Fractons topological phases
&
Foliated field theories

Masazumi Honda

(本多正純)

based on collaborations with

Hiromi Ebisu (RIKEN iTHEMS), Taiichi Nakanishi (YITP)

& Soichiro Shimamori (Osaka U.)

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Bridging analytical and numerical methods
for quantum field theory @ECT*, Trento

This workshop:

Bridging analytical & numerical methods

for

Quantum field theory

This talk (?): (in the context of quantum computing)

Quantum field theory

for

Bridging analytical & numerical methods

⇓ long (?) term

numerical methods for QFT (QFT→QC→QFT)

Fracton topological phases

| |

quasi-particle excitations w/ mobility constraints

(∼anyons)

Fracton topological phases

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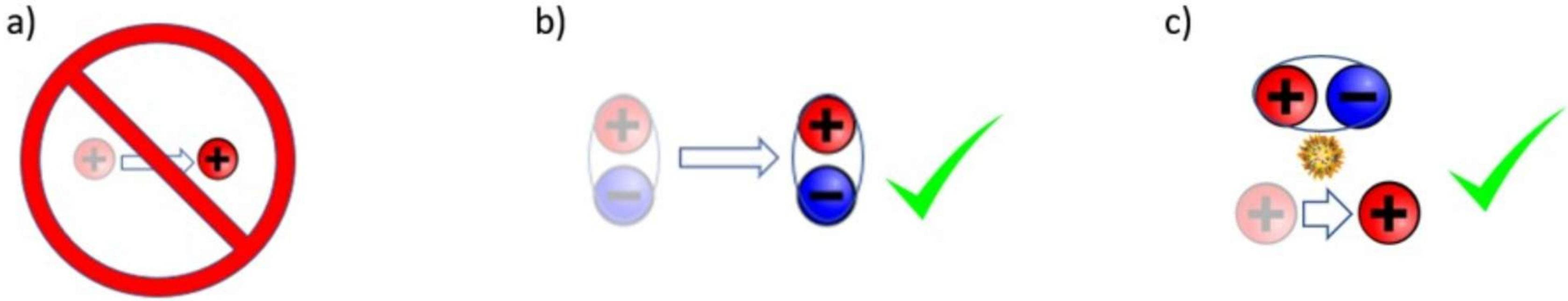
quasi-particle excitations w/ mobility constraints

(~anyons)

Ex.) particles w/ only dipole mobility [cf. review: Pretko-Chen-You '20]

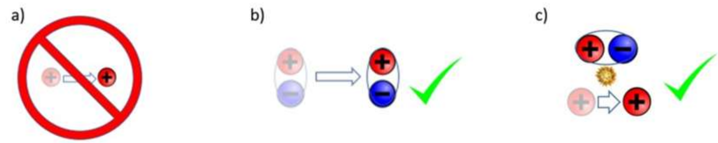


realized in systems w/ conserved dipole charge $\left(\sim \int x\rho(x)\right)$

∃recent attentions in the contexts of

condensed matter, quantum info., high energy

Fracton topological phases



<u>Condensed matter</u>

exotic phases of matter

<u>Quantum information</u>

lattice models have large ground state degeneracy

⇩

quantum error correction & quantum hard disk

(~extension of toric code)

<u>High energy physics</u>

new class of symmetries & field theories

(~modulated symmetries, foliated field theories)

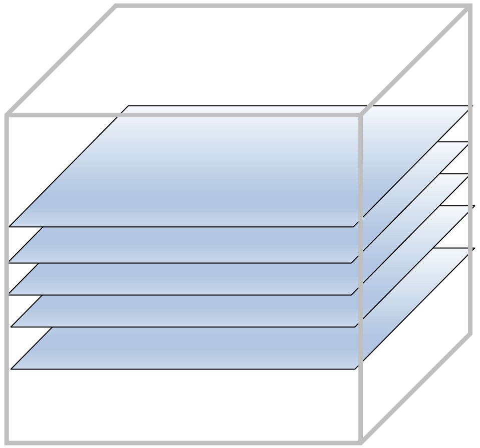
Main results

[Ebisu-MH-Nakanishi]

QFT understanding of fractonic lattice models

- Low energy effective theory = Foliated QFT

[cf. Slagle-Aasen-Williamson '18, etc...]



not coupled to metric
but to foliation
(=decomposition to submfd.)

- gauging modulated symmetry → Foliated QFT

- interpretation from a topological term

(Dijkgraaf-Witten twist term)

Plan

1. Introduction

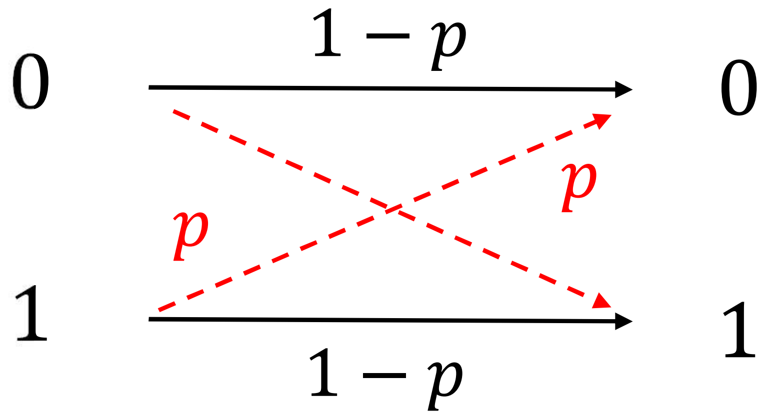
2. Review of toric code

3. Fracton topological phases

4. Summary & Outlook

Errors in classical computers

Computer interacts w/ environment ➡ error/noise

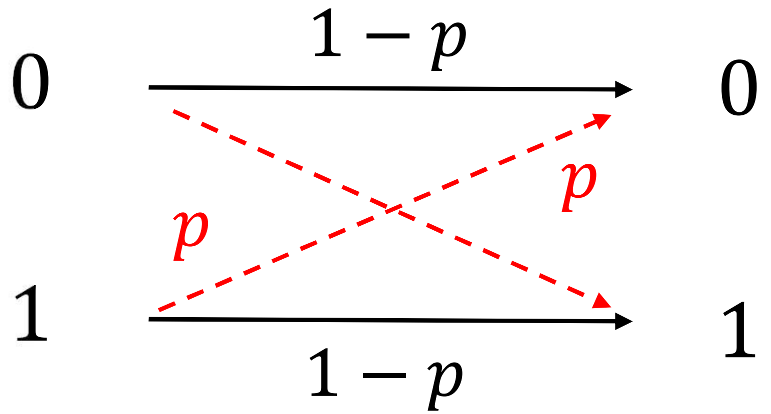


Suppose we send a bit but have “error” in probability p

Errors in classical computers

Computer interacts w/ environment → error/noise

one bit →



Suppose we send a bit but have "error" in probability p

A simple way to correct errors:

① Duplicate the bit (encoding): $0 \rightarrow 000, \quad 1 \rightarrow 111$

② Error detection & correction by "majority voting":

$$001 \rightarrow 000, \quad 011 \rightarrow 111, \quad \text{etc...}$$

→ $P_{\text{failed}} = 3p^2(1-p) + p^3$ (improved if $p < 1/2$)

Errors in quantum computers

Computer interacts w/ environment ⇒ error/noise

- Unknown unitary operators are multiplied:

(in addition to decoherence & measurement errors)

$$|\psi\rangle \xrightarrow{\text{error!}} U|\psi\rangle$$

not only bit flip!

