Electroweak Structure of Nuclei

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EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

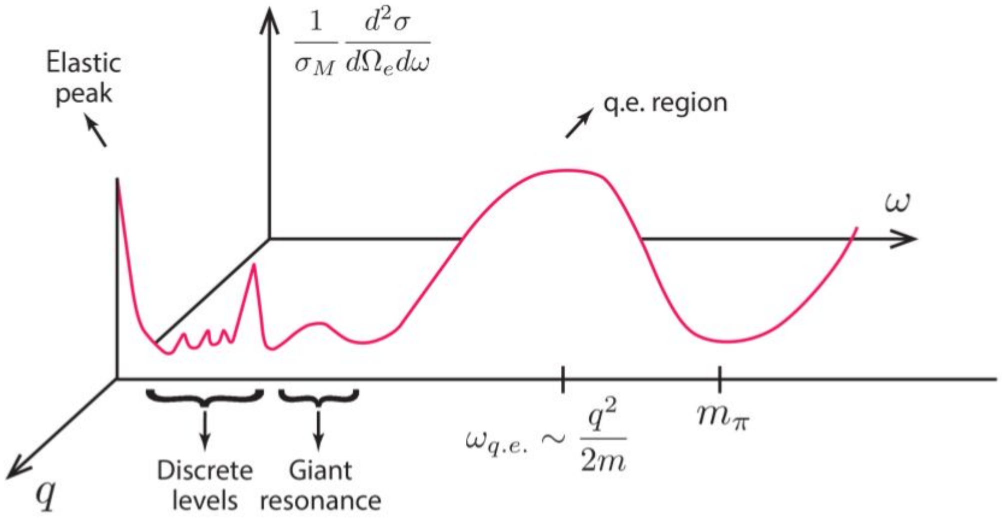
New perspectives in the charge radii determination for light nuclei

FRIB Theory Alliance

INFN Istituto Nazionale di Fisica Nucleare

EURO-LABS EUROPEAN LABORATORIES FOR ACCELERATOR BASED SCIENCES

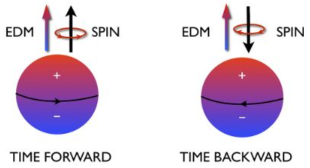
Electron-Nucleus Scattering Cross Section



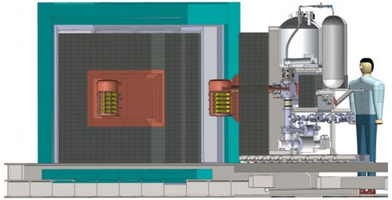
Energy and momentum transferred (ω,q)

Current and planned experimental programs rely on theoretical calculations at different kinematics

Ground States' Electroweak Moments, Form Factors, Radii



Neutrinoless Double Beta Decay, Muon-Capture



Accelerator Neutrino Experiments, Lepton-Nucleus XSecs

T2K

(ω,q)~0 MeV

ω~few MeVs q~0 MeV

ω~few MeVs q~10^2 MeV

ω~tens of MeVs

ω~10^2 MeV

FRIB

RIKEN NiSHiNA CENTER

GANIL

Electromagnetic Decay, Beta Decay, Double Beta Decay & inverse processes

JINA-CEE

Nuclear Rates for Astrophysics

DUNE

Jefferson Lab

Strategy

Validate the Nuclear Model against available data for strong and electroweak observables

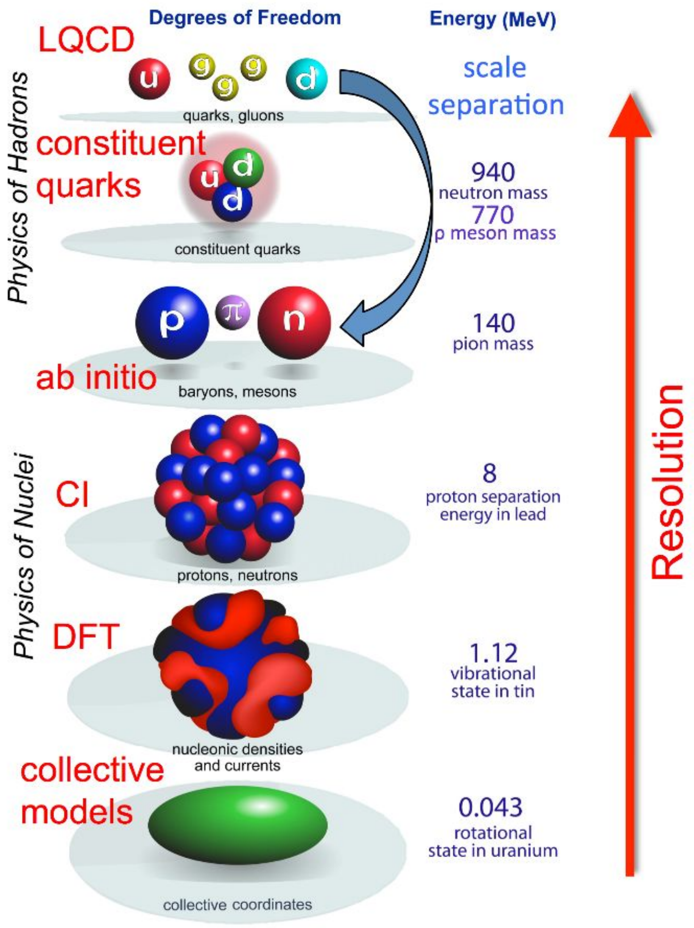
- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- Electron-Nucleus Scattering Cross Sections, ...

Use attained information to make (accurate) predictions for BSM searches and precision tests

- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- Neutrino-Nucleus Scattering Cross Sections, ...
- ...

From Quarks to Nuclei

- Nuclei are complex systems made of interacting protons and neutrons, which in turns are composite objects made of interacting constituent quarks
- All fundamental forces are at play in nuclei
- **EFTs** low-energy approximations of QCD whose d.o.f. are bound states of QCD (e.g., protons, neutrons, pions, …); used to construct many-nucleon interactions and currents
- Accurate inputs at the single- and few-nucleon level are required (e.g., from **LQCD**)

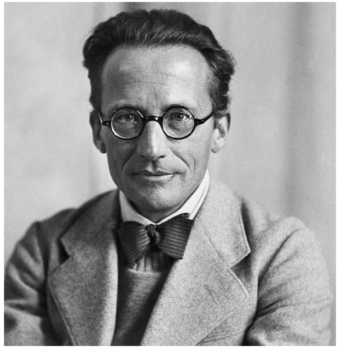


Microscopic (or *ab initio*) Description of Nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

- Accurate understanding of the interactions/correlations between nucleons in **paris**, **triplets**, ... (**two- and three-nucleon forces**)
- Accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated nucleon-pairs, … (**one- and two-body electroweak currents**)
- **Computational methods** to solve the many-body nuclear problem of strongly interacting particles



Erwin Schrödinger

$$H\Psi = E\Psi$$

Many-body Nuclear Problem

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, s_1, s_2, \ldots, s_A, t_1, t_2, \ldots, t_A)$$

Ψ are spin-isospin vectors in 3A dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components

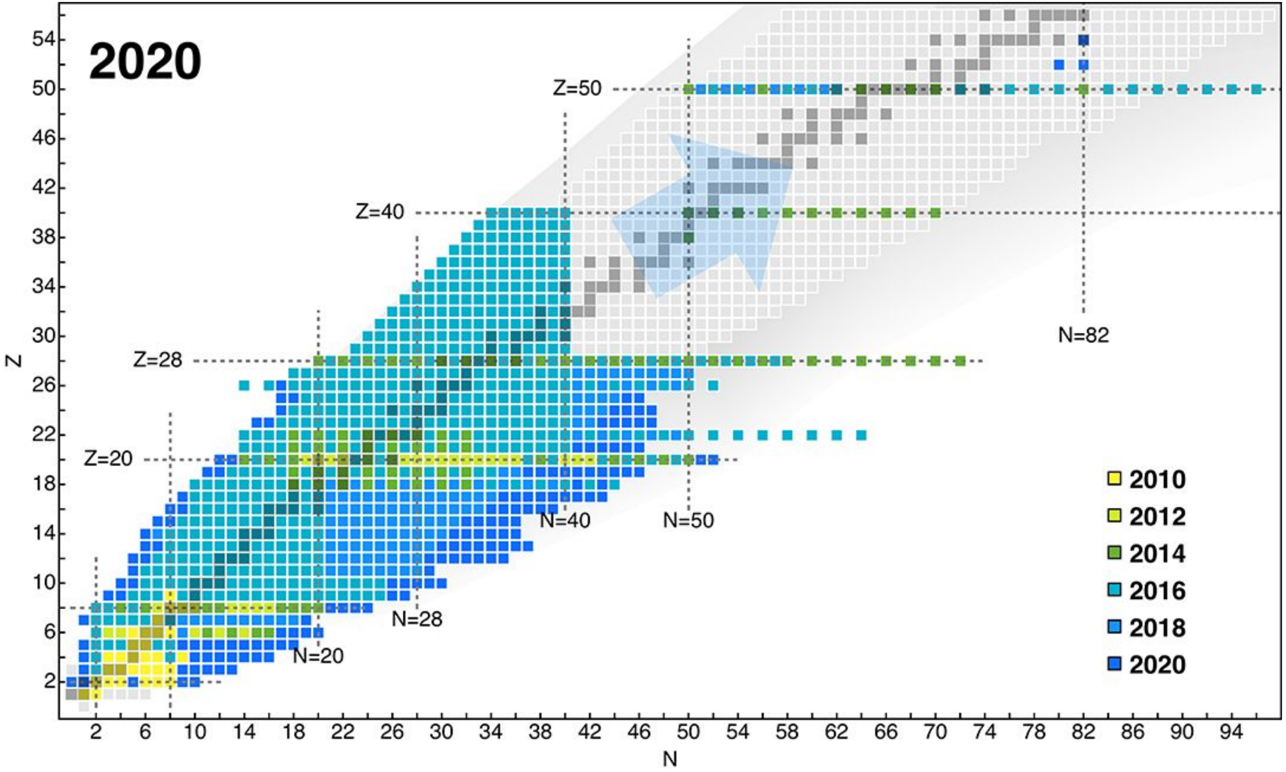
Develop Computational Methods to solve (numerically) exactly or within approximations that are under control the many-body nuclear problem

http://exascaleage.org/np/

^{4}He : 96
^{6}Li : 1280
^{8}Li : 14336
^{12}C : 540572

$$H\Psi = E\Psi$$

Current Status



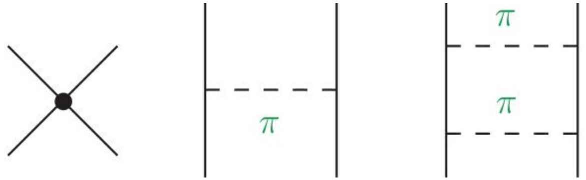
H. Hergert
Front. Phys.
07 October 2020

Many-body Nuclear Interactions

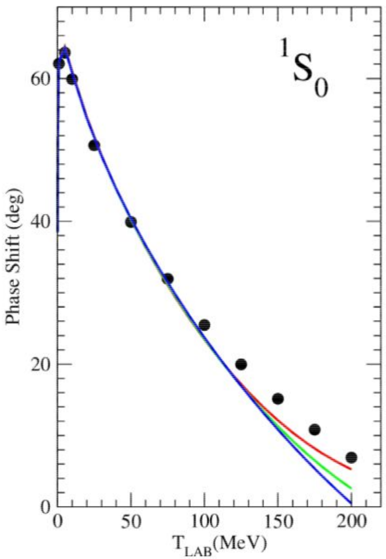
Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



Contact term: short-range
Two-pion range: intermediate-range $r \propto (2\, m_\pi)^{-1}$
One-pion range: long-range $r \propto m_\pi^{-1}$



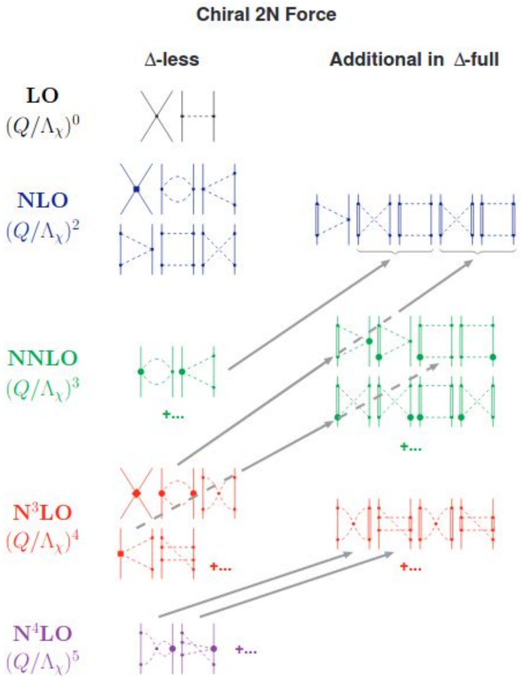
SP et al. PRC80(2009)034004

Hideki Yukawa

AV18+UIX; AV18+IL7
Wiringa, Schiavilla, Pieper *et al*.

chiral πNΔ
N3LO+N2LO Piarulli *et al*. **Norfolk Models**

Norfolk Two- and Three-body Potentials



Norfolk Chiral Potentials

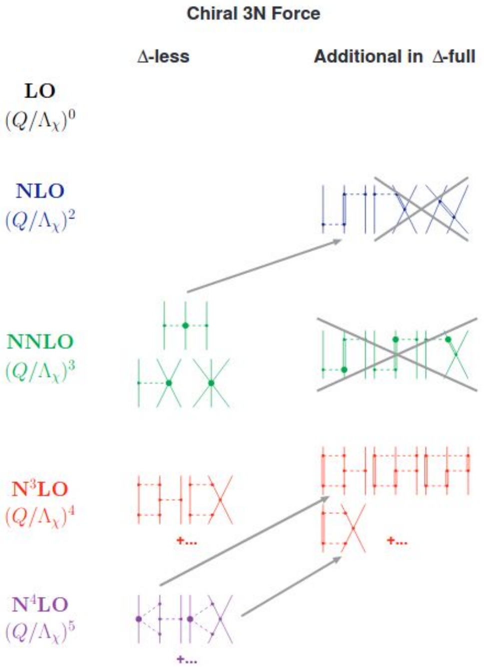
NV2: two-body

26 LECs fitted to np and pp Granada database (2700-3700 data points; lab energies up to 125-200 MeV) with a chi-square/datum ~1

