Peculiar phase transitions in three dimensional gauge systems

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Based on joint works with A. Pelissetto, E. Vicari, et al.
see Physics Reports **1133**, 1 (2025), 2410.05823 for a review

Bridging analytical and numerical methods for quantum field theory
Trento, 25^{th}-29^{th} August 2025

Outline

What I mean by "peculiar"

When no gauge degrees of freedom are present

If the Hamiltonian of a statistical system is local and a spontaneous symmetry breaking generates a continuous phase transition, universal properties of the transition are encoded in the ϕ^4 QFT with the proper symmetry breaking pattern.

What happens when gauge fields are present? What is (if any) the role of the gauge fields in the effective description of the transition?

More precisely: can we write the effective theory using only gauge invariant composite fields or do we need gauge fields also in the effective theory? ("beyond LGW paradigm" Senthil et al. PRB **70**, 144407 (2004))

Different points of view on the same problem

Statistical mechanics point of view: can we find statistical models whose critical behaviour is described by a gauge EFT?

Finite temperature field theory point of view: which EFT provides the correct description of the finite T transition of a 4D gauge field theory?

Constructive field theory point of view: can we define nonperturbatively three dimensional gauge theories?

Renormalization group in field theory point of view: do IR stable RG fixed points exist in three dimensional gauge theories?

Finite-T chiral transition in massless QCD

Standard approach to investigate the universal properties of the finite-T chiral transition Pisarski, Wilczek PRD, **29**, 338 (1984)

1. Model the 4D finite-T transition by a 3D effective field theory.
2. Assume that the chiral transition is governed by the gauge-invariant order parameter (the $N_f \times N_f$ chiral condensate matrix).
3. Write down the most general Φ^4 effective Lagrangian for the order parameter compatible with the chiral symmetry breaking pattern.
4. If IR-stable FPs of the RG-flow exist the chiral transition can be continuous of the given FP universality classes or discontinuos.
   If no IR-stable FPs exist, discontinuous transition.

Finite-T chiral transition in massless QCD

The gauge invariant order parameter is $\Phi_{ij} = \langle \bar{\Psi}_i (1+\gamma_5) \Psi_j \rangle$.

Global symmetry:

$$U_V(1) \times U_A(1) \times [SU(N_f)/Z_{N_f}]_L \times [SU(N_f)/Z_{N_f}]_R \quad \text{classic}$$
$$U_V(1) \times Z^A_{N_f} \times [SU(N_f)/Z_{N_f}]_L \times [SU(N_f)/Z_{N_f}]_R \quad \text{anomaly}$$

with SSB pattern

$$[U_L(N_f) \times U_R(N_f)]/U_V(1) \to U_V(N_f)/U_V(1) \quad \text{classic}$$
$$[SU_L(N_f) \times SU_R(N_f)]/Z^V_{N_f} \to SU_V(N_f)/Z^V_{N_f} \quad \text{anomaly}$$

Effective lagrangian

$$\mathcal{L} = \mathrm{Tr}(\partial_\mu \Phi^\dagger)(\partial_\mu \Phi) + r_0 \mathrm{Tr}\Phi^\dagger\Phi + u_0(\mathrm{Tr}\Phi^\dagger\Phi)^2 + v_0 \mathrm{Tr}(\Phi^\dagger\Phi)^2 +$$
$$+ w_0(\det\Phi^\dagger + \det\Phi) +$$
$$+ x_0(\mathrm{Tr}\Phi^\dagger\Phi)(\det\Phi^\dagger + \det\Phi) + y_0\left[(\det\Phi^\dagger)^2 + (\det\Phi)^2\right]$$

2^{nd} and 3^{rd} lines $\leftrightarrow$ anomalous terms; for $N_f > 2$ the 3^{rd} line is irrelevant.

Finite-T chiral transition in massless QCD

Expectations obtained by studying the RG flow of the effective theory
Pisarski, Wilczek PRD, **29**, 338 (1984), Pelissetto, Vicari PRD **88**, 105018 (2013)

	with anomaly	without anomaly
$N_f = 2$	$O(4)$ or 1^{st}	$U_L(2) \times U_R(2) \to U_V(2)$ or 1^{st}
$N_f > 2$	1^{st}	1^{st}

Verifying these results by LQCD simulations is extremely challenging and the results are quite disappointing:

- for $N_f = 2$ we have no real consensus on the order of the transition: different groups using different discr. got different results
- for $N_f > 2$ everybody see 1^{st} order but the critical value of the mass seems to approach zero with the lattice spacing

(see e.g. Sharma 1901.07190 for a review)

Elitzur theorem: use and misuse

In a nonperturbatively regularized gauge-invariant theory any local operator that transforms under a nontrivial irreducible representation of the gauge group has vanishing expectation value.
Elitzur PRD **12**, 3978 (1975); De Angelis, de Falco, Guerra PRD **17**, 1624 (1978)

Consequence: the order parameter which characterize a thermodynamic phase has to be gauge invariant

What we assumed: the EFT which describes the critical behavior can be written by using only local gauge invariant composite fields.

