

Symmetry of massless Dirac fermions

- In even spacetime D , a free massless Dirac fermion ψ has the symmetry

$$G_\psi = \frac{U(1)_V \times U(1)_A}{\mathbb{Z}_2}$$

$$U(1)_V : \psi \rightarrow e^{i\alpha} \psi$$

$$U(1)_A : \psi \rightarrow e^{i\Gamma\beta} \psi$$

$$\{\Gamma, \gamma^\mu\} = 0$$

- **Mixed 't Hooft anomaly:** turning on a $U(1)_V$ background gauge field A_μ breaks $U(1)_A$, and vice versa.
 - This is the origin of the **ABJ anomaly**.

Massless fermions of the lattice

Nielsen-Nin

Naive lattice fermions

Wilson, ...

- Replacing the continuum derivatives by naive lattice finite difference operators gives the naive lattice fermion discretization.
- $\mathcal{D}$ invertible except at each corner of the Brillouin zone, leading to 2^D massless fermions.

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