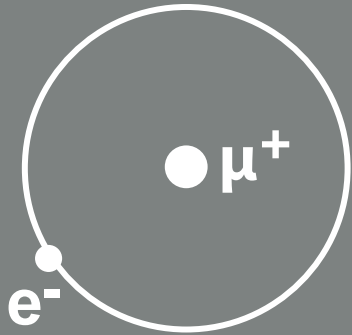
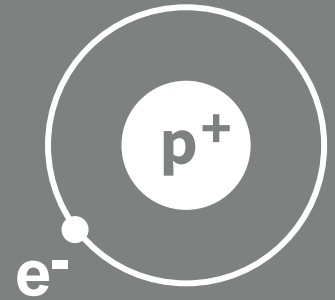


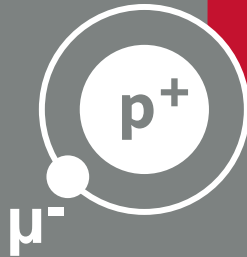


Finite-size Effects & New Physics



Sotiris Pitelis

in collaboration with F. Hagelstein,
V. Lensky and V. Pascalutsa



31.07.25



JOHANNES GUTENBERG
UNIVERSITÄT MAINZ



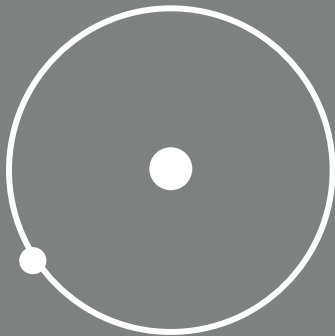
ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

Emmy
Noether-
Programm

DFG Deutsche
Forschungsgemeinschaft

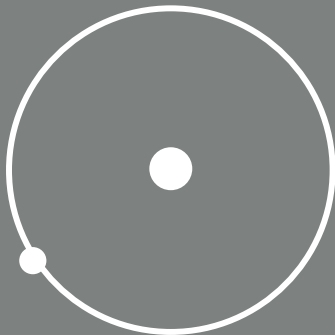


A brief outline



A brief outline

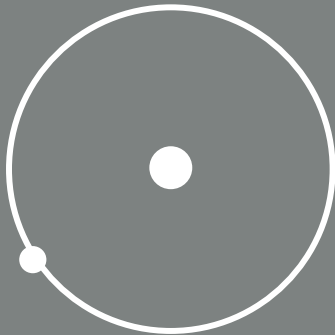
An
illustration



A brief outline

An
illustration

The
standard
model...

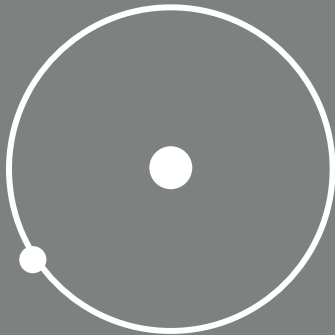


A brief outline

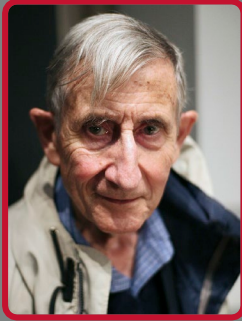
An
illustration

The
standard
model...

...and
beyond



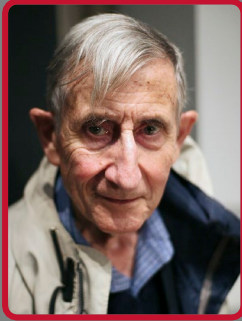
Frontiers of New Physics Searches



“If you look for nature’s secrets in only one direction, you are likely to miss the most important secrets...”

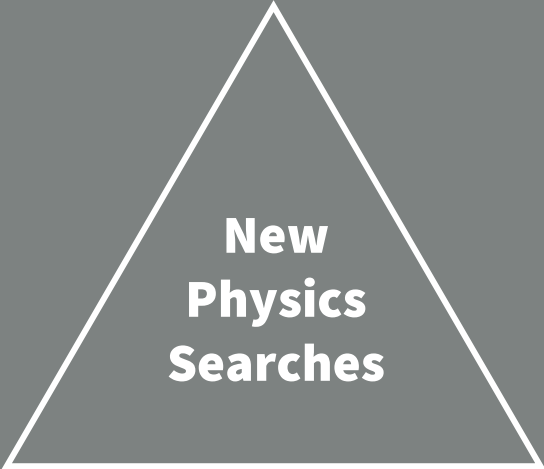
-Freeman Dyson

Frontiers of New Physics Searches



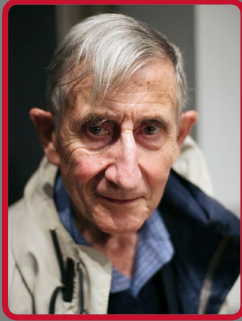
*“If you look for nature’s secrets in only one direction,
you are likely to miss the most important secrets...”*

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A large white triangle is centered on the slide. Inside the triangle, the text 'New Physics Searches' is written in a bold, white, sans-serif font, arranged in three lines.

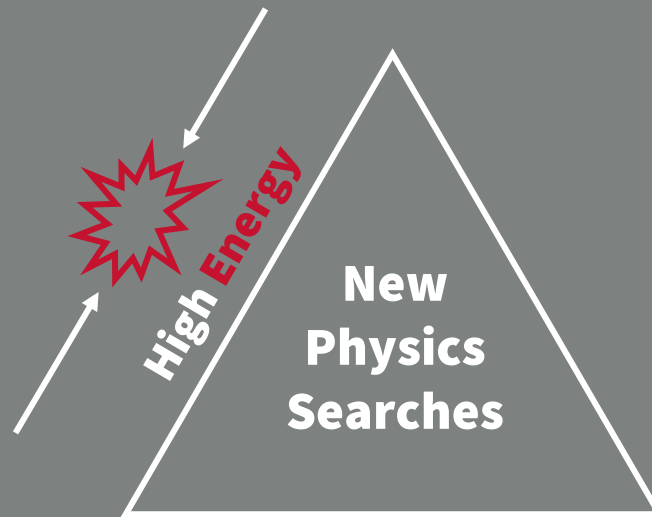
**New
Physics
Searches**

Frontiers of New Physics Searches

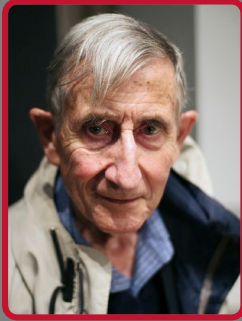


“If you look for nature’s secrets in only one direction, you are likely to miss the most important secrets...”

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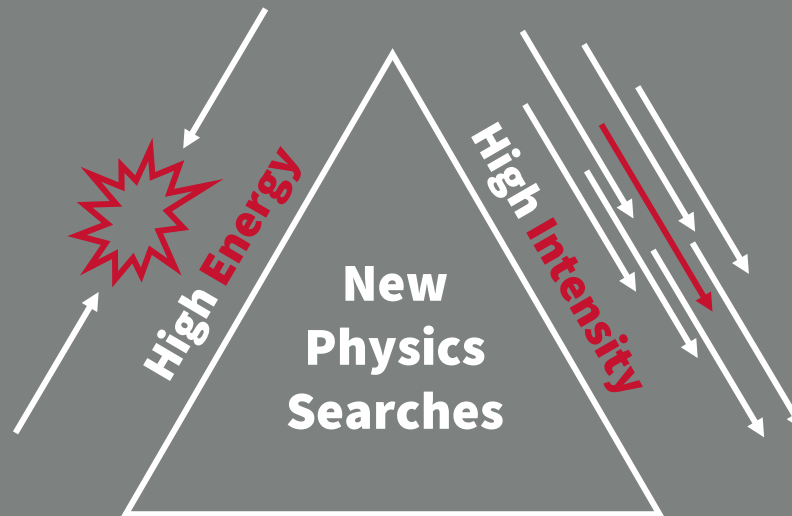


Frontiers of New Physics Searches

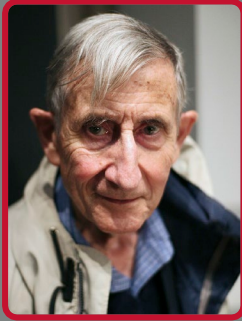


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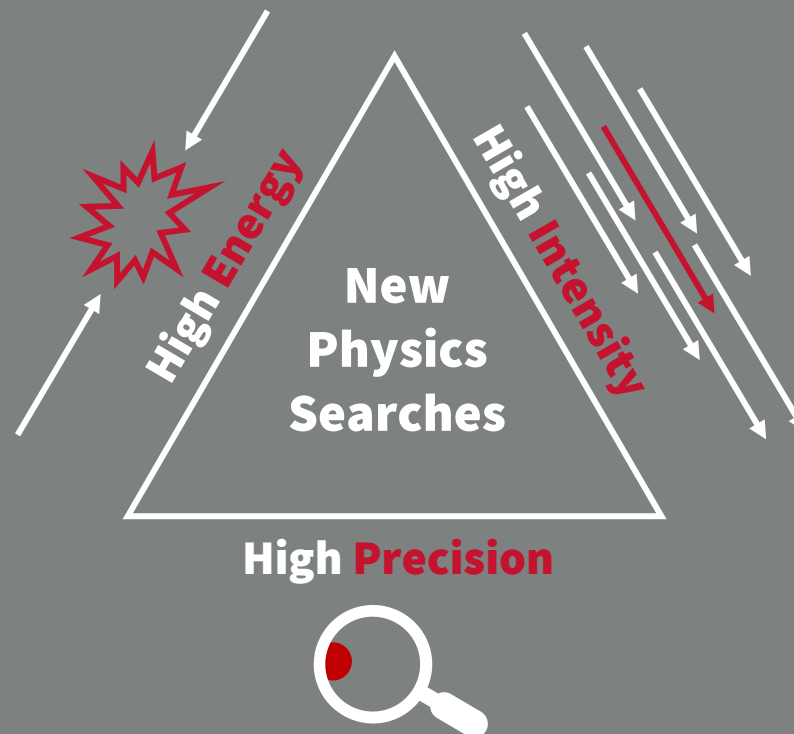


Frontiers of New Physics Searches

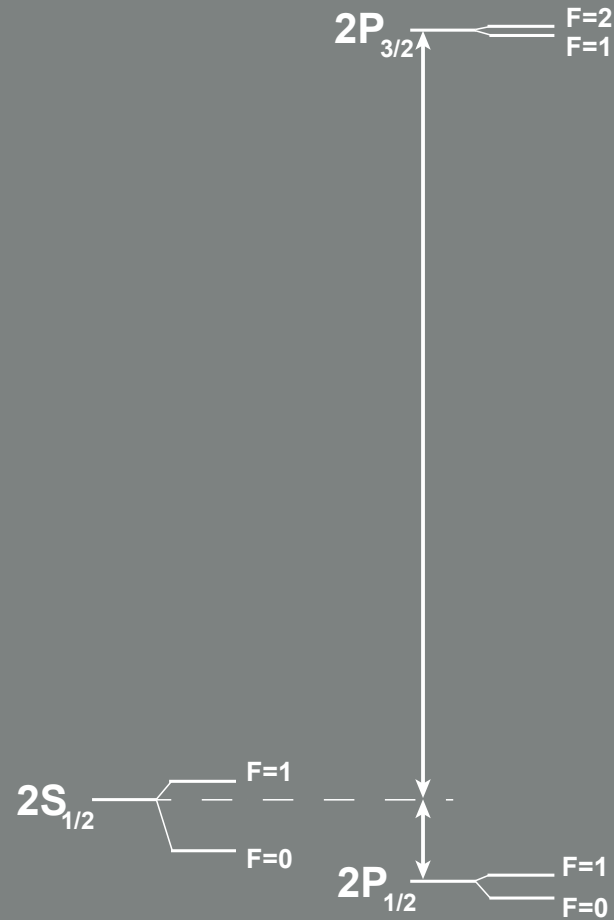
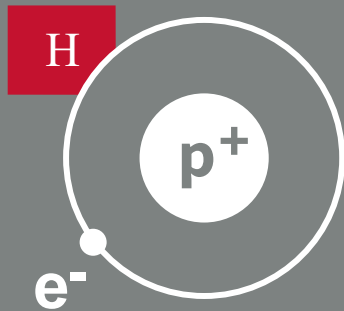


“If you look for nature’s secrets in only one direction, you are likely to miss the most important secrets...”

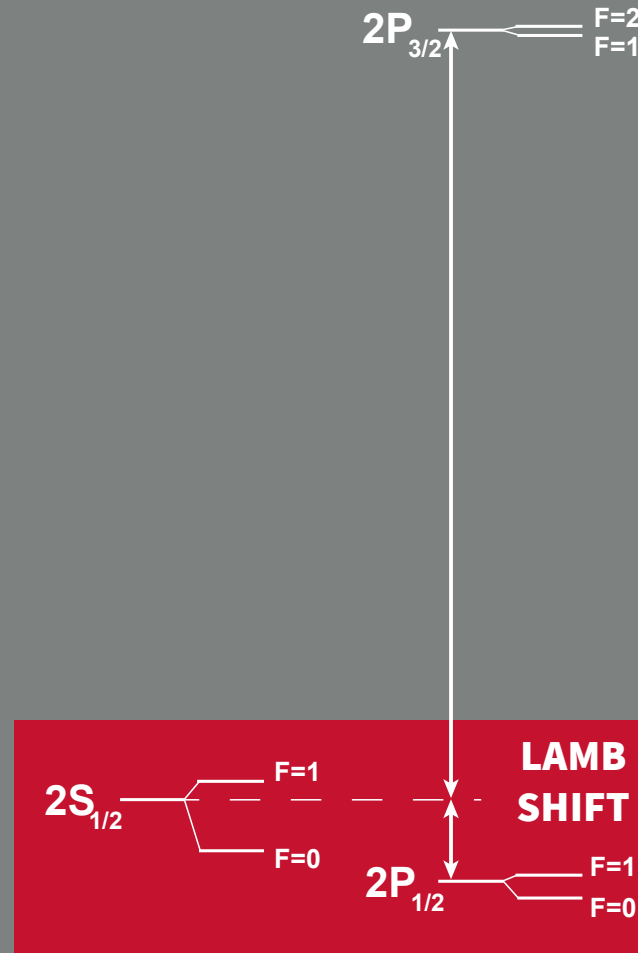
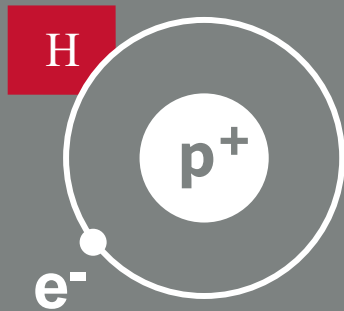
-Freeman Dyson



