The spectrum of open confining strings in the large-N_c limit

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Bridging analytical and numerical methods for quantum field theory

Agenda

Motivation

String Theory

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Our Goals

- Studying the spectrum of open fluxtubes in large N_c limit and look for deviation from string models.
- Looking for possible world-sheet axion states.
- Why large N_c? how large? $N_c = 3, 4, 5, 6$.
-

Eight very excited spectra and one possible axion in SU(3) lattice gauge theory

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We compute the spectra of flux tubes formed between a static quark antiquark pair up to a significant number of excitations and for eight symmetries of the flux tubes, up to Δ_u, using pure $SU(3)$ gauge lattice QCD in 3+1 dimensions. To accomplish this goal, we use a large set of appropriate operators, an anisotropic tadpole improved action, smearing techniques, and solve a generalized eigenvalue problem. Moreover, we compare our results with the Nambu-Goto string model to evaluate possible tensions which could be a signal for novel phenomena. Especially, we provide evidence for the coupling of a massive particle, say an axion, to the Σ_g^-, Σ_u^-, and Σ_u^{-*} flux tube with approximate masses $2.25\sqrt{\sigma}$, $1.85\sqrt{\sigma}$, $3.30\sqrt{\sigma}$, respectively.

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Nambu-Goto String model

$$S = -\sigma \int \mathrm{d}^2 x \sqrt{-\det h_{\alpha\beta}}, \quad h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu \tag{1}$$

$$E_N^{\mathrm{NG}}(R) = \sqrt{\sigma^2 R^2 + 2\pi\sigma \left(N - \frac{D-2}{24} \right)}, \tag{2}$$

$$V(R) = \sigma R - \frac{\pi}{12R} + \dots, \tag{3}$$

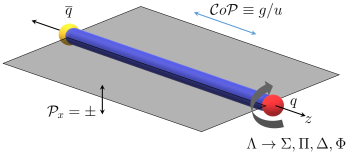
Effective string models

$$S_{EST} = \sigma RT + \frac{\sigma}{2} \int \mathrm{d}^2 x \partial_\alpha X_i . \partial^\alpha X^i + \dots \tag{4}$$

$$E_N(R) \quad = \quad E_N^{\mathrm{NG}}(R) + \frac{\bar{b}_2 \pi^3 \sqrt{\sigma}}{(R\sqrt{\sigma})^4} \left(B_N - \frac{D-2}{60} \right) + \dots \tag{5}$$

[Bonati *et al.*(2021)Bonati, Caselle, and Morlacchi]

Symmetries of the open fluxtube



String states in term of eigenmode are expressed:

$$\prod_{m=1}^{\infty} \left((a_{m^+})^{n_{m^+}} (a^{\dagger}_{m^-})^{n_{m^-}} \right) |0\rangle, \tag{6}$$

$$N = \sum_{m=1}^{\infty} m(n_{m^+} + n_{m^-}), \quad \Lambda = \left| \sum_{m=1}^{\infty} (n_{m^+} - n_{m^-}) \right|, \quad \eta_{CP} = (-1)^N. \tag{7}$$

Excitation	Symmetry	State	State
$N=0$	Σ_g^+	$\lvert 0\rangle$	
$N=1$	Π_u	$a_{1+}^\dagger \lvert 0\rangle$	$a_{1-}^\dagger \lvert 0\rangle$
$N=2$	$\Sigma_g^{+\prime}$	$a_{1+}^\dagger a_{1-}^\dagger \lvert 0\rangle$	
	Π_g	$a_{2+}^\dagger \lvert 0\rangle$	$a_{2-}^\dagger \lvert 0\rangle$
	Δ_g	$(a_{1+}^\dagger)^2 \lvert 0\rangle$	$(a_{1-}^\dagger)^2 \lvert 0\rangle$
$N=3$	Σ_u^+	$(a_{1+}^\dagger a_{2-}^\dagger + a_{1-}^\dagger a_{2+}^\dagger)\lvert 0\rangle$	
	Σ_u^-	$(a_{1+}^\dagger a_{2-}^\dagger - a_{1-}^\dagger a_{2+}^\dagger)\lvert 0\rangle$	
	Π_u'	$a_{3+}^\dagger \lvert 0\rangle$	$a_{3-}^\dagger \lvert 0\rangle$
	Π_u''	$(a_{1+}^\dagger)^2 a_{1-}^\dagger \lvert 0\rangle$	$a_{1+}^\dagger (a_{1-}^\dagger)^2 \lvert 0\rangle$
	Δ_u	$a_{1+}^\dagger a_{2+}^\dagger \lvert 0\rangle$	$a_{1-}^\dagger a_{2-}^\dagger \lvert 0\rangle$
	Φ_u	$(a_{1+}^\dagger)^3 \lvert 0\rangle$	$(a_{1-}^\dagger)^3 \lvert 0\rangle$

