Emergent thermalization in the massive Schwinger model

Adrien Florio

AF, D. Frenklakh, K. Ikeda, D. Kharzeev, V. Korepin, S. Shi, K. Yu
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AF, D. Frenklakh, S. Grieninger, D. Kharzeev, A. Palermo, S. Shi
arXiv: 2506.14983

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

UNIVERSITÄT BIELEFELD
Faculty of Physics

Bridging analytical and numerical methods for quantum field theory

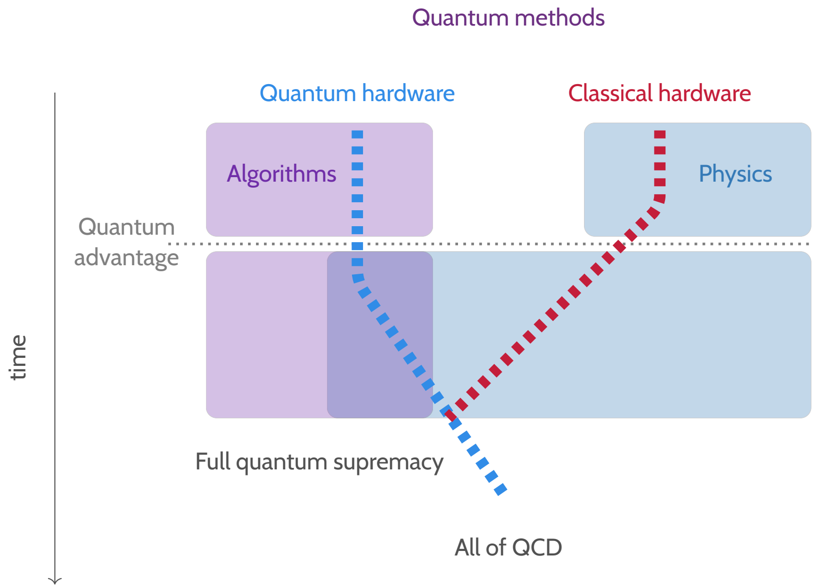
Line world

Relativistic field theories in $1+1$ dimensions

↙ ↘

Opportunity for co-design quantum comp.

Learn about physics



Quantum methods
Quantum hardware
Classical hardware
Algorithms
Physics
Quantum advantage
time
Full quantum supremacy
All of QCD

Schwinger model $\leftrightarrow$ Electro~~magnetism~~ in $1+1D$

Model: $S = \int \mathrm{d}x\mathrm{d}t \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\left(\not{D}+m\right)\psi\right]$

Electric field

Vector potential

$$H = \int \mathrm{d}x \left[\frac{1}{2}E^2 + \bar{\psi}\left(-i\gamma^1\partial_1 + gA^1\gamma_1 + m\right)\psi\right]$$

Fermion

Gauss law: $\partial_x E = \bar{\psi}\gamma^0\psi$ Temp. gauge: $A_0 = 0$

Properties:

- **Confinement:** $\partial_x E = \delta(x) \leftrightarrow E(x) = \Theta(x) \leftrightarrow$ linearly rising confining force
- **Chiral anomaly:** $\partial_\mu j_5^\mu|_{m=0} = \frac{1}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} = \frac{1}{2\pi}E$
- **Chiral condensate:** $\langle\bar{\psi}\psi\rangle \neq 0$
- **Duality:** $m = 0 \rightarrow$ free *massive* boson, $m \neq 0$ interacting massive boson

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No dynamical gauge fields

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String breaking in the Schwinger model

Look at string breaking

En.$\sim m + m + \alpha l_1$

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En.$\sim$ $m + m + \alpha l_1$

En.$\sim$ $m + m + \alpha l_2$

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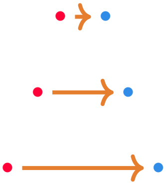

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En.$\sim m + m + \alpha l_3$

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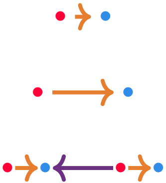

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when $\alpha l_3 > 2m$

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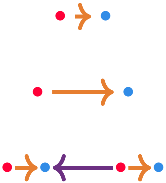

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Screen field by creating particles!

