The mass of the Baryon Junction in $(2+1)d$

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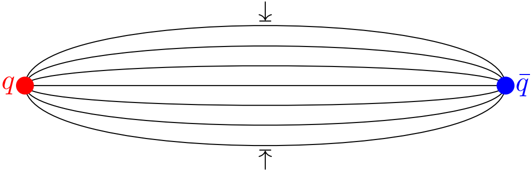
Outline

1. Effective string theory of the color flux tube: Nambu-Gotō string and beyond
2. Baryons in the EST
3. Open string channel: Baryon Junction mass determination on the lattice
4. Closed string channel: High temperature results and comparison with Svetitsky–Yaffe mapping

Flux tube as a string

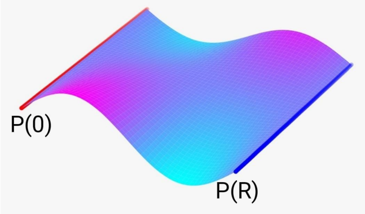
Confinement is related to the formation of **chromoelectric flux tubes** connecting color sources



Effective String Theory provides an accurate description of confining strings in Yang-Mills theories

Polyakov loop correlator:

$$G(R) \sim \int DXe^{-S_{EFT}(X)} \equiv Z_{EST}$$



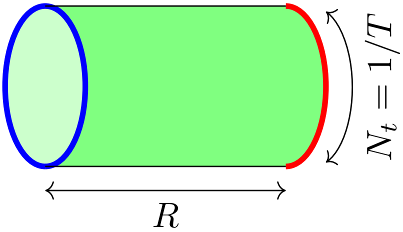
Nambu-Gotō string

Nambu-Gotō (NG) string model:

$$S_{NG} = \sigma \int_{\Sigma} d^2\xi \sqrt{g}$$

The Polyakov loop correlator at large distance is dominated by:

$$G(R) \sim K_0(E_0 R), \qquad E_0 = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$



EST corrections beyond Nambu-Gotō

To obtain the correct EST describing the gauge theory, it is essential to go **beyond the NG approximation**...

$$S_{EST} = \int d^2\xi [\sigma + \gamma_1 \mathcal{R} + \gamma_2 \mathcal{K}^2 + \gamma_3 \mathcal{K}^4 + \dots]$$

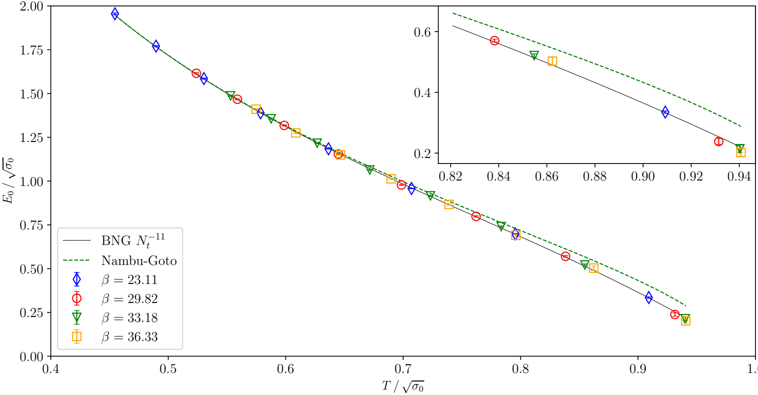
- $\mathcal{R}$ is topological invariant
- $\mathcal{K}^2$is proportional to EoM $\rightarrow$ can be eliminated

Low-energy universality of the EST $\Longrightarrow$ First correction BNG can only appear at the order $1/N_t^7$
[O. Aharony, Z. Komargodski, 1906.08098]

S-matrix approach$\Longrightarrow$First two correction ($1/N_t^7$ and $1/N_t^9$ terms) are controlled by the same parameter, next parameter appears at $1/N_t^{11}$ order.
[J.E. Miró et al, 1906.08098]

