Nuclear radii from muonic atoms spectroscopy

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New perspectives in the charge radii determination for light nuclei
ETC*, July 30, 2025

Outline

"Live fast, die young!"

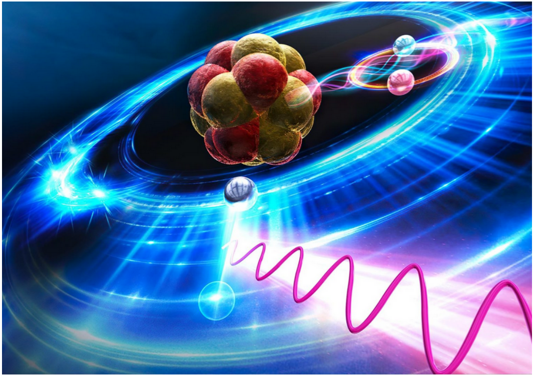
https://www.particlezoo.net

Disclaimer

"... you can tell how advanced a physical theory is by the number of interacting bodies it can't handle. Newton's law of gravity runs into problems with three bodies. General relativity has difficulty dealing with two bodies. Quantum theory is overextended for one body, and quantum field theory runs into trouble with no bodies - the vacuum."

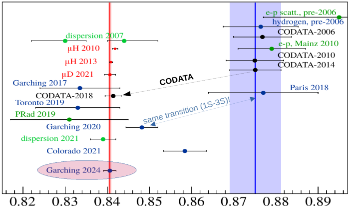
From "The Great mathematical problems" by Ian Stewar

Fig source: https://scitechdaily.com/



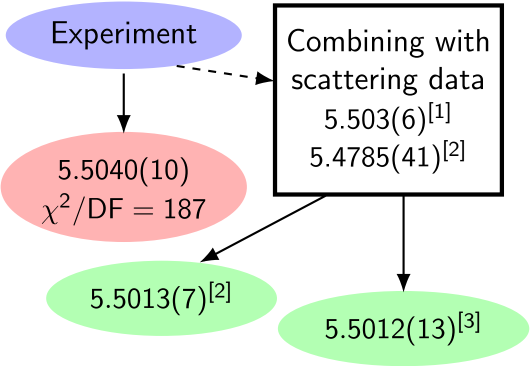
Muonic anomalies

Proton radius puzzle



From: Randolf Pohl, Monday

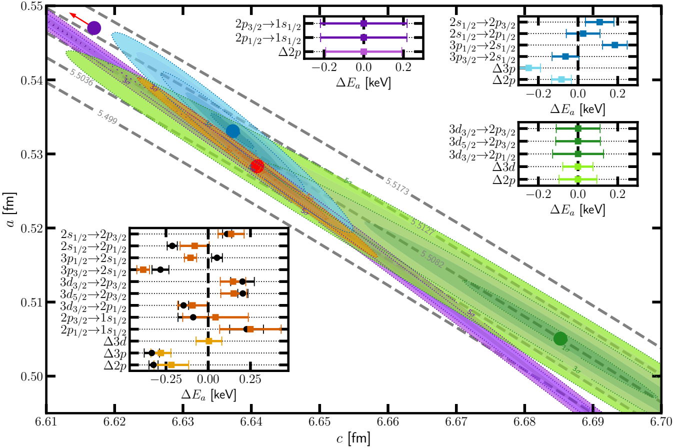
Muonic fine-structure anomaly, Pb



Observed in ^{208}Pb, ^{120}Sn, and ^{90}Zr

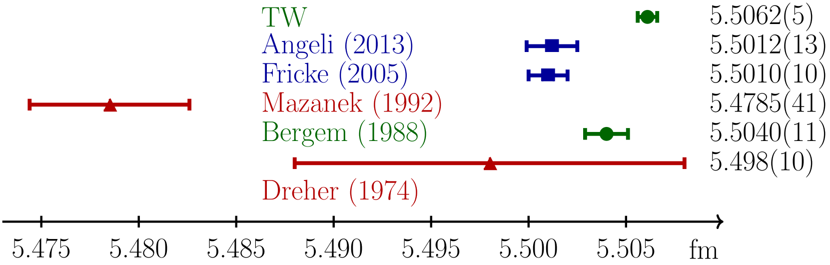
[1] Bergem *et al.*, PRC **37**, 2821 (1988)
[2] Fricke and Bernhardt, At. Data Nucl. Data Tables **60**, 177 (1995)
[3] Angeli and Marinova, At. Data Nucl. Data Tables **99**, 69 (2013)

Results for ^{208}Pb



Purple: $2p_{1,3/2} - 1s_{1/2}$, green: $3d_{3,5/2} - 2p_{1,3/2}$,
blue: $3p_{1,3/2} - 2s_{1/2}$ and $2s_{1/2} - 2p_{3,1/2}$, red: total