Excitation	Symmetry	State	State
$N=4$	$\Sigma_g^{+\prime}$	$a_{2+}^\dagger a_{2-}^\dagger \|0\rangle$	
	$\Sigma_g^{+\prime\prime}$	$(a_{1+}^\dagger)^2 (a_{1-}^\dagger)^2 \|0\rangle$	
	$\Sigma_g^{+(\mathrm{iv})}$	$(a_{1+}^\dagger a_{3-}^\dagger + a_{1-}^\dagger a_{3+}^\dagger) \|0\rangle$	
	Σ_g^{-}	$(a_{1+}^\dagger a_{3-}^\dagger - a_{1-}^\dagger a_{3+}^\dagger) \|0\rangle$	
	$\Pi_g^{\prime}$	$a_{4+}^\dagger \|0\rangle$	$a_{4-}^\dagger \|0\rangle$
	$\Pi_g^{\prime\prime}$	$(a_{1+}^\dagger)^2 a_{2-}^\dagger \|0\rangle$	$(a_{1-}^\dagger)^2 a_{2+}^\dagger \|0\rangle$
	$\Pi_g^{\prime\prime\prime}$	$a_{1+}^\dagger a_{1-}^\dagger a_{2+}^\dagger \|0\rangle$	$a_{1+}^\dagger a_{1-}^\dagger a_{2-}^\dagger \|0\rangle$
	$\Delta_g^{\prime}$	$a_{1+}^\dagger a_{3+}^\dagger \|0\rangle$	$a_{1-}^\dagger a_{3-}^\dagger \|0\rangle$
	$\Delta_g^{\prime\prime}$	$(a_{2+}^\dagger)^2 \|0\rangle$	$(a_{2-}^\dagger)^2 \|0\rangle$
	$\Delta_g^{\prime\prime\prime}$	$(a_{1+}^\dagger)^3 a_{1-}^\dagger \|0\rangle$	$a_{1+}^\dagger (a_{1-}^\dagger)^3 \|0\rangle$
	Φ_g	$(a_{1+}^\dagger)^2 a_{2+}^\dagger \|0\rangle$	$(a_{1-}^\dagger)^2 a_{2-}^\dagger \|0\rangle$
	Γ_g	$(a_{1+}^\dagger)^4 \|0\rangle$	$(a_{1-}^\dagger)^4 \|0\rangle$

Table: Low-lying string states for an open string with fixed end Ref. [Juge *et al.*()Juge, Kuti, and Morningstar]

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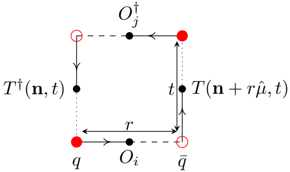
Action

- Wilson Anisotropic Action

$$S_{\text{Wilson}} = \beta \left(\frac{1}{\xi} \sum_{x,s>s'} W_{s,s'} + \xi \sum_{x,s} W_{s,t} \right), \tag{8}$$

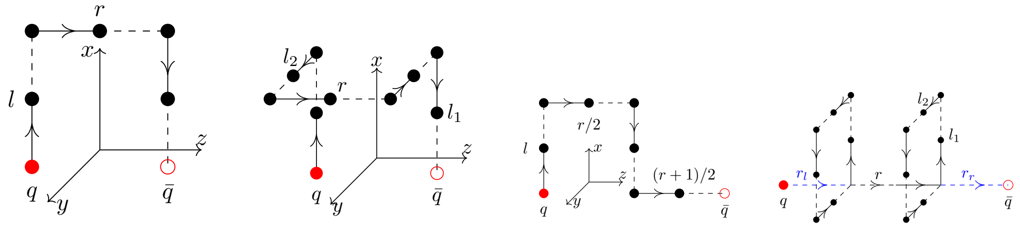
- Wilson Loop



$$\mathcal{C}_{j,i} = \langle O_j(r,t) O_i(r,0) \rangle \tag{9}$$

- O_i operators are replaced by operators of desired symmetries.