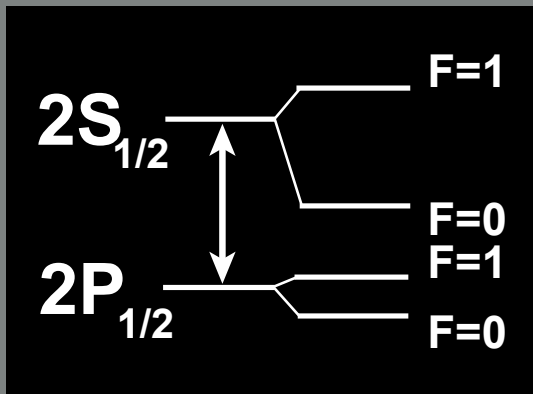
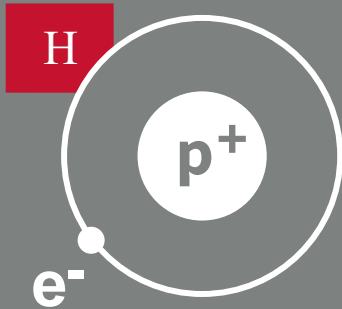
Precision Atomic Spectroscopy



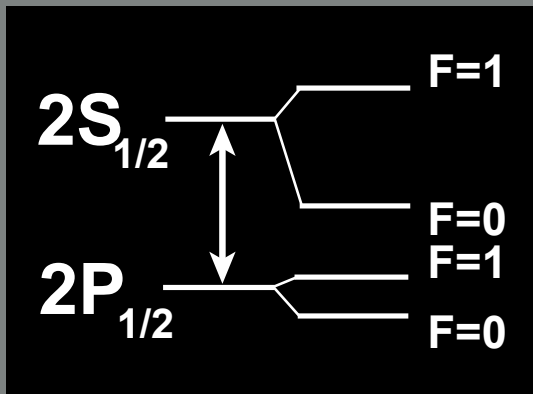
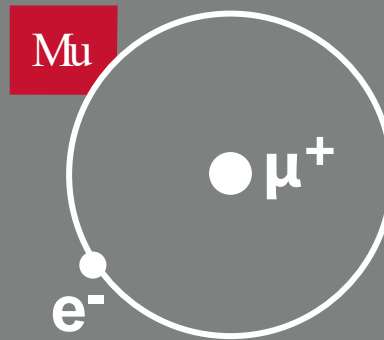
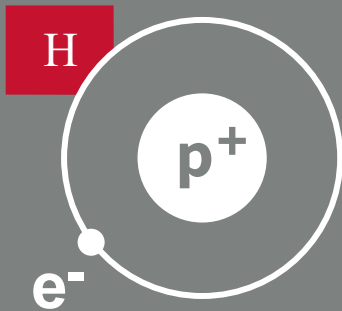
Precision Atomic Spectroscopy



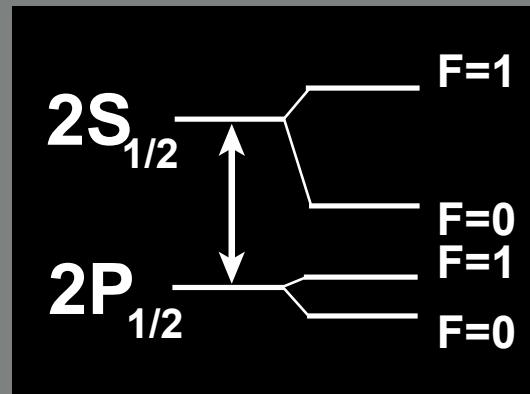
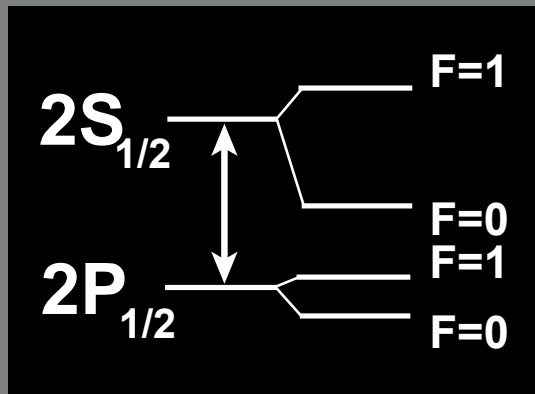
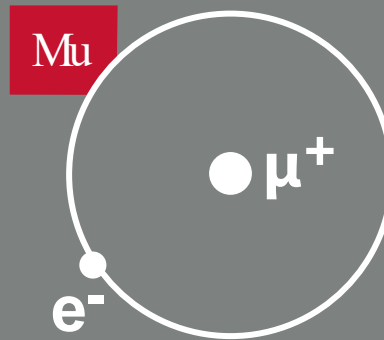
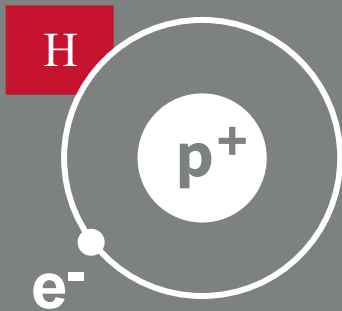
Precision Atomic Spectroscopy



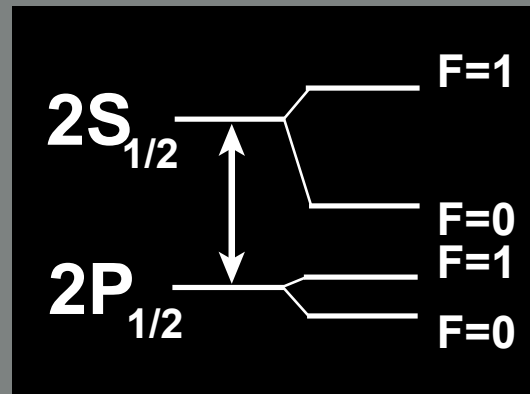
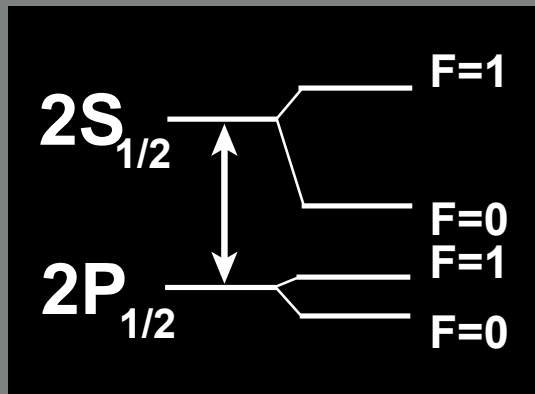
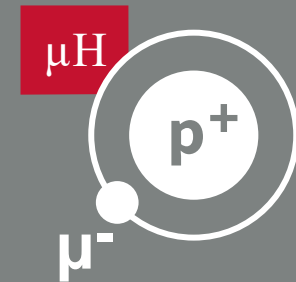
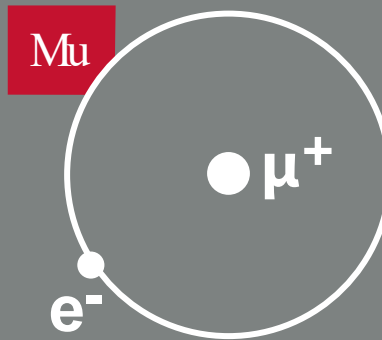
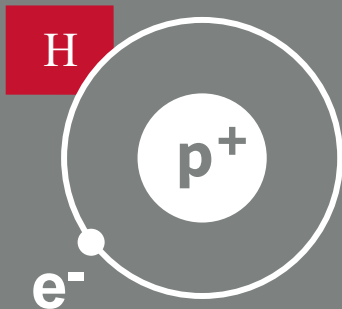
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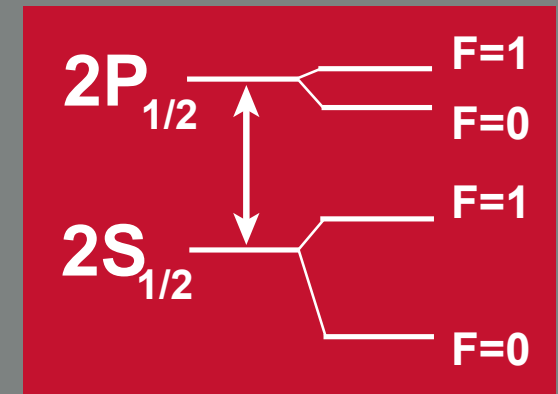
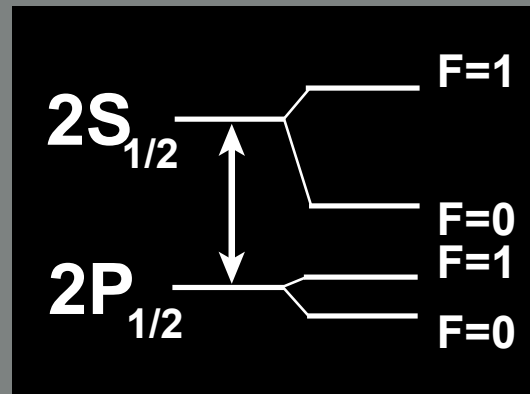
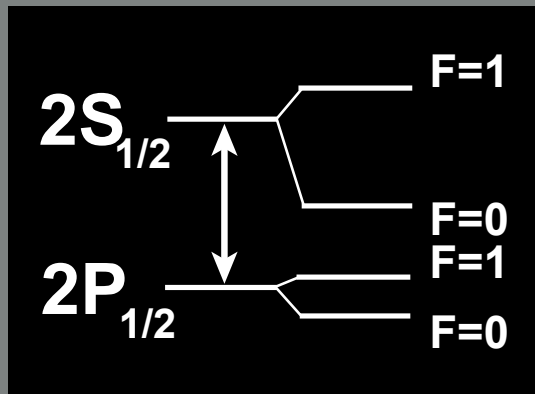
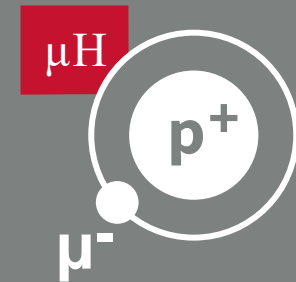
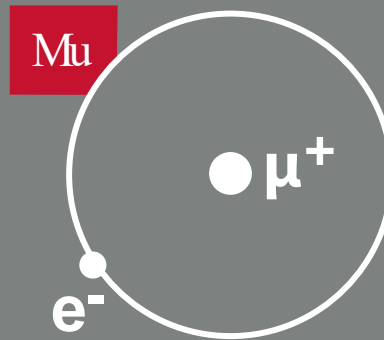
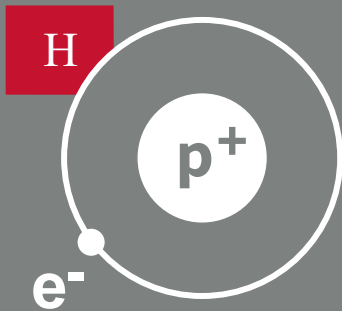
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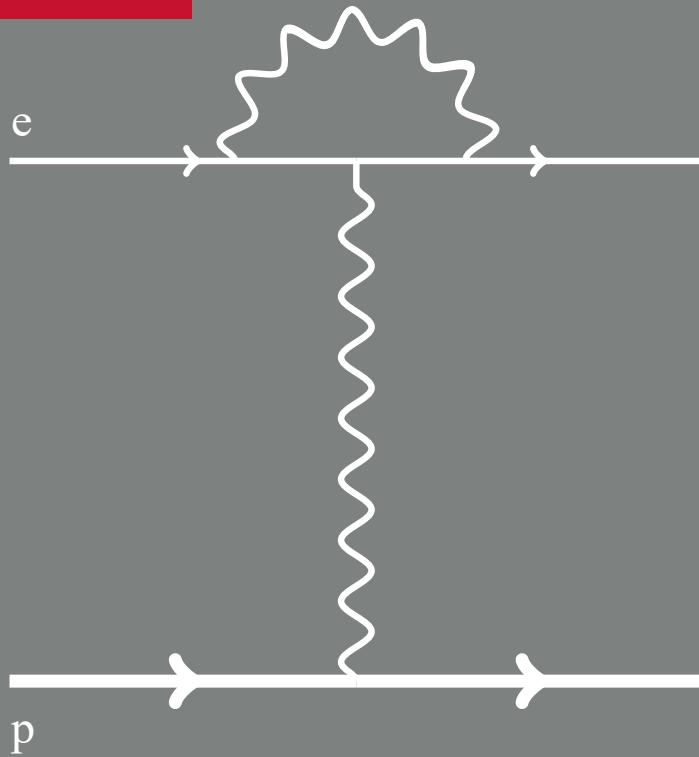
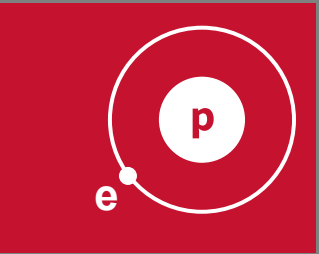
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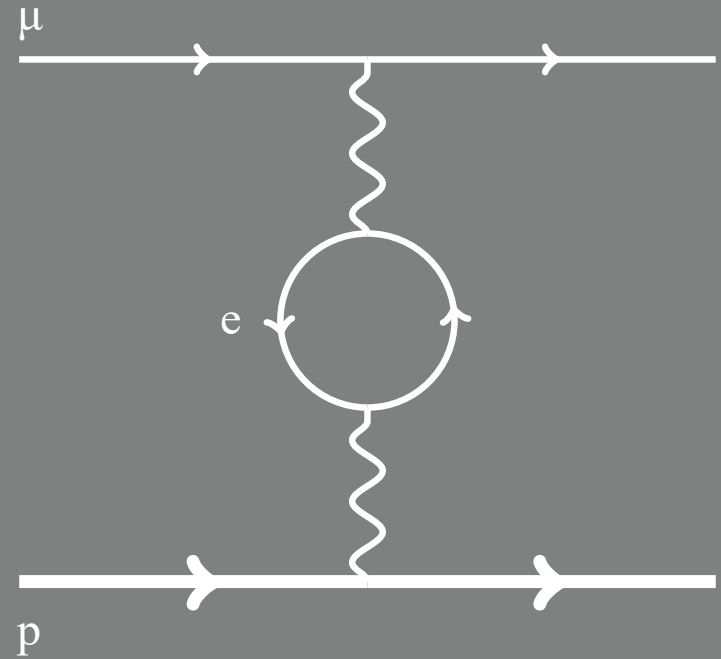
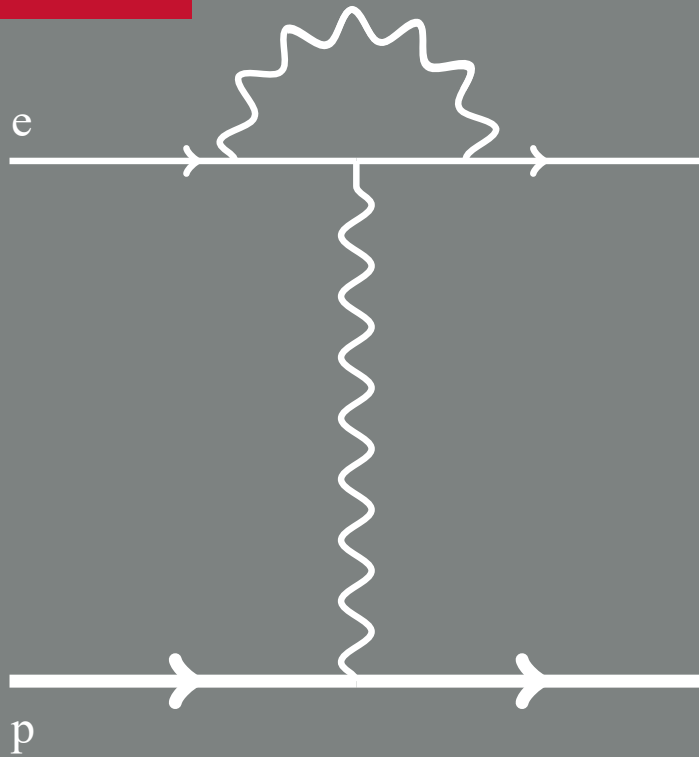
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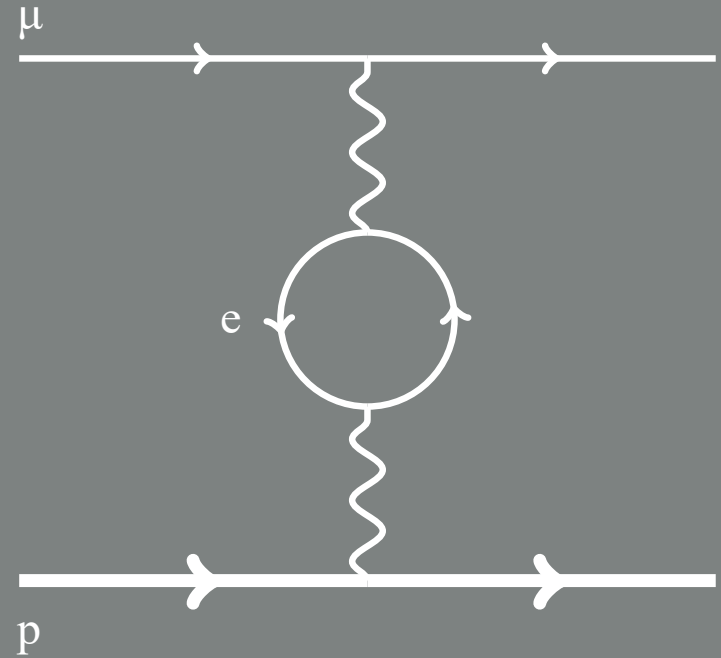
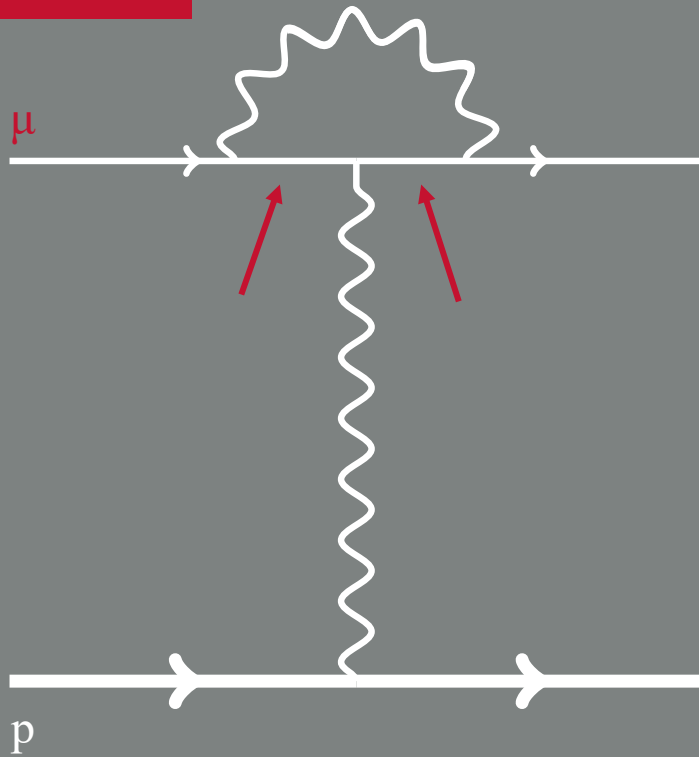
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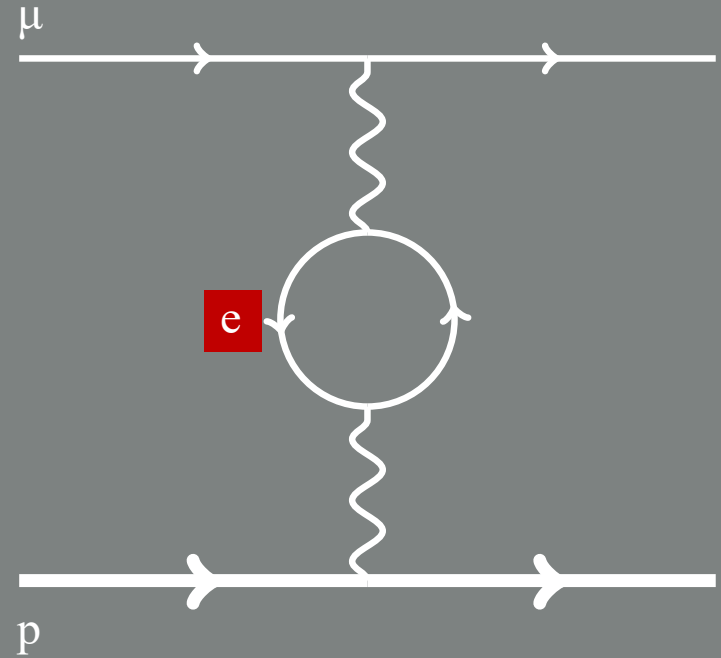
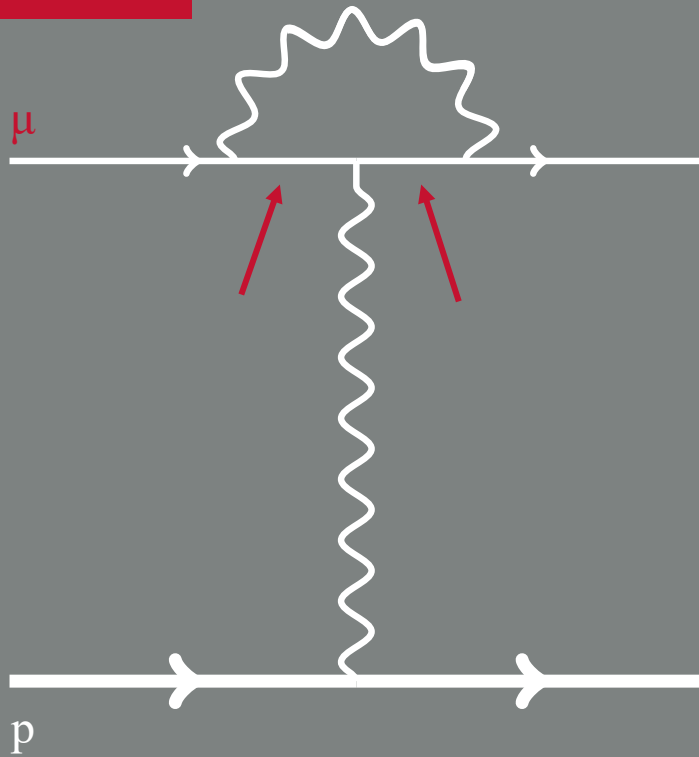
Leading QED Corrections



