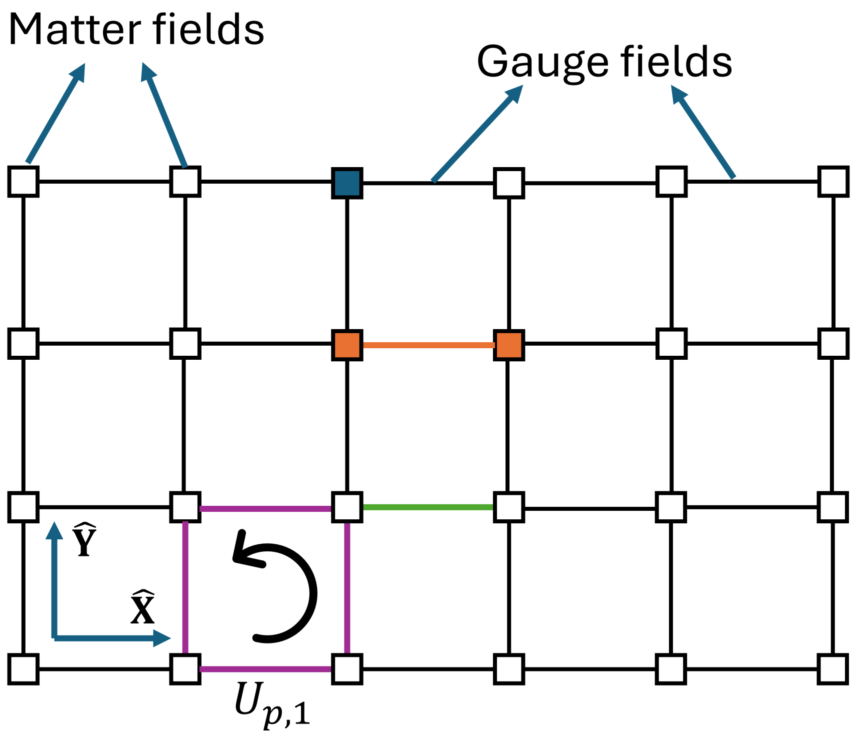
Quantum simulations of lattice gauge theories



Quantum simulations of many-body physics

1. Go beyond classical methods
2. Hardware and software benchmark
3. Synthetic quantum systems

Quantum simulations of lattice gauge theories



$$H = H_{\text{hop}} + H_M + H_E + H_B$$

Symmetry group G:

$$\Theta_n(g) H \Theta_n^\dagger(g) = H \ \ \forall n \in \mathbb{Z}^d, \forall g \in G$$

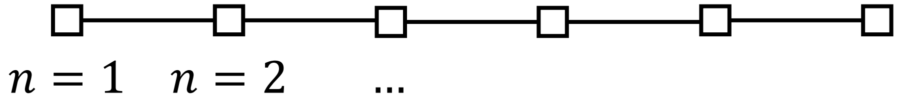
$$H_{\text{hop}} = J \sum_n \psi_n^\dagger U_{n,n+1} \psi_{n+1} + H.c.$$

$$H_M = m \sum_n \psi_n^\dagger \psi_n$$

$$H_E = g^2 \sum_l E_l^2$$

$$H_B = \frac{1}{g^2} \sum_p \mathrm{Re}(\mathrm{Tr}(U_{p,1} U_{p,2} U_{p,3}^\dagger U_{p,4}^\dagger))$$

Quantum simulations of lattice gauge theories



Gauss's Law for the electric field $\nabla \cdot \boldsymbol{E} = -\rho \;\Rightarrow$

Local symmetry: $\Theta_{g,n} = \mathbb{I} \otimes \mathbb{I} \ldots \Theta^{R}_{g;n-1,n} \Theta^{Q}_{g,n} \Theta^{L}_{g;n,n+1} \ldots \mathbb{I} \otimes \mathbb{I}$

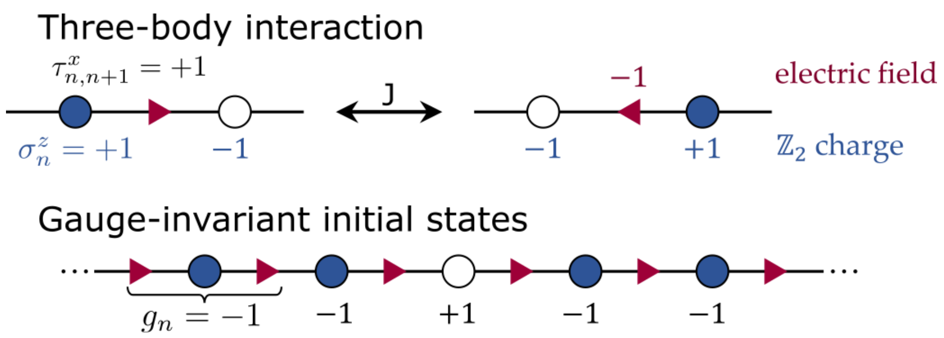
$\Theta^{L}_{g} = \Theta^{R}_{g}$ for Abelian groups

$[H, \Theta_{g,n}] = 0 \; \forall g, n$

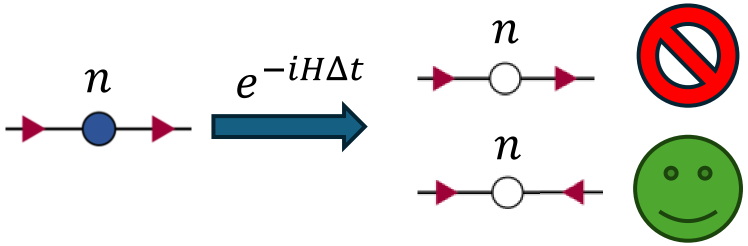
$\Theta_{g,n} |\psi\rangle_{\text{phys}} = \alpha_{g,n} |\psi\rangle_{\text{phys}}$

The set of phases $\alpha_{g,n}$ defines the gauge sector

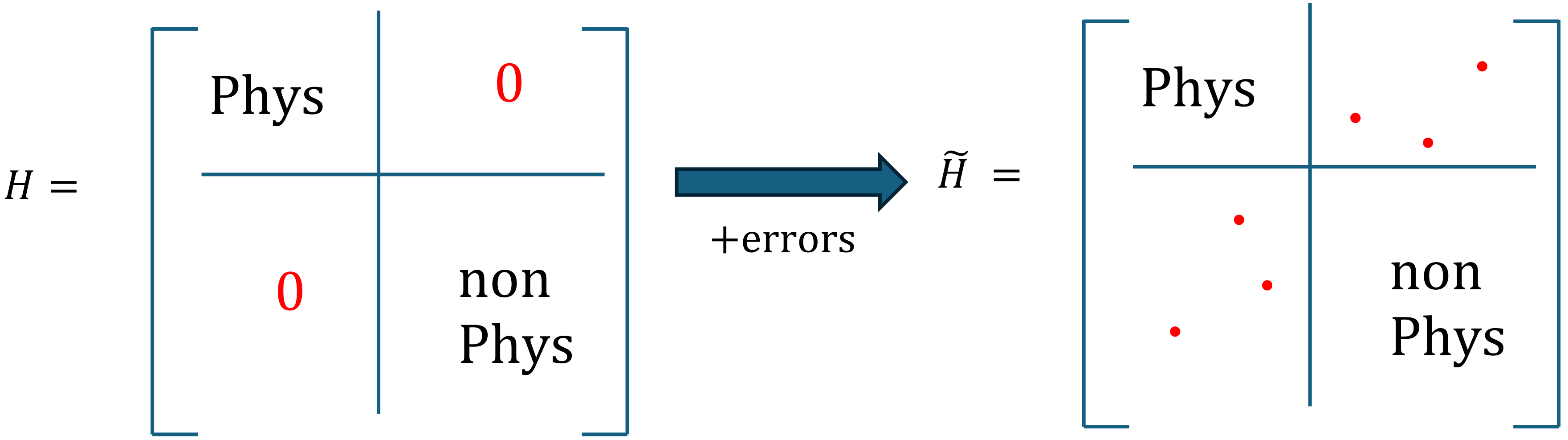
Z_2 LGT in one dimension

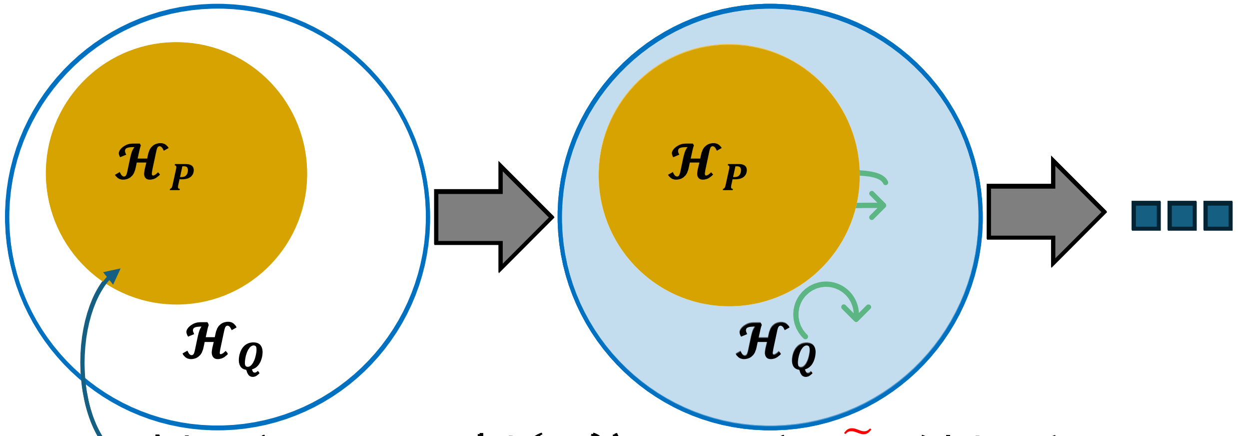


Local gauge operator $\Theta_n = -\tau^{x}_{n-1,n} \sigma^{z}_{n} \tau^{x}_{n,n+1}$



Quantum simulations of lattice gauge theories: time evolution





$|\psi_{\text{init}}\rangle$

$|\psi(\Delta t)\rangle = \exp(-i\widetilde{H}\Delta t)|\psi_{\text{init}}\rangle$

$H = \sum H_j$

$\sim \prod_j \exp(-iH_j\Delta t)$

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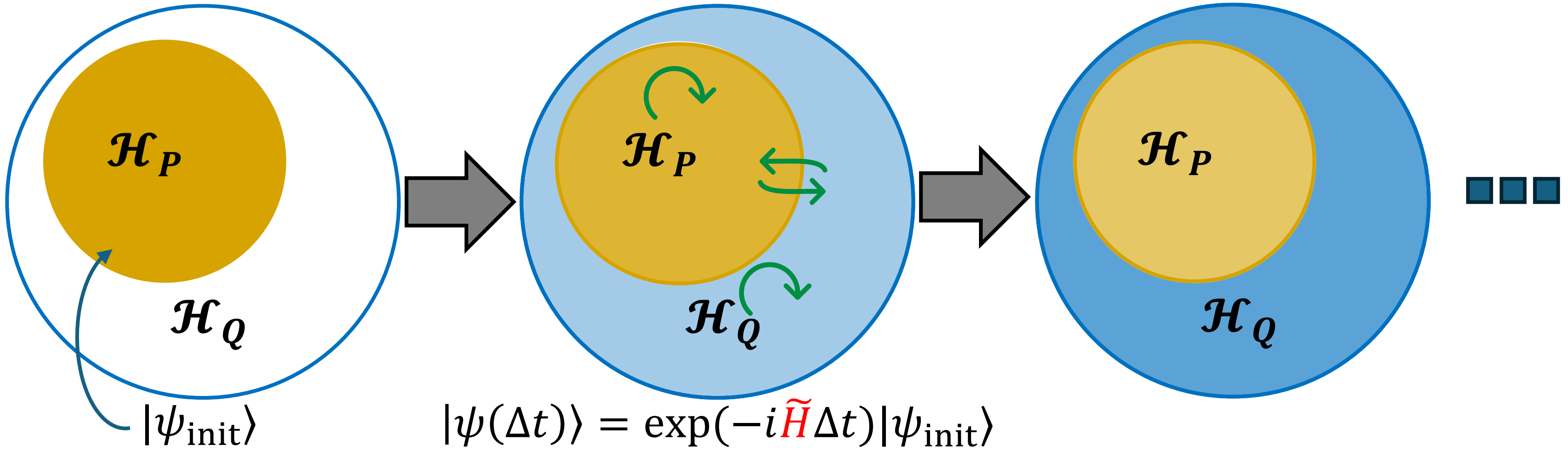


How do we detect and suppress errors?