NV3: three-body

2 LECs

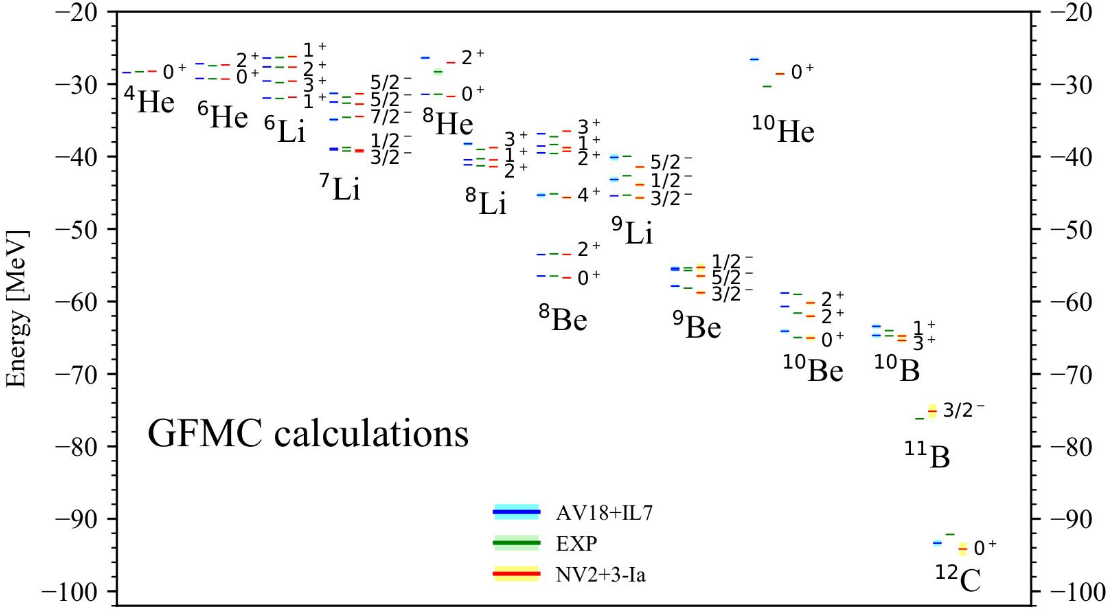
Piarulli *et al*. PRC91(2015) PRC94(2016)



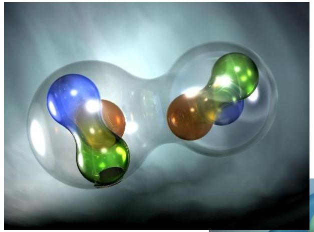
Figs. credit Entem and Machleidt Phys.Rept.503(2011)1

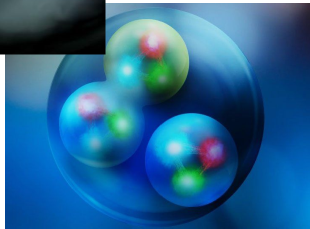
8 Models depending on the fitting strategy adopted for the LECs

Energies

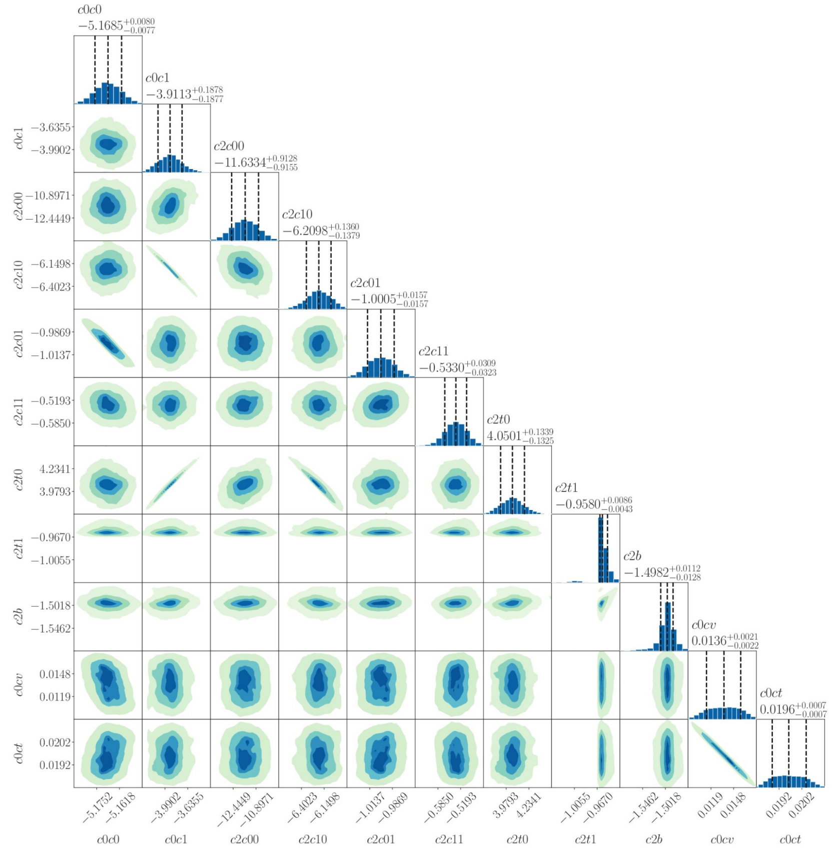


Piarulli *et al.* PRL120(2018)052503





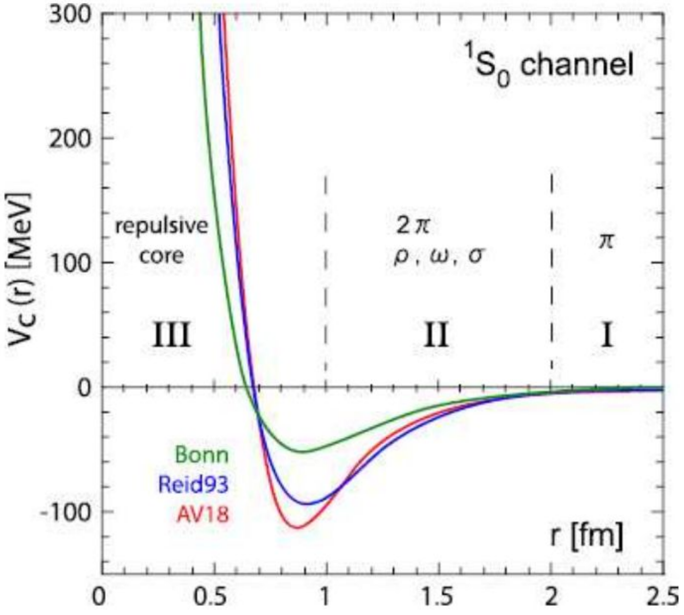
Optimization of Nuclear Two-body Interactions



Development and Optimization of two-body interactions based on Bayesian methods

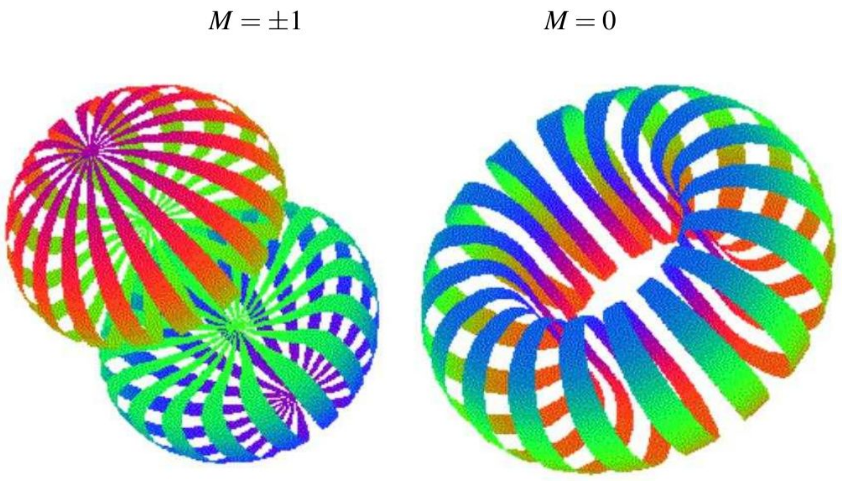
Jason Bub *et al.* arxiv:2408.02480 (2024)

Nucleon-Nucleon Potential



Aoki *et al.* Comput.Sci.Disc.1(2008)015009

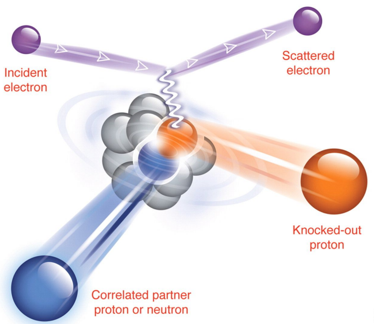
The Deuteron

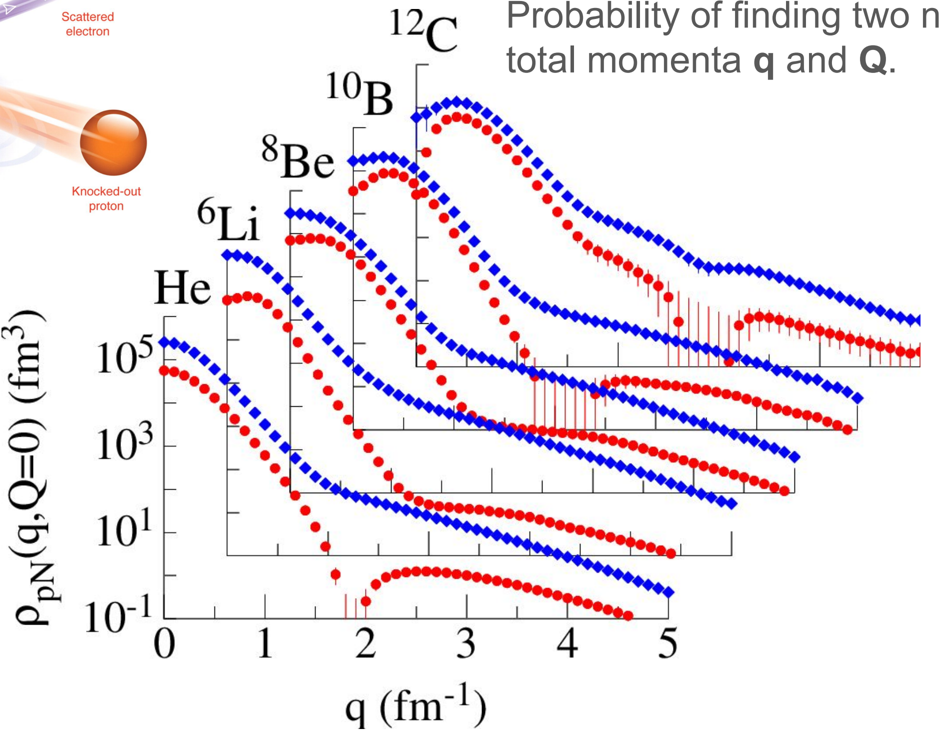


Constant density surfaces for a polarized deuteron in the $M = \pm 1$ (left) and $M = 0$ (right) states

Carlson and Schiavilla Rev.Mod.Phys.70(1998)743

Two-nucleon correlations & momentum distributions





pp-pairs; np-pairs

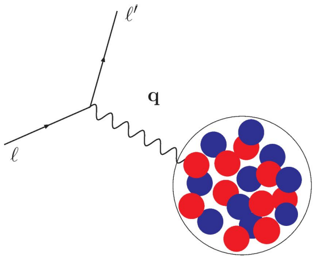
Probability of finding two nucleons with relative and total momenta **q** and **Q**.

Tensor correlations lead to large differences in the np versus pp distributions.

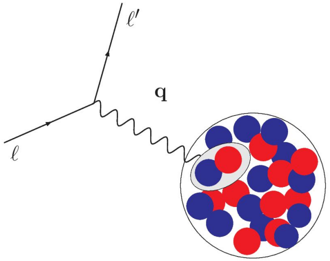
These differences are observed in A(e, e′np) and A(e, e′pp) reactions.

Schiavilla Carlson Wiringa Pieper
PRL98(2007) & PRC89(2014)

Many-body Nuclear Electroweak Currents



one-body



two-body

Nuclear Charge Operator

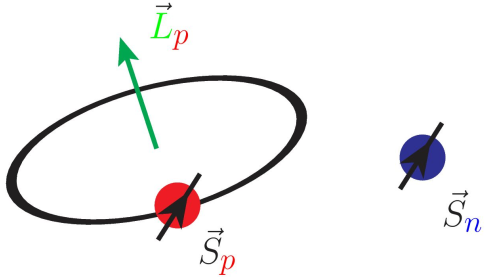
$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$

- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

$$\mathbf{q} \cdot \mathbf{j} = [\, H,\, \rho\,] = [t_i + v_{ij} + V_{ijk},\, \rho\,]$$



Magnetic Moment: Single Particle Picture

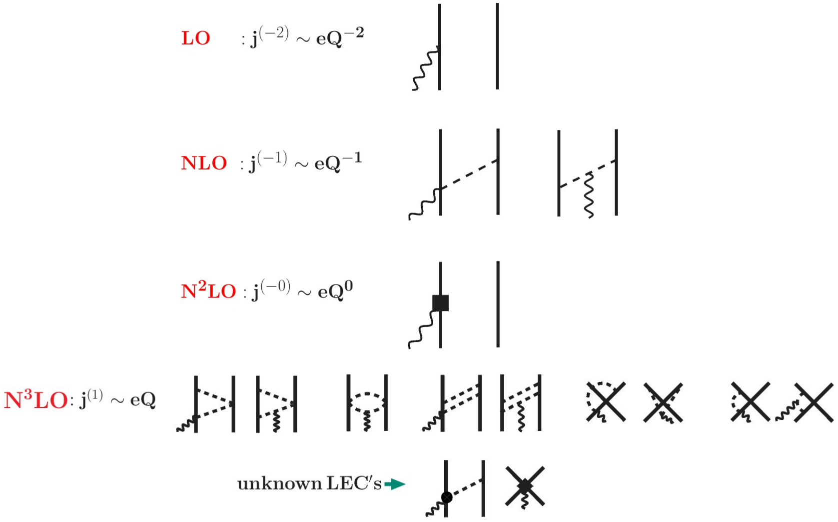
Many-body Currents

- **Meson Exchange Currents** (MEC)

Constrain the MEC current operators by imposing that the current conservation relation is satisfied with the AV18 two-body potential

- **Chiral Effective Field Theory Currents**

Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (**LECs**), need to be **determined by either fits to experimental data or by Lattice QCD calculations**

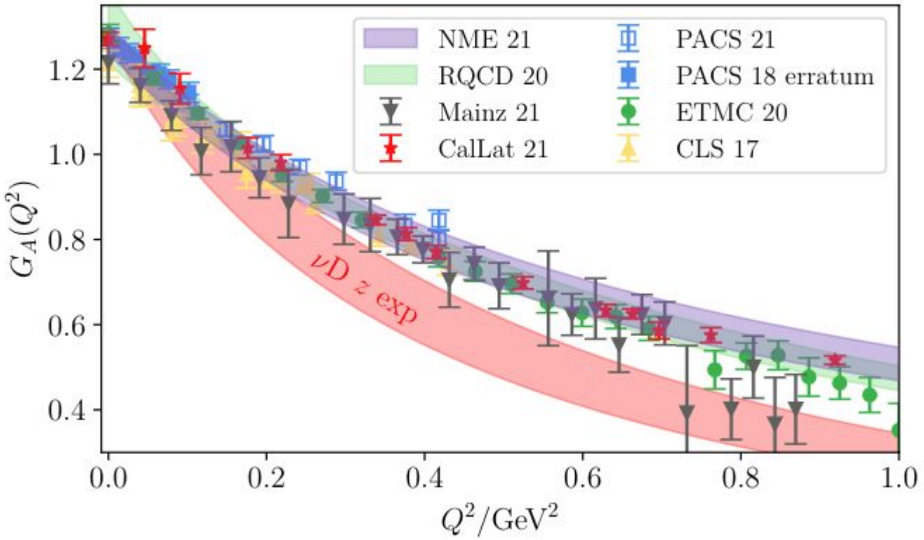


Electromagnetic Current Operator

SP *et al.* PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001, PRC87(2013)014006
Park *et al.* NPA596(1996)515, Phillips (2005)
Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

LQCD for single- and few-nucleon properties

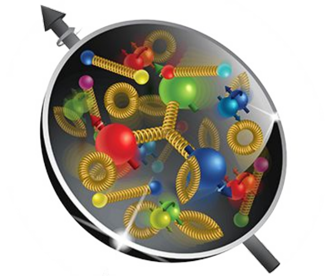
Microscopic approaches rely on accurate inputs at the single- and few-nucleon level from experimental data (where available) and Lattice QCD theoretical calculations.