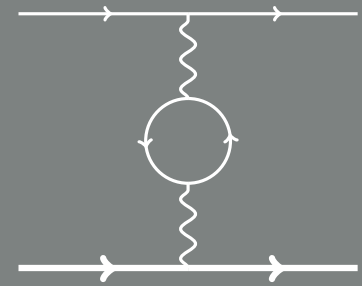
Leading QED Corrections



Leading QED Corrections



Electronic Vacuum Polarization in μH

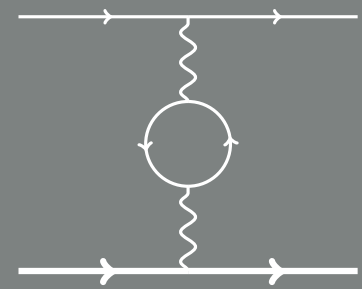


DOI: 10.1103/RevModPhys.96.015001

Sec.	Order	Correction	μH
III.A	$\alpha (Z \alpha)^2$	eVP ⁽¹⁾	205.007 38
III.A	$\alpha^2 (Z \alpha)^2$	eVP ⁽²⁾	1.658 85
III.A	$\alpha^3 (Z \alpha)^2$	eVP ⁽³⁾	0.007 52
III.B	$(Z, Z^2, Z^3) \alpha^5$	light by light eVP	−0.000 89(2)
III.C	$(Z \alpha)^4$	recoil	0.057 47
III.D	$\alpha (Z \alpha)^4$	relativistic with eVP ⁽¹⁾	0.018 76
III.E	$\alpha^2 (Z \alpha)^4$	relativistic with eVP ⁽²⁾	0.000 17

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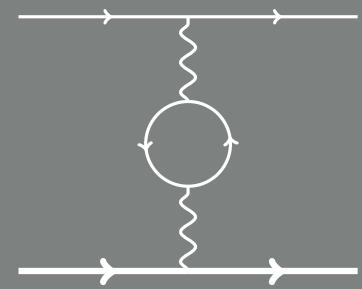
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		•	
		•	
		•	
III	E_{QED}	point nucleus	206.034 4(3)

Electronic Vacuum Polarization in μH

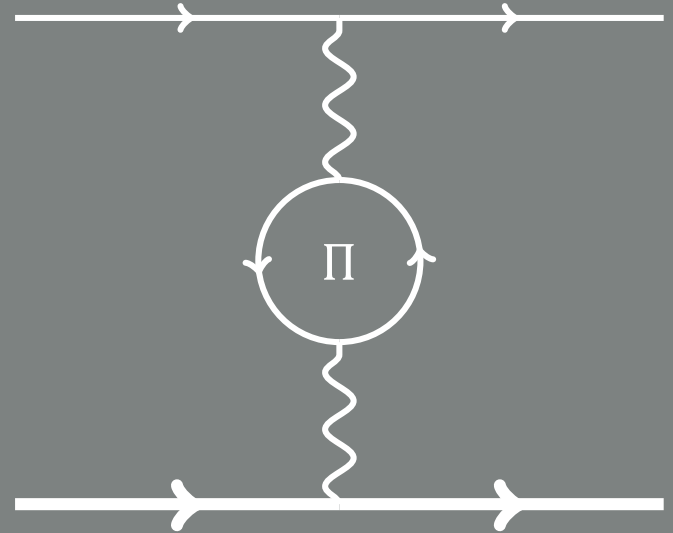


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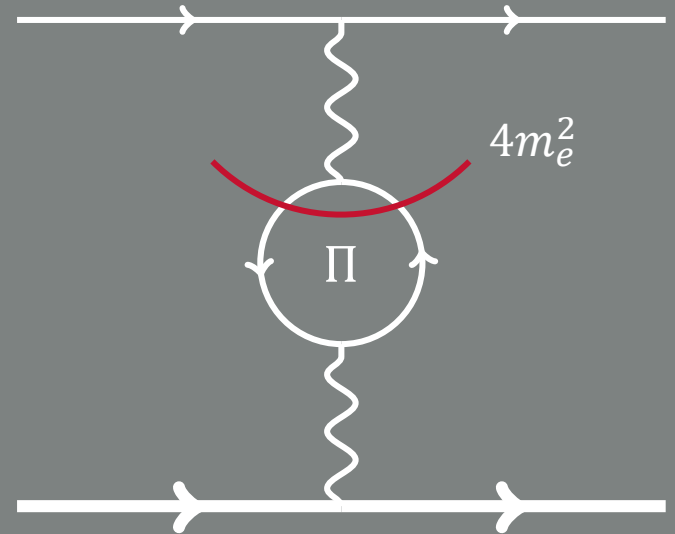
Dispersive Calculation of the eVP Contribution

$$E_{2P-2S}^{\langle \text{eVP} \rangle} = - \frac{(Z\alpha)^4 m_r^3}{2\pi} \int_{4m_e^2}^{\infty} dt \frac{\alpha \text{Im}\Pi(t)}{(\sqrt{t} + Z\alpha m_r)^4}$$



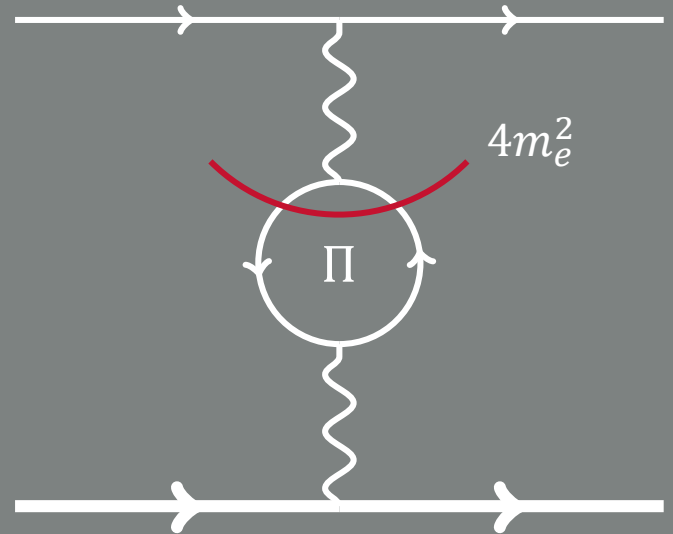
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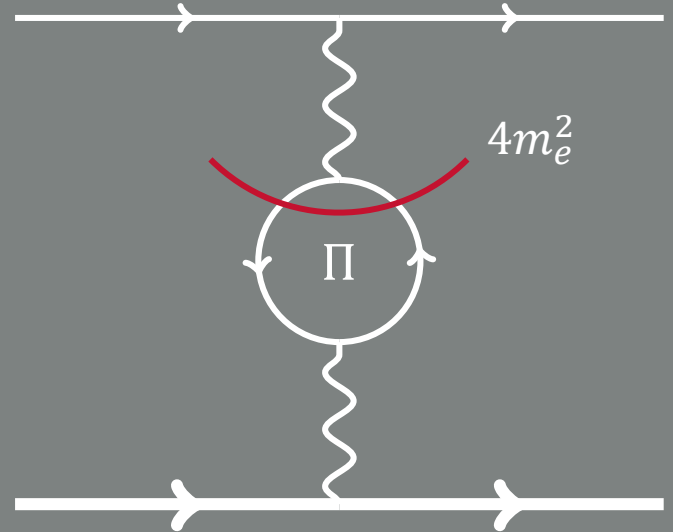
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 &= - \frac{\alpha(Z\alpha)^4 m_r^3}{4m_e^2} \frac{1}{2\pi} \int_1^{\infty} dt \frac{\text{Im}\Pi(4m_e^2 t)}{\left(\sqrt{t} + \frac{m_r}{2m_e} Z\alpha\right)^4}
 \end{aligned}$$



