

A new method for measuring high-spin glueball states on the lattice

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Outline

- 1 Motivation
- 2 Conventional methods
 - The path-finder method
- 3 The matrix method
- 4 Preliminary results
 - $U(1)$
 - $SU(2)$
- 5 High spin states

Regge trajectories

For $2 \rightarrow 2$ scattering, $\mathcal{A}(s, t) \sim s^{\alpha(t)}$
($s \gg |t|$) has poles where $\alpha(t) \in \mathbb{N}$

Bound states at these points with
 $\alpha(m_i^2) = J_i$

Experimentally, $\alpha(t) = \alpha(0) + \alpha' t$

For ρ meson,

- $\alpha_\rho(0) \approx 0.5$
- $\alpha'_\rho \approx 1 \text{ GeV}^{-2}$

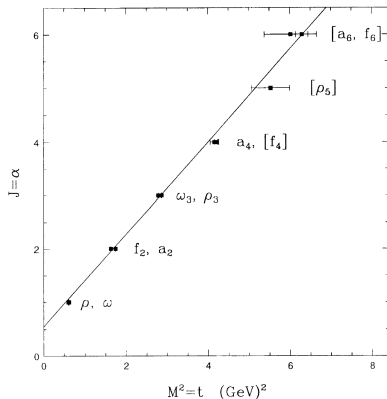


Figure: Chew & Frautschi (1961, 1962)

The Pomeron (in 3+1D)

$p - p$ and $p - \bar{p}$ scattering show another trajectory with no bound states
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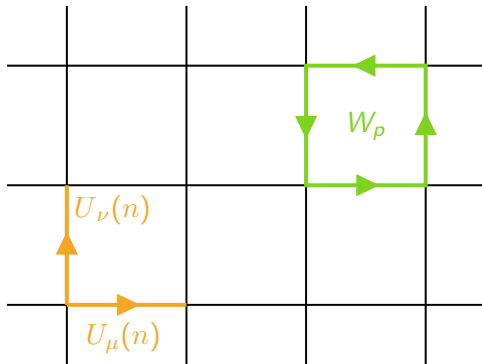
$\alpha_P(0) > 1 \Leftrightarrow$ exchange of vacuum quantum numbers
(Pomeranchuk, 1956)

Glueballs are only candidates in QCD that could lie on the Pomeron trajectory

This project:

Aims to find accurate masses for high- J glueballs and see if they lie on the Pomeron trajectory

Lattice field theory - a brief reminder

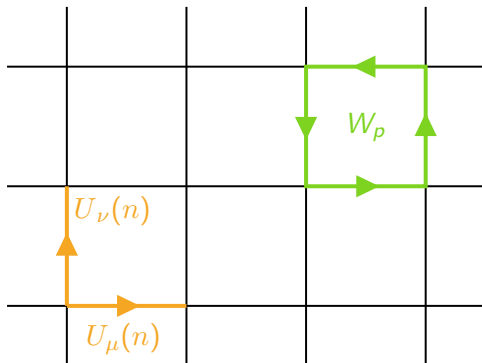


$$W_p = \prod_{l \in p} U_l$$

$$S = \beta \sum_p \text{Re Tr}(W_p)$$

$$\mathcal{Z} = \int \prod_l dU_l e^{-S[U]}$$

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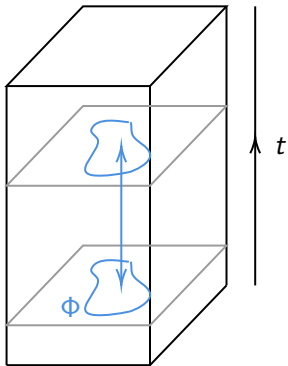
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We work in 2+1D, although we hope to generalise to 3+1D in the future

Measuring glueball states

Use Cabbibo-Marinari heatbath to generate field configurations

Measure 2-point functions of closed-loop operators $\Phi(t)$

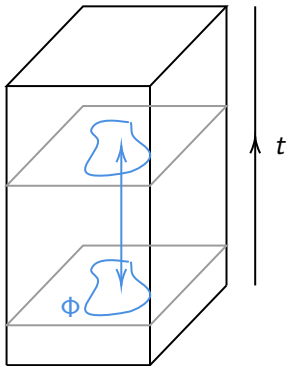


$$\langle \Phi(t) \Phi(0) \rangle = \sum_{\text{states } \alpha} |c_\alpha|^2 e^{-m_\alpha t}$$

