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Institute for Theoretical Physics

Classical vs. quantum simulations of lattice gauge theories

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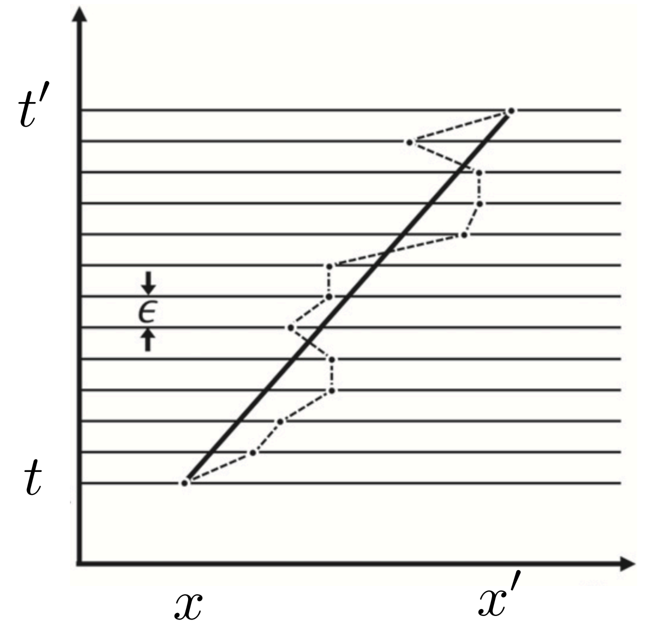
ECT* Trento, Bridging analytical and numerical methods for quantum field theory, 28 August 2025

Non-perturbative treatment of Lattice Gauge Theories

- Systematic method for computing hadronic observables from first principles

[K. Wilson, "Confinement of quarks" Phys. Rev.D. 10 (8): 2445–245 (1974)]

Path Integral



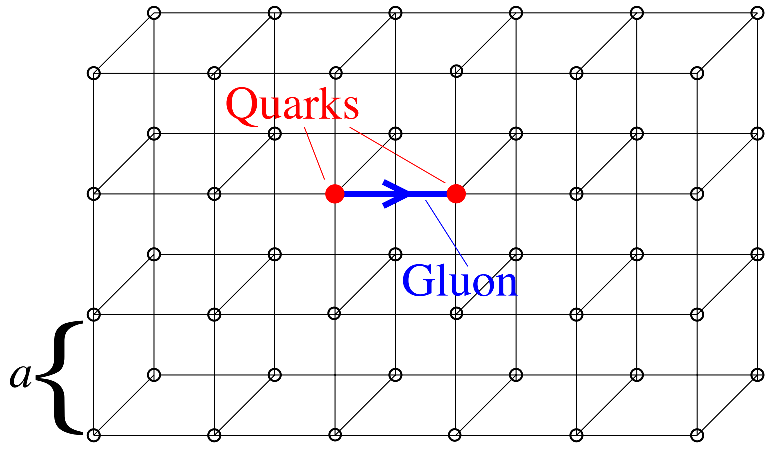
[Credit:Feynman, Hibbs, McGraw-Hill, NY (1965)]

+

Euclidean space-time

$\epsilon \longrightarrow -i\, a$

+



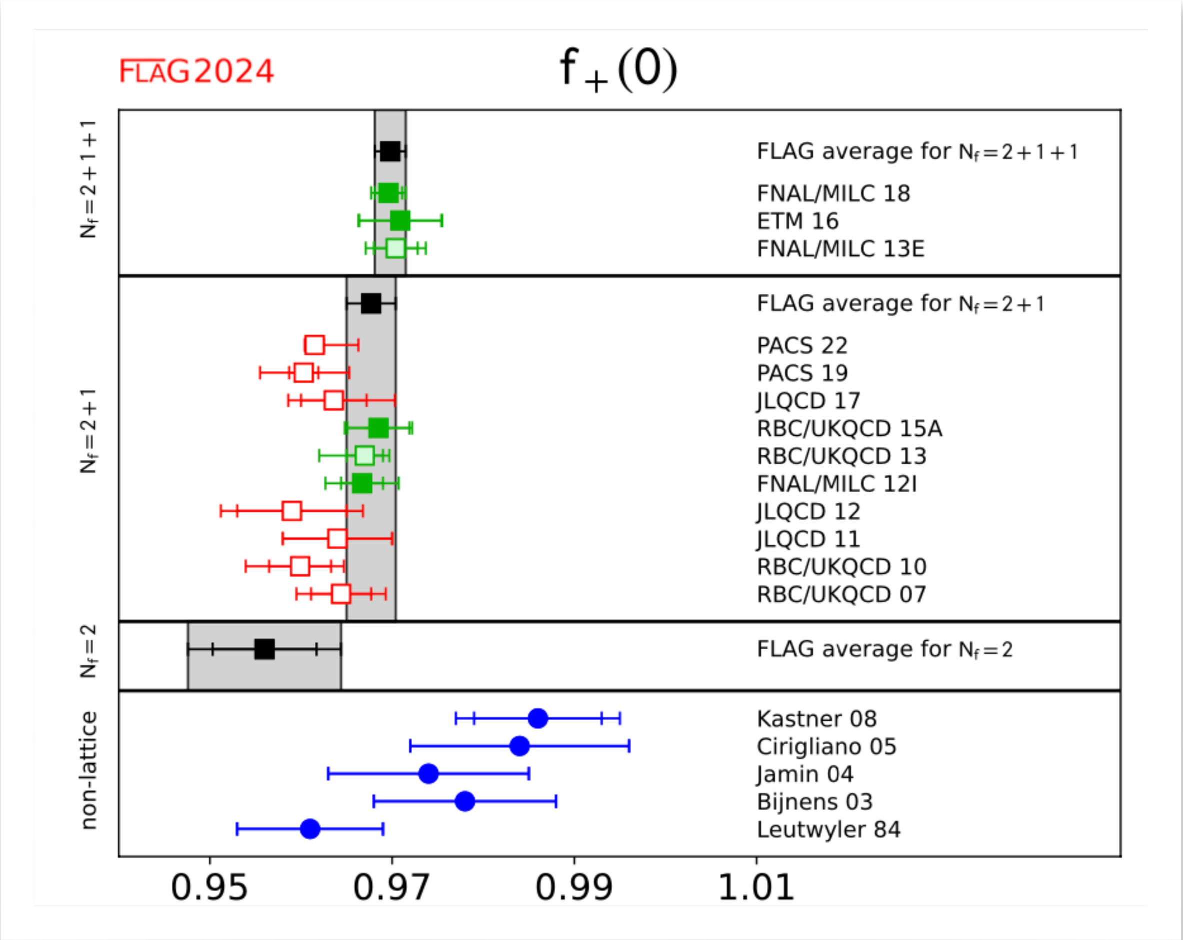
=

Lattice QCD

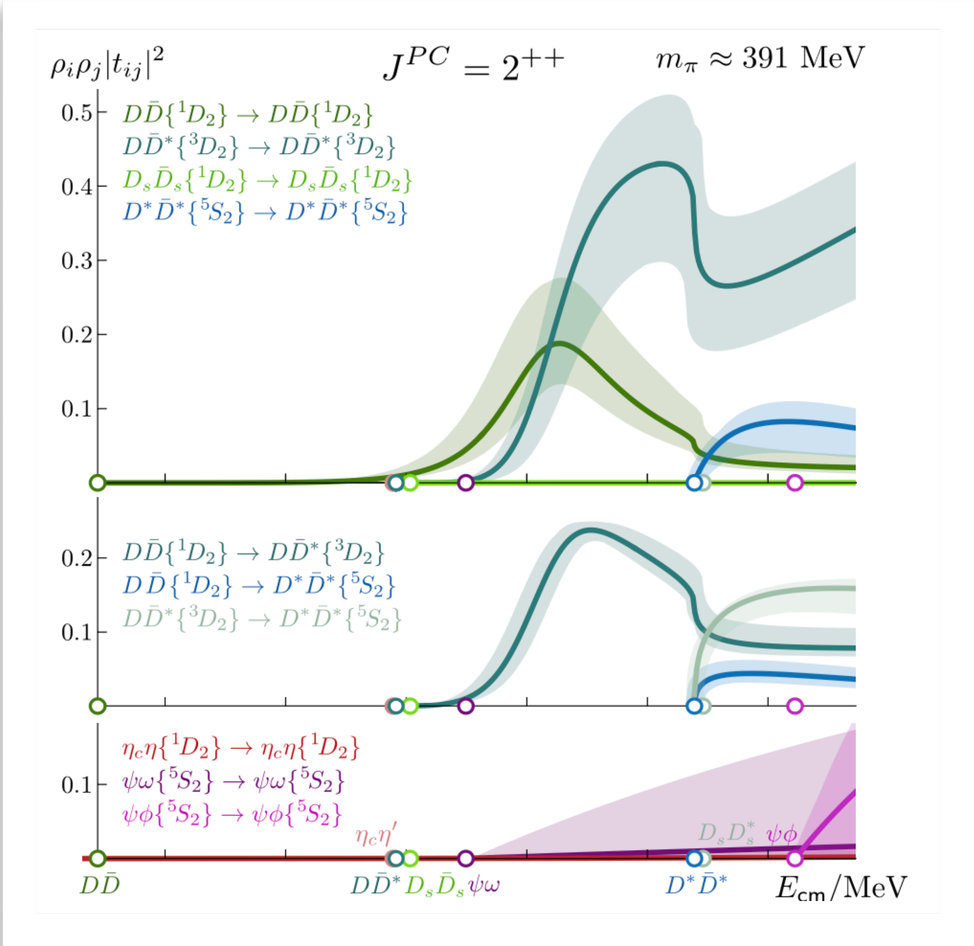
$$\mathscr{L}^E_{QCD} = \frac{1}{2g} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=u,d,s,...} \overline{\psi}_f \left\{ \gamma_\mu (\partial_\mu + i A^a_\mu T^a) + m_f \right\} \psi_f$$

$$S^E_{QCD} = \int d^4x \left[\frac{1}{2g} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=u,d,s,...} \overline{\psi}_f \left\{ \gamma_\mu (\partial_\mu + i A^a_\mu T^a) + m_f \right\} \psi_f \right] = S_G + S_F$$

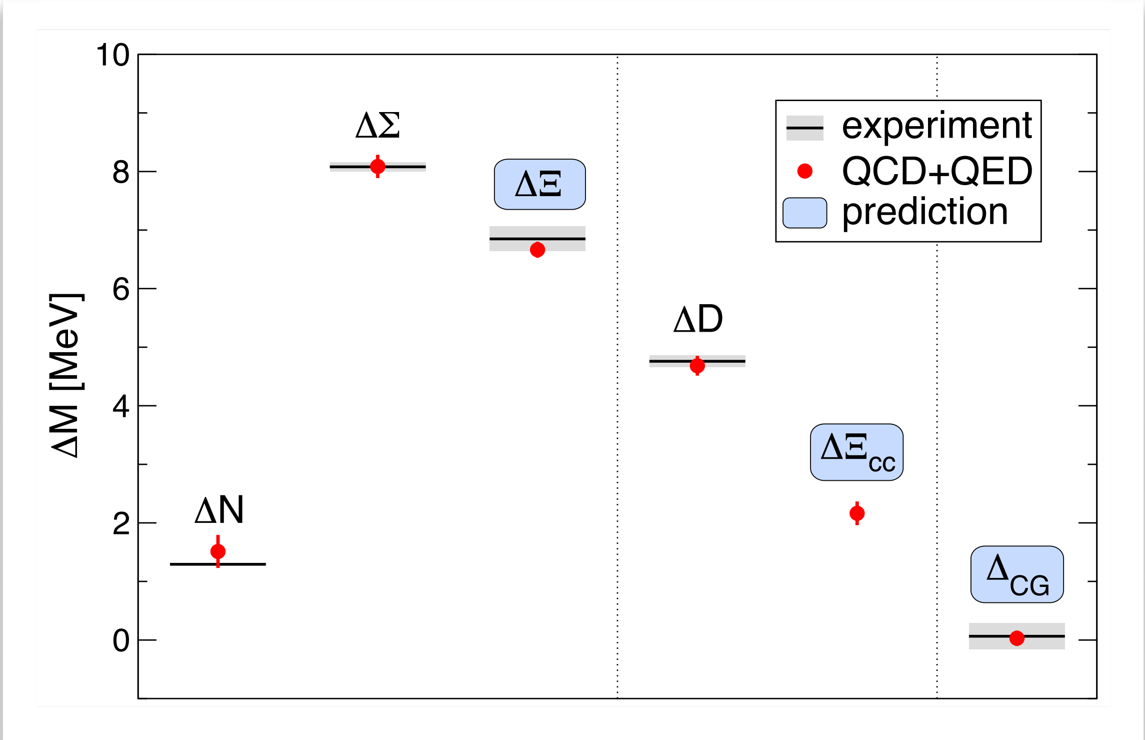
Precision Lattice QCD calculations



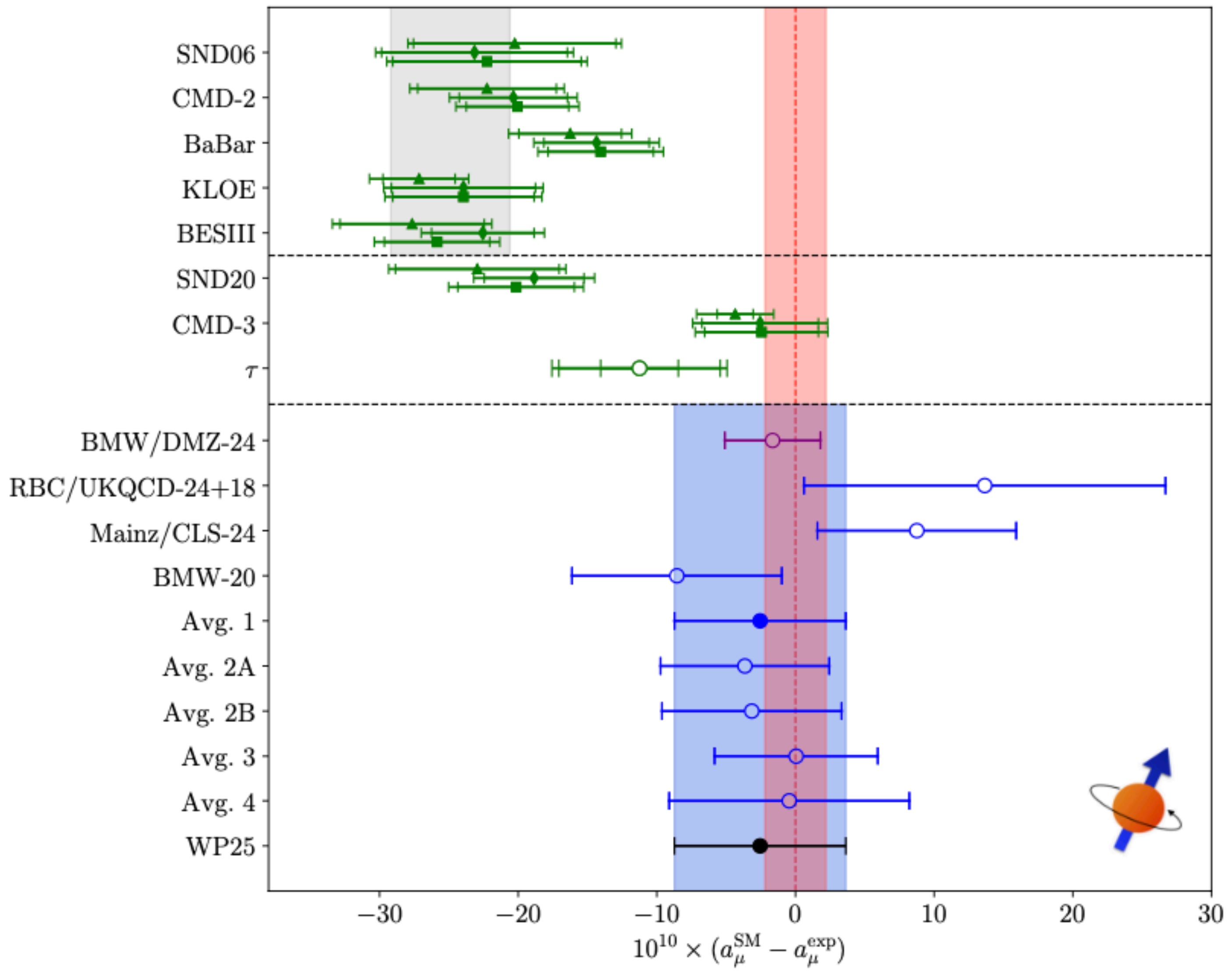
FLAG24 [Aoki et al. 2411.04268]



Charmonium Resonances in Coupled-Channel Scattering [Wilson, Thomas, Dudek, Edwards, 2309.14070]

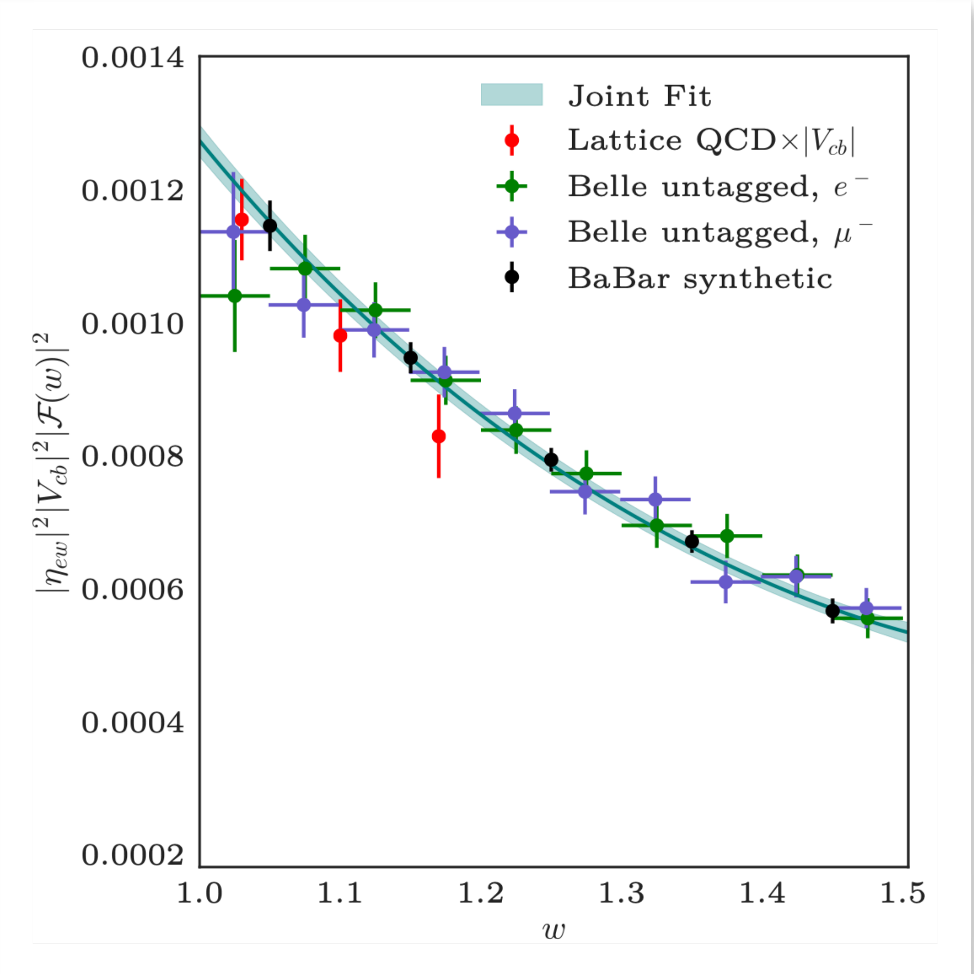


M_p-M_n [Borsanyi et al. 1406.4088]