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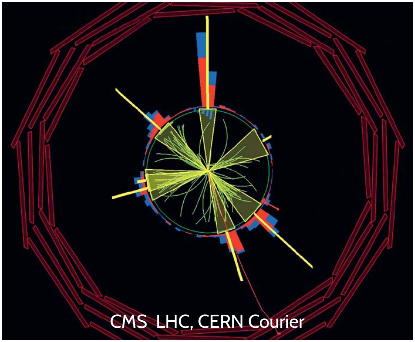

Motivation: QCD jets

Set-up

Electric field → E^2; Vector potential → A^1, A_1; Fermion → $\bar\psi$, ψ

$$H(t) = \int \mathrm{d}x \left[\tfrac{1}{2}E^2 + \bar\psi\left(-i\gamma^1\partial_1 + gA^1\gamma_1 + m\right)\psi + j^1_{ext}(t)A_1\right]$$

External charges: $j^1_{ext}(t) = g\left(\delta(x+t) + \delta(x-t)\right)\theta(t)$

2 point charges moving apart at speed of light

Idea: • Find $|vac\rangle_{t<0}$

• Compute $|\psi(t)\rangle = e^{-i\int_0^t \mathrm{d}t' H(t')}|vac\rangle_{t<0}$

see also [Casher, Kogut, Susskind, '74], [Kharzeev, Loshaj, '12, '13], [Berges, Hebenstreit, '14], [Batini, Kuhn, Berges, Floerchinger, '24], [Janik, Nowak, Rams, Zahed, '25]

In practice

- Staggered fermions χ_n
- Integrate out E: $\partial_1 E = \rho + \rho_{ext}$
- Use tensor networks (MPS + DMRG + TDVP)

Elevator pitch: tensor networks

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state of the art $N^d \sim O(40)$

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- Ground states entanglement entropy S_E follow area law: $S_E \propto N^{d-1}$
- # d.o.f. $\sim \exp(S_E)$

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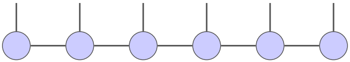
Attempt 2: Encode area law in an efficient representation of $|\psi\rangle$

$\leftrightarrow$

optimal truncation from $2^{N^d} \rightarrow 2^{N^{d-1}}$ d.o.f.

Matrix product states (MPS):



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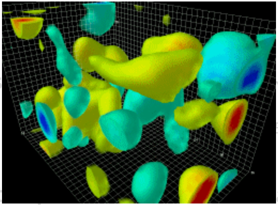
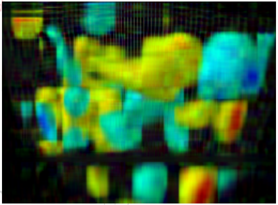


Singular Value Decomposition

(SVD)

$\longrightarrow$



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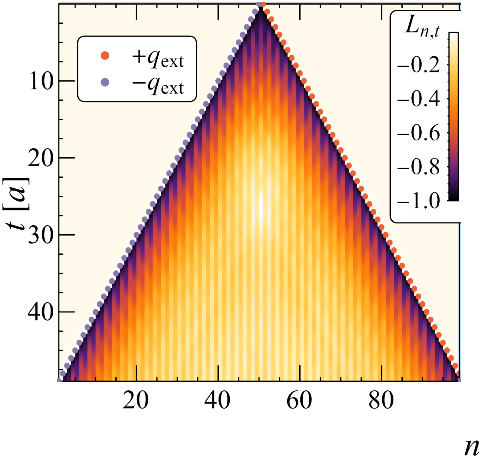
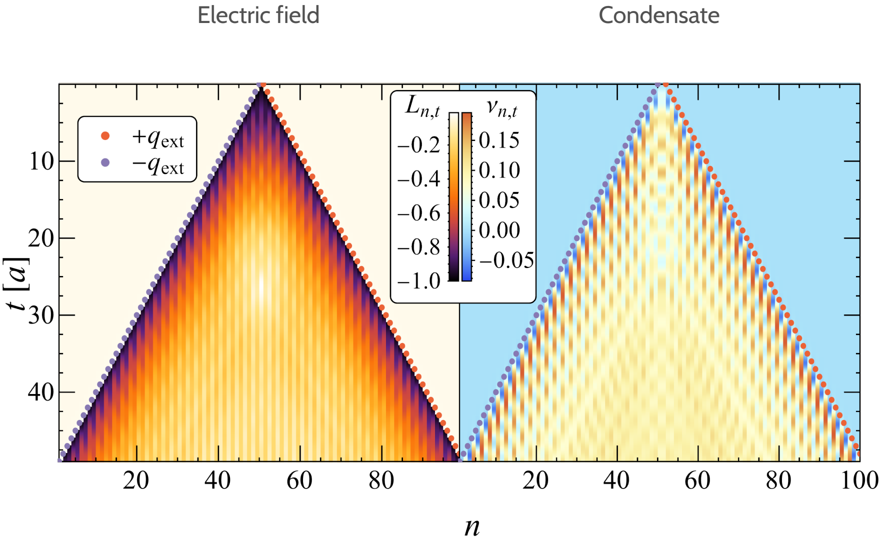
Electric field