To assume no gauge degrees of fredom in the EFT is ≈equivalent to assume non critical gauge degrees of freedom at the transition. Whether this is true or false is model dependent.

U(1) gauge model with SU(N) global symmetry

We consider models with scalar matter fields: lattice simulations much simpler than in the fermionic case, possibility to obtain high-accuracy results to compare with continuous EFT results.

Direct test of LGW or beyond LGW behavior by comparing critical exponents with theoretical expectatons

Field content of the lattice model

Model 1	Model 2
scalar field $\boldsymbol{z_x} \in \mathbb{C}^{\mathrm{N}}$, $|\boldsymbol{z_x}| = 1$	scalar field $\boldsymbol{z_x} \in \mathbb{C}^{\mathrm{N}}$, $|\boldsymbol{z_x}| = 1$
noncompact gauge field $A_{\boldsymbol{x},\mu} \in \mathbb{R}$	compact gauge field $\lambda_{\boldsymbol{x},\mu} \in U(1)$

In both the cases the symmetries of the scalar field are:
Local invariance: $\boldsymbol{z_x} \to G_{\boldsymbol{x}} \boldsymbol{z_x}$ with $G_{\boldsymbol{x}} \in U(1)$
Global invariance $\boldsymbol{z_x} \to M \boldsymbol{z_x}$ with $M \in SU(N)$

If gauge modes are not critical

Gauge invariant order parameter of the lattice model:

$$Q_{\boldsymbol{x}}^{ab} = z_{\boldsymbol{x}}^{a} \bar{z}_{\boldsymbol{x}}^{b} - \frac{1}{N}\delta^{ab} \ , \quad a, b = 1, \ldots, N$$

Gauge invariant order parameter to write the field theory: $Q(\boldsymbol{x})$ traceless $N \times N$ Hermitian and transforming as $Q(\boldsymbol{x}) \to MQ(\boldsymbol{x})M^{\dagger}$

Most general 4^{th}-order polynomial consistent with the global symmetry:

$$\mathrm{Tr}(\partial_{\mu} Q)^2 + r\,\mathrm{Tr}\,Q^2 + w\,\mathrm{Tr}\,Q^3 + u\,(\mathrm{Tr}\,Q^2)^2 + v\,\mathrm{Tr}\,Q^4$$

For $N_f > 2$: $\mathrm{Tr}\,Q^3 \neq 0$ and a first order phase transition is expected.

For $N_f = 2$: $\mathrm{Tr}\,Q^3 = 0$, the two quartic terms become equivalent and we obtain the standard ϕ^4 theory of the 3D O(3) universality class using $Q(\boldsymbol{x}) = \boldsymbol{\phi} \cdot \boldsymbol{\sigma}$

If gauge modes are critical

Natural to expect the continuum Abelian Higgs model as EFT

$$|D_\mu \phi|^2 + r\,\phi^\dagger \cdot \phi + \frac{1}{6} u\,(\phi^\dagger \cdot \phi)^2 + \frac{1}{4g^2}\,F_{\mu\nu}^2$$

Using four loops computation Ihrig et al. PRB **100**, 134507 (2019) got a IR-stable charged FP for $N > N_{cr}(\varepsilon)$ with

$$N_{cr}(\varepsilon) = N_4\left[1 - 1.752\,\varepsilon + 0.789\,\varepsilon^2 + 0.362\,\varepsilon^3 + O(\varepsilon^4)\right]\ , \quad N_4 \approx 184$$

Resumming and taking into account 2d: IR-stable FP found for $N > N_{cr}^* = 12.2(3.9)$ in 3d.