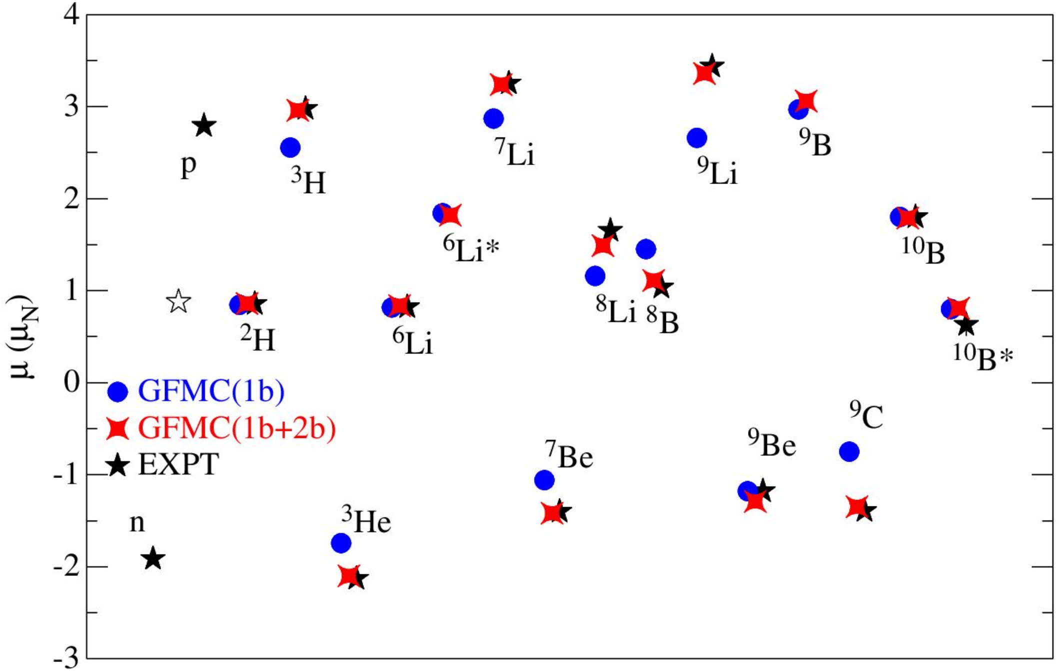
Snowmass WP: Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators; arXiv:2203.09030, Meyer, Walker-Loud, Wilkinson (2022)

Building blocks of ab initio nuclear approaches:

Nucleonic form factors
Transition form factors
Pion production amplitudes
Two-nucleon couplings (strong and EW)
…

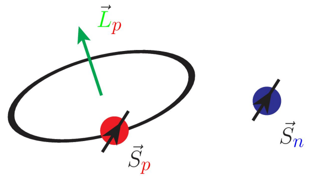


Magnetic Moments of Light Nuclei



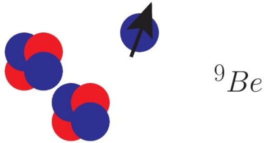
SP *et al*. PRC87(2013)035503

Single particle picture

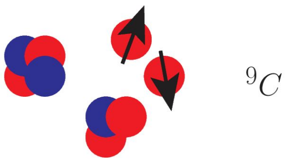


$$\mu_N(1\mathrm{b}) = \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

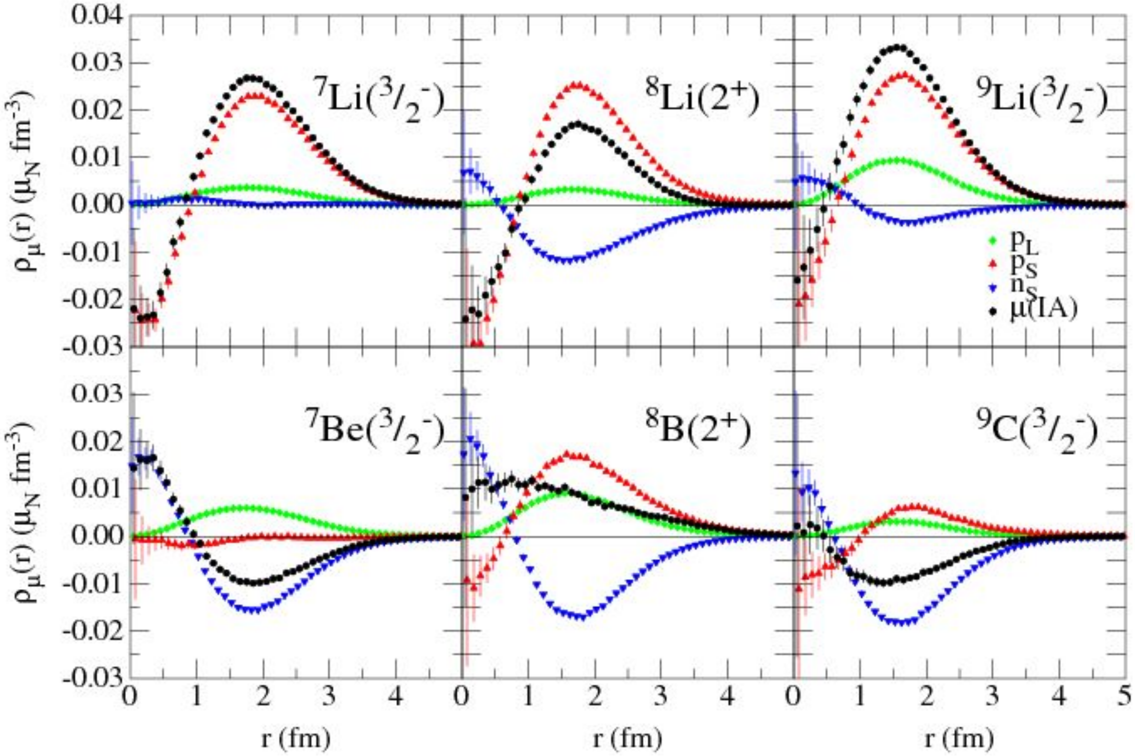
Small two-body current effects



Large two-body current effects



One-body magnetic density

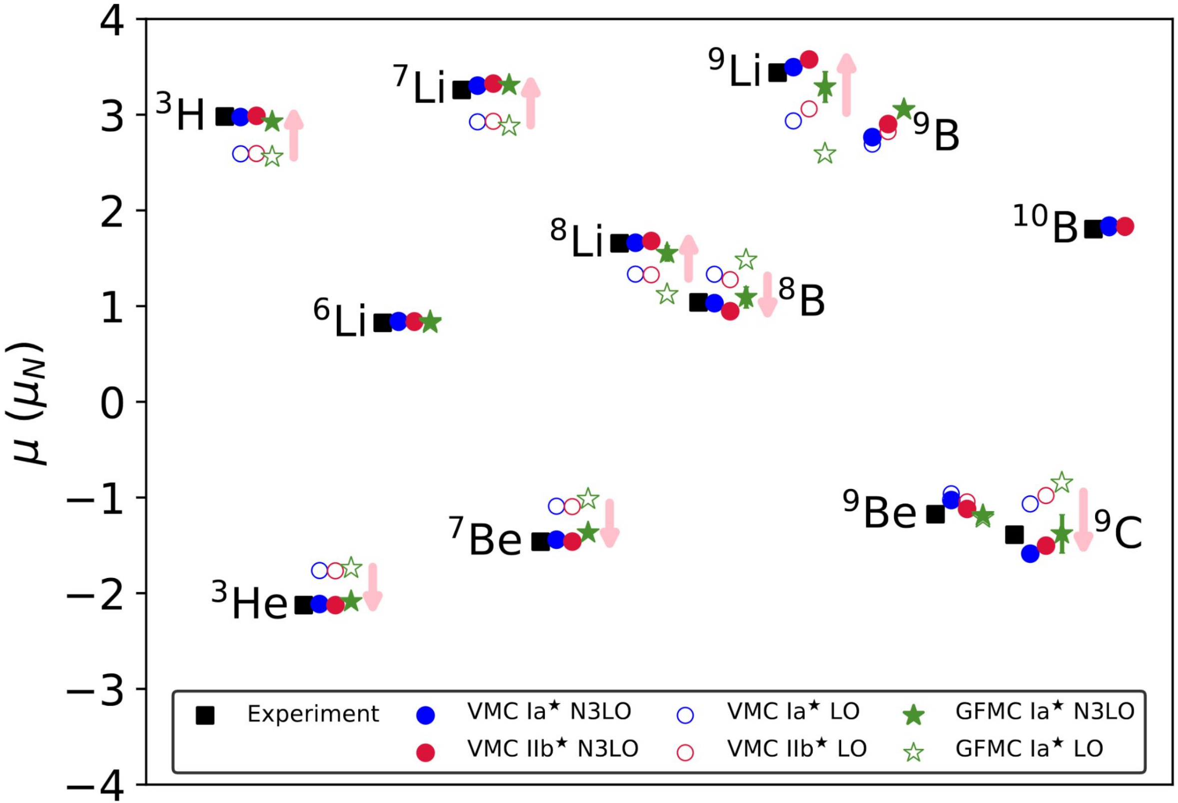


$$\mu^{1b} \propto \int \rho_M^{1b}(r)dr$$

r single particle coordinate from the c.m.

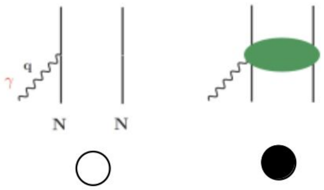
$$\mu_i = \mu_N \left[(L_i + g_p S_i)\frac{1+\tau_{i,z}}{2} + g_n S_i \frac{1-\tau_{i,z}}{2} \right]$$

Magnetic moments in light nuclei

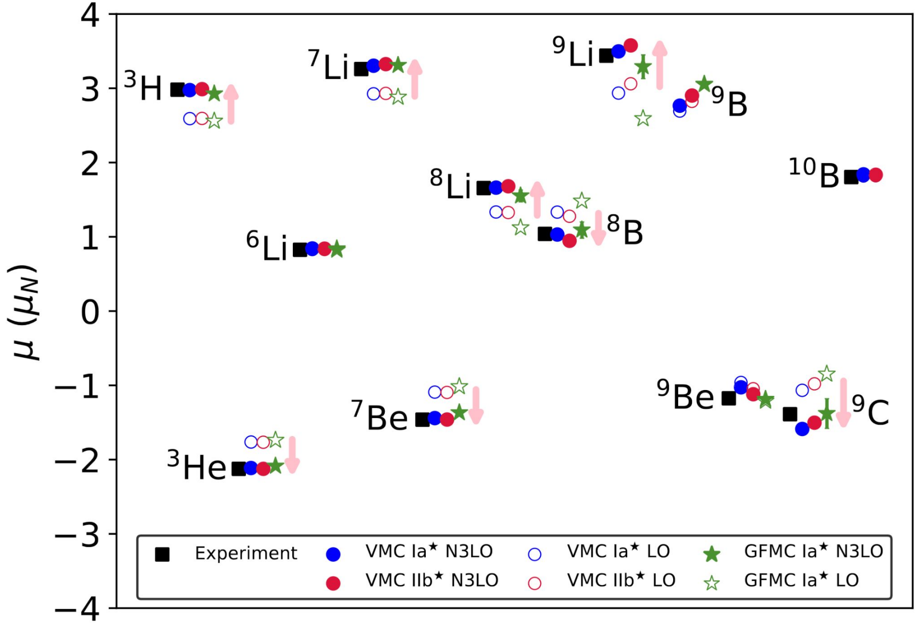


Chambers-Wall, King, Gnech et al. PRL 2024 [2407.03487]

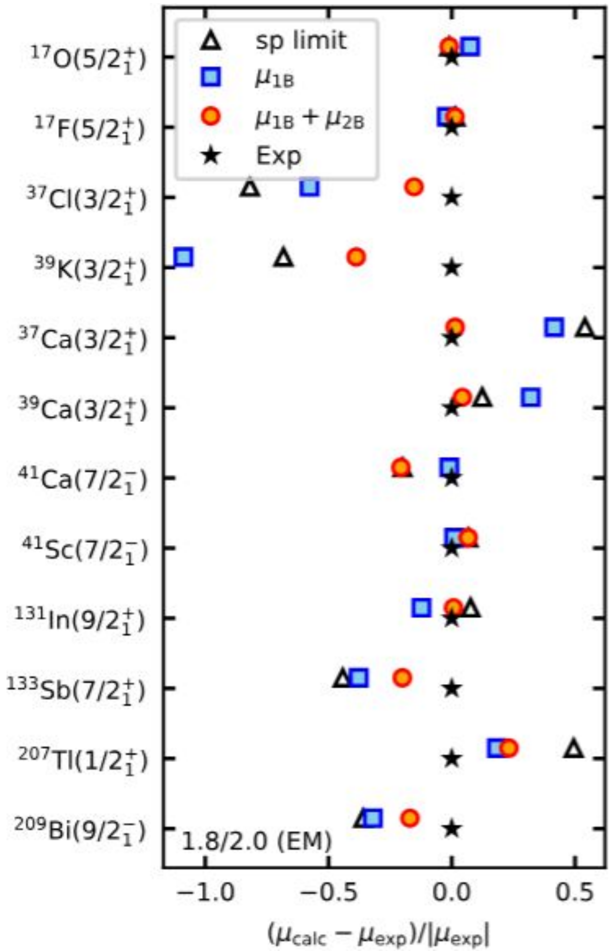
Based on Norfolk interactions and one- plus two-body currents



Magnetic moment



Chambers-Wall, King, Gnech *et al.* PRL 2024 2407.03487



Miyagi *et al.* PRL 132 (2024)

Elastic scattering

Cross section

$$\frac{d\sigma}{d\Omega} = 4\pi\sigma_M f_{\mathrm{rec}}^{-1} \left[\frac{Q^4}{q^4} F_L^2(q) + \left(\frac{Q^2}{2q^2} + \tan^2\theta_e/2\right) F_T^2(q)\right]$$