$N = 100,\ g \cdot a = 0.5,\ m \cdot a = 0.25$



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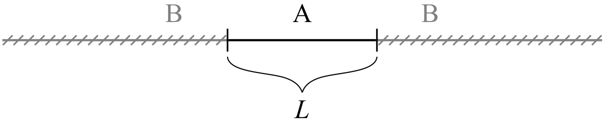
Relaxation of condensate



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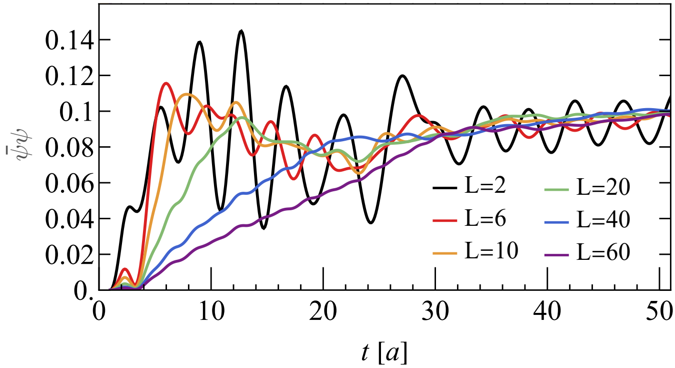

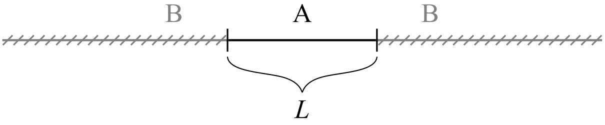

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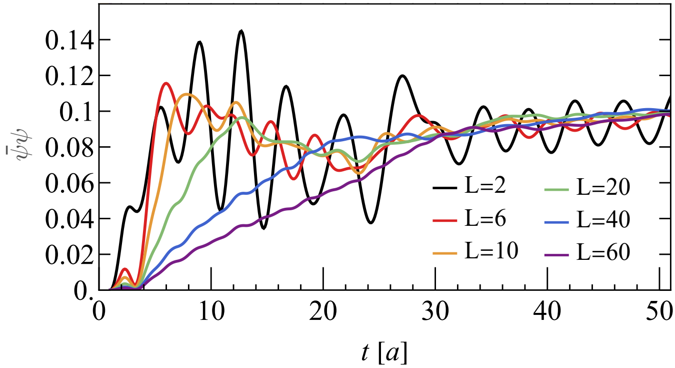
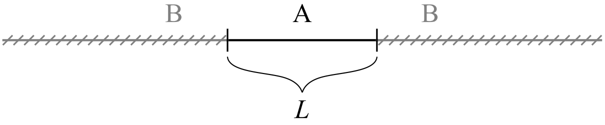
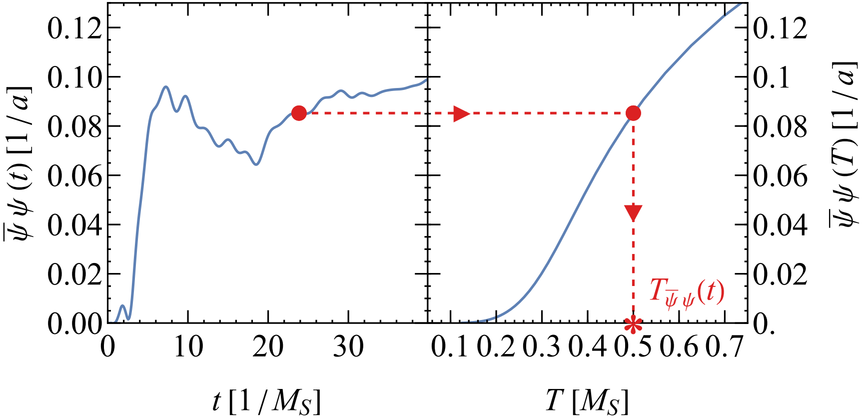




Effective thermalization?

