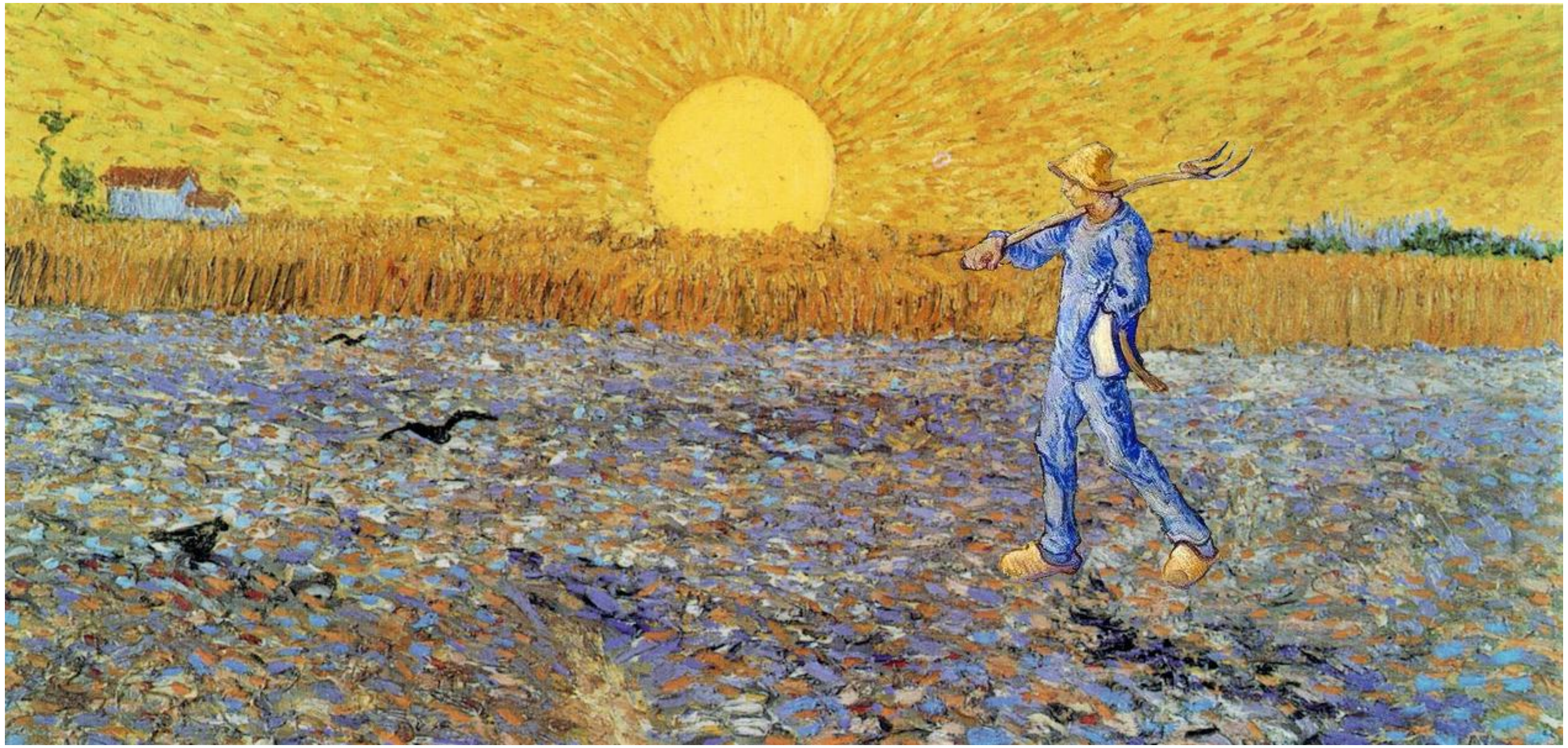
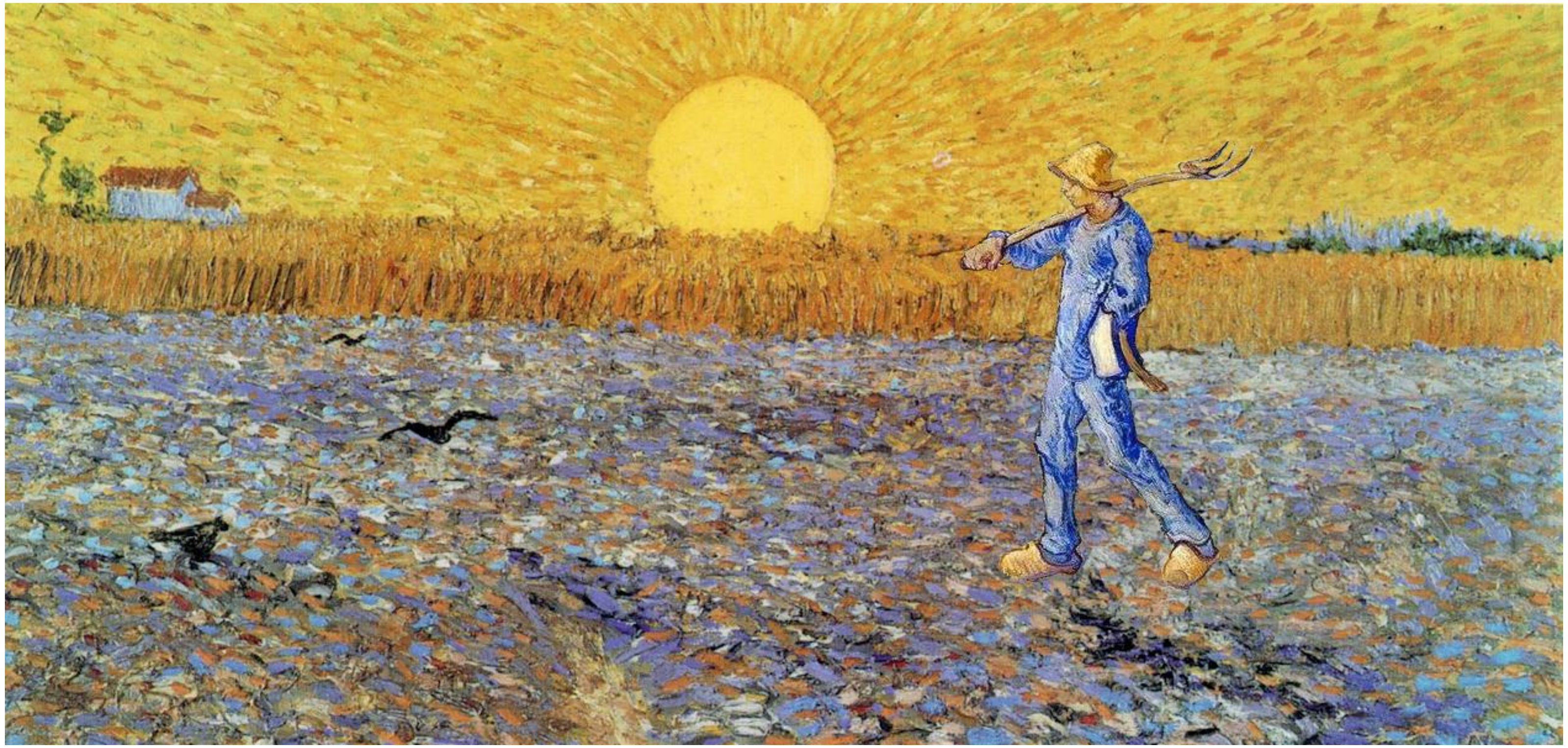
3D U(1) confining strings beyond effective string theory



Alessandro Mariani
University of Turin (Italy)
26 August 2025

3D U(1) confining strings beyond effective string theory



Alessandro Mariani
University of Turin (Italy)
26 August 2025

Closely related to:
Talk by Aharony
Wednesday 3:30pm

Abelian gauge theory in three dimensions

Continuum action: $S = \int d^3x \left(-\frac{1}{4} F_{\mu\nu} F_{\mu\nu}\right)$

$$Z = \int DA\, e^{-S}$$

Abelian gauge theory in three dimensions

Continuum action: $S = \int d^3x \left(-\frac{1}{4} F_{\mu\nu} F_{\mu\nu}\right)$

$$Z = \int DA\, e^{-S}$$

Dependence on the gauge group (UV completion): $\mathbb{R}$ vs $\mathrm{U}(1)$

$U(1)$ vs $\mathbb{R}$ gauge theory in three dimensions

Two possible gauge groups: $\mathbb{R}$ and $\mathrm{U}(1)$. Continuum $L = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu}$

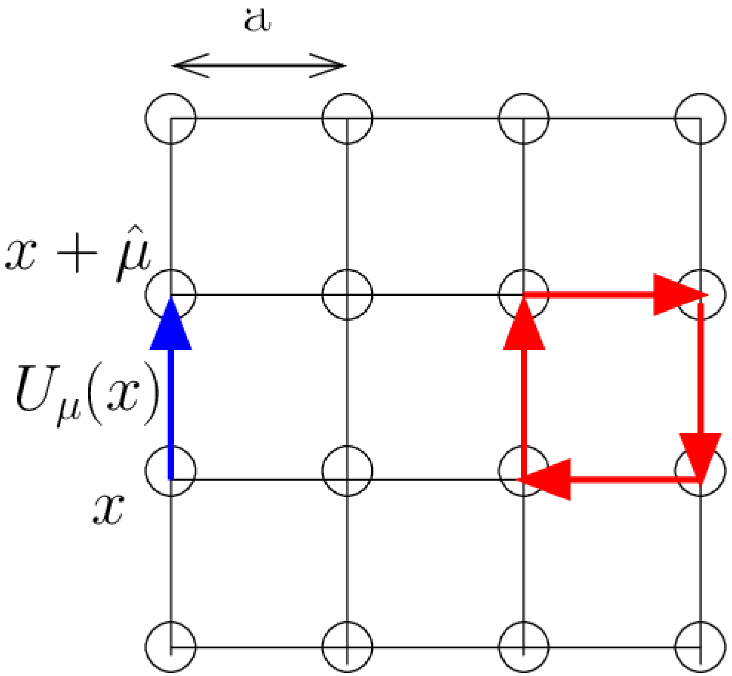
Gauge group $\mathbb{R}$. Field variable $A_\mu(x) \in \mathbb{R}$.