- have to detect errors & act "inverse of errors" to recover w/o destroying states

- need more qubits as in the classical case

Lattice model as Quantum Error Correction

Encoding

physical qubits $\sim$ total Hilbert sp.

logical qubits $\sim$ vacuum Hilbert sp. (w/ degeneracy)

Error detection & recovery

excitation of energy = signal of errors

⇨ recover by map to "nearest" vacuum

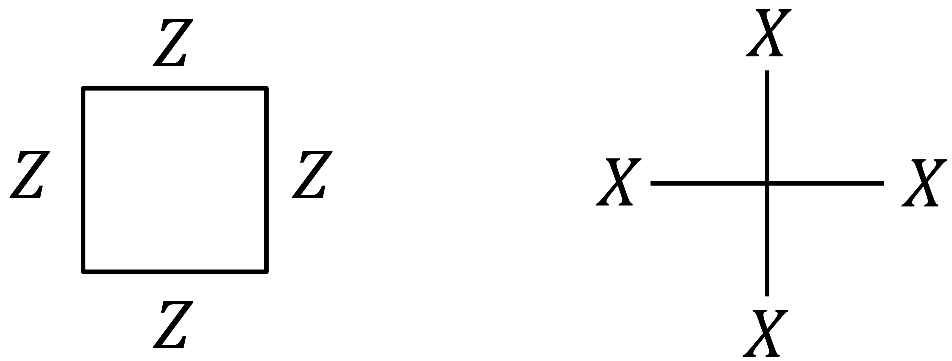
Toric code = canonical example (next slides)

Toric code

[Kitaev ’97]

Consider 2d periodic square lattice and put qubits on edges

$$H = -J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} Z_e - J \sum_{\text{vertex}} \prod_{e | \partial e = \text{vertex}} X_e \qquad (J > 0)$$



All the terms are commuting → ground states:

$$\prod_{e \in \partial(\text{face})} Z_e \, |g\rangle = |g\rangle, \qquad \prod_{e | \partial e = \text{verte}} X_e \, |g\rangle = |g\rangle$$

Ground state degeneracy on torus

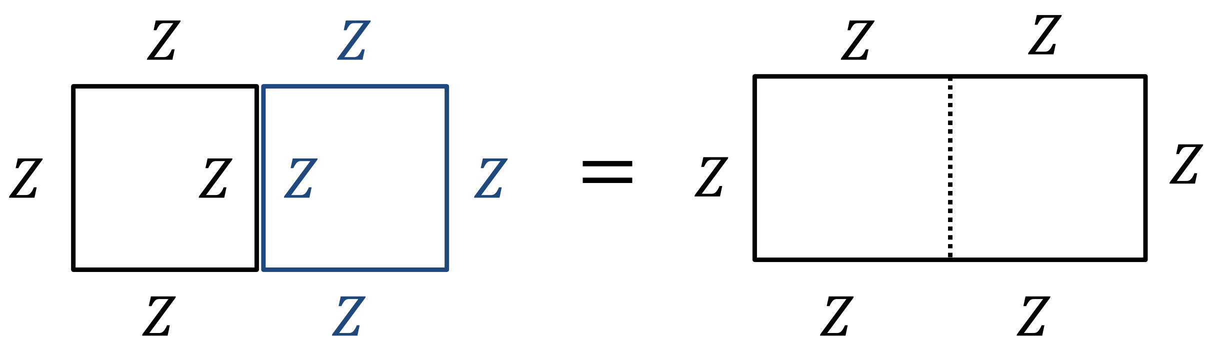
Ground states:

$$\prod_{e\in\partial(\mathrm{face})} Z_e\,|g\rangle = |g\rangle,\quad \prod_{e|\partial e=\mathrm{vertex}} X_e\,|g\rangle = |g\rangle$$

not all the conditions are independent

$$\prod_{\mathrm{faces}}\ \prod_{e\in\partial(\mathrm{face})} Z_e = 1,\qquad \prod_{\mathrm{vertices}}\ \prod_{e|\partial e=\mathrm{vertex}} X_e = 1$$

(ex.)



)

Degeneracy:

$$\#(\mathrm{GSD}) = 2\times 2 = 4 \qquad (2^{2g} \text{ for genus } g)$$

Another viewpoint : operator counting

$$\prod_{e\in\partial(\text{face})} Z_e\,|g\rangle = |g\rangle, \qquad \prod_{e|\partial e=\text{vertex}} X_e\,|g\rangle = |g\rangle$$

Q1. operators commuting w/ the conditions?

⇨ loop ops. : $\prod_{e\in\text{loop}} Z_e$, $\prod_{e\in\text{dual loop}} X_e$

Q2.

Another viewpoint : operator counting

$$\prod_{e\in\partial(\text{face})} Z_e \, |g\rangle = |g\rangle, \qquad \prod_{e|\partial e=\text{vertex}} X_e \, |g\rangle = |g\rangle$$

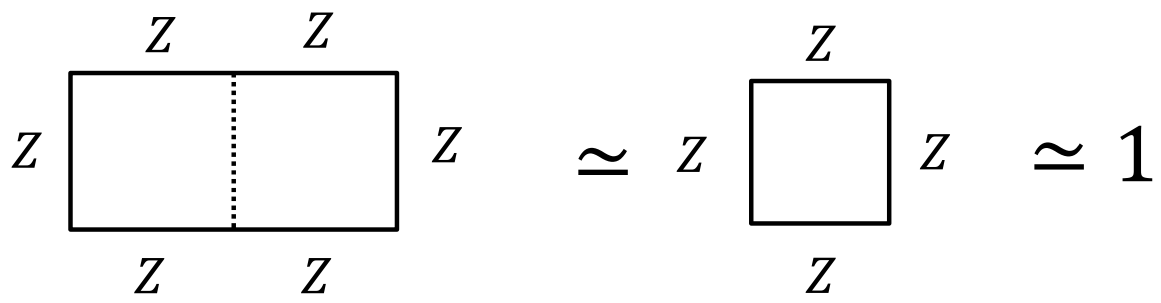
Q1. operators commuting w/ the conditions?

