Intrinsic Width of the Flux Tube in 2 + 1 dimensions

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Trento ECT*, 25 / 8 / 2025

Outline

1. Introduction
   - Effective String Theory
   - Flux tube profile with Stochastic Normalizing Flows
   - Flux tube profile on the lattice
2. Low temperature results
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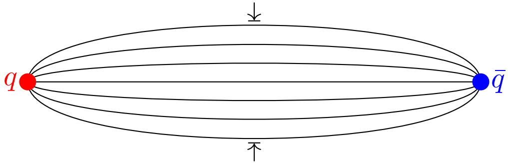
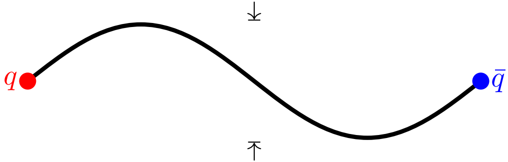
   - Non-Gaussianity in SU(2)
   - Intrinsic width
3. High temperature results
   - The Svetitsky-Yaffe mapping
   - The temperature-dependent intrinsic width
4. Conclusions

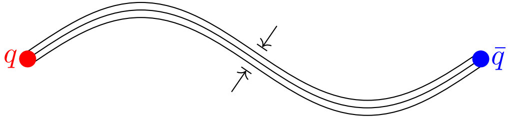
Flux Tube as a string



Chromo-Electric and Chromo-Magnetic fields concentrated in a "tube"



In Effective String Theory (EST) vibration of the string $\Longrightarrow$ finite width of flux tube

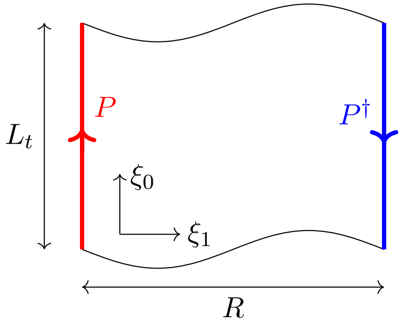


Intrinsic width:
width of the flux tube NOT due to string fluctuations

Predictions from Effective String Theory

The width of the flux tube can be computed in EST

$$w^2_{\text{EST}} \sim \int \mathcal{D}X(\xi_0, \xi_1) X^2 e^{-S_{\text{EST}}[X]}$$



At **low temperature** Lüscher-Munster-Weisz Nucl. Phys. B **180** (1981):

$$\sigma\, w^2_{\text{EST}} = \frac{1}{2\pi} \log\left(\frac{R}{R_0}\right) + \dots$$

At **high temperature** Caselle-Allais [0812.0284]:

$$\sigma\, w^2_{\text{EST}} = \frac{R}{4L_t} + \frac{1}{2\pi} \log\left(\frac{L_t}{L_0}\right) + \dots$$

No prediction about an intrinsic width, but always involve a **length scale** (R_0 and L_0)

More on EST in many talks!

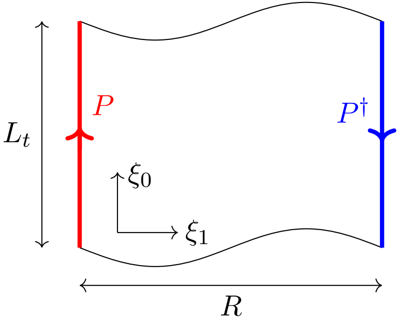
The string profile with Stochastic Normalizing Flows

$$\mathcal{Z}_{\text{EST}} = \int \mathcal{D}X(\xi_0, \xi_1) e^{-S_{\text{EST}}}$$

Given $S_{\text{EST}}[X]$ it is in principle possible to sample configurations of X

For the Nambu-Gotō action, standard Monte Carlo method are not efficient

However **Stocastic Normalizing Flows** (SNF) have proven effective
More on SNF in the talk by Elia Cellini



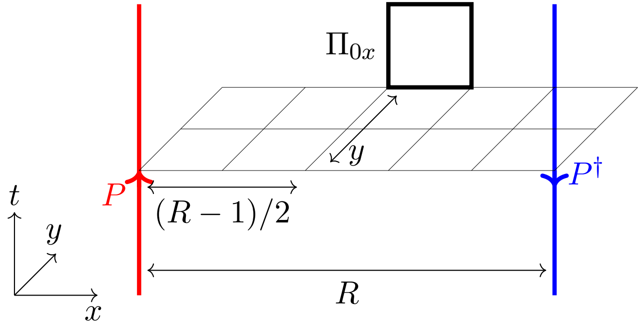
The profile of the string was studied with this method (Caselle-Cellini-Nada [2409.15937])
No deviation from a Gaussian profile were found for pure Nambu-Gotō action
A **positive Binder cumulat** was found adding terms beyond Nambu-Gotō