Effective temperature?



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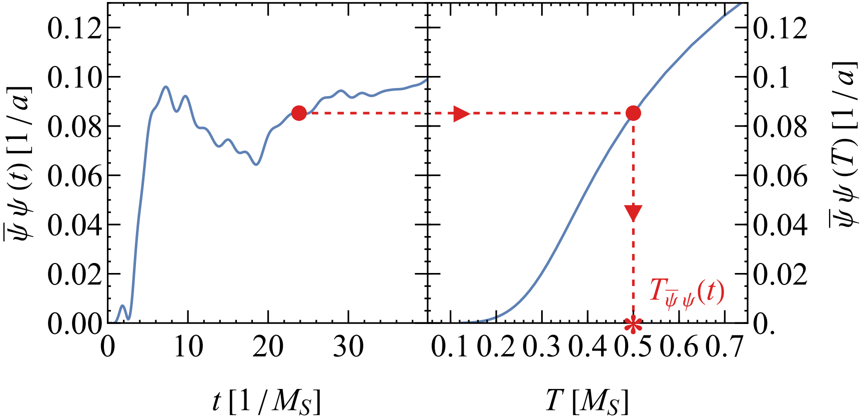

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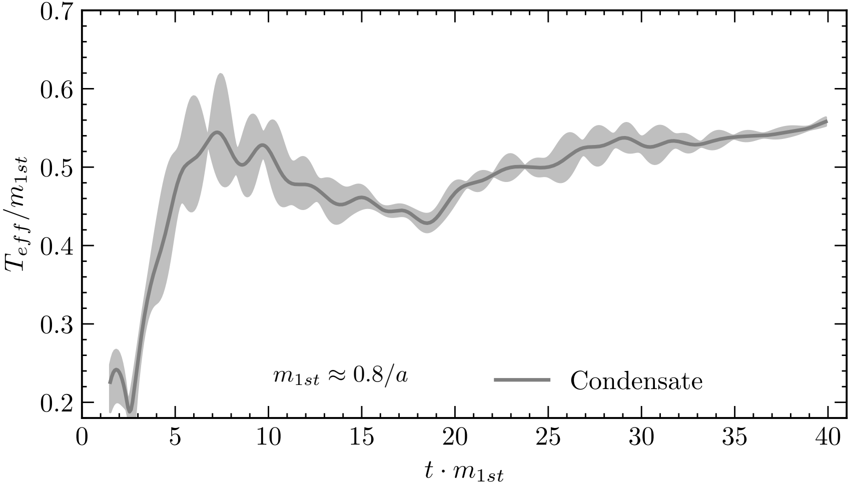

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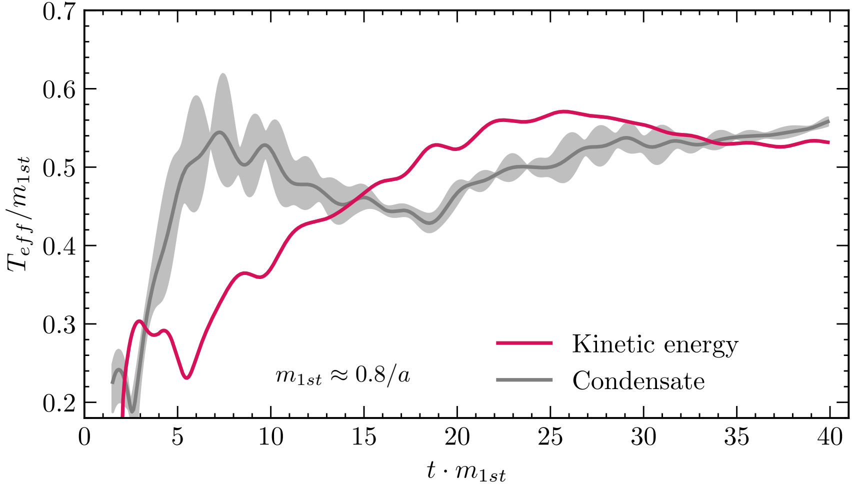