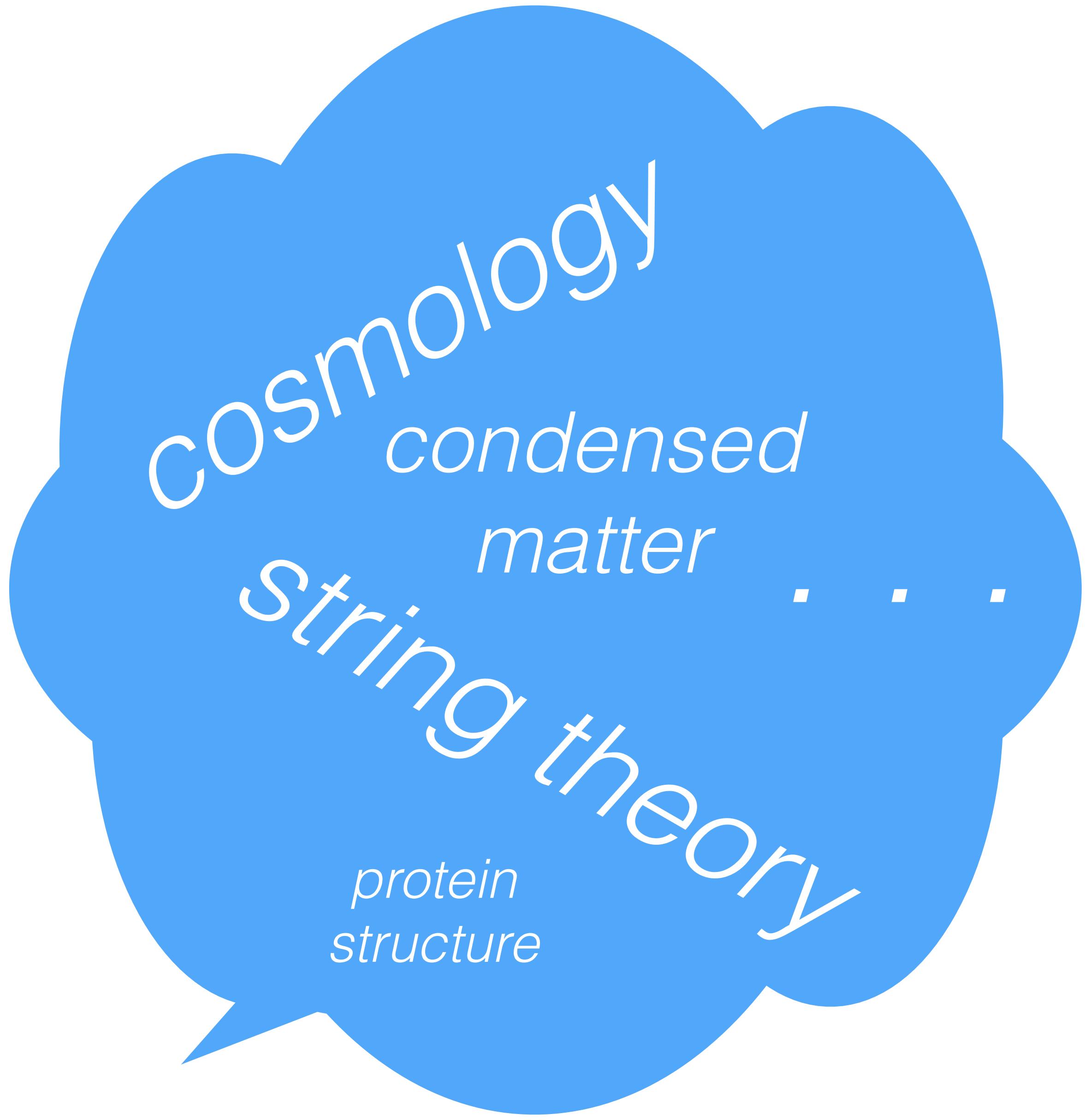


[Muon g-2 theory Initiative, $(g-2)_\mu$ in the SM: an update, arXiv:2505.21476 [hep-ph]]

NEW!



$|V_{cb}|$ from $B \to D^* l\nu$ [Bazavov et al. 2021, arxiv:2105.14019]



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Classical Lattice QCD Simulation

(1) Generate ensembles of field configurations using Monte Carlo

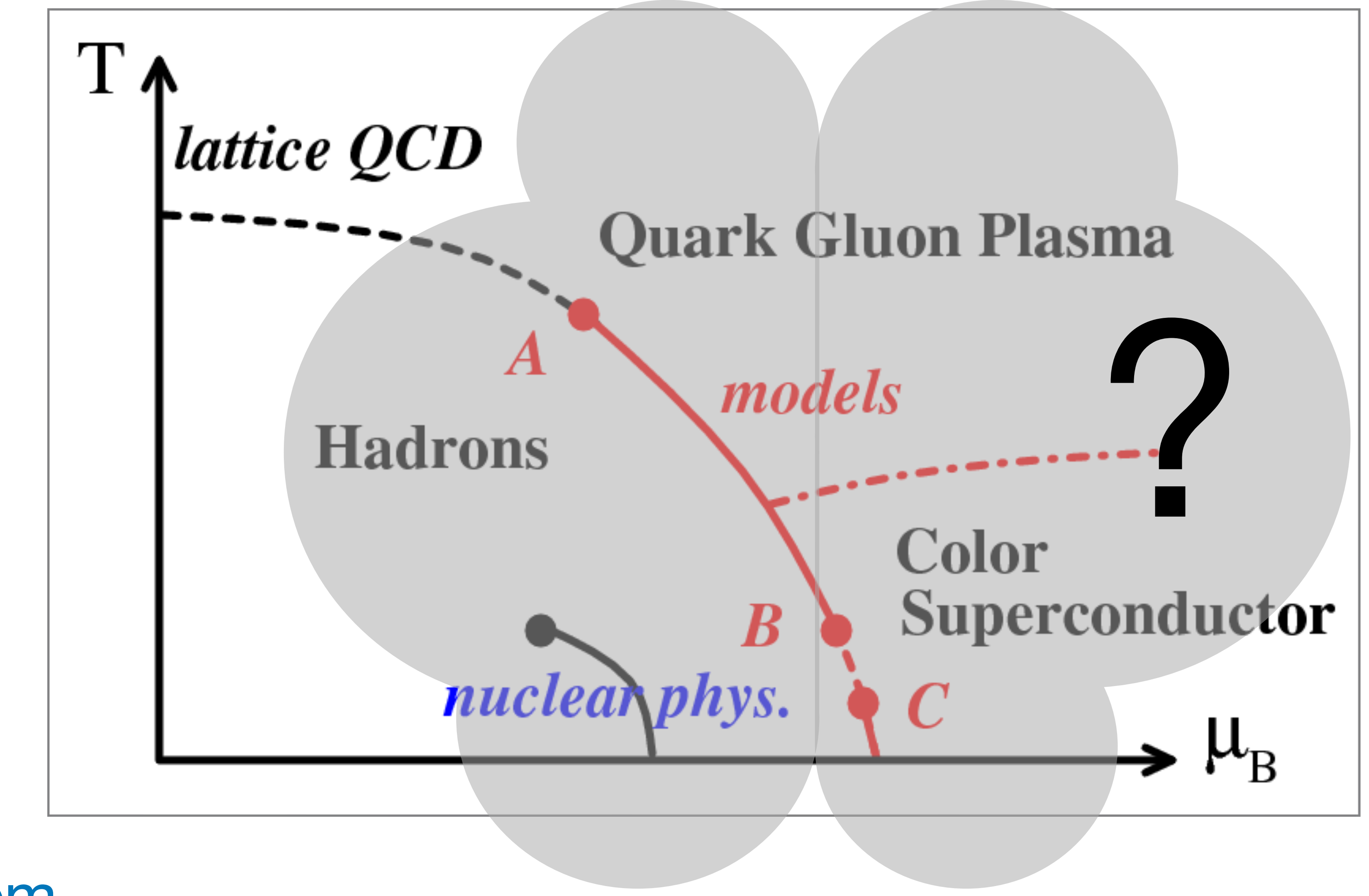
(2) Average over a set of configurations: $$\langle O[A]\rangle \approx \bar{O} = \frac{1}{N_{cnfg}} \sum_{i=1}^{N_{cnfg}} O[A] + \mathcal{O}(\frac{1}{\sqrt{N_{cnfg}}})$$

- Compute correlation function of fields, extract Euclidean matrix elements or amplitudes
- Computational cost dominated by quarks: inverses of large, sparse matrix

(3) Extrapolate to continuum, infinite volume, physical quark masses (now directly accessible)

Many regimes we don’t (yet) know how to simulate

… real time dynamics, finite baryonic densities, θ terms



[C. Sasaki, Nucl.Phys.A830 (2009)]

$$\langle O[A,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int D[A] \det D[A] e^{-S_G[A]}$$

Complex Weight W → Sign Problem

$$\langle O\rangle = \frac{\sum_C O_C \cdot W_C}{\sum_C W_C} = \frac{\sum_C O_C \cdot \mathrm{sign}(W_C)\cdot |W_C|}{\sum_C \mathrm{sign}(W_C)\cdot |W_C|}$$

$$= \frac{\langle \mathrm{sign}\cdot O\rangle_{|W|}}{\langle \mathrm{sign}\rangle_{|W|}}$$

- Quantum simulations for fundamental understanding of the phases of matter?
- Analytic and classical algorithmic approaches are equally needed

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Quantum (LGT) Simulation

(1) Initial state of a quantum many body system: $|\psi_0\rangle$ [state preparation algorithms]

(2) Time evolution: [Trotter, Proc. Am. Math. Soc. 10 (1959), Suzuki Commun. Math. Phys. 51 (1976)]

- $|\psi_T\rangle = \mathcal{T}\{e^{-i\int_0^T dt\, H(t)}\}|\psi_0\rangle \approx \underbrace{U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)}_{M \text{ steps}}|\psi_0\rangle$
- Suzuki-Trotter decomposition $U(\tau) = e^{-iH(\tau)\delta t}$, $\delta t = \frac{T}{M}$, T finite, $\int$ discretized

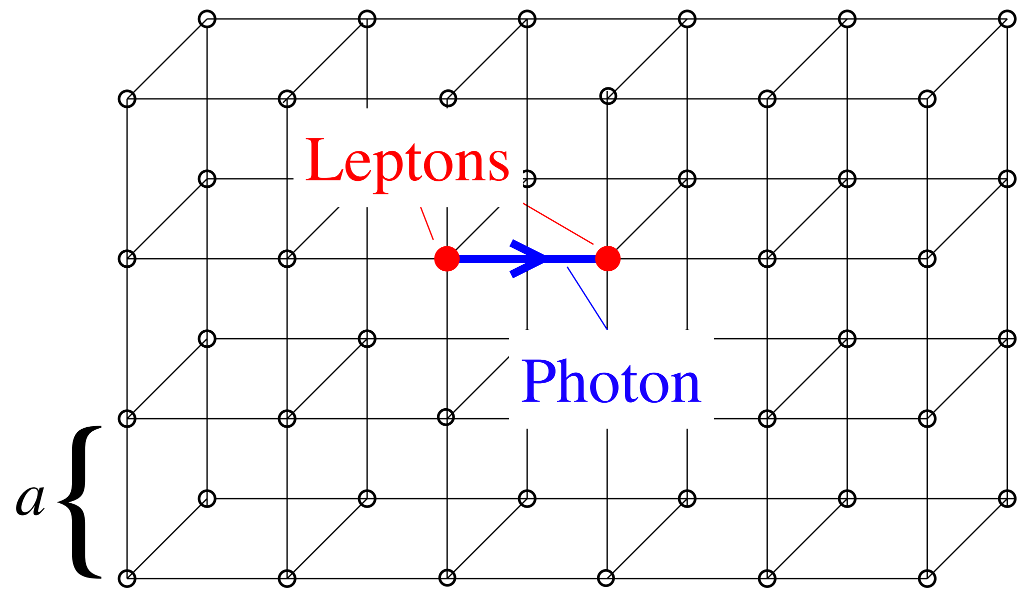
(3) Measure observables [correlation functions, entanglement entropy, scattering processes, …] NEW!
[Davoudi et al., arXiv:2507.15840]

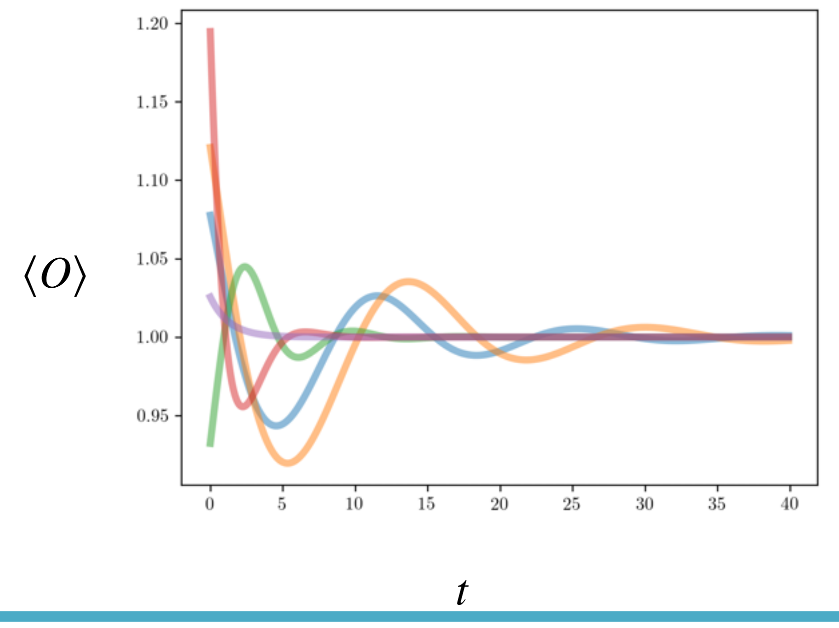
(4) Extrapolate to continuum, infinite volume [precision fronteer; futuristic]

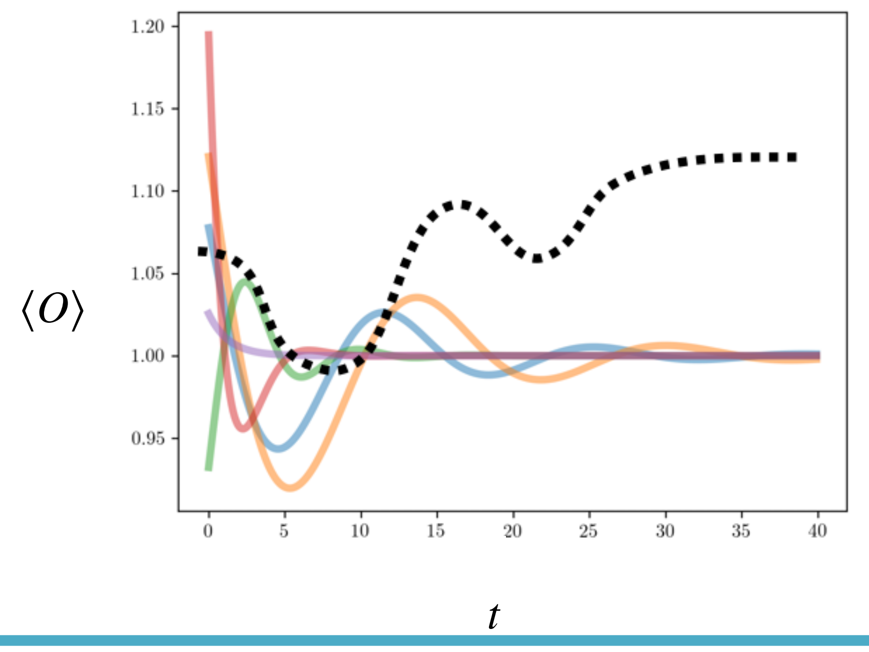
Real-time dynamics of QED_2 and QED_3

[O. Aharony, Wed. 15.30]

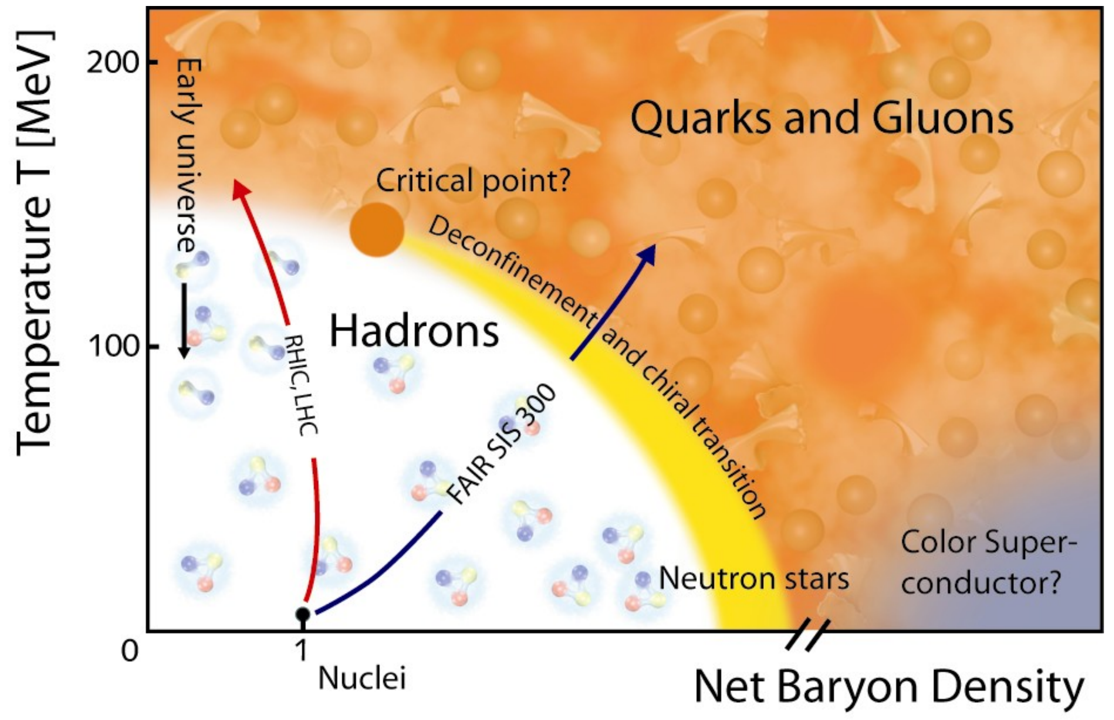
1. (How) do gauge theories thermalize? $[\mathrm{QED}_3]$







2. What can we learn from quantum simulations about the phase diagram of QED_2 ?



[Credit: Peter Senger (p.senger@gsi.de)]

Toy model for QCD in 3+1D: QED in 1+1D

Schwinger model [Schwinger, Physical Review 125, 397 (1962)]