Partition function: $Z = \int_{\mathbb{R}} \prod_{x,\mu} dA_\mu(x)\, e^{-S}$

Lattice action: $S = -\beta \sum_{x,\mu,\nu} F_{\mu\nu} F_{\mu\nu}$

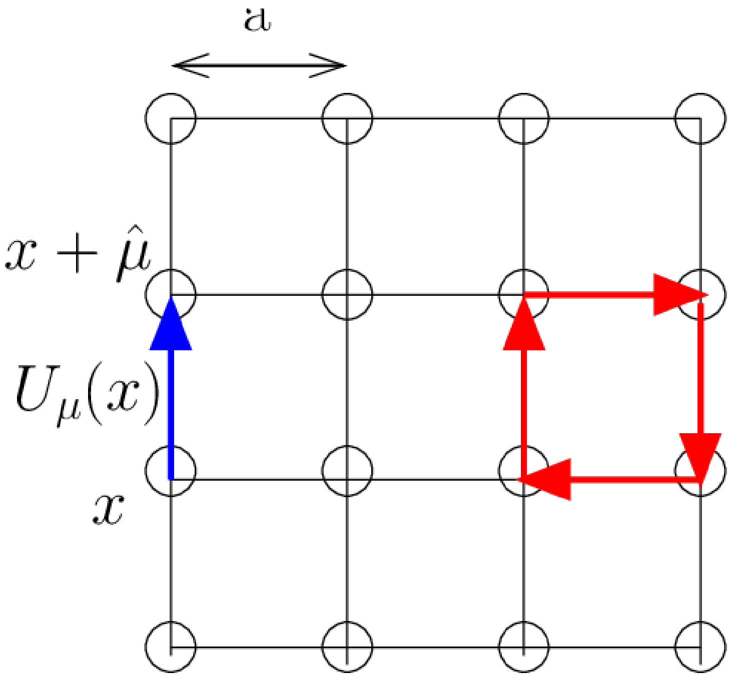
$F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$

Discrete derivative



$U(1)$ vs $\mathbb{R}$ gauge theory in three dimensions

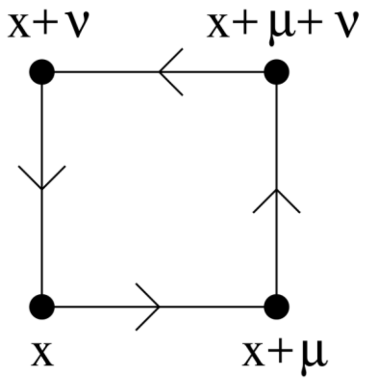
Two possible gauge groups: $\mathbb{R}$ and $\mathrm{U}(1)$. Continuum $L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$



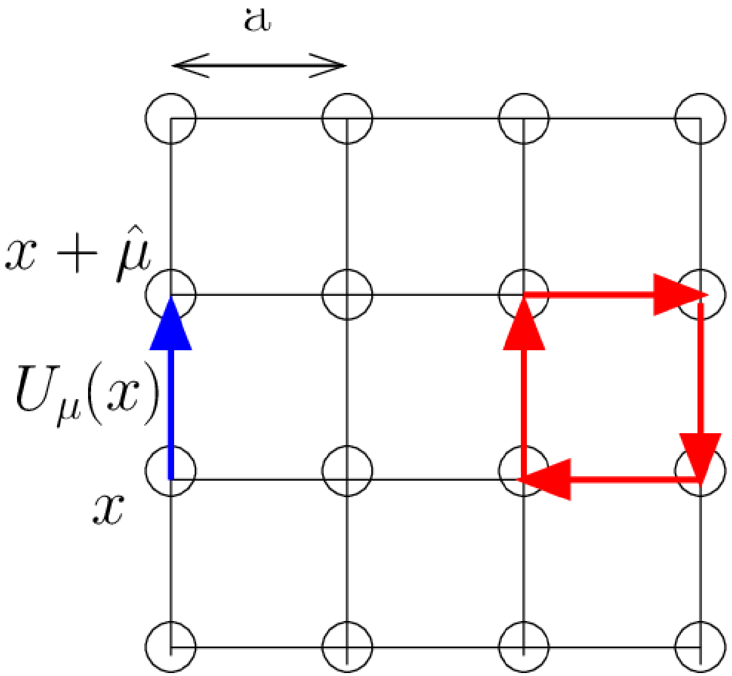
Gauge group $U(1)$. Field variable $U_\mu(x) = e^{iaA_\mu(x)} \in \mathrm{U}(1)$.

Partition function: $Z = \int_{U(1)} \prod_{x,\mu} dU_\mu(x)\, e^{-S}$

Action: $S = -\beta \sum_{x,\mu,\nu} U_\mu(x)U_\nu(x+\mu)U_\mu(x+\nu)^{-1}\, U_\nu(x)^{-1}$



$U(1)$ vs $\mathbb{R}$ gauge theory in three dimensions



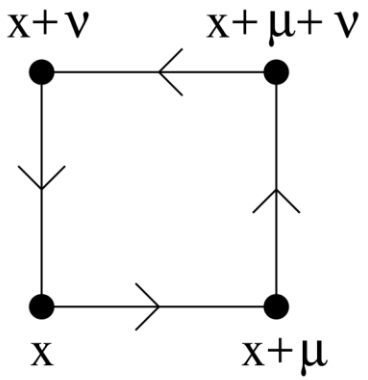
Two possible gauge groups: $\mathbb{R}$ and $\mathrm{U}(1)$. Continuum $L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$

As $a \to 0$, get continuum action: $S \approx \int d^3x \left(-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}\right)$

Gauge group $U(1)$. Field variable $U_\mu(x) = e^{iaA_\mu(x)} \in \mathrm{U}(1)$.

Partition function: $Z = \int_{U(1)} \prod_{x,\mu} dU_\mu(x)\, e^{-S}$

Action: $S = -\beta \sum_{x,\mu,\nu} U_\mu(x) U_\nu(x+\mu) U_\mu(x+\nu)^{-1}\, U_\nu(x)^{-1}$



Duality transformation

On the lattice, can do **exact** change of variables:

In 3D: **gauge theories** are dual to **scalar theories**

$$\mathrm{Z}_1 = \int DA\, e^{-S} = \int D\phi\, e^{-S'} = Z_2$$

Strong to weak coupling:

$$\beta \leftrightarrow 1/\beta$$

Duality transformation

On the lattice, can do **exact** change of variables:

In 3D: **gauge theories** are dual to **scalar theories**

$$Z_1 = \int DA\, e^{-S} = \int D\phi\, e^{-S'} = Z_2$$

Strong to weak coupling:

$$\beta \leftrightarrow 1/\beta$$

$\mathbb{R}$ gauge theory dual to a massless **real** free scalar $S' = -\frac{1}{\beta}\sum_{x,\mu}\big(\partial_\mu \phi(x)\,\big)^2$

Duality transformation

On the lattice, can do **exact** change of variables:

Strong to weak coupling:

$$\beta \leftrightarrow 1/\beta$$

In 3D: **gauge theories** are dual to **scalar theories**

$$Z_1 = \int DA\, e^{-S} = \int D\phi\, e^{-S'} = Z_2$$

$\mathbb{R}$ gauge theory dual to a massless **real** free scalar $S' = -\frac{1}{\beta}\sum_{x,\mu}\left(\partial_\mu \phi(x)\right)^2$ Boring! 🥱

Duality transformation

On the lattice, can do **exact** change of variables:

In 3D: **gauge theories** are dual to **scalar theories**

$$Z_1 = \int DA\, e^{-S} = \int D\phi\, e^{-S'} = Z_2$$

Strong to weak coupling:

$$\beta \leftrightarrow 1/\beta$$

$\mathbb{R}$ gauge theory dual to a massless **real** free scalar $S' = -\frac{1}{\beta}\sum_{x,\mu}\left(\partial_\mu \phi(x)\right)^2$ Boring! 🥱

$U(1)$ gauge theory dual to an **integer-valued** scalar $S' = -\frac{1}{\beta}\sum_{x,\mu}\left(\partial_\mu h_x\right)^2$

$$\boxed{h_x \in \mathbb{Z}}$$

Duality transformation

Strong to weak coupling:

$$\beta \leftrightarrow 1/\beta$$

On the lattice, can do **exact** change of variables:

In 3D: **gauge theories** are dual to **scalar theories**

$$Z_1 = \int DA\, e^{-S} = \int D\phi\, e^{-S'} = Z_2$$

$\mathbb{R}$ gauge theory dual to a massless **real** free scalar $S' = -\frac{1}{\beta}\sum_{x,\mu}\left(\partial_\mu \phi(x)\right)^2$

Boring! 🥱

$U(1)$ gauge theory dual to an **integer-valued** scalar $S' = -\frac{1}{\beta}\sum_{x,\mu}\left(\partial_\mu h_x\right)^2$

$$\boxed{h_x \in \mathbb{Z}}$$

Weird but Interesting! 🤔

3D U(1): properties and duality

Analytical predictions: [Polyakov, Göpfert & Mack]

$$Z = \prod_{x} \sum_{h_x \in \mathbb{Z}} \exp\left[-\frac{1}{2\beta} \sum_{x,\mu} \left(\partial_\mu h_x\right)^2\right]$$

3D U(1): properties and duality

$$Z = \prod_x \sum_{h_x \in \mathbb{Z}} \exp\left[-\frac{1}{2\beta}\sum_{x,\mu}\left(\partial_\mu h_x\right)^2\right]$$

Analytical predictions: [Polyakov, Göpfert & Mack]

Confinement for all values of the bare coupling β (rigorous proof).

Static charges are confined: non-zero **string tension** σ.

Spectrum has a **mass gap** m.

3D U(1): properties and duality

$$Z = \prod_{x} \sum_{h_x \in \mathbb{Z}} \exp\left[-\frac{1}{2\beta} \sum_{x,\mu} \left(\partial_\mu h_x\right)^2\right]$$

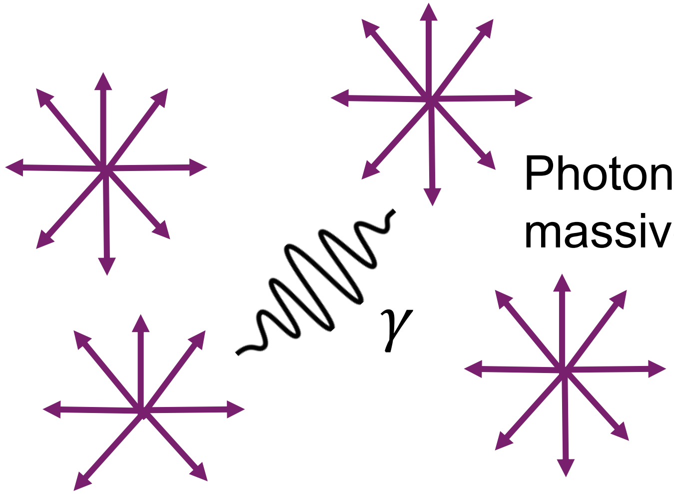
Analytical predictions: [Polyakov, Göpfert & Mack]

Confinement for all values of the bare coupling β (rigorous proof).

Static charges are confined: non-zero **string tension** σ.

Spectrum has a **mass gap** m.

Confinement via topology, i.e. **monopoles**



Photons are packed into massive **photoballs**.

3D U(1): properties and duality

$$Z = \prod_{x} \sum_{h_x \in \mathbb{Z}} \exp\left[-\frac{1}{2\beta} \sum_{x,\mu} \left(\partial_\mu h_x\right)^2 \right]$$

↓

Replace $h_x \in \mathbb{Z}$ with $\Phi_x \in \mathbb{R}$ and compute corrections systematically for large β:

$$S = \frac{1}{4\pi^2 \beta} \sum_{x} \left[\frac{1}{2} (\nabla \Phi_x)^2 + 8\pi^2 \beta \, e^{-2\pi^2 v_0 \beta} \cos(\Phi_x) \right]$$

Get a Sine-Gordon model, with a potential generated by monopole configurations.