Dispersive Calculation of the eVP Contribution

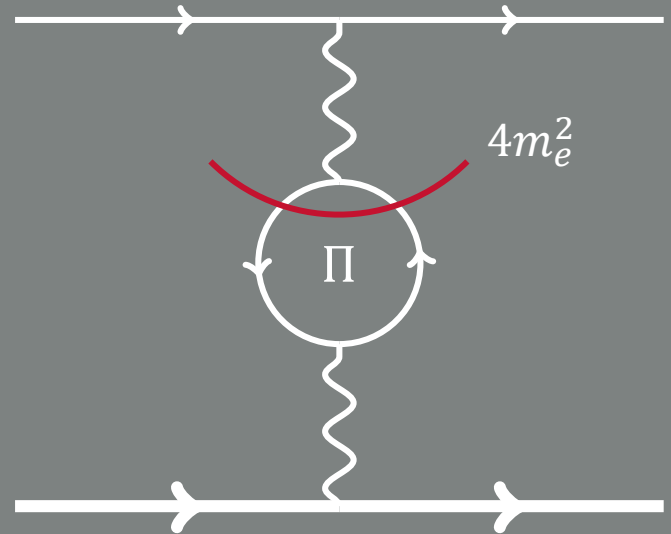
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 &= - \frac{\alpha(Z\alpha)^4 m_r^3}{4m_e^2} \frac{1}{2\pi} \int_1^{\infty} dt \frac{\text{Im}\Pi(4m_e^2 t)}{\left(\sqrt{t} + \frac{m_r}{2m_e} Z\alpha\right)^4}
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$$= - \frac{\alpha(Z\alpha)^4 m_r^3}{4m_e^2} \frac{1}{2\pi} \int_1^{\infty} dt \frac{\text{Im}\Pi(4m_e^2 t)}{\left(\sqrt{t} + \frac{m_r}{2m_e} Z\alpha\right)^4}$$



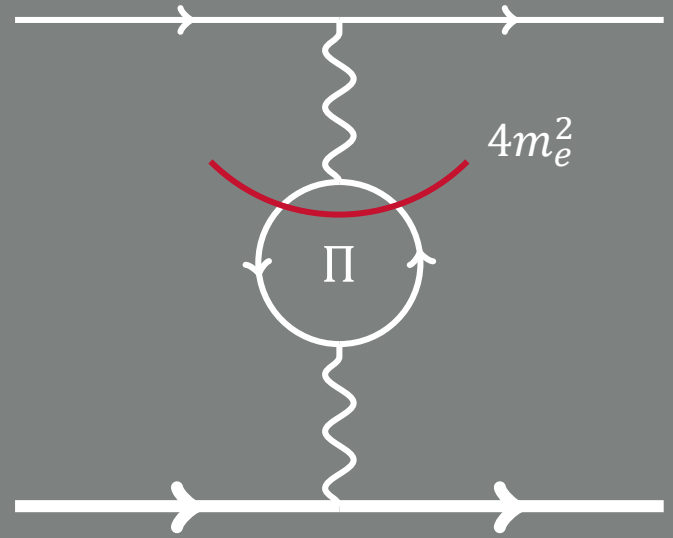
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$\alpha(Z\alpha)^2$?

$$= - \frac{\alpha(Z\alpha)^4 m_r^3}{4m_e^2} \frac{1}{2\pi} \int_1^{\infty} dt \frac{\text{Im}\Pi(4m_e^2 t)}{\left(\sqrt{t} + \frac{m_r}{2m_e} Z\alpha\right)^4}$$

κ



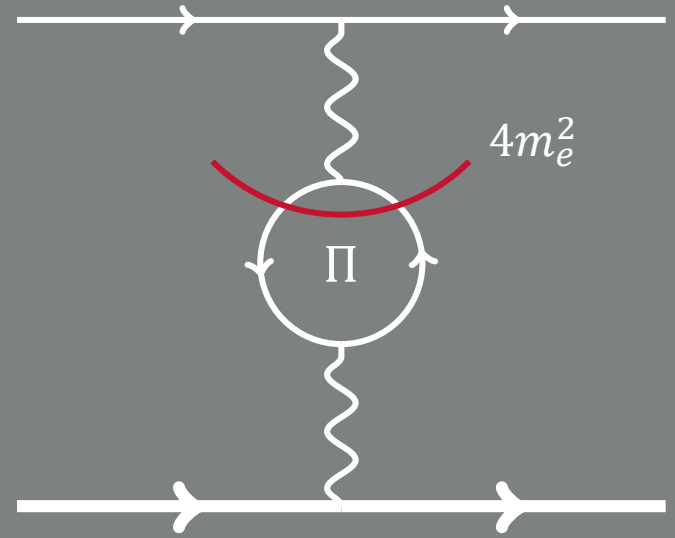
Dispersive Calculation of the eVP Contribution

$$E_{2P-2S}^{(eVP)} = - \frac{(Z\alpha)^4 m_r^3}{2\pi} \int_{4m_e^2}^{\infty} dt \frac{\alpha \operatorname{Im}\Pi(t)}{(\sqrt{t} + Z\alpha m_r)^4}$$

$$= - \frac{\alpha(Z\alpha)^4 m_r^3}{4m_e^2} \frac{1}{2\pi} \int_1^{\infty} dt \frac{\operatorname{Im}\Pi(4m_e^2 t)}{\left(\sqrt{t} + \frac{m_r}{2m_e} Z\alpha\right)^4}$$

$$= \alpha(Z\alpha)^2 \kappa^2 m_r$$

$$\times \frac{1}{3\kappa^5} \left(1 + \frac{\kappa(2\kappa^6 - 13\kappa^4 + 44\kappa^2 - 24)}{12\pi(1 - \kappa^2)^2} - \frac{15\kappa^4 - 20\kappa^2 + 8}{4\pi(1 - \kappa^2)^{\frac{5}{2}}} \operatorname{ArcCos} \kappa \right)$$



Dispersive Calculation of the eVP Contribution

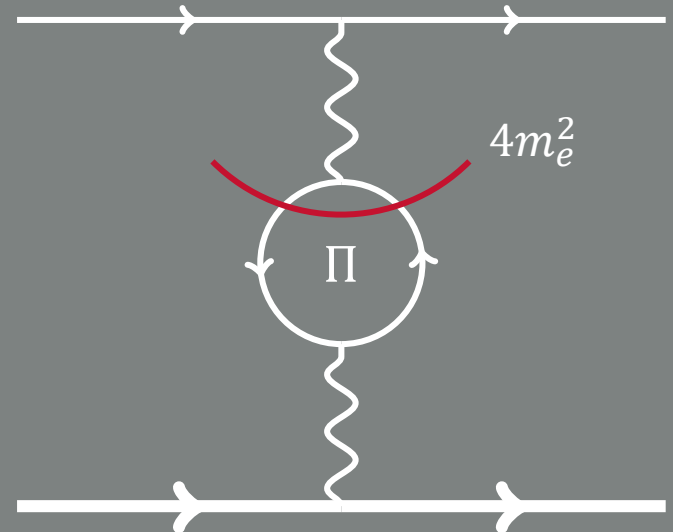
$$E_{2P-2S}^{(eVP)} = - \frac{(Z\alpha)^4 m_r^3}{2\pi} \int_{4m_e^2}^{\infty} dt \frac{\alpha \operatorname{Im}\Pi(t)}{(\sqrt{t} + Z\alpha m_r)^4}$$

?

$$= - \frac{\alpha(Z\alpha)^4 m_r^3}{4m_e^2} \frac{1}{2\pi} \int_1^{\infty} dt \frac{\operatorname{Im}\Pi(4m_e^2 t)}{\left(\sqrt{t} + \frac{m_r}{2m_e} Z\alpha\right)^4}$$

κ

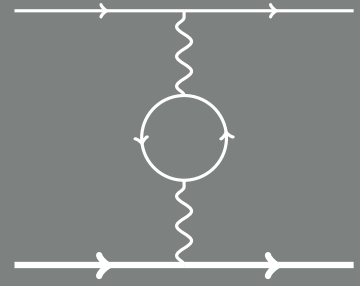
$$= \alpha(Z\alpha)^2 \kappa^2 m_r$$



$$\times \frac{1}{3\kappa^5} \left(1 + \frac{\kappa(2\kappa^6 - 13\kappa^4 + 44\kappa^2 - 24)}{12\pi(1 - \kappa^2)^2} \frac{15\kappa^4 - 20\kappa^2 + 8}{4\pi(1 - \kappa^2)^{\frac{5}{2}}} \operatorname{ArcCos} \kappa \right)$$

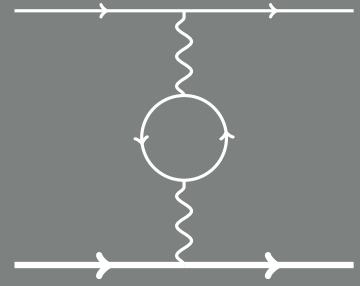
(almost) constant as κ changes

The eVP Contribution Across Different Systems



$$E_{2P-2S}^{(eVP)} \propto \alpha (Z\alpha)^2 \kappa^2 m_r, \quad \kappa = \frac{m_r}{2m_e} Z\alpha$$

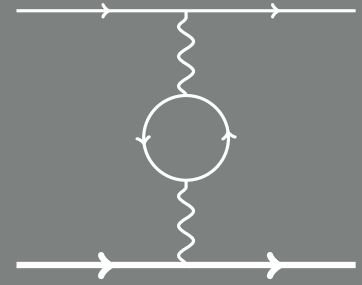
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System	m_r [MeV]	κ	$\kappa^2 m_r$ [MeV]
Mu	0.509	$0.498 Z\alpha$	6.71×10^{-6}

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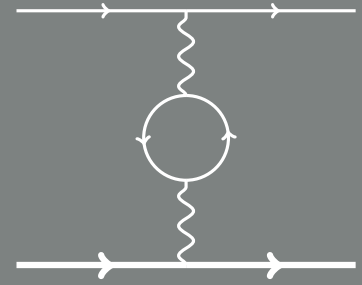


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$$Z\alpha \simeq \frac{1}{137}$$

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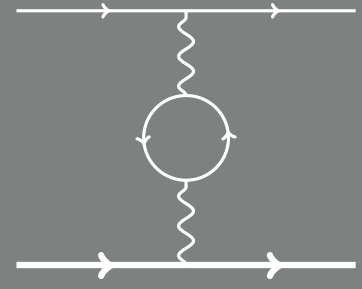


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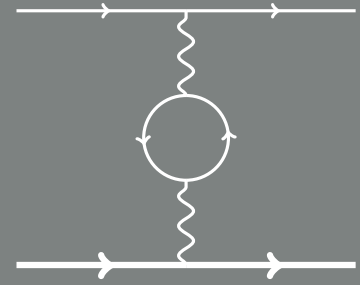


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The eVP Contribution Across Different Systems



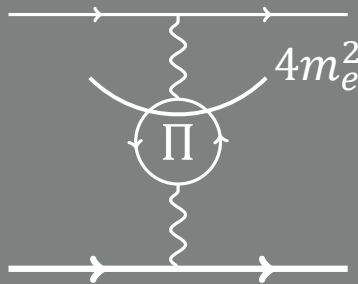
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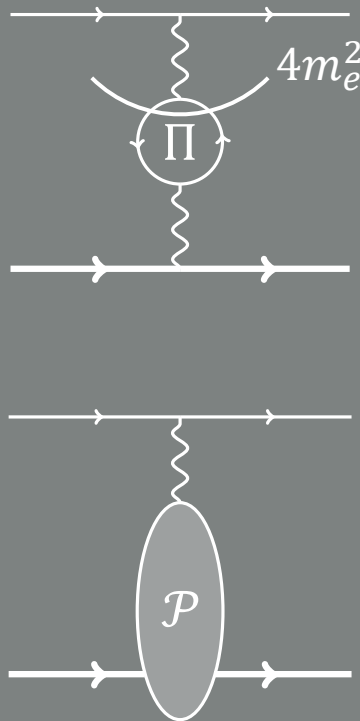
$$\alpha (Z\alpha)^2 !$$

One-Photon-Exchange Potentials



$$E_{2P-2S}^{\langle \text{eVP} \rangle} = - \frac{(Z\alpha)^4 m_r^3}{2\pi} \int_{4m_e^2}^{\infty} dt \frac{a \operatorname{Im}\Pi(t)}{(\sqrt{t} + Z\alpha m_r)^4}$$

One-Photon-Exchange Potentials

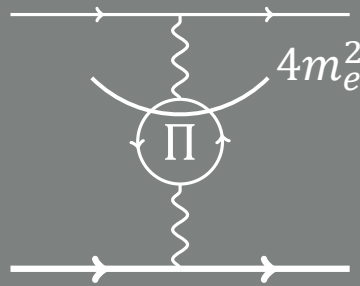


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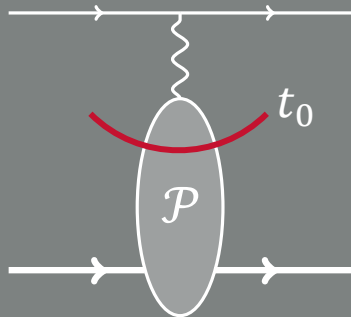


$$E_{2P-2S}^{\langle \text{OPE} \rangle} = - \frac{(Z\alpha)^4 m_r^3}{2\pi} \int_{t_0}^{\infty} dt \frac{\operatorname{Im} \mathcal{P}(t)}{(\sqrt{t} + Z\alpha m_r)^4}$$

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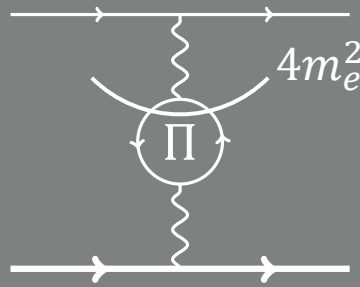


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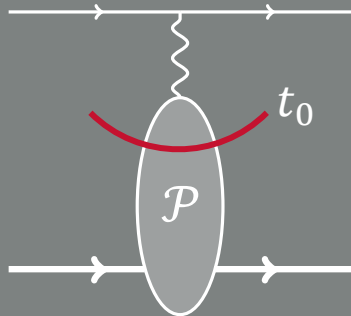