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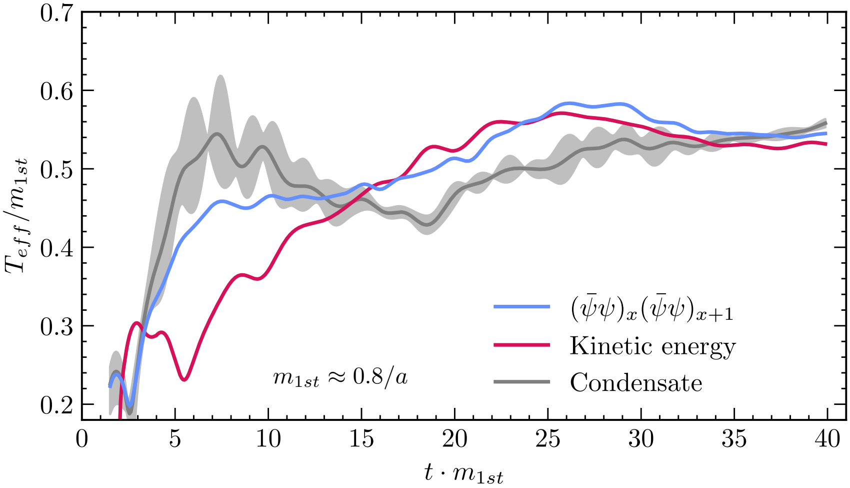
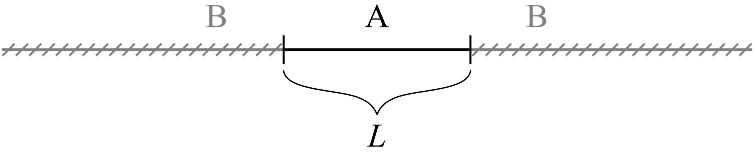


Area and volume law of entanglement



Gapped ground states: area law

Thermal states: volume law

Ent. entropy: $S = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

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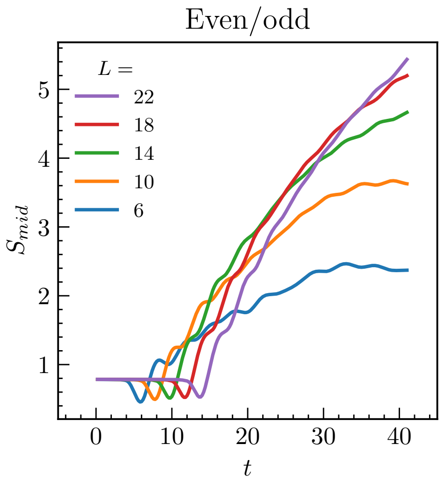

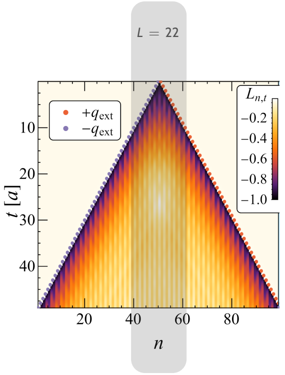

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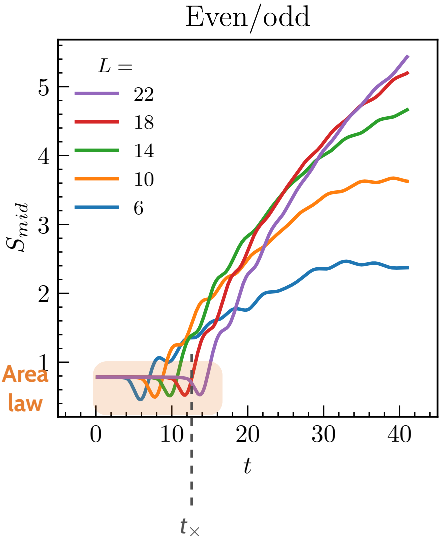

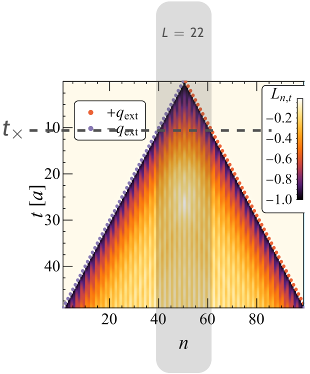

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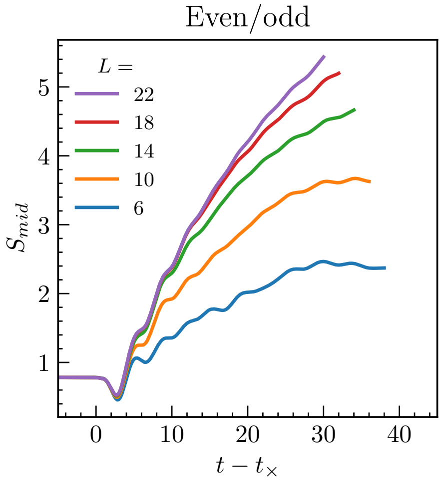