Chiral symmetry, confinement, string breaking …

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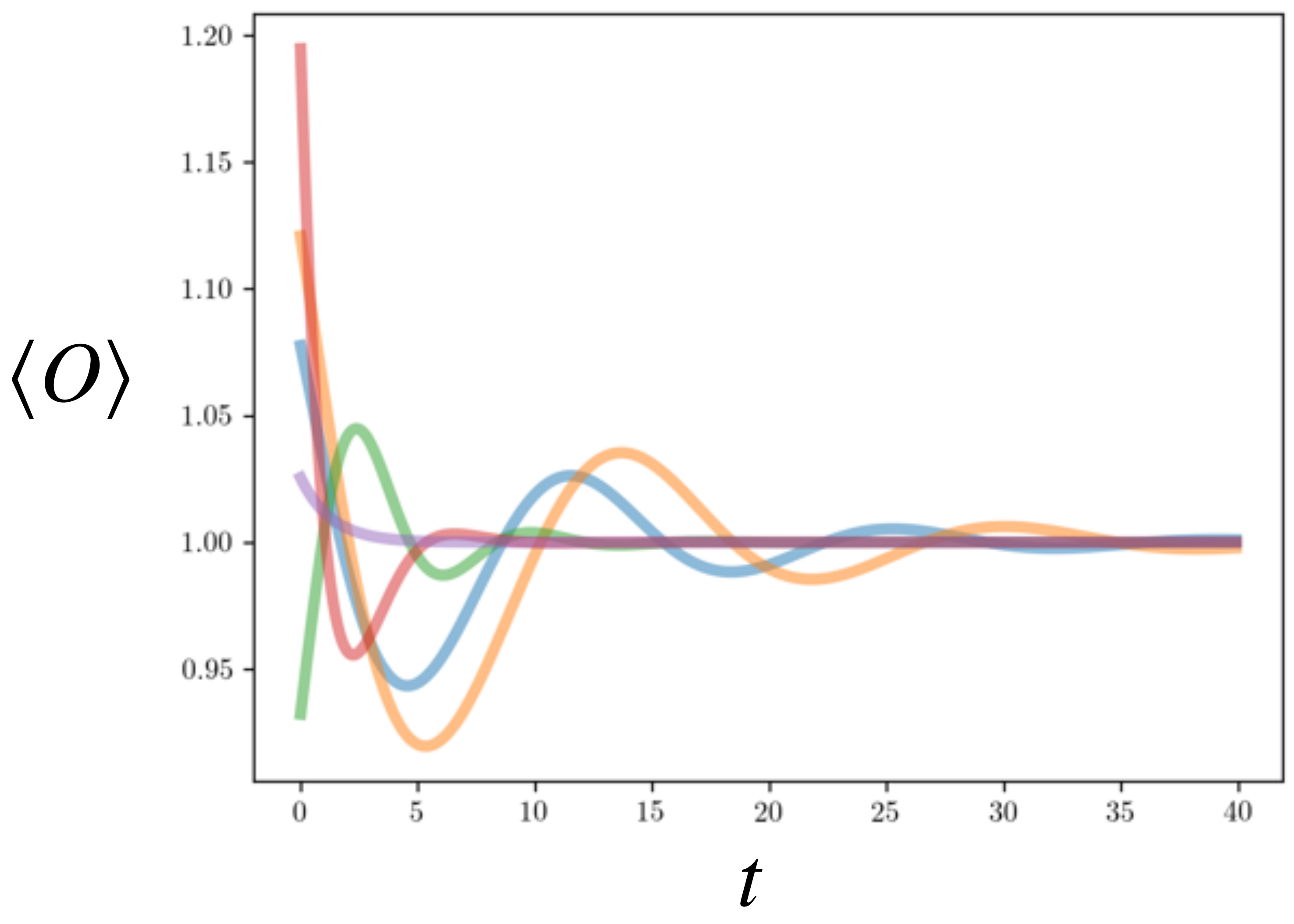
Eigenstate Thermalization Hypothesis (ETH)

- Local observables are expected to thermalize

$$\mathcal{O}(t) = \langle \psi(t) | \mathcal{O} | \psi(t) \rangle = \sum_n |c_n|^2 \mathcal{O}_{nn} + \sum_{m \neq n} c_m^* c_n \, e^{i(E_m - E_n)t} \, \mathcal{O}_{mn}$$



- ETH [von Neumann(1929), Deutsch. PRA (1991), Srednicki PRE (1994)] bounds the fluctuations

$$\mathcal{O}_{mn} = \mathcal{O}(\overline{E}) \, \delta_{mn} + e^{-S(\overline{E})/2} f_{\mathcal{O}}(\overline{E}, \omega) \, R_{mn}$$

$\overline{E} = (E_m + E_n)/2$; $\omega = E_m - E_n$; $f_{\mathcal{O}}$ — smooth function; R_{mn} — random variable

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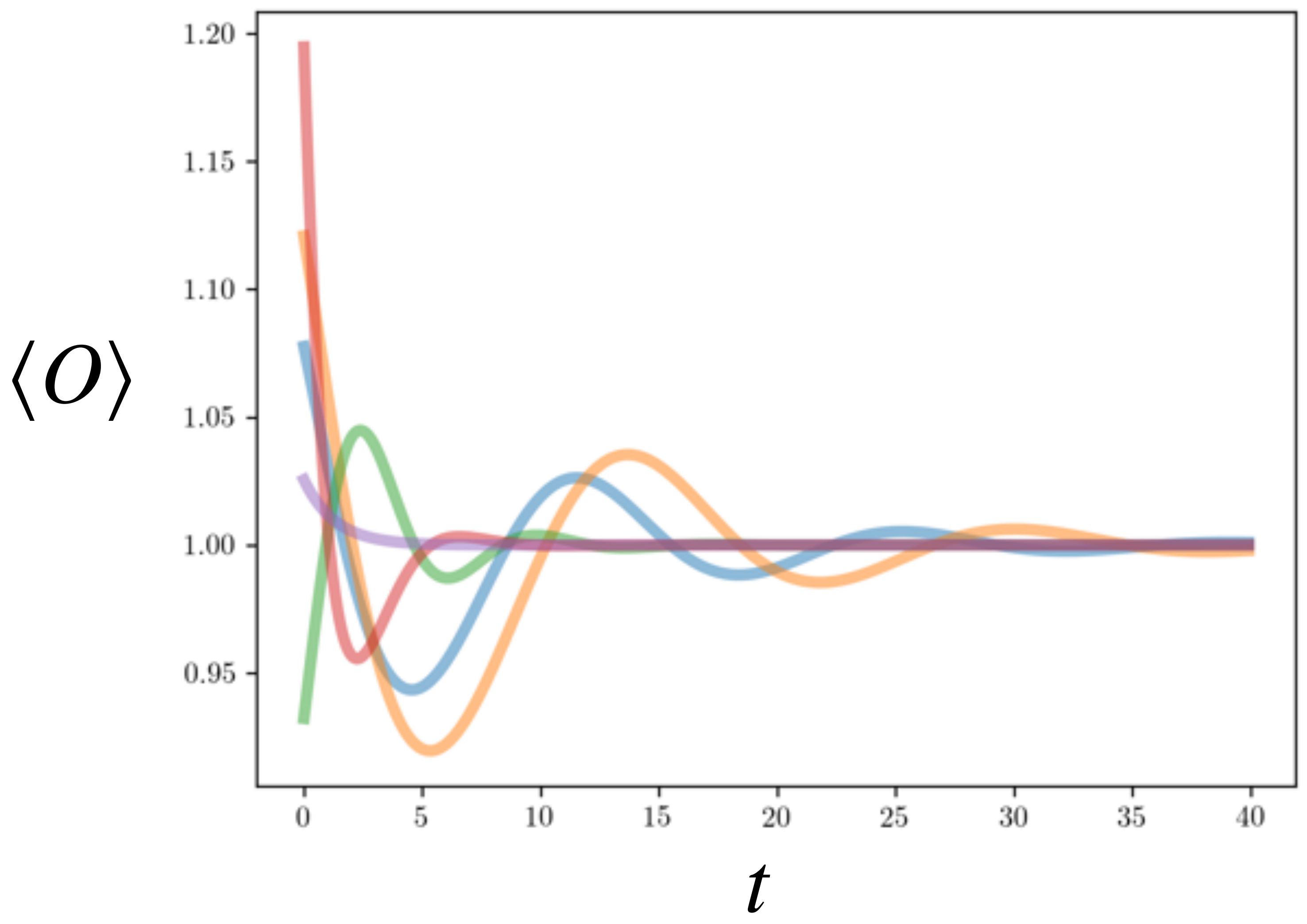
Thermalization of isolated quantum systems

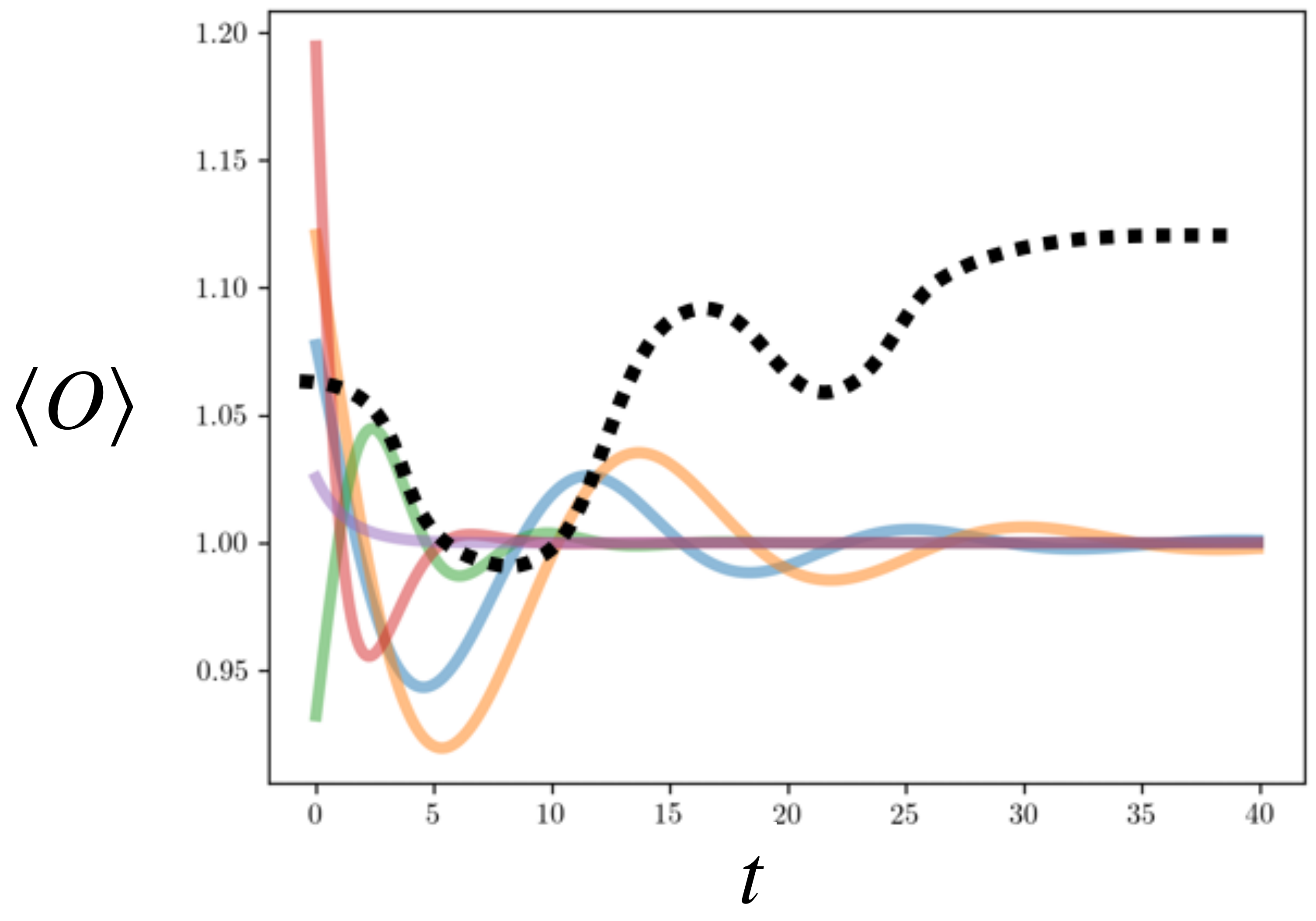


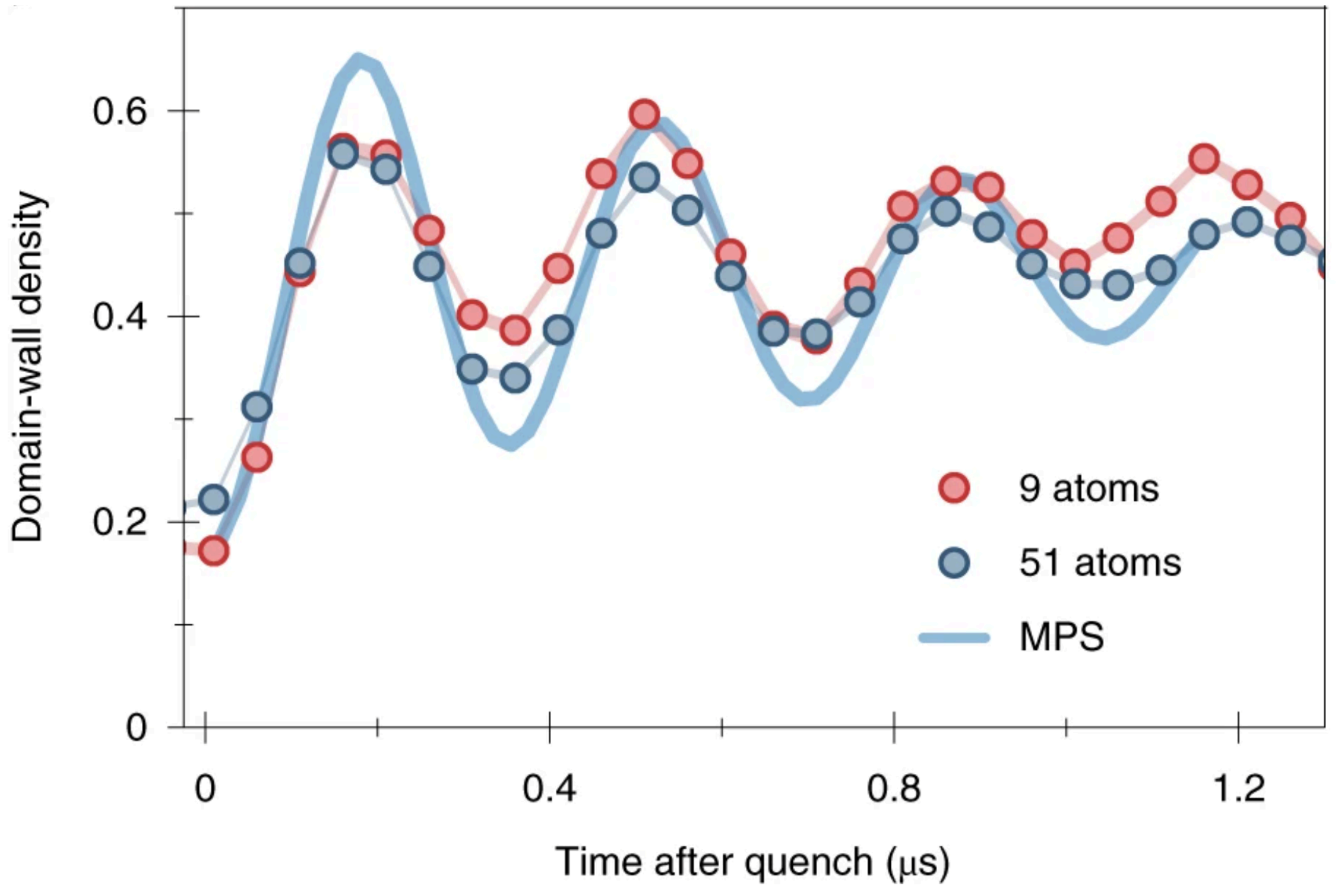
Exceptions to thermalization:

- Integrable models
- Many Body Localization
- Fragmentation
- *Quantum Many-Body Scars*: special initial conditions avoid thermalization [Turner et al.,Nature Physics (2018))]

- Ergodicity: single fragment Hilbert space; Hamiltonian connects all basis states
- Strong breaking of ETH: integrable models, disordered systems; exponentially many fragments; $\frac{n}{N_{tot}} \sim e^{-V}$
- QMBS: weak breaking of ETH, (many) anomalous high-energy states; $\frac{n}{N_{tot}} \to 1$ as $V \to \infty$.