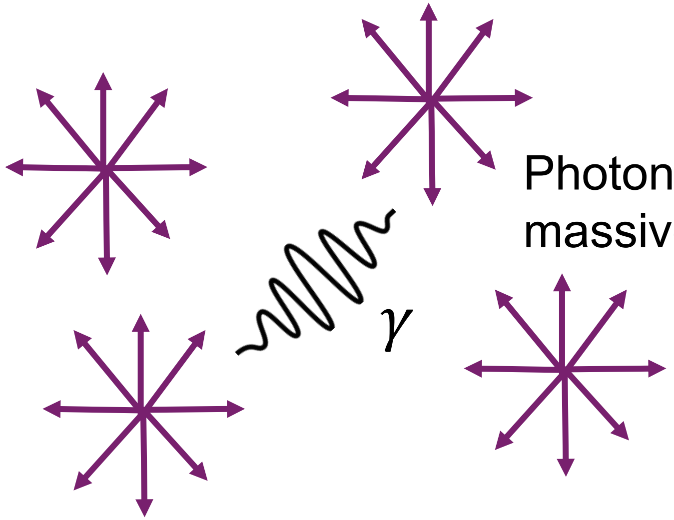
Analytical predictions: [Polyakov, Göpfert & Mack]

Confinement for all values of the bare coupling β (rigorous proof).

Static charges are confined: non-zero **string tension** σ.

Spectrum has a **mass gap** m.

Confinement via topology, i.e. **monopoles**



Photons are packed into massive **photoballs**.

Mass gap prediction

$$Z = \prod_{x} \sum_{h_x \in \mathbb{Z}} \exp\left[-\frac{1}{2\beta} \sum_{x,\mu} \left(\partial_\mu h_x\right)^2 \right]$$

Replace $h_x \in \mathbb{Z}$ with $\Phi_x \in \mathbb{R}$ and compute corrections systematically for large β:

$$S = \frac{1}{4\pi^2\beta} \sum_x \left[\frac{1}{2} (\nabla\Phi_x)^2 + 8\pi^2\beta\, e^{-2\pi^2 v_0 \beta} \cos(\Phi_x) \right]$$

Analytical predictions: [Polyakov, Göpfert & Mack]

Confinement for all values of the bare coupling β (rigorous proof).

Static charges are confined: non-zero **string tension** σ.

Spectrum has a **mass gap** m.

Get mass gap by expanding cosine and reading off the quadratic term:

$$a^2 m^2 = 8\pi^2\beta\, e^{-2\pi^2 v_0 \beta}$$

String tension prediction

$$Z = \prod_x \sum_{h_x \in \mathbb{Z}} \exp\left[-\frac{1}{2\beta}\sum_{x,\mu}\left(\partial_\mu h_x - j_{x,\mu}\right)^2\right]$$

Insert Wilson loop source

Replace $h_x \in \mathbb{Z}$ with $\Phi_x \in \mathbb{R}$ and compute corrections systematically for large β:

$$S = \frac{1}{4\pi^2\beta}\sum_x\left[\frac{1}{2}(\nabla\Phi_x)^2 + 8\pi^2\beta\, e^{-2\pi^2 v_0 \beta}\cos(\Phi_x - \delta j_x)\right]$$

Analytical predictions: [Polyakov, Göpfert & Mack]

Confinement for all values of the bare coupling β (rigorous proof).

Static charges are confined: non-zero **string tension** σ.

Spectrum has a **mass gap** m.

String tension prediction

Analytical predictions: [Polyakov, Göpfert & Mack]

Confinement for all values of the bare coupling β (rigorous proof).

Static charges are confined: non-zero **string tension** σ.

Spectrum has a **mass gap** m.

$$Z = \prod_x \sum_{h_x \in \mathbb{Z}} \exp\left[-\frac{1}{2\beta}\sum_{x,\mu}\left(\partial_\mu h_x - j_{x,\mu}\right)^2\right]$$

Insert Wilson loop source

Replace $h_x \in \mathbb{Z}$ with $\Phi_x \in \mathbb{R}$ and compute corrections systematically for large β:

$$S = \frac{1}{4\pi^2\beta}\sum_x \left[\frac{1}{2}(\nabla\Phi_x)^2 + 8\pi^2\beta\, e^{-2\pi^2 v_0 \beta}\cos(\Phi_x - \delta j_x)\right]$$

Semiclassical approximation:

$$Z[j] \approx \exp(-S[\Phi_{cl}]) \approx \exp(-\sigma \text{ Area})$$

Get string tension:

$$a^2\sigma = \frac{2}{\pi^2}\frac{am}{\beta}$$

Continuum limit

$$a^2m^2 = 8\pi^2\beta\, e^{-2\pi^2 v_0 \beta}$$

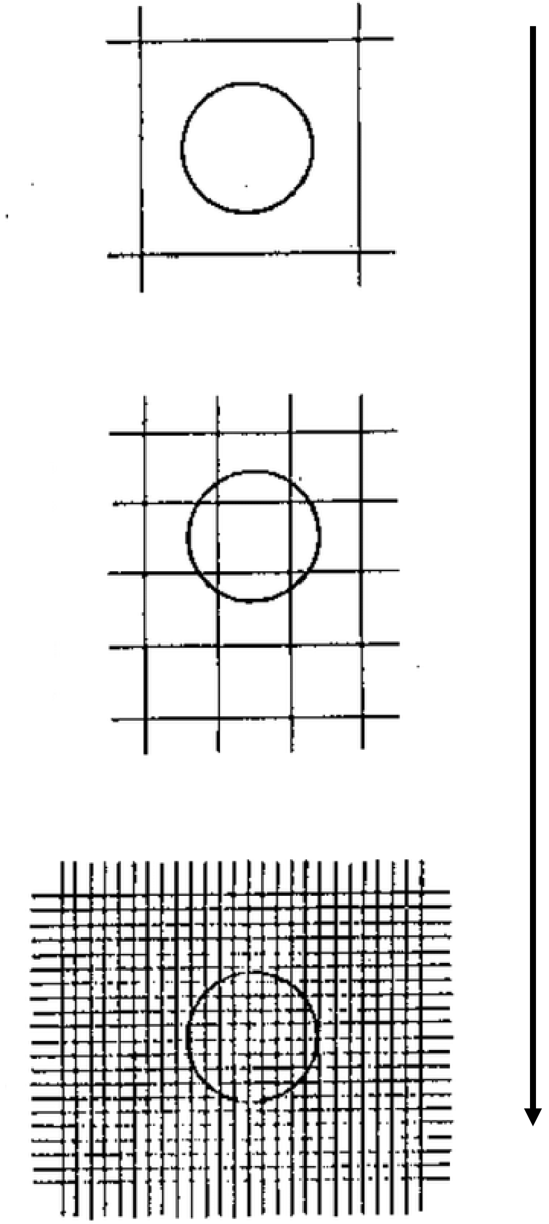
$$a^2\sigma = c\,\frac{am}{\beta}$$

Note that in the continuum limit $\beta \to \infty$:

$$\frac{m^2}{\sigma} \to 0$$

Inequivalent length scales, different continuum limits.

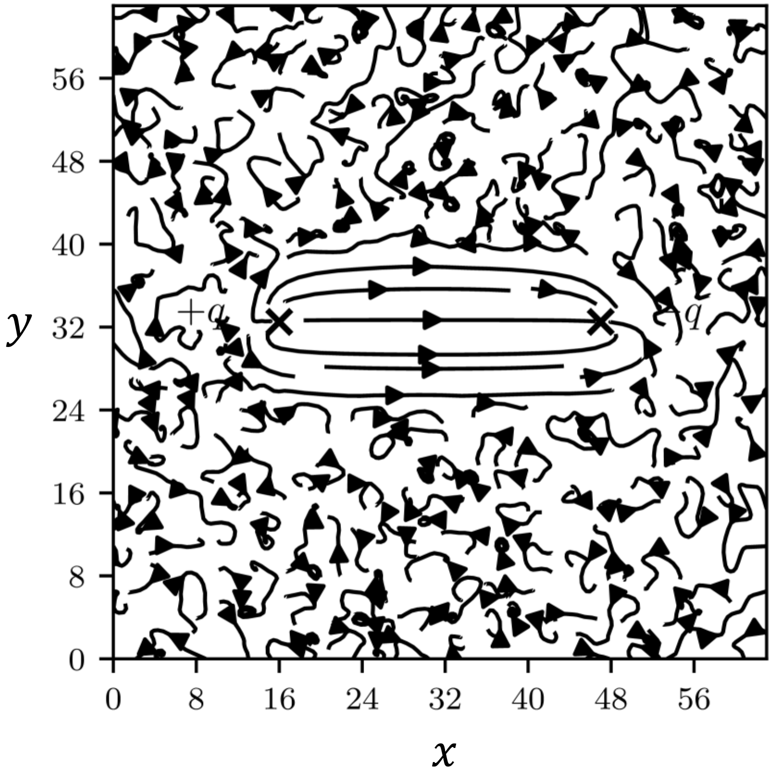
Holding σ fixed the mass **the mass goes to zero**.