⇨ loop ops. : $\prod_{e\in\text{loop}} Z_e$, $\prod_{e\in\text{dual loop}} X_e$

Q2. independent operators among them?

loop ops. along topologically the same paths are equivalent up to actions of the ops. in the conditions

ex.)



⇨ loop ops. along the nontrivial cycles

#(independent ops.)$= 4 =$ #(GSD)

First excited states

$$\left(H = -J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} Z_e - J \sum_{\text{vertex}} \prod_{e | \partial e = \text{vertex}} X_e \right)$$

- Z_e, X_e are anti-commuting w/ two of the terms

- This is still true for

$$\prod_{e \in C} Z_e \,, \qquad \prod_{e \in \tilde{C}} X_e$$

where $C, \tilde{C}$: simply connected open path in (dual) lattice

First excited states:

First excited states

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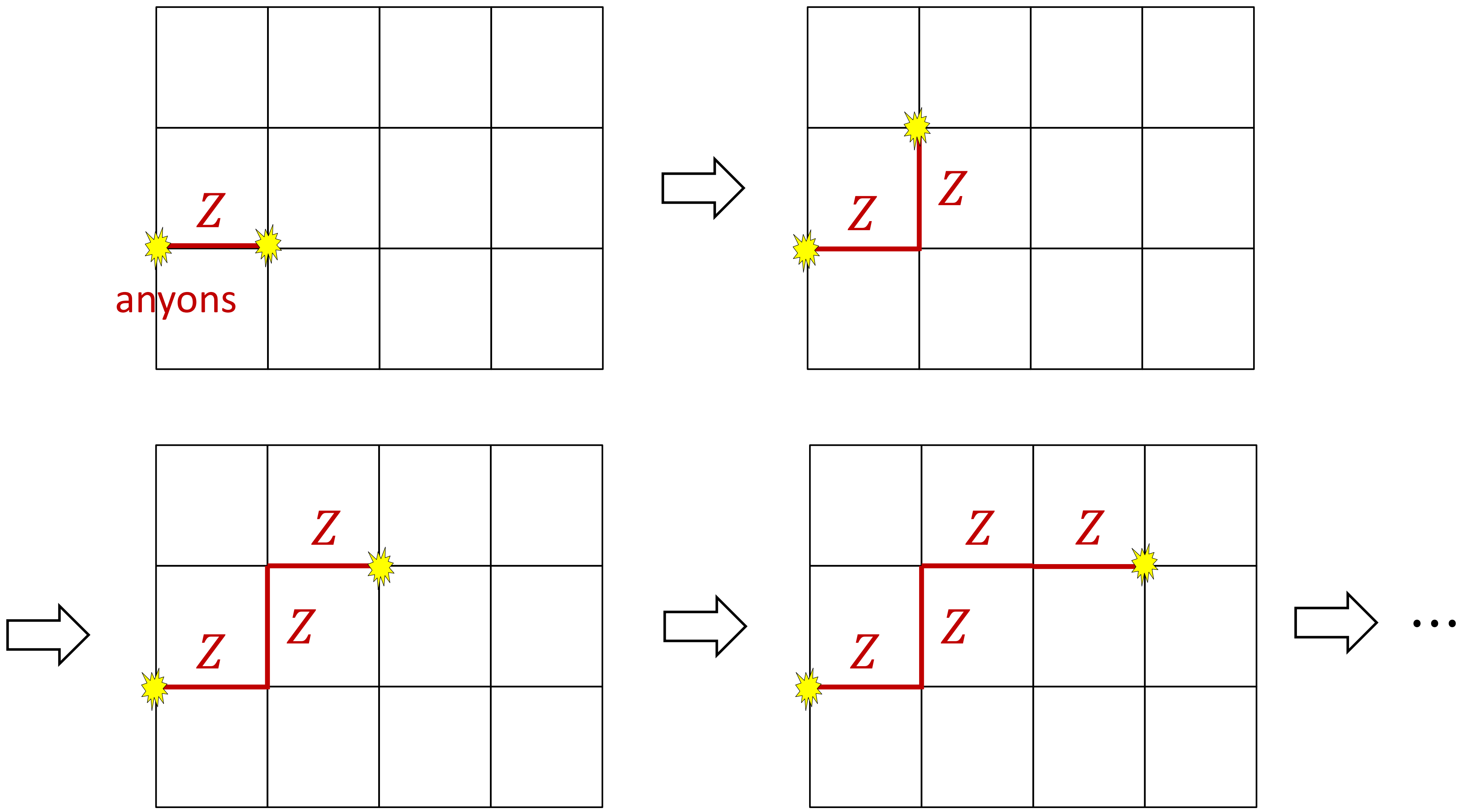
First excited states:

$$\prod_{e \in C} Z_e \, |g\rangle, \qquad \prod_{e \in \tilde{C}} X_e \, |g\rangle$$

corresponding to anyons

Mobility of anyons



Anyons can move in the whole bulk w/o changing energy

Error correction viewpoint

- physical qubits $=$ qubits on edges

- logical qubits $=$ ground states

- stabilizer conditions:

$$\prod_{e\in\partial(\text{face})} Z_e \,|g\rangle = |g\rangle, \quad \prod_{e|\partial e=\text{vertex}} X_e \,|g\rangle = |g\rangle$$

- logical ops. $=$ loop ops. along the nontrivial cycles

- error $=$ ops. giving excitation

- $2g$ (g:genus) logical qubits by #(edges) physical qubits

Toric code as a Z_2 lattice gauge theory

Z_2 gauge theory on 2d square lattice: $(U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$

$$H = g^2 \sum_e \Pi_e - J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} U_e$$

Gauss law: $(\Pi_e U_{e'} \Pi_e^\dagger = -\delta_{ee'} U_e)$

$$\prod_{e | \partial e = \text{vertex}} \Pi_e \, |\text{phys}\rangle = |\text{phys}\rangle$$

Ground state for $g = 0$:

$$\prod\nolimits_{e | \partial e = \text{verte}} U_e \, |\text{ground}\rangle = |\text{ground}\rangle$$