How do operators look like?



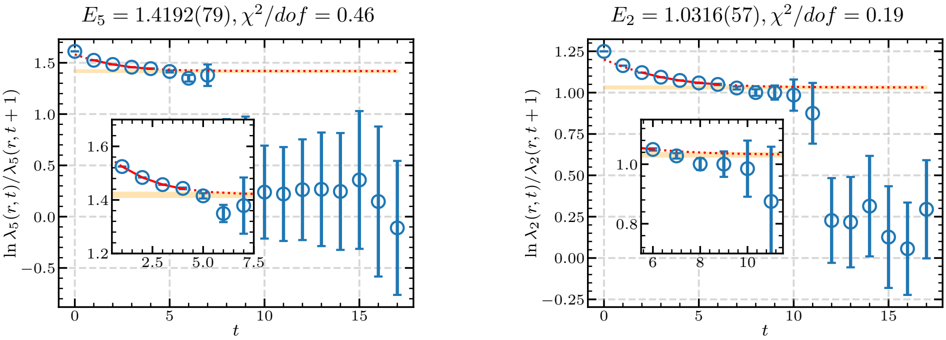
(a) Σ_g^+, Π_u, and Δ_g. (b) Σ_g^-. (c) Σ_u^+, Π_g, and Δ_u. (d) Σ_u^-.

Figure: These operators were already sieved to have lower energy for the ground state, and less degeneracy in the spectrum. arXiv:2303.15152

- The code is open source (`https://github.com/nmrcardoso/sun`) and runs on GPUs.

Generalized Eigenvalue Problem

$$\mathcal{C}(r,t)\vec{\nu}_n = \lambda_n(r,t)\mathcal{C}(r,t_0)\vec{\nu}_n$$
$$E_n = \ln \frac{\lambda_n(r,t)}{\lambda_n(r,t+1)} \tag{10}$$

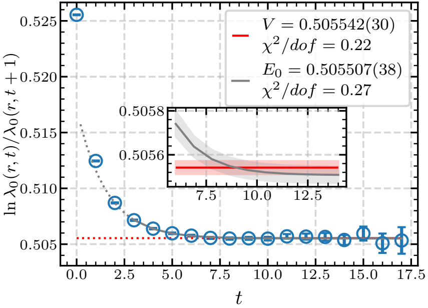


$$\langle W(r,t)\rangle = \sum_{n=0}^{\infty} A_n e^{-E_n(r)t}, \tag{11}$$

$$\frac{\langle W(r,t)\rangle}{\langle W(r,t+1)\rangle} = e^{E_0}\,\frac{1 + A_1 e^{-(E_1-E_0)t} + \dots}{1 + A_1 e^{-(E_1-E_0)(t+1)} + \dots}. \tag{12}$$

$$E_0(r,t) = \lim_{t\to\infty} \ln \frac{\langle W(r,t)\rangle}{\langle W(r,t+1)\rangle}. \tag{13}$$

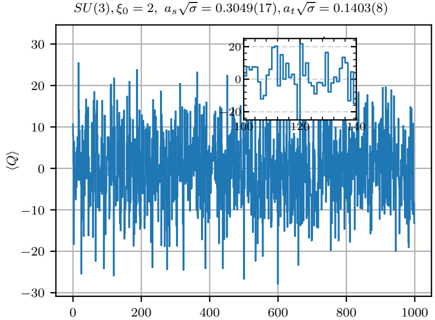
$$\ln \frac{\langle W(r,t)\rangle}{\langle W(r,t+1)\rangle} = E_0 + \ln \frac{1 + A_1 e^{-\delta E t}}{1 + A_1 e^{-\delta E(t+1)}}. \tag{14}$$

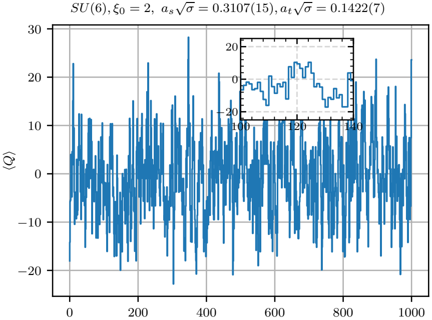


Topological charge and critical slowing down

$$Q = \frac{1}{32\pi^2} \int d^4x\, \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[G_{\mu\nu}(x) G_{\rho\sigma}(x)\right], \quad \text{We use Clover representation.} \qquad (15)$$

- The code is open source (`https://github.com/nmrcardoso/sun`) and runs on GPUs.