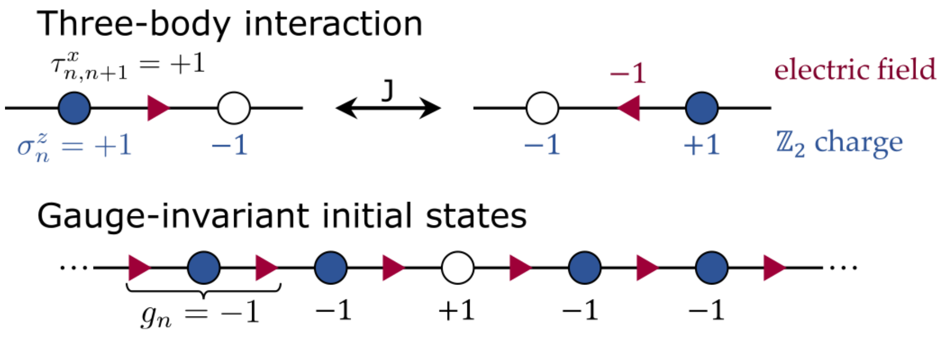
Abelian LGT:

- Post-selection: check if the final state satisfies local symmetries;
- Effective Hamiltonian: energy penalty
- Engineered dissipation: stochastic driving;

Non-Abelian LGT:

- Local symmetry generators do not commute
- Post-selection?

Z_2 LGT in one dimension and coherent error



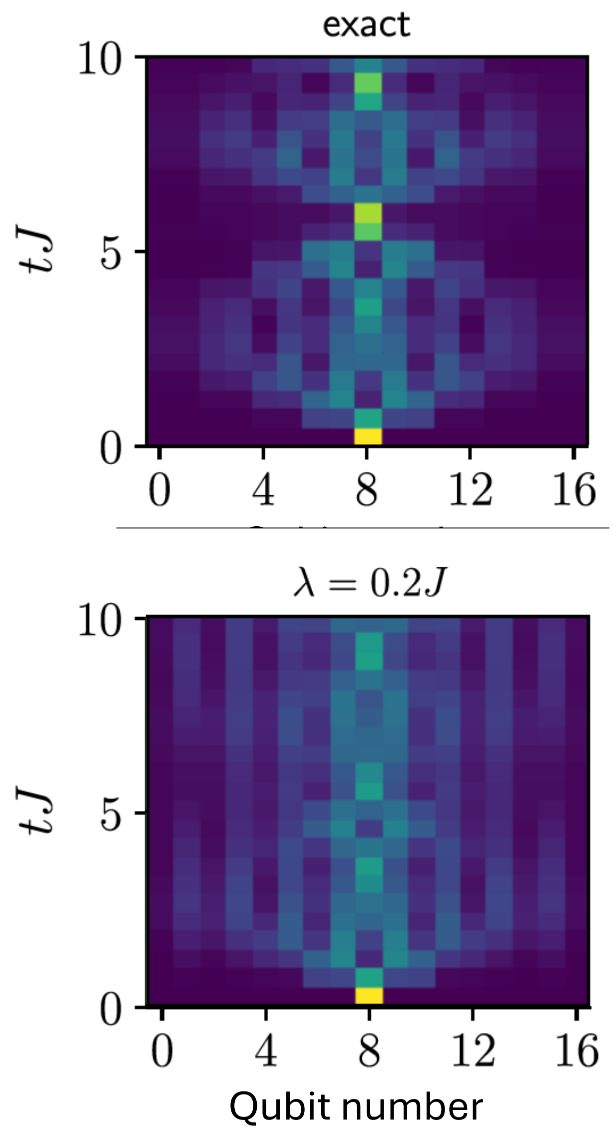
Local gauge charge $\widehat{\Theta}_n = -\tau^x_{n-1,n}\sigma^z_n\tau^x_{n,n+1}$ $\quad [\widehat{H}_0, \widehat{\Theta}_n] = 0$

Gauge-symmetry breaking coherent error

$$\widehat{H}_{\mathrm{err}} = \lambda \sum_n (\sigma^+_n \sigma^-_{n+1} + \mathrm{h.c.}) + \lambda \sum_n \tau^z_{n,n+1}$$

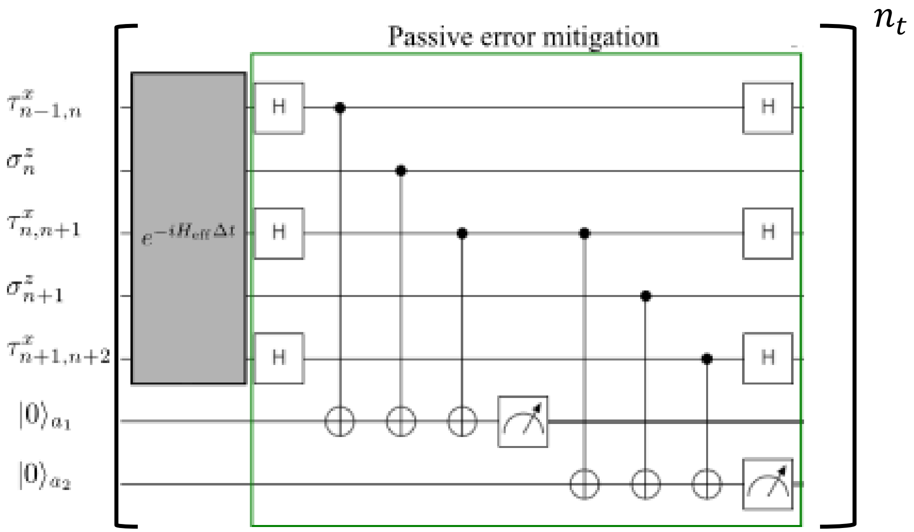
$$[\widehat{H}_{\mathrm{err}}, \widehat{\Theta}_n] \neq 0$$

Digital quantum simulation



Measurement-induced gauge protection in digital quantum simulations

Dynamical post-selection approach



- At each Trotter step $\widehat{\Theta}_n$ is encoded in an auxiliary qubit
- Requires $O(N)$ auxiliary qubits
- No reset of $|a\rangle$

$|\psi(t)\rangle_{\text{phys}} \Rightarrow |\psi(t+\Delta t)\rangle = \alpha|\psi(t+\Delta t)\rangle_{\text{phys}} + \beta|\psi\rangle_{\text{np}}$

Coupling with the auxiliary $|a\rangle$

$|\psi, a\rangle = \alpha|0\rangle_a|\psi\rangle_{\text{phys}} + \beta|1\rangle_b|\psi\rangle_{\text{np}}$

Measure $|a\rangle$

$$|\tilde{\psi}(t+\Delta t)\rangle = \begin{cases} |\psi(t+\Delta t)\rangle_{\text{phys}} & \text{with probability } |\alpha|^2 \rightarrow \text{keep} \\ |\psi\rangle_{\text{np}} & \text{with probability } |\beta|^2 \rightarrow \text{discard} \end{cases}$$

Equation for the density matrix in the continuous time limit

$$\dot{\rho} = -i\hbar[\hat{H}, \rho] + \frac{1}{2\tau}\sum_n \hat{G}_n \rho \hat{G}_n^\dagger - \frac{1}{2}\{\hat{G}_n^\dagger \hat{G}_n, \rho\} = \mathcal{L}\rho$$

Time between measurements

Measurement-induced gauge protection in digital quantum simulations

$$\dot{\rho} = -i\hbar[\hat{H}, \rho] + \gamma \sum_n \hat{G}_n \rho \hat{G}_n^\dagger - \frac{1}{2}\{\hat{G}_n^\dagger \hat{G}_n, \rho\} = \mathcal{L}\rho$$

Possible implementations:

- Engineered dissipation [1]
- Random gauge transformations [2]
- Continuous measurements
- Continuous limit for DPS

[1] Stanningel et al. PRL **112** (2014)
[2] Lamm et al. arxiv:2005.12688

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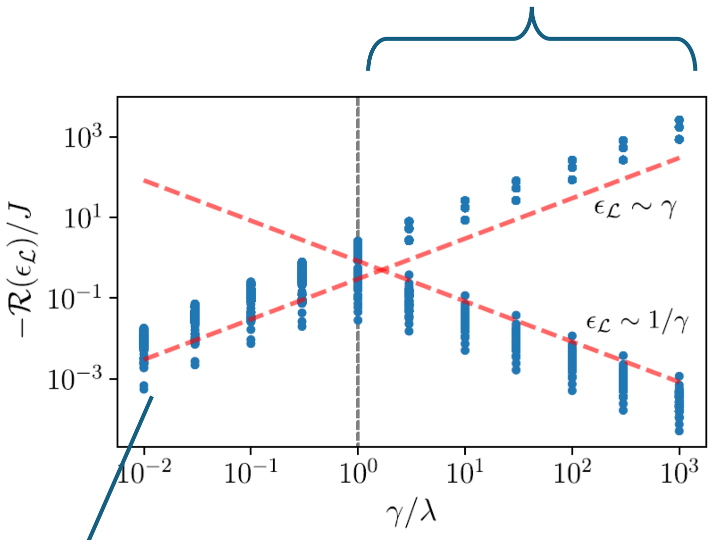
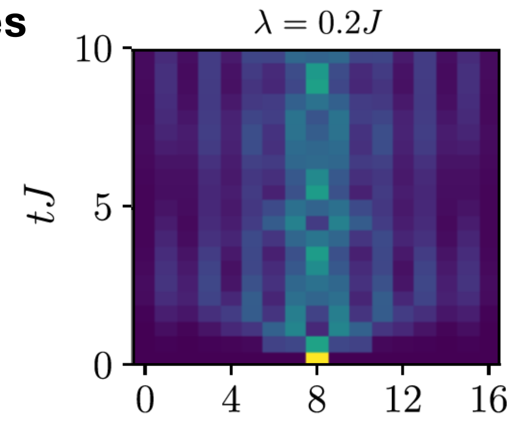
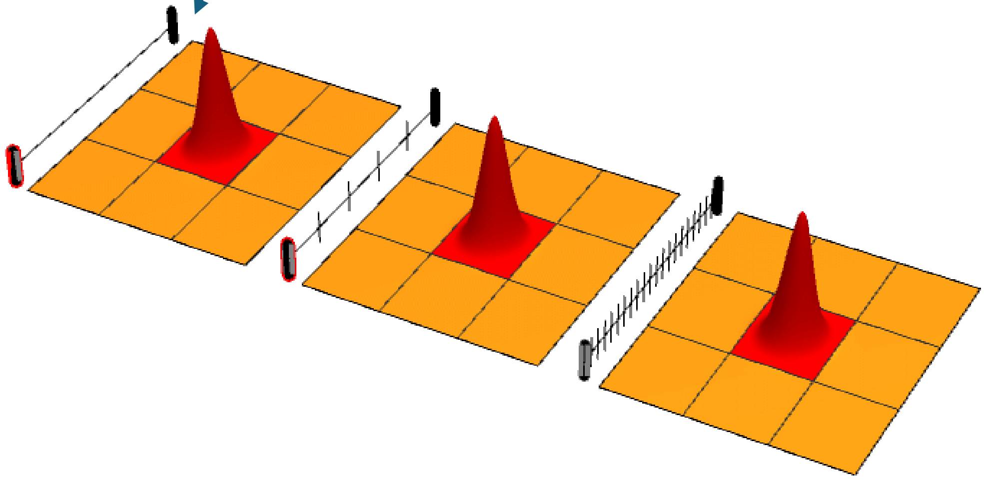
Quantum Zeno transition between protected and chaotic phases

Quantum Zeno regime



Dark states





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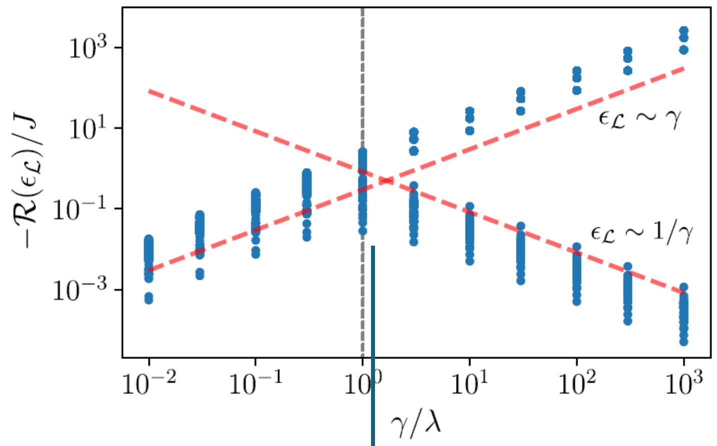