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$$\langle \Phi(t) \Phi(0) \rangle = \sum_{\text{states } \alpha} |c_\alpha|^2 e^{-m_\alpha t}$$

Choose Φ to maximise $|c_\alpha|^2$
Extract m_α by looking for plateaus in effective mass:

$$m_{\text{eff}} = -\log \left(\frac{\langle \Phi(t) \Phi(0) \rangle}{\langle \Phi(t-1) \Phi(0) \rangle} \right)$$

To extract excited states, first do a variational calculation over basis of operators

Conventional glueball operators

Multiply link variables is a closed loop:

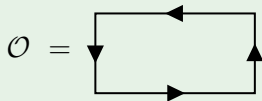
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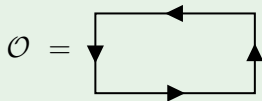


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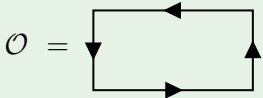
To define spin, combine rotations and parity inversions to sit in irrep of lattice group (D4)

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Example

$$\Phi_{\pm} = \text{Re} \left\{ \text{Diagram 1} \pm \text{Diagram 2} \right\}$$

Diagram 1: A square loop with arrows pointing clockwise (top-left to top-right, top-right to bottom-right, bottom-right to bottom-left, bottom-left to top-left).

Diagram 2: A square loop with arrows pointing counter-clockwise (top-left to top-right, top-right to bottom-right, bottom-right to bottom-left, bottom-left to top-left).

Φ_+ projects onto $0^+, 4^+, \dots$

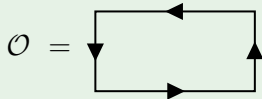
Φ_- projects onto $2^+, 6^+, \dots$

Conventional glueball operators

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Example



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Example

$$\Phi_{\pm} = \text{Re} \left\{ \text{Loop 1} \pm \text{Loop 2} \right\}$$

Diagram 1: A square loop with arrows pointing left (top), up (right), right (bottom), and down (left).

Diagram 2: A square loop with arrows pointing right (top), down (right), left (bottom), and up (left).

Φ_+ projects onto $0^+, 4^+, \dots$

Φ_- projects onto $2^+, 6^+, \dots$

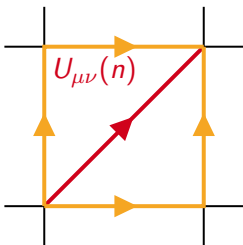
Need loops which can be rotated by $\theta < \pi/2$

Method I - the path finder method

Used to calculate even $J = 0, \dots, 6$ first by H Meyer (arXiv:hep-lat/0306019), and recently upto $J = 8$ by P Conkey, S Dubovsky and M Teper (arXiv:1909.07430)

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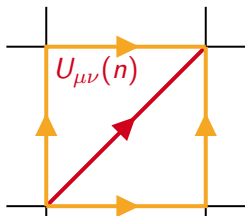


Construct diagonal links:

$$U_{\mu\nu}(n) = \mathcal{U}(U_{\mu}(n)U_{\nu}(n + \mu) + U_{\nu}(n)U_{\mu}(n + \nu))$$

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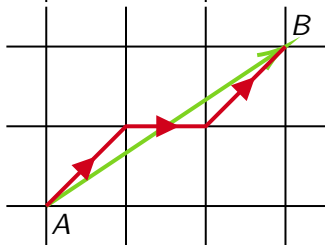
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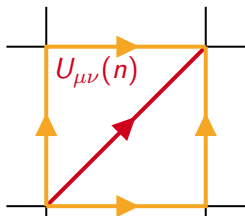
Path find to construct edges \vec{AB}



Use edges to construct operators which can approximately be rotated by $\theta < \pi/2$

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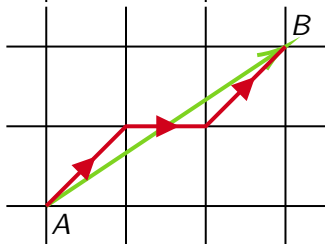
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Use edges to construct operators which can approximately be rotated by $\theta < \pi/2$