Ensemble

Ensemble	N_c	β	ξ_0	ξ_r	lattice	$a_s\sqrt{\sigma}$	$a_t\sqrt{\sigma}$	$\langle Plaquette \rangle$	$\chi^{1/4}/\sqrt{\sigma}$	#config
$W_{3,2}$	3	5.9	2	2.1735	$24^3 \times 48$	0.3049(17)	0.1403(8)	0.595665(6)	0.4239(4)	1000
$W_{4,2}$	4	10.7	2	2.1825	$24^3 \times 48$	0.3493(9)	0.1601(4)	0.571288(5)	0.4232(6)	1000
$W_{5,2}$	5	17.2	2	2.1830	$24^3 \times 48$	0.3006(5)	0.1377(2)	0.575065(4)	0.4021(5)	1000
$W_{6,2}$	6	24.9	2	2.1850	$24^3 \times 48$	0.3107(15)	0.1422(7)	0.568885(3)	0.3943(6)	1000
$W_{3,4}$	3	5.7	4	4.5385	$24^3 \times 96$	0.4024(16)	0.0887(4)	0.617109(6)	0.4395(1)	1000
$W_{4,4}$	4	10.4	4	4.5663	$24^3 \times 96$	0.4445(3)	0.0973(1)	0.599735(5)	0.4477(7)	1000
$W_{5,4}$	5	16.5	4	4.5766	$24^3 \times 96$	0.4484(1)	0.0980(0)	0.593900(4)	0.4415(10)	1000
$W_{6,4}$	6	24.0	4	4.5811	$24^3 \times 96$	0.4333(2)	0.0946(0)	0.592902(4)	0.4434(10)	1000

Table: Details of the ensembles generated using the anisotropic Wilson action. ξ_r is the renormalized anisotropy factor, computed using the method described in Ref. [Garcia Perez and van Baal(1997)]. All ensembles have undergone 100 times multihit smearing in the temporal direction and APE smearing ($\alpha = 0.3$, $n_s = 20$) in the spatial directions. For $SU(3)$ the value of topological suceptibility [Athenodorou and Teper(2021)] reported as $\chi^{1/4}/\sqrt{\sigma} = 0.4246(36)$.

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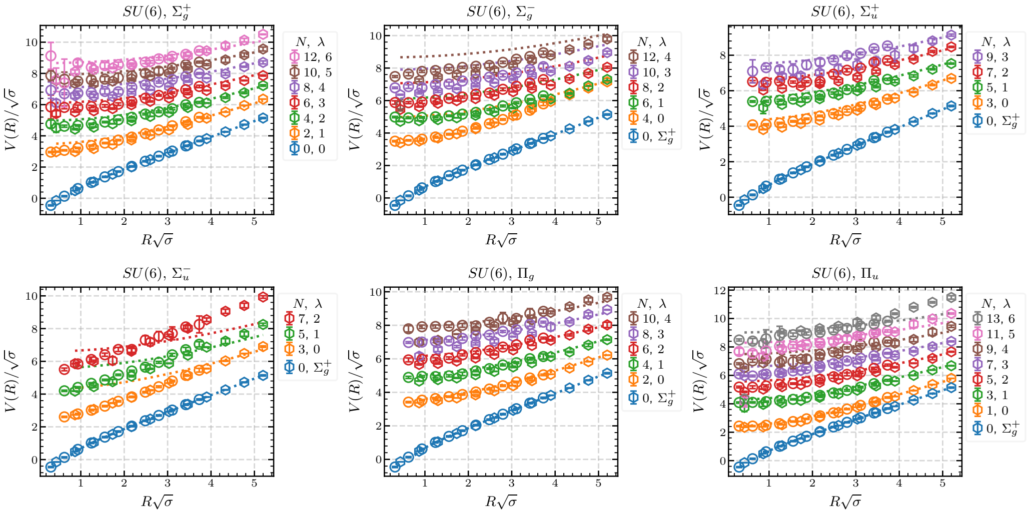
Spectra