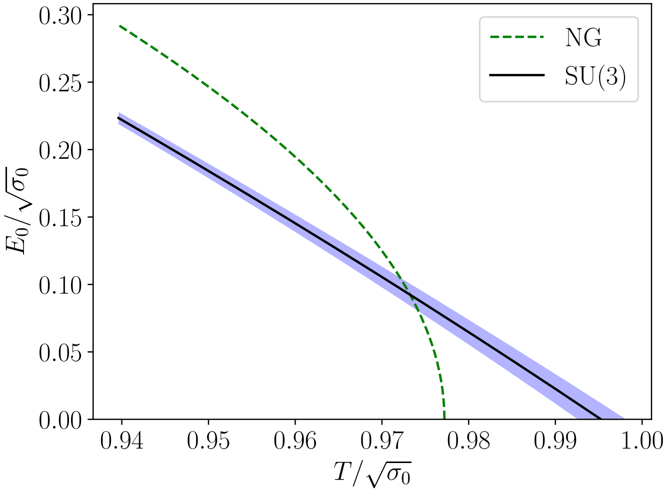
SU(3) numerical results

[M.Caselle et al, 2407.10678]



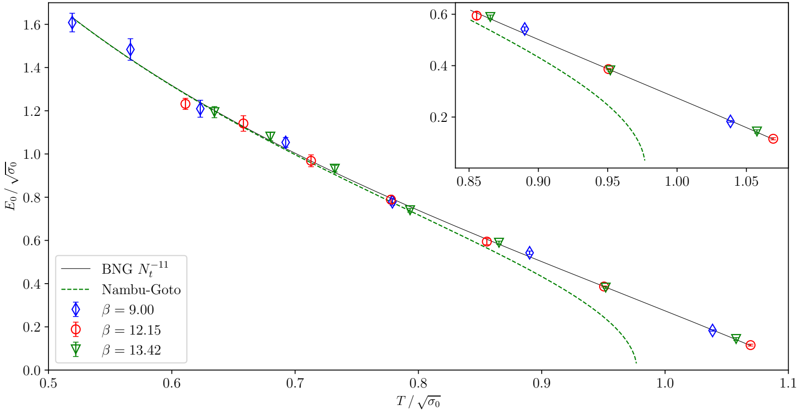
Combined best fits of the SU(3) data including all terms up to $1/N_t^{11}$

SU(3) numerical results



$T_{E_0=0} = 0.995(5)\sqrt{\sigma}$
compatible with $T_c = 0.9890(31)\sqrt{\sigma}$ found in [J. Liddle, M. Teper, 0803.2128]

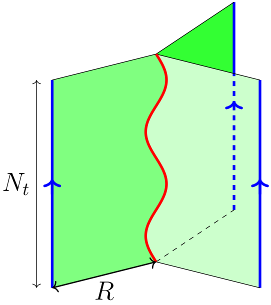
SU(2) numerical results



Baryon Junction: N confining flux tubes that meet at a common point

- Calculations are more complex than in the standard $\langle P^{+}(0)P(R)\rangle$ setting
- In the $SU(3)$ theory the EFT of three static quarks positioned at the vertices of a triangle is determined by M and σ up to $\mathcal{O}(1/R^2)$

  [Z. Komargodski, S. Zhong, 2405.12005]



Baryon Junction mass

1. Sign and magnitude of M play a crucial role in the understanding of EST and of colour confinement.
   - **M>0**: Confining strings are weakly coupled in the long-distance regime, perturbative stability and unitarity
   - **M<0**: Strong coupling, perturbative instability and possible violations of unitarity
2. In the closed channel, the junction can be seen as an interaction vertex
3. Involved in models of exotic hadrons
4. Can be used to test existing models based on the AdS/QCD duality

Open string channel ($N_t \gg R$)

$$\langle P(x_1)\, P(x_2)\, P(x_3)\rangle = A_{open}(N_t)\, e^{-N_t E_0(R)}$$

With:

- $A_{open}(N_t) = A(N_t) e^{-N_t M}$
- $E_0(R) = 3R\sigma - \frac{(d-2)\pi}{16R} - \frac{(d+2)M\pi}{144\sigma R^2} + \mathcal{O}(1/R^3)$

[O. Jahn, P. de Forcrand, 0309115, 0502039][M. Pfeuffer et al, 0810.1649]