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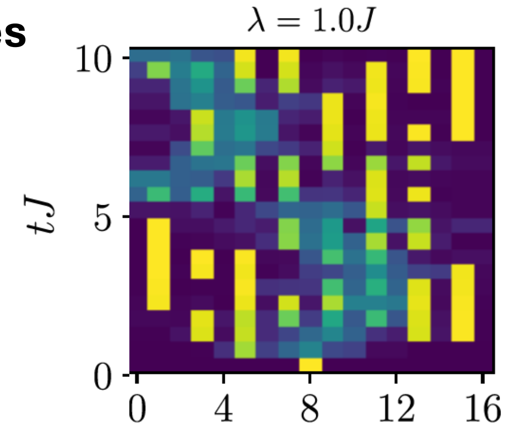

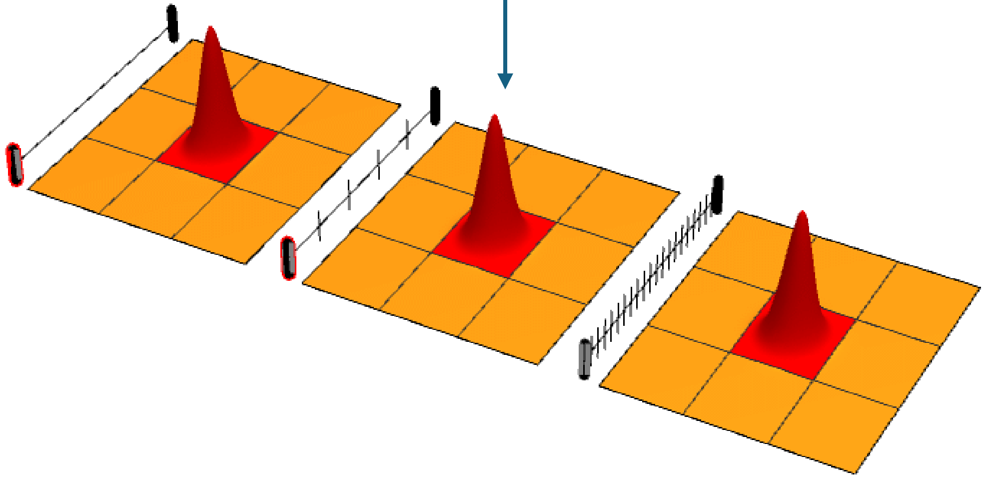

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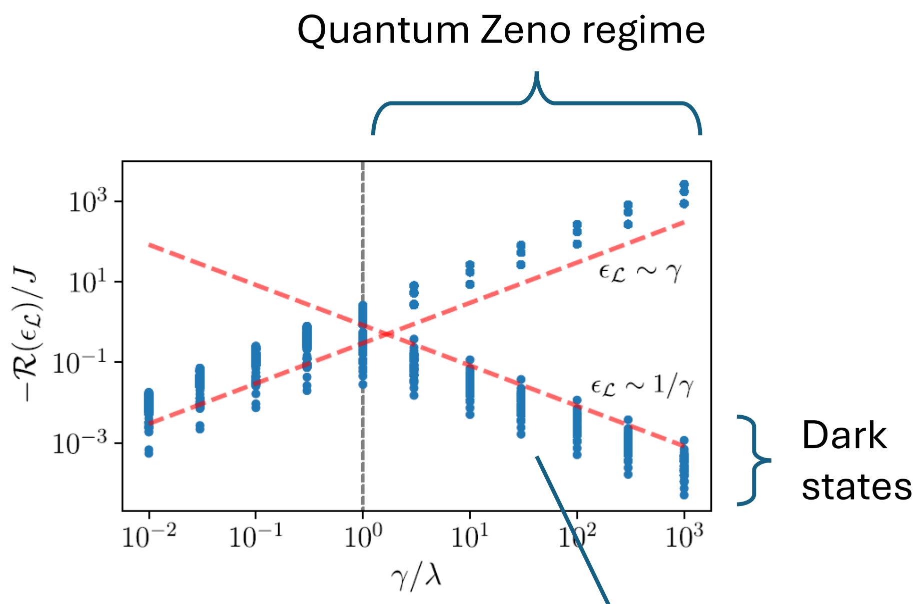

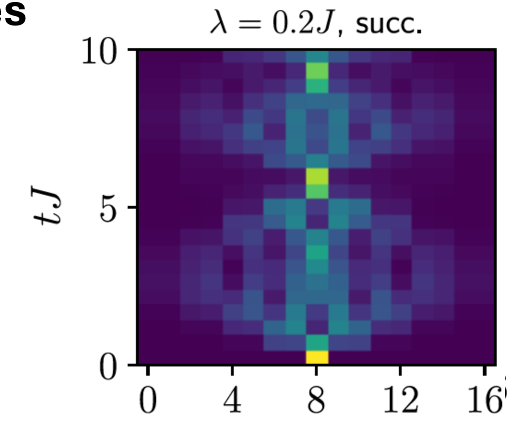

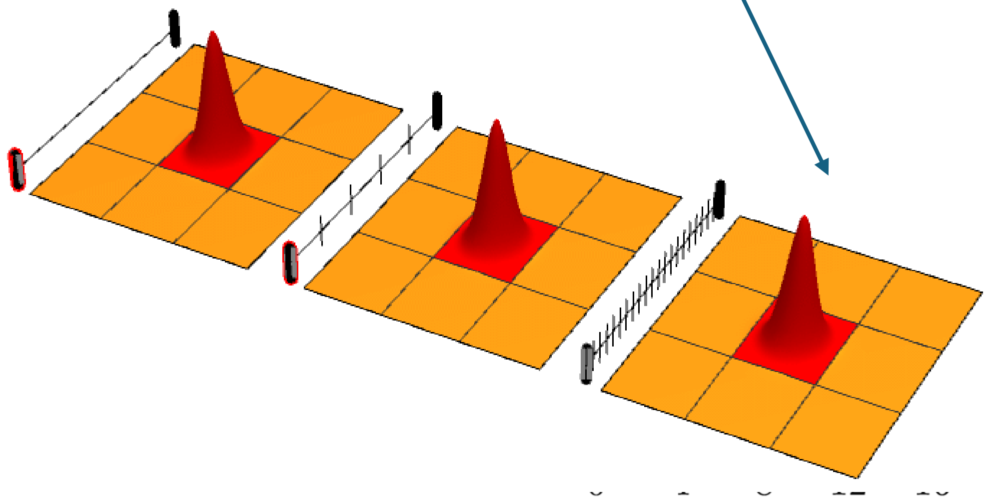

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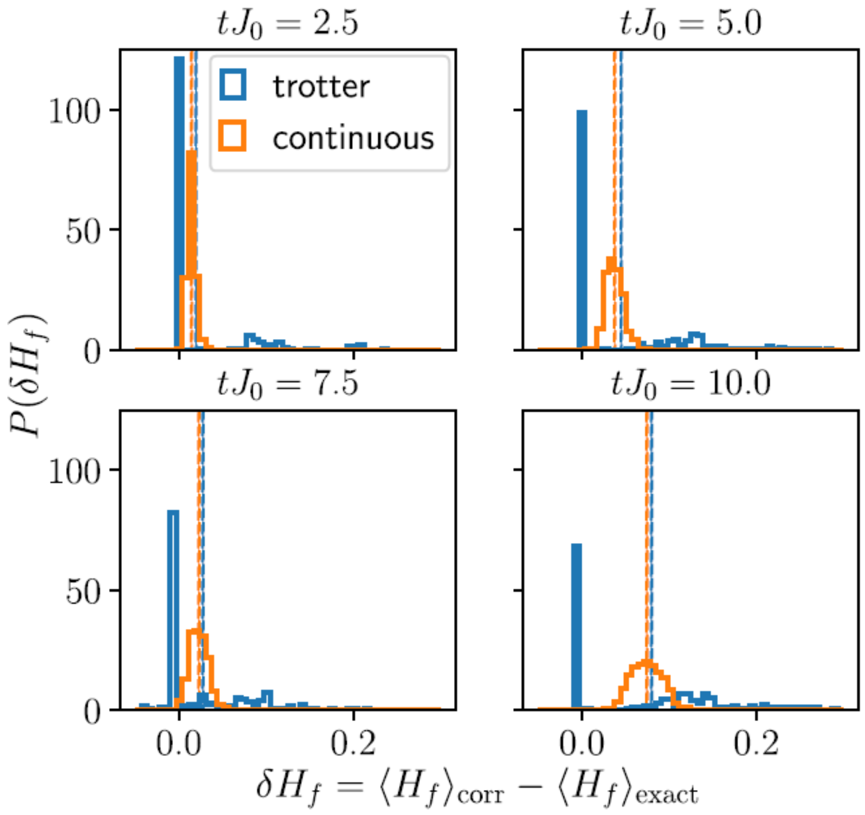

Quantum Zeno transition between protected and chaotic phases

Importance of unraveling: same ensemble average, different stochastic trajectories

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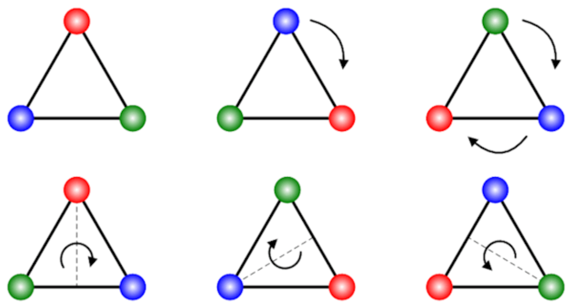
Nonabelian LGT: benchmark on D_3



D_3 gauge symmetry group
Smallest discrete nonabelian group ⇒ “fits” Ca trapped ion qudit platform*.

2+1 dimensional square lattice , pure gauge:

$$\hat{H}_0 = \boxed{-\frac{1}{g^2}\sum_p \mathcal{R}\left[\mathrm{Tr}\left(\hat{U}^j_{p_1}\hat{U}^j_{p_2}\hat{U}^{j\dagger}_{p_3}\hat{U}^{j\dagger}_{p_4}\right)\right]} + \hat{H}_E$$

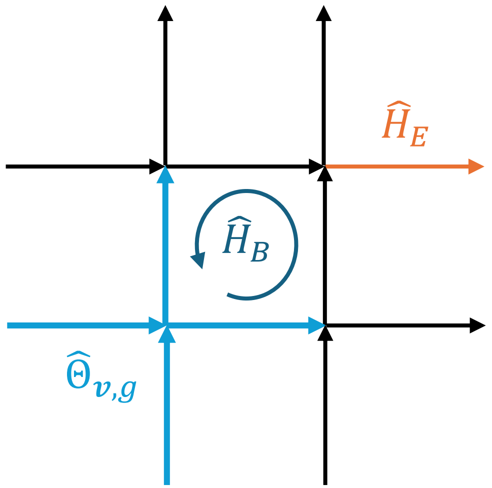
$\hat{H}_B$

$$\left[\hat{H}_B, \hat{H}_E\right] \neq 0$$

Gauge transformation on vertex $\boldsymbol{v}$

$$\hat{\Theta}_{v,g} = \hat{\Theta}^L_{3,g}\hat{\Theta}^L_{4,g}\hat{\Theta}^R_{1,g}\hat{\Theta}^R_{2,g}, \quad \boxed{\left[\hat{\Theta}_{v,g}, \hat{\Theta}_{v,h}\right] \neq 0 \text{ but } \Pi_s\left[\hat{\Theta}_{v,g}, \hat{\Theta}_{v,h}\right]\Pi_s = 0}$$

$$\hat{\Theta}_{v,g}\left|\psi_{\text{phys}}\right\rangle = \left|\psi_{\text{phys}}\right\rangle \quad \forall g \in G, \boldsymbol{v}$$



*Ringbauer et al, Nat Phys 18 (2022)

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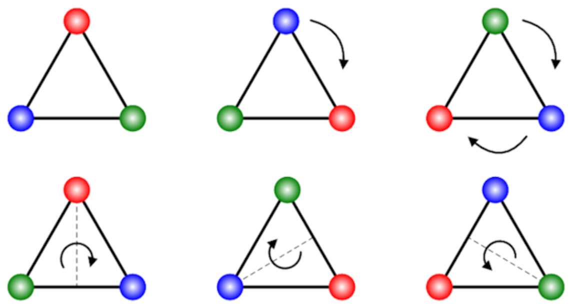