Large N computation: Halperin et al., PRL **32**, 292 (1974)

$$\nu = 1 - 48/(\pi^2 N) + O(N^{-2})\,,$$

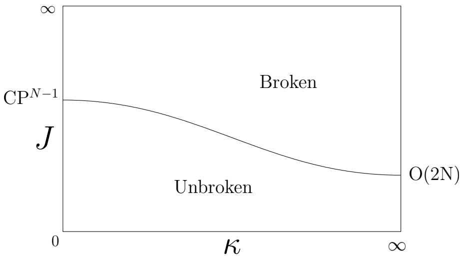
Anom. dim. of $Q^{ab} = \phi^a \bar{\phi}^b - \frac{|\phi|^2}{N}\delta^{ab}$ field Irkhin et al PRB **54**, 11953 (1996)

$$\eta_q = 1 - 32/(\pi^2 N) + O(N^{-2})\,,$$

$U(1)$ compact

$$H = -JN \sum_{\boldsymbol{x},\mu} 2\,\mathrm{Re}\,(\lambda_{\boldsymbol{x},\mu}\, \bar{\boldsymbol{z}}_{\boldsymbol{x}} \cdot \boldsymbol{z}_{\boldsymbol{x}+\mu}) - \kappa \sum_{\boldsymbol{x},\mu>\nu} 2\,\mathrm{Re}\,(\lambda_{\boldsymbol{x},\mu}\, \lambda_{\boldsymbol{x}+\mu,\nu}\, \bar{\lambda}_{\boldsymbol{x}+\nu,\mu}\, \bar{\lambda}_{\boldsymbol{x},\nu})$$

- for $\kappa = 0$ it reduces to a standard discretization of the CP^{N-1} model
- for $\kappa \to \infty$ the model reduces to $O(2N)$ (unstable with respect to κ in the ε-exp of Abelian Higgs FT)
- for $J = 0$ the theory is confining for all $\kappa > 0$ Polyakov PLB **59**, 82 (1975)
- for $J \to \infty$ the system is completely polarized, no transition in κ



Numerical evidence: O(3) transition for $N = 2$, dicontinuous for $N > 2$ for all values of κ.

Pelissetto, Vicari PRE **100**, 042134 (2019)

Lattice observables

$$Q_{\boldsymbol{x}}^{ab} = z_{\boldsymbol{x}}^{a} \bar{z}_{\boldsymbol{x}}^{b} - \frac{1}{N}\delta^{ab}$$

On a L^3 lattice with periodic b.c. we can define the two point function

$$G(\boldsymbol{x}-\boldsymbol{y}) = \langle \mathrm{Tr}\,(Q_{\boldsymbol{x}} Q_{\boldsymbol{y}})\rangle,$$

from which we get the susceptibility χ, the (2^{nd} momentum, finite volume) correlation lenght ξ and the Binder cumulant U

$$\chi = \sum_{\boldsymbol{x}} G(\boldsymbol{x}) \ , \quad \xi^2 = \frac{1}{4\sin^2(\pi/L)} \frac{\widetilde{G}(0) - \widetilde{G}(\boldsymbol{p}_m)}{\widetilde{G}(\boldsymbol{p}_m)}$$

$$U = \frac{\langle \mu_2^2 \rangle}{\langle \mu_2 \rangle^2}, \quad \mu_2 = \frac{1}{V^2} \sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{Tr}\, Q_{\boldsymbol{x}} Q_{\boldsymbol{y}} \ ,$$

where $\boldsymbol{p}_m = (2\pi/L, 0, 0)$ or permutations.

Finite Size Scaling of lattice observables

U and $R_\xi = \xi/L$ are RG invariants, close to the transition they scale as

$$R(\beta, L) = f_R(X) + L^{-\omega} g_R(X) + \cdots$$

where $X = (\beta - \beta_c)L^{1/\nu}$ and f_R is universal (for given b.c. and lattice aspect ratios) up to an arbitrary rescaling of X.

For the susceptibility we have also the anomalous dimension

$$\chi(\beta, L) \simeq L^{2-\eta_q} f_\chi(X) + \cdots$$

Writing U as a function of R_ξ we get

$$U(\beta, L) = F_U(R_\xi) + O(L^{-\omega}),$$

where F_U is universal and complely fixed by b.c. Convenient for parameter free comparison of different models.

$U(1)$ non-compact

The Hamiltonian takes the form ($\lambda_{\boldsymbol{x},\mu} \equiv e^{iA_{\boldsymbol{x},\mu}}$, $A_{\boldsymbol{x},\mu} \in \mathbb{R}$ is the fundamental variable)

$$H = -JN \sum_{\boldsymbol{x},\mu} 2\,\mathrm{Re}\,(\lambda_{\boldsymbol{x},\mu}\,\bar{\boldsymbol{z}}_{\boldsymbol{x}} \cdot \boldsymbol{z}_{\boldsymbol{x}+\mu}) + \frac{\kappa}{2} \sum_{\boldsymbol{x},\mu>\nu} (\Delta_\mu A_{\boldsymbol{x},\nu} - \Delta_\nu A_{\boldsymbol{x},\mu})^2 \,.$$

where $\Delta_\mu A_{\boldsymbol{x}} \equiv A_{\boldsymbol{x}+\mu} - A_{\boldsymbol{x}}$. Gauge invariance: $A_{\boldsymbol{x},\mu} \to A_{\boldsymbol{x},\mu} + \Delta_\mu \alpha_{\boldsymbol{x}}$, $\boldsymbol{z}_{\boldsymbol{x}} \to e^{i\alpha_{\boldsymbol{x}}} \boldsymbol{z}_{\boldsymbol{x}}$.