[Z. Komargodski, S. Zhong, 2405.12005]

$$\Downarrow$$

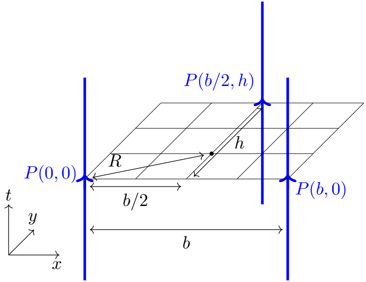
Space dimension $d = 2$ case particularly effective to measure the junction mass M

Baryon Junction mass on the lattice

We consider $N_t \gg R$ in the low temperature regime ($T/T_c \simeq 0.18$)

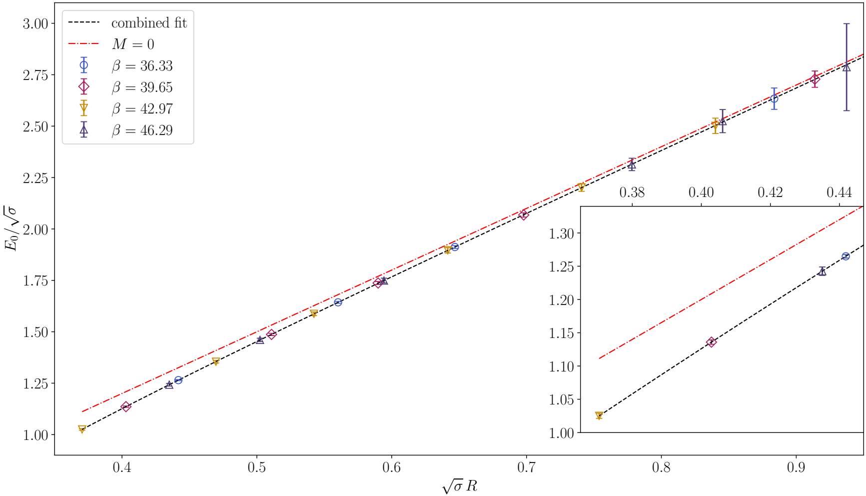
$$G^{(3)}(R) = Ae^{-N_t E_0}$$

$$E_0 = 3R(\sigma a^2) - \frac{M\pi}{36(\sigma a^2)R^2}$$



β	R_{min}/a	R_{max}/a	σa^2	σa^2 from lit.	$M/\sqrt{\sigma}$
36.33	4.64	12.68	0.009022(53)	0.009344(17)	0.142(13)
39.65	4.64	13.93	0.007518(23)	0.007831(58)	0.137(6)
42.97	4.64	13.93	0.006378(18)	0.006645(59)	0.129(5)
46.29	5.87	15.17	0.005460(27)	0.005716(59)	0.133(12)

Results



$$\frac{M}{\sqrt{\sigma}} = 0.1355(36)$$

Closed string channel ($N_t \ll R$, $N_t\sqrt{\sigma} \gg 1$)

$$\langle P(x_1)P(x_2)P(x_3)\rangle = A_{closed}(N_t)\left(\frac{N_t}{R}\right)^{\frac{2d-3}{2}} e^{-3RE_0(N_t)}$$

With:

- $E_0(N_t) = \sigma N_t - \frac{\pi(2d-3)}{6N_t}$
- $A_{closed}(N_t) = A(N_t)\, e^{-MN_t\left[1-\frac{\pi(d-2)}{18\sigma N_t^2}\right]}$