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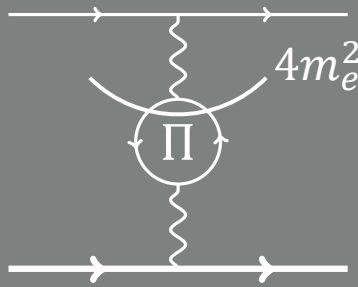
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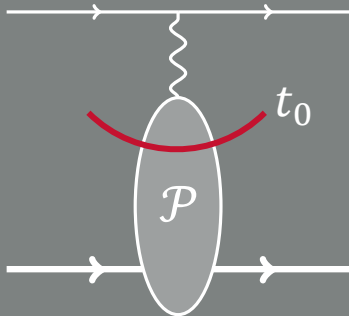
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One-Photon-Exchange Potentials



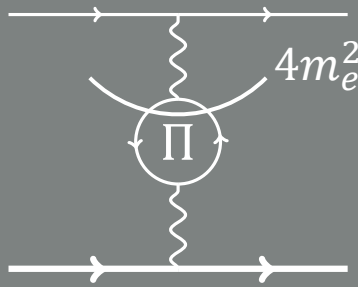
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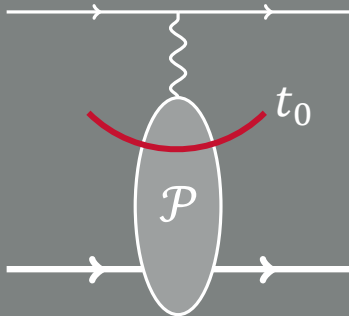
$$E_{2P-2S}^{\langle \text{OPE} \rangle} = - \frac{(Z\alpha)^4 m_r^3}{2\pi} \int_{t_0}^{\infty} dt \frac{\text{Im}\mathcal{P}(t)}{(\sqrt{t} + Z\alpha m_r)^4}$$

$$\left. \begin{array}{l} \kappa = \frac{m_r}{\sqrt{t_0}} Z\alpha \\ \text{larger } m_r \\ \text{smaller } t_0 \end{array} \right\} \text{larger } \kappa \rightarrow \text{enhancement!}$$

One-Photon-Exchange Potentials



$$E_{2P-2S}^{\langle \text{eVP} \rangle} = - \frac{(Z\alpha)^4 m_r^3}{2\pi} \int_{4m_e^2}^{\infty} dt \frac{a \operatorname{Im}\Pi(t)}{(\sqrt{t} + Z\alpha m_r)^4}$$



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$$\kappa = \frac{m_r}{\sqrt{t_0}} Z\alpha$$

larger m_r
smaller t_0 }

Contributions can be enhanced depending on the system's m_r !

Finite-Size Corrections

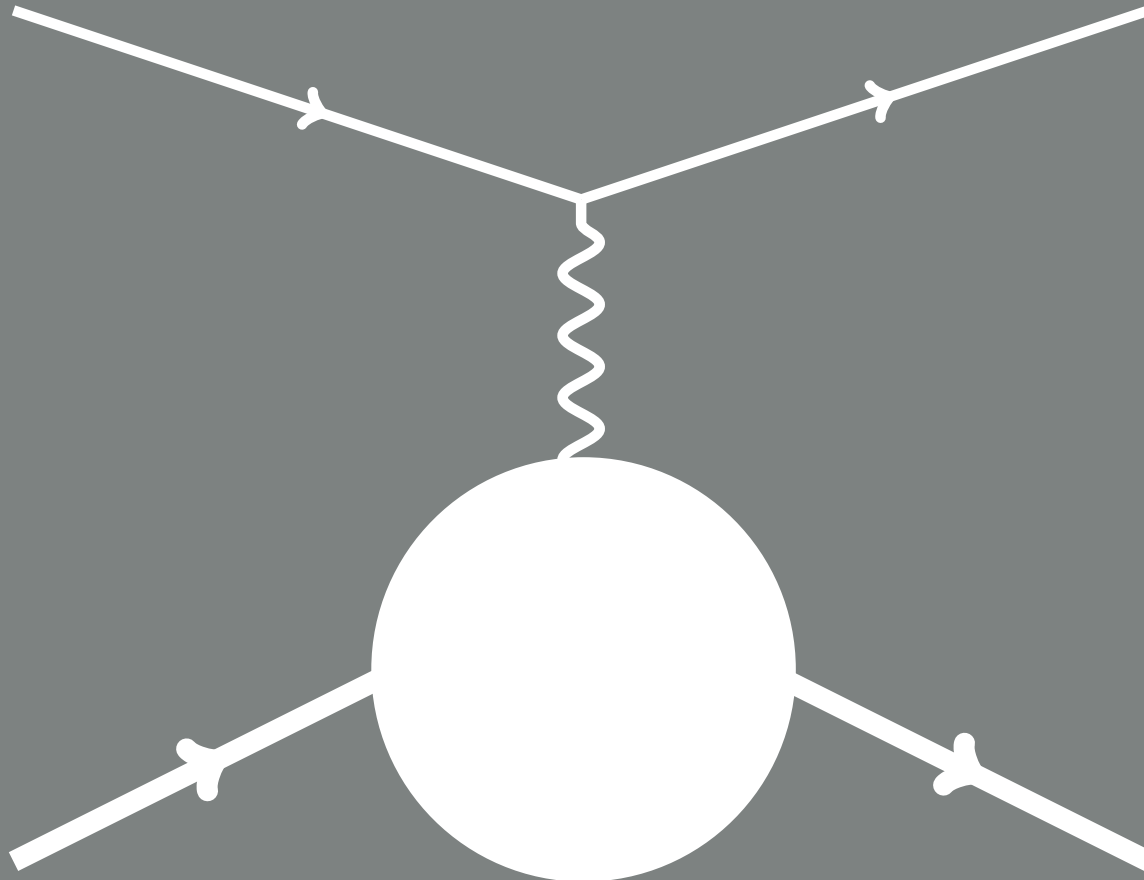
$$E_{2P-2S}^{\langle \text{FS} \rangle} = \int_0^\infty dQ w(Q) G_E(Q^2)$$

DOI:10.1103/PhysRevA.91.040502

Finite-Size Corrections

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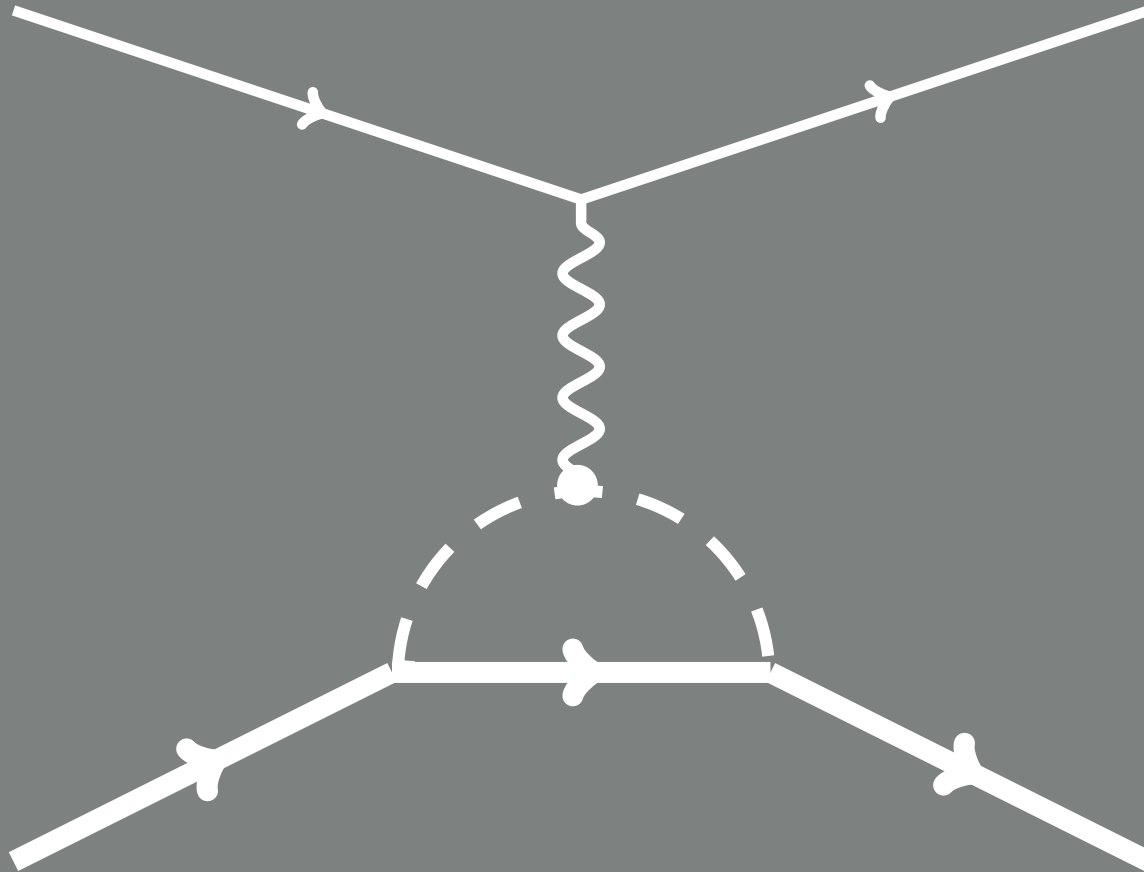
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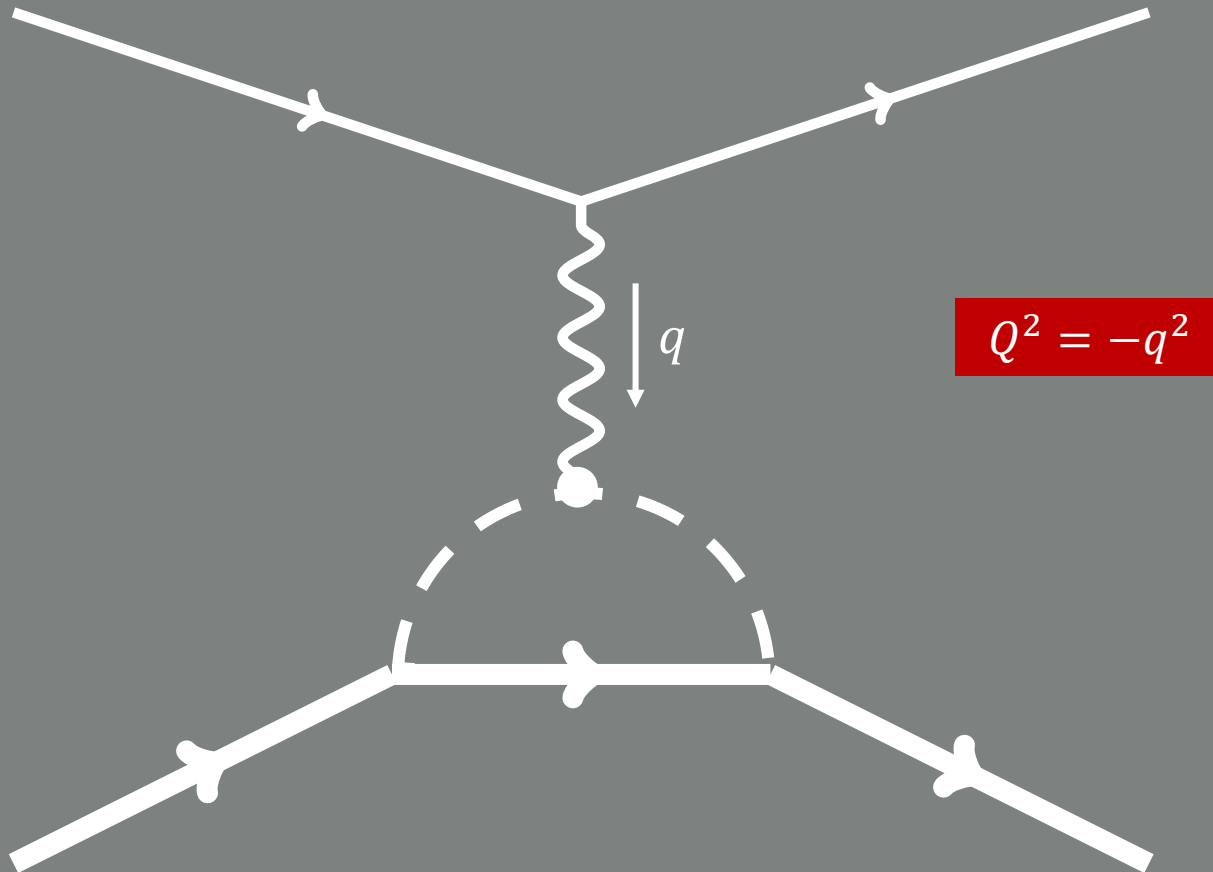
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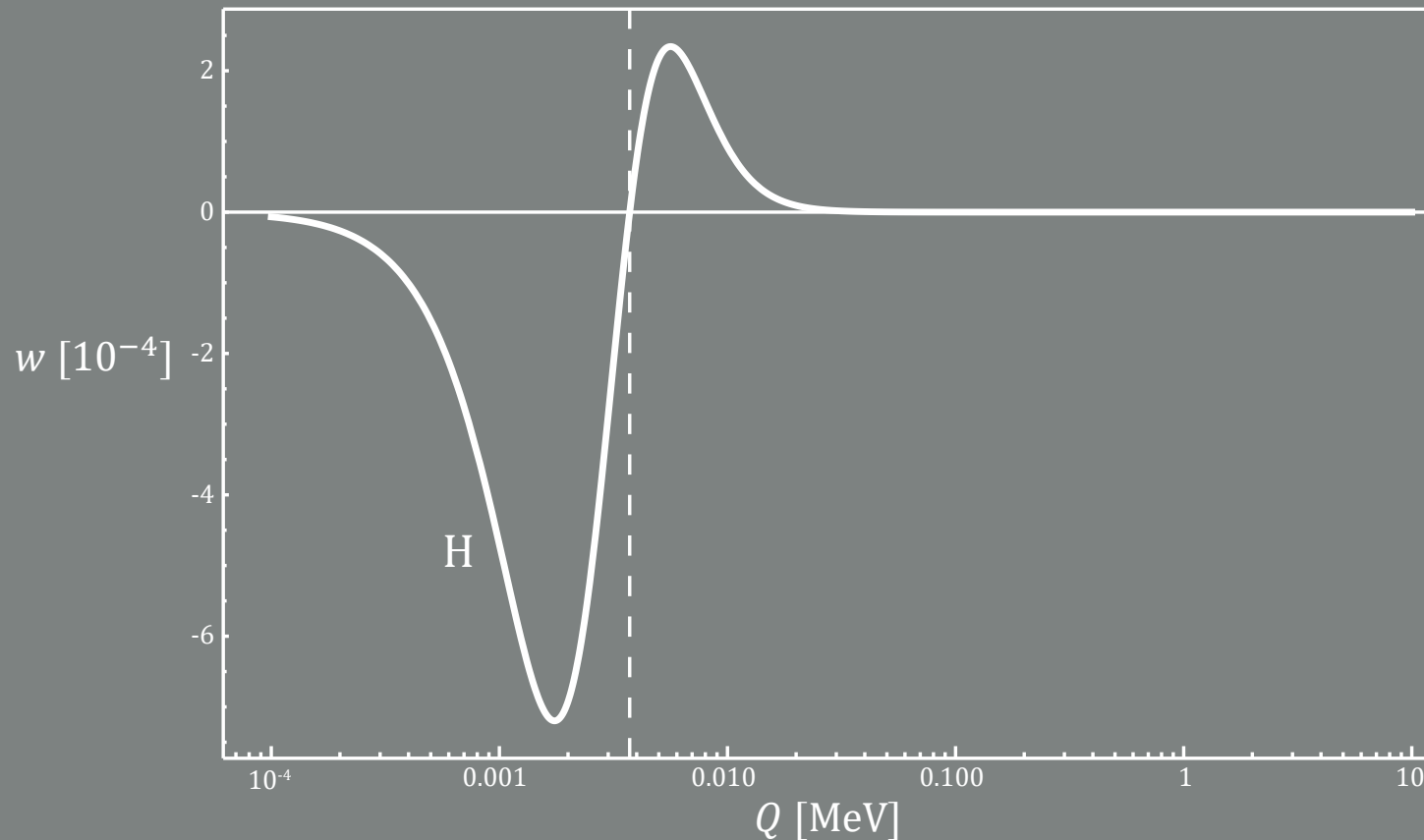
$$E_{2P-2S}^{\langle \text{FS} \rangle} = \int_0^\infty dQ \, w(Q) G_E(Q^2)$$