Technical problem: even on a finite lattice gauge invariant correlators are generically not well defined. When using periodic b.c. the "center" symmetry $A_{\boldsymbol{x},\mu} \to A_{\boldsymbol{x},\mu} + 2\pi n_\mu$ ($n_\mu \in \mathbb{Z}$) prevents Polyakov loops $P_{\boldsymbol{x},\mu} = \sum_j A_{\boldsymbol{x}+j\mu,\mu}$ from thermalizing. C^* b.c. explicitly breaks this symmetry and makes everything well defined.

Bonati, Pelissetto, Vicari PRB **103**, 085104 (2021)

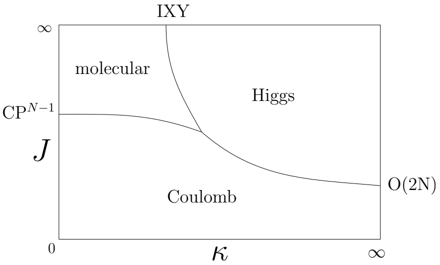
$U(1)$ non-compact: limiting cases

$$H = -JN \sum_{\boldsymbol{x},\mu} 2\,\mathrm{Re}\,(\lambda_{\boldsymbol{x},\mu}\, \bar{\boldsymbol{z}}_{\boldsymbol{x}} \cdot \boldsymbol{z}_{\boldsymbol{x}+\mu}) + \frac{\kappa}{2} \sum_{\boldsymbol{x},\mu>\nu} (\Delta_\mu A_{\boldsymbol{x},\nu} - \Delta_\nu A_{\boldsymbol{x},\mu})^2$$

- For $\kappa = 0$ lattice CP^{N-1} model: O(3) for $N = 2$ and 1^{st} for $N > 2$
- For $\kappa = \infty$ the model reduces to $O(2N)$ (unstable with respect to κ in the ε-exp of Abelian Higgs FT)
- For $J = 0$: free (lattice) photons
- For $J = \infty$ we have $\boldsymbol{z}_{\boldsymbol{x}} = \lambda_{\boldsymbol{x},\mu} \boldsymbol{z}_{\boldsymbol{x}+\mu}$ and repeated use of this relation implies $\lambda_{\boldsymbol{x},\mu}\, \lambda_{\boldsymbol{x}+\mu,\nu}\, \bar{\lambda}_{\boldsymbol{x}+\nu,\mu}\, \bar{\lambda}_{\boldsymbol{x},\nu} = 1$. This means $A_{\boldsymbol{x},\mu} = 2\pi n_{\boldsymbol{x},\mu}$ and we get a SOS-like model (with variables on links and plaquette interaction), which can related to XY by duality.

  Dasgupta, Halperin PRL **47**, 1556 (1981), Neuhaus et al. PRB **67**, 014525 (2003)

$U(1)$ non-compact: phase diagram ($N > 1$)

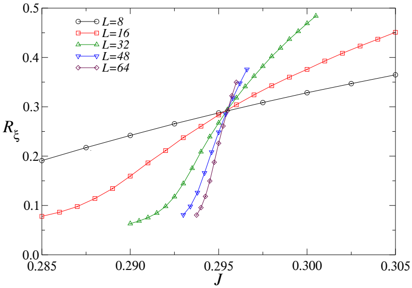


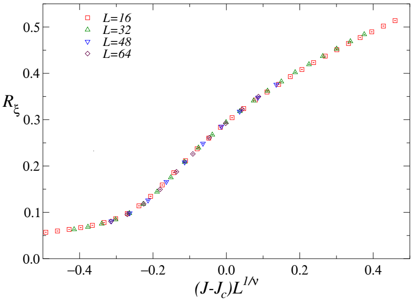
Coulomb	SU(N) unbroken, long range gauge
molecular	SU(N) broken, long range gauge
Higgs	SU(N) broken, short range gauge

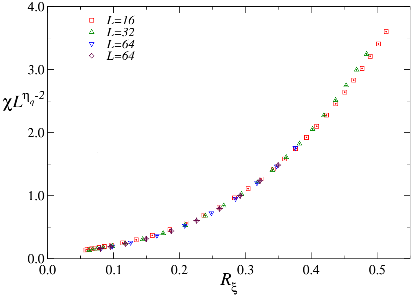
Coulomb-molecular: O(3) for $N = 2$, discontinuous for $N > 2$,
molecular-Higgs: IXY or discontinuous,
Coulomb-Higgs: possibly something unusual could happen.