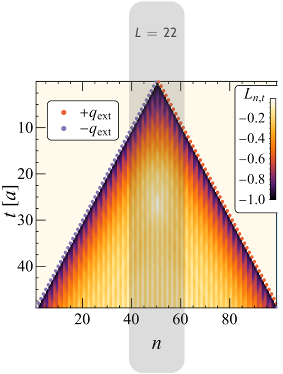

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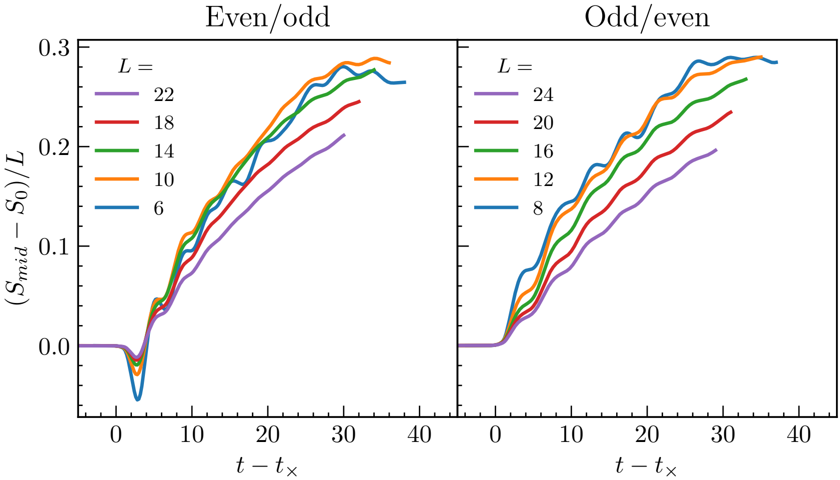

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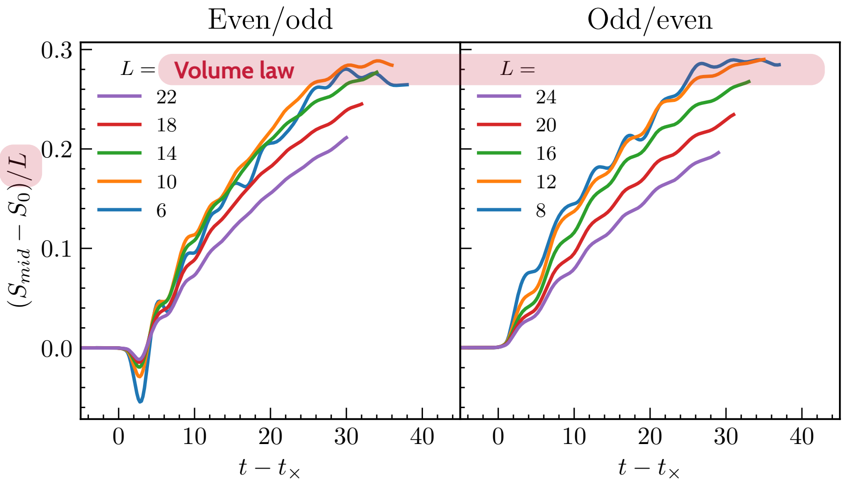
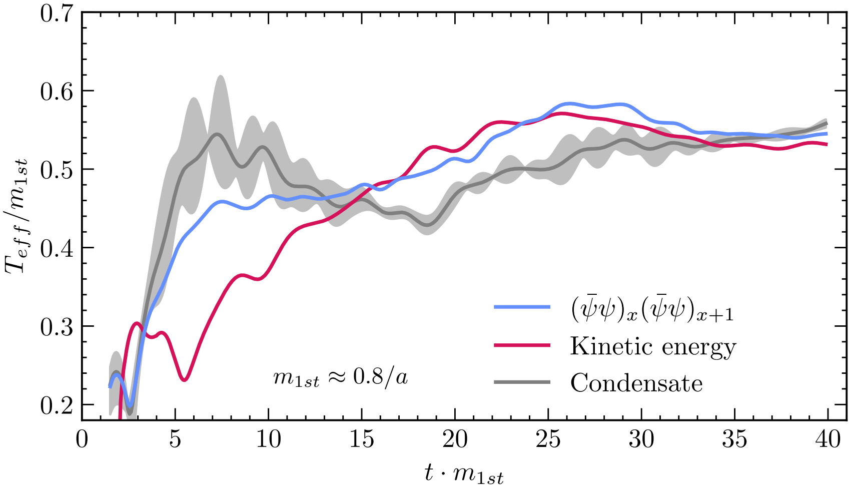