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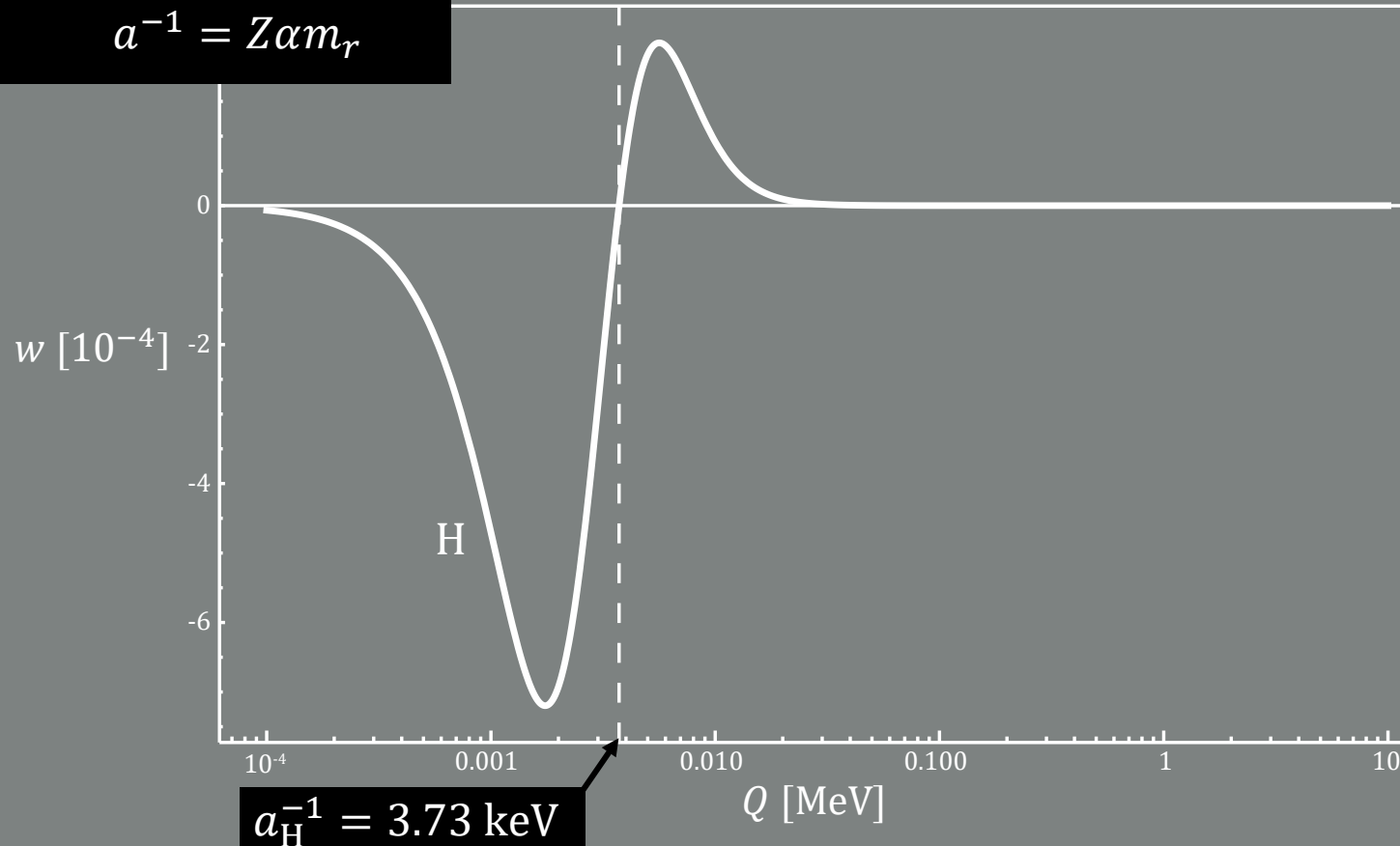
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Inv. Bohr radius:

$$a^{-1} = Z\alpha m_r$$

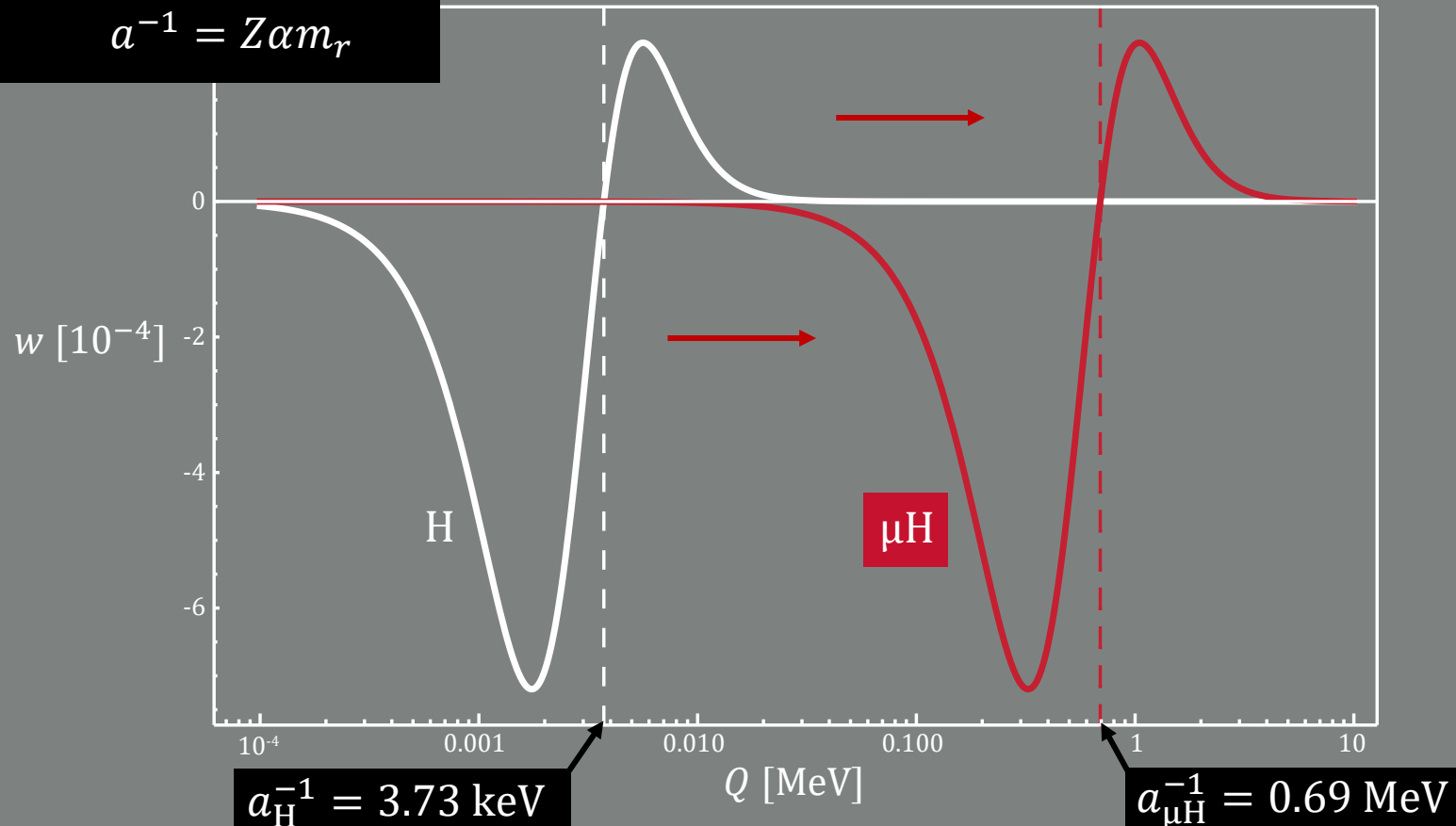


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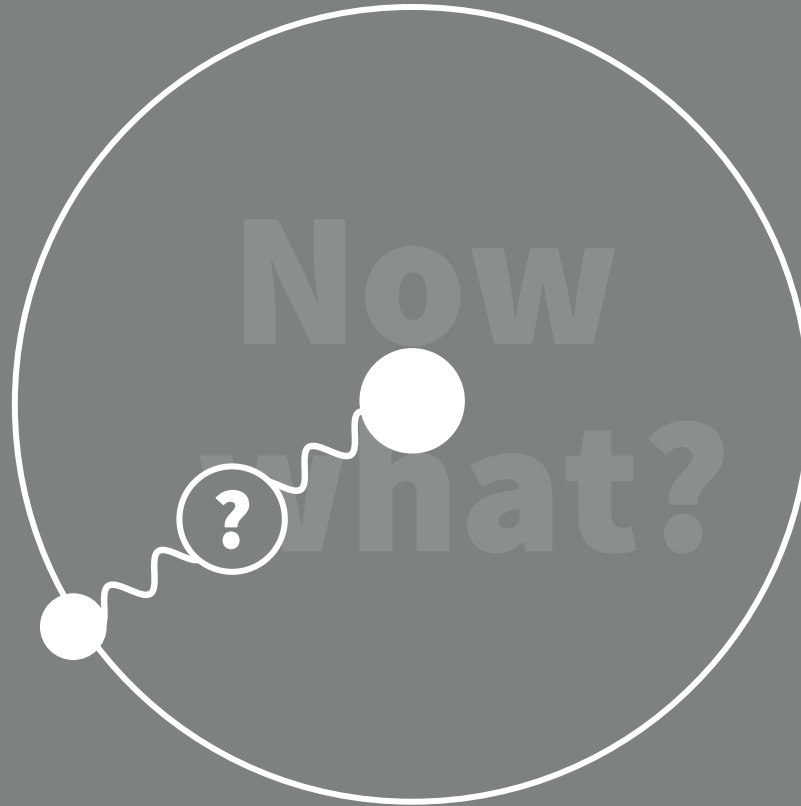
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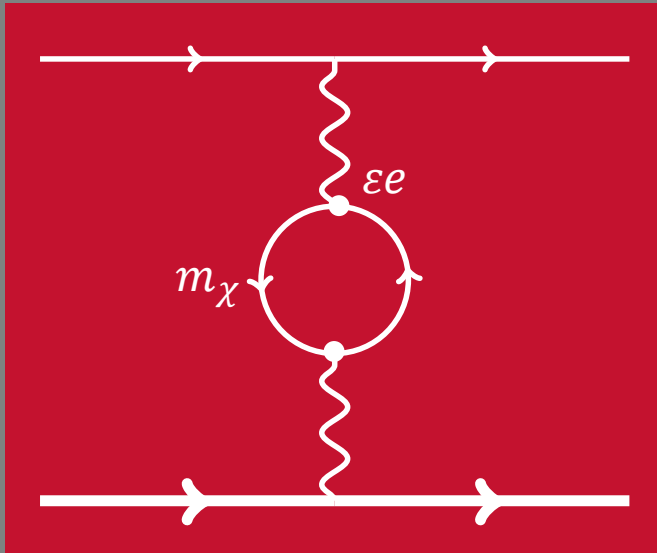
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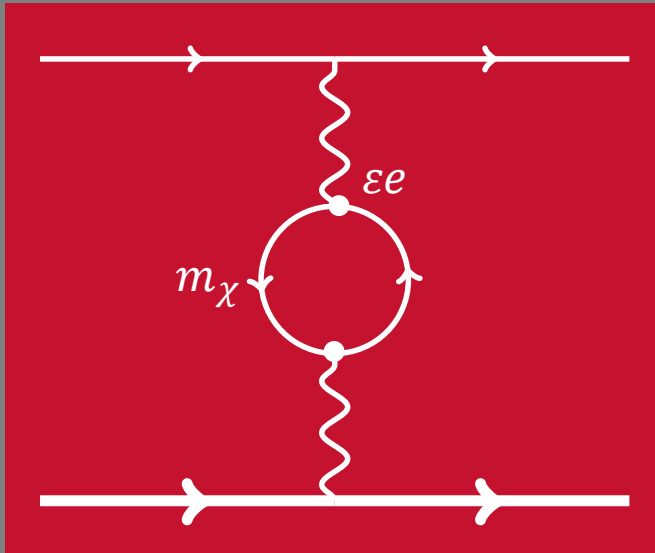
**Now
what?**