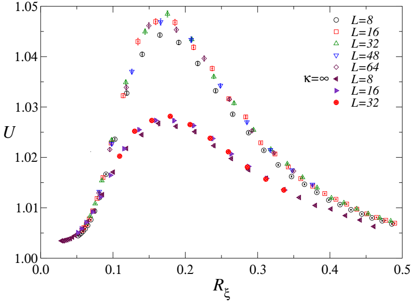
For $N = 2$ Coulomb-Higgs is discontinuous
Motrunich, Vishwanath, 0805.1494, Kuklov et al PRL **101**, 050405 (2008)

$U(1)$ non-compact: Coulomb-Higgs line $N = 25$









$U(1)$ non-compact: Coulomb-Higgs line

N	ν	ν_{LN}	η_q	$\eta_{q,LN}$
25	0.802(8)	0.805	0.883(7)	0.870
15	0.721(3)	0.676	0.815(10)	0.784
10	0.64(2)	0.514	0.74(2)	0.678

Bonati, Pelissetto, Vicari PRB **103**, 085104 (2021)

Numerical results consistent (for large $N \ldots$) with large N results for the charged FP of the Abelian Higgs model

$$\nu_{LN} = 1 - 48/(\pi^2 N) + O(N^{-2}) \; ; \quad \eta_{q,LN} = 1 - 32/(\pi^2 N) + O(N^{-2})$$

Halperin et al., PRL **32**, 292 (1974); Irkhin et al PRB **54**, 11953 (1996)

On the lattice cont. trans. for $N > N_\ell$ and numerically $4 < N_\ell < 10$.

IR fixed point of Abelian Higgs model exists for $N > N_{cr}$ and $N_\ell \geq N_c$.

If we assume $N_\ell = N_{cr}$ then $N_{cr} = 7 \pm 2$.

The compact higher charge model

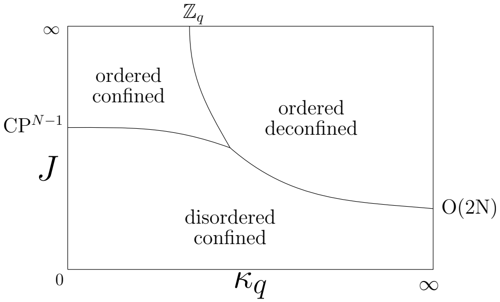
Same as the compact model but $\lambda_{\mathbf{x},\mu} \to \lambda^q_{\mathbf{x},\mu}$ with $q \in \mathbb{N}$:

$$H = -JN \sum_{\mathbf{x},\mu} 2\,\mathrm{Re}\,(\lambda^q_{\mathbf{x},\mu}\,\bar{\mathbf{z}}_{\mathbf{x}} \cdot \mathbf{z}_{\mathbf{x}+\mu}) - \kappa_q \sum_{\mathbf{x},\mu>\nu} 2\,\mathrm{Re}\,(\lambda_{\mathbf{x},\mu}\,\lambda_{\mathbf{x}+\mu,\nu}\,\bar{\lambda}_{\mathbf{x}+\nu,\mu}\,\bar{\lambda}_{\mathbf{x},\nu})$$

For $J = \infty$ we have $\mathbf{z}_{\mathbf{x}} = \lambda^q_{\mathbf{x},\mu}\mathbf{z}_{\mathbf{x}+\mu}$ and repeated use of this relation implies $(\lambda_{\mathbf{x},\mu}\,\lambda_{\mathbf{x}+\mu,\nu}\,\bar{\lambda}_{\mathbf{x}+\nu,\mu}\,\bar{\lambda}_{\mathbf{x},\nu})^q = 1$, up to a gauge choice we have $\lambda_{\mathbf{x},\mu} \in \mathbb{Z}_q$ with Wilson action.