In identification (U-basis)~(computational basis),
this is the same condition as the toric code

Low energy effective field theory

BF theory (2+1d topological field theory):

$$\mathcal{L} = \frac{N}{2\pi} b \wedge da \propto \epsilon_{ijk} b^i \partial^j a^k \quad (a, b\text{: gauge fields, } N \in \boldsymbol{Z})$$

Low energy effective field theory

BF theory (2+1d topological field theory):

$$\mathcal{L} = \frac{N}{2\pi} b \wedge da \propto \epsilon_{ijk} b^i \partial^j a^k \quad (a, b\text{: gauge fields}, N \in \boldsymbol{Z})$$

Nontrivial gauge invariant ops.:

$$W = \exp\left[i \oint_C a \right] \in \boldsymbol{Z_N} \quad \text{(C: topologically nontrivial cycle)}$$

Ground state degeneracy:

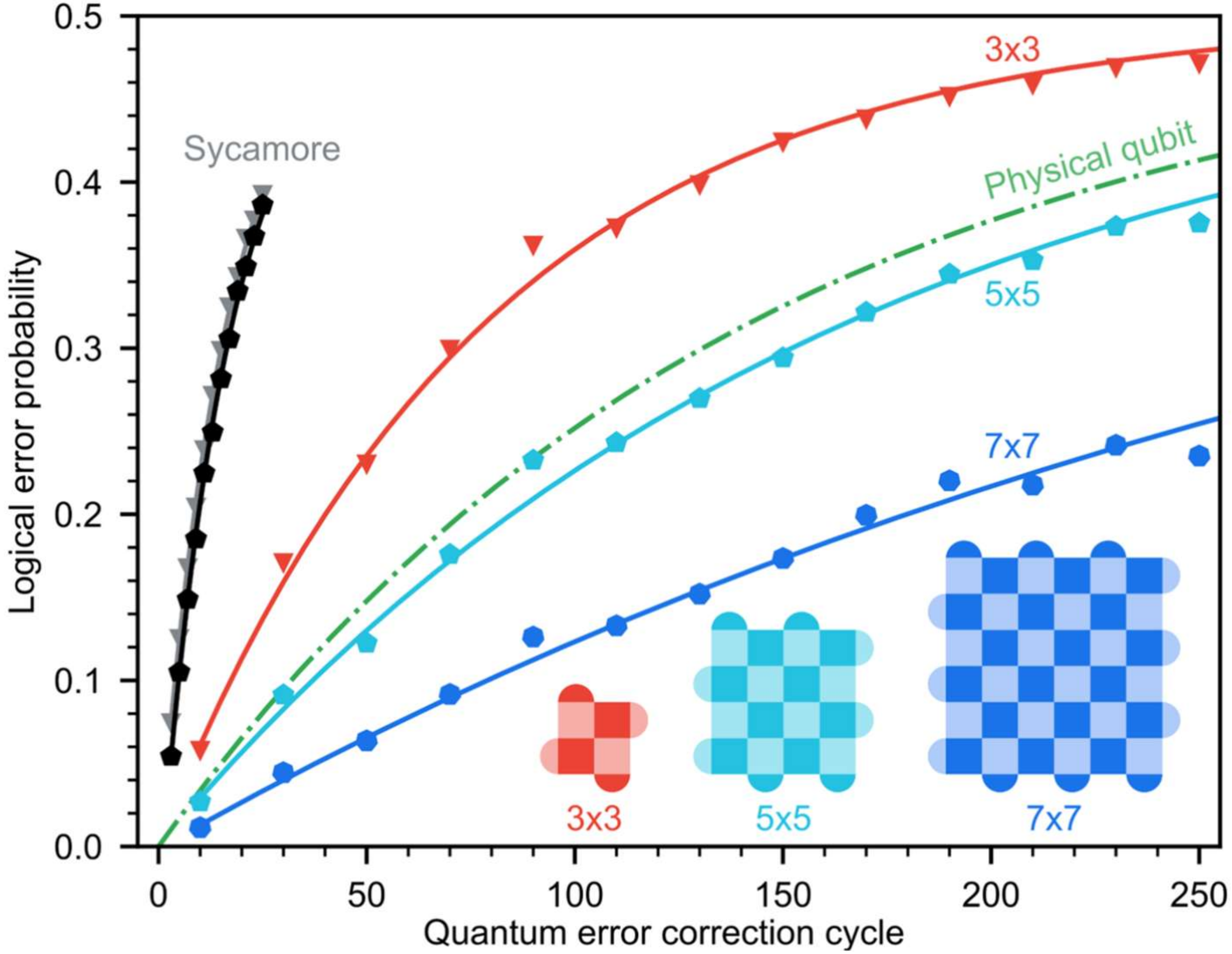
$$\#(\text{GSD}) = N^{2g}$$

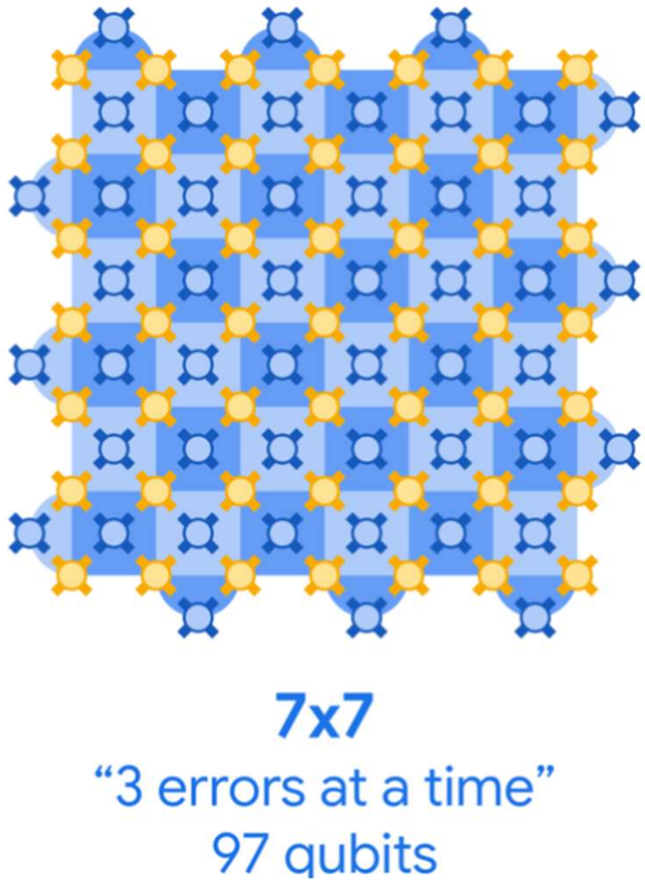
the same as Z_N generalization of the toric code

($N = 2, g = 1$ in the standard toric code on torus)

Realization on real devise

[Google Quantum AI ’24]





Article

Quantum error correction below the surface code threshold

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Generalization of toric code?