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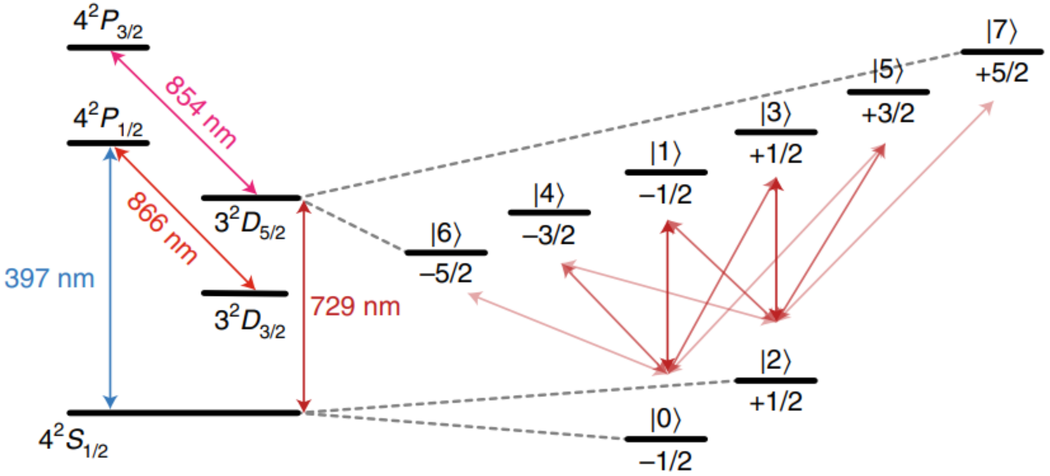

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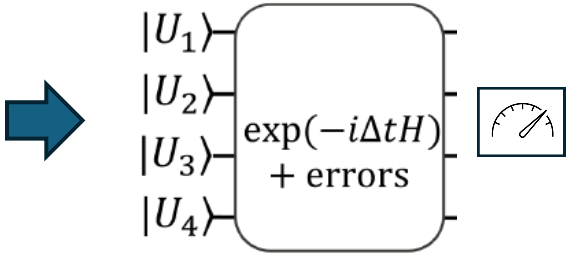
D_3 Dynamical post-selection

What do we measure?

Computational basis

- ✓ Plaquette operator
- ✕ Local gauge charge



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Eigenbasis of $\widehat{\Theta}_{\boldsymbol{v},g}$

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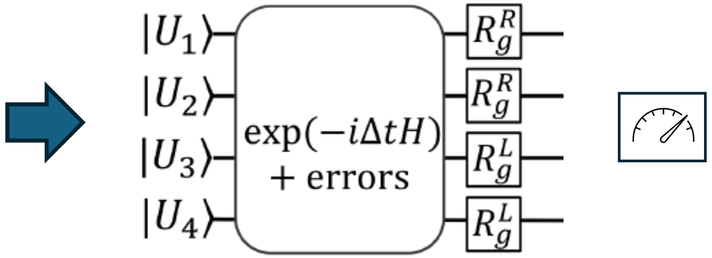

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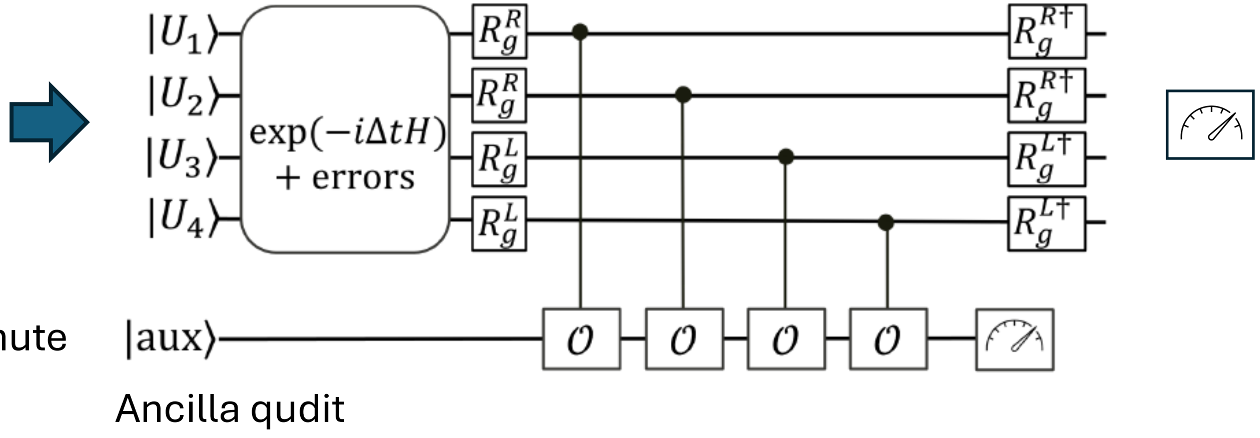
Dynamical post-selection

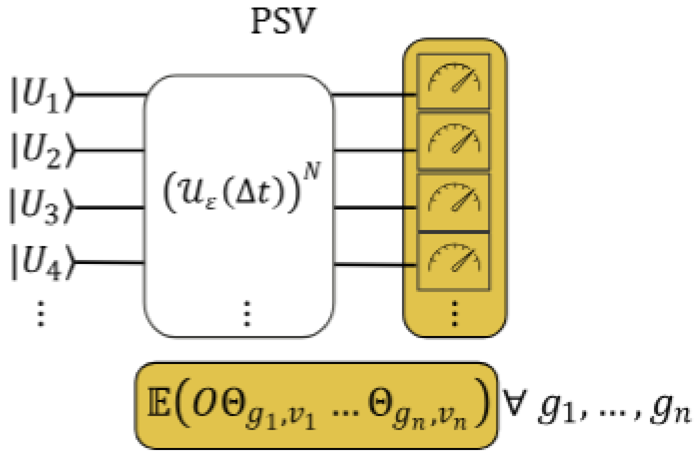


D_3 post-Processed Symmetry Verification (PSV)

Π_s = projector on gauge-symmetry sector

ρ = density matrix after noisy evolution

$$\rho_s = \frac{\Pi_s \rho \Pi_s}{\mathrm{Tr}[\Pi_s \rho]}$$



Symmetry-projected expectation value of gauge-invariant observable

$$\mathrm{Tr}(O\rho_s) = \frac{\mathrm{Tr}[O\Pi_s\rho\Pi_s]}{\mathrm{Tr}[\Pi_s\rho]} = \frac{\mathrm{Tr}[O_s\rho]}{\mathrm{Tr}[\Pi_s\rho]}, \qquad \text{with } O_s = \Pi_s O \Pi_s = \Pi_s O$$

Discrete groups

$$\Pi_s = \prod_{v\in \mathrm{V}} \left[\frac{1}{|G|}\sum_{g\in \mathrm{G}} \Theta_{g,v}\right] = \frac{1}{|G|^{n_v}} \sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \prod_{v\in \mathrm{V}} \Theta_{g_v,v}$$

$$\mathrm{Tr}(O\rho_s) = \frac{\mathrm{Tr}[O\Pi_s\rho]}{\mathrm{Tr}[\Pi_s\rho]} = \frac{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho O \Pi_{v\in \mathrm{V}} \Theta_{g_v,v}]}{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho \Pi_{v\in \mathrm{V}} \Theta_{g_v,v}]}$$

→ Effective group symmetrization by averaging over multiple observables

Numerical Results

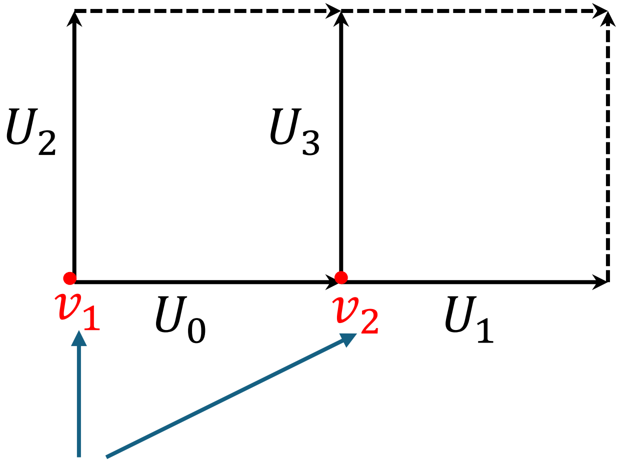
Noise model: random unitaries close to the identity

$$|\psi(t)\rangle = \exp(-iHt)\,|\psi(0)\rangle$$
$$\cong \left[\mathcal{U}_{\varepsilon(\gamma)} \exp\left(-\frac{iH_E t}{N}\right) \exp\left(-\frac{iH_B t}{N}\right)\right]^N |\psi(0)\rangle$$

$$\hat{H}_0 = -\frac{1}{g^2}\sum_p \mathcal{R}\left[\mathrm{Tr}\left(\hat{U}^j_{p_1}\hat{U}^j_{p_2}\hat{U}^{j\dagger}_{p_3}\hat{U}^{j\dagger}_{p_4}\right)\right] + \hat{H}_E$$

Two plaquettes with PBC

$$\dim\mathcal{H}_{\mathrm{tot}} = 6^4 = 1296, \qquad \dim\mathcal{H}_{\mathrm{phys}} = 49$$

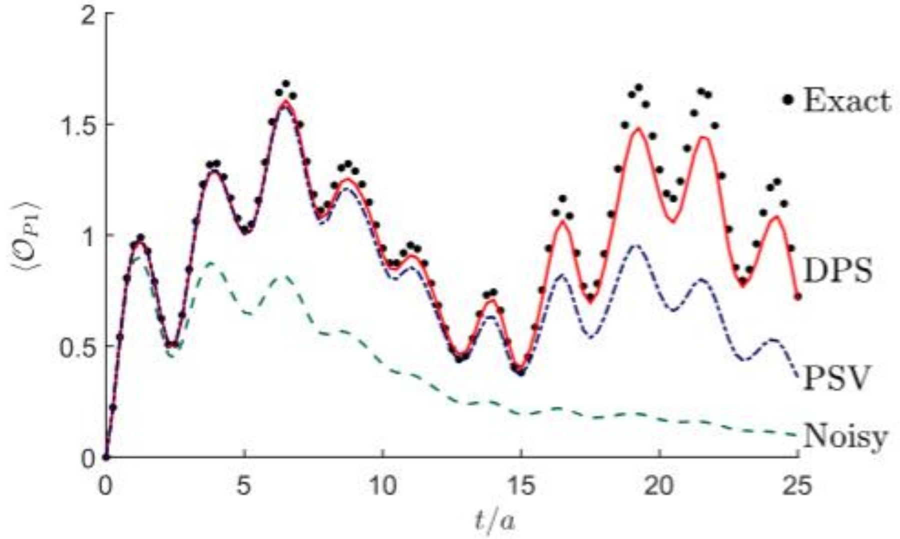


Two vertices

$$\hat{\Theta}_{v_1,g} = \hat{\Theta}^L{}_{0,g}\hat{\Theta}^L{}_{2,g}\,\hat{\Theta}^R{}_{0,g}\hat{\Theta}^R{}_{1,g}$$
$$\hat{\Theta}_{v_2,g} = \hat{\Theta}^L{}_{1,g}\hat{\Theta}^L{}_{3,g}\,\hat{\Theta}^R{}_{0,g}\hat{\Theta}^R{}_{3,g}$$

Quench protocol:

- **DPS:** Each trotter step, measure **one** local charge
- **PSV:** 16 independent observables to sample



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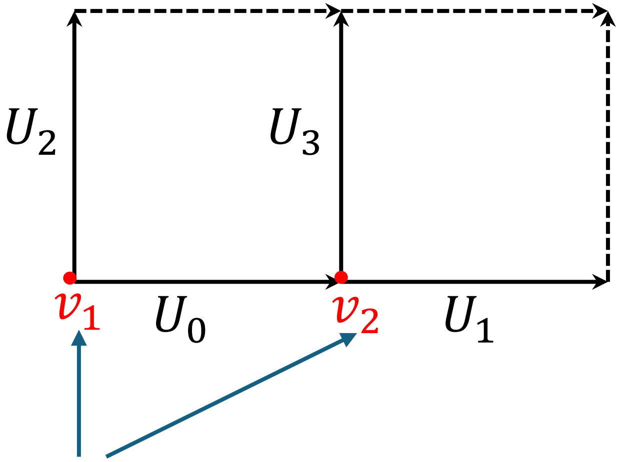