The DC-OD transition for $q \geq 2$ is equivalent to the Coulomb-Higgs transition of the noncompact model

Bonati, Pelissetto, Vicari
PRE **102**, 062151 (2020)
PRB **105**, 085112 (2022)



The relation compact higher charge – noncompact

If we define in the compact higher charge case $A_{\mathbf{x},\mu}$ by $\lambda_{\mathbf{x},\mu} = e^{iA_{\mathbf{x},\mu}/q}$

$$H_c = -JN \sum_{\mathbf{x},\mu} 2\,\mathrm{Re}\left(\bar{\mathbf{z}}_{\mathbf{x}} \cdot e^{iA_{\mathbf{x},\mu}}\, \mathbf{z}_{\mathbf{x}+\hat{\mu}}\right)$$
$$- 2\kappa_q \sum_{\mathbf{x},\mu>\nu} \mathrm{Re}\exp\left[-\frac{i}{q}(\Delta_{\hat{\mu}} A_{\mathbf{x},\nu} - \Delta_{\hat{\nu}} A_{\mathbf{x},\mu})\right]$$

and for $q \to \infty$, $\kappa_q \to \infty$ with $\kappa = 2\kappa_q/q^2$ fixed we recover the noncompact formulation.

Relation of the $q \to \infty$ limit with the monopole suppression to be further investigated Murthy, Sachdev NPB **344** 557 (1990), Motrunich, Vishwanath PRB **70**, 075104 (2004) Pelissetto, Vicari PRE **101** 062136 (2020), Bonati, Pelissetto, Vicari JSM **2206**, 063206 (2022)

Incidentally: this explains the behaviour $\kappa_c^{(q)} \propto q^2$ of the critical coupling of 3d Z_q LGT Borisenko et al. NPB **879**, 80 (2014), Neuhaus et al. PRB **67**, 014525 (2003)

Lattice SU(N_c) model

Lattice fields: scalar fields $\Phi_{\boldsymbol{x}}^{af}$, with $a = 1, \ldots, N_c$, $f = 1, \ldots N_f$ (constraint $\mathrm{Tr}(\Phi_{\boldsymbol{x}}^\dagger \Phi_{\boldsymbol{x}}) = 1$), gauge fields $U_{\boldsymbol{x},\mu} \in \mathrm{SU}(N_c)$

Lattice Hamiltonian:

$$H = -J N_f \sum_{\boldsymbol{x},\mu} \mathrm{Re}\,\mathrm{Tr}\,\Phi_{\boldsymbol{x}}^\dagger\, U_{\boldsymbol{x},\mu}\,\Phi_{\boldsymbol{x}+\hat{\mu}} + \frac{v}{4}\sum_{\boldsymbol{x}} \mathrm{Tr}[(\Phi_{\boldsymbol{x}}^\dagger \Phi_{\boldsymbol{x}})^2] - \frac{\gamma}{N_c}\sum_{\boldsymbol{x},\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\,[U_{\boldsymbol{x},\mu}\,U_{\boldsymbol{x}+\hat{\mu},\nu}\,U_{\boldsymbol{x}+\hat{\nu},\mu}^\dagger\,U_{\boldsymbol{x},\nu}^\dagger]$$

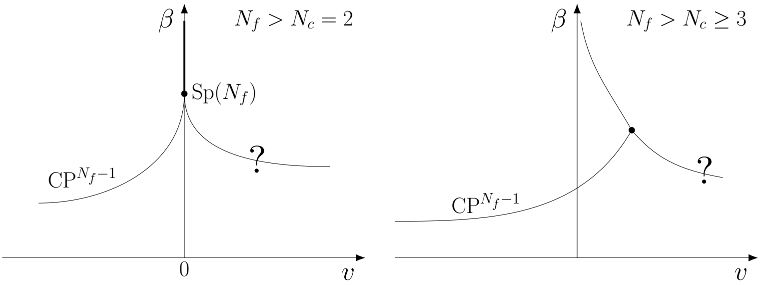
Local symmetry: $U_{\boldsymbol{x},\mu} \to G_{\boldsymbol{x}} U_{\boldsymbol{x},\mu} G_{\boldsymbol{x}+\hat{\mu}}^\dagger$, $\Phi_{\boldsymbol{x}} \to G_{\boldsymbol{x}} \Phi_{\boldsymbol{x}}$, $G_{\boldsymbol{x}} \in \mathrm{SU}(N_c)$

Global symmetry: $\Phi_{\boldsymbol{x}} \to \Phi_{\boldsymbol{x}} M$ with $M \in \mathrm{SU}(N_f)$ (if $N_f < N_c$) or $M \in \mathrm{U}(N_f)/\mathbb{Z}_{N_c}$ (if $N_f \geq N_c$).
If $N_c = 2$ and $v = 0$: symmetry enhancement to $\mathrm{Sp}(N_f) \subset \mathrm{SU}(2N_f)$.