Entanglement and thermal entropy



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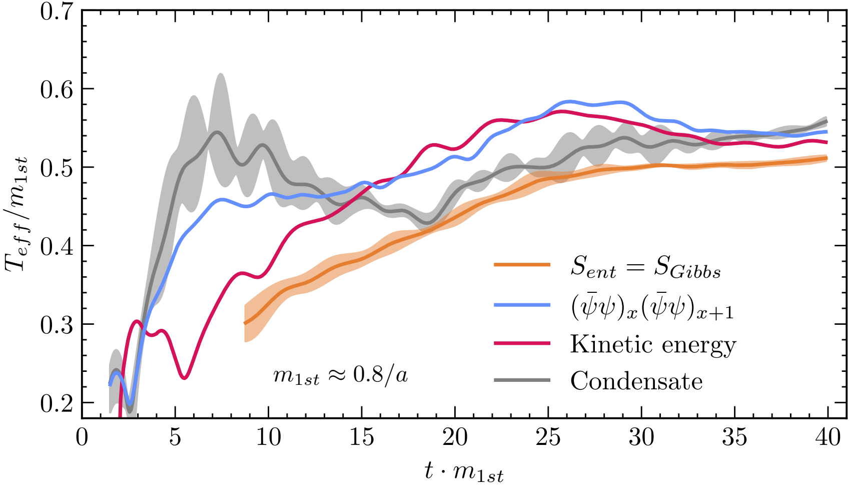
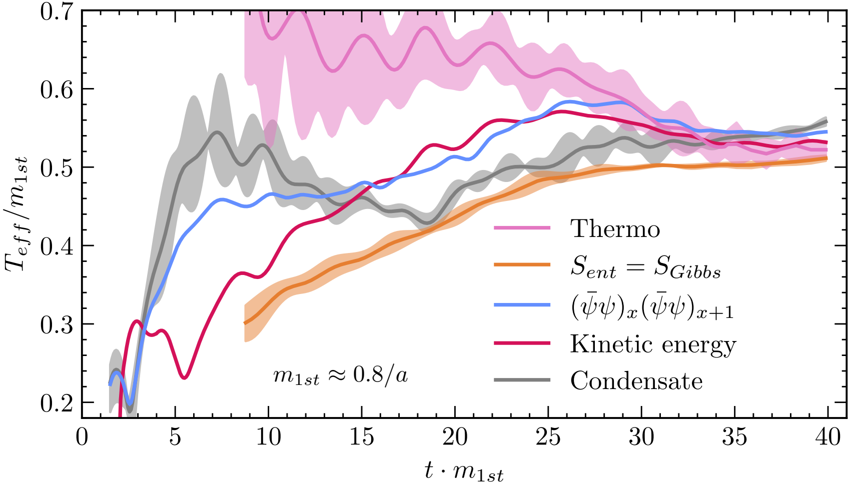


Thermodynamics

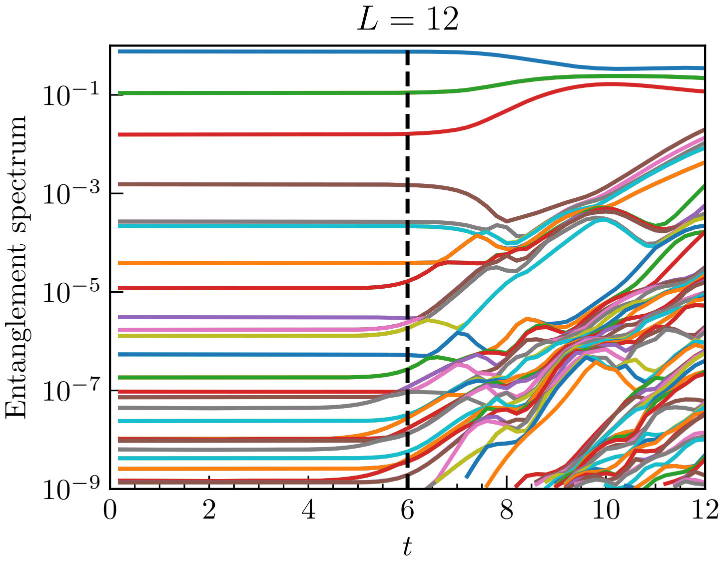
$$s = \frac{\epsilon+p}{T}$$

Ideal fluid: $\epsilon = T_{00}, p = \frac{1}{3}T^i_i$

Thermodynamics



Outlook: understand spectrum rearangement?