$SU(6),\ \Sigma_g^+$
$SU(6),\ \Sigma_g^-$
$SU(6),\ \Sigma_u^+$
$SU(6),\ \Sigma_u^-$
$SU(6),\ \Pi_g$
$SU(6),\ \Pi_u$
$V(R)/\sqrt{\sigma}$
$R\sqrt{\sigma}$
$N,\ \lambda$
12, 6
10, 5
8, 4
6, 3
4, 2
2, 1
0, 0
$N,\ \lambda$
12, 4
10, 3
8, 2
6, 1
4, 0
$0,\ \Sigma_g^+$
$N,\ \lambda$
9, 3
7, 2
5, 1
3, 0
$0,\ \Sigma_g^+$
$N,\ \lambda$
7, 2
5, 1
3, 0
$0,\ \Sigma_g^+$
$N,\ \lambda$
10, 4
8, 3
6, 2
4, 1
2, 0
$0,\ \Sigma_g^+$
$N,\ \lambda$
13, 6
11, 5
9, 4
7, 3
5, 2
3, 1
1, 0
$0,\ \Sigma_g^+$

Spectra

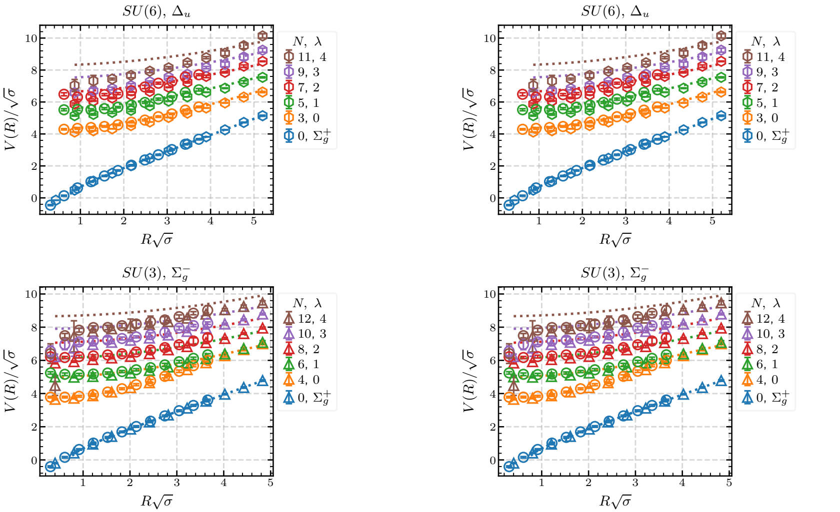
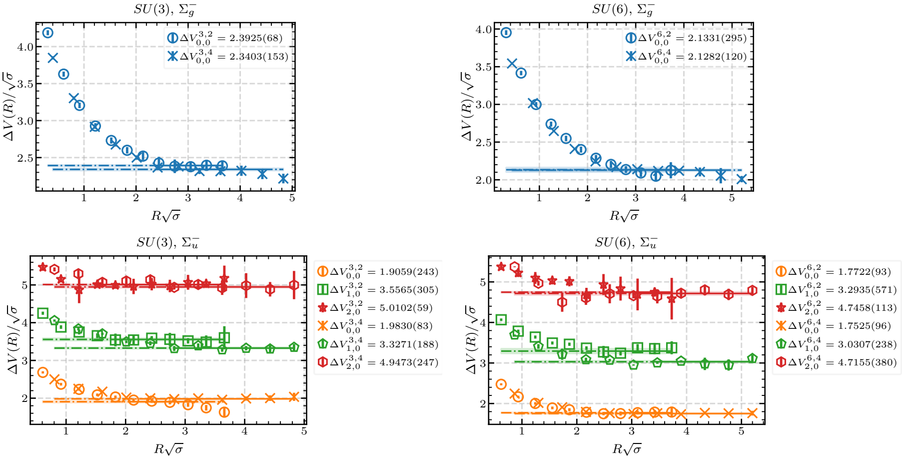


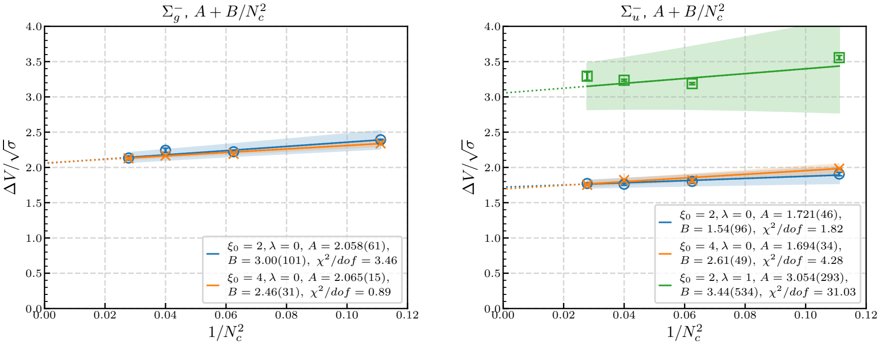
Figure: $SU(6)$ spectra, [Sharifian *et al.*(2025)Sharifian, Athenodorou, and Bicudo]