Properties of toric code:

- ground states $=$ logical qubits
- degeneracy of ground states $= 2^{2g}$ (g: genus of space)
- low energy effective theory $=$ topological gauge theory
- logical gates $=$ gauge invariant operators
- quasi particle $=$ anyon

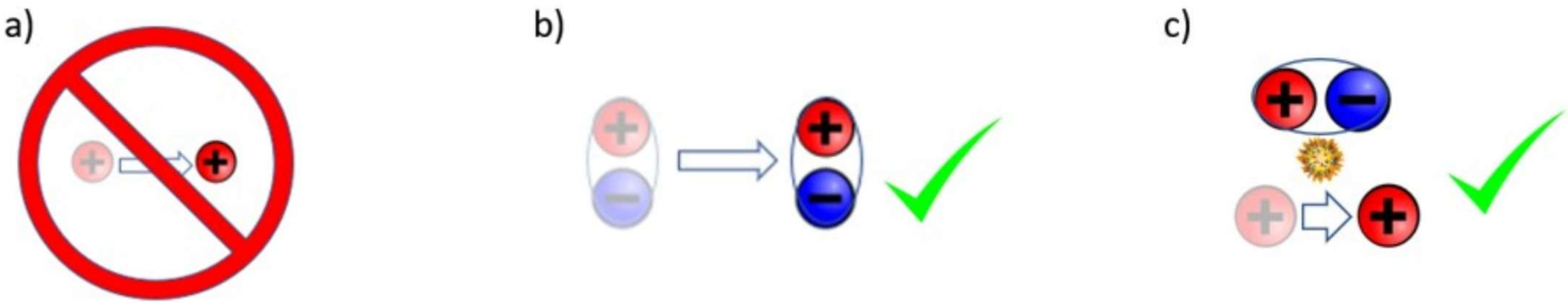
Generalization w/ larger degeneracy? ⇨ Fractons!

Fractons?

[cf. Pretko-Chen-You ’20]

Fracton = (quasi) particle w/ mobility constraints

e.g. particles w/ only dipole mobility



realized in systems w/ conserved dipole charge

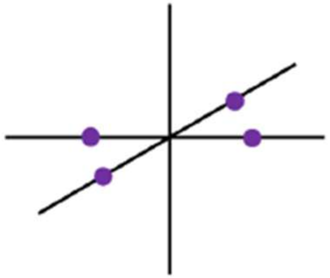
$$\left(\int x\rho(x)\right)$$

$\Downarrow$

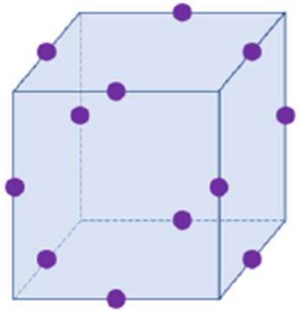
Generalizing toric code in this direction?

Ex.1) X-cube model (3+1d lattice model)

[cf. Pretko-Chen-You '20]



$$A_v^z = \prod_{+_{xy}} Z$$



$$B_c = \prod_{\partial c} X$$

(qubits on edges)

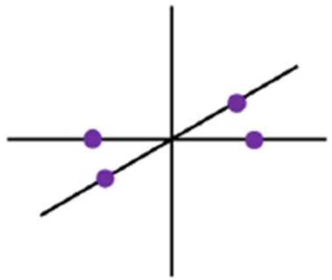
Hamiltonian:

$$H = -\sum_v (A_v^x + A_v^y + A_v^z) - \sum_c B_c$$

Ground states:

Ex.1) X-cube model (3+1d lattice model)

[cf. Pretko-Chen-You '20]



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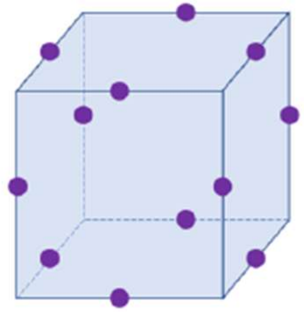

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Hamiltonian:

$$H = -\sum_v (A_v^x + A_v^y + A_v^z) - \sum_c B_c$$

Ground states:

$$A_v^x|g\rangle = A_v^y|g\rangle = A_v^z|g\rangle = B_c|g\rangle = |g\rangle$$

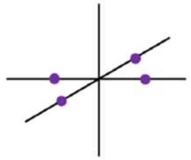
Taking into account overlapped constraints,

$$\log_2 \#(\mathrm{GSD}) = 2L_x + 2L_y + 2L_z - 3$$

size dependence!

First excited states

$$H = -\sum_v (A_v^x + A_v^y + A_v^z) - \sum_c B_c$$



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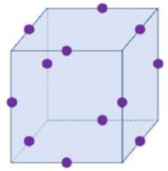
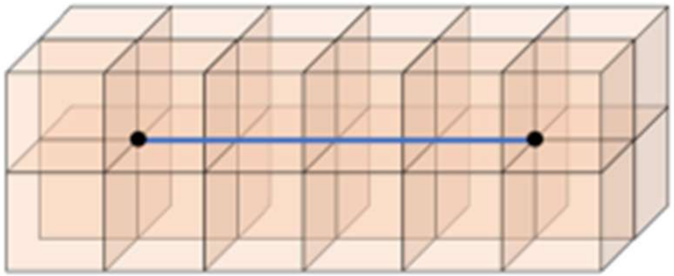


$B_c = \prod_{\partial c} X$

Ops. to create first excited states:

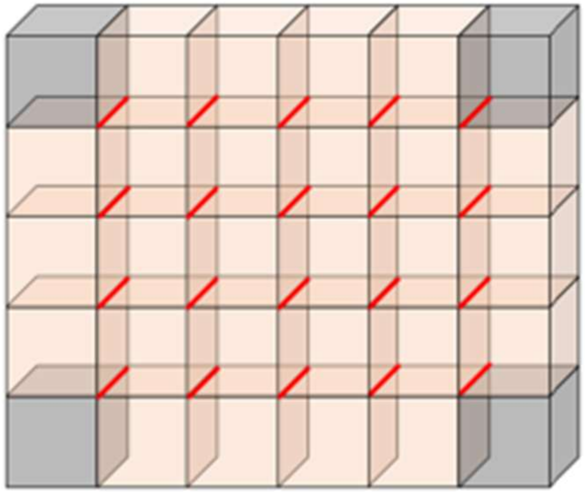
[cf. Pretko-Chen-You '20]



X

$\sim$ open line op.