The flux tube profile on the lattice

We study a three point function that reproduces the **Chromo-Electric field** in the presence of a **quark-anti-quark** pair

Lattice operators:

- **Polyakov loop**
- **Polyakov loop (inverse)**
- **Plaquette**



We consider the field midway between the two sources

Strongest signal:
temporal-longitudinal plaquette

$$\text{profile}(y) = \frac{\langle P\, P^\dagger\, \Pi \rangle}{\langle P\, P^\dagger \rangle} - \langle \Pi \rangle$$

The case of U(1)

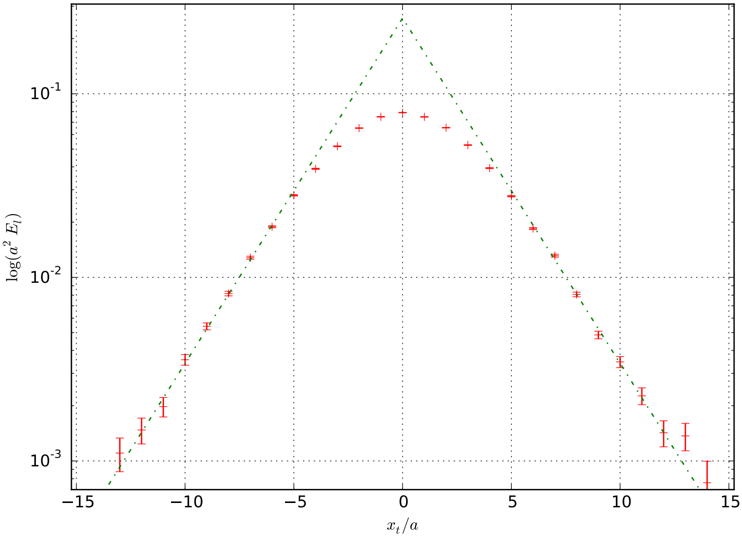
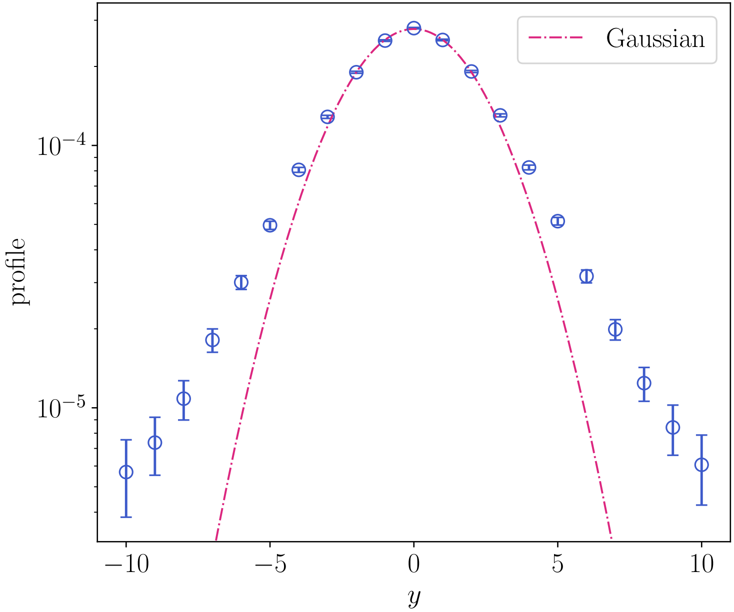


Figure from Caselle-Panero-Vadacchino [1601.07455]

Evident **deviation from a Gaussian**

This deviation from a Gaussian profile was recently explained (Aharony-Barel-Sheaffer [2412.01313])