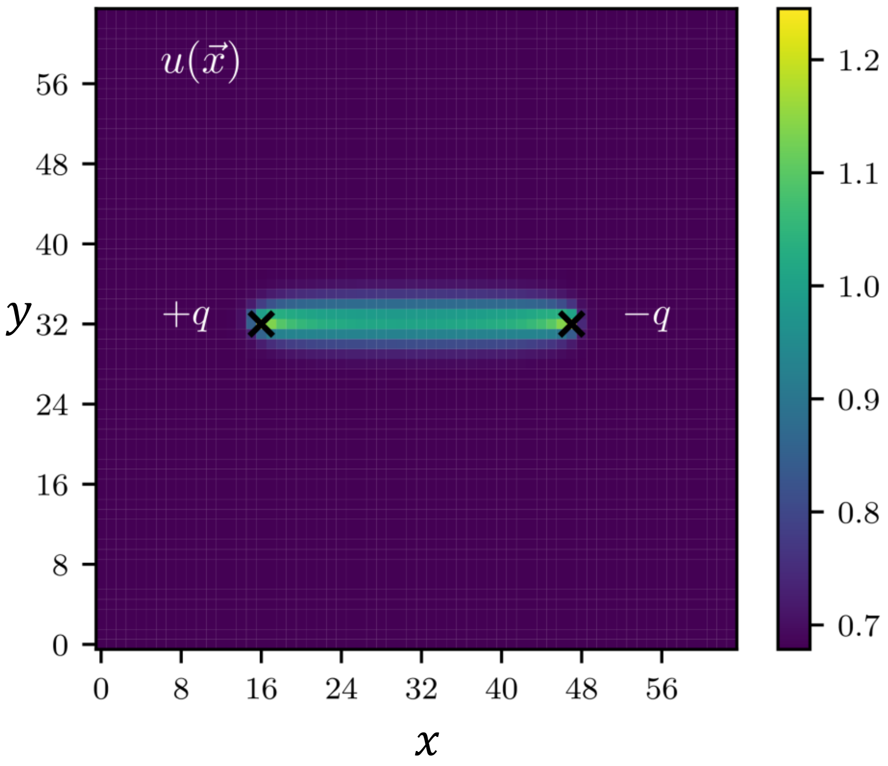
The confining string for $\mathrm{U}(1)$ in 3D

Static electrons are **confined**: (pictures are time-averaged)

Electric field:



Energy density:

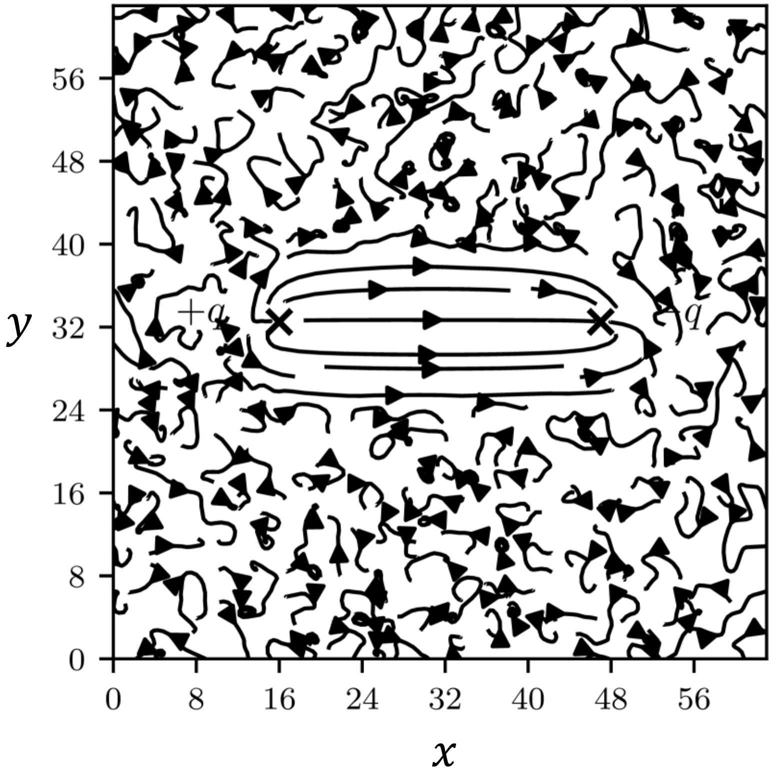


$$V(R) = \sigma R \ + O\left(\frac{1}{R}\right)$$

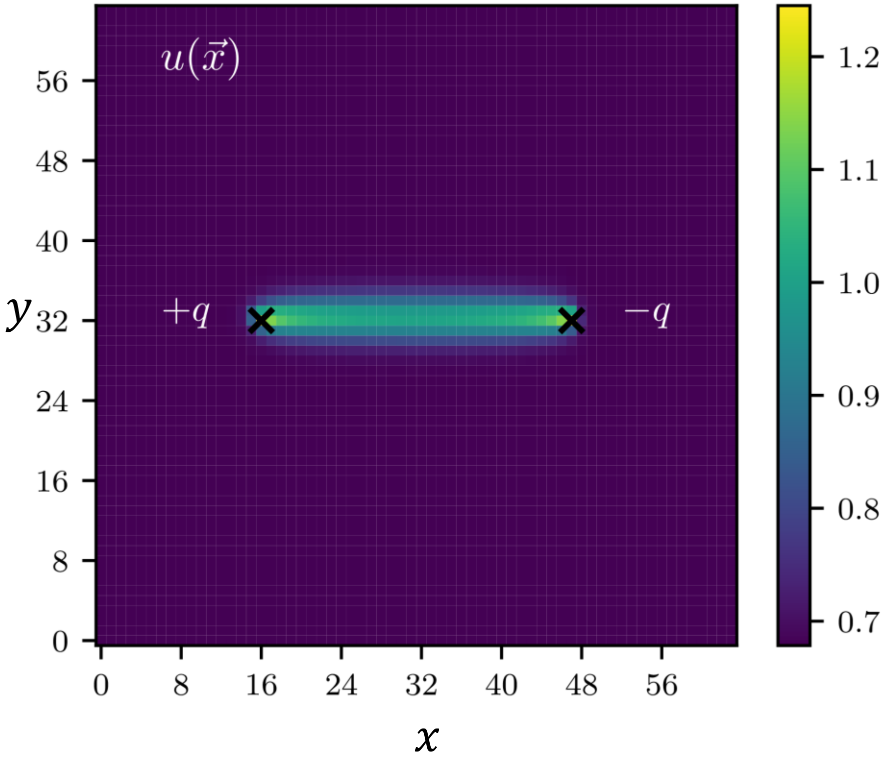
The confining string for $\mathrm{U}(1)$ in 3D

Static electrons are **confined**: (pictures are time-averaged)

Electric field:



Energy density:



$$V(R) = \sigma R + O\left(\frac{1}{R}\right)$$

Subleading corrections?

Effective string theory (EST)

[Luscher ’81, etc.]

Parametrize:
- t time
- s string arclength

Relevant field:
- $\xi(t,s)$ transverse displacement

Effective string theory (EST)

[Luscher ’81, etc.]

Parametrize:
- t time
- s string arclength

Relevant field:
- $\xi(t,s)$ transverse displacement

Effective theory: string breaks **transverse translational invariance**

$\longrightarrow$ write down derivative expansion for ξ (Goldstone bosons)

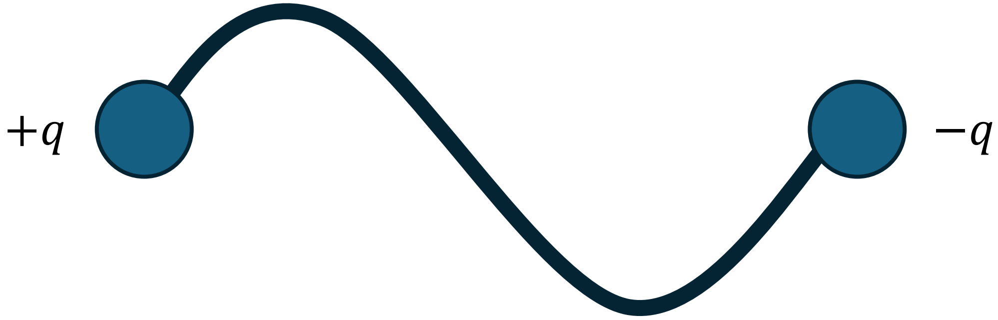
$$S_{EST} = \int dt\, ds\, \left[c_0 \partial_\mu \xi\, \partial^\mu \xi + c_1 \partial_\mu \partial^\mu \xi \partial_\nu \partial^\nu \xi + c_2 \left(\partial_\mu \xi\, \partial^\mu \xi\right)^2 + \cdots\right]$$

Effective string theory (EST)

[Luscher, Weisz, Aharony, Mayer, Gorbenko, Gliozzi, Meineri, etc]

Parametrize:
- t time
- s string arclength

Relevant field:
- $\xi(t,s)$ transverse displacement



Effective theory: string breaks **transverse translational invariance**

$\longrightarrow$ write down derivative expansion for ξ (Goldstone bosons)

$$S_{EST} = \int dt\, ds \left[c_0 \partial_\mu \xi\, \partial^\mu \xi + c_1 \partial_\mu \partial^\mu \xi \partial_\nu \partial^\nu \xi + c_2 \left(\partial_\mu \xi\, \partial^\mu \xi\right)^2 + \cdots\right] =$$

$$= \int dt\, ds \underset{\nearrow}{\sqrt{\det g}} \left[\sigma + \gamma_1 R + \gamma_2 K^2 + \gamma_3 K^4 + \cdots\right]$$

Nambu-Goto

$X^\alpha = (t, s, \xi)$
$g_{\mu\nu} = \partial_\mu X^\alpha \partial_\nu X_\alpha$ induced metric
R Ricci scalar
K extrinsic curvature

Effective string theory (EST)

$$S_{EST} = \int dt\, ds\, \sqrt{\det g}\,[\sigma + \gamma_1 R + \gamma_2 K^2 + \gamma_3 K^4 + \cdots]$$

Nambu-Goto

Concrete predictions for the ground state energy:

Nambu-Goto:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

Nambu-Goto + γ corrections:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{32\pi^6\gamma_3}{225\sigma^3 N_t^7} - \frac{64\pi^7\gamma_3}{675\sigma^4 N_t^9} - \frac{\frac{2\pi^8\gamma_3}{45} + \frac{32768\pi^{10}\gamma_5}{3969}}{\sigma^5 N_t^{11}}\,.$$

[Elias Mirò et al ’19]

Effective string theory (EST)

$$S_{EST} = \int dt\, ds\, \sqrt{\det g}\, [\sigma + \gamma_1 R + \gamma_2 K^2 + \gamma_3 K^4 + \cdots]$$

Nambu-Goto

Concrete predictions for the ground state energy:

Nambu-Goto:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

Nambu-Goto + γ corrections:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{32\pi^6 \gamma_3}{225\sigma^3 N_t^7} - \frac{64\pi^7 \gamma_3}{675\sigma^4 N_t^9} - \frac{\frac{2\pi^8\gamma_3}{45} + \frac{32768\pi^{10}\gamma_5}{3969}}{\sigma^5 N_t^{11}} \, .$$

[Elias Mirò et al ’19]

$$\tilde{\gamma}_n = \gamma_n + (-1)^{(n+1)/2} \frac{1}{n 2^{3n-1}}$$

$$\tilde{\gamma}_3 \geq 0 \, ,$$

$$\tilde{\gamma}_5 \geq 4\tilde{\gamma}_3^2 - \frac{1}{64}\tilde{\gamma}_3 \, .$$

[Dubovsky et al ’12]

Effective string theory (EST)

$$S_{EST} = \int dt\, ds \sqrt{\det g}\, [\sigma + \gamma_1 R + \gamma_2 K^2 + \gamma_3 K^4 + \cdots]$$

Nambu-Goto

Concrete predictions for the ground state energy:

Nambu-Goto:

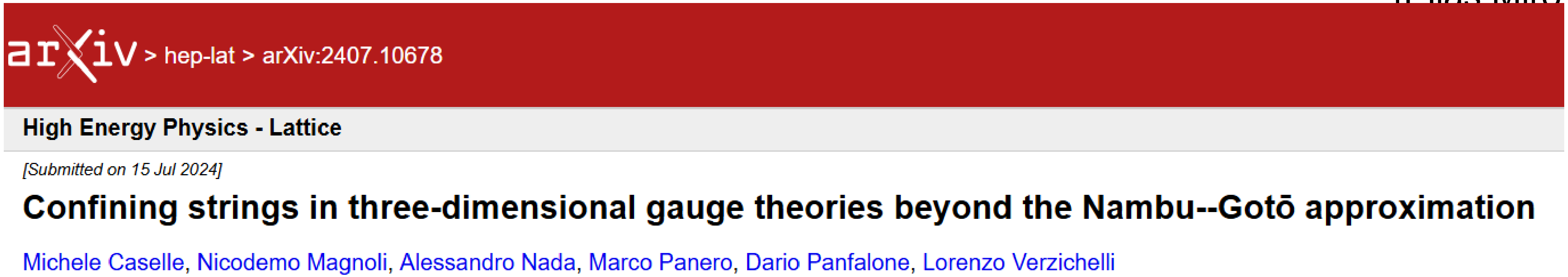
$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

Nambu-Goto + γ corrections:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{32\pi^6\gamma_3}{225\sigma^3 N_t^7} - \frac{64\pi^7\gamma_3}{675\sigma^4 N_t^9} - \frac{\frac{2\pi^8\gamma_3}{45} + \frac{32768\pi^{10}\gamma_5}{3969}}{\sigma^5 N_t^{11}} .$$

Works well for non-Abelian gauge theories:

[Elias Mirò et al '19]



arXiv > hep-lat > arXiv:2407.10678

High Energy Physics - Lattice

[Submitted on 15 Jul 2024]

Confining strings in three-dimensional gauge theories beyond the Nambu--Gotō approximation

Michele Caselle, Nicodemo Magnoli, Alessandro Nada, Marco Panero, Dario Panfalone, Lorenzo Verzichelli

Effective string theory (EST)

[Luscher '81, etc.]

Parametrize:
- t time
- s string arclength

Relevant field:
- $\xi(t, s)$ transverse displacement

Effective theory: string breaks **transverse translational invariance**

$\longrightarrow$ write down derivative expansion for ξ (Goldstone bosons)

Does EST apply to U(1) in 3D?

Key assumption: below the mass gap scale, the only degrees of freedom are the above Goldstone bosons.

This breaks down for 3D U(1). If we hold σ fixed, then $\frac{m}{\sqrt{\sigma}} \to 0$ in the continuum.

Aharony, Barel, Sheaffer '24

See:
Talk by Aharony
Wednesday 3:30pm

Recall how we computed the string tension:

$$S = \frac{1}{4\pi^2\beta}\sum_x\left[\frac{1}{2}(\nabla\Phi_x)^2 + 8\pi^2\beta\, e^{-2\pi^2 v_0\beta}\cos(\Phi_x - \overset{\text{source}}{\delta j_x})\right]$$

Semiclassical approximation:

$$Z[j] \approx \exp(-S[\Phi_{cl}]) \approx \exp(-\sigma\ \mathrm{Area})$$

Get string tension:

$$a^2\sigma = \frac{2}{\pi^2}\frac{am}{\beta}$$

Aharony, Barel, Sheaffer ‘24

See:
Talk by Aharony
Wednesday 3:30pm

Recall how we computed the string tension:

source

$$S = \frac{1}{4\pi^2\beta}\sum_x \left[\frac{1}{2}(\nabla\Phi_x)^2 + 8\pi^2\beta\, e^{-2\pi^2 v_0 \beta}\cos(\Phi_x - \delta j_x)\right]$$

Semiclassical approximation:

$$Z[j] \approx \exp(-S[\Phi_{cl}]) \approx \exp(-\sigma\ \mathrm{Area})$$

Get string tension:

$$a^2\sigma = \frac{2}{\pi^2}\frac{am}{\beta}$$

Idea: go beyond semiclassical. Write

$$\Phi = \Phi_{cl} + \hat{\Phi}$$

$$S[\Phi] = S[\Phi_{cl}] + \Delta S\big[\hat{\Phi}\big] + \cdots$$

quantize these modes

Aharony, Barel, Sheaffer '24

See:
Talk by Aharony
Wednesday 3:30pm

Recall how we computed the string tension:

source

$$S = \frac{1}{4\pi^2\beta}\sum_x \left[\frac{1}{2}(\nabla\Phi_x)^2 + 8\pi^2\beta\, e^{-2\pi^2 v_0 \beta}\cos(\Phi_x - \delta j_x)\right]$$

$$\downarrow$$

Semiclassical approximation:

$$Z[j] \approx \exp(-S[\Phi_{cl}]) \approx \exp(-\sigma\,\text{Area})$$

Get string tension:

$$a^2\sigma = \frac{2}{\pi^2}\frac{am}{\beta} - \frac{(am)^2}{4\pi}$$

Idea: go beyond semiclassical. Write

$$\Phi = \Phi_{cl} + \widehat{\Phi}$$

$$S[\Phi] = S[\Phi_{cl}] + \Delta S[\widehat{\Phi}] + \cdots$$

quantize these modes

$$E_0(N_t) = \sigma N_t - \frac{\pi}{6N_t} + \frac{1}{\pi N_t}\mathrm{Li}_2(e^{-mN_t})$$

this is the mass gap

Possible fit forms

1) Nambu-Goto: $$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

2) Nambu-Goto + γ corrections: $$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{32\pi^6\gamma_3}{225\sigma^3 N_t^7} - \frac{64\pi^7\gamma_3}{675\sigma^4 N_t^9} - \frac{\frac{2\pi^8\gamma_3}{45} + \frac{32768\pi^{10}\gamma_5}{3969}}{\sigma^5 N_t^{11}} .$$

3) Rigid string: $$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{\widetilde{m}}{\pi} \sum_{n=1}^{\infty} \frac{K_1(n\widetilde{m}N_t)}{n}$$

4) Aharony, Barel, Sheaffer: $$E_0(N_t) = \sigma N_t - \frac{\pi}{6N_t} + \frac{1}{\pi N_t}\mathrm{Li}_2(e^{-mN_t})$$

Possible fit forms

Expansion in $1/N_t$

1) Nambu-Goto:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

2) Nambu-Goto + γ corrections:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{32\pi^6\gamma_3}{225\sigma^3 N_t^7} - \frac{64\pi^7\gamma_3}{675\sigma^4 N_t^9} - \frac{\frac{2\pi^8\gamma_3}{45} + \frac{32768\pi^{10}\gamma_5}{3969}}{\sigma^5 N_t^{11}} .$$

3) Rigid string:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{\tilde{m}}{\pi} \sum_{n=1}^{\infty} \frac{K_1(n\tilde{m}N_t)}{n}$$

4) Aharony, Barel, Sheaffer:

$$E_0(N_t) = \sigma N_t - \frac{\pi}{6N_t} + \frac{1}{\pi N_t}\mathrm{Li}_2(e^{-mN_t})$$

Expansion in $\frac{m^2}{\sigma} \sim (am)\beta$

Ground state energy of the string

Based on: M. Caselle, A. Mariani, *The finite temperature ground state energy of the confining string in three-dimensional U(1) gauge theory*, JHEP 06 (2025) 010. arXiv:2503.05354.

Ground state energy of the string

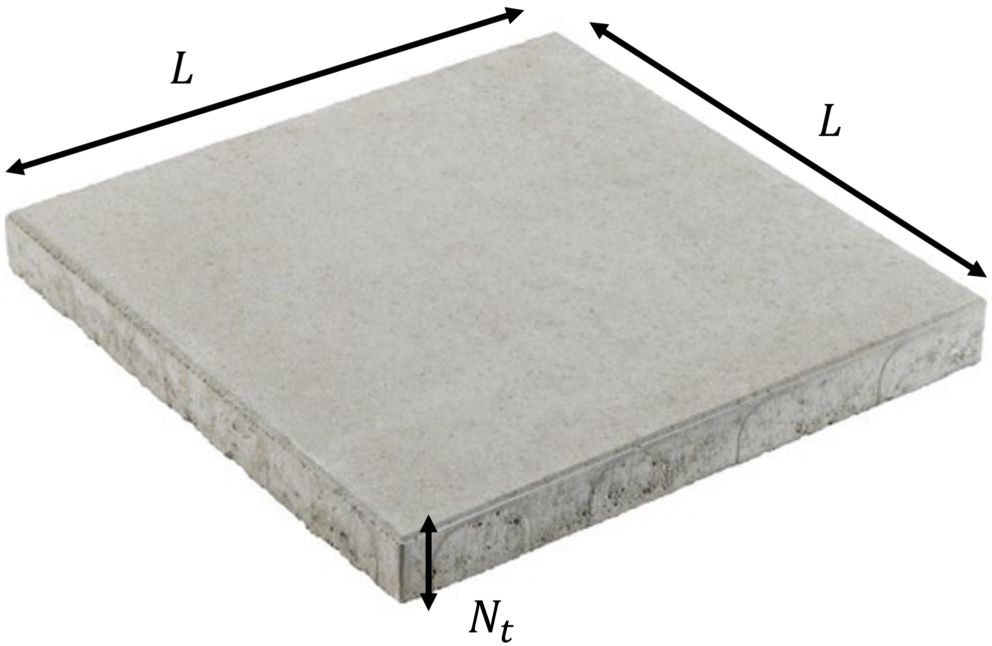
Based on: M. Caselle, A. Mariani, *The finite temperature ground state energy of the confining string in three-dimensional U(1) gauge theory*, JHEP 06 (2025) 010. arXiv:2503.05354.

Quantum-classical correspondence gives

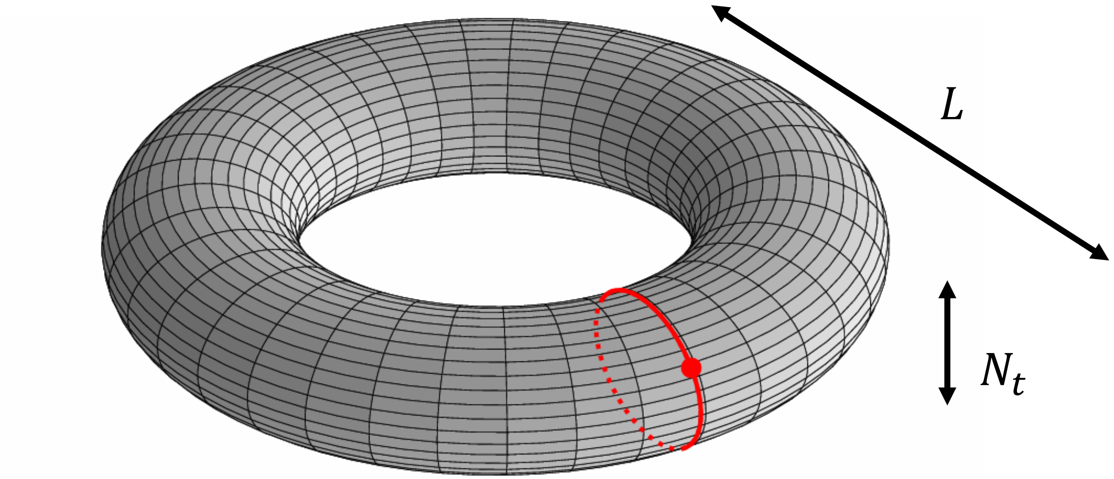
$$T = \frac{1}{aN_t}$$

with $N_t \ll L$.