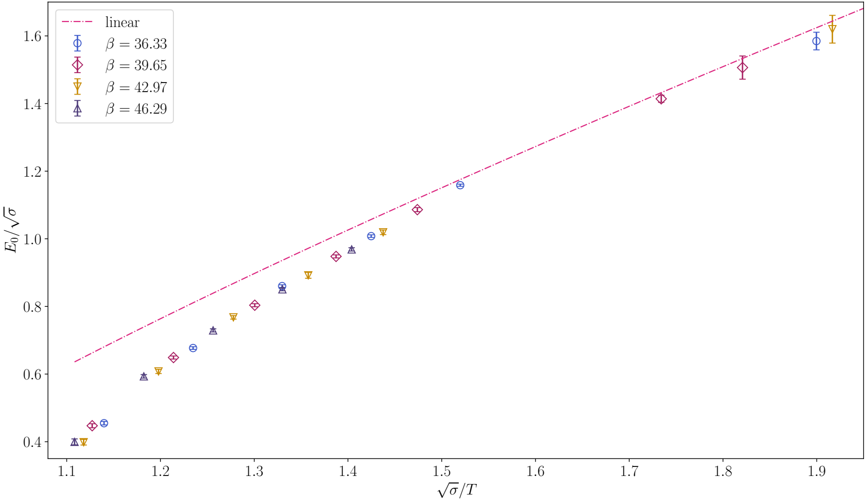
In the high-T regime of the theory, near the critical temperature T_c, N_t is small and $N_t\sqrt{\sigma} \sim 1$. Higher order terms of the Nambu-Goto action become important, but we have no EST result for these terms

- Fixing $d = 2$:

$$\langle P(x_1)\, P(x_2)\, P(x_3)\rangle \sim \frac{e^{-3RE_0(N_t)}}{\sqrt{R}}$$

$$E_0(N_t) = \sigma N_t \left(1 - \frac{\pi}{6\sigma N_t^2}\right)$$

High Temperature results ($0.75 < \frac{T}{T_c} < 0.9$)



Svetitsky–Yaffe mapping

In the vicinity of the deconfinement point:

$SU(3)$ LGT in $(2+1)d \Longleftrightarrow$ Three-state Potts model in $2d$

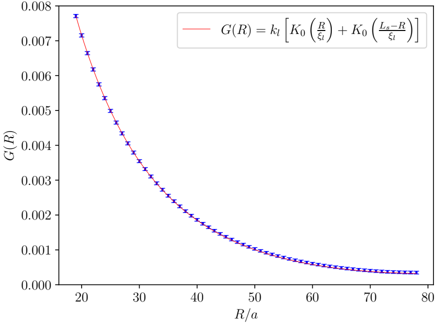
- Spin operator $\Longleftrightarrow$ Polyakov loop
- Energy operator $\Longleftrightarrow$ Plaquette
- Ordered phase of the spin model $\Longleftrightarrow$ Deconfined phase of the gauge theory

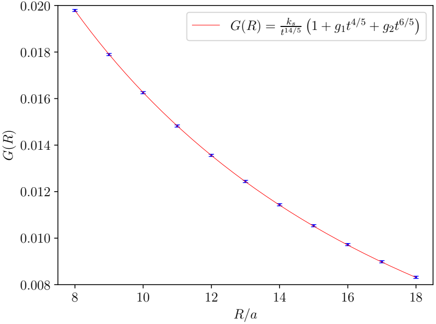
Predictions for both the short- and long-distance behaviour of the two- and three-point correlators

Svetitsky–Yaffe mapping

This conjecture was already tested for the $SU(2)$ and for the $SU(3)$ gauge theory in $(2+1)d$ [M.Caselle, et al, 2407.10678, 2109.06212]

E_0 extracted from the short-range fits of $\langle P^+ P \rangle$ is in good agreement with the large distance fit using the EST prediction





Correlators in the spin model

[M. Caselle, G. Delfino, P. Grinza, O. Jahn, N. Magnoli, 0511168]

- Large distance limit (recall: $K_0(x) \sim \frac{e^{-x}}{\sqrt{x}}$):

$$\langle s^+(0)s(R)\rangle \sim K_0(mR)$$
$$\langle s(x_1)s(x_2)s(x_3)\rangle \sim K_0(mR_Y)$$

- The mass scale governing the two-point and the three-point functions coincides
- Agrees with EST in the closed string channel if $m \longleftrightarrow E_0$