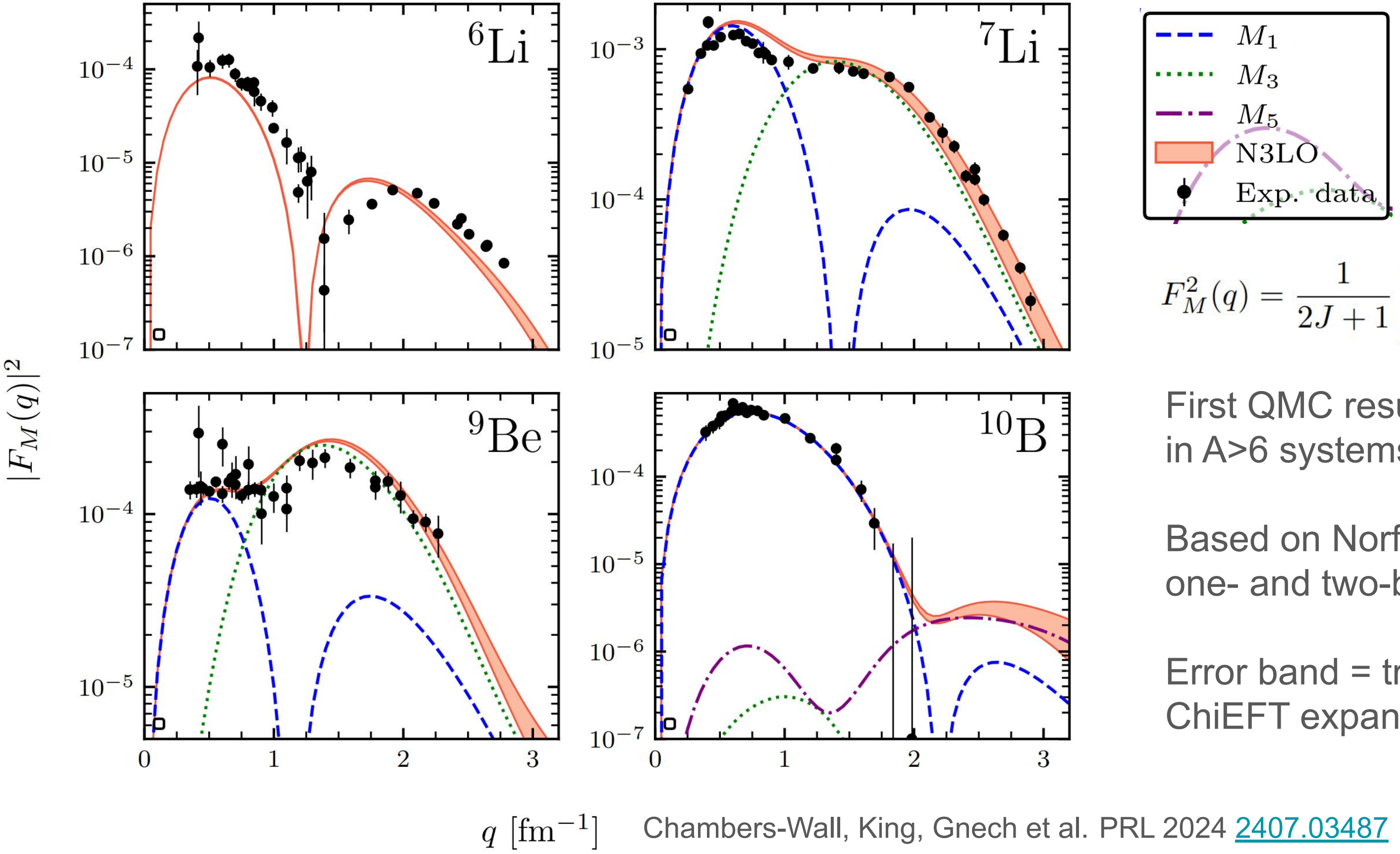
$$F_T^2(q) = F_M^2(q) = \frac{1}{2J_i+1}\sum_{L=1}^{\infty} |\langle J_f||M_L(q)||J_i\rangle|^2$$

$$F_L^2(q) = \frac{1}{2J_i+1}\sum_{L=0}^{\infty} |\langle J_f||C_L(q)||J_i\rangle|^2$$

Magnetic and Charge Form Factors

$\langle JJ|\mathrm{j}_y(q\hat{\boldsymbol{x}})|JJ\rangle$ $\langle J_f M|\rho^\dagger(q)|J_i M\rangle$

Magnetic form factors: comparison with the data



$$F_M^2(q) = \frac{1}{2J+1} \sum_{L=1}^{\infty} |\langle J||M_L(q)||J\rangle|^2$$

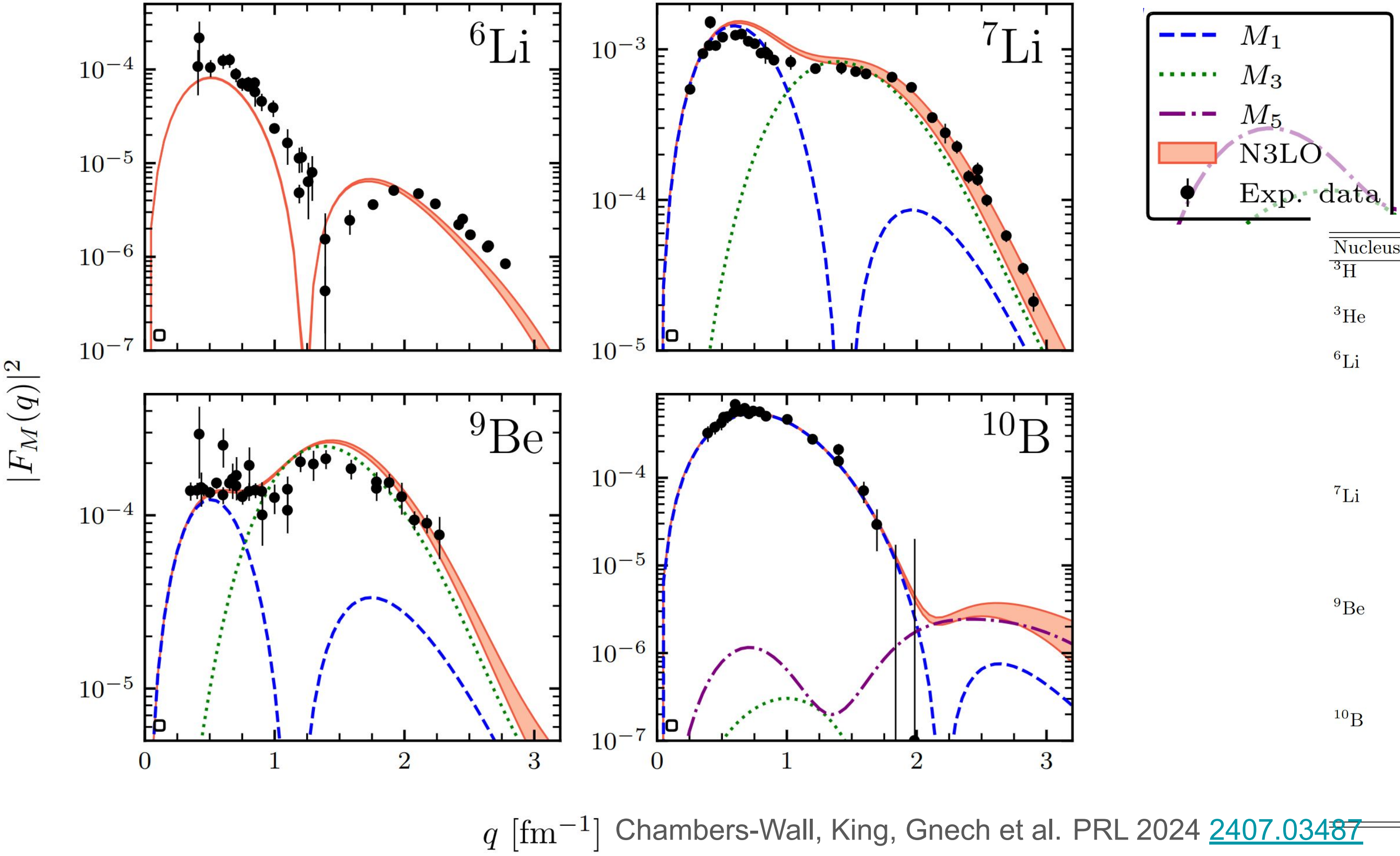
First QMC results for form factors in A>6 systems.

Based on Norfolk interactions and one- and two-body currents.

Error band = truncation error in the ChiEFT expansion.

Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487

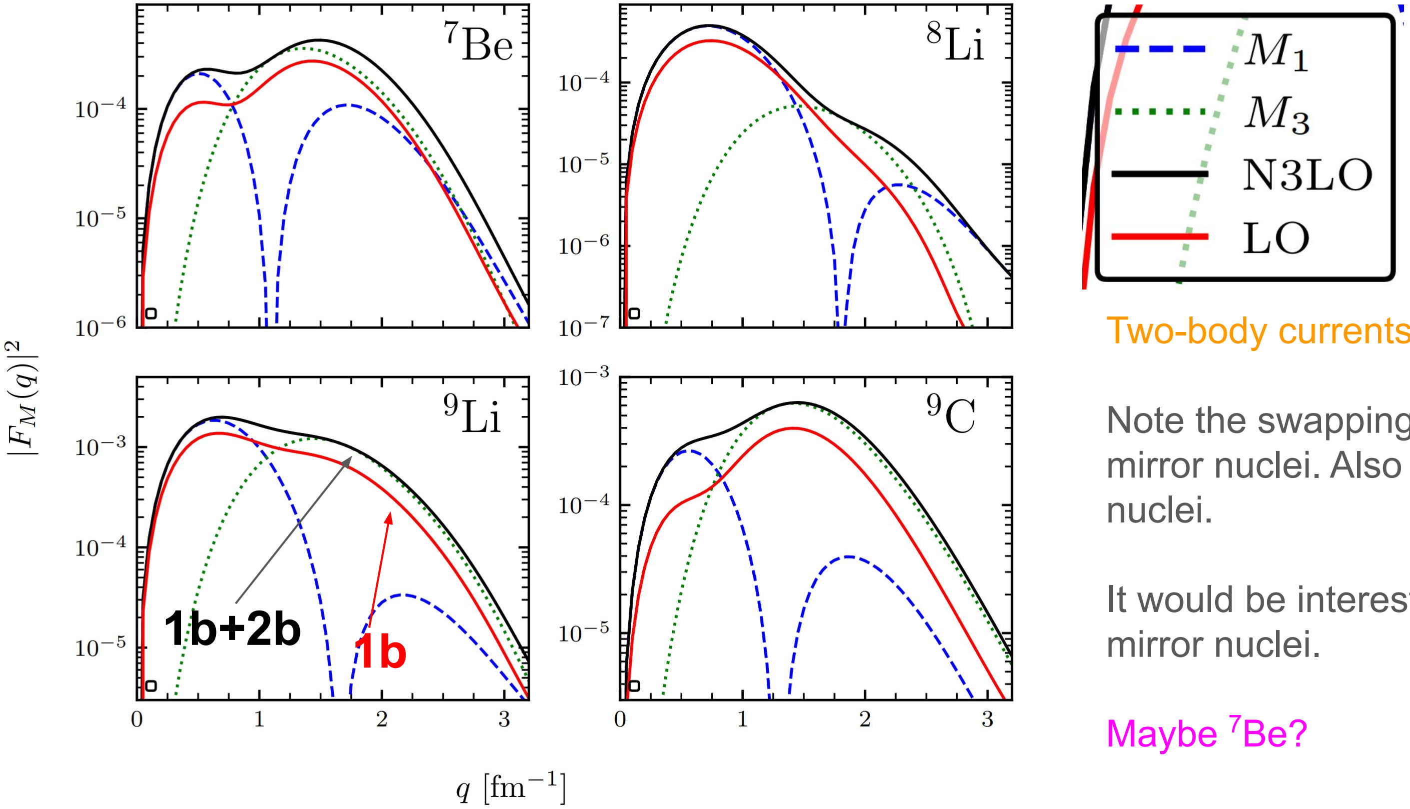
Magnetic form factors: comparison with the data



Chambers-Wall, King, Gnech et al. PRL 2024 [2407.03487](https://arxiv.org/abs/2407.03487)

Nucleus	Reference	Data type	ratio/method
^{3}H	Sick 2001 [89]	N	1
^{3}He	Sick 2001 [89]	N	1
^{6}Li	Peterson 1962 [90]	N	Eq. (C2)
	Goldemberg 1963 [91]	N	Eq. (C2)
	Rand 1966 [92]	N	Eq. (C1)
	Lapikas 1978 [93]	D	$1/4\pi$
	Bergstrom 1982 [94]	N	$Z^2/4\pi$
^{7}Li	Peterson 1962 [90]	N	Eq. (C2)
	Goldemberg 1963 [91]	N	Eq. (C2)
	Van Niftrik 1971 [95]	D	Eq. (C1)
	Lichtenstadt 1983 [96]	N	$Z^2/4\pi$
^{9}Be	Goldemberg 1963 [91]	N	Eq. (C2)
	Vanpraet 1965 [98]	N	Eq. (C1)
	Rand 1966 [92]	N	Eq. (C1)
	Lapikas 1975 [97]	N	Eq. (C2)
^{10}B	Goldemberg 1963 [91]	N	Eq. (C2)
	Goldemberg 1965 [100]	N	Eq. (C2)
	Vanpraet 1965 [98]	N	Eq. (C1)
	Rand 1966 [92]	N	Eq. (C1)
	Lapikas 1978 [93]	D	$1/4\pi$

Magnetic form factors: predictions



Two-body currents provide 40-60%.