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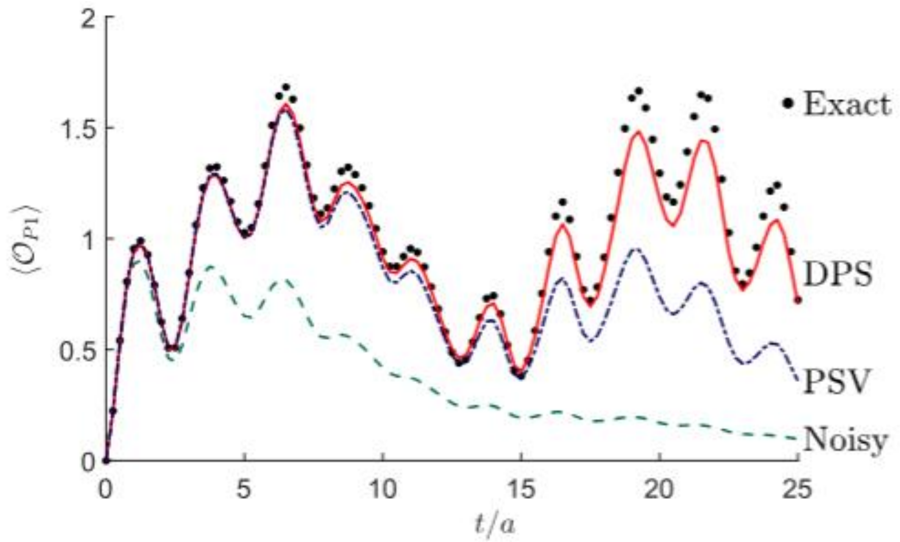

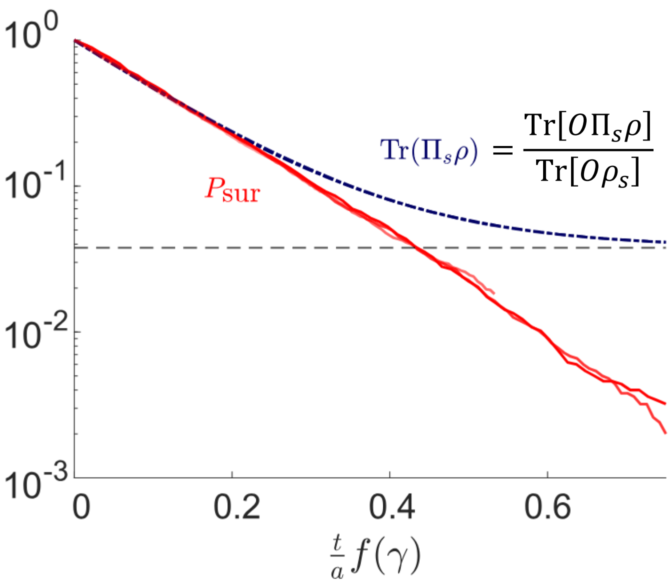

Conclusions & outlook

Abelian: PhysRevB.111.094315 non-Abelian: *Quantum* 9 (2025) 1802

Two post-selection approaches general symmetries, tested for non-Abelian systems

- Dynamical post selection
  - Mid-circuit measurements
  - Entangling gates
  - Measurements and reset are slow

→

- Optimize measurement strategies
- Local observable may not require full gauge invariance

- Post-processed symmetry verification
  - "Cheap" extra circuitry
  - Exponential number of observables

→

- Identify commensurate observables

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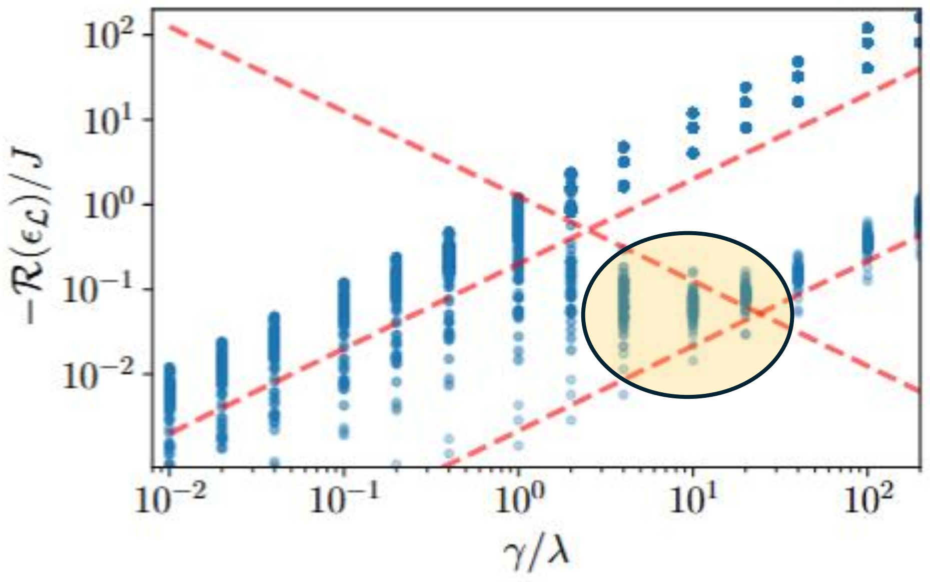
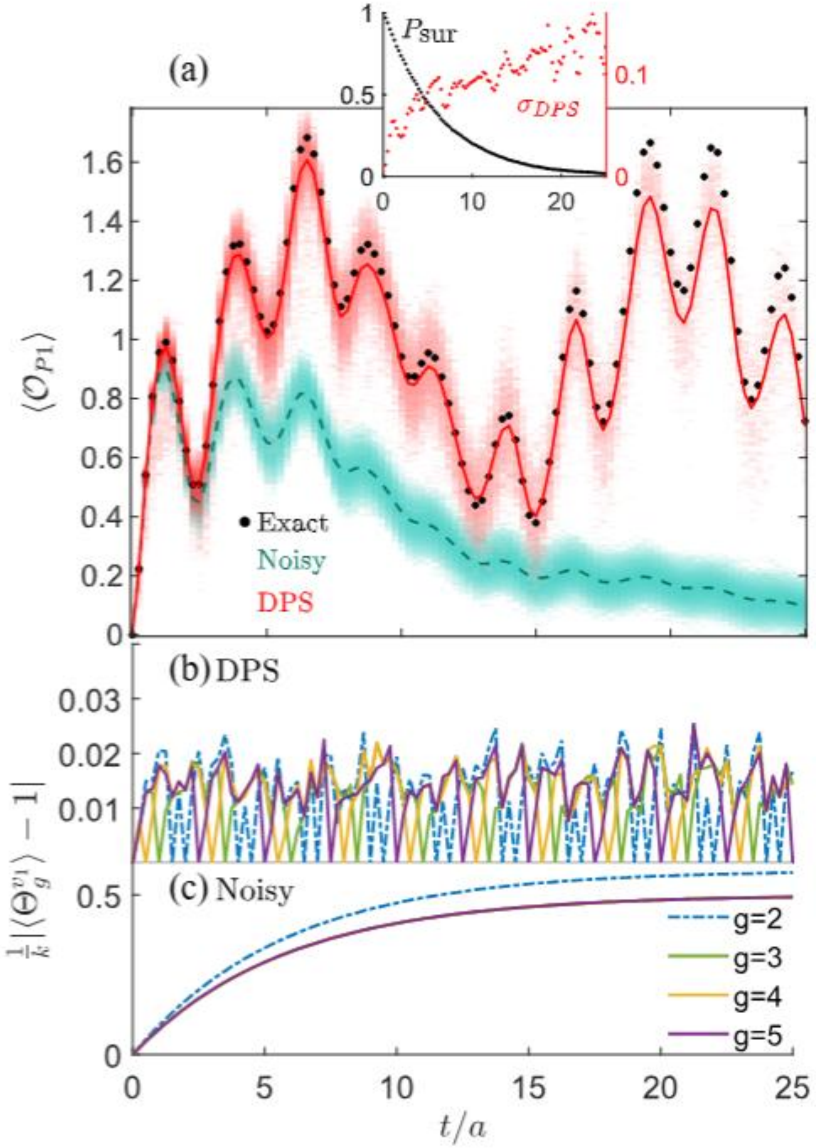
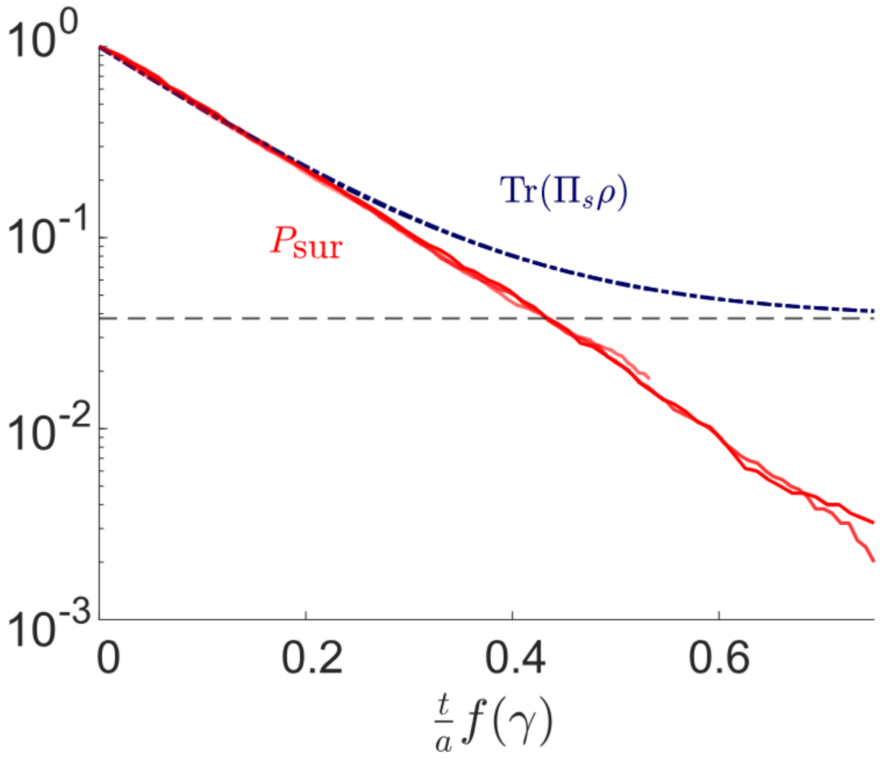
When we measure $\hat{\Theta}_n = -\tau^z_{n-1,n}\sigma^z_n\tau^z_{n,n+1}$,
we apply small random rotations around the **x** and **z** axes.



Numerical Results

$$\mathrm{Tr}(O\rho_s) = \frac{\mathrm{Tr}[O\Pi_s\rho]}{\mathrm{Tr}[\Pi_s\rho]} = \frac{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho O \Pi_{v\in \mathrm{V}}\Theta_{g_v,v}]}{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho \Pi_{v\in \mathrm{V}}\Theta_{g_v,v}]}$$





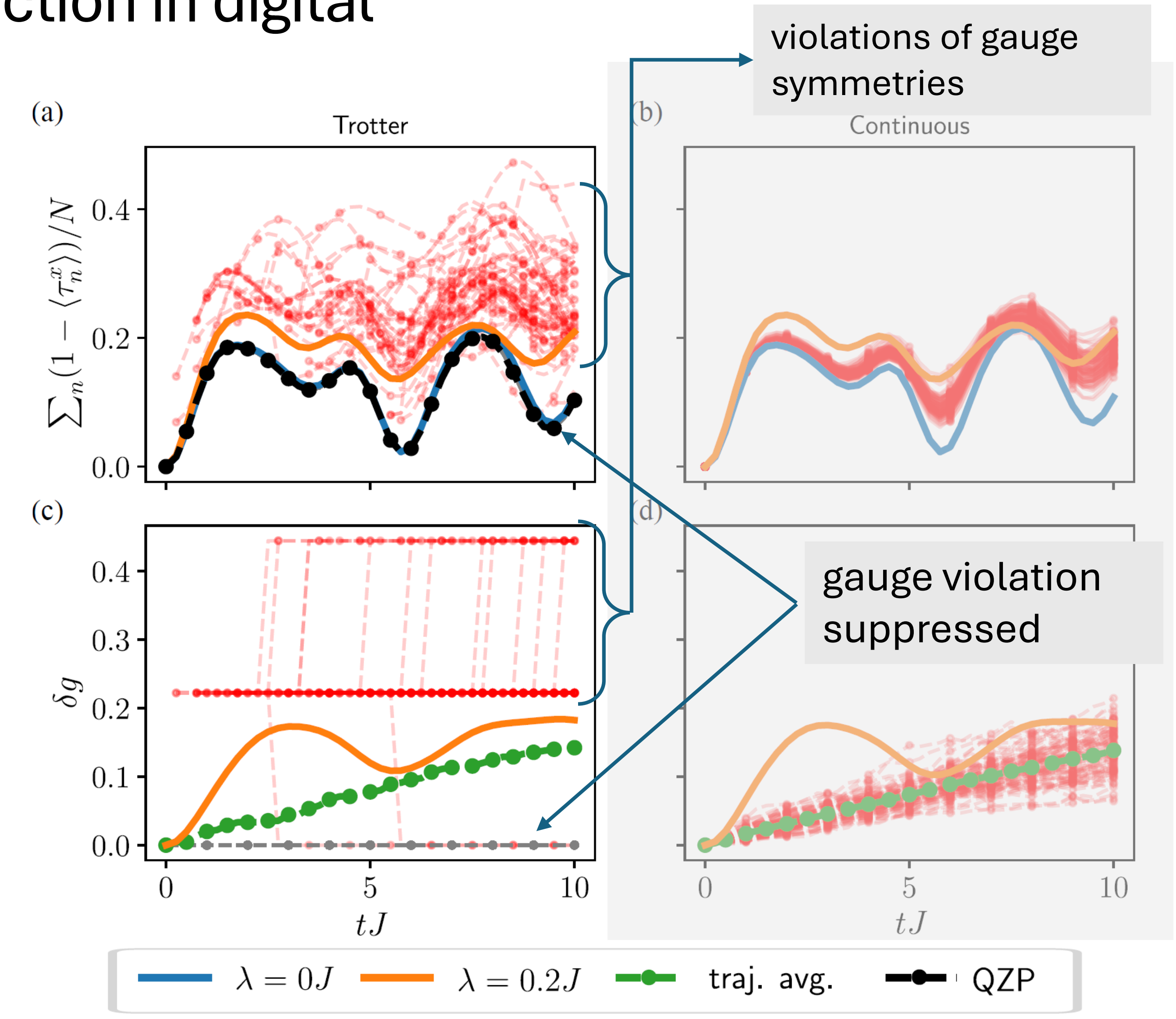
Measurement-induced gauge protection in digital quantum simulations