- Fairly cheap
- Quickly becomes cumbersome for high spins

Method II - the matrix method

Add non-dynamical scalar field to $S[U]$ (later we will need to use staggered fermions instead):

$$S[U] \supset \sum_{n,\mu} \phi(n)^\dagger U(n, n+\mu) \phi(n+\mu) + h.c.$$

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where we define:

$$(M_0)_{ij} = \begin{cases} U(i, j) & \text{if } i, j \text{ nearest neighbours} \\ 0 & \text{otherwise} \end{cases}$$

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M_{ij}^{-1} gives lattice scalar propagator from site $i \rightarrow j$

We can also view M_{ij}^{-1} as a sum over paths:

$$\begin{aligned} M^{-1} &= (M_0 - \alpha)^{-1} \\ &= -\frac{1}{\alpha} \sum_{n=0}^{\infty} \frac{M_0^n}{\alpha^n} \end{aligned}$$

$(M_0^n)_{ij}$ contains all paths $i \rightarrow j$ in exactly n steps

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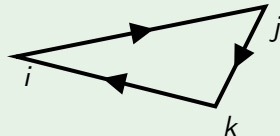
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Construct operators as closed loops of propagators

Example

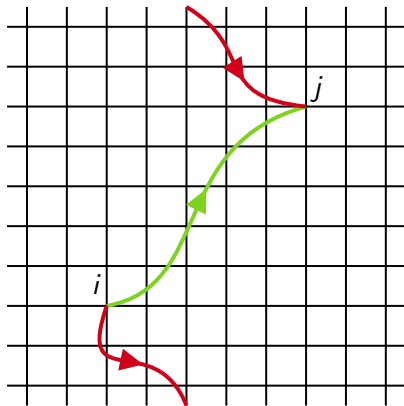
$$\mathcal{O} = \text{Tr}(M_{ij}^{-1} M_{jk}^{-1} M_{ki}^{-1})$$

=

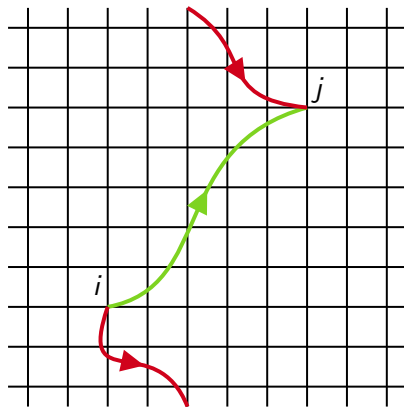


Flux tube contributions

M_{ij}^{-1} includes contributions
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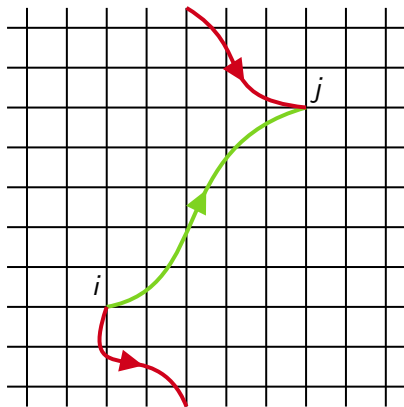


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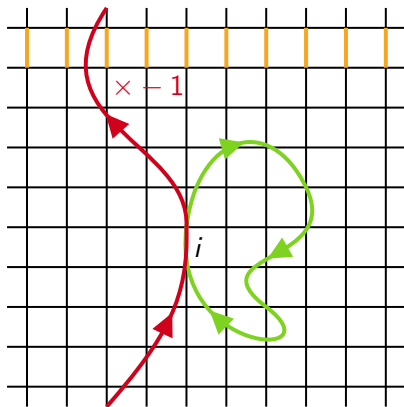
Example

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Therefore closed loops will project onto flux tube states

Flux tubes lighter than glueball states (on the volumes we are interested in), so will contaminate the spectrum