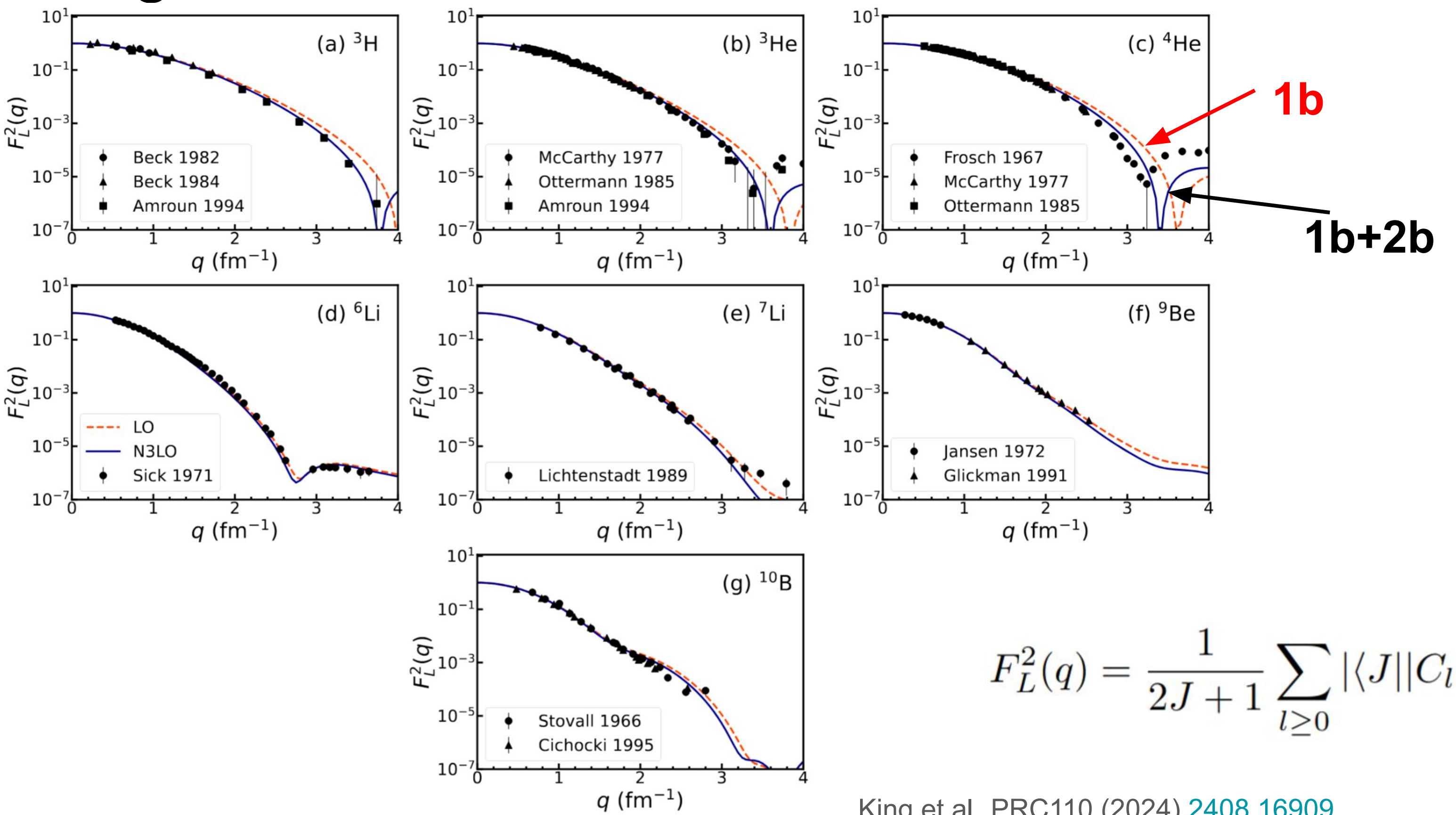
Note the swapping of M1 and M3 in mirror nuclei. Also observed in A=7 nuclei.

It would be interesting to have data for mirror nuclei.

Maybe ^{7}Be?

Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487

Charge form factors



$$F_L^2(q) = \frac{1}{2J+1} \sum_{l \geq 0} |\langle J || C_l(q) || J \rangle|^2$$

King et al. PRC110 (2024) 2408.16909

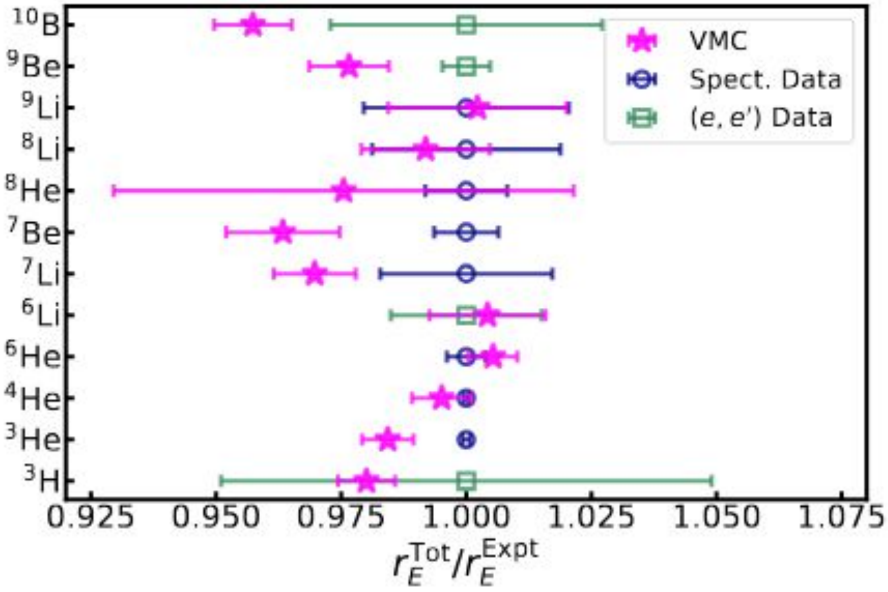
Charge radii

Extracted from low-momentum transfer behavior of form factor.

$$\frac{1}{Z}\langle JJ|\rho(q\hat{\mathbf{z}})|JJ\rangle \approx 1 - \frac{1}{6} r_E^2 q^2 + \mathcal{O}(q^4)$$

Accounts for two-body correlations, finite size/nucleon level corrections via nucleonic form factors.

Agreement of ~5% or better.



King *et al*. submitted to PRC 2025

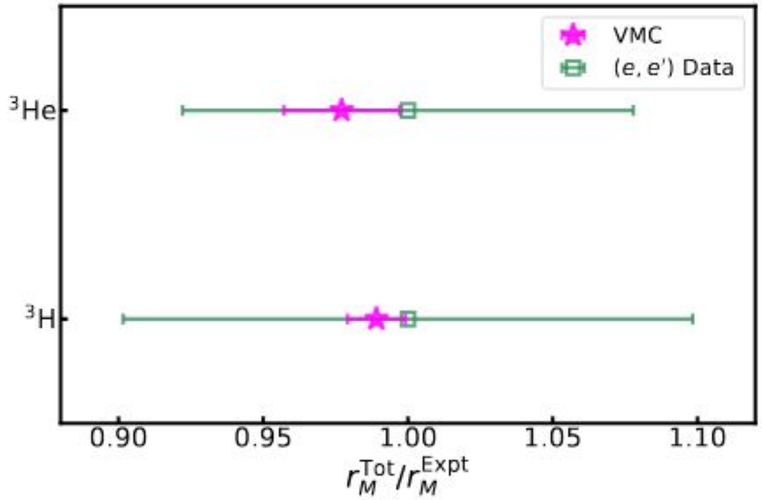
Magnetic radii

Extracted from low-momentum transfer behavior of form factor.

$$-i\frac{2m}{q\mu}\langle JJ|\mathrm{j}_y(q\hat{\mathbf{x}})|JJ\rangle \approx 1-\frac{1}{6}r_M^2q^2+\mathcal{O}(q^4)$$

Accounts for two-body currents, finite size/nucleon level corrections via nucleonic form factors.

Limited data, predictions available for A up to 10.



King *et al*. submitted to PRC 2025

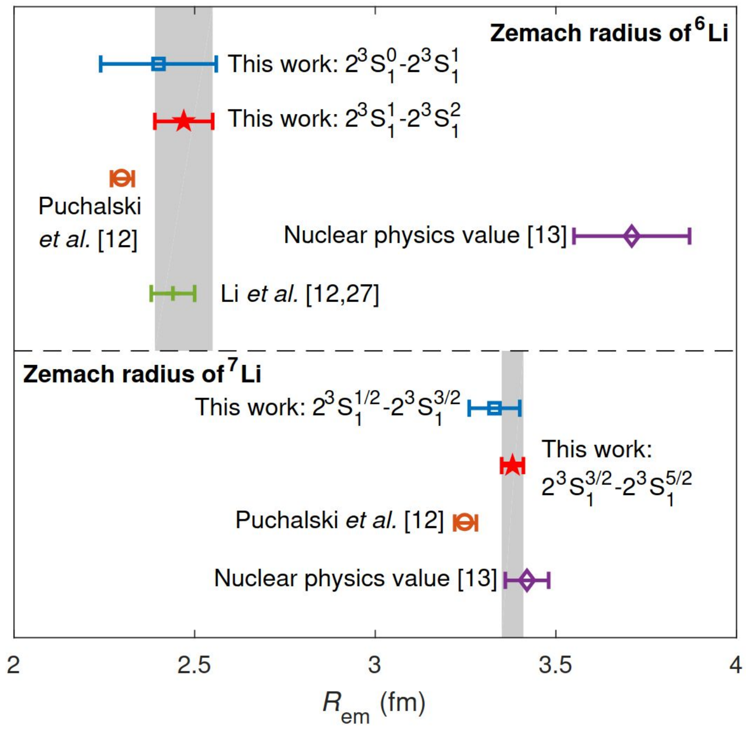
Electromagnetic radii: Tables

	r_E^{LO} (fm)	r_E^{Tot} (fm)	Expt. (fm)
$^3\mathrm{H}(\frac{1}{2}^+;\frac{1}{2})$	1.69(1)	1.72(1)	1.755(86) [45]
$^3\mathrm{He}(\frac{1}{2}^+;\frac{1}{2})$	1.90(1)	1.92(1)	1.9506(14) [46]
$^4\mathrm{He}(0^+;0)$	1.64(1)	1.67(1)	1.67824(83) [47]
$^6\mathrm{He}(0^+;1)$	2.07(1)	2.07(1)	2.059(8) [48]
$^6\mathrm{Li}(1^+;0)$	2.58(3)	2.60(3)	2.589(39) [49]
$^7\mathrm{Li}(\frac{3}{2}^-;\frac{1}{2})$	2.35(2)	2.37(2)	2.444(42) [49]
$^7\mathrm{Be}(\frac{3}{2}^-;\frac{1}{2})$	2.53(2)	2.55(3)	2.647(17) [50]
$^8\mathrm{He}(0^+;2)$	1.97(1)	1.91(9)	1.958(16) [48]
$^8\mathrm{Li}(2^+;1)$	2.32(2)	2.32(3)	2.339(44) [51]
$^8\mathrm{Be}(0^+;0)$	2.53(2)	2.55(2)	–
$^8\mathrm{B}(2^+;1)$	2.63(3)	2.67(4)	–
$^8\mathrm{C}(0^+;2)^\dagger$	2.88(4)	2.91(5)	–
$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2})$	2.25(2)	2.25(4)	2.245(46) [49]
$^9\mathrm{Be}(\frac{3}{2}^-;\frac{1}{2})$	2.45(2)	2.46(2)	2.519(12) [52]
$^9\mathrm{B}(\frac{3}{2}^-;\frac{1}{2})^\dagger$	2.55(2)	2.59(3)	–
$^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})^\dagger$	2.67(3)	2.70(4)	–
$^{10}\mathrm{B}(3^+;0)$	2.45(2)	2.47(2)	2.58(7) [53]

	r_M^{LO} (fm)	r_M^{Tot} (fm)	Expt (fm)
$^3\mathrm{H}(\frac{1}{2}^+;\frac{1}{2})$	1.88(2)	1.82(1)	1.840(181) [45]
$^3\mathrm{He}(\frac{1}{2}^+;\frac{1}{2})$	2.02(3)	1.92(2)	1.965(153) [45]
$^6\mathrm{Li}(1^+;0)$	3.32(10)	3.32(10)	–
$^7\mathrm{Li}(\frac{3}{2}^-;\frac{1}{2})$	2.89(7)	2.99(29)	–
$^7\mathrm{Be}(\frac{3}{2}^-;\frac{1}{2})$	3.42(11)	3.37(31)	–
$^8\mathrm{Li}(2^+;1)$	2.22(2)	2.31(1)	–
$^8\mathrm{B}(2^+;1)$	3.04(4)	3.25(2)	–
$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2})$	2.80(7)	2.87(31)	–
$^9\mathrm{Be}(\frac{3}{2}^-;\frac{1}{2})$	3.34(7)	3.28(7)	–
$^9\mathrm{B}(\frac{3}{2}^-;\frac{1}{2})^\dagger$	2.80(9)	2.82(12)	–
$^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})^\dagger$	3.34(7)	3.14(30)	–
$^{10}\mathrm{B}(3^+;0)$	2.33(2)	2.33(2)	–

King *et al*. submitted to PRC 2025

Zemach & Elastic Contribution to TPE



$$\langle R_Z \rangle = -\frac{4}{\pi\mu}\int_0^\infty \frac{dq}{q^2}\left[F_C(q^2)F_M(q^2) - 1\right]$$

$$\langle R_E^3 \rangle_{(2)} = \frac{48}{\pi}\int_0^\infty \frac{dq}{q^4}\left[F_C^2(q^2) - 1 + \frac{q^2\langle R_E^2 \rangle}{3}\right]$$

XQ Qi *et al*. Phys. Rev. Lett. **125**, 183002

Interactions with neutrinos: beta decay

$$(Z,N) \rightarrow (Z+1,N-1)+e+\bar{\nu}_e$$

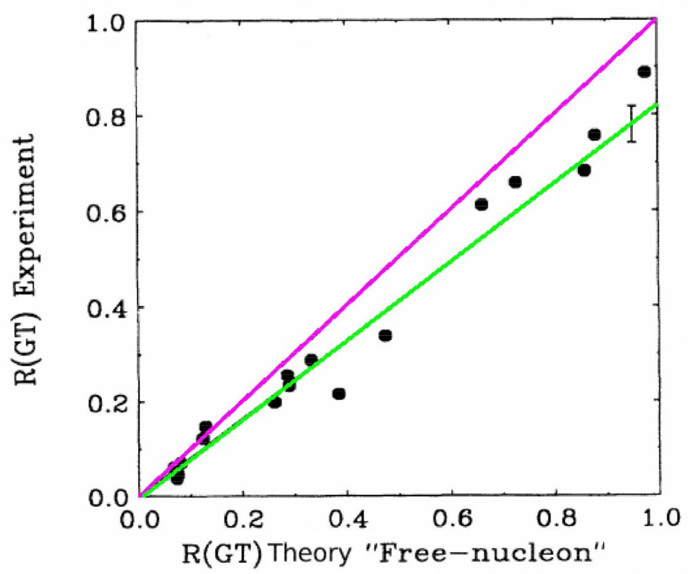
$$\Gamma_\beta \propto |M_\beta|^2 = |M_{\rm F}|^2 + \frac{g_A^2}{g_V^2}|M_{\rm GT}|^2$$

Gamow-Teller transitions (GT) allow to test the axial currents in nuclei. In the single-particle picture (q=0)

$$\mathrm{GT} = \sum_k \sigma_k \tau_{k\pm}$$

The systematic theoretical overpredition of GT matrix elements by truncated nuclear models is explained by nucleonic correlations and currents.