$$\dot{\rho} = -i\hbar[\hat{H},\rho] + \gamma \sum_n \hat{G}_n \rho \hat{G}_n^\dagger - \frac{1}{2}\{\hat{G}_n^\dagger \hat{G}_n, \rho\} = \mathcal{L}\rho$$

Possible implementations:

- Continuous measurements
- Continuous limit for DPS

Digital vs Analog: same ensemble average, different stochastic trajectories



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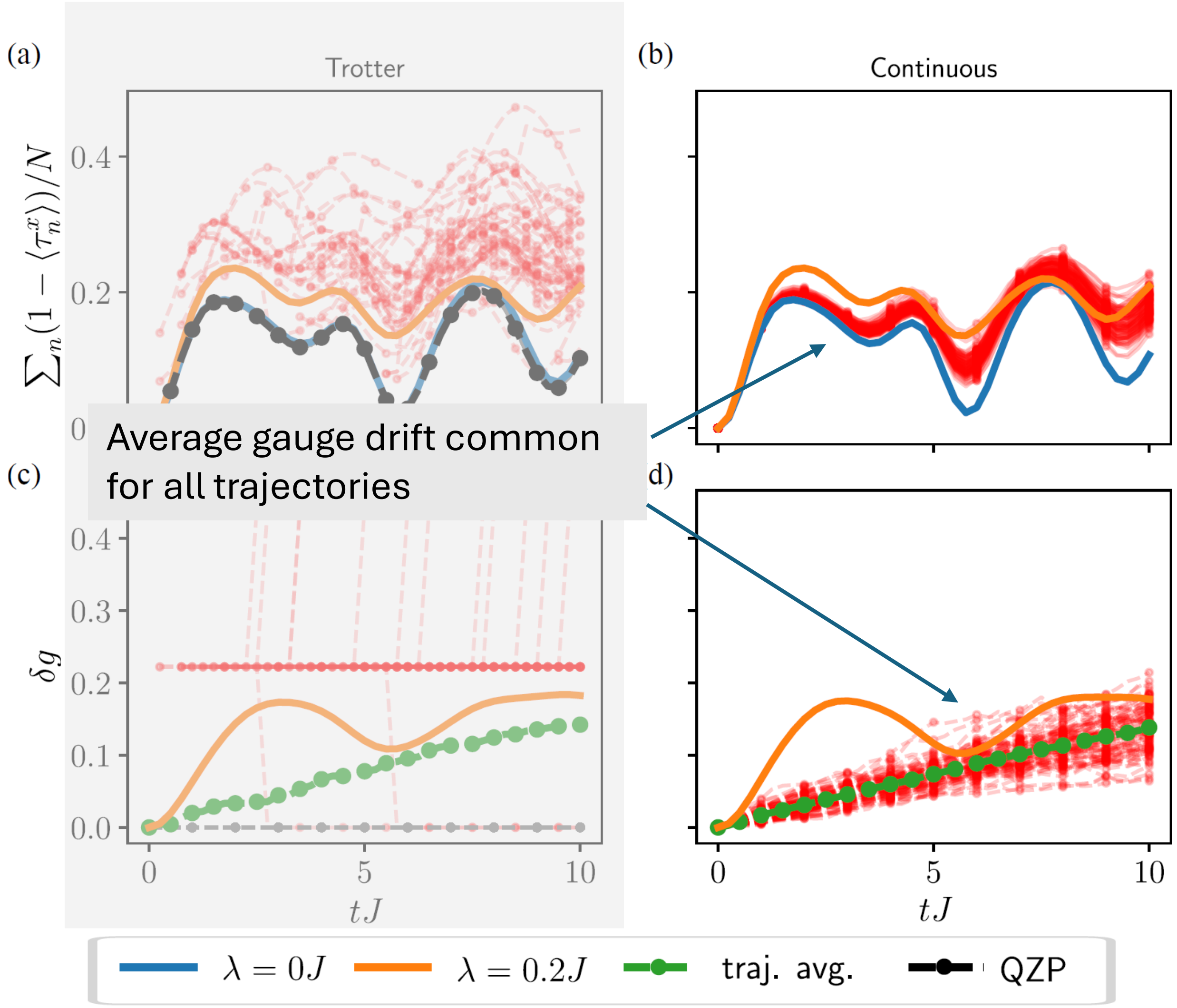

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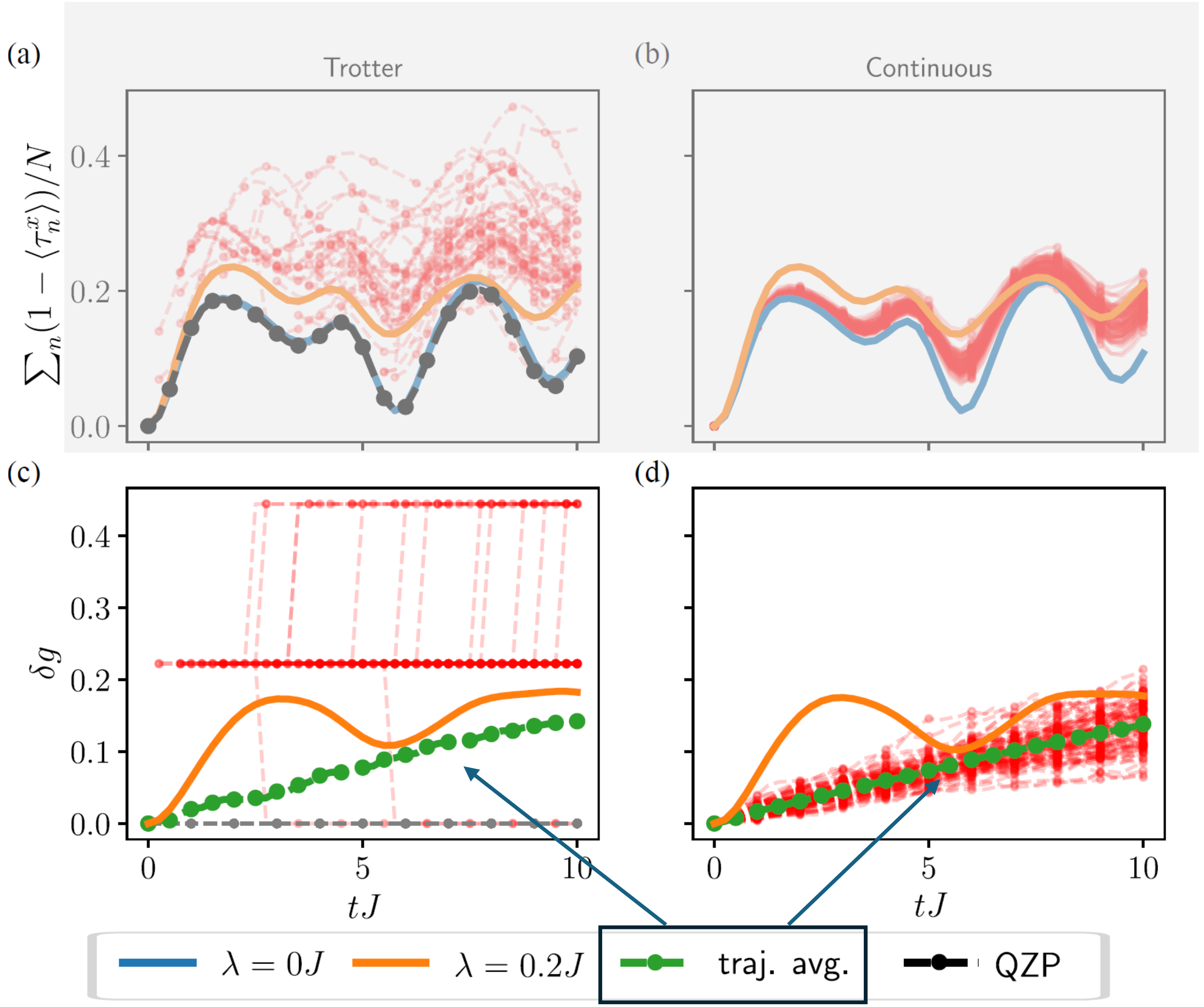


Non-Abelian LGT: post-Processed Symmetry Verification (PSV)

The projection of a noisy outcome ρ of a simulation onto the symmetry-preserving subspace (defined by Π_s) is

$$\rho_s = \frac{\Pi_s \rho \Pi_s}{\mathrm{Tr}[\Pi_s \rho]}$$

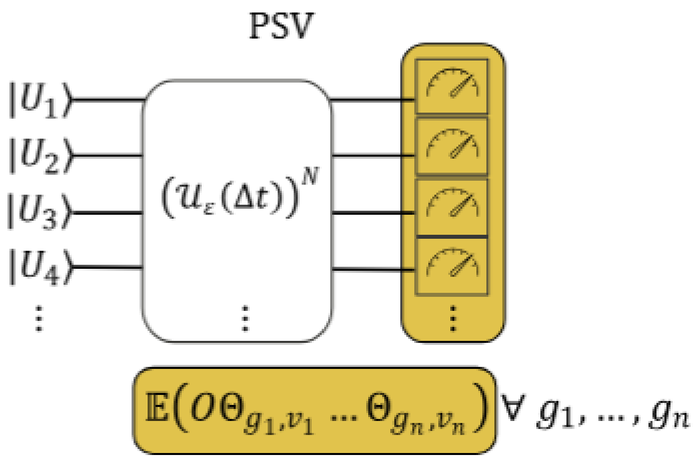
The expectation value of an observable w.r.t. the symmetry-projected state reads

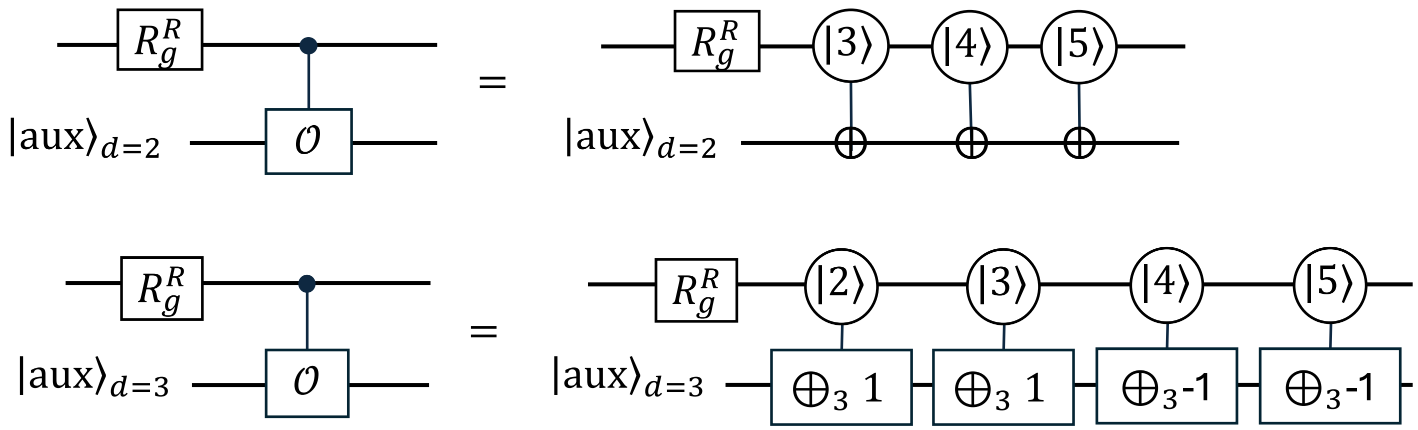
$$\mathrm{Tr}(O\rho_s) = \frac{\mathrm{Tr}[O\Pi_s\rho\Pi_s]}{\mathrm{Tr}[\Pi_s\rho]} = \frac{\mathrm{Tr}[O_s\rho]}{\mathrm{Tr}[\Pi_s\rho]}, \text{with } O_s = \Pi_s O \Pi_s$$