Non-Gaussianity of the flux tube



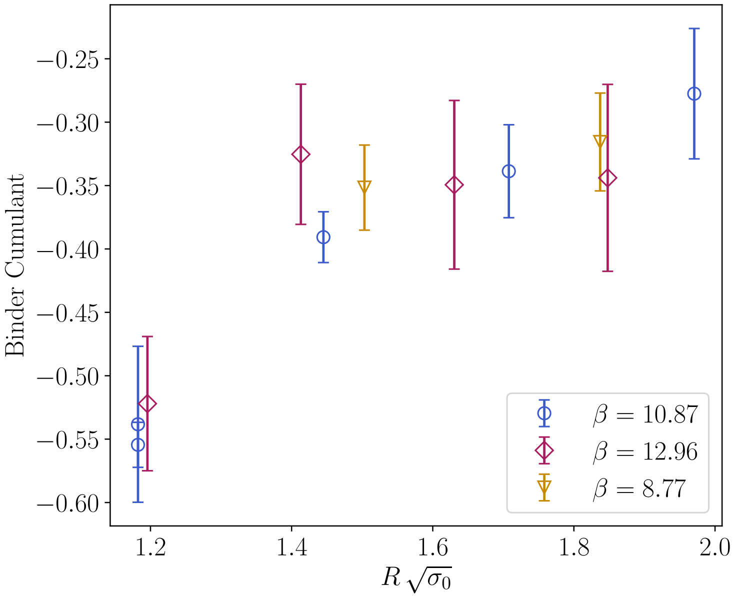
SU(2) in $D = 2 + 1$

$$\beta = 10.87$$
$$L_t = 30$$
$$T/T_c = 0.23$$

$$a\sqrt{\sigma} = 0.01728(23)$$
$$R_c/a = 3.9$$
$$R/a = 9$$

Data obtained with a 2-level algorithm as in Gliozzi-Pepe-Wiese [1010.1373]

Estimation of the Binder cumulant



Interpolating our data, we extracted the "Binder cumulant":

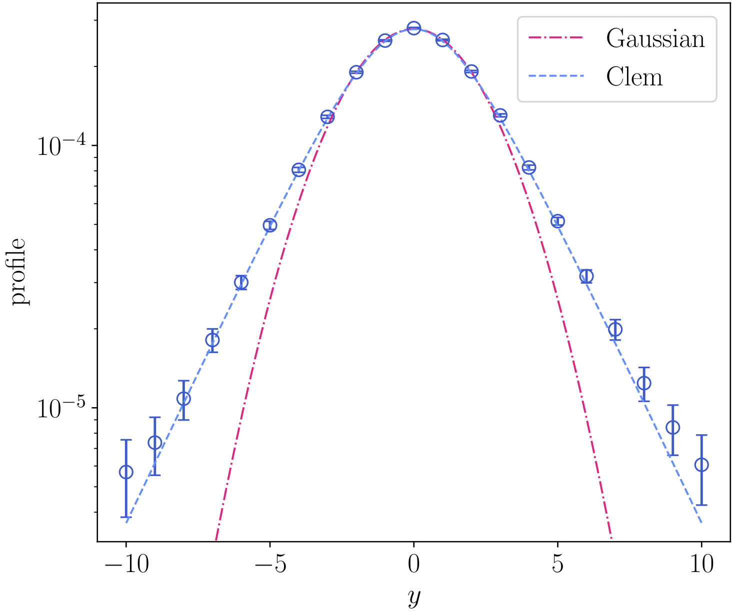
$$1 - \frac{\mu_4}{3{\mu_2}^2}$$

where $\mu_n = \int dy\, p(y)\, y^n$

A Gaussian would have vanishing Binder cumulant

A **negative Binder cumulant** is incompatible with the results from direct simulations of the EST

The Clem formula



SU(2) in $D = 2 + 1$

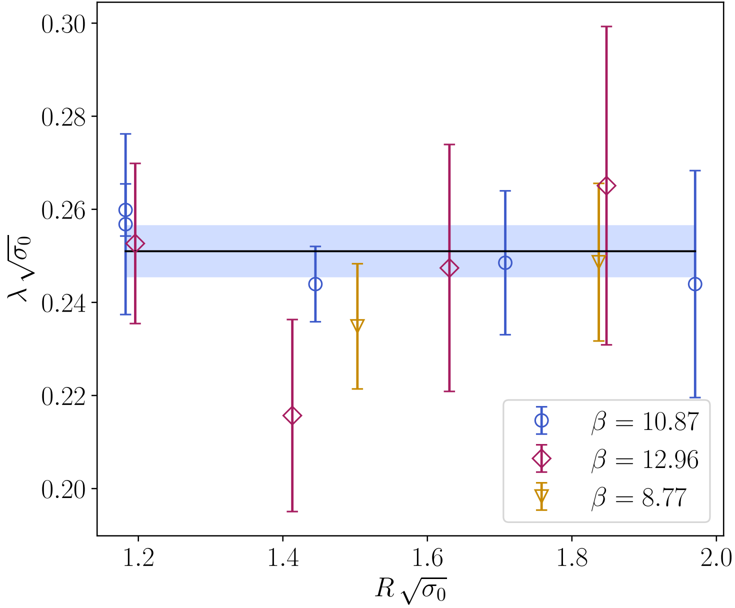
$$\beta = 10.87$$
$$L_t = 30$$
$$T/T_c = 0.23$$