Compared with old data



- Mild tension with Bergem
- 3-4σ with Angeli, Fricke
- $\chi^2/\mathrm{DoF} = 9.5$: factor of 20 improvement
- Evidence against the existence of the anomaly

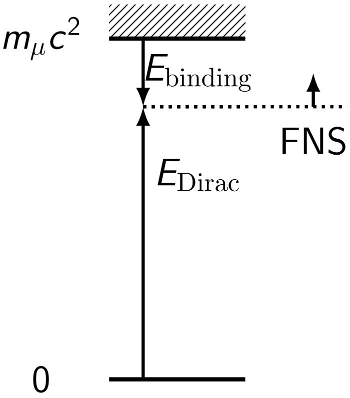
Normalized fit quality $\chi^2/\mathrm{DoF} = 1$: value 5.5062(17) fm
reduced model-dependence

Muonic atoms anomalies: H vs Pb

Fig: CERN / Sandbox Studio

Fig: The Simpsons

Dirac value and nuclear size



- Muons are close to the nucleus, relativistic → Dirac equation
- $m_\mu c^2 \approx 100$ MeV
  $E_{\mathrm{Dirac}} \approx 80$ MeV
  $E_{\mathrm{binding}} \approx 20$ MeV
- Extended nucleus: sphere, Fermi, deformed Fermi

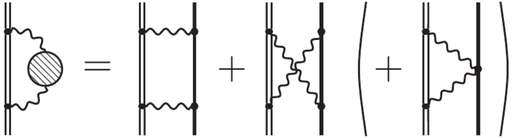
$$\rho_{a,c,\beta}(r_\mu, \vartheta_\mu) = \frac{N}{1 + e^{[r - c(1+\beta Y_{20}(\vartheta_\mu))]/a}}$$

- $\Delta E_{\mathrm{FNS}} \approx 10$ MeV

Simple formula → QED on a grid for Fermi distribution parameters

Our improvements

Nuclear polarization

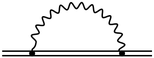


- field-theory approach, including transverse part
- state-of-art muonic and nuclear input, model dependence
- $0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and 1^+ excitation modes
- 4252 eV $\rightarrow$ 5712 eV

Relativistic recoil

- rigorous QED calculations
- $\Delta E_{\mathrm{rec}} = 385^* \rightarrow 3902$ eV

Self energy

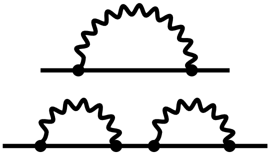


- $\Delta E_{\mathrm{SE}} = 3270(160)$, $3373 \rightarrow 3225(15)$ eV

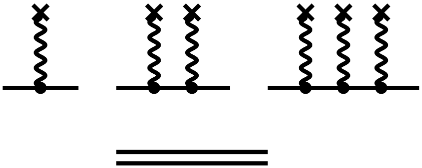
How can we do the calculations?

Perturbation theory:

in α



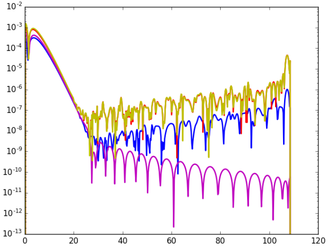
in αZ



The two approaches can be independent or complimentary!

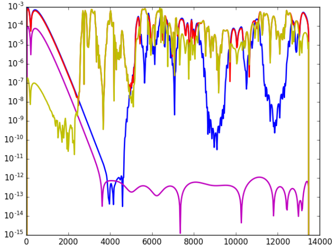
Complications

All-order in αZ: Fury picture



$Z = 92$

total=0pot+1pot+2pot



$Z = 1$

NRQED:

expansions in α, αZ and rms

III.D	$\alpha(Z\alpha)^4$	Relativistic with $eVP^{(1)}$
III.E	$\alpha^2(Z\alpha)^4$	Relativistic with $eVP^{(2)}$
III.F	$\alpha(Z\alpha)^4$	$\mu SE^{(1)} + \mu VP^{(1)}$, LO
III.G	$\alpha(Z\alpha)^5$	$\mu SE^{(1)} + \mu VP^{(1)}$, NLO
III.H	$\alpha^2(Z\alpha)^4$	$\mu VP^{(1)}$ with $eVP^{(1)}$
III.I	$\alpha^2(Z\alpha)^4$	$\mu SE^{(1)}$ with $eVP^{(1)}$
III.J	$(Z\alpha)^5$	Recoil
III.K	$\alpha(Z\alpha)^5$	Recoil with $eVP^{(1)}$

very fast too many terms:
feasible for $Z > 10$?..