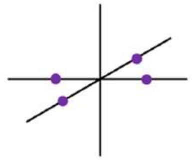
Z



$\sim$ open membrane op.

Mobility constraints on "anyons"

$$H = -\sum_v (A_v^x + A_v^y + A_v^z) - \sum_c B_c$$



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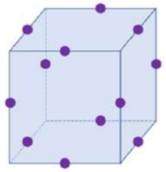
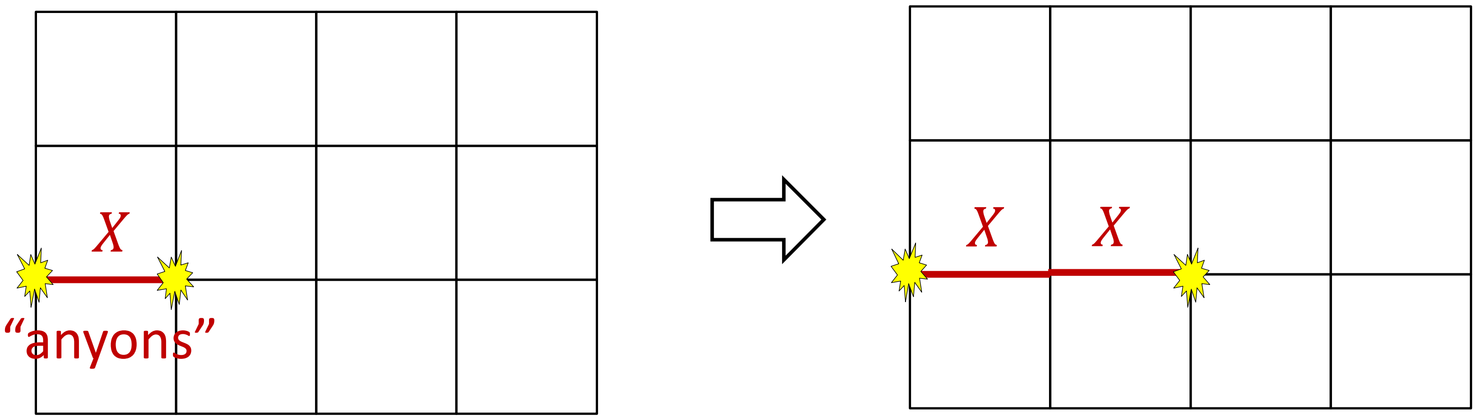
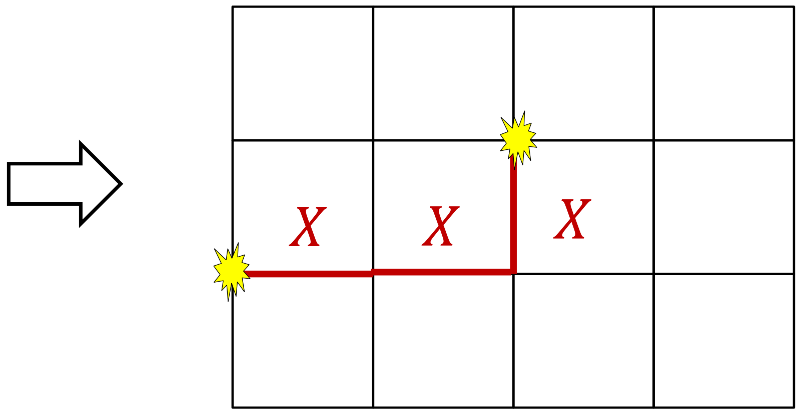


$B_c = \prod_{\partial c} X$



no energy loss

(not-commuting w/ 4 terms)



energy loss!

(not-commuting w/ 6 terms)

"Anyons" can move in a straight way w/o changing energy but consume energy to curve

X-cube model:

$$H = -\sum_v (A_v^x + A_v^y + A_v^z) - \sum_c B_c$$

Properties:

- ground state degeneracy

$$\log_2 \#(\mathrm{GSD}) = 2L_x + 2L_y + 2L_z - 3$$

- there are quasi-particle excitations like anyons but they have constrained mobilities

$^\exists$several lattice models w/ similar properties

X-cube model:

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Properties:

- ground state degeneracy

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- there are quasi-particle excitations like anyons
  but they have constrained mobilities

$^\exists$several lattice models w/ similar properties

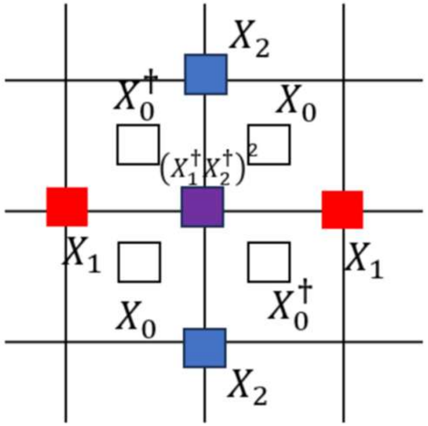
This may imply new types of

"topological" phases, symmetries & field theories?

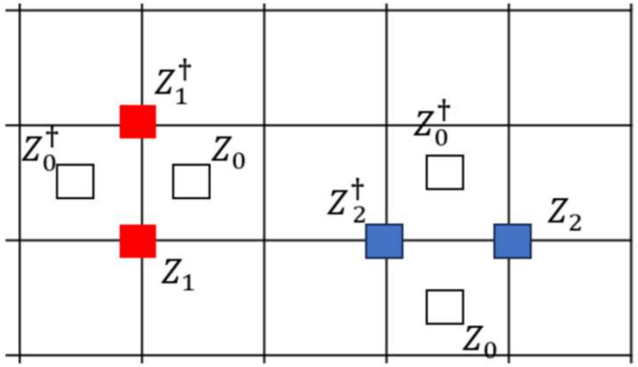
Ex.2) "dipolar" $\boldsymbol{Z_N}$ toric code

[Pace-Wen '22]

2+1d lattice w/ $\boldsymbol{Z_N} \times \boldsymbol{Z_N}$ d.o.f. on sites & $\boldsymbol{Z_N}$ on dual sites



$V_{(\hat{x},\hat{y})}$



$P_{(\hat{x},\hat{y}+1/2)}$ $Q_{(\hat{x}+1/2,\hat{y})}$

Hamiltonian:

$$H_{dip} := -\sum_{\hat{x},\hat{y}} \left[V_{(\hat{x},\hat{y})} + P_{(\hat{x},\hat{y}+1/2)} + Q_{(\hat{x}+1/2,\hat{y})} \right] + (\text{h.c.}) \ .$$