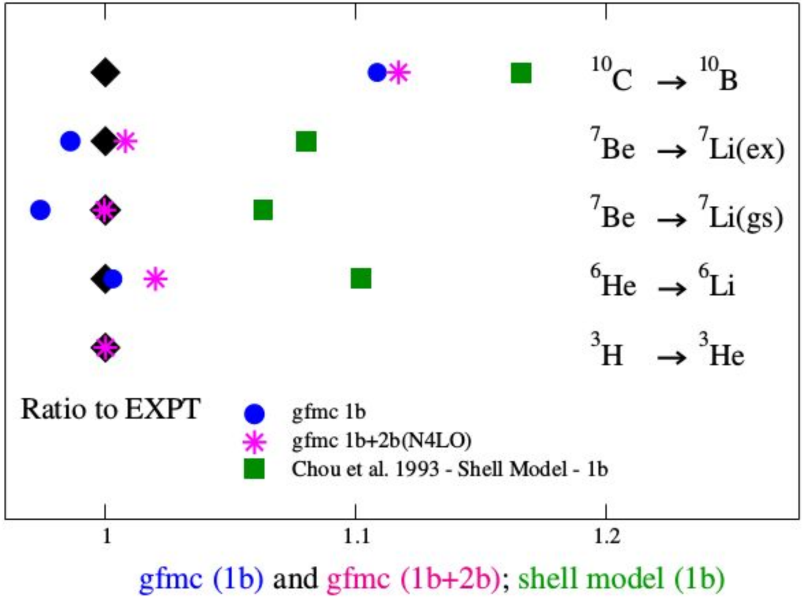
Gamow-Teller Matrix Elements Theory vs Expt



in $3 \leq \mathrm{A} \leq 18 \longrightarrow g_A^{\rm eff} \simeq 0.80\, g_A$

Chou *et al.* PRC47(1993)163

Beta decay in light nuclei



Based on AV18+IL7 and one- plus two-body axial currents j_5

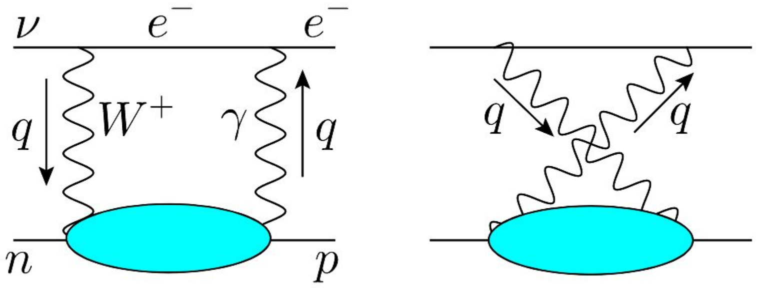
$$\mathrm{RME(GT)} = \frac{\sqrt{2J_f+1}}{g_A} \frac{\langle J_f M | j^z_{\pm,5}(\mathbf{q}\rightarrow 0) | J_i M\rangle}{\langle J_i M, 10 | J_f M\rangle}$$

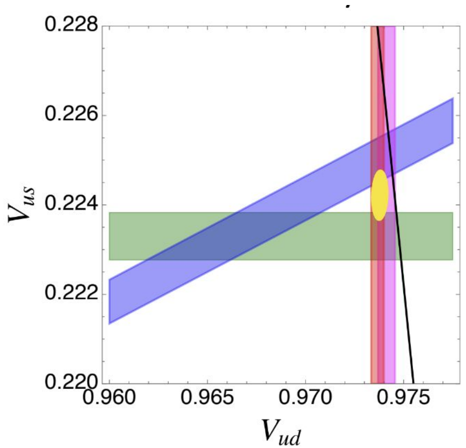
In light nuclei many-nucleon correlations in the wave functions improve agreement with the data.

SP *et al*. PRC97(2018)022501

$*$ data from TUNL, Suzuki *et al.* PRC67(2003)044302, Chou *et al.* PRC47(1993)163

Superallowed beta decay and CKM unitarity



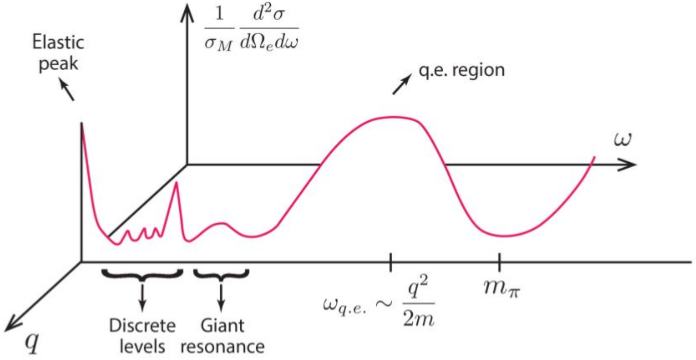


LRP23

Superallowed beta decay used to test CKM unitarity

Radiative corrections receive contributions from the QE region and require the evaluations of nuclear responses

$$\frac{\log 2}{ft} = \frac{G_F^2 m_e^5 |V_{ud}|^2}{\pi^3}(1 + \Delta_R^V + \delta_R' + \delta_{NS} - \delta_C)$$



Lepton-Nucleus scattering: Inclusive Processes

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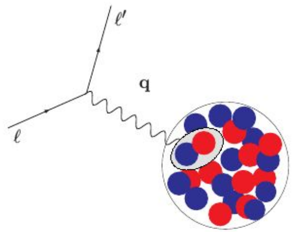
Longitudinal response induced by the charge operator $O_L = \rho$
Transverse response induced by the current operator $O_T = \boldsymbol{j}$
5 Responses in neutrino-nucleus scattering

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$

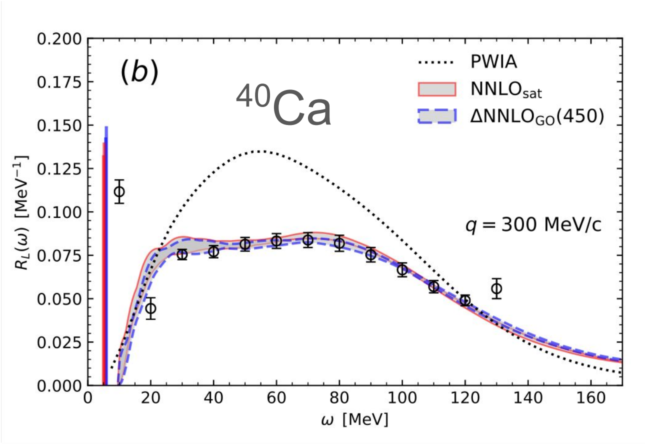


For a recent review on QMC, SF methods see
Rocco *Front. In Phys.*8 (2020)116

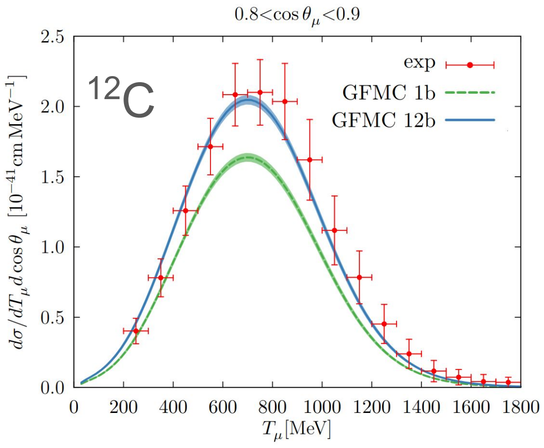
Inclusive Cross Sections with Integral Transforms

Exploit integral properties of the response functions and closure to avoid explicit calculation of the final states (Lorentz Integral Transform **LIT**, **Euclidean**, …)

$$S(q,\tau) = \int_0^\infty d\omega\, K(\tau,\omega) R_\alpha(q,\omega)$$

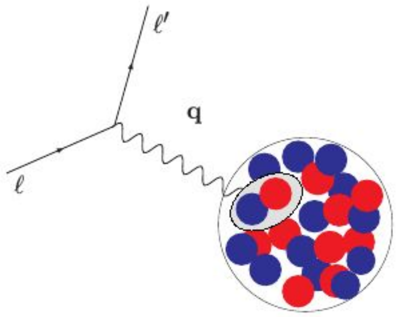


Sobczyk et al, PRL127 (2021)



Lovato et al. PRX10 (2020)

Short-Time-Approximation



Short-Time-Approximation:

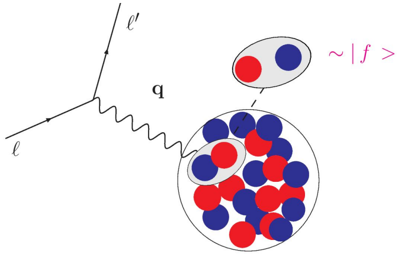
- Based on Factorization
- Retains two-body physics
- Correctly accounts for interference

$$R(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_0)t} \langle 0|O^\dagger e^{-iHt} O|0\rangle$$

$$O_i^\dagger e^{-iHt} O_i + O_i^\dagger e^{-iHt} O_j + O_i^\dagger e^{-iHt} O_{ij} + O_{ij}^\dagger e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i<j} v_{ij}$$

Short-Time-Approximation



Short-Time-Approximation:

- Based on Factorization
- **Retains two-body physics**
- Response functions are given by the **scattering from pairs of fully interacting nucleons** that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions ∝ Cross Sections

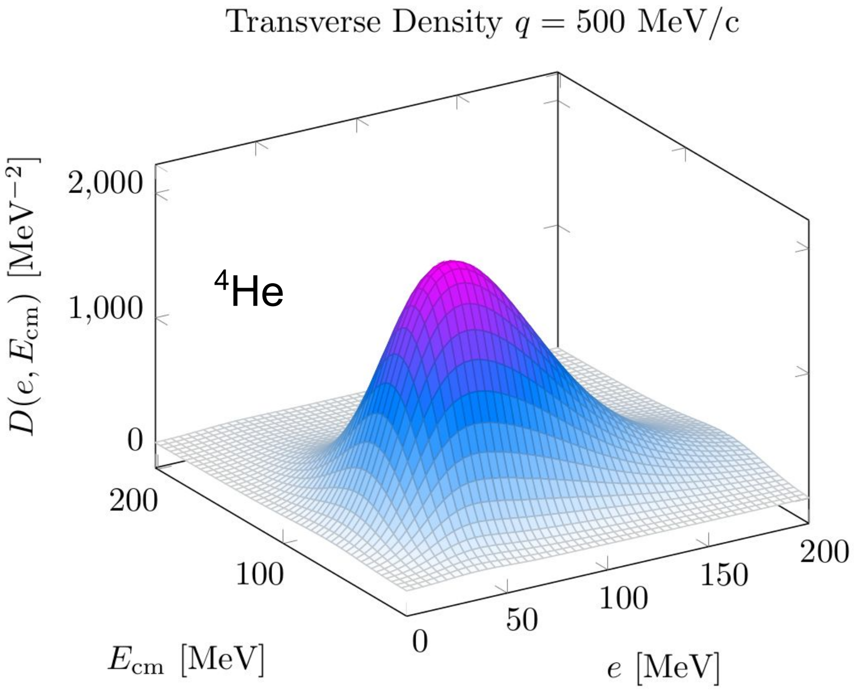
$$R_\alpha(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) |\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$

Response ***Densities***

$$R(q,\omega) \sim \int \delta\left(\omega + E_0 - E_f\right) \, dP' \, dp' \mathcal{D}(p', P'; q)$$

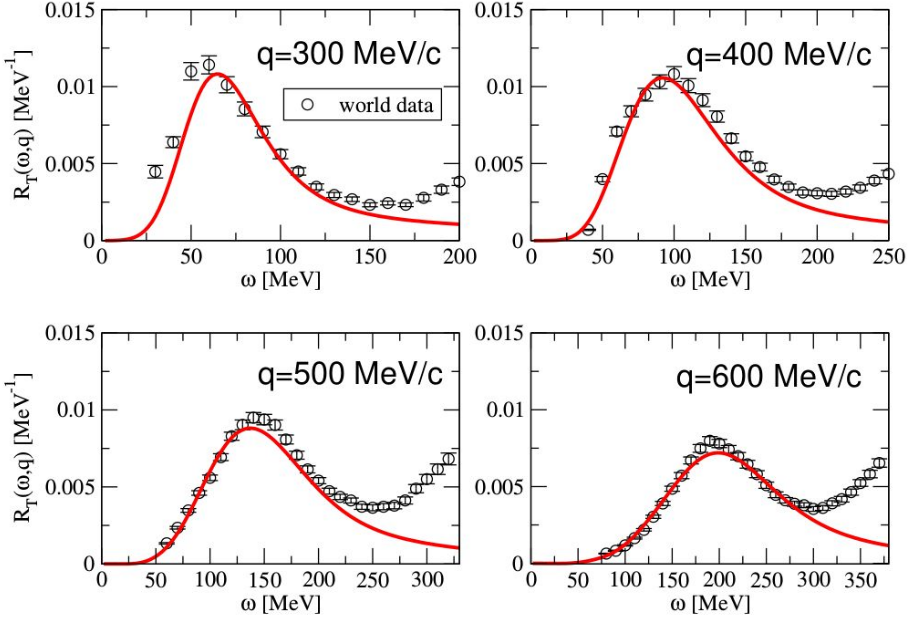
P' and *p'* are the CM and relative momenta of the struck nucleon pair

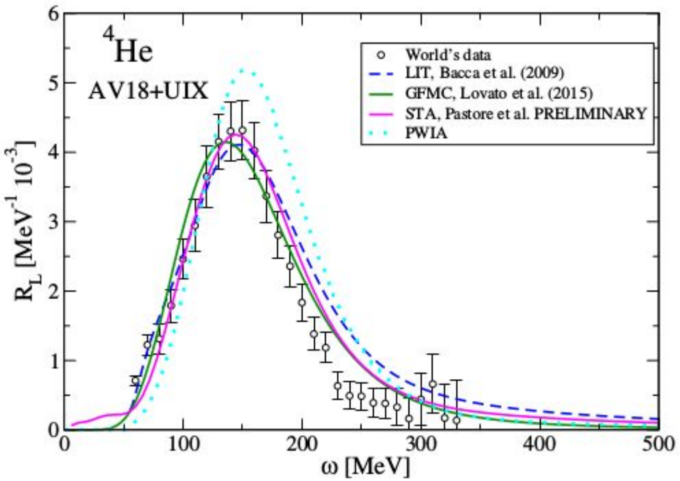
Transverse Response Density: *e*-^{4}He scattering



SP *et al.* PRC101(2020)044612

Helium-4: data & model dependence

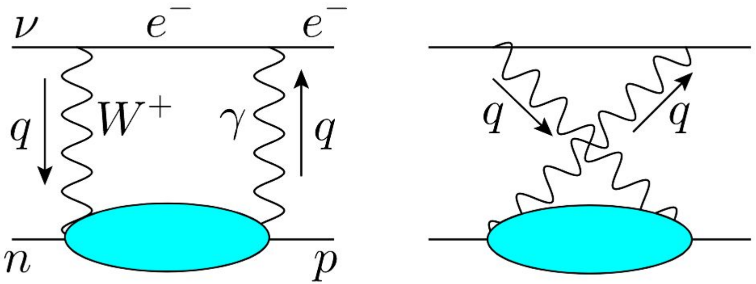




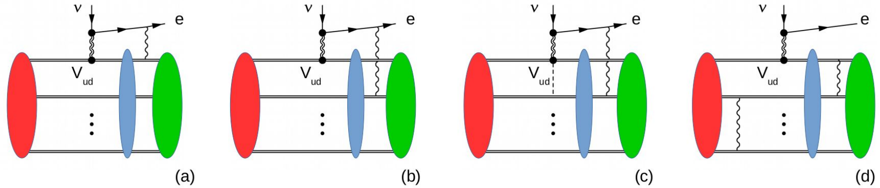
Benchmark in ^{4}He

SP *et al.* PRC101(2020)044612

EFT approach



$$\frac{\log 2}{ft} = \frac{G_F^2 m_e^5 |V_{ud}|^2}{\pi^3}(1 + \Delta_R^V + \delta_R' + \delta_{NS} - \delta_C)$$