Removing flux tube contributions



For simplicity, consider an

$SU(2)$ theory and

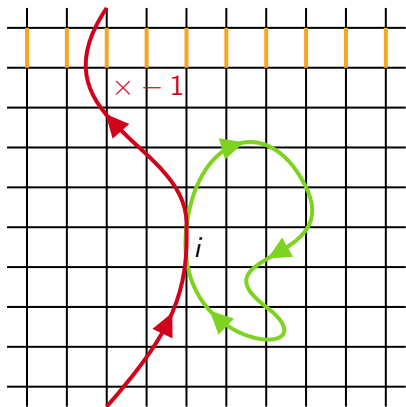
$$\mathcal{O} = \text{Tr}(M_{ii}^{-1})$$

Winding paths charged under
 \mathbb{Z}_2 1-form symmetry

Apply \mathbb{Z}_2 transformation:

$$U_y(x, y = L) \mapsto -U_y(x, y = L)$$

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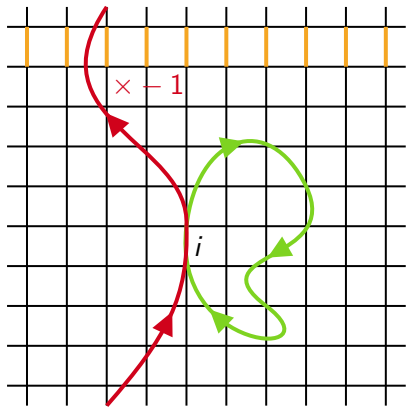
$$U_y(x, y = L) \mapsto -U_y(x, y = L)$$

$$\mathcal{O} =$$

$$\text{Tr}(M(0, 0)_{ii}^{-1} + M(1, 0)_{ii}^{-1})$$

contains no y-winding paths

Removing flux tube contributions



$\mathcal{O} = \text{Tr}(M(0,0)_{ii}^{-1} + M(1,0)_{ii}^{-1} + M(0,1)_{ii}^{-1} + M(1,1)_{ii}^{-1})$ also removes x-winding paths

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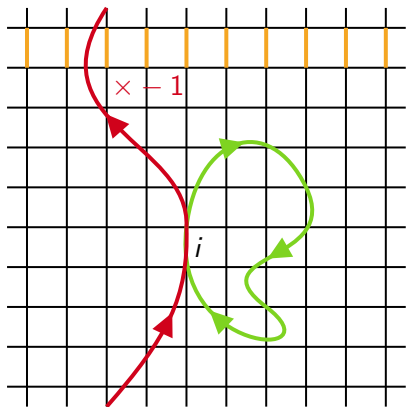
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In $SU(N)$, take the combination $\mathcal{O} = \text{Tr} \left(\sum_{k_x, k_y=0}^{N-1} M(k_x, k_y)_{ii}^{-1} \right)$

This is in principle generalisable to multi-site operators

For simplicity, consider an

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Matrix inversion performance

We use a standard version of the well-studied conjugate gradient algorithm
Efficiency determined by κ

$$\kappa = \frac{\lambda_{\max}}{\lambda_{\min}}$$

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Recall $M = M_0 - \alpha$

Denote $\{\lambda_i\} \in \mathbb{R}$ as eigenvalues of M_0

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If any $\lambda_i \approx \alpha$, CG fails!