$$a\sqrt{\sigma_0} = 0.01728(23)$$
$$R_c/a = 3.9$$
$$R/a = 9$$

Clem:

$$A\, K_0\left(\sqrt{y^2 + \xi^2}/\lambda\right)$$

The intrinsic width



$$A\ K_0\left(\sqrt{y^2+\xi^2}/\lambda\right)$$

We analyzed profiles for different values of R and of the lattice spacing

The value of λ **is constant**
$\lambda\sqrt{\sigma_0} = 0.251(6)$

In this plot:
data at $T/T_c = 0.23$

we obtain a compatible result at $T/T_c = 11$

Logarithmic broadening and the R_0 scale

$$A\,K_0\left(\sqrt{y^2+\xi^2}/\lambda\right)$$

The total width of the flux tube is an increasing function of both λ and ξ

ξ increase with the distance R between the sources, driving the logarithmic broadening:

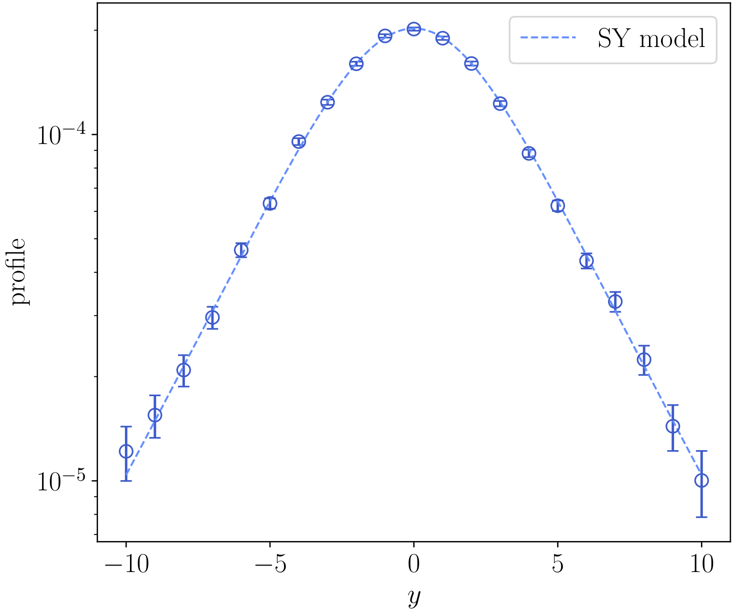
$$\sigma\,w^2_{\mathsf{EST}} = \frac{1}{2\pi}\log\left(\frac{R}{R_0}\right) + \ldots$$

This was studied numerically (Gliozzi-Pepe-Wiese [1006.2252]), finding $R_0\sqrt{\sigma_0} = 0.364(3)$

A numerical coincidence?

Extrapolating the width of the string at the smallest meaningful distance in the EST, R_c, one finds $w_{\mathsf{min}} = 0.233(3)$, which is close to our estimation of $\lambda = 0.251(6)$

High temperature



SU(2) in $D = 2 + 1$

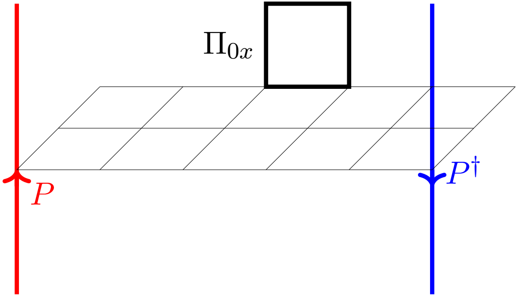
$$\beta = 10.87$$
$$L_t = 10$$
$$T/T_c = 0.70$$

$$a\sqrt{\sigma_0} = 0.01728(23)$$
$$R_c/a = 3.9$$
$$R/a = 11$$

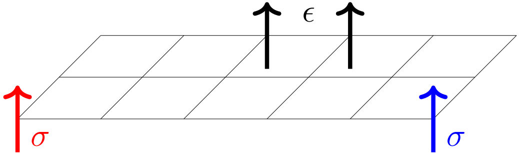
The fitting model is based on the **Svetitsky-Yaffe mapping**

For $\mathrm{SU}(2)$ in $D = 2 + 1$, the deconfinement phase transition is second order $\implies$
Universality allows to map correlators of the gauge theory into correlators of the Ising model

This is a particular case of the Svetitsky-Yaffe (SY) mapping,
from a gauge theory in D dimensions to a spin model in $D - 1$ dimensions