Cirigliano, Mereghetti, Dekens, et al. *Phys.Rev.C* 110 (2024)

EFT approach to radiative corrections

In χEFT the calculation of can be reduced to the calculation of matrix elements of two- and three-body transition operators between the wave functions of initial and final states

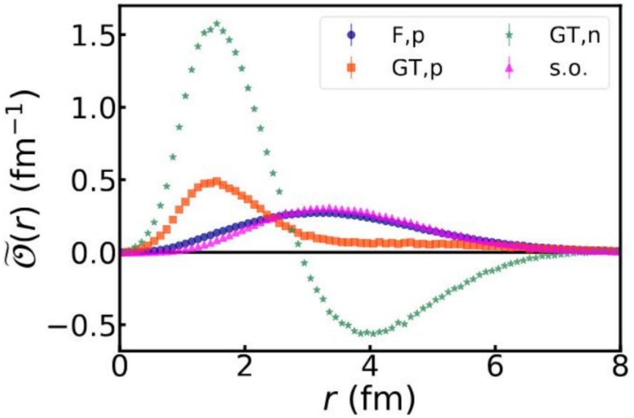
$$\delta_{\rm NS}^{(0)} = \frac{2}{g_V(\mu_\pi) M_{\rm F}^{(0)}} \sum_{N=n,p} \left[\alpha \left(M_{{\rm GT},N}^{\rm mag} + M_{{\rm T},N}^{\rm mag} + M_{{\rm GT},N}^{\rm CT} + M_{{\rm so},N} \right) + \alpha^2 M_{{\rm F},N}^{+} \right]$$

^{10}C(0+) -> ^{10}B(0+) β-decay

In an effective field theory approach:

$$\delta_{\rm NS} = \sum_{m,n,i} \alpha^m E_0^n c_{m,n} M^i_{m,n}$$

Can also evaluate: $M = \int drC(r)$



GFMC: $\delta_{\rm NS} = -4.05(38) \times 10^{-3} - -4.10(77) \times 10^{-3}$

Hardy and Towner: $\delta_{\rm NS} = -4.0(5) \times 10^{-3}$

Gennari et al PRL **134,** 012501: $\delta_{\rm NS} = -4.22(32) \times 10^{-3}$

Los Alamos NATIONAL LABORATORY

In collaboration with: **Mereghetti (LANL)**, Carlson (LANL), Flores (WUSTL), Gandolfi (LANL), Pastore (WUSTL), Piarulli

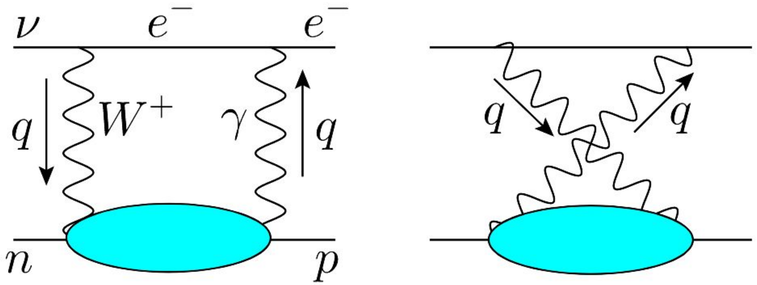
Courtesy of Garrett King

Quantum Monte Carlo calculations for next-generation electroweak physics experiments

Garrett King

Next-generation ab initio nuclear theory
ECT*, Trento, Italy
7/17/2025

Ties to two photon exchange (TPE)

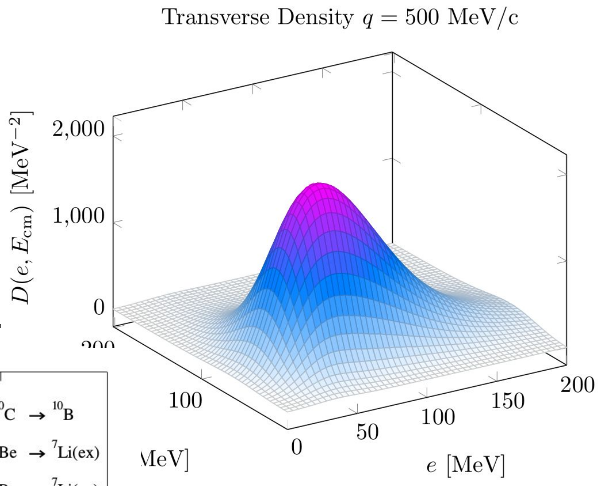


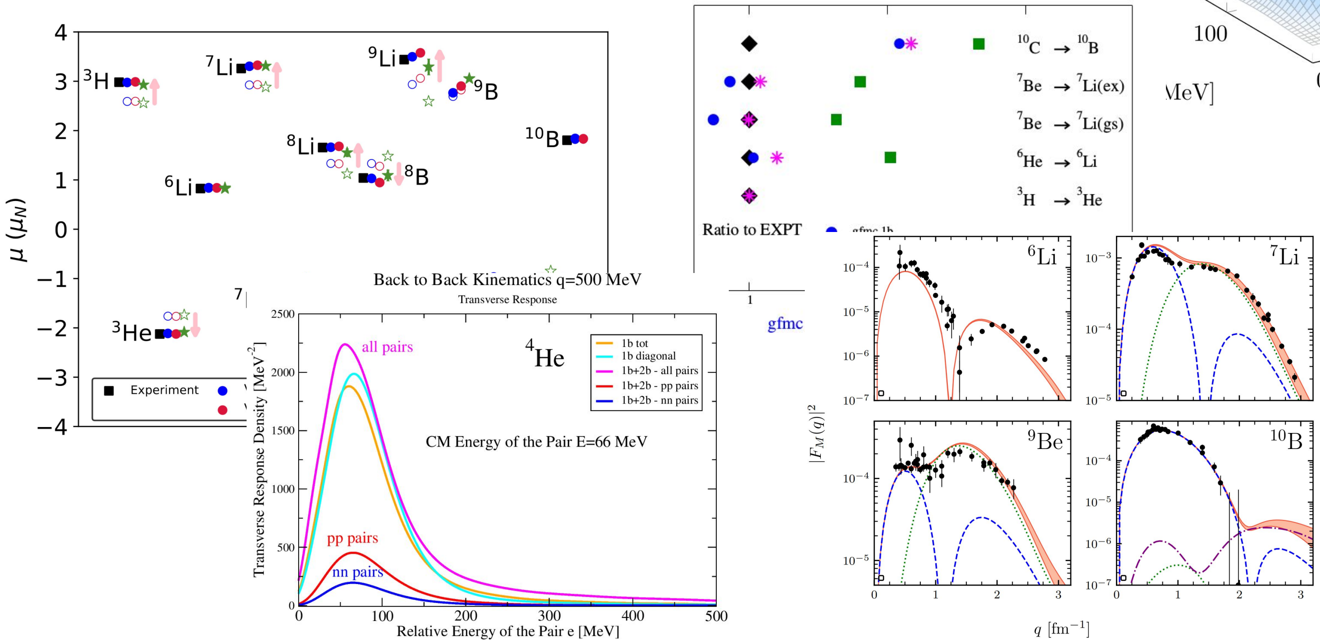
Cirigliano, Mereghetti, Dekens, et al. *Phys.Rev.C* 110 (2024) 5, 055502

The EFT approach can be applied to determine the TPE contribution of relevance to muonic physics

Summary

Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where many-body effects play an essential role to explain available data.





Close **collaborations** between **NP**, **LQCD**, **Pheno**, **Hep**, **Comp**, **Expt**, … are required to progress e.g., NP is represented in the Snowmass process

It's a very exciting time!

Graham Chambers-Wall
(WashU GS)

Garrett King
(LANL PD)

Lorenzo Andreoli
(ODU/JLab PD)

King *et al.* PRC 110 (2024) 5, 054325; *Ann.Rev.Nucl.Part.Sci. 74* (2024) 343
Chambers-Wall, Gnech, King *et al*. PRL 133 (2024) 21, 212501; PRC 110 (2024) 5, 054316
Andreoli *et al*. *PRC 110* (2024) 6, 064004

Collaborators

WashU: **Bub Chambers-Wall Flores Novario Piarulli Weiss**
LANL: Carlson Gandolfi Hayes **King** Mereghetti
JLab+ODU: **Andreoli Gnech** Schiavilla
ANL: McCoy Lovato Wiringa
UW/INT: Cirigliano Dekens
Pisa U/INFN: Kievsky Marcucci Viviani
Salento U: Girlanda
Huzhou U: Dong Wang
Fermilab: Gardiner Betancourt Rocco
MIT: Barrow

Washington University in St. Louis
Arts & Sciences

U.S. DEPARTMENT OF ENERGY
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Nuclear Computational Low-Energy Initiative
A SciDAC-5 Project

NTNP

ASCR Leadership Computing Challenge
ALCC
2024-2025 Awards

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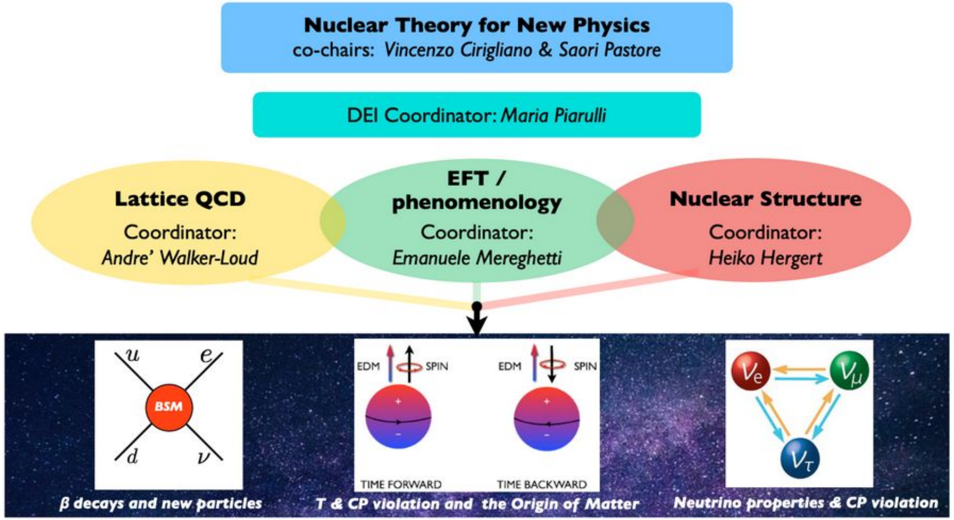
Nuclear Theory for New Physics NP&HEP TC

Nuclear Theory for New Physics

- About Us
- Commitment to Diversity
- Funding Acknowledgement



Funding Acknowledgement

We are funded in part through The Department of Energy, Office of Science, Office of Nuclear Physics and the Office of High Energy Physics

NTNP

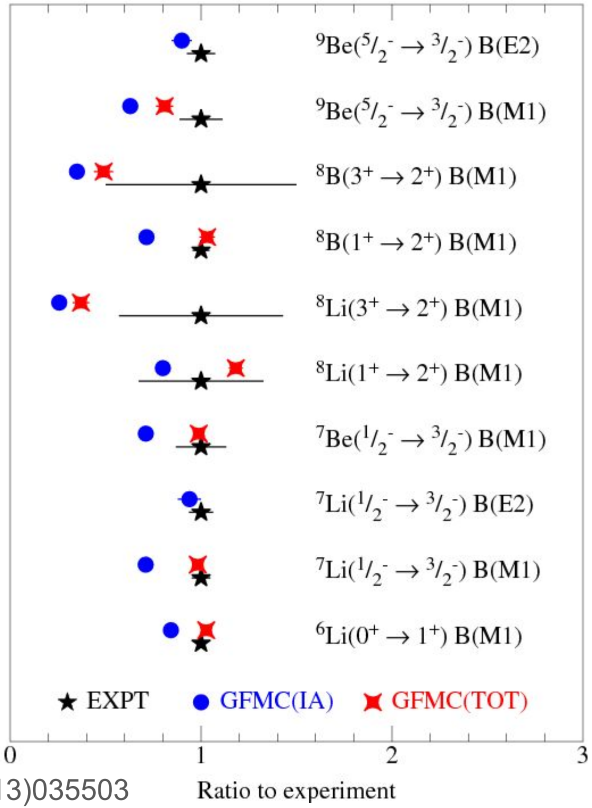
Snowmass:
Topical groups and Frontier Reports, Whitepapers, …

LRP:
White papers, 2301.03975, FSNN, …

Electromagnetic transitions

Two-body electromagnetic currents bring the theory in agreement with the data

~ 60 − 70% of total two-body current is due to one-pion-exchange currents



SP *et al*. PRC87(2013)035503

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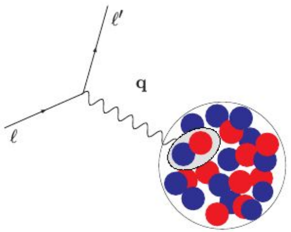

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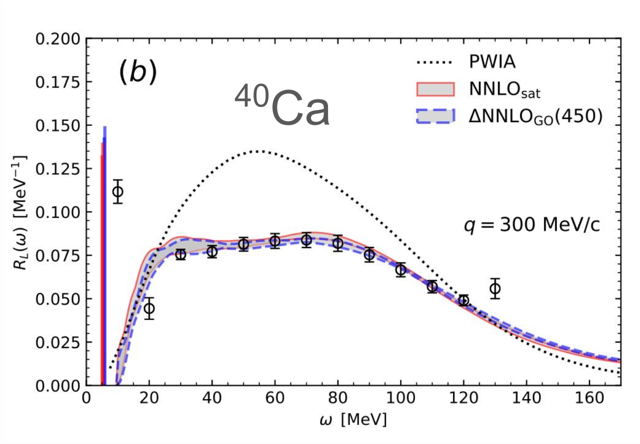
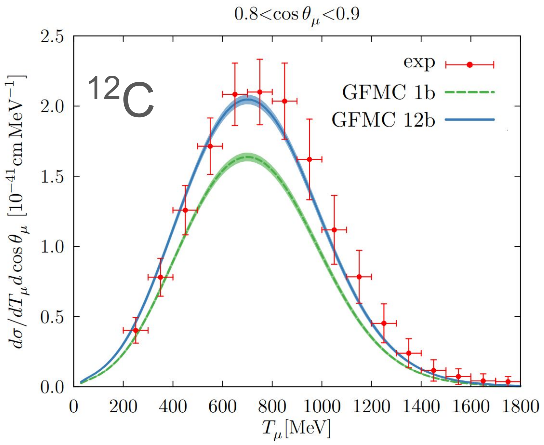


Sobczyk et al, PRL127 (2021)