Dark Matter Fermion



Dark Matter Fermion



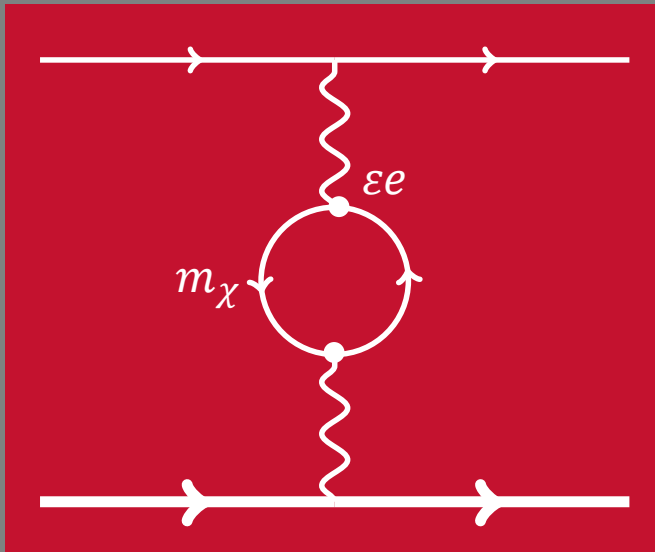
Lamb Shift Measurements

Mu: 4.3309(105) μeV

H: 4.37483(1) μeV

μH : 202.3706(23) meV

Dark Matter Fermion

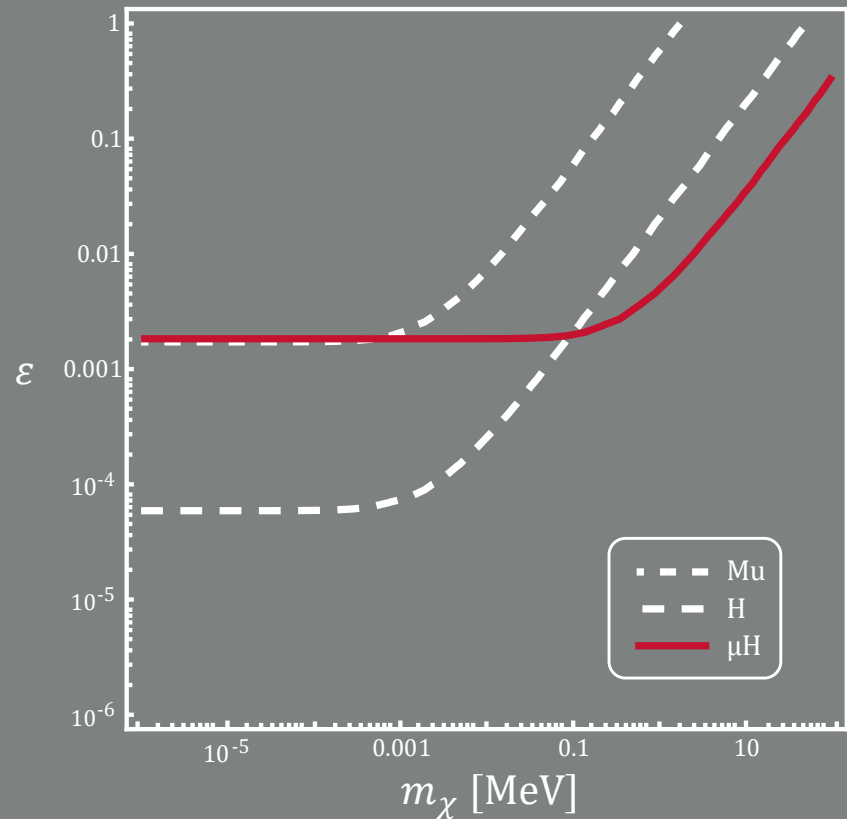


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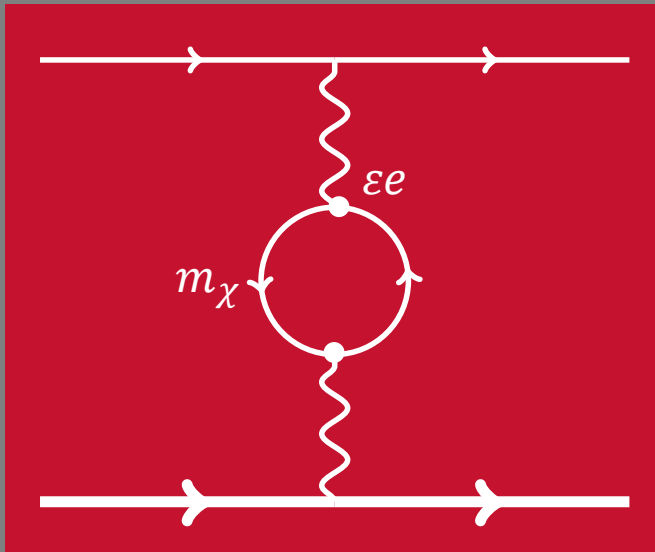
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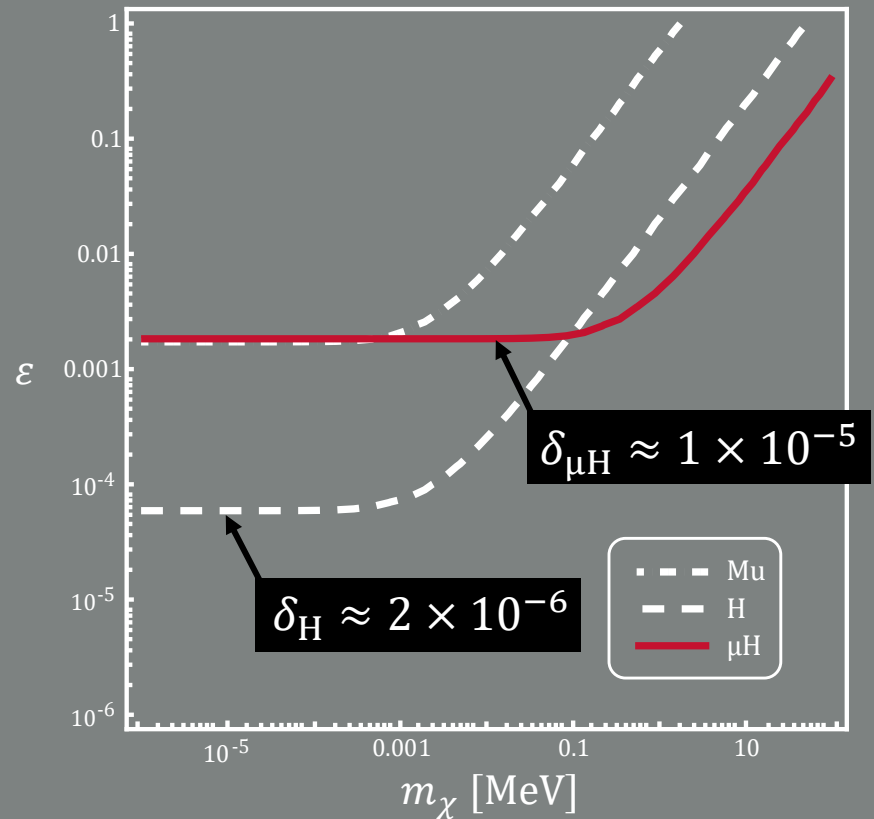


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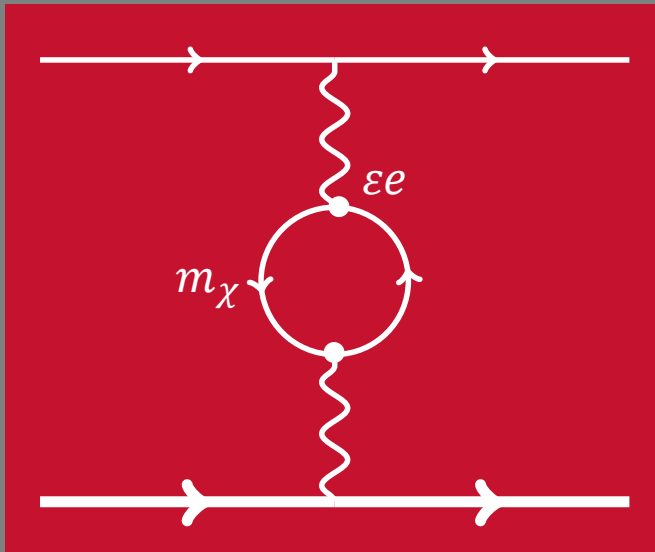
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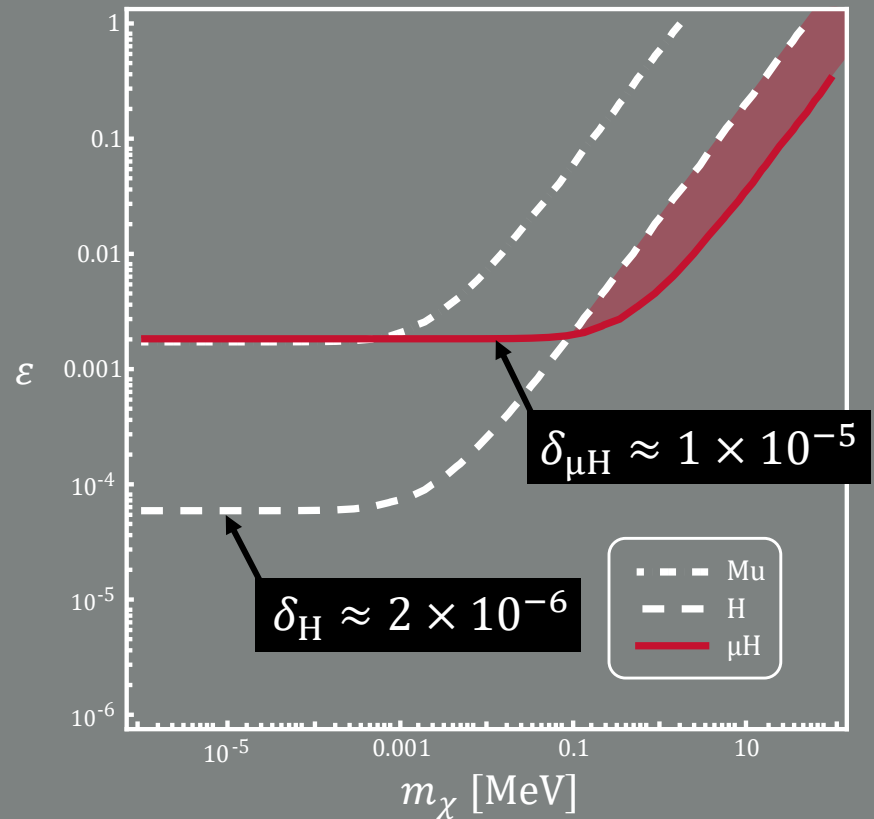


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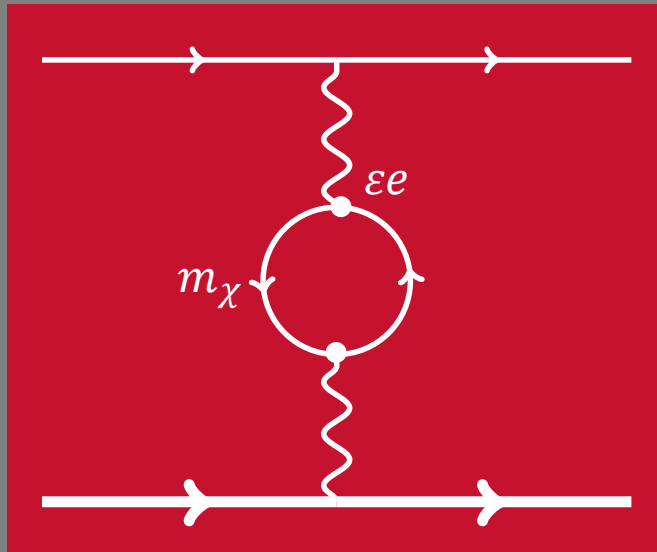
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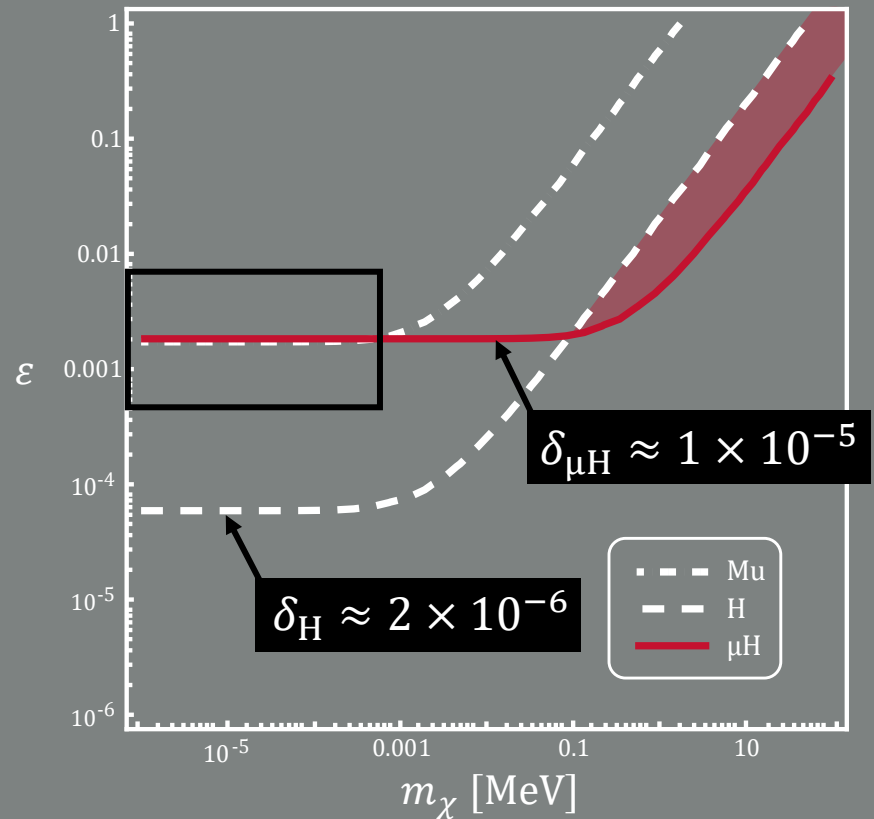