Ground states:

Ex.2) “dipolar” $\boldsymbol{Z_N}$ toric code

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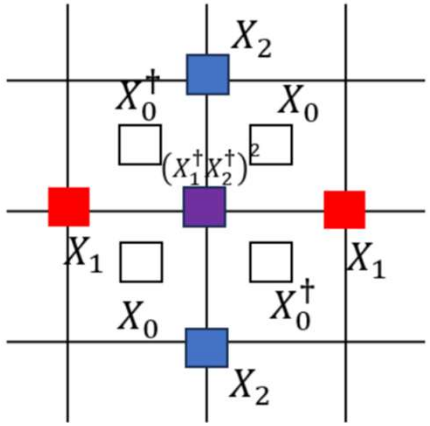

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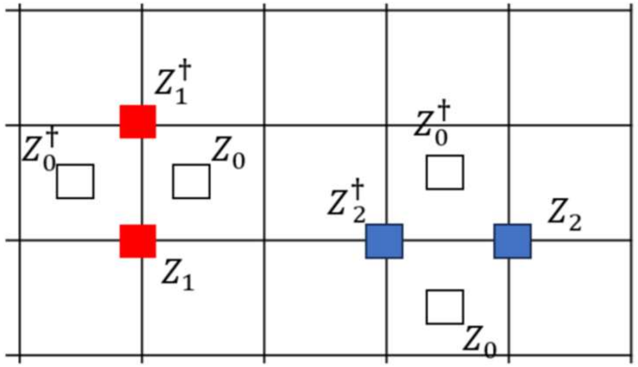

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Ground states:

$$V_{(\hat{x},\hat{y})}|g\rangle = P_{(\hat{x},\hat{y}+1/2)}|g\rangle = Q_{(\hat{x}+1/2,\hat{y})}|g\rangle = |g\rangle$$

$$GSD = N^3 \times \gcd(N, L_x) \times \gcd(N, L_y) \times \gcd(N, L_x, L_y).$$

Symmetries behind fraction-like theory

Modulated symmetry:

$$\Phi(t, x_1, \cdots, x_d) \to e^{i\theta(t, x_1, \cdots, x_d)} \Phi(t, x_1, \cdots, x_d)$$

$\theta(t, x_1, \cdots, x_d)$: not arbitrary function unlike gauge trans.

Symmetries behind fraction-like theory

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$\theta(t, x_1, \cdots, x_d)$: not arbitrary function unlike gauge trans.

- subsystem symmetry (e.g. X-cube model)

$$\theta(t, x_1, \cdots, x_d) = \theta(x_1, \cdots, x_{d-p})$$

- multi-pole symmetry (e.g. “dipolar” toric code)

$\theta(t, x_1, \cdots, x_d)$: finite order polynomial of space

.
.
.

Constructing effective QFT for dipole case

Dipole sym. algebra:

$Q_I \sim \int d^2x\, x^I \rho(x)$

$$[P_I, Q] = 0, \quad [P_I, Q_J] = \delta_{IJ} Q \qquad (I = x, y)$$

⇩ gauging consistent w/ the algebra

Constructing effective QFT for dipole case

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⇩ gauging consistent w/ the algebra

Dipole gauge trans. :

[Ebisu-MH-Nakanishi '23]

$$a \to a + d\Lambda + \sigma_I dx_I, \quad A^I \to A^I + d\sigma_I$$

⇩ write down BF-like gauge inv. action

Constructing effective QFT for dipole case

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⇩ write down BF-like gauge inv. action

Foliated BF theory:

$e^x = dx, e^y = dy$

$$\mathcal{L}_{dip} = \frac{N}{2\pi} a \wedge db + \sum_{I=x,y} \frac{N}{2\pi} A^I \wedge dc^I + \frac{N}{2\pi} A^I \wedge b \wedge e^I.$$

partially topological

Foliated BF theory

BF theory (for toric code):

$$\mathcal{L}_{TC} = \frac{N}{2\pi} b \wedge f = \frac{N}{2\pi} b \wedge da,$$

Foliated BF theory (for dipolar toric code):

$$\boxed{\mathcal{L}_{dip} = \frac{N}{2\pi} a \wedge db + \sum_{I=x,y} \frac{N}{2\pi} A^I \wedge dc^I + \frac{N}{2\pi} A^I \wedge b \wedge e^I .}$$

$$e^x = dx, e^y = dy$$

Ground state degeneracy on torus:

$$\boxed{GSD = N^3 \times \gcd(N, L_x) \times \gcd(N, L_y) \times \gcd(N, L_x, L_y).}$$

Generalization of toric code?

Properties of fractonic generalization: [cf. Ebisu-MH-Nakanishi]

- ground states = logical qubits
- degeneracy of grounds states = size dependent
- low energy theory = partially topological gauge theory
  (“foliated BF theory”)
- logical gates = gauge invariant operators
- quasi particles = anyons w/ mobility constraints

But (w/ the same number of physical qubits)

should be less tolerant to errors than toric code

→ what’s more detailed properties as codes? (to be studied)

QFT as a generator of error correcting code?

Toric code

[Kitaev '97]

- Lattice model interpreted as error correction (QEC)
- Low energy effective theory = QFT (BF theory)

QFT ↔ Lattice model ↔ QEC

Idea : if we get something new in one of them,
then try to fill the other parts

ex.) "Dipolar" generalization of toric code [Pace-Wen '22]

⇩

corresponds to a "layer" of BF theory w/ some rule

[Ebisu-MH-Nakanishi '23]

interesting to find new class of QFTs w/ similar properties

Comment (1/3): higher form generalization

[Ebisu-MH-Nakanishi '24]

$$\mathscr{L} = \frac{N}{2\pi}\left[b^{(d-p)} \wedge f^{(p+1)} + \sum_I c^{I(d-p)} \wedge F^{I(p+1)}\right]$$
$$= \frac{N}{2\pi}\left[b^{(d-p)} \wedge \left(da^{(p)} + (-1)^p \sum_I A^{I(p)} \wedge e^I\right) + \sum_I c^{I(d-p)} \wedge dA^{I(p)}\right]$$

gauge trans. :