Thermalization of isolated quantum systems





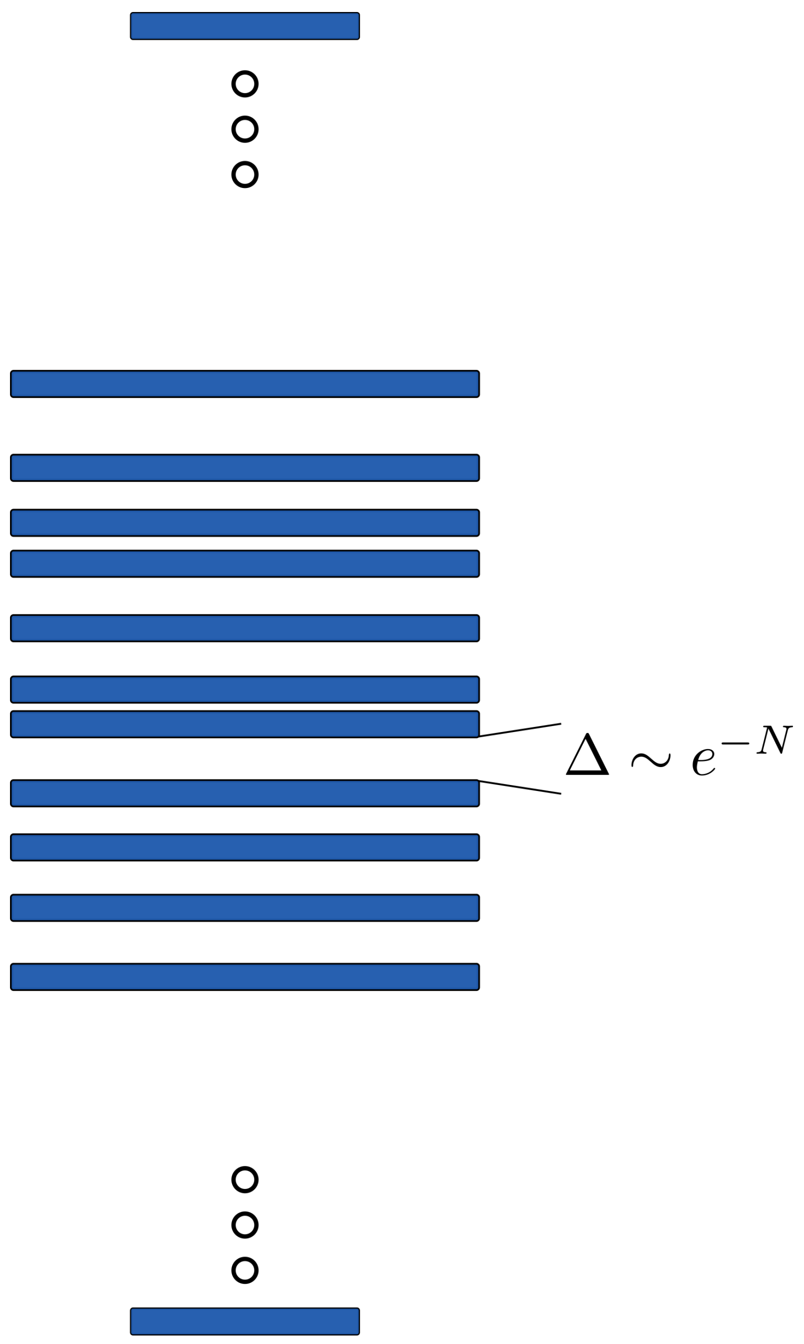
[Bernien et al. Nature 551 (2017)]

- Quantum Many-Body Scars: special initial conditions avoid thermalization [Turner et al.,Nature Physics (2018), Pakrouski et al. PRL 125, 230602 (2020)]
- Fundamental questions that might be answered on quantum simulators
- Real-time evolution; severe sign problems [Troyer, Wiese PRL 94 170201(2005)]
- Signatures found in analog quantum simulations with Rydberg atom arrays [Bernien et al. Nature (2017)]

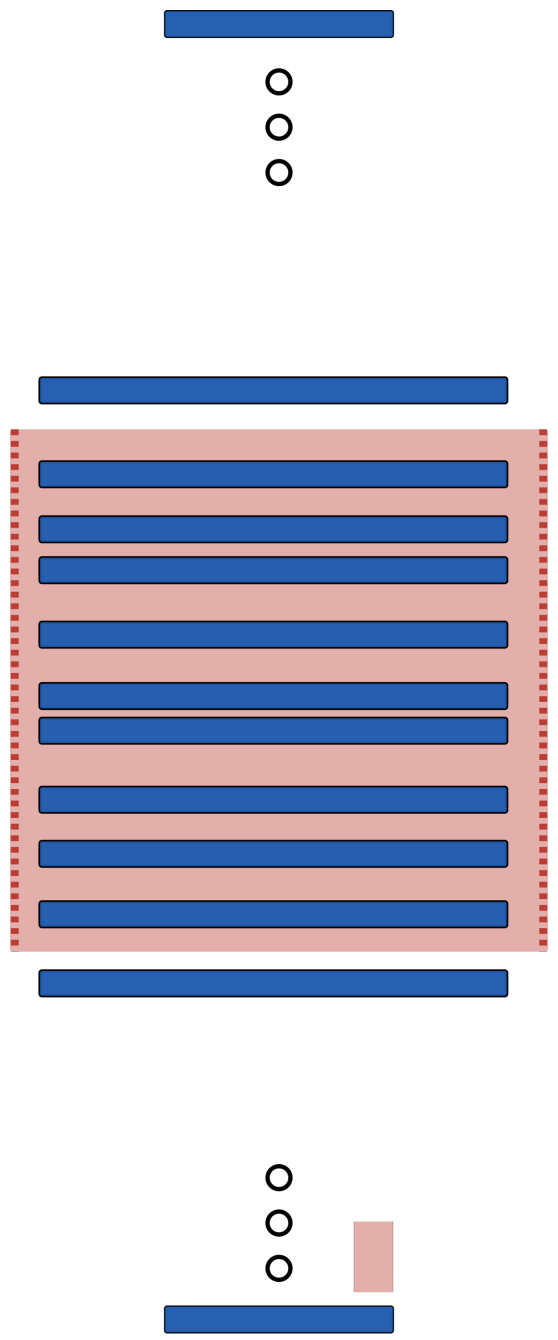
Eigenstate Thermalization Hypothesis (ETH)

Effect of small perturbations in parameters to the spectrum of the Hamiltonian:

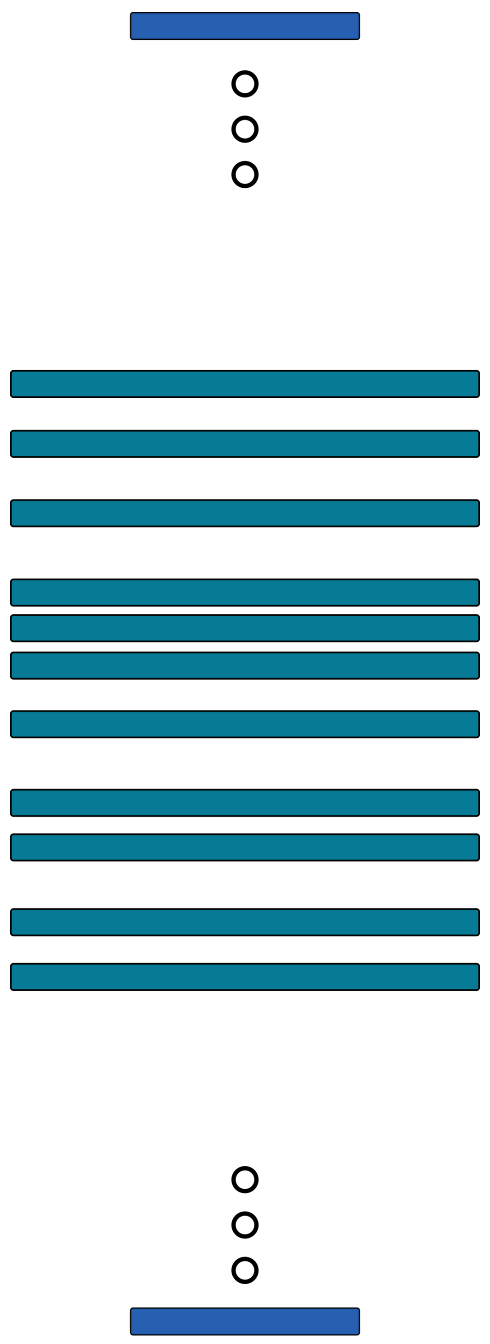
$$H \longrightarrow H + \delta H$$



Mid-spectrum states have exponentially small gaps



Small δH mix many of mid-spectrum states



Highly entangled mid-spectrum states

[Credit: J. Pinto Barros]

Motivation: real-time dynamics of QED_3

- Quantum systems with many degrees of freedom: fast/slow thermalization [von Neumann(1929), Deutsch. PRA (1991), Srednicki PRE (1994)]

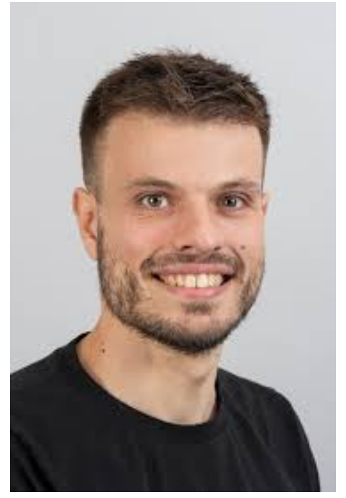
- Under which conditions can thermalization be evaded?

- Weak and strong ergodicity breaking in gauge theories

Motivation: real-time dynamics of QED_3

- Quantum systems with many degrees of freedom: fast/slow thermalization [von Neumann(1929), Deutsch. PRA (1991), Srednicki PRE (1994)]
- Under which conditions can thermalization be evaded?
- Weak and strong ergodicity breaking in gauge theories
- Quantum Many-Body Scars in simple gauge/fermionic theories [Pakrouski et al. PRL 125 (2020), Mukherjee et al. PRB(2021), Banerjee, Sen PRL (2021), Surace et al. PRX (2021), Delacretaz et al. JHEP (2022), Desaules et al. PRB (2023), Srdinsek et al. PRL (2024), Calajo et al. (2024)…]
- Probed for small volumes and small spins due to severe sign problems in real-time dynamics simulations
- Analytic construction of ergodicity breaking states for arbitrary volumes and spins

Thea Budde



Joao Pinto Barros

[Budde, MKM, Pinto Barros, PRD 110, 094506, arXiv:2403.08892]

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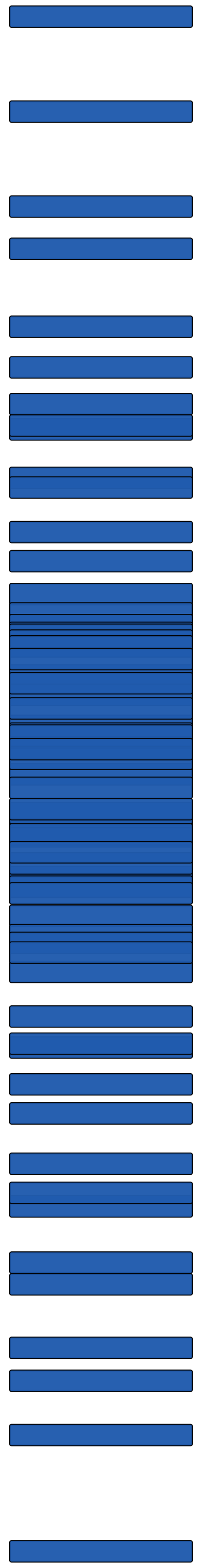
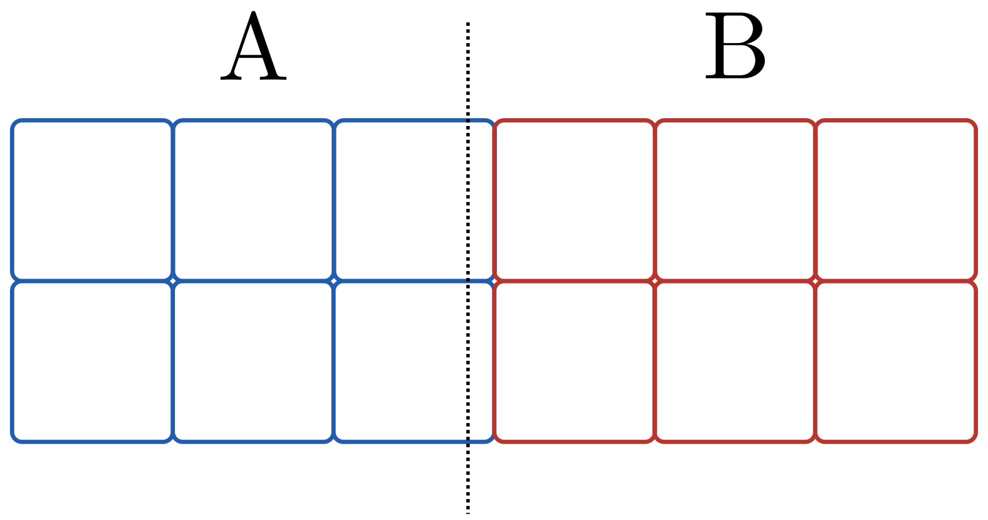
Entanglement Entropy

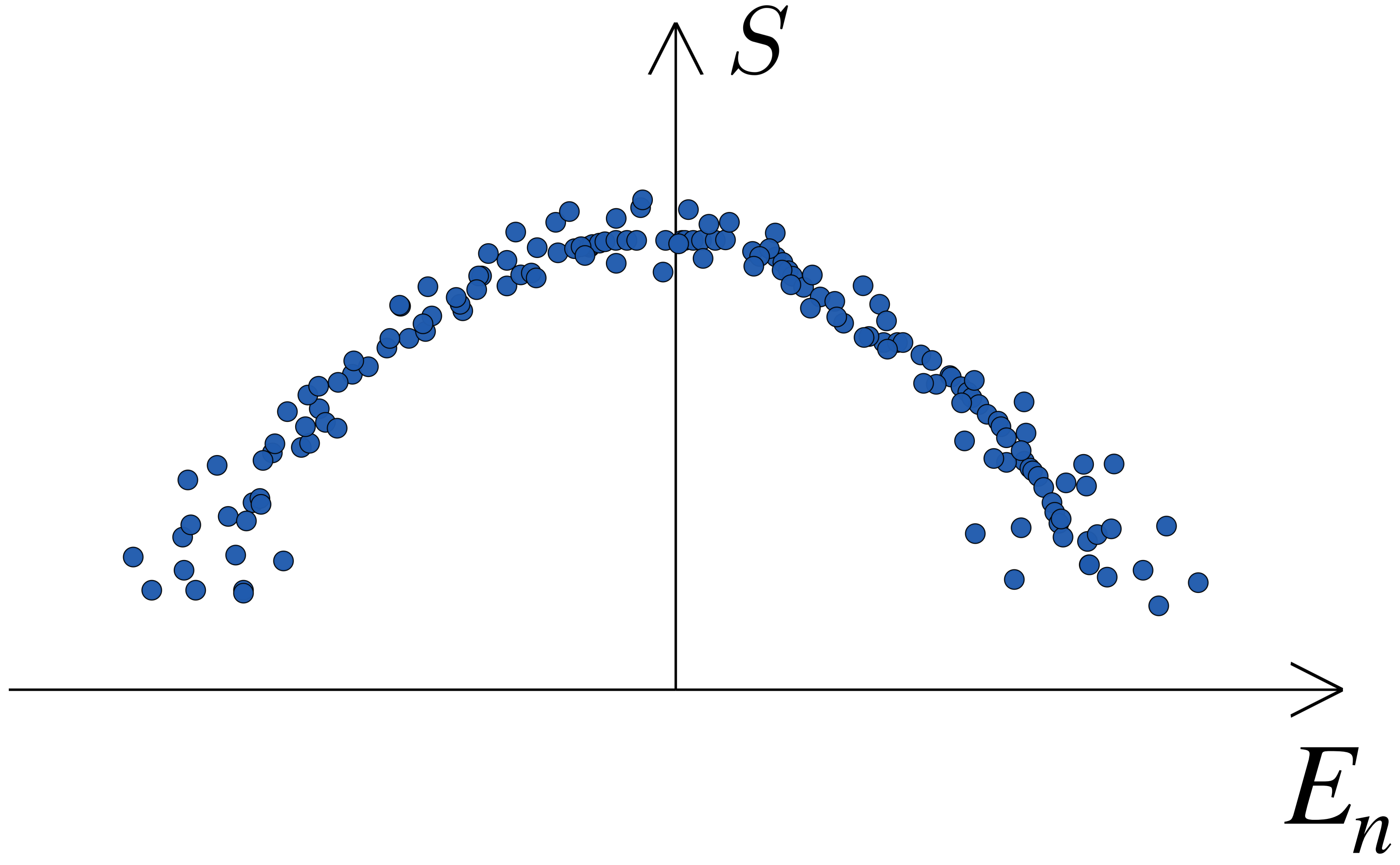


Bipartite entanglement entropy:

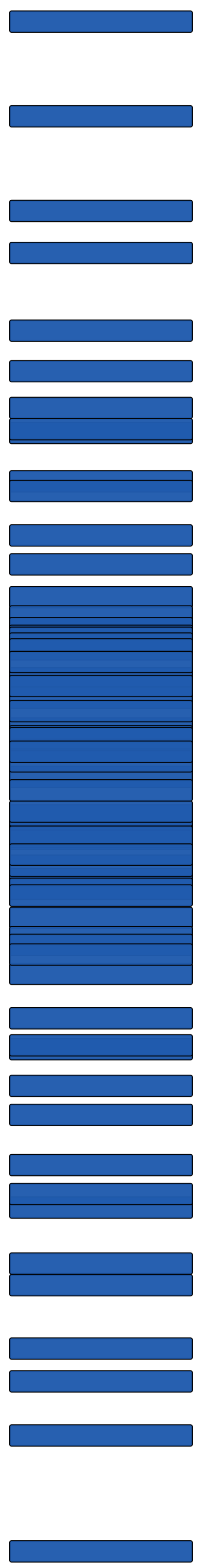


Compute the trace: $\rho_A = \mathrm{tr}_B \rho$

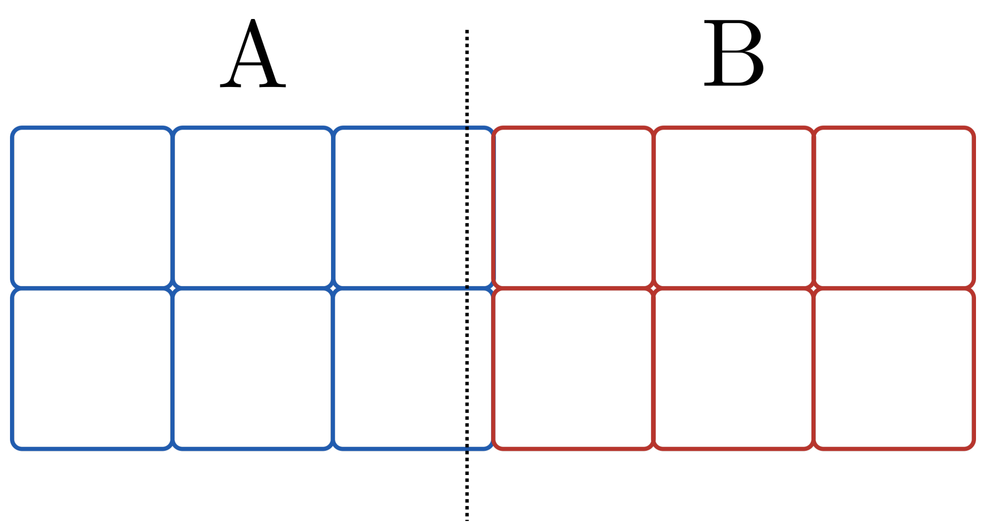
Compute entanglement entropy: $S = -\mathrm{tr} \rho_A \log \rho_A$



Entanglement Entropy



Bipartite entanglement entropy:



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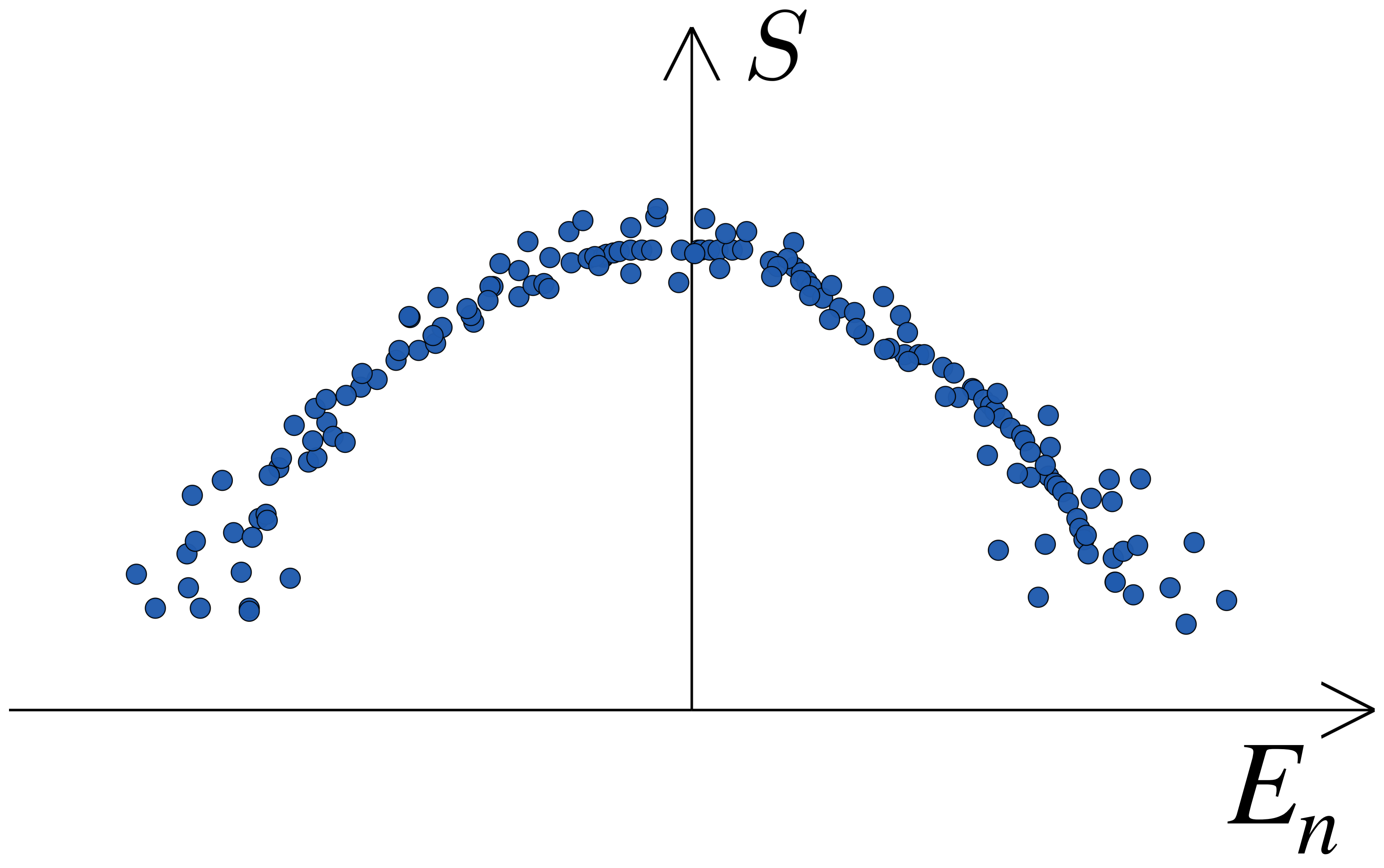
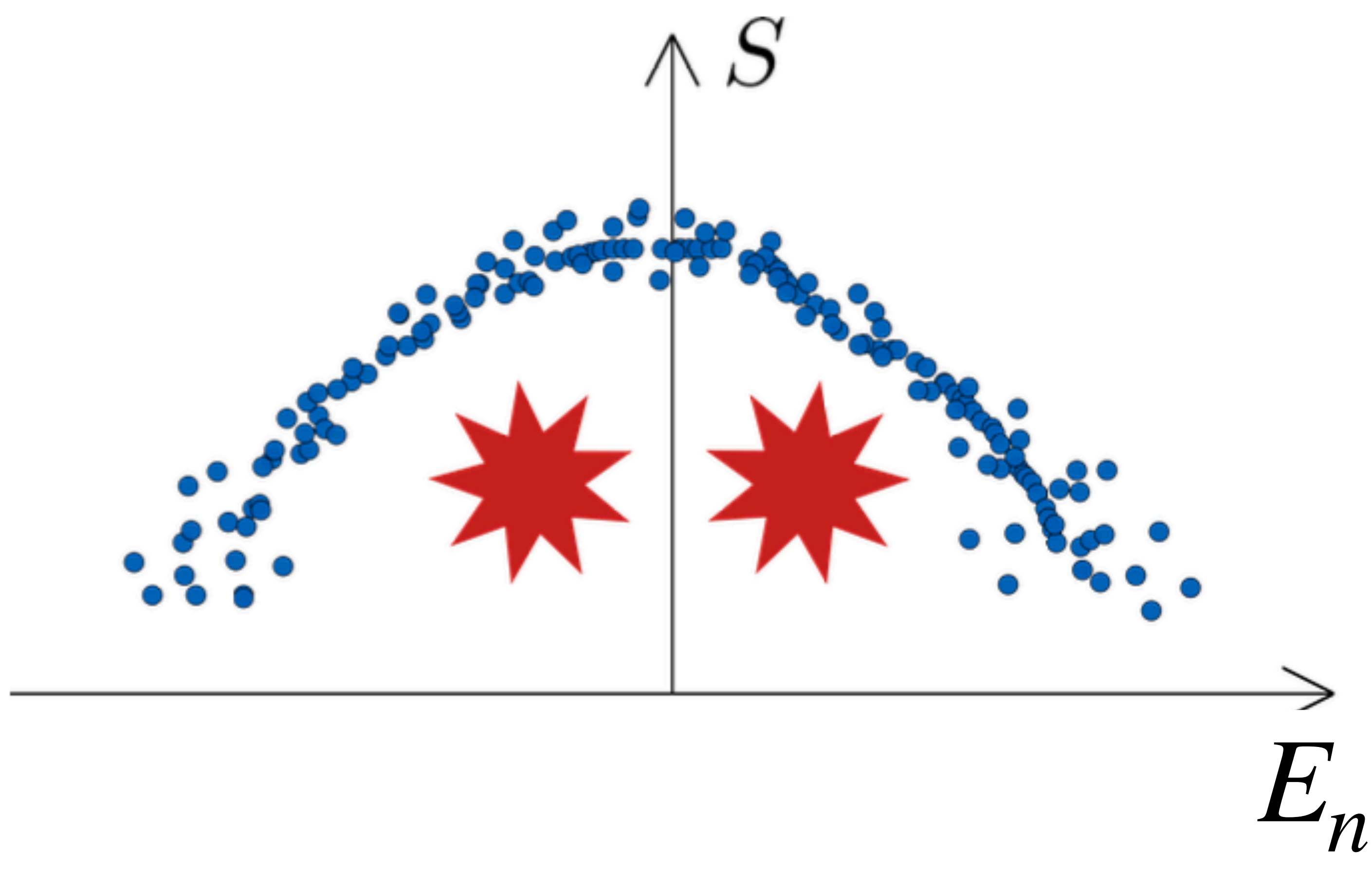




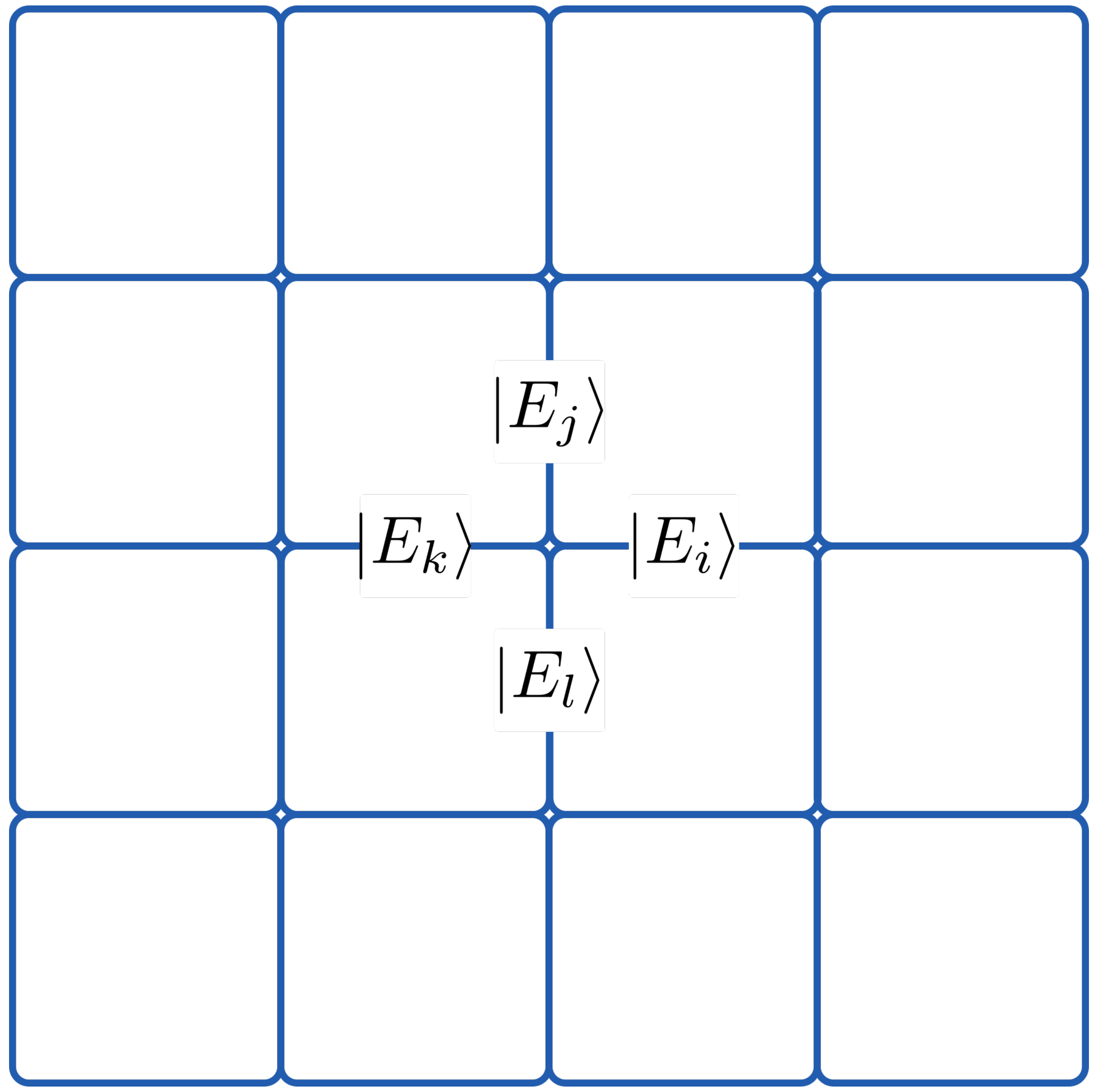
QMBS: entanglement measure shows anomalous behaviour

Hamiltonian of U(1) gauge theory in 2+1D

- Quantum spins: continuous gauge symmetries with discrete link operators

- Finite Hilbert space at each link: (2S + 1)-dim.

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$



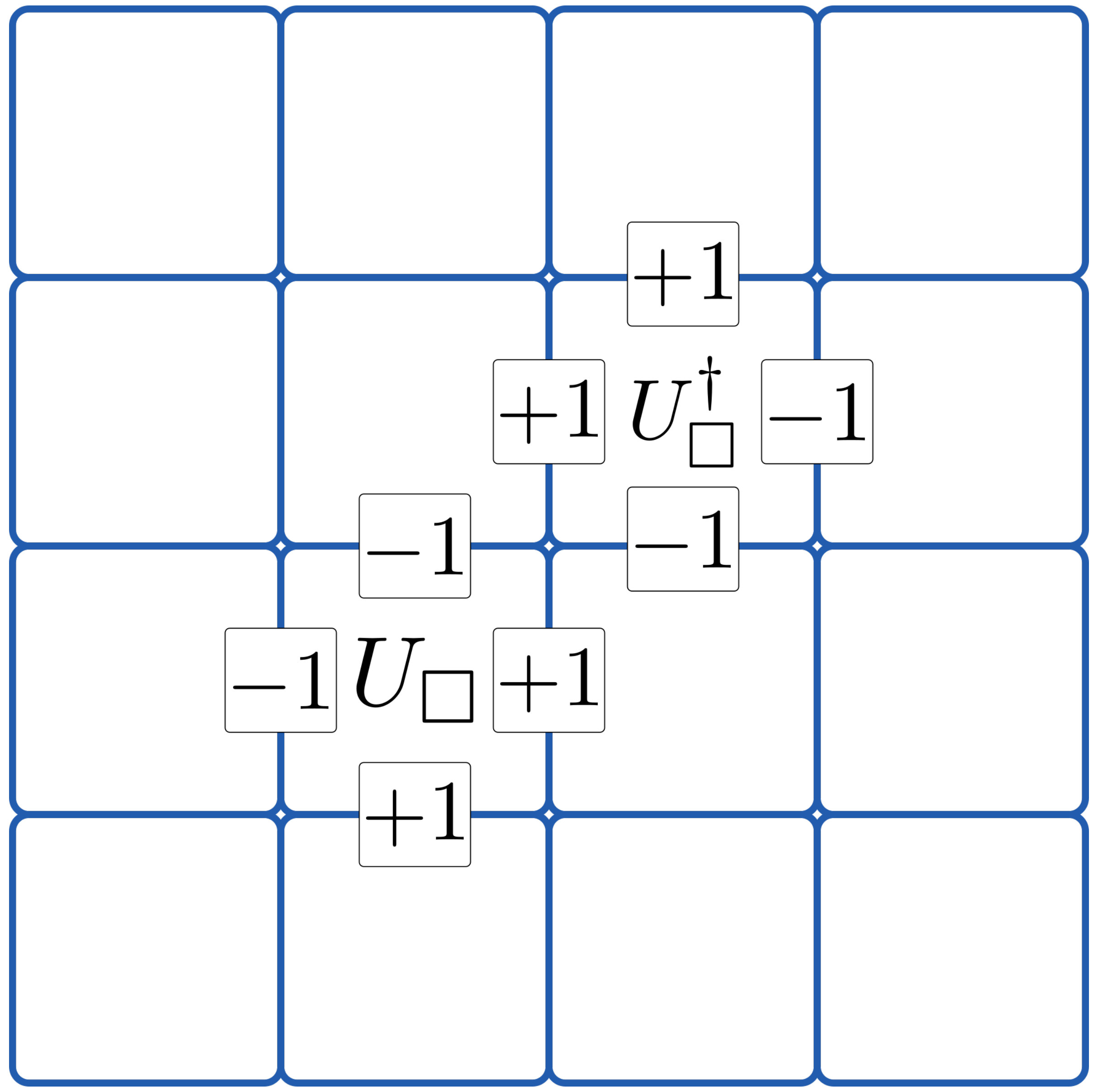
$E_n \in \mathbb{Z}$

U_n unitary raising operator

$U_i \left|E_i\right\rangle = \left|E_i + 1\right\rangle$

Gauss' Law

$E_i - E_k + E_j - E_l = 0$

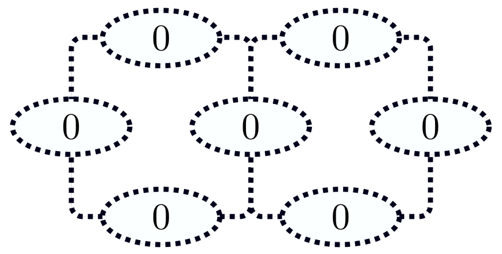


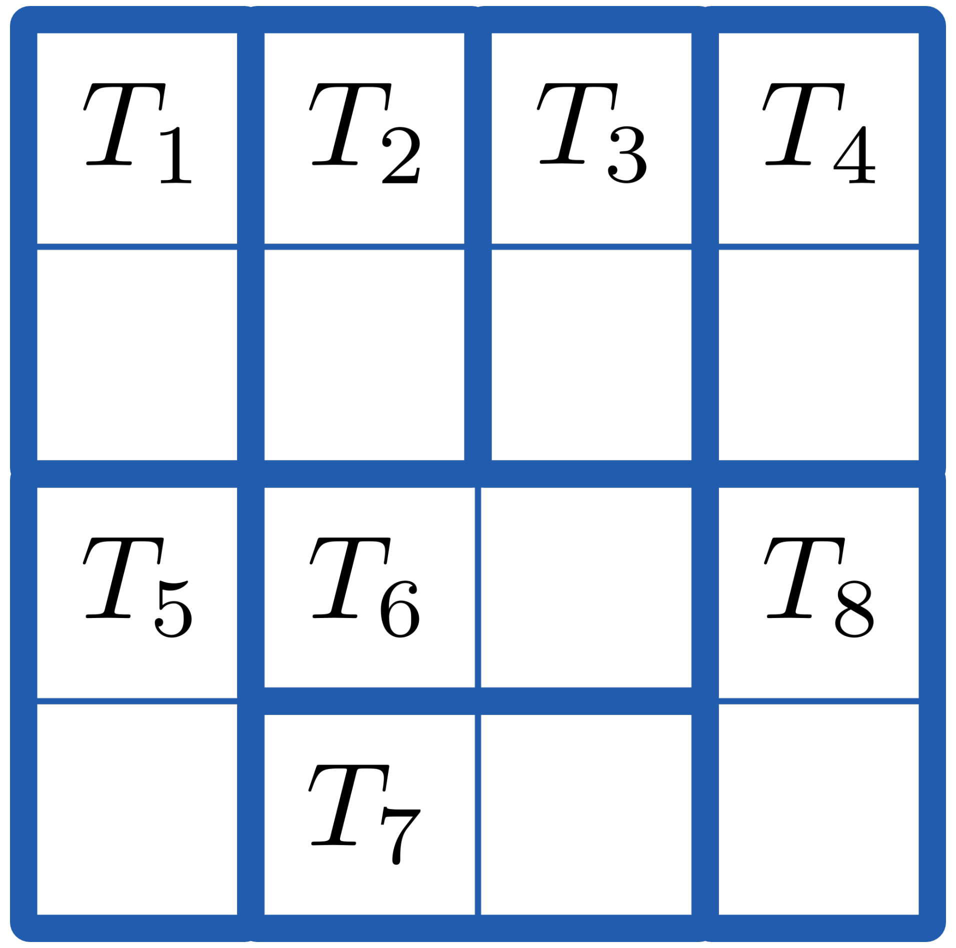
[Credit: J. Pinto Barros]

TLM: Scars for arbitrary volumes and spins

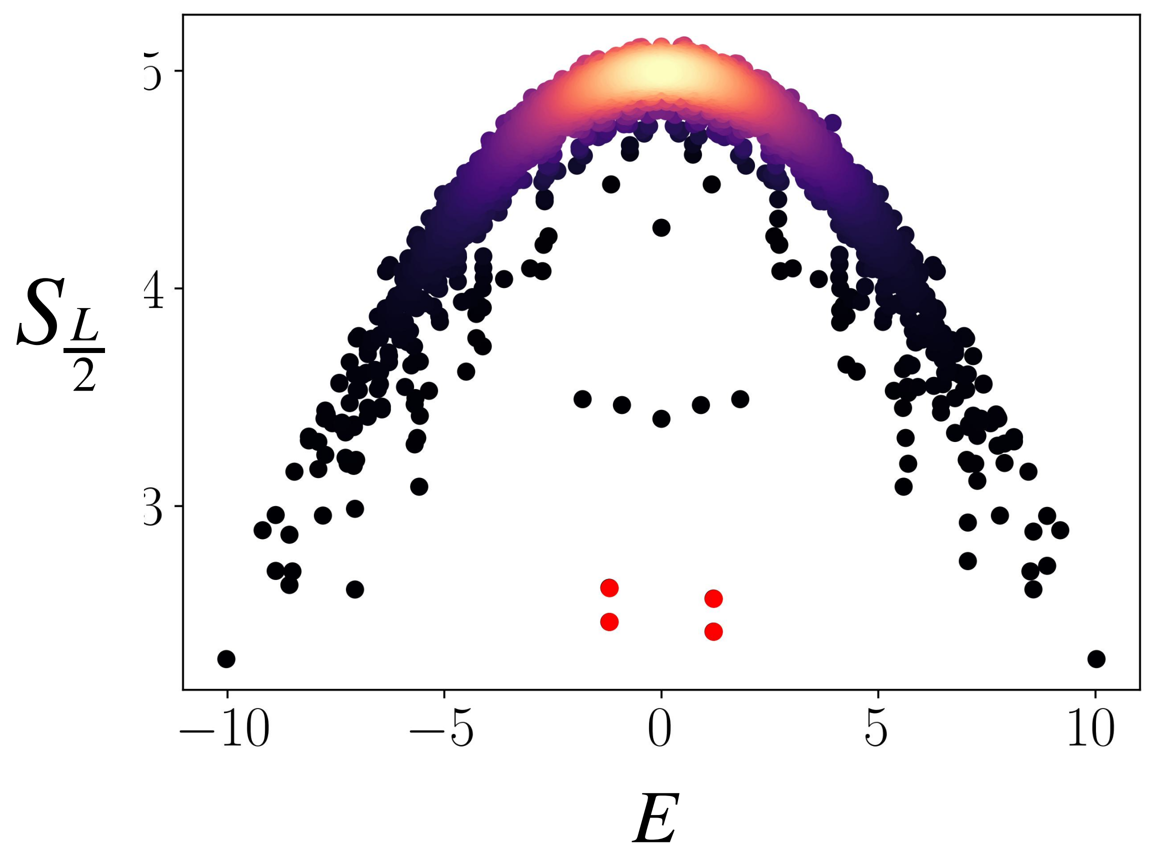
[Budde, MKM, Pinto Barros, PRD 110, 094506 (2024), arXiv:2403.08892]