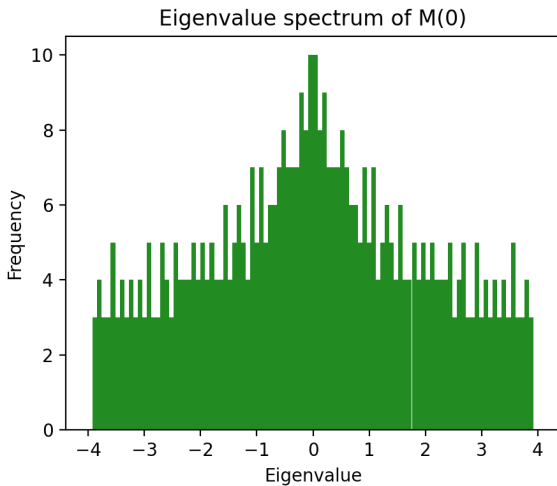


Figure: Eigenvalue spectrum of M_0

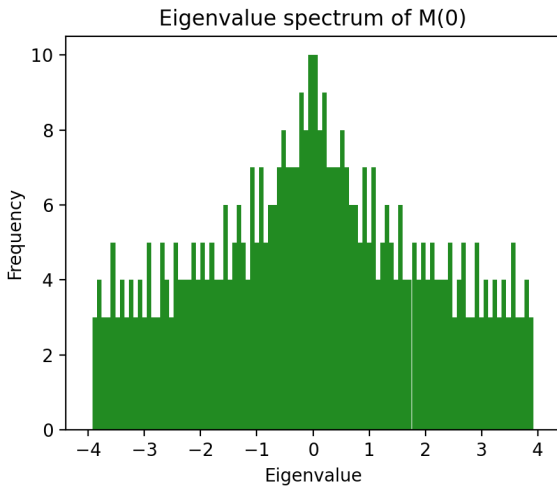


Figure: Eigenvalue spectrum of M_0

CG fails for $\alpha < 4$

Fermionic propagators

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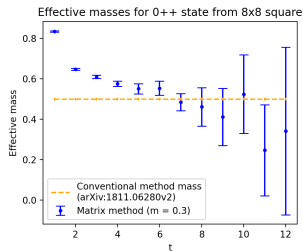
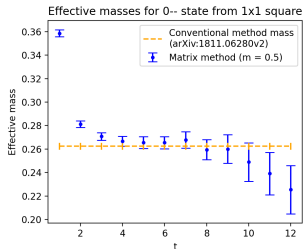
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However, some paths subtracted in M_{ij}^{-1}

$U(1)$ results

$$\beta = 2.2, 22^2 36$$

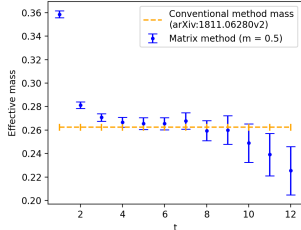


$U(1)$ results

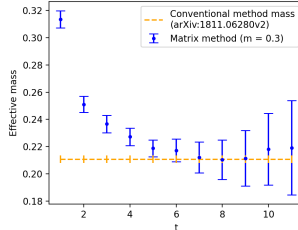
$$\beta = 2.2, 22^2 36$$

$$\beta = 2.3, 30^2 36$$

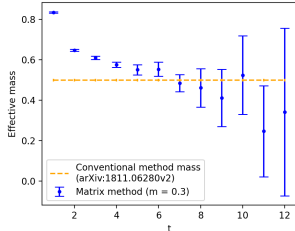
Effective masses for 0^{--} state from 1×1 square



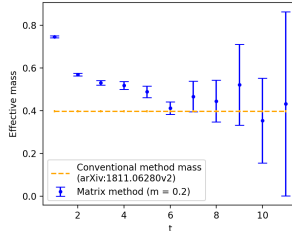
Effective masses for 0^{--} state from 5×5 square



Effective masses for 0^{++} state from 8×8 square

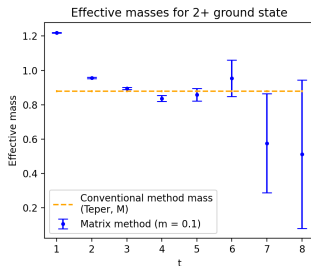
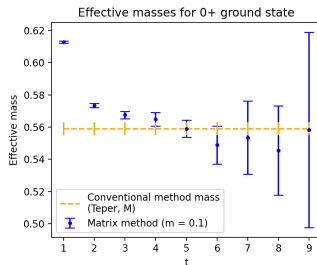
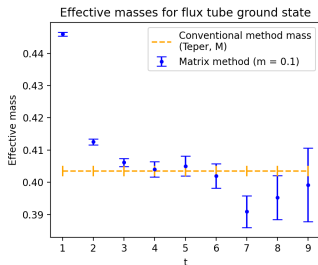


Effective masses for 0^{++} state from 9×9 square

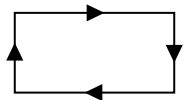


$SU(2)$ results

$$\beta = 12.0, 30^2 36$$

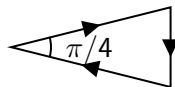


Higher spin states on the lattice



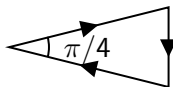
\rightarrow irreps of $D_4 \subset O(2)$ $\rightarrow J = 0, \dots, 2$