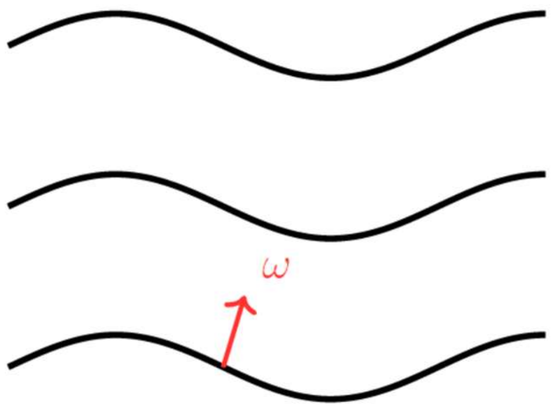
$$c^{I(d-p)} \to c^{I(d-p)} + d\chi^{I(d-p-1)} + (-1)^{d-p}\sigma^{(d-p-1)} \wedge e^I, \quad b^{(d-p)} \to b^{(d-p)} + d\sigma^{(d-p-1)}$$

$$GSD = N^{K(d,p)} \times \prod_{1\leq i_1<i_2\cdots<i_p\leq d} \gcd(N, L_{i_1}, L_{i_2}, \cdots, L_{i_p}) \times \prod_{1\leq i_1<i_2\cdots<i_{p+1}\leq d} \gcd(N, L_{i_1}, L_{i_2}, \cdots, L_{i_{p+1}})$$

$^\exists$mixed 't Hooft anomaly btw p-form & $(d-p)$-form syms.
(and its anomaly inflow argument)

Comment (2/3): redundancy of normal field

[Ebisu-MH-Nakanishi-Shimamori '24]



ω & $c(x)\omega$ give the same foliation structure

→ Ideally foliated QFT should have

$$\omega \sim c(x)\omega$$

But,

foliated BF theories don't have the redundancy

Recently we constructed

a QFT w/ the redundancy based on a characteristic class involving foliation called Godbillon-Vey class:

$$S[b, c, \lambda, \phi] = \frac{k}{2\pi} \int_{M^3} \left[b \wedge \mathrm{d}c - \lambda \wedge (\mathrm{d}\phi \wedge \omega - \omega \wedge b) \right]$$

Comment (3/3): fractonic "chiral" fermion

(d+1)-dim. fermion w/ subsystem sym.: [MH-Nakanishi '22]

$$\mathcal{L}_\psi := \frac{\mu_0}{2}\left[i\psi_+(\partial_t - \alpha\partial_\#^d)\psi_+ + i\psi_-(\partial_t + \alpha\partial_\#^d)\psi_-\right] \qquad \text{(d:odd)}$$

($\psi_\pm$: 1-component Grassmann, $\partial_\# \coloneqq \partial_1 \cdots \partial_d$)

"Naive" lattice fermion:

$$H_{\text{naive}} = i\beta\sum_{\vec{n}} c_{\vec{n}}\tilde{\Delta}_\#^d c_{\vec{n}}, \qquad \left(\tilde{\Delta}_\#^d := \prod_{i=1}^d \tilde{\Delta}_\#^i, \quad \tilde{\Delta}_\#^i f_{\vec{n}} := f_{\vec{n}+e_i} - f_{\vec{n}-e_i}.\right)$$

In momentum basis,

$$H_{\text{naive}} = \beta\sum_{\vec{k}}\left(\prod_{i=1}^d \sin(k_i a)\right) b_{-\vec{k}} b_{\vec{k}}.$$

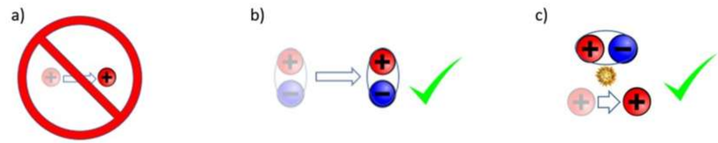
# of zero modes:

$$2\sum_{i=1}^d \frac{V}{L_i} - 4\sum_{1\le i<j\le d}\frac{V}{L_iL_j} + \cdots + 2^{d-2}\sum_{1\le i<j\le d} L_iL_j - 2^{d-1}\sum_{i=1}^d L_i + 2^d$$

(desired: $\sum_{i=1}^d \frac{V}{L_i} - \sum_{1\le i<j\le d}\frac{V}{L_iL_j} + \cdots + \sum_{1\le i<j\le d} L_iL_j - \sum_{i=1}^d L_i + 1$)

Summary & Outlook

Fracton topological phases



<u>Condensed matter</u>

exotic phases of matter

<u>Quantum information</u>

lattice models have large ground state degeneracy

⇩

quantum error correction & quantum hard disk

(∼extension of toric code)

<u>High energy physics</u>

new class of symmetries & field theories

(∼modulated symmetries, foliated field theories)

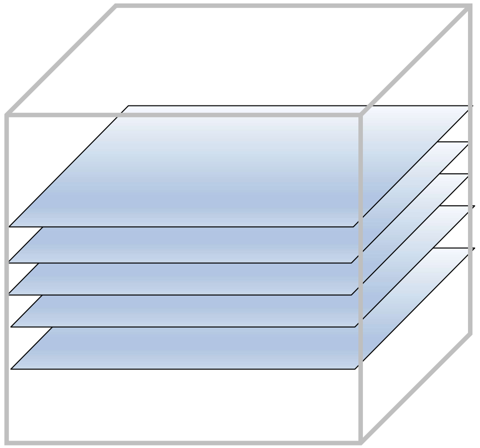
Summary

[Ebisu-MH-Nakanishi]

QFT understanding of fractonic lattice models

- Low energy effective theory = Foliated QFT

[cf. Slagle-Aasen-Williamson ’18, etc…]



not coupled to metric
but to foliation
(=decomposition to submfd.)

- gauging modulated symmetry → Foliated QFT

- interpretation from a topological term

(Dijkgraaf-Witten twist term)

Outlook

- quantum information properties of fractonic lattice models
- thermalization of fractons?
- interacting theories
- Lattice regularization
- more general modulated symmetries?
  (e.g. non-abelian, non-invertible) [cf. Cao-Lee-Yamazaki-Zheng '23, Furukawa '25, etc...]
- exploring dualities btw foliated QFT & tensor gauge theories
  [cf. Ohmori-Shimamura, etc...]
- classification of partially topological theories
  [cf. viewpoint from characteristic class involving foliation: Ebisu-MH-Nakanishi-Shimamori '24]
- SUSY observables & foliation? [cf. Closset-Dumitrescu-Festuccia-Komargodski '13]
- topological string w/ "flavor" brane & foliation?
  [cf. Aganagic-Costello-McNamara-Vafa '17, Aharony-Feldman-MH '19]

Thanks!