The lattice looks like this:



- $N_t =$ no. lattice sites in the **time** direction
- $L =$ no. lattice sites in the **space** directions



Ground state energy of the string

Based on: M. Caselle, A. Mariani, *The finite temperature ground state energy of the confining string in three-dimensional U(1) gauge theory*, JHEP 06 (2025) 010. arXiv:2503.05354.

Quantum-classical correspondence gives

$$T = \frac{1}{aN_t}$$

with $N_t \ll L$.

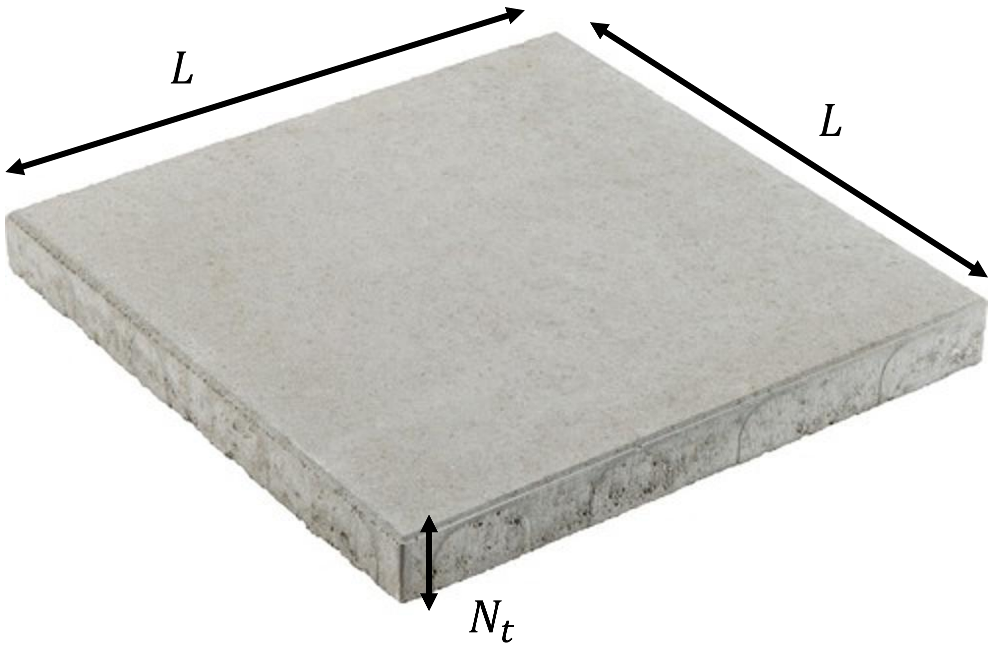
We work in the **closed string channel**, we measure the string ground state energy:

$$\boxed{E_0(N_t) = \sigma N_t + \cdots}$$

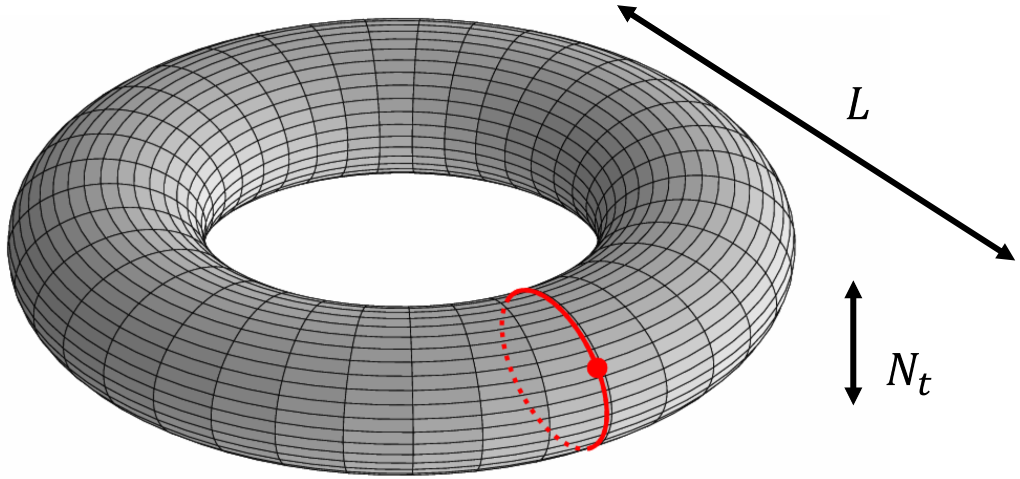
(not the interquark potential)

This avoids boundary contributions.

The lattice looks like this:



- $N_t =$ no. lattice sites in the **time** direction
- $L =$ no. lattice sites in the **space** directions



Ground state energy of the string

Based on: M. Caselle, A. Mariani, *The finite temperature ground state energy of the confining string in three-dimensional U(1) gauge theory*, JHEP 06 (2025) 010. arXiv:2503.05354.

Quantum-classical correspondence gives

$$T = \frac{1}{aN_t}$$

with $N_t \ll L$.

We work in the **closed string channel**, we measure the string ground state energy:

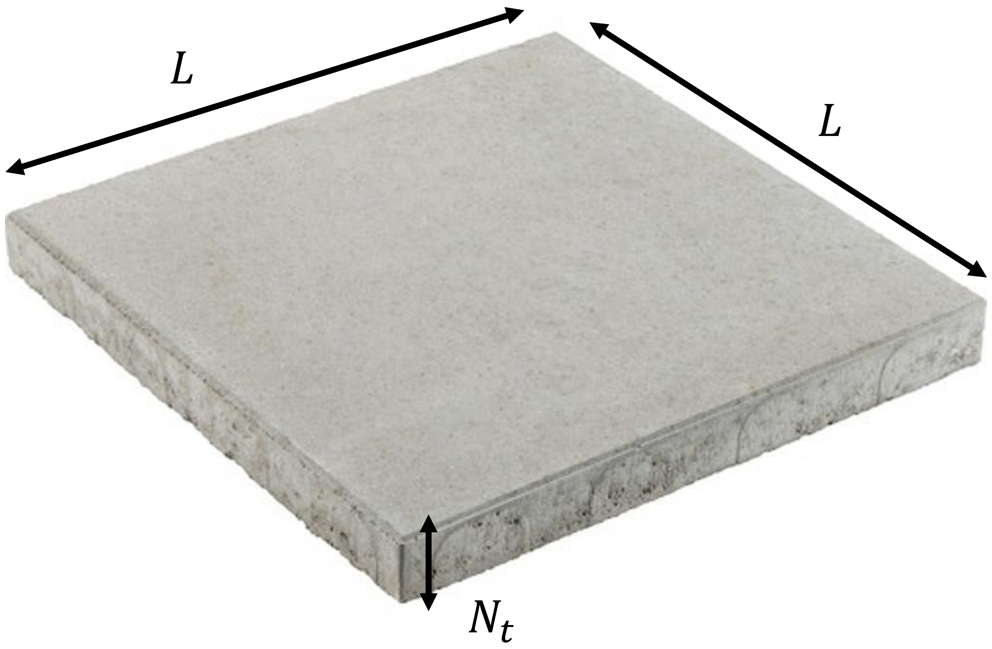
$$\boxed{E_0(N_t) = \sigma N_t + \cdots}$$

Subleading corrections?

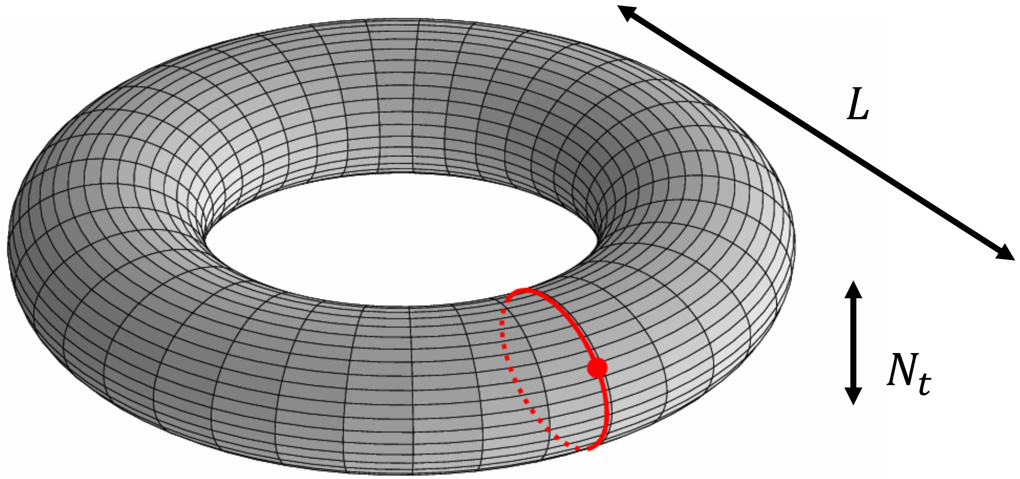
(not the interquark potential)

This avoids boundary contributions.

The lattice looks like this:



- $N_t =$ no. lattice sites in the **time** direction
- $L =$ no. lattice sites in the **space** directions



Numerical calculation

1) Compute Polyakov loop correlator

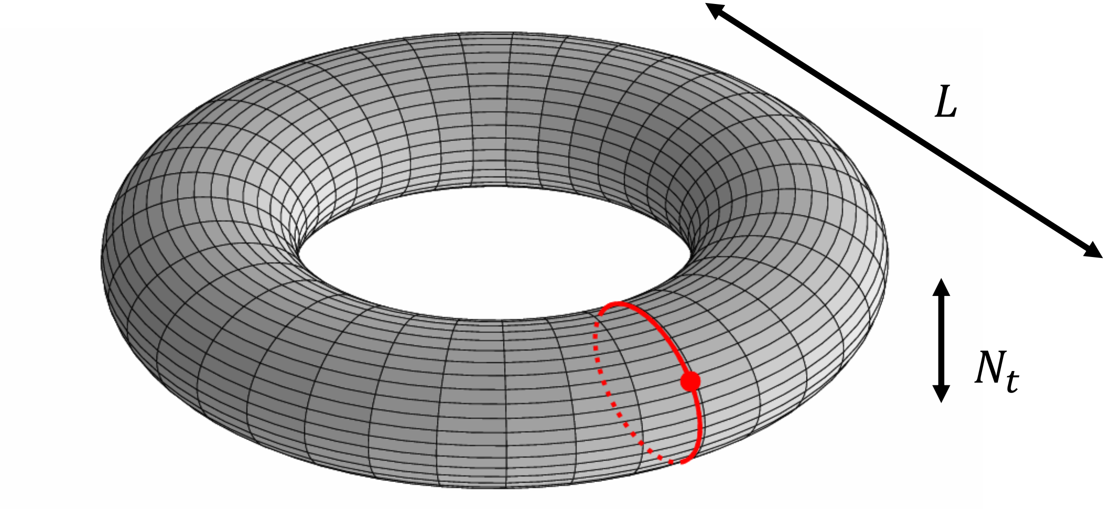
$$\langle P(x)P(x+R)^* \rangle$$

2) Extract energy by fitting to theoretical prediction:

$$\langle P(x)P(x+R)^* \rangle \sim K_0(E_0 R)$$

Get $E_0(N_t)$.