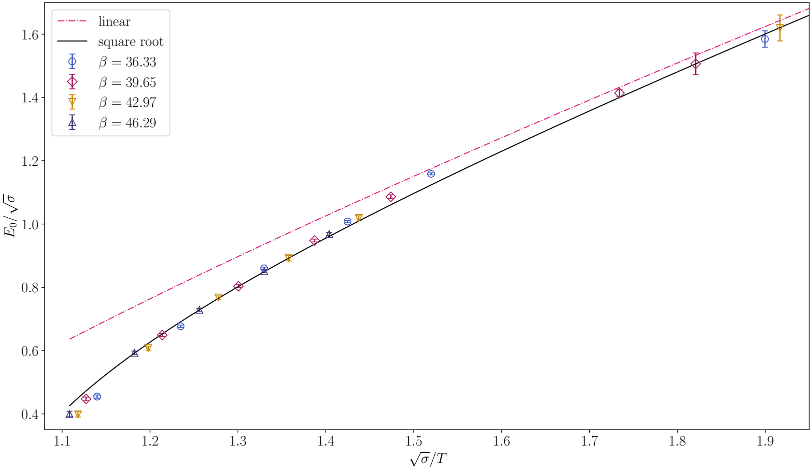
This suggests that:

$$E_0(N_t) = \sigma N_t \left(1 - \frac{\pi}{6\sigma N_t^2}\right) \longrightarrow E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

Comparison with Svetitsky–Yaffe prediction

EST first order calculation (linear): $E_0 = \sigma N_t \left(1 - \frac{\pi}{6\sigma N_t^2}\right)$

S-Y mapping (square root) : $E_0 = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$



Conclusion and outlook

- The determination of the baryon junction mass is relevant from the theoretical and phenomenological perspectives
- Lattice data shows excellent agreement with an independent description based on the S-Y mapping
- σ extracted from $\langle P^+ P \rangle$ should coincide with that extracted from $\langle PPP \rangle$
- What happens in $3+1$ dimensions and in full QCD?
- What is the shape of the flux tube in the vicinity of the baryon junction?

Closed string channel ($N_t \ll R$, $N_t\sqrt{\sigma} \gg 1$)

Fixing $d = 2$:

$$\langle P(x_1)\, P(x_2)\, P(x_3)\rangle \sim \frac{e^{-3RE_0(N_t)}}{\sqrt{R}}$$

- $E_0(N_t) = \sigma N_t \left(1 - \frac{\pi}{6\sigma N_t^2}\right)$

Similar to the two-point correlator:

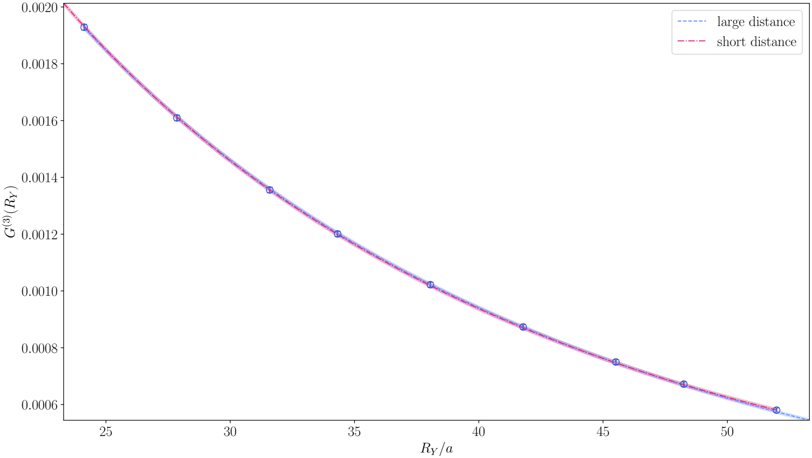
$$\langle P^+(0) P(R)\rangle \sim \frac{e^{-RE_{P+P}(N_t)}}{\sqrt{R}}$$

- $E_{P+P}(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} \simeq \sigma N_t \left(1 - \frac{\pi}{6\sigma N_t^2} + \cdots\right)$

High Temperature results $(0.75 < \frac{T}{T_c} < 0.9)$

Large distance: $G^{(3)}(R_Y) = A_3\, K_0(R_Y E_0)$

Short distance : $G^{(3)}(r) = \frac{A_0}{r^{2/5}}\left(c_1 + c_2 r^{4/5} + b r^{6/5}\right), \quad r = E_0 R_Y$



High Temperature results