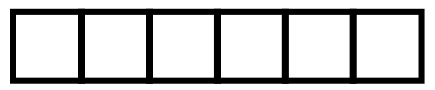
$$|\psi_s^{(i,T)}\rangle = \frac{1}{(S+1)^{|T|/2}} \prod_{(n,n')\in T} \left(\sum_{k=0}^{S} (-1)^k (U_{\square n})^{i-S+k} (U_{\square n'})^{i-k} \right) |\mathbf{0}\rangle$$

$|\mathbf{0}\rangle \equiv$ 



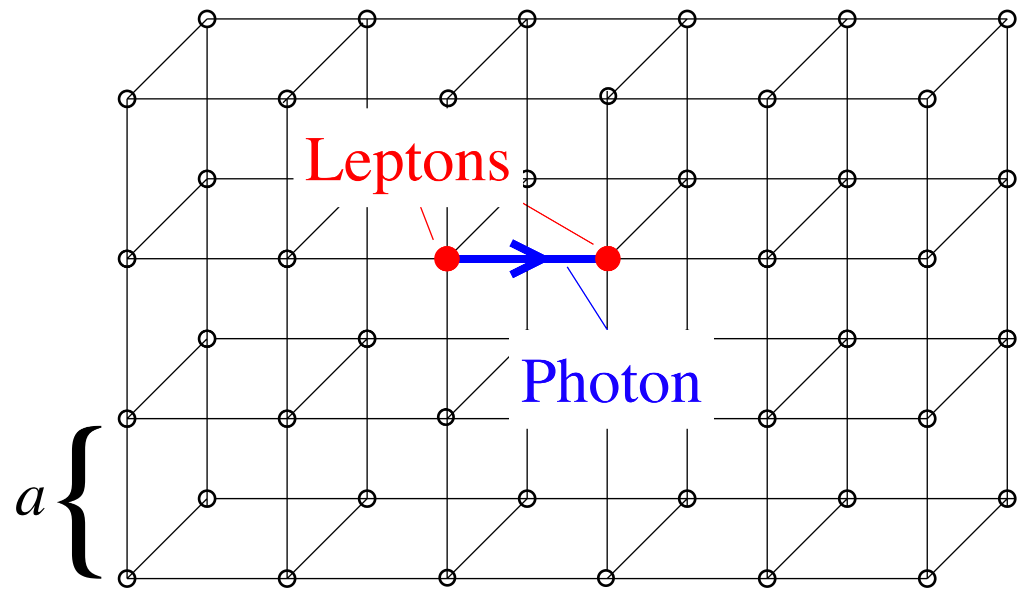
Entanglement entropy for a $S=2$ ladder

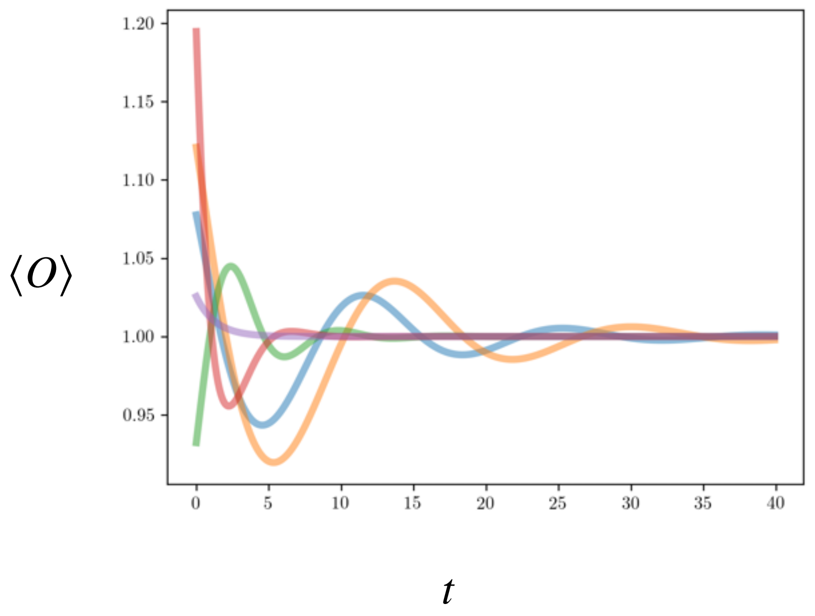


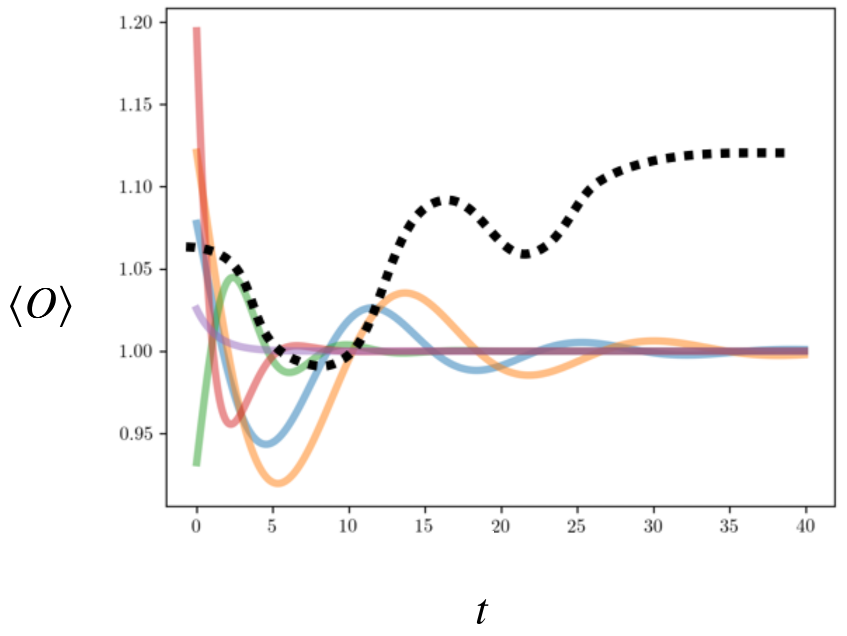
 **lattice**

Real-time dynamics of QED_2 and QED_3

1. (How) do gauge theories thermalize? $[\mathrm{QED}_3]$

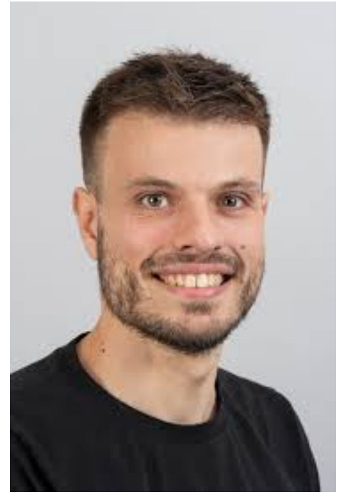






2. What can we learn from quantum simulations about the phase diagram of QED_2 ?

Matteo D'Anna



Joao Pinto Barros

Toy model for QCD in 3+1D: QED in 1+1D

Schwinger model [Schwinger, Physical Review 125, 397 (1962)]

Chiral symmetry, confinement, string breaking …

[D'Anna, MKM, Pinto Barros, PRD 111, 094514, arXiv:2411.01079]

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Quantum Electrodynamics in 1+1D

- Schwinger Model (QED2) Lagrangian with topological θ angle

[Schwinger, Physical Review 125, 397 (1962)]

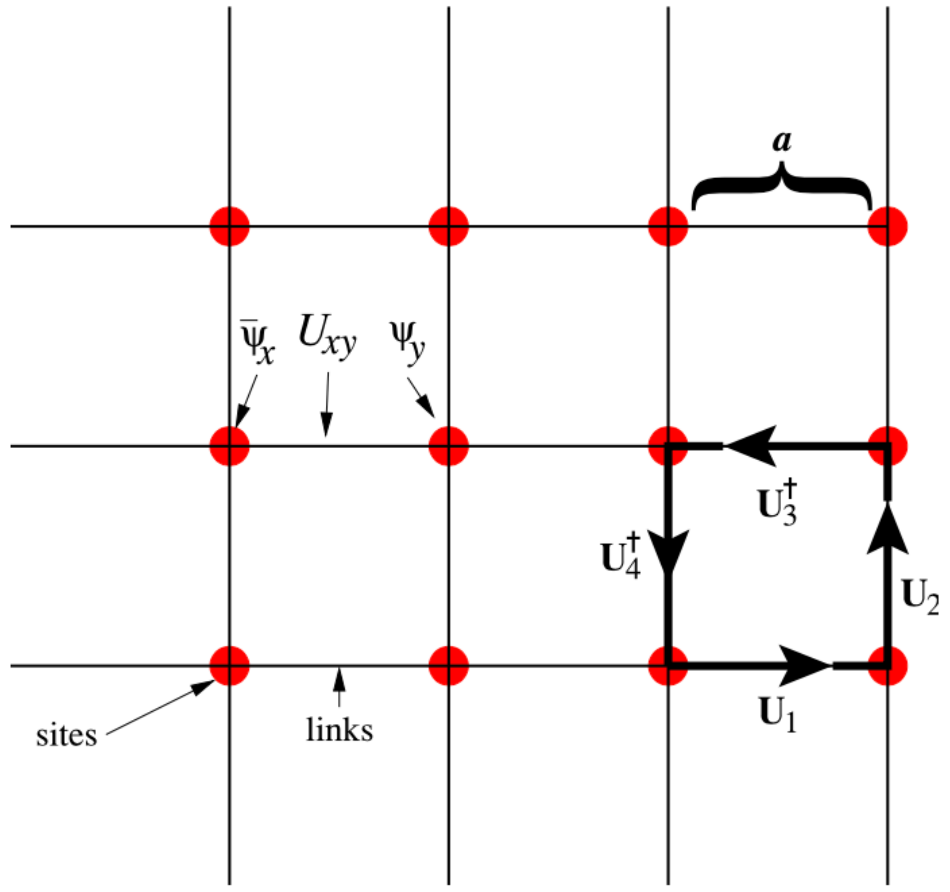
$$\mathscr{L}_{QED2} = -\frac{1}{4} \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \frac{g\theta}{4\pi}\varepsilon_{\mu\nu}\mathrm{F}^{\mu\nu} + \mathrm{i}\bar{\psi}\gamma^{\mu}\left(\partial_{\mu} + \mathrm{i}gA_{\mu}\right)\psi - m\bar{\psi}\psi$$

- Lattice Schwinger Model Hamiltonian:

$$H_{QED2} = -\mathrm{i}\sum_{n=1}^{N-1}\left(w - (-1)^{n}\frac{m}{2}\sin\theta\right)\left(\chi_n^{\dagger}\chi_{n+1} - \chi_{n+1}^{\dagger}\chi_n\right) + m\cos\theta\sum_{n=1}^{N}(-1)^{n}\chi_n^{\dagger}\chi_n + J\sum_{n=0}^{N-1}L_n^2,$$

[Kogut, Susskind, Phys. Rev. D 11, 395 (1975)]

$$L_n = \sum_{k=1}^{n}\left(\chi_k^{\dagger}\chi_k - \frac{1-(-1)^k}{2}\right)$$ Gauss' law



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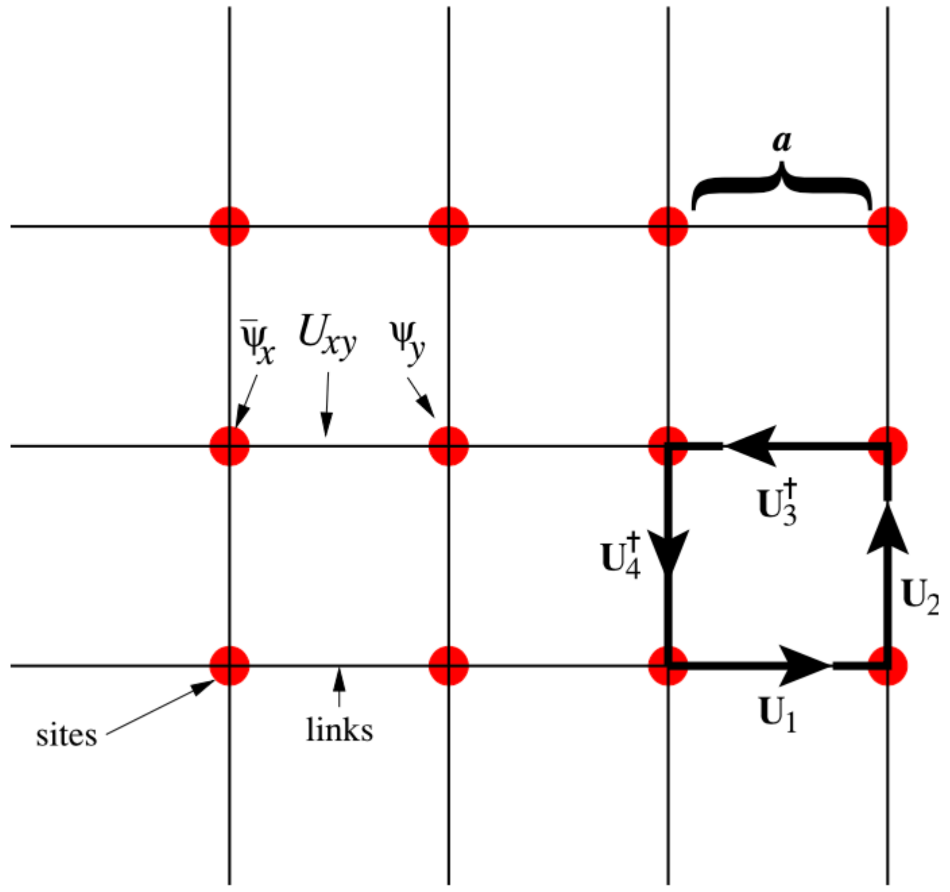
[Kogut, Susskind, Phys. Rev. D 11, 395 (1975)]

$$L_n = \sum_{k=1}^{n}\left(\chi_k^{\dagger}\chi_k - \frac{1-(-1)^k}{2}\right) \quad \text{Gauss' law}$$



- Digital Quantum Simulations of QED_2: non-exhaustive list

[Martinez et al. Nature 534, 516–519 (2016)]
[Klco et al. Phys. Rev. A 98, 032331 (2018)]
[Kokail et al. Nature 569, 355–360 (2019)]
[Jong et al. Phys. Rev. D 106, 054508 (2022)]
[Nguyen et al. Quantum 3, 020324 (2022)]
[Chakraborty et al. Phys. Rev. D 105, (2022) 94503]
[Farell et al., PRX Quantum 5, 020315(2024)]
[Ghim et al., 2404.14788]
[Guo et al, 2407.15629]
[Bazavov et al.,2411.00243,PRD 111, 074515 (2025)] . . .

Symmetries of the QED_2 Hamiltonian

- Charge conservation:

$$Q = \frac{g}{2}\sum_n Z_n \qquad \left[Q, H_{QED2}\right] = 0$$

- Full (θ, m) phase diagram inaccessible to conventional MC simulations
- Phase transition for $\theta = \pi$ and $m/g > m_c/g \approx 0.33$

[Coleman, Annals of Physics 101 (1976) 239]

[Coleman, Jackiw, Susskind, Annals of Physics 93 (1975) 267]

[Thompson, Siopsis, Quantum Science&Technology (2021)7]

Symmetries of the QED_2 Hamiltonian

- Charge conservation:

$$Q = \frac{g}{2} \sum_n Z_n \qquad \left[Q, H_{QED2}\right] = 0$$

- Full (θ, m) phase diagram inaccessible to conventional MC simulations
- Phase transition for $\theta = \pi$ and $m/g > m_c/g \approx 0.33$

[Coleman, Annals of Physics 101 (1976) 239]

[Coleman, Jackiw, Susskind, Annals of Physics 93 (1975) 267]

[Thompson, Siopsis, Quantum Science&Technology (2021)7]

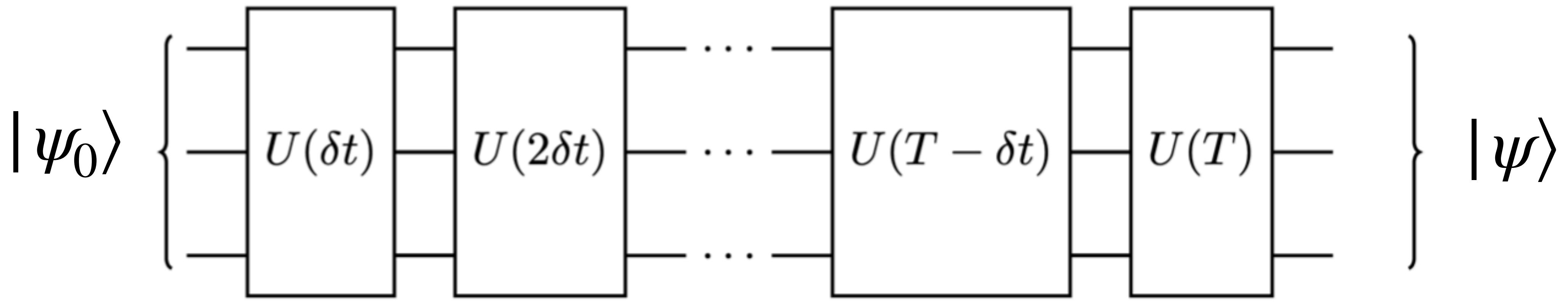
- QED_2 has a ground state in nonzero charge sector for $\theta \in [0.8\pi, 1.5\pi]$ and certain values of m
- String breaking not accessible with state preparation algorithms that keep Q fixed

→ Multi-Q Adiabatic State desirable for efficient state preparation

Adiabatic State Preparation (I)

- Ground state preparation for H_{QED2}

1. **Initial state $|\psi_0\rangle$: ground state of $H_A(0) \equiv H_0$**



2. **"Evolve" $H_A(t)$: $H_A(T) \equiv H_{QED2}$**

Adiabatic Theorem:

$H_A(t)$ gapped, unique ground state $\implies$

$$|\psi_T\rangle = \lim_{T\to\infty} \mathcal{T}\{e^{-i\int_0^T dt\, H_A(t)}\}|\psi_0\rangle$$

3. **Ground state at time T approximated by**

$$|\psi_T\rangle = \mathcal{T}\{e^{-i\int_0^T dt\, H_A(t)}\}|\psi_0\rangle \approx \underbrace{U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)}_{M \text{ steps}}|\psi_0\rangle$$