Lattice order parameter: $Q_{\boldsymbol{x}}^{fg} = \sum_a \bar{\Phi}_{\boldsymbol{x}}^{af} \Phi_{\boldsymbol{x}}^{ag} - \frac{1}{N_f}\delta^{fg}$, $Q_{\boldsymbol{x}} \to M^\dagger Q_{\boldsymbol{x}} M$ under the global symmetry.

Lattice SU(N_c) model: phase diagram

An analysis of the minimum energy configurations suggests something unusual to exist only for $N_f > N_c$, as also confirmed by numerical simulations. Bonati, Franchi, Pelissetto, Vicari PRE **104**, 064111 (2021)



Phase diagrams for fixed γ and $\beta = J$.

Theoretical expectations

The SU(N_c) gauge field theory

$$\mathcal{L} = \frac{1}{4g_0^2}\mathrm{Tr}\,F_{\mu\nu}^2 + \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] + \frac{r}{2}\,\mathrm{Tr}\,\Phi^\dagger\Phi + \\ + \frac{u_0}{4}(\mathrm{Tr}\,\Phi^\dagger\Phi)^2 + \frac{v_0}{4}\mathrm{Tr}[\,(\Phi^\dagger\Phi)^2]$$

has an IR stable charged FP for $v > 0$ if $N_f > N_f^*$: $N_f^* = 375.4 + O(\varepsilon)$ if $N_c = 2$, $N_f^* = 638.9 + O(\varepsilon)$ if $N_c = 3$.

Bonati, Franchi, Pelissetto, Vicari PRE **104**, 064111 (2021)

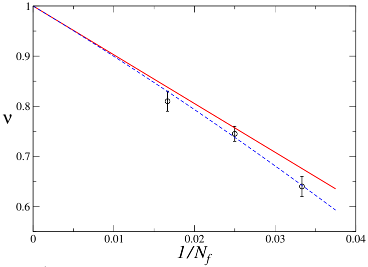
In the large N_f expansion at fixed N_c Hikami, PrThPhys **64**, 1425 (1980)

$$\nu = 1 - \frac{38N_c}{\pi^2 N_f} + O(N_f^{-2})$$

If gauge fieds are noncritical one obtain the same effective FT of the U(1) gauge model.

Numerical results for $N_c = 2$ and large N_f

Continuous phase transitions are observed at $\gamma = 1, v = 1$ for $N_c \gtrsim 30$
Bonati, Pelissetto, Soler Calero, Vicari PRD **110**, 094504 (2024)



Solid line: $\nu = 1 - 9.727/N_f$ from Hikami, PrThPhys **64**, 1425 (1980)
Dashed line $\nu = 1 - 9.727/N_f + a/N_f^2$ with $a \approx -30$.

Anomalous dimension of the lattice order parameter: $\eta_q \approx 1 - 5/N_f$ for $N_f \gtrsim 40$. No analytical computation of this quantity to our knowledge.

The $\mathbb{Z}_2$ gauge O(N) model

Lattice model field content:
scalar field $\boldsymbol{s_x} \in \mathbb{R}^N$ (with $|\boldsymbol{s_x}|^2 = 1$), gauge field $\sigma_{\boldsymbol{x},\mu} \in \mathbb{Z}_2$

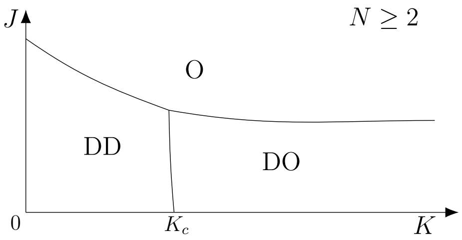
Lattice Hamiltonian

$$H = -JN \sum_{\boldsymbol{x},\mu} \sigma_{\boldsymbol{x},\mu} \boldsymbol{s_x} \cdot \boldsymbol{s}_{\boldsymbol{x}+\hat{\mu}} - K \sum_{\boldsymbol{x},\mu>\nu} \sigma_{\boldsymbol{x},\mu} \sigma_{\boldsymbol{x}+\hat{\mu},\nu} \sigma_{\boldsymbol{x}+\hat{\nu},\mu} \sigma_{\boldsymbol{x},\nu}$$

Boundaries of the phase diagram
$K = 0$: we sum on $\sigma_{\boldsymbol{x},\mu}$ and we get the 3d RP^{N-1} model (for $N \geq 2$)
$K = \infty$: up to a gauge transformation we get the O(N) model
$J = 0$: the 3d $\mathbb{Z}_2$ gauge model, dual of Ising Wegner JMP **12**, 2259 (1971)
$J = \infty$: the system is completely polarized and there is no transition

For small K gauge fields are noncritical and for small J scalar fields are noncritical: lines of RP^{N-1} and $\mathbb{Z}_2$ types enter the bulk of the phase diagram Fradkin, Shenker PRD **19**, 3682 (1979)

The $\mathbb{Z}_2$ gauge O(N) model: phase diagram



The existence of the line coming from $K \to \infty$ is less trivial.