Irrep	Dimension	Continuum glueball content (J^P)
A_1	1	$0^+, 4^+, \dots$
A_2	1	$0^-, 4^-, \dots$
B_1	1	$2^+, 6^+, \dots$
B_2	1	$2^-, 6^-, \dots$
E	2	$1^\pm, 3^\pm, \dots$



$$\rightarrow \text{irreps of } D8 \subset O(2) \rightarrow J = 0, \dots, 4$$

Irrep	Dimension	Continuum glueball content (J^P)
A_1	1	$0^+, 8^+, \dots$
A_2	1	$0^-, 8^-, \dots$
B_1	1	$4^+, 12^+, \dots$
B_2	1	$4^-, 12^-, \dots$
E_1	2	$1^\pm, 7^\pm, \dots$
E_2	2	$2^\pm, 6^\pm, \dots$
E_3	2	$3^\pm, 5^\pm, \dots$


 $\rightarrow \text{irreps of } D8 \subset O(2) \rightarrow J = 0, \dots, 4$

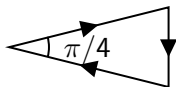
Irrep	Dimension	Continuum glueball content (J^P)
A_1	1	$0^+, 8^+, \dots$
A_2	1	$0^-, 8^-, \dots$
B_1	1	$4^+, 12^+, \dots$
B_2	1	$4^-, 12^-, \dots$
E_1	2	$1^\pm, 7^\pm, \dots$
E_2	2	$2^\pm, 6^\pm, \dots$
E_3	2	$3^\pm, 5^\pm, \dots$

0^+ states: add all rotations

4^+ states: alternating sum

$$\Phi = \sum_{n=1}^8 \mathcal{O} \left(\theta = \frac{n\pi}{4} \right)$$

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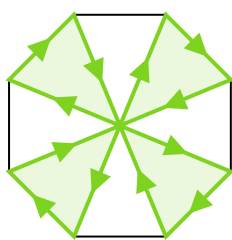
\rightarrow irreps of $DN \subset O(2) \rightarrow J = 0, \dots, N/2$

Symmetry breaking

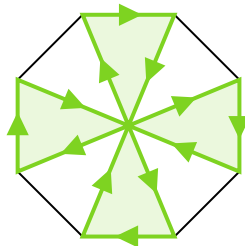
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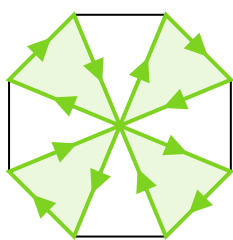
Block 1



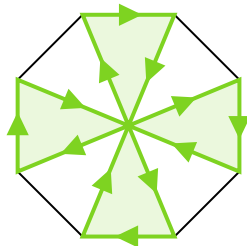
Block 2

Symmetry breaking

There is a soft $D8 \rightarrow D4$ symmetry breaking induced by the lattice



Block 1



Block 2

Decompose trial unbroken 4^+ operator:

$$\begin{aligned}\Phi &= \sum_{n=1}^8 (-1)^n \mathcal{O}\left(\frac{n\pi}{4}\right) \\ &= - \underbrace{\sum_{n=1,3,5,7} \mathcal{O}\left(\frac{n\pi}{4}\right)}_{\Phi_1} + \underbrace{\sum_{n=2,4,6,8} \mathcal{O}\left(\frac{n\pi}{4}\right)}_{\Phi_2}\end{aligned}$$

Define weight vector \vec{w}_0 :

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For higher spins, choose \vec{w} such that:

- $\langle \Phi[\vec{w}] \rangle = 0$
- $\max(\vec{w} \cdot \vec{w}_0)$

$$\Rightarrow \vec{w} = \vec{w}_0 - \frac{\vec{w}_0 \cdot \vec{\Phi}}{\vec{\Phi} \cdot \vec{\Phi}} \vec{\Phi}$$

Conclusions and Outlook

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Questions for the audience...

- How can we increase the speed of the Conjugate Gradient algorithm?
- Can we exploit the symmetries of staggered fermions to improve/speed up the calculation?