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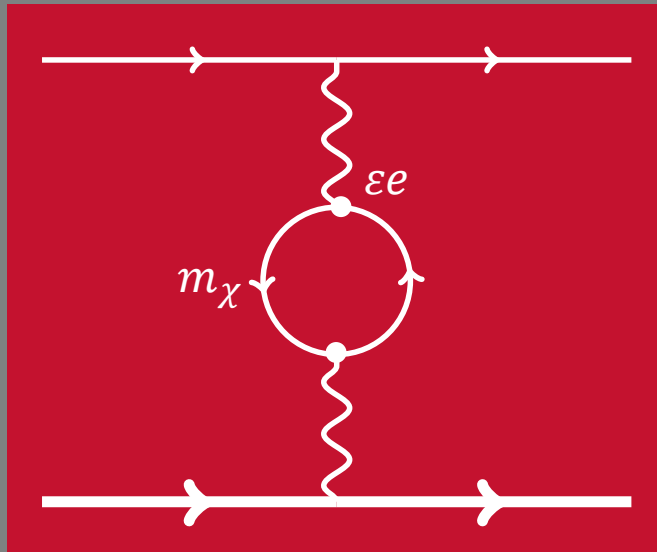
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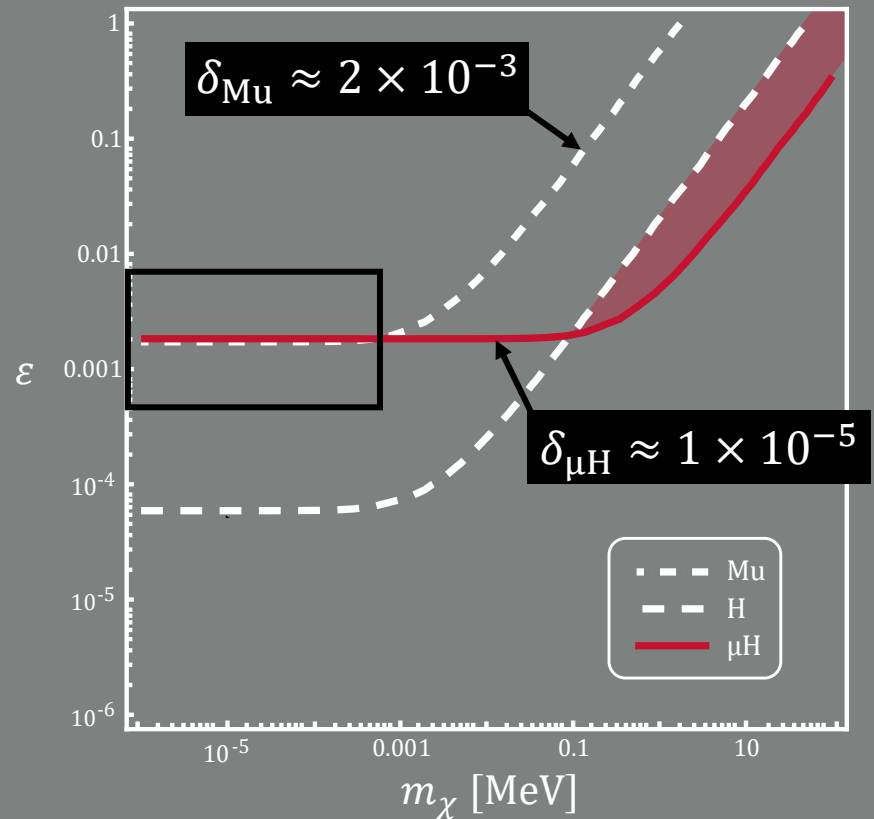


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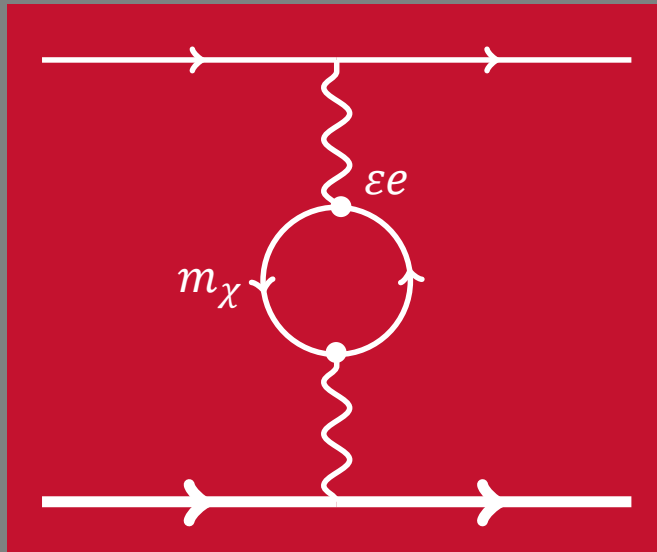
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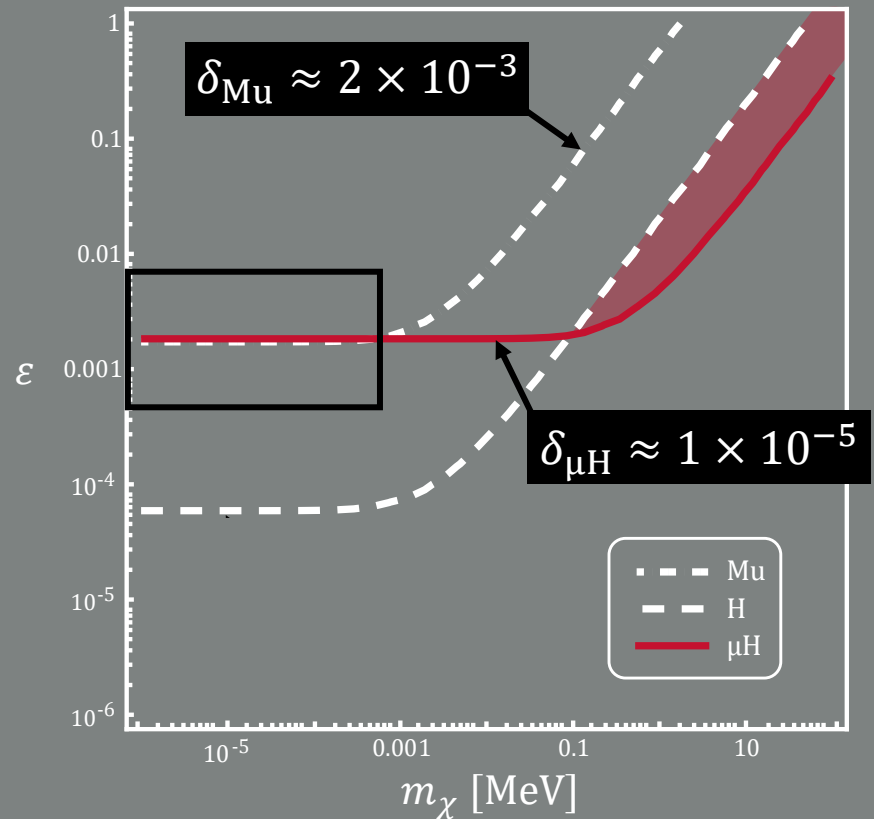
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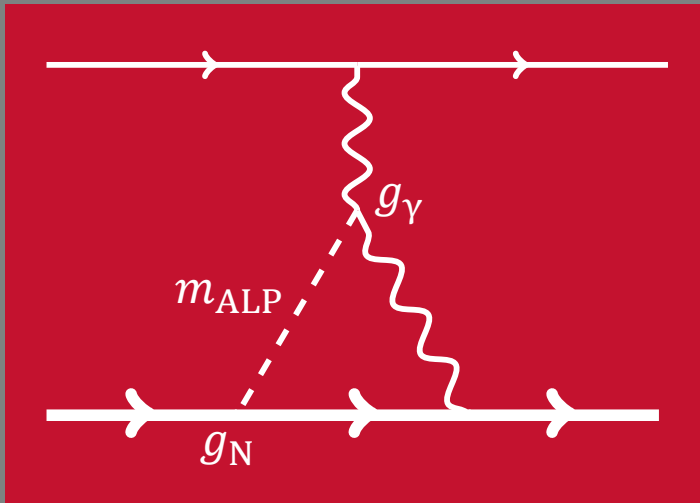
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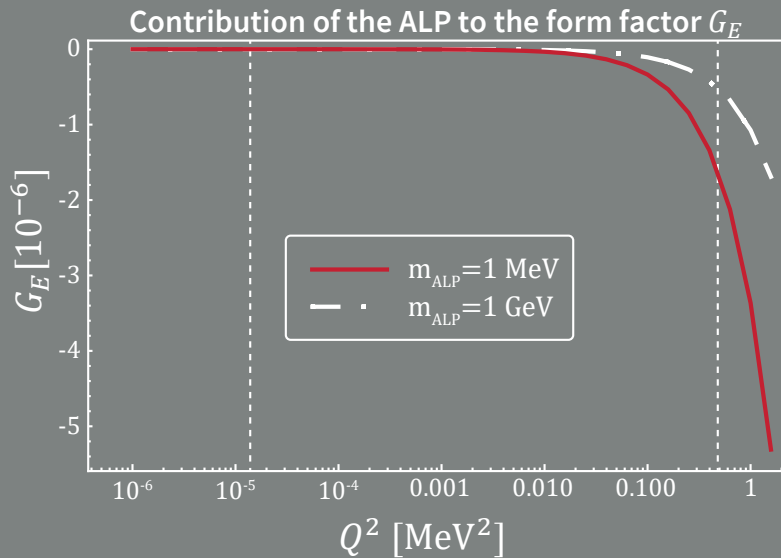
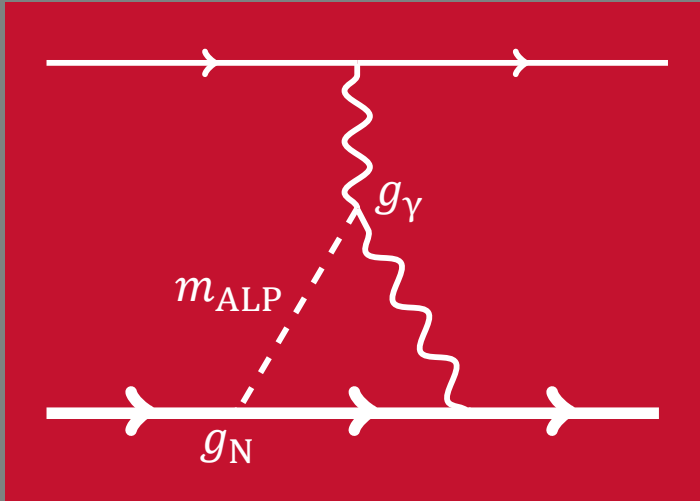
Sensitivity depends on experimental precision, Bohr radius, BSM parameters



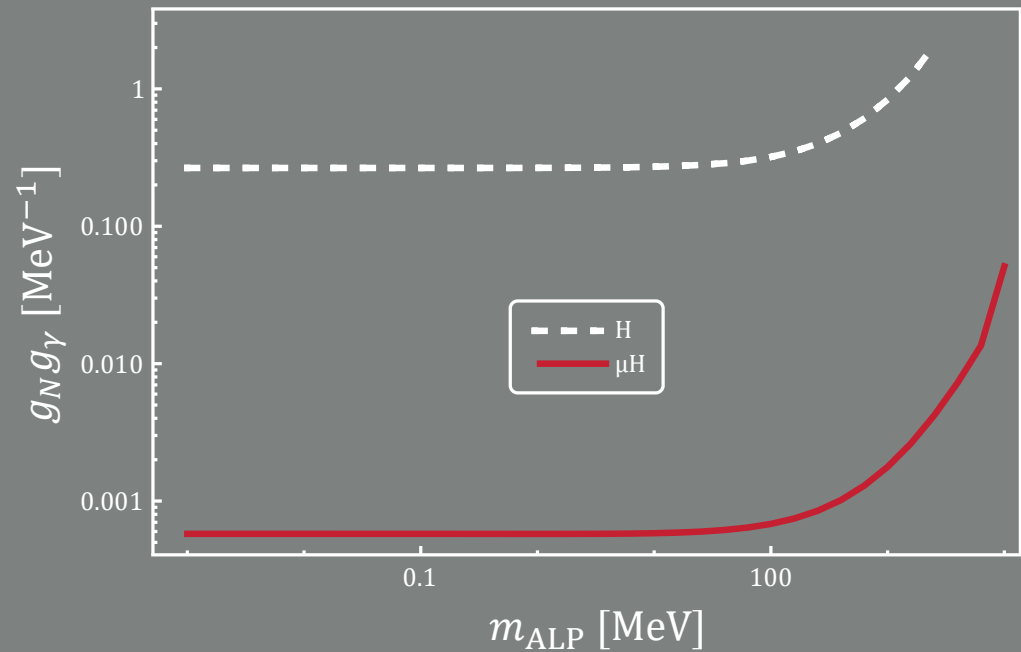
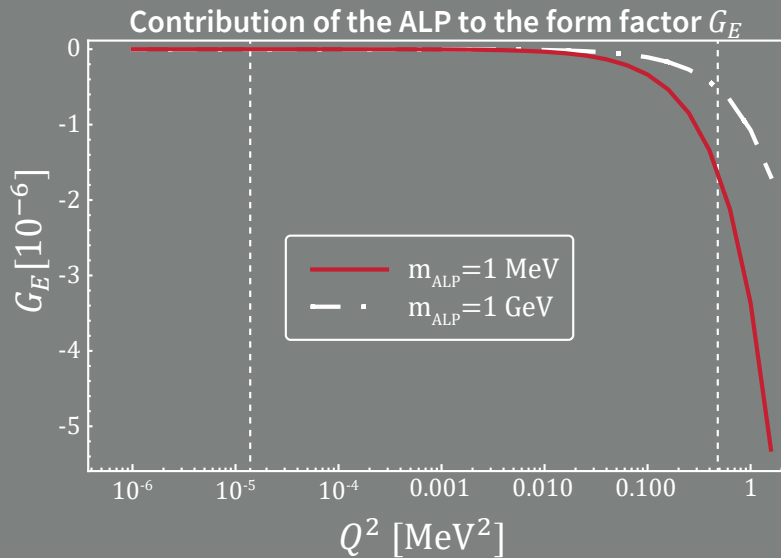
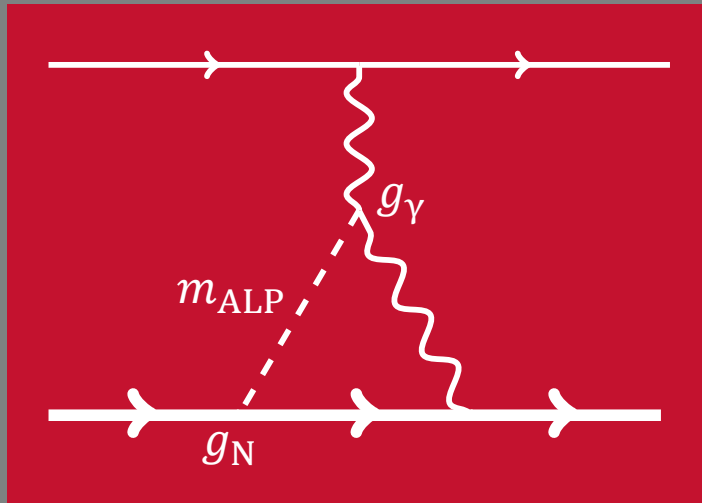
Axion-Like Particle



Axion-Like Particle

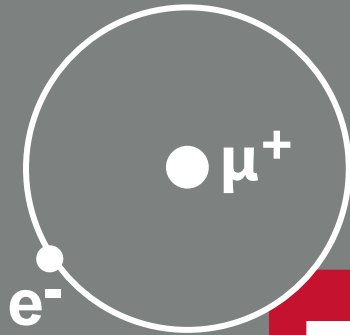


Axion-Like Particle



Conclusions

- Contributions to spectroscopy observables may be enhanced depending on the bound state
- Need to be careful with finite-size contributions that can't be seen in scattering
- Precision atomic spectroscopy holds potential for New Physics searches
- Sensitivity to New Physics depends on energy transition, experimental precision, (exotic) atom, BSM model
- Variety of (exotic) systems with different scales



Thank you!