Axions



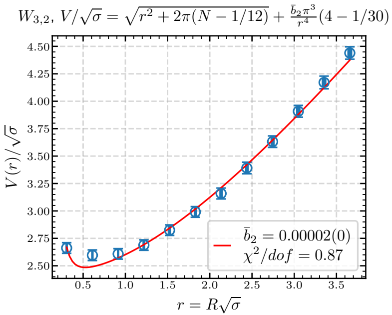
$SU(3)$, Σ_g^-
$\Delta V_{0,0}^{3,2} = 2.3925(68)$
$\Delta V_{0,0}^{3,4} = 2.3403(153)$
$SU(6)$, Σ_g^-
$\Delta V_{0,0}^{6,2} = 2.1331(295)$
$\Delta V_{0,0}^{6,4} = 2.1282(120)$
$SU(3)$, Σ_u^-
$\Delta V_{0,0}^{3,2} = 1.9059(243)$
$\Delta V_{1,0}^{3,2} = 3.5565(305)$
$\Delta V_{2,0}^{3,2} = 5.0102(59)$
$\Delta V_{0,0}^{3,4} = 1.9830(83)$
$\Delta V_{1,0}^{3,4} = 3.3271(188)$
$\Delta V_{2,0}^{3,4} = 4.9473(247)$
$SU(6)$, Σ_u^-
$\Delta V_{0,0}^{6,2} = 1.7722(93)$
$\Delta V_{1,0}^{6,2} = 3.2935(571)$
$\Delta V_{2,0}^{6,2} = 4.7458(113)$
$\Delta V_{0,0}^{6,4} = 1.7525(96)$
$\Delta V_{1,0}^{6,4} = 3.0307(238)$
$\Delta V_{2,0}^{6,4} = 4.7155(380)$
$\Delta V(R)/\sqrt{\sigma}$
$R\sqrt{\sigma}$

Axions



- In $\Sigma_g^- \to m_{\text{axion}}/\sqrt{\sigma} = \{2.058(61),\ 2.065(15)\}$
- In $\Sigma_u^- \to m_{\text{axion}}/\sqrt{\sigma} = \{1.721(46), 1.694(34)\}$,$m_{\text{axion}}/\sqrt{\sigma} = 3.054(293)$
- In literature $m_{\text{axion}}/\sqrt{\sigma} = 1.65(2)$
  [Athenodorou *et al.*(2024)Athenodorou, Dubovsky, Luo, and Teper].

Boundary term



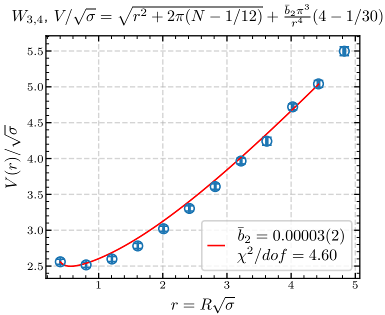


Figure: Reported values for $\tilde{b}_2 = 0.020(3), 0.025(5)$, [Bonati *et al.*(2021)Bonati, Caselle, and Morlacchi]

Conclusions

- Open flux-tube spectra for $SU(3\text{–}6)$ at two anisotropies: small a-effects; clean $N_c \to \infty$.
- Ergodic ensembles up to $N_c = 6$ from topological charge histories.
- Large-R: Nambu–Goto/Arvis describes most states; no significant $O((R\sqrt{\sigma})^{-4})$ boundary term seen.
- Σ_u^-, Σ_g^-: clear non-NG behaviour $\Rightarrow$ massive worldsheet modes persisting at large N_c.
- Lightest "axion" mass matches closed-string excitation.
- Interpretation: worldsheet axion(s) or longitudinal wave.

Open questions & next steps

- Develop TBA for *open* strings to count/characterize massive modes.
- Shorter R; non-perturbative anisotropy;
- Broader operator basis; systematic study of topology effects.
- Explore finite-temperature flux-tube spectra.

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