- the Polyakov loops are mapped into spins
- the Plaquette is mapped into the energy



More about the SY mapping in the talk by Dario Panfalone

SY prediction for the profile

Knowing the spin-spin-energy correlator Caselle-Grinza [1207:6523]
we derive a model for the profile at high temperature $T \lesssim T_c$

$$\mathrm{profile}(y) = A \frac{\pi\, R}{2\, K_0\left(\frac{1}{2}R/\lambda\right)}\; \frac{\exp\left(-\sqrt{y^2 + R^2/4}/\lambda\right)}{y^2 + R^2/4}$$

Where R is not a free parameter, but the distance between the Polyakov loops
A and λ should not depend on R

One less free parameter than the Clem formula $A(R)\; K_0\left(\sqrt{y^2 + \xi^2}/\lambda\right)$

SY mapping predicts λ to be the same length scale such that

$$\langle P(0)\, P^\dagger(R)\rangle \sim \exp\left(-\tfrac{1}{2}R/\lambda\right)$$

The intrinsic width in the SY prediction

$$\text{profile}(y) = A \frac{\pi\, R}{2\, K_0\left(\frac{1}{2} R/\lambda\right)}\; \frac{\exp\left(-\sqrt{y^2+R^2/4}/\lambda\right)}{y^2+R^2/4}$$

$$\langle P(0)\, P^\dagger(R)\rangle \sim \exp\left(-\tfrac{1}{2} R/\lambda\right) = \exp(-\sigma(L_t)\, L_t\, R),$$

Where $\sigma(L_t)$ is the temperature dependent string tension

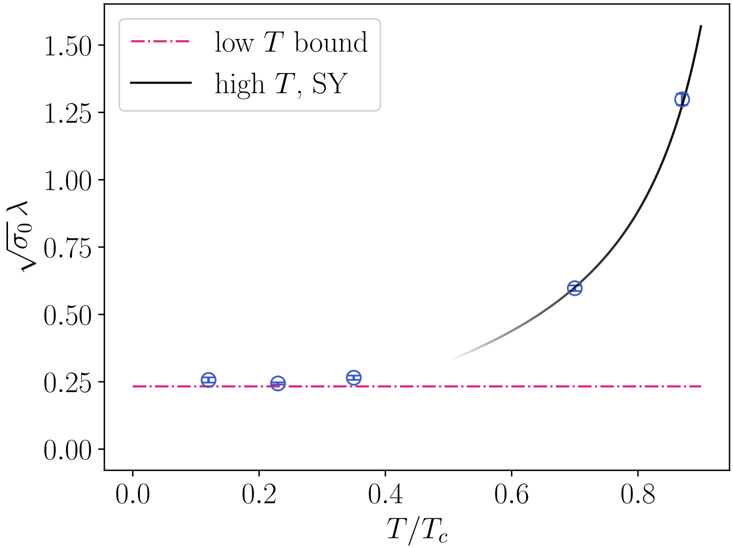
$$\sigma(L_t) = \sigma_0 \sqrt{1 - \frac{\pi}{3\sigma_0 L_t^2}} + \text{Beyond Nambu-Got\=o}$$

(About the corrections Beyond Nambu-Gotō, see Caselle *et al.* [2109.06212] and [2407.10678])

We obtain a prediction about the intrinsic width

$$\lambda = \frac{1}{2\, \sigma(L_t)\, L_t}$$

The temperature-dependent intrinsic width



At low T: the intrinsic width appears **constant**

Numerical results are compatible with the width extrapolated at R_c

Approaching the critical temperature: $\sigma(L_t) \to 0$

The SY predicts a **divergence in λ**

Some remarks

- The profile of the flux tube deviates sensitively from a Gaussian
- The deviation seems to be characteristic of the gauge theory, not of the EST
- At every temperature we analyzed, we identified an exponential tails of the profile
- The characteristic length of the tail does not depend on the distance between the sources
- We identify this scale as the intrinsic width of the flux tube
- At low temperature the intrinsic width is a constant
- At high temperature it diverges according to the SY mapping prediction

Outlook and WIP

- Extend this work to other gauge groups ($\mathrm{SU}(N)$)

- Compare the large N result for the intrinsic width to holographic predictions

- Study with similar technique a "baryon" instead of a "meson"

- Extend the study to theories in $D = 3 + 1$

Some Questions

- Profile of a different string? *E.g.* the rigid string?
- Prediction for the profile from holography? Finite N?
- Possible analytical understanding of the intrinsic width for $\mathrm{SU}(2)$?

Thank you for your attention!