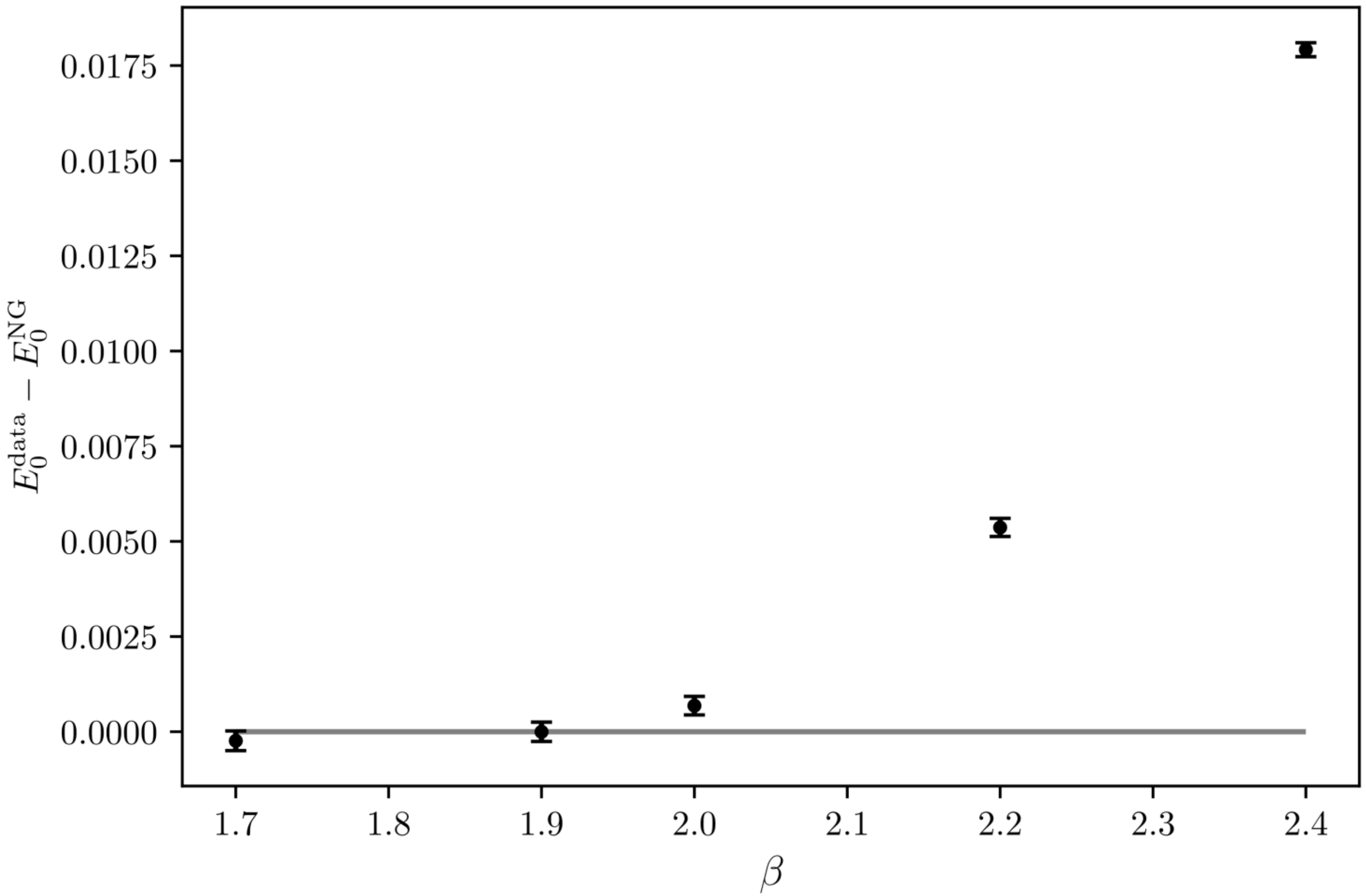
We use the **snake algorithm**.



Polyakov loop:

$$P(x) = \prod_{t=0}^{N_t-1} U_0(x,t)$$

Comparing Nambu-Goto with numerical data: fixed $N_t = 10$

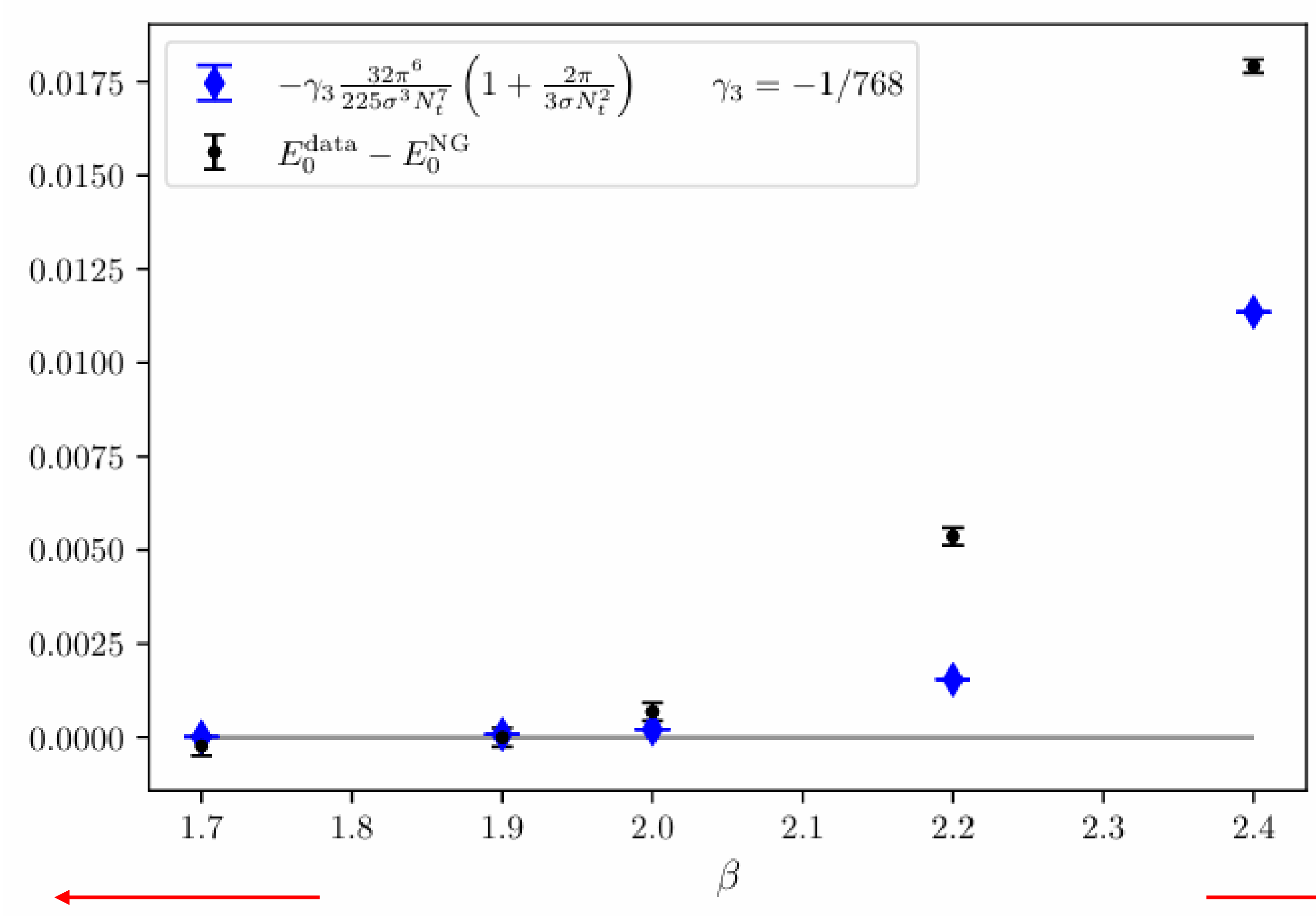


- Agreement at small β, disagreement at larger β.
- Difference is positive, i.e. data lies above Nambu-Goto.
- Therefore incompatible with rigid string.
- Can it be accounted for by γ_3?

← "small m"

"large m" →

Comparing Nambu-Goto with numerical data: fixed $N_t = 10$



“small m”

“large m”

- Agreement at small β, disagreement at larger β.
- Difference is positive, i.e. data lies above Nambu-Goto.
- Therefore incompatible with rigid string.
- Can it be accounted for by γ_3?
- Also incompatible with γ_3.

Possible fit forms

✓ small β

✗ larger β

1) Nambu-Goto:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

2) ✗ Nambu-Go - γ corrections:

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{32\pi^6\gamma_3}{225\sigma^3 N_t^7} - \frac{64\pi^7\gamma_3}{675\sigma^4 N_t^9} - \frac{\frac{2\pi^8\gamma_3}{45} + \frac{32768\pi^{10}\gamma_5}{3969}}{\sigma^5 N_t^{11}} \, .$$