Since for discrete groups

$$\Pi_s = \prod_{v\in \mathrm{V}} \frac{1}{|G|} \sum_{g\in \mathrm{G}} \Theta_{g,v} = \frac{1}{|G|^{n_v}} \sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \prod_{v\in \mathrm{V}} \Theta_{g_v,v}$$

$$\mathrm{Tr}(O\rho_s) = \frac{\mathrm{Tr}[O\Pi_s\rho]}{\mathrm{Tr}[\Pi_s\rho]} = \frac{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho O \Pi_{v\in \mathrm{V}} \Theta_{g_v,v}]}{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho \Pi_{v\in \mathrm{V}} \Theta_{g_v,v}]}$$





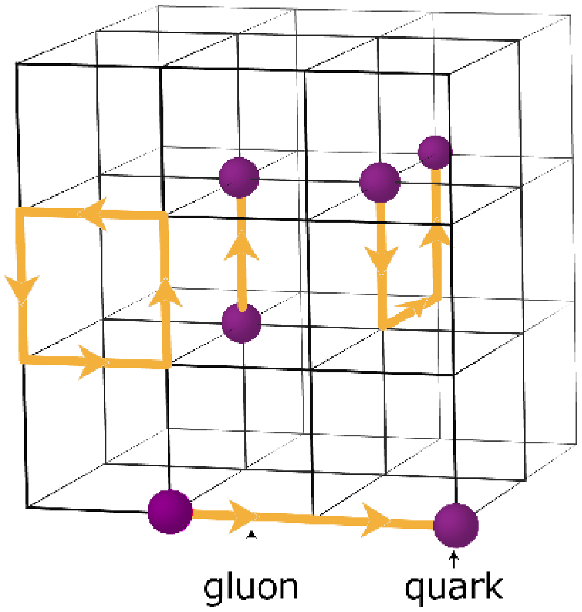
g is a reflection

$R_g^{R\dagger}\widehat{\Theta}_g^R R_g^R = \mathrm{diag}(1,1,1,-1,-1,-1)$

g is a rotation

$R_g^{R\dagger}\widehat{\Theta}_g^R R_g^R =$
$\mathrm{diag}(1,1,e^{2\pi i/3},e^{2\pi i/3},e^{-2\pi i/3},e^{-2\pi i/3})$

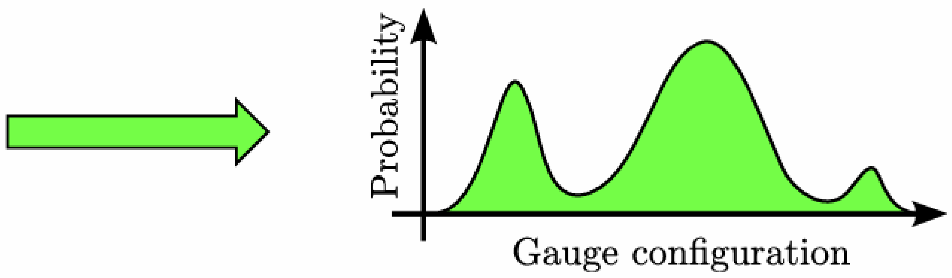
Lattice QCD



Credit: Lattice QCD GPU Inverters on ROCm Platform

- QCD Lagrangian on a discrete spacetime grid, Wick rotation to Euclidean time;
- Observables are calculated using the Path Integral formalism;
- Monte Carlo methods for probability distribution of gauge configurations.

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z_0} \int \mathcal{D}[\bar{\psi}, \psi]\mathcal{D}[A] e^{-\int d^4 x_E \mathcal{L}_E} \mathcal{O}$$



Lattice QCD

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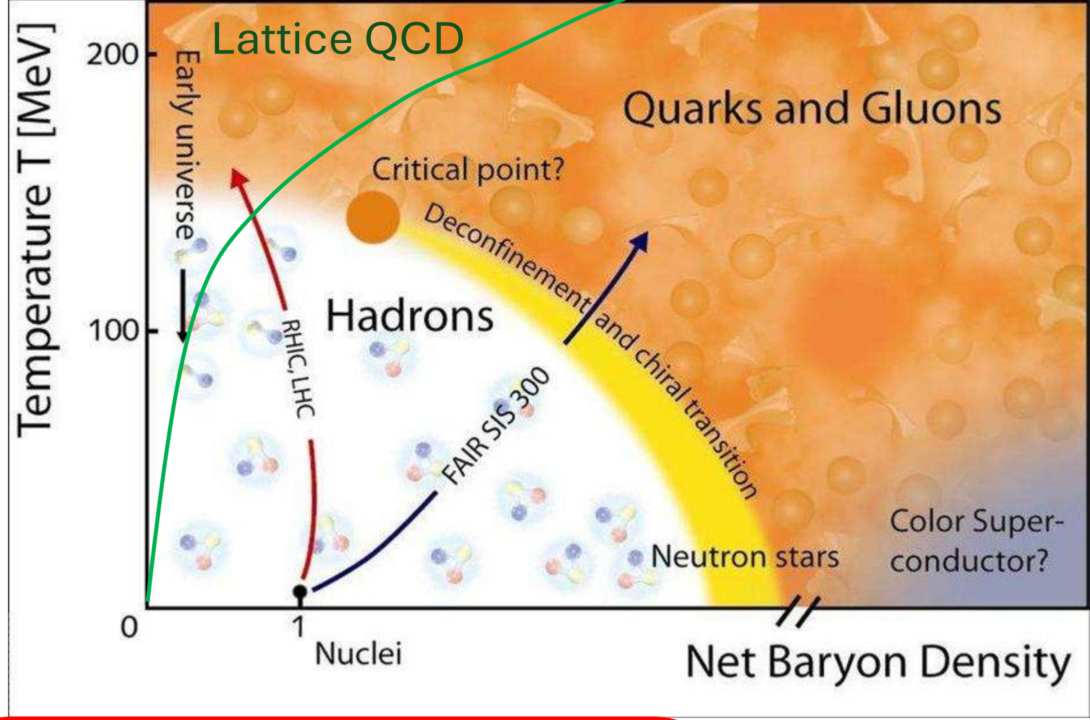


Successes

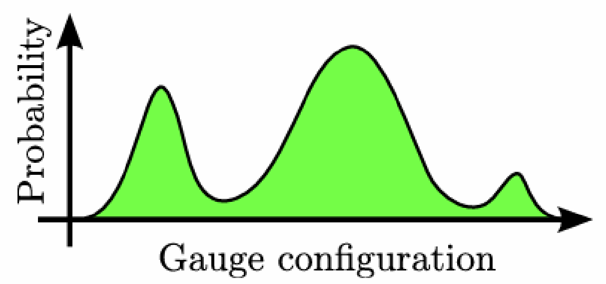
- ✓ Hadron spectrum and exotic states
- ✓ Hadron form factors
- ✓ Quark masses and the strong coupling costant
- ✓ Decay rates and low energy constants
- ✓ Two- and three-body scattering amplitudes

Shortcomings

- ✗ QCD phase diagram:
  Sign problem:
  Statistical weights are not positive
- ✗ Euclidean time:
  Real time evolution of system
- ✗ Many-body processes are harder to obtain

Lattice QCD

$$\langle \hat{O} \rangle = \frac{1}{Z_0} \int \mathcal{D}[\bar{\psi}, \psi] \mathcal{D}[A] e^{-\int d^4 x_E \mathcal{L}_E} \mathcal{O}$$



Hamiltonian Formulation

$$\langle \hat{O}(t) \rangle = \langle 0 | e^{iHt} \hat{O}(0) e^{-iHt} | 0 \rangle$$

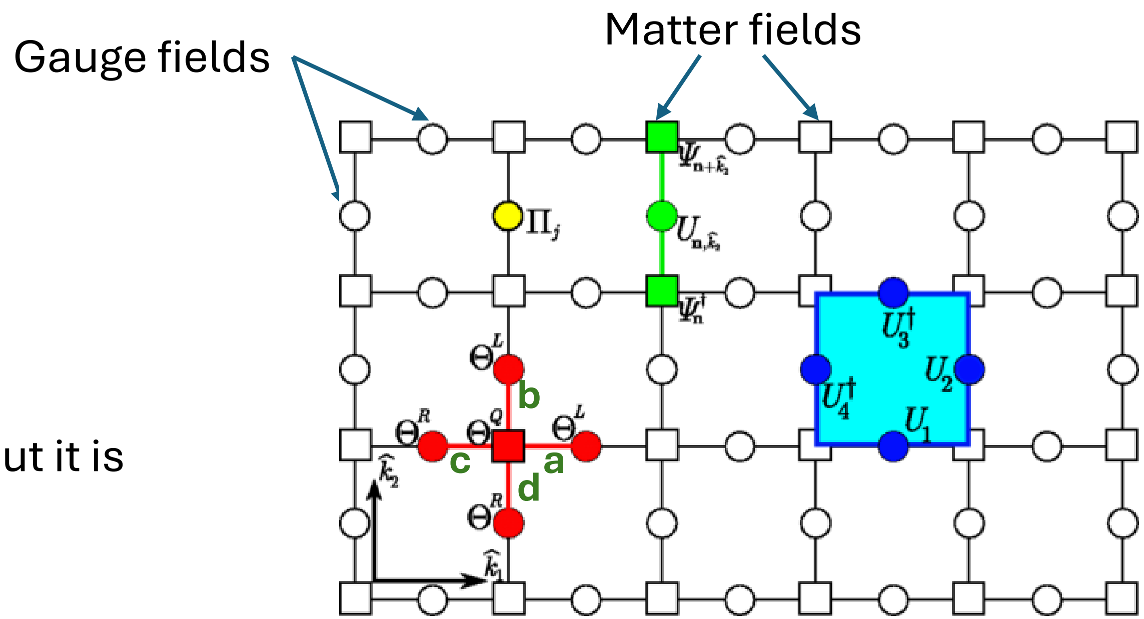
1. No sign problem
2. Both real- and imaginary-time evolution
3. Many-body processes and scattering
4. Hilbert space scales exponentially with the system size

Quantum simulation

Shortcomings

- QCD phase diagram:
  Sign problem:
  Statistical weights are not positive
- Euclidean time:
  Real time evolution of system
- Many-body processes are harder to obtain

Quantum simulations of lattice gauge theories DA RIFARE



Zohar and Burrello, PRD, 2015

Local symmetry, eg $\nabla \cdot \boldsymbol{E} = -\rho$

How we store information

$$|\psi\rangle = |\ldots \mathrm{U_a U_b U_c U_d \Phi_Q} \ldots\rangle$$

The dimension of the Hilbert space grows exponentially, but it is partitioned in gauge sectors described by **local gauge symmetries**.

Symmetry group $g \in G$, group elements represent Gauge fields on lattice links.