In U(1) and SU(N_c) gauge theories: the O($2N_f$) FP at $K \to \infty$ was unstable wrt gauge perturbations. In this case no continuous fields, and the number of -1 plaquettes is exp suppressed for large K.

Natural (bold?) guess: O(N) line comes from $K = \infty$ Sachdev RepProgPhys. **82**, 014001 (2019). Note that $\langle \boldsymbol{s_x} \cdot \boldsymbol{s_y} \rangle = \delta_{\boldsymbol{x},\boldsymbol{y}}$ due to gauge invariance, and $\langle \boldsymbol{s_x} \, \mathrm{string} \, \boldsymbol{s_y} \rangle$ is exp decaying Frohlich, Morchio, Strocchi NPB **190**, 553 (1981)

The $\mathbb{Z}_2$ gauge O(2) model

Lattice order parameter for RP^{N-1}: $Q^{ab}_{\boldsymbol{x}} = s^a_{\boldsymbol{x}} s^b_{\boldsymbol{x}} - \frac{1}{2}\delta^{ab}$ (real, symmetric, traceless)

Effective field theory for RP^{N-1}:

$$\mathcal{L} = \mathrm{Tr}(\partial_\mu Q)^2 + r\,\mathrm{Tr}(Q^2) + w\,\mathrm{Tr}(Q^3) + u(\mathrm{Tr}(Q^2))^2 + v\,\mathrm{Tr}(Q^4)$$

reduces to the O(2) model for $N = 2$.

We thus expect:
on the left a XY transition
on the right a $XY^\times$ transition (XY without vector order parameter)
Sachdev RepProgPhys. **82**, 014001 (2019)

How to distinguish them? By the anomalous dimension of the lattice operator $Q_{\boldsymbol{x}}$: on the left it should be the anomalous dimension of the fundamental O(2) field, on the right the anomalous dimension of the spin 2 op. in O(2) field theory Bonati, Pelissetto, Vicari PRB **109**, 235121 (2024)

The $\mathbb{Z}_2$ gauge O(2) model: numerics

$K_c = 0.761413292(11)$ (K_c of the $\mathbb{Z}_2$ gauge model)

For $K = 0.5$ we found $\chi_Q \sim L^{\kappa_v}$ consistent with

$$\kappa_v = 3 - 2Y_{V,XY} = 1.96182(2) \; , \quad Y_{V,XY} = \frac{d-2+\eta}{2} = 0.519088(2)$$

Chester at al. JHEP **06**, 142 (2020) (see also Hasenbusch 2507.19265)

For $K = 0.8$ we found $\chi_Q \sim L^{\kappa_t}$ consistent with

$$\kappa_t = 3 - 2Y_{T,XY} = 0.5274(2) \; , \quad Y_{T,XY} = 1.23629(11)$$

In both the cases correlations diverge with $\nu = \nu_{XY} = 0.6717(1)$

Clear evidence of an universality class whose effective field theory admits a LGW description, but the order parameter field entering the effective Lagrangian is not gauge invariant

The gauge variant order parameter can be exposed by using a gauge-fixing
Bonati, Pelissetto, Vicari PRB **109**, 235121 (2024), PRB **110**, 125109 (2024)

Conclusion / Something for the future

4 transition classes identified so far: LGW, GT, $\mathrm{LGW}^{\times}$, topological.

Large N_f: higher order computation of the critical exponents for U(1) and ν for $SU(N_c)$, at least leading order for η_q in the $SU(N_c)$ case

High-order perturbation theory: estimate of N_f^* for $SU(N_c)$

Conformal bootstrap: independent estimates of N_f^* and critical exponents

For the $q \geq 2$ U(1) case: possibility of studying confinement or effective string properties in the large N limit? Spectrum? Relation with the noncompact model?

Consequences of explicit gauge symmetry braking? Nonperturbative gauge fixing? Nonlocal order parameters for topological transitions?

NonAbelian discrete gauge groups? Defects?

Thank you for your attention