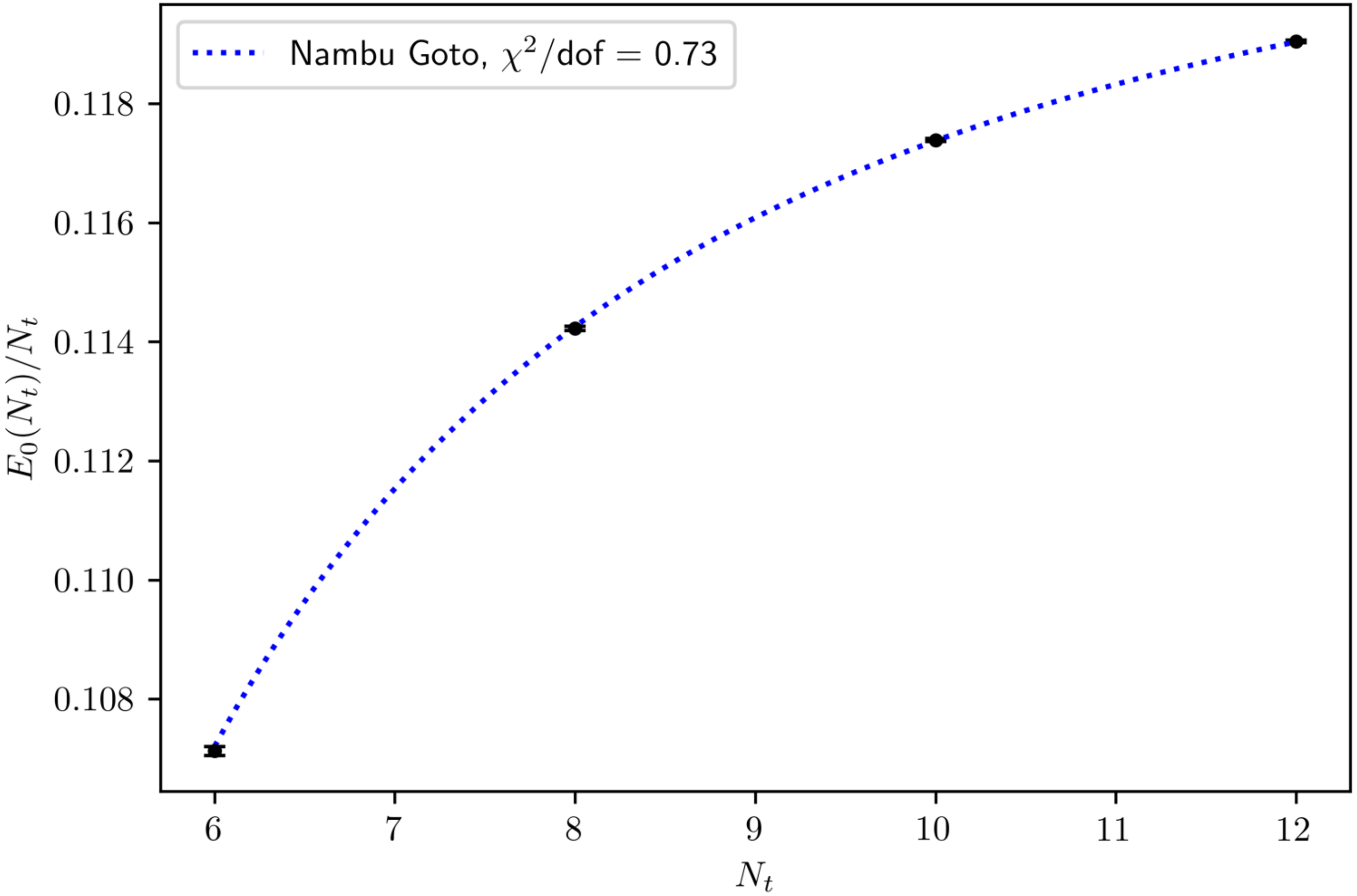
3) ✗ Rigid strin

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} - \frac{\widetilde{m}}{\pi} \sum_{n=1}^{\infty} \frac{K_1(n\widetilde{m}N_t)}{n}$$

4) Aharony, Barel, Sheaffer:

$$E_0(N_t) = \sigma N_t - \frac{\pi}{6N_t} + \frac{1}{\pi N_t}\mathrm{Li}_2(e^{-mN_t})$$

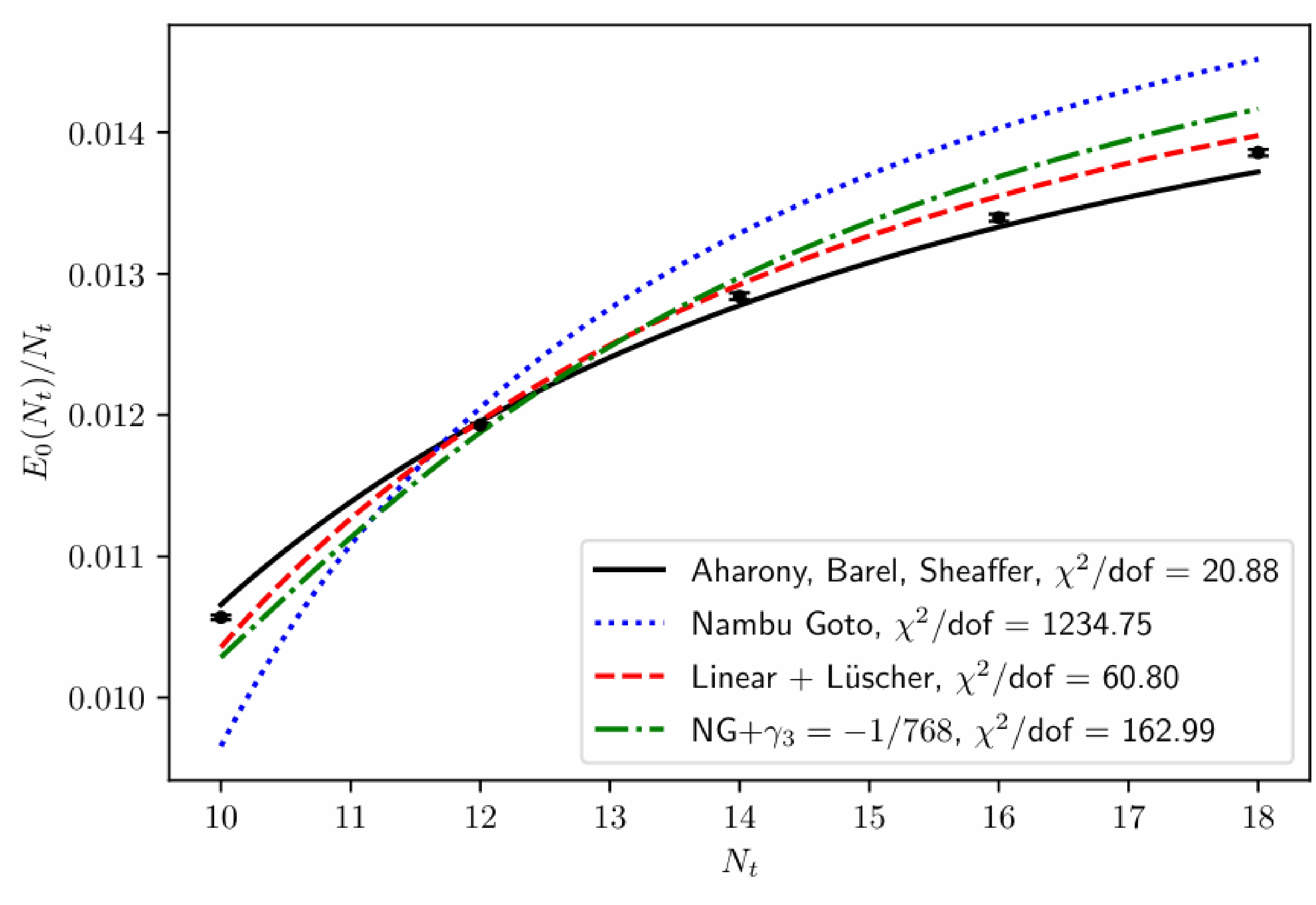
Comparing with numerical data: fixed $\beta = 1.7$



- Fit $E_0(N_t)$ vs N_t keeping the string tension as a fit parameter.

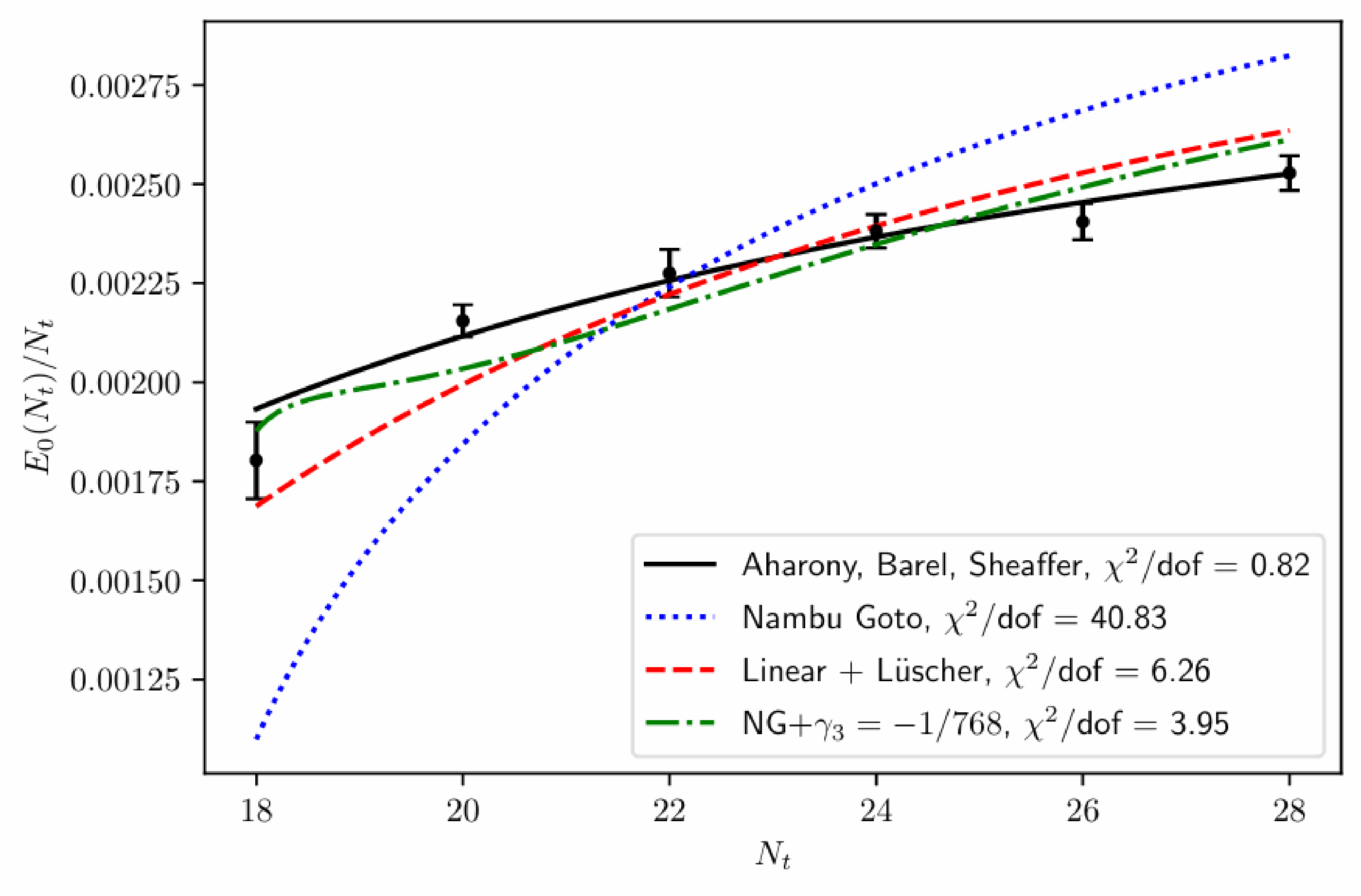
- For this β, Nambu-Goto works perfectly (as expected).

Comparing with numerical data: fixed $\beta = 2.4$



- No agreement

Comparing with numerical data: fixed $\beta = 3.0$



- Aharony, Barel, Sheaffer is clearly preferred.

Summary

3D U(1) has remarkably different confinement properties. Effective string theory does not always apply.

Keeping the string tension fixed, we find good agreement with Nambu-Goto at low β, and with Aharony, Barel, Sheaffer at large β.