Recoil with reduced mass

Schödinger equation:

$$H\psi(\mathbf{r}_1, \mathbf{r}_2) = E\psi(\mathbf{r}_1, \mathbf{r}_2)$$

$$\left(-\frac{\hbar^2}{2m_e}\nabla_e - \frac{\hbar^2}{2m_N}\nabla_N + V(|\mathbf{r}_e - \mathbf{r}_N|)\right)\psi(\mathbf{r}_e, \mathbf{r}_N) = E\psi(\mathbf{r}_e, \mathbf{r}_N)$$

$$\{\mathbf{r}_e, \mathbf{r}_N\} \to \left\{\mathbf{r} = \mathbf{r}_e - \mathbf{r}_N, \mathbf{R} = \frac{m_e\mathbf{r}_e + m_N\mathbf{r}_N}{m_e + m_N}\right\}$$

$$\left(-\frac{\hbar^2}{2\mu}\nabla + V(r)\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Dirac equation:

$$H_D = \alpha\mathbf{p} + \beta m + V$$

Relativistic recoil

PHYSICAL REVIEW LETTERS **130,** 053002 (2023)

QED Theory of the Nuclear Recoil with Finite Size

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$$\Delta E_{\mathrm{rec,LO}} = \langle a | \frac{\mathbf{p}^2}{2m} | a \rangle$$

$$\Delta E_{\mathrm{rec,HO}} = \text{some other integral}$$

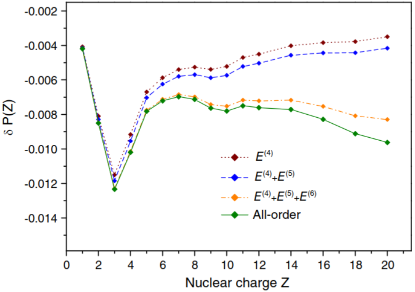

FIG. 1. Finite-size nuclear recoil correction for the $1s$ state of H-like ions, in terms of function $\delta P = E_{\mathrm{recfns}}/[(m^2/M)(Z\alpha)^5/\pi]$.

Polarization or polarizability?

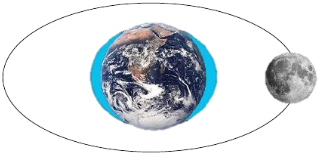

Image source: www.universetoday.com

- nucleus plus muon together
- "measure of entanglement"
- sometimes can be separated (approximately)

Nuclear polarization $\approx$ Nuclear polarizability $\times$ muonic factor

Nuclear polarization: light vs heavy

2nd order PT $\Delta E_p = \sum_{N\neq 0} \langle 0' | \Delta H_c | N \rangle \left[\sum_n \frac{|n\rangle\langle n|}{\epsilon_0 - \epsilon_n - \omega_N} \right] \langle N | \Delta H_c | 0' \rangle$ *Ericson, Hüfner 1972* *Friar 1977*

Perturbation: transition induced by Coulomb interaction $\Delta H_c(\vec{r}) = -\alpha \int \frac{d^3\vec{r}_N}{|\vec{r} - \vec{r}_N|} \hat{\rho}(\vec{r}_N)$

First approximation:

nucleus much smaller than atom

nuclear energy splittings much larger than atomic energy $\left[\sum_n \frac{|n\rangle\langle n|}{\cancel{\epsilon_0 - \epsilon_n} - \omega_N} \right]$

Npol-induced potential - δ-function at origin; relativistic treatment of nuclear system

From: Misha Gorshteyn, Monday

- overlap of WF is significant $\Rightarrow$ no spacial factorization
- nuclear and muonic energy scales are the same $\Rightarrow$ no selection
- αZ is not small $\Rightarrow$ Furry picture

	NP	Natalia et al 2504.19977		MG
^{208}Pb ΔE_{1S}:		5.7(6) keV	vs.	4.9(7) keV

Outline

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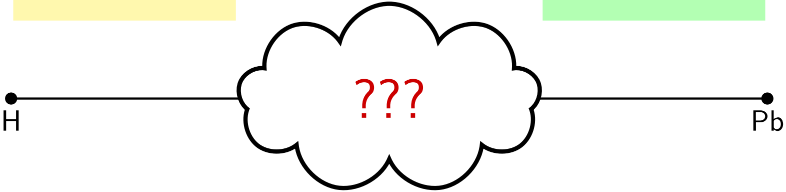
Outlook

NRQED:
α, αZ, rms
many terms

Furry QED:
α
numerics



2p-1s: one line
no sensitivity to 2pF
model-independent

2p3 and 2p1-1s
can determine a, c
model-dependent

Workshop

Precise nuclear charge radii and beyond (Z>10 ;-)

January 26 – 30, 2026 @ MPIK, Heidelberg