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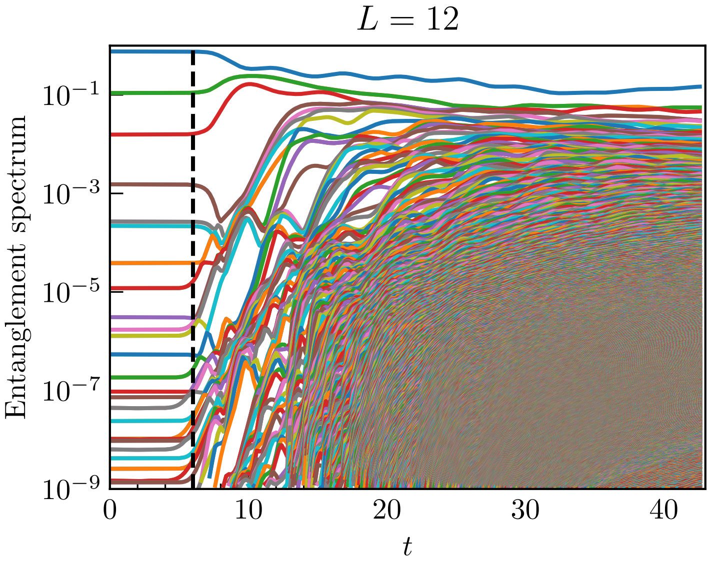
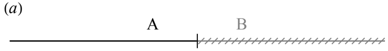


Outlook: thermalization time and entanglement propagation

Mid-lattice entanglement



Stop the external particles at t_{end}

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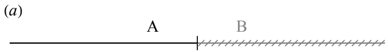

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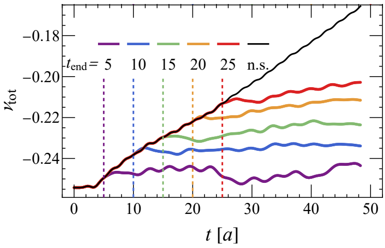

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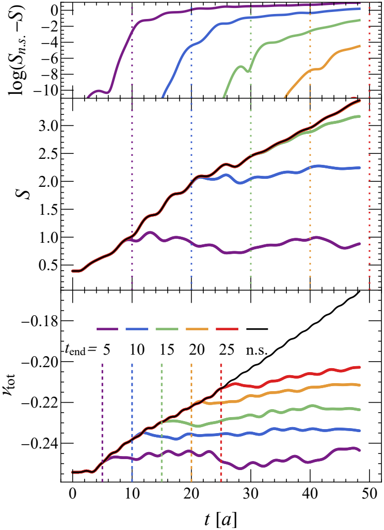

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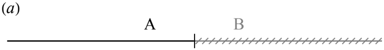

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- The Schwinger model can still teach us some physics
- Direct observation of quantum properties of string breaking and onset of thermalization
- Entanglement entropy becomes statistical entropy
- Be creative with low-dimensional models!

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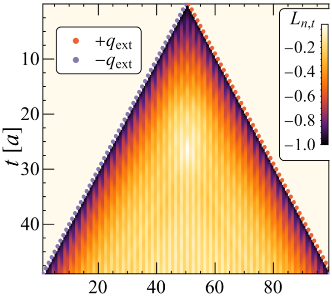

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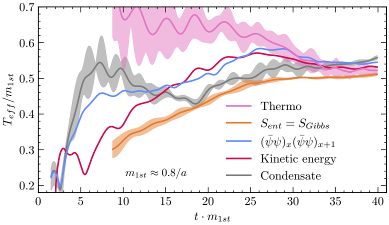

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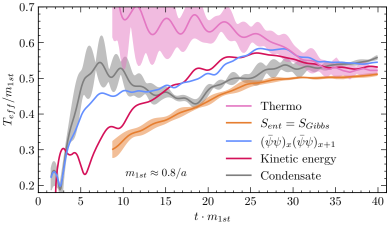

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