➡ $U(\tau) = e^{-iH_A(\tau)\delta t}$, $\delta t = \dfrac{T}{M}$, T finite, $\int$ discretized

➡ Applied to $QED2$ [Chakraborty et al. Physical Review D 105, (2022) 094503]

[Ghim, Honda, arXiv:2404.14788]

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Adiabatic State Preparation (II)

- Fixed-Q ASP Algorithm (A1): single charge sector of H_{QED2}

$$H_{A1}(t) = H_{QED2}\Big|_{m=m(t),w=w(t),\theta=\theta(t)}$$

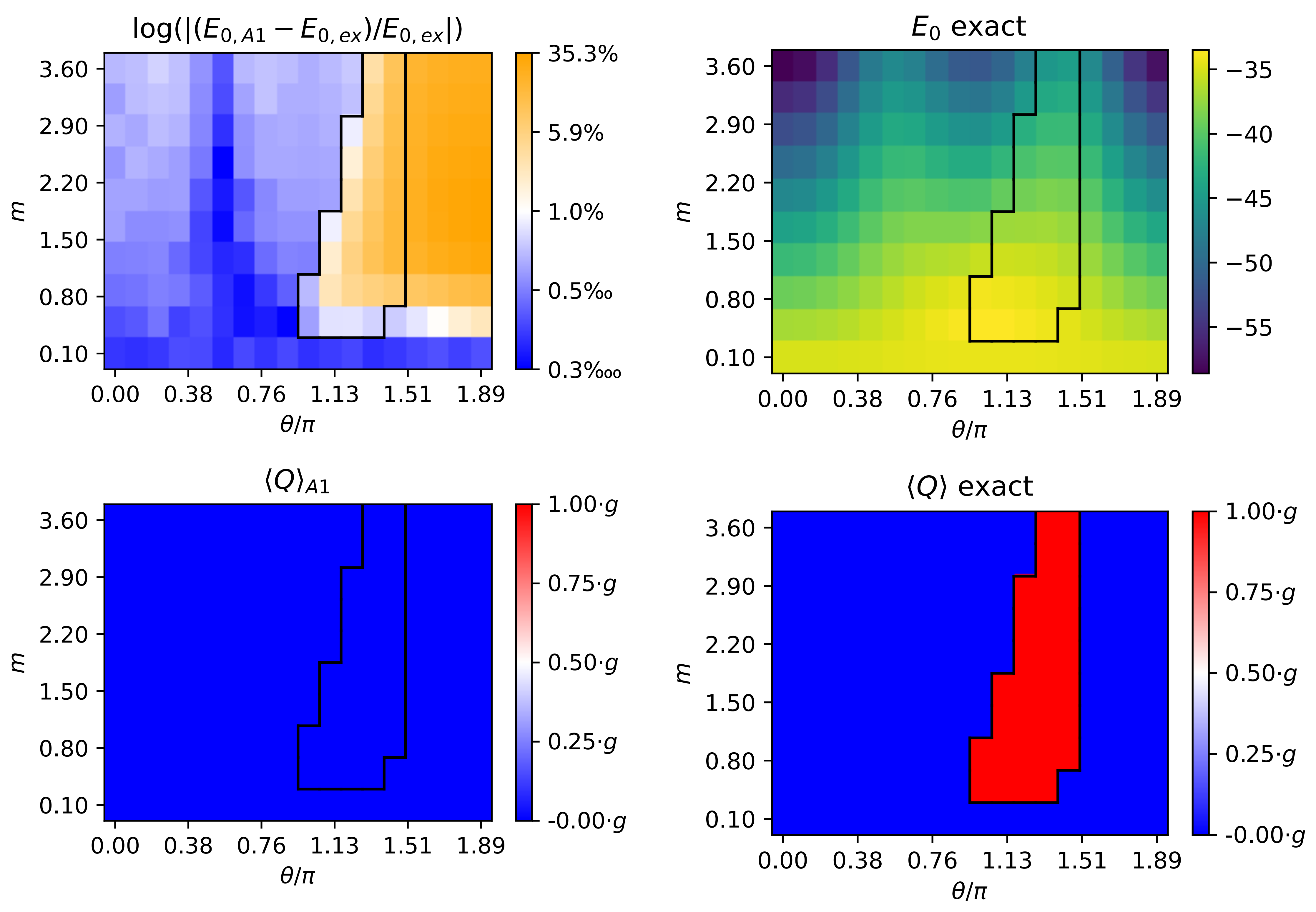
$$m(t) = m_0\left(1-\frac{t}{T}\right) + m\frac{t}{T}; \qquad w(t) = w\frac{t}{T}; \qquad \theta(t) = \theta\frac{t}{T}$$

- Charge remains constant:

  ➡ $[Q, H_{A1}(t)] = 0\ ,\ \forall t$

- Challenging to determine the ground state for all (θ, m)

Fixed-Q ASP Algorithm: **Exact Diagonalization:**



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Adiabatic State Preparation (III)

- Multi-Q algorithm: arbitrary charge sector of H_{QED2}

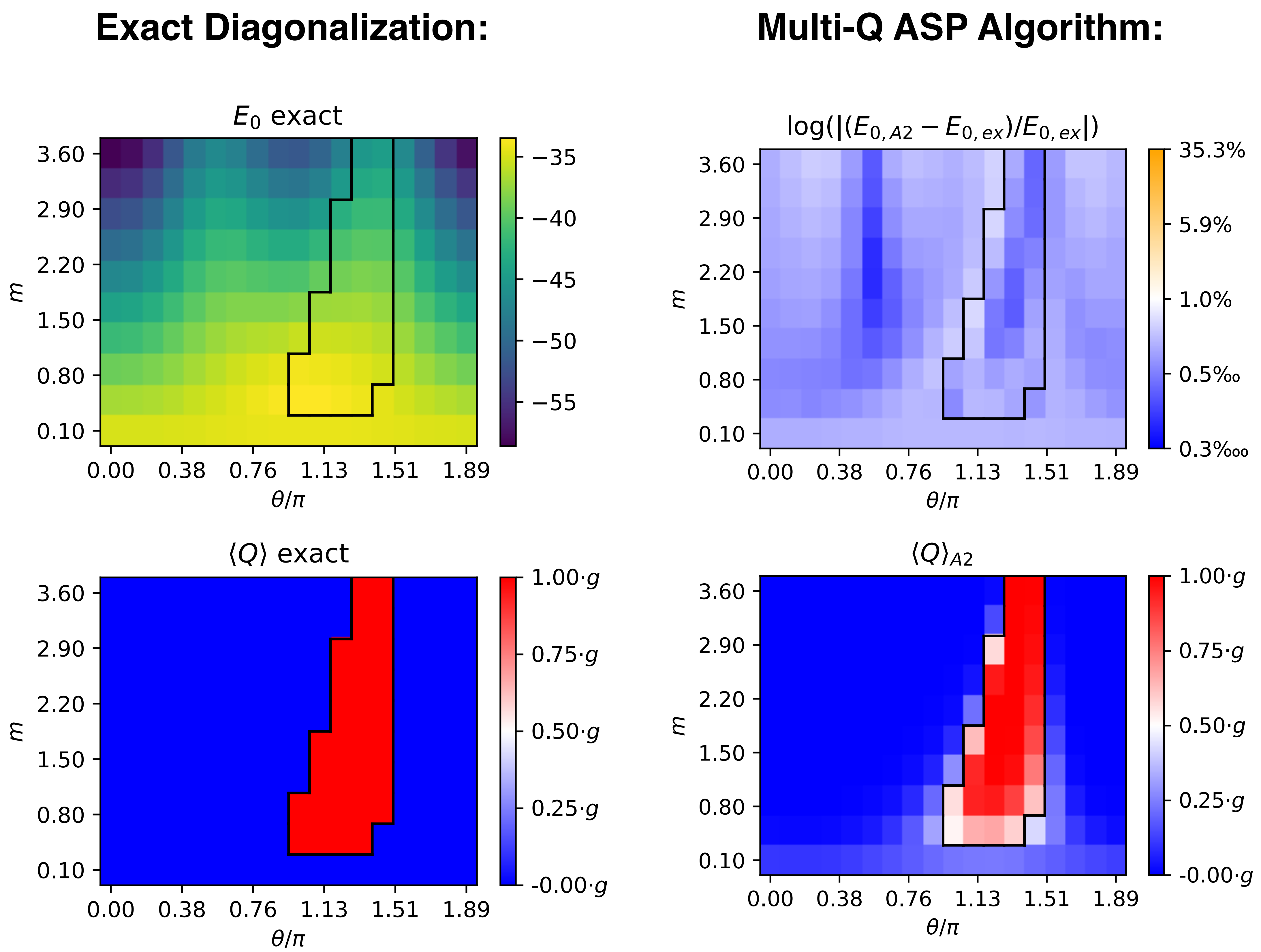
$$H_{A2}(t) = \left(1 - \frac{t}{T}\right) \beta \sum_{n=1}^{N} f(n) X_n + \frac{t}{T} H_{QED2} \quad \text{for} \quad f(n) \in \{-1,1\}$$

- Several choices for $f(n)$ probed, best thus far: $f(n) = (-1)^n$

➡ $\beta \approx \frac{|E_0|}{N}$

➡ Total charge no longer a symmetry of $H_{A2}(t)$

➡ Ground state of $H_{A2}(0)$ mixes states within different charge sectors



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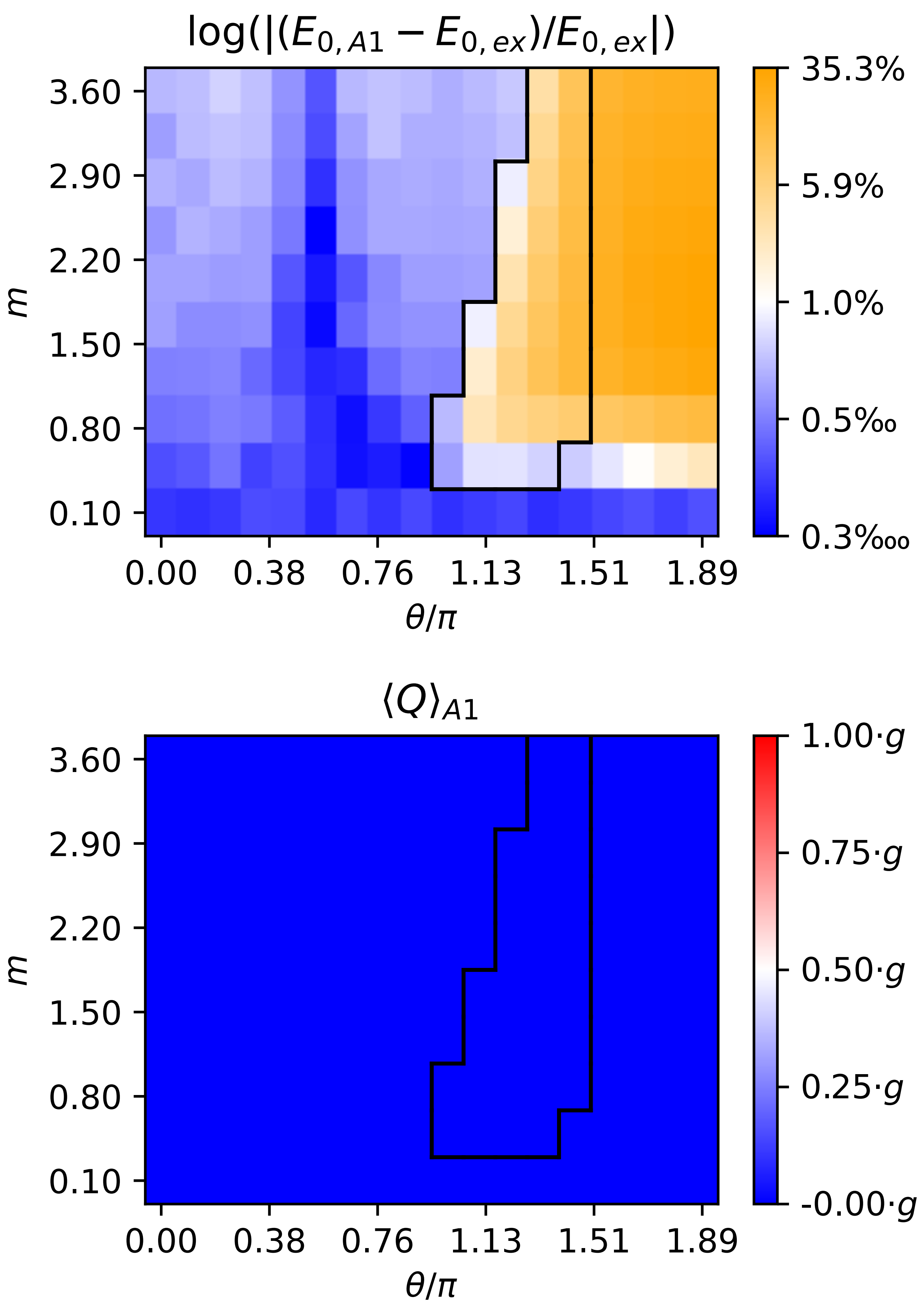
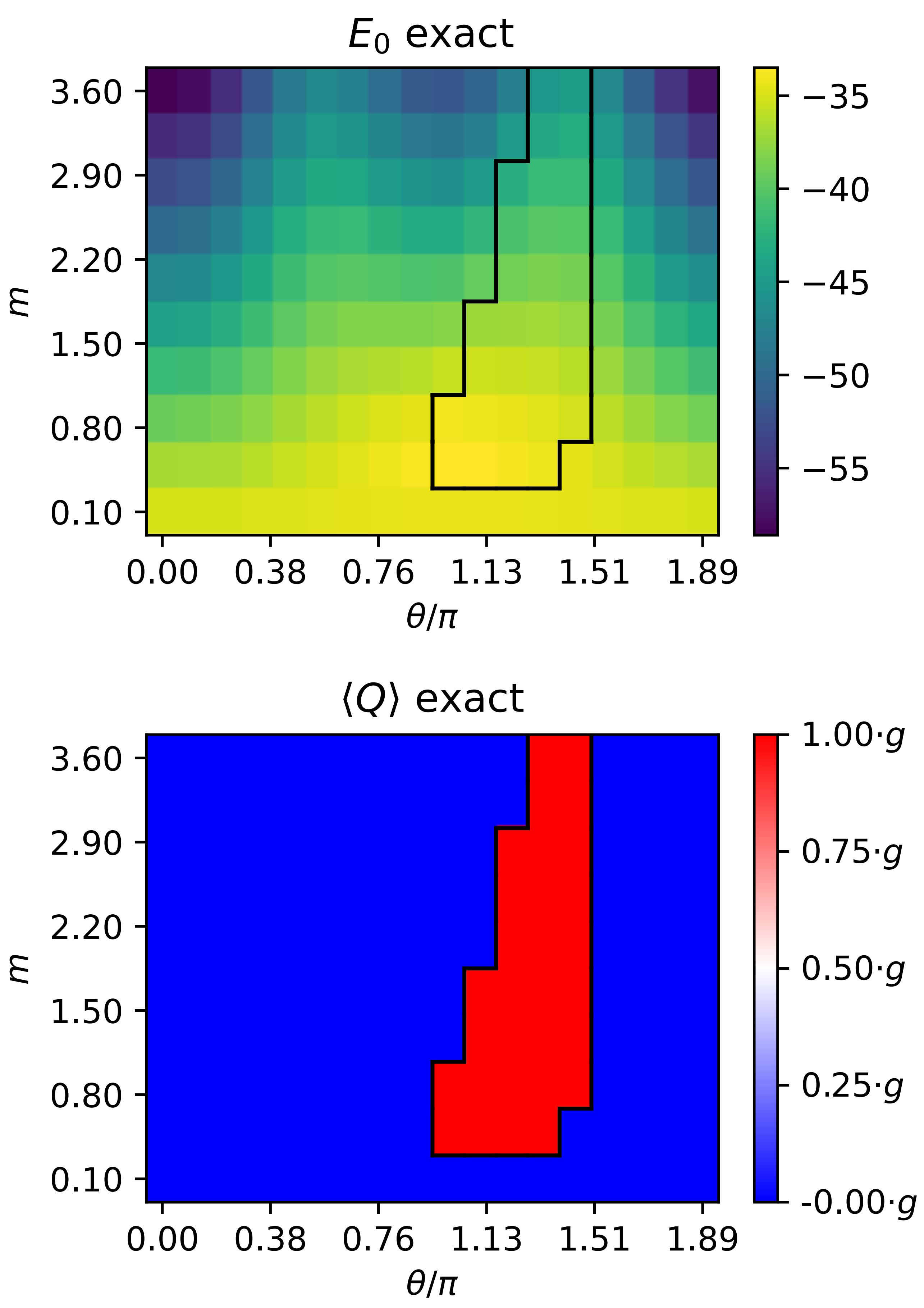
Comparison of State Preparation Algorithms

[D'Anna, MKM, Pinto Barros, PRD 111, 094514 (2025), arXiv:2411.01079]

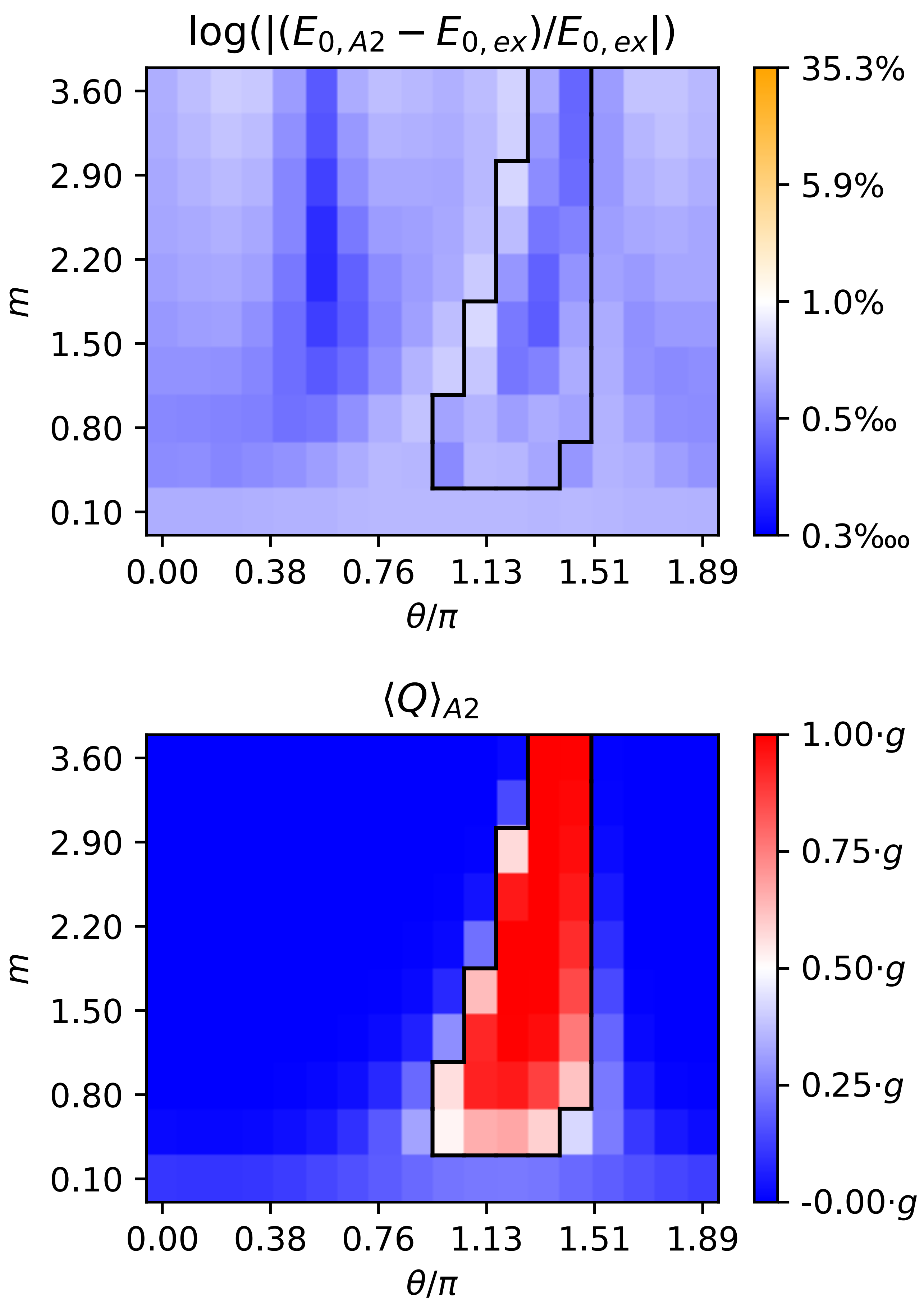
Fixed-Q ASP Algorithm:



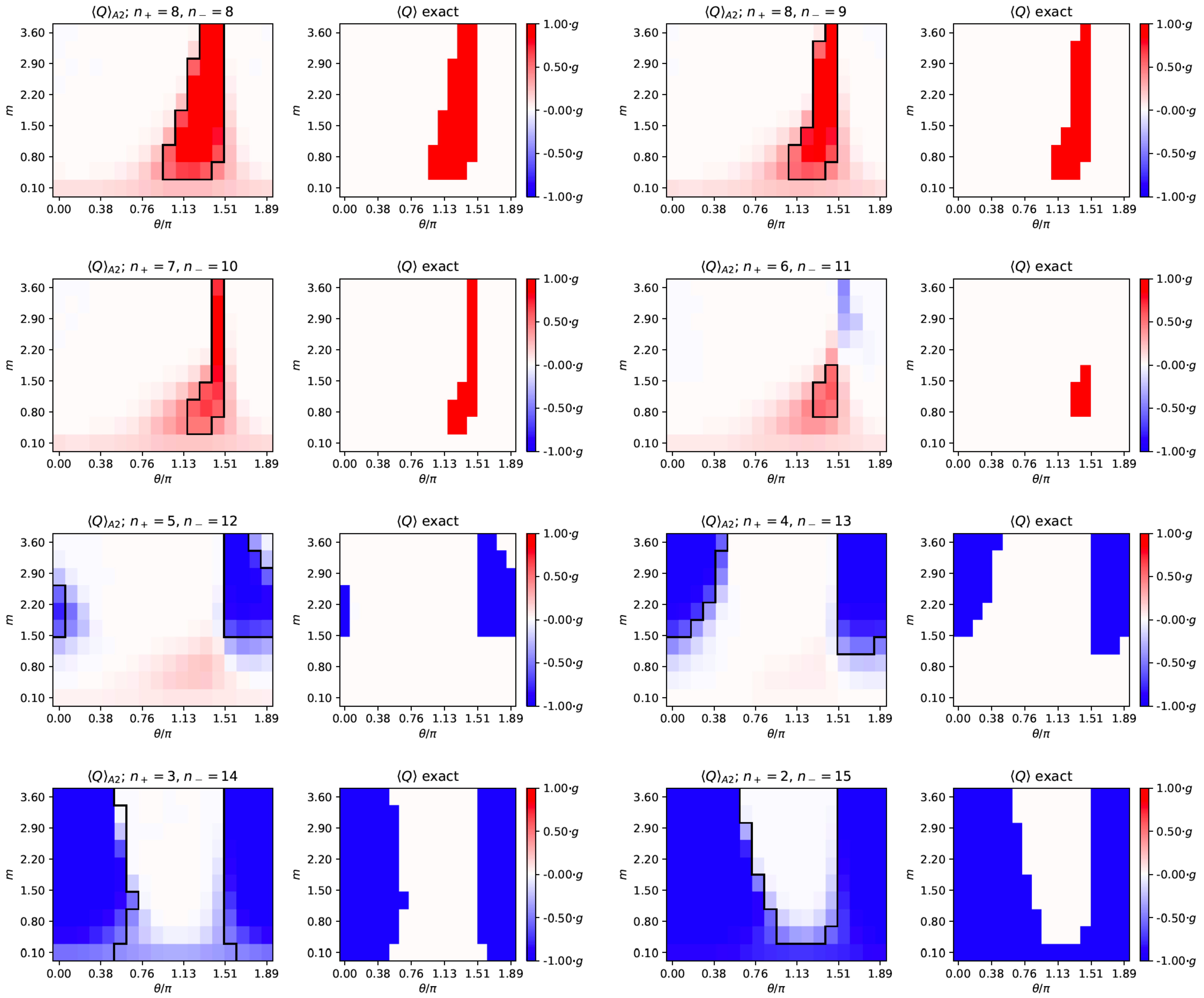
Exact Diagonalization:



Multi-Q ASP Algorithm:



$\langle Q \rangle_{A2}$ in the presence of static charges



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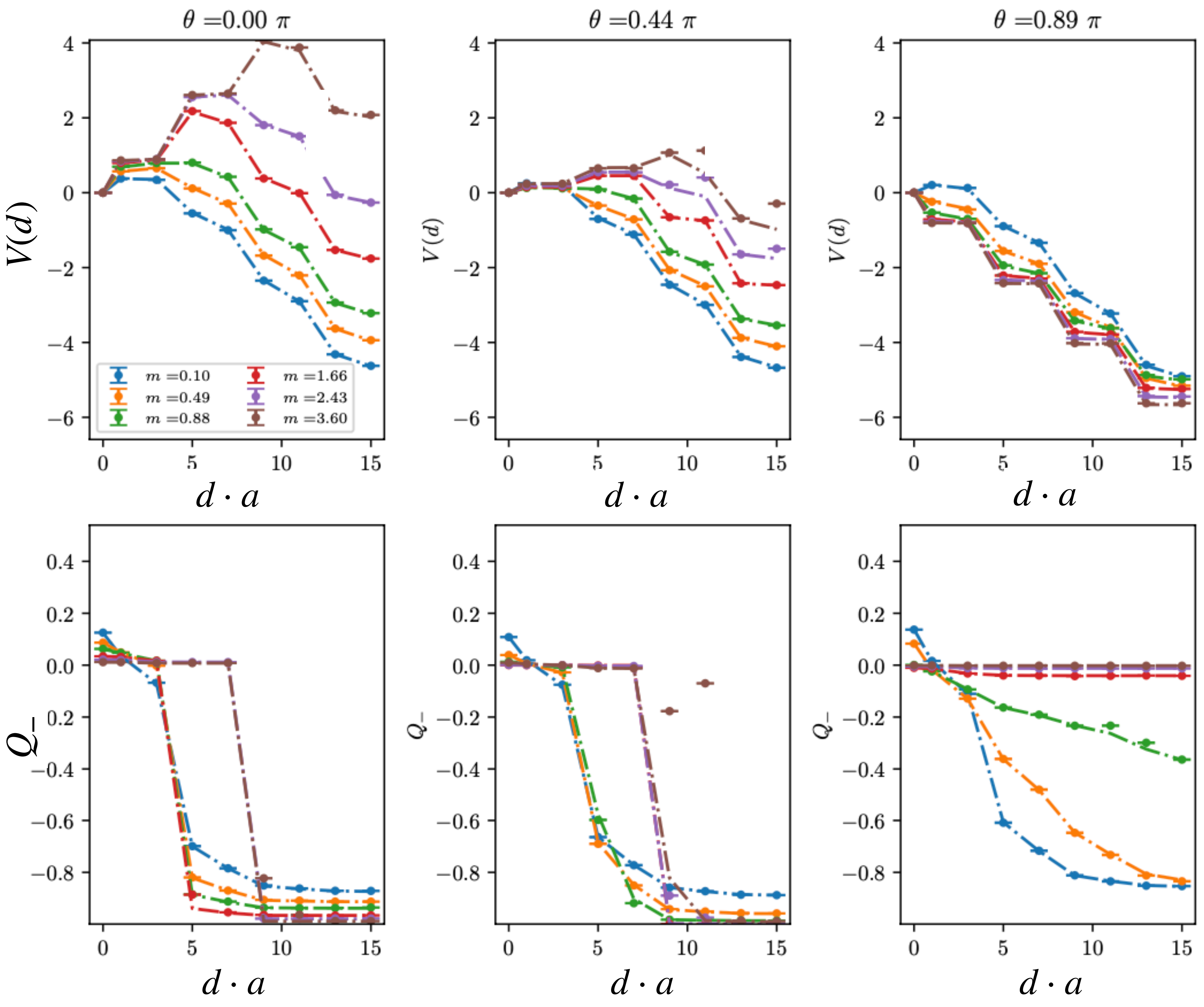
String Breaking in QED_2 (I)

[D'Anna, MKM, Pinto Barros, PRD 111, 094514 (2025), arXiv:2411.01079]



- Multi-charge ASP enables studies of confinement of charge in QED2
- Notation from [Buyens et. al. Physical Review X 6, 041040 (2016)] $V(d) = E_0(d) - E_0(0)$; $Q_- = \frac{g}{2}\sum_{n=1}^{N/2} Z_n$

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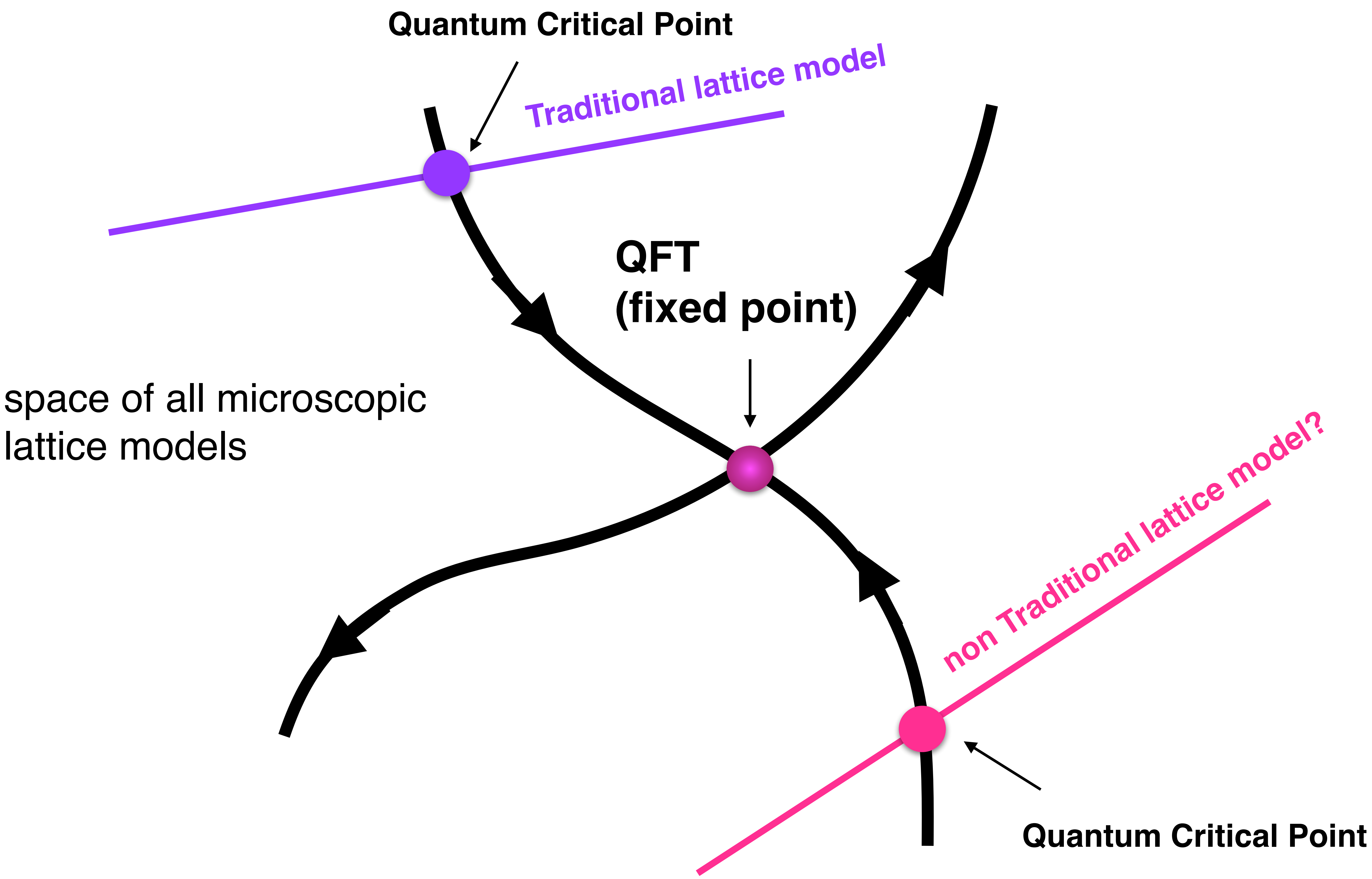
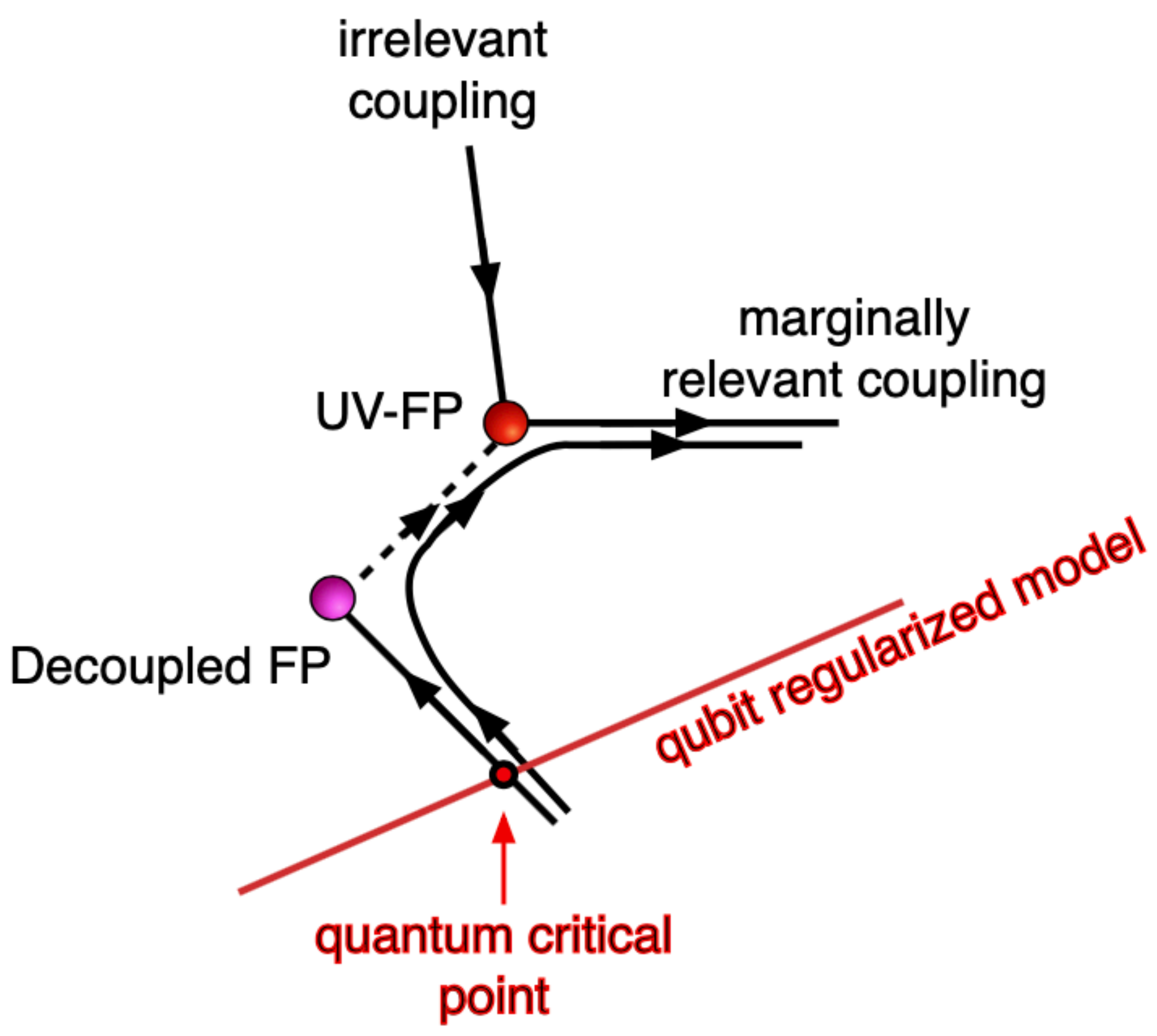
Wilson's Renormalization Group



- Exploring alternate regularizations compatible with quantum simulations of gauge theories
- e.g. qubit regularized models [Bhattacharya et al. PRL126 (2021), 172001, Zhou et al. PRD 105, 054510 (2022), …]

Wilson's RG working in an unexpected way



- At the decoupled FP: two differently qubit regularized models describe the physics of a critical system containing two decoupled theories
- When a small non-zero coupling is introduced between the theories, the long distance physics flows towards the desired universal physics of the UV-FP theory
- Non-traditional qubit regularized model gives the cont. physics of the XY model [Maiti et al. PRL 132 (2024) 4]

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Summary & Outlook (I)

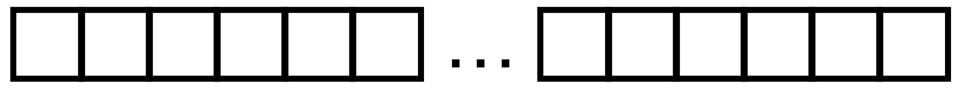
- Lattice QCD mature, but phase diagrams, real time dynamics of LGTs intractable from first principles: Hamiltonian formulations

- Quantum Many-Body Scars challenge foundational aspects of thermalization; initial-value problems in QFT

- Scars occur in many-body quantum systems including simple gauge theories; analytic construction for any spin S now possible $[\mathrm{QED}_3]$

- Algorithms for phase diagrams of gauge theories: test-bed example QED_2, much progress in the field [https://indico.cern.ch/event/1519819/overview]

Summary & Outlook (II)

- Constructed scars in 2+1D amenable for experimental realization 
- Can the proposed mechanism be generalized for other models? [Osborne, McCulloch, Halimeh 2403.08858, Calajo et al. Phys. Rev. Research 7, 013322 (2025)]
- Are the scars present in the continuum limit?
- Systematic way to construct efficient state preparation algorithms [Gottesman-Knill (1998), Bravyi, Kitaev, PRA 71, 022316 (2005), Chernyshev, Robin, Savage 2411.04203]
- Implications for high energy physics, condensed matter theory, nuclear physics, … [Berges, Heller, Mazeliauskas, Venugopalan. Rev. Mod. Phys (2021)]

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