Form of local symmetries

$$\Theta_{g,v} = \mathbb{I}\otimes\mathbb{I} \ldots \Theta^L_{g,\mathbf{a}}\Theta^L_{g,\mathbf{b}}\Theta^R_{g,\mathbf{c}}\Theta^R_{g,\mathbf{d}}\Theta^Q_g \ldots \mathbb{I}\otimes\mathbb{I}, \;\; g \in G, \text{unitary, not Hermitian}$$

$$[H, \Theta_{g,v}] = 0$$

$$\Theta_{g,v}|\psi\rangle_{\text{phys}} = 1|\psi\rangle_{\text{phys}}$$

Measurement-induced gauge protection in digital quantum simulations

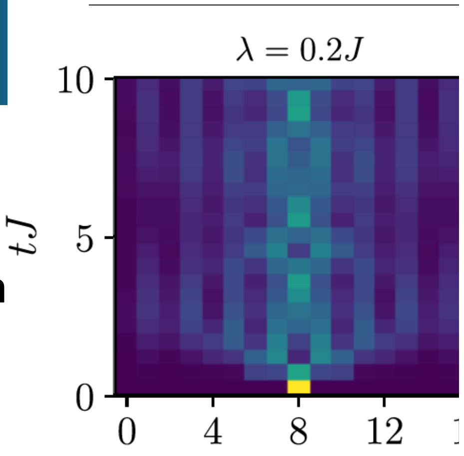
Taking the continuous time limit

$$\dot{\rho} = -i\hbar[\hat{H},\rho] + \gamma \sum_n \hat{\Theta}_n \rho \hat{\Theta}_n^\dagger - \frac{1}{2}\{\hat{\Theta}_n^\dagger \hat{\Theta}_n, \rho\} = \mathcal{L}\rho$$

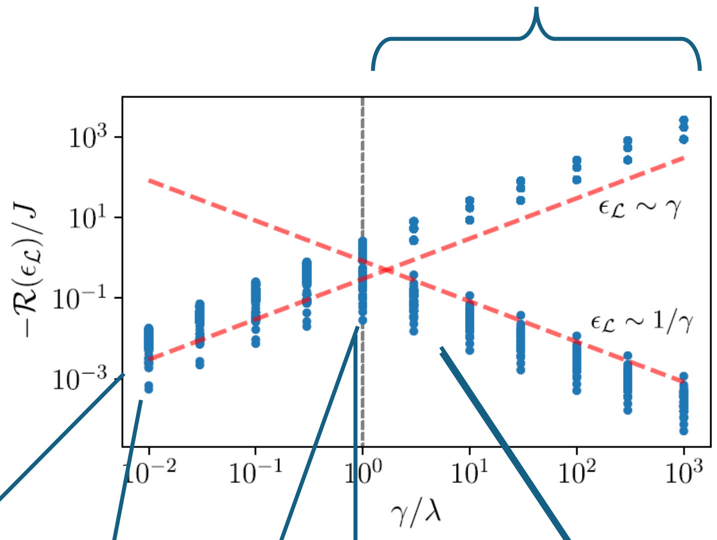
Possible implementations:

- Engineered dissipation [1]
- Random gauge transformations [2]
- Continuous measurements
- Continuous limit for DPS

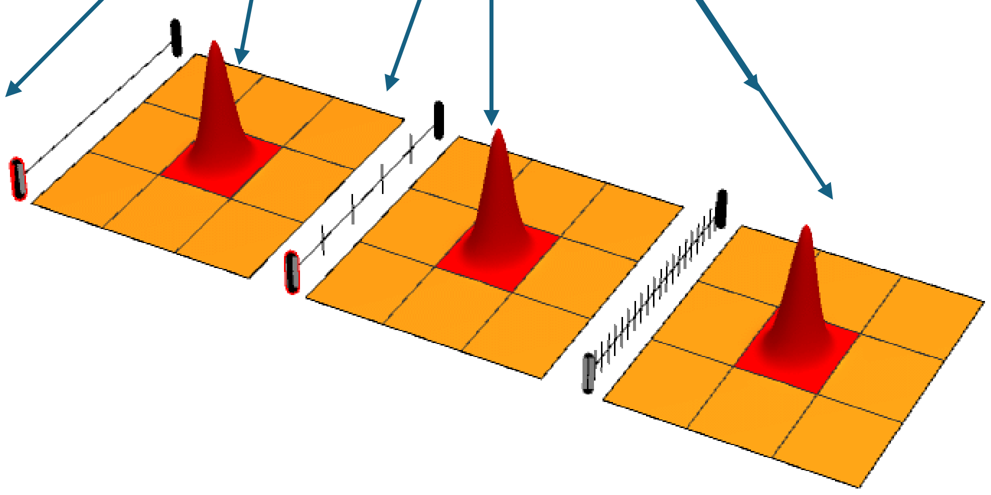
Quantum Zeno transition between protected and chaotic phases



Quantum Zeno regime



Dark states, stabilized by measurements



By JozumBjada - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=112625244

[1] Stanningel et al. PRL **112** (2014)
[2] Lamm et al. arxiv:2005.12688

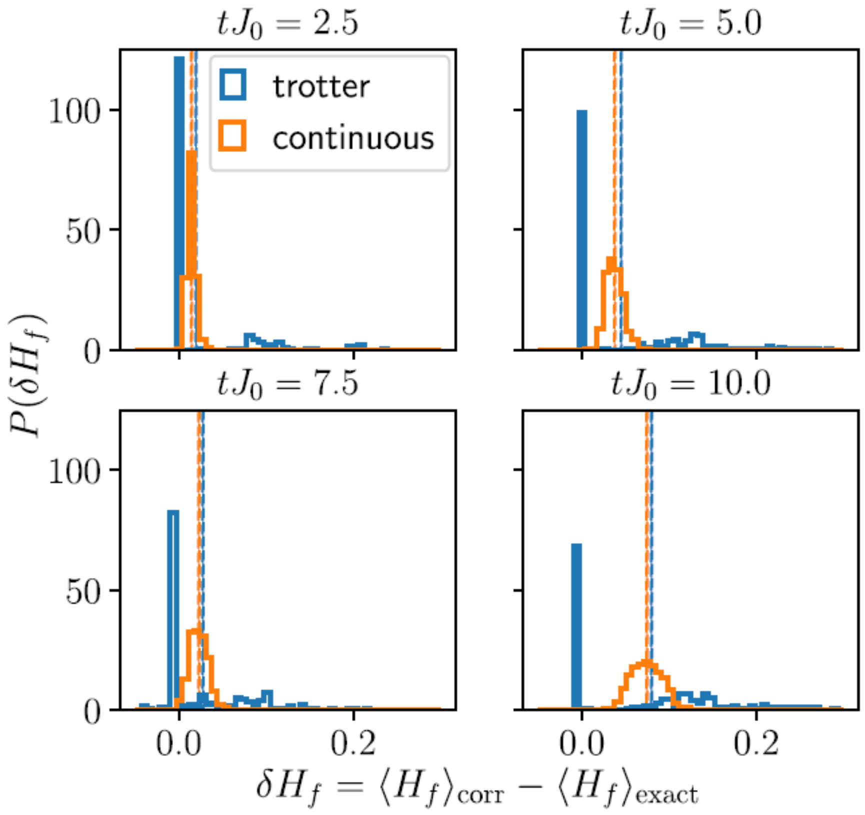
Measurement-induced gauge protection in digital quantum simulations

$$\dot{\rho} = -i\hbar[\hat{H}, \rho] + \gamma \sum_n \hat{G}_n \rho \hat{G}_n^\dagger - \frac{1}{2}\{\hat{G}_n^\dagger \hat{G}_n, \rho\} = \mathcal{L}\rho$$

Possible implementations:

- Continuous measurements
- Continuous limit for DPS

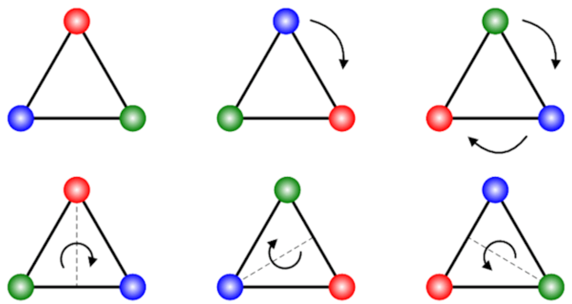
Digital vs Analog: same ensemble average, different stochastic trajectories



Non-Abelian LGT: D_3 and a qudit approach

D_3 gauge symmetry group

- Smallest discrete nonabelian group ⇒ "fits" Ca trapped ion qudit platform.
- LGTs: natural application for qudit quantum hardware. Qudits efficiently represent high-dimensional gauge fields: local operations act on a single gauge field ($|G| \leq d$);



Non-Abelian LGT

D_3 gauge symmetry group

2+1-dimensional square lattice, pure gauge:

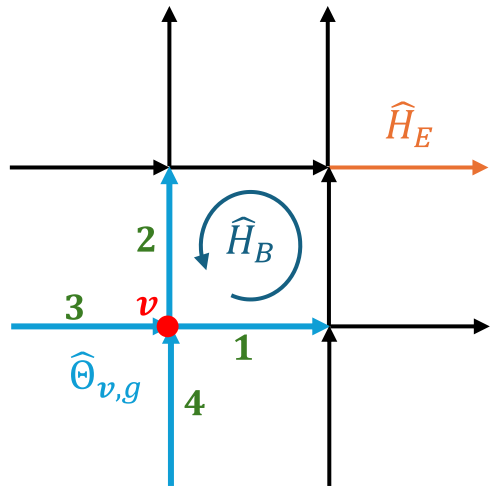
$$\hat{H}_0 = \underbrace{-\frac{1}{g^2}\sum_p \mathcal{R}\left[\mathrm{Tr}\left(\hat{U}^{j}_{p_1}\hat{U}^{j}_{p_2}\hat{U}^{j\dagger}_{p_3}\hat{U}^{j\dagger}_{p_4}\right)\right]}_{\hat{H}_B} + \hat{H}_E$$

$$\left[\hat{H}_B, \hat{H}_E\right] \neq 0$$

Gauge transformations on vertex $\boldsymbol{v}$

$$\hat{\Theta}_{\boldsymbol{v},g} = \hat{\Theta}^L_{\mathbf{1},g}\hat{\Theta}^L_{\mathbf{2},g}\hat{\Theta}^R_{\mathbf{3},g}\hat{\Theta}^R_{\mathbf{4},g}, \qquad \left[\hat{\Theta}_{\boldsymbol{v},g}, \hat{\Theta}_{\boldsymbol{v},h}\right] \neq 0$$

$$\hat{\Theta}_{\boldsymbol{v},g}\left|\psi_{\text{phys}}\right\rangle = \left|\psi_{\text{phys}}\right\rangle \quad \forall g \in G, \boldsymbol{v}$$



Non-Abelian LGT: Dynamical Post Selection (DPS)

What do we measure?

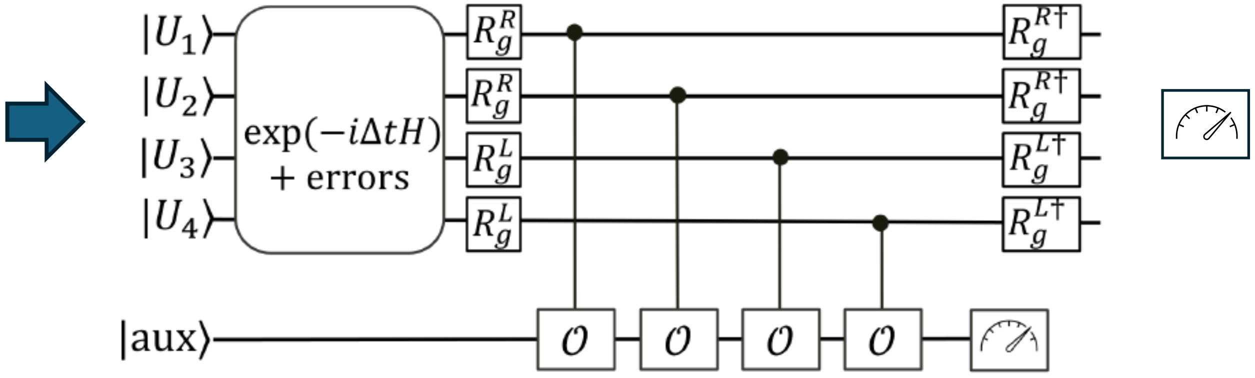
Computational basis

- ✓ Plaquette operator
- ✗ Local gauge charge

Eigenbasis of $\widehat{\Theta}_{v,g}$

- ✗ Plaquette operator
- ✓ Local gauge charge

Digital post-selection



auxiliary qudit

Non-Abelian LGT: post-Processed Symmetry Verification (PSV)

The projection of a noisy outcome ρ of a simulation onto the symmetry-preserving subspace (defined by Π_s) is

$$\rho_s = \frac{\Pi_s \rho \Pi_s}{\mathrm{Tr}[\Pi_s \rho]}$$

We can find the following expression

$$\mathrm{Tr}(O\rho_s) = \frac{\mathrm{Tr}[O\Pi_s\rho]}{\mathrm{Tr}[\Pi_s\rho]} = \frac{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho O \Pi_{v\in \mathrm{V}} \Theta_{g_v,v}]}{\sum_{\boldsymbol{g}\in \mathrm{G}^{\times n_v}} \mathrm{Tr}[\rho \Pi_{v\in \mathrm{V}} \Theta_{g_v,v}]}$$

The projection onto the gauge sector is computed by sampling several observables to construct the ratio.