Hamiltonian Lattice Gauge Theories:

Status, Novel Developments and Applications

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(Non-Abelian) Gauge theories

abundant in nature

- induces interactions in the SM
- most interesting:
  Quantum Chromodynamics
  - asymptotically free
  - confining
- not solvable analytically

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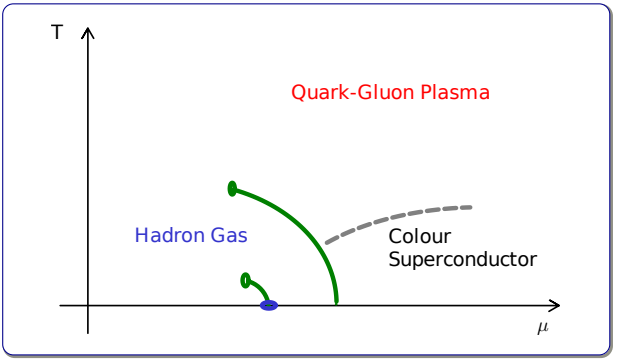

The (notorious) Challenges

Gauge quantum field theories share some or all of the challenges:

- **curse of dimensionality**
  → exponentially growing Hilbert space in quantum systems

- **strongly coupled**
  → non-perturbative methods required

- **QFTs need regularisation and renormalisation**
  → Lattice regularisation

State-of-the-Art: Stochastic Simulations

- Monte Carlo simulation of Euclidean path integral on the lattice

$$\int \mathcal{D}U \;\; \det(D[U])\, e^{-S_G[U]}$$

[Wilson (1974), ...]

- fermion determinant $\det(D)$, Euclidean gauge action S_G
- MC beats the curse of dimensionality
- highly successful for equilibrium physics
  - hadron spectrum
  - matrix elements
  - but also scattering amplitudes

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Remarks

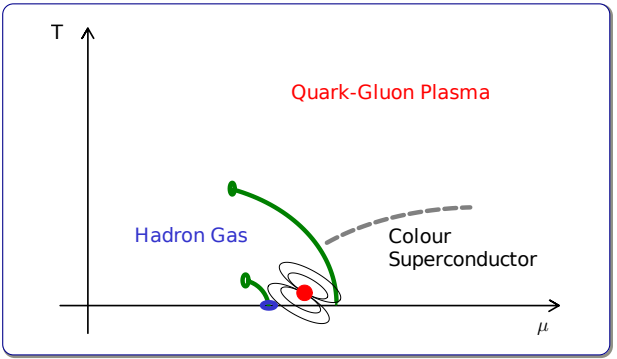
- formulation gauge invariant to machine precision
- "non-local" in fermion determinant
- the biggest challenge is $\det(D)$
- gauge dynamics almost trivial

Where MC fails...

- at $\mu \neq 0$ lattice action becomes complex

$\Rightarrow$ "sign-problem" hinders MC

- real-time evolution impossible due to Euclidean space time
- non-equilibrium dynamics complicated due to statistical approach



Hamiltonian Lattice Gauge Theories

Hamiltonian Formulation

- real-time evolution possible
- non-equilibrium dynamics accessible
- Hamiltonian known since long

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Curse of dimensionality?

$\rightarrow$ Tensor Network States or Quantum Computers

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Curse of dimensionality?

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Some Challenges

- additional constraint: Gauss' Law
- ∞-many gauge dofs $\mapsto$ finite no. computational states
- initial state preparation
- approximate time evolution

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Remarks II

- biggest challenge seems to be the gauge field dynamics!?
- fermions “simpler”

Workshop Goals

Given the rapid evolution of the topic, we would like to

- assess the status of the field,
- discuss novel developments,
- identify future directions and challenges,
- bridge towards phenomenologically interesting applications.

Workshop Structure

- **Monday**: Efficient Formulations for Simulations
  keynote by **Martin Savage**
- **Tuesday**: QC and applications
  keynote by **Christian Bauer**
- **Wednesday**: Tensor Network States
  keynote by **Stefan Kühn**
- **Thursday**: Quantum Hardware and beyond
  keynote by **Enrico Rinaldi**
- **Friday**: Summary by the organisers and discussions

Digitisation of Gauge Theories

Questions:

- which currently available gauge field digitisation is most promising for simulations with quantum computers and/or tensor network methods?
- Wilson's formulation? Quantum Link models? more?
- is a breaking of gauge invariance acceptable and if so, to what level?
- are these relevant questions ten years from now?

Tensor Network States vs. Quantum Computing

Questions:

- what are the methods and algorithms available for quantum and tensor network gauge field simulations in the Hamiltonian formulation?
- what are the advantages and disadvantages of quantum and tensor network methods, i.e. in which situation is their application preferable?
- is there a multi-grid approach possible in the Hamiltonian formalism?

Phenomenological Applications

Questions:

- which phenomenologically relevant applications are in reach for current quantum devices and tensor network algorithms?
- can the results from Hamiltonian simulations and large scale Monte Carlo simulations be combined to obtain physically relevant results for experiments?

More theoretical

Questions:

- how to perform non-perturbative renormalisation in the Hamiltonian formulation?
- can non-perturbative improvement be implemented?

We wish all of us a fruitful workshop!