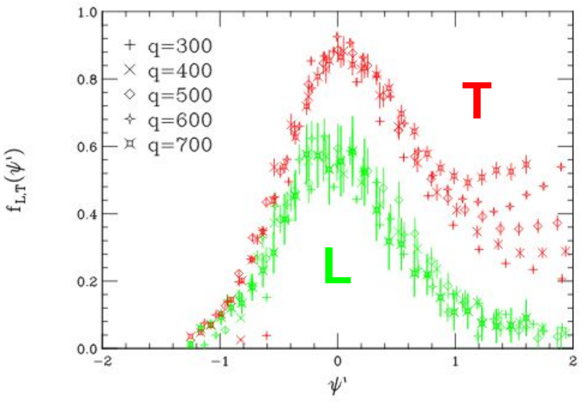


Lovato et al. PRX10 (2020)

Lepton-Nucleus scattering: Data

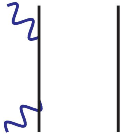
Transverse Sum Rule

$$S_T(q) \propto \langle 0|\mathbf{j}^\dagger\ \mathbf{j}|0\rangle \propto \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{1b}|0\rangle + \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{2b}|0\rangle + \ldots$$



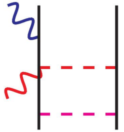
^{4}He Electromagnetic Data
Carlson *et al.* PRC65(2002)024002

Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term



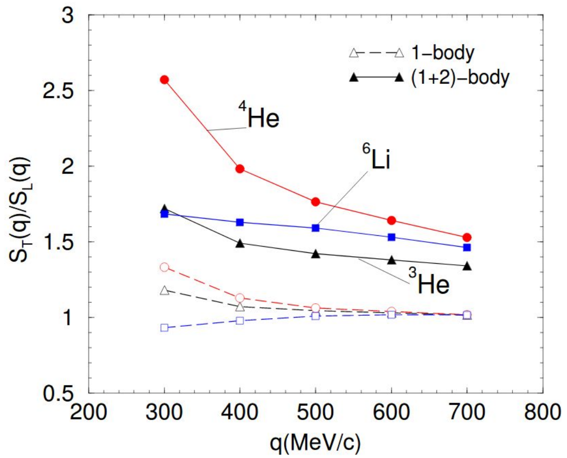
$\langle \mathbf{j}_{1b}^\dagger\ \ \mathbf{j}_{1b}\rangle\ > 0$

Leading one-body term



$\langle \mathbf{j}_{1b}^\dagger\ \mathbf{j}_{2b}\ v_\pi\rangle \propto \langle v_\pi^2\rangle\ \ > 0$

Interference term

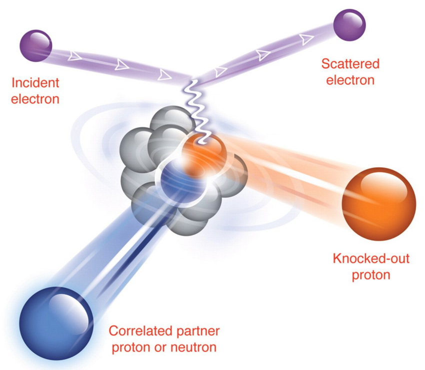


Transverse/Longitudinal Sum Rule
Carlson *et al.* PRC65(2002)024002

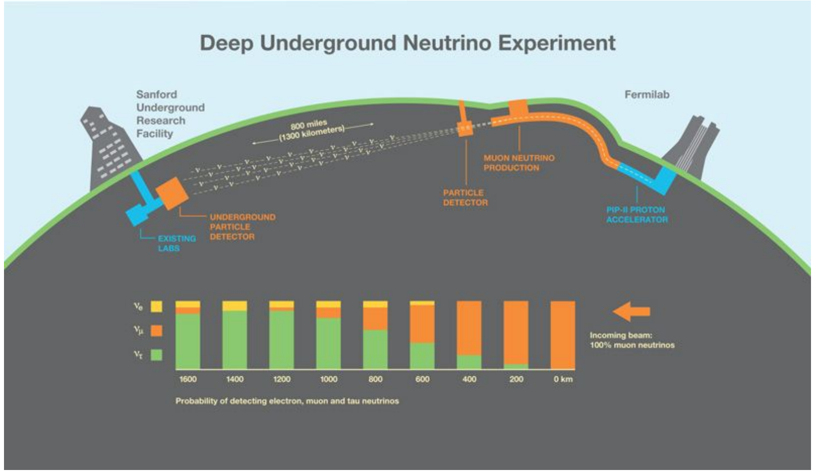
Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



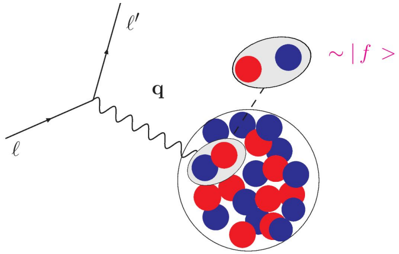
Subedi et al. Science320(2008)1475



Stanford Lab article

e4u collaboration

Short-Time-Approximation



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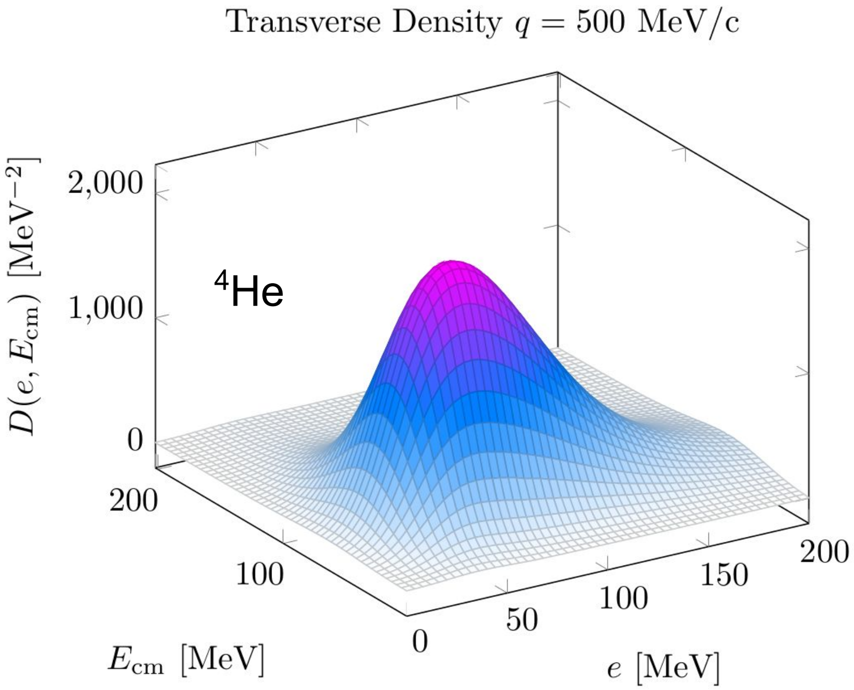
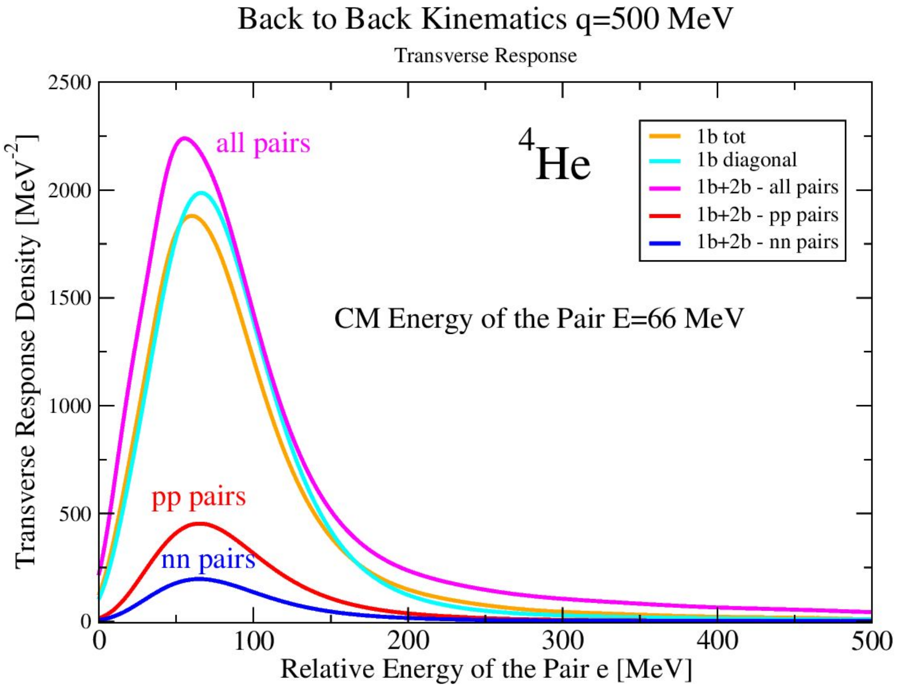
Transverse Response Density: e-^{4}He scattering



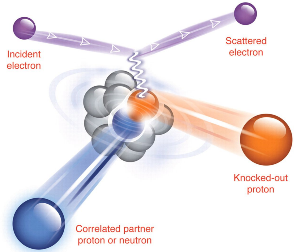
SP *et al.* PRC101(2020)044612

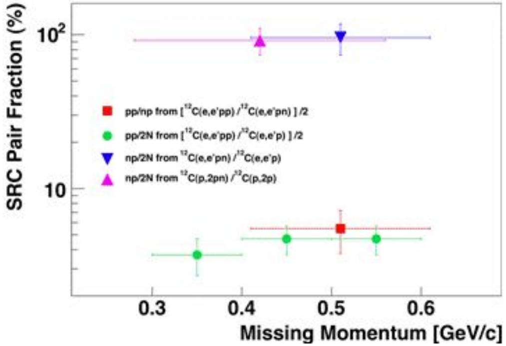
e-^{4}He scattering in the back-to-back kinematic



SP *et al.* PRC101(2020)044612

- pp pairs
- nn pairs
- all pairs 1body
- all pairs tot

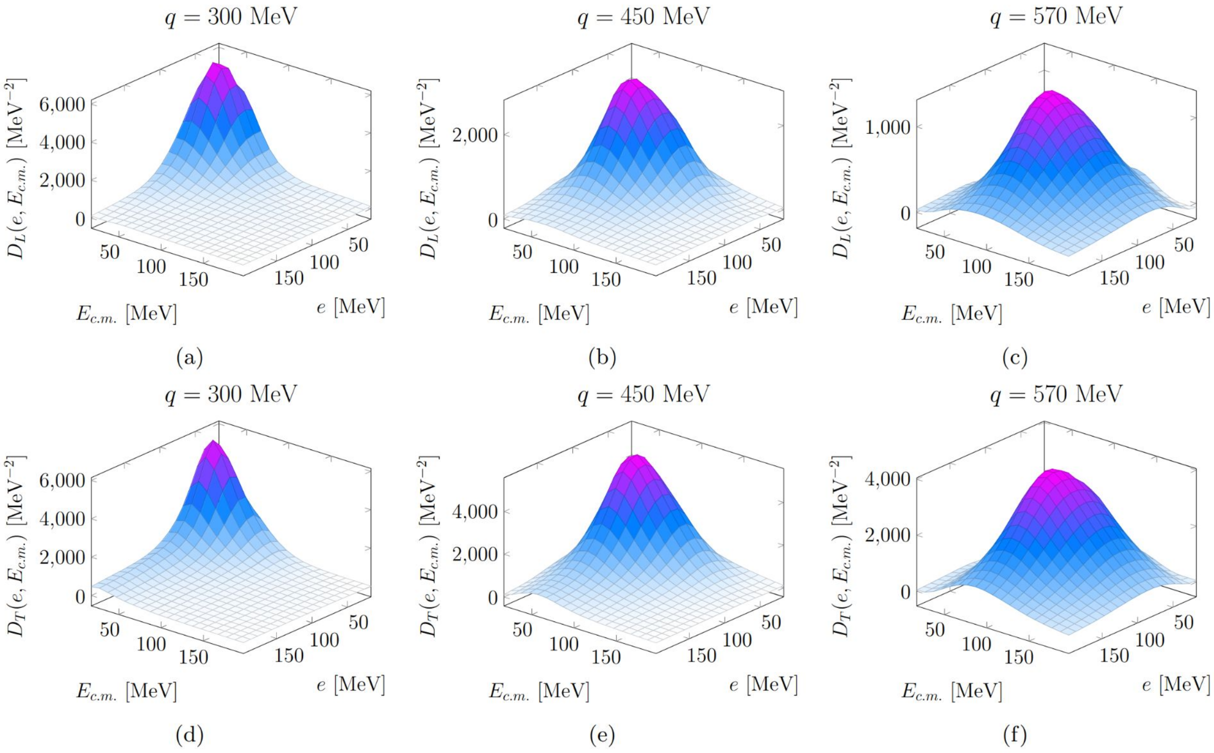




Subedi et al. Science320(2008)1475

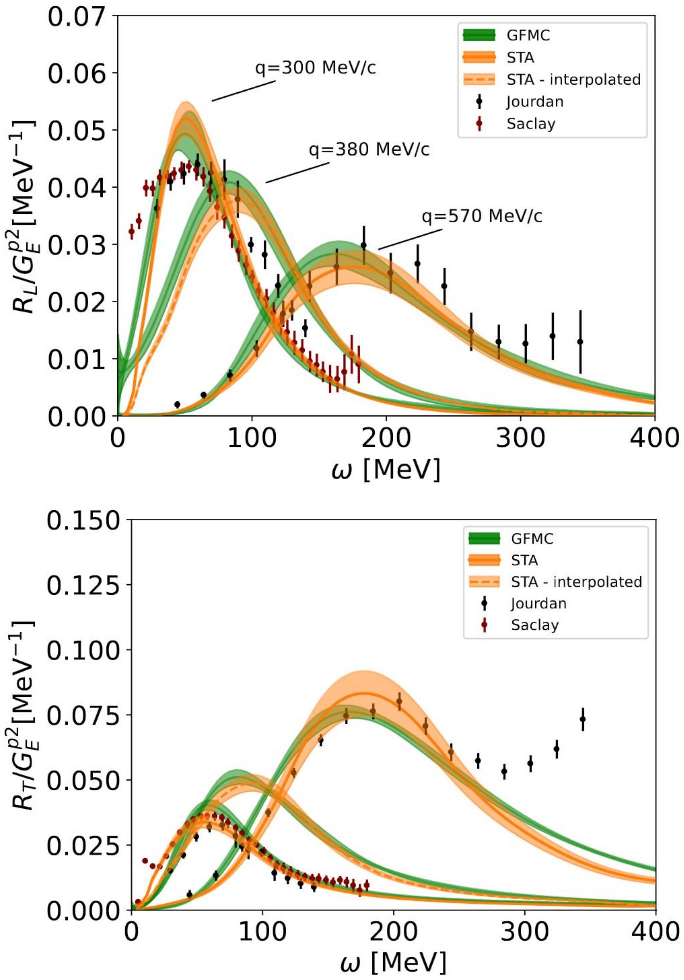
Electromagnetic vertex included

^{12}C Response Densities



Andreoli *et al.* *Phys.Rev.C* 110 (2024) 6, 064004 arXiv:2407.06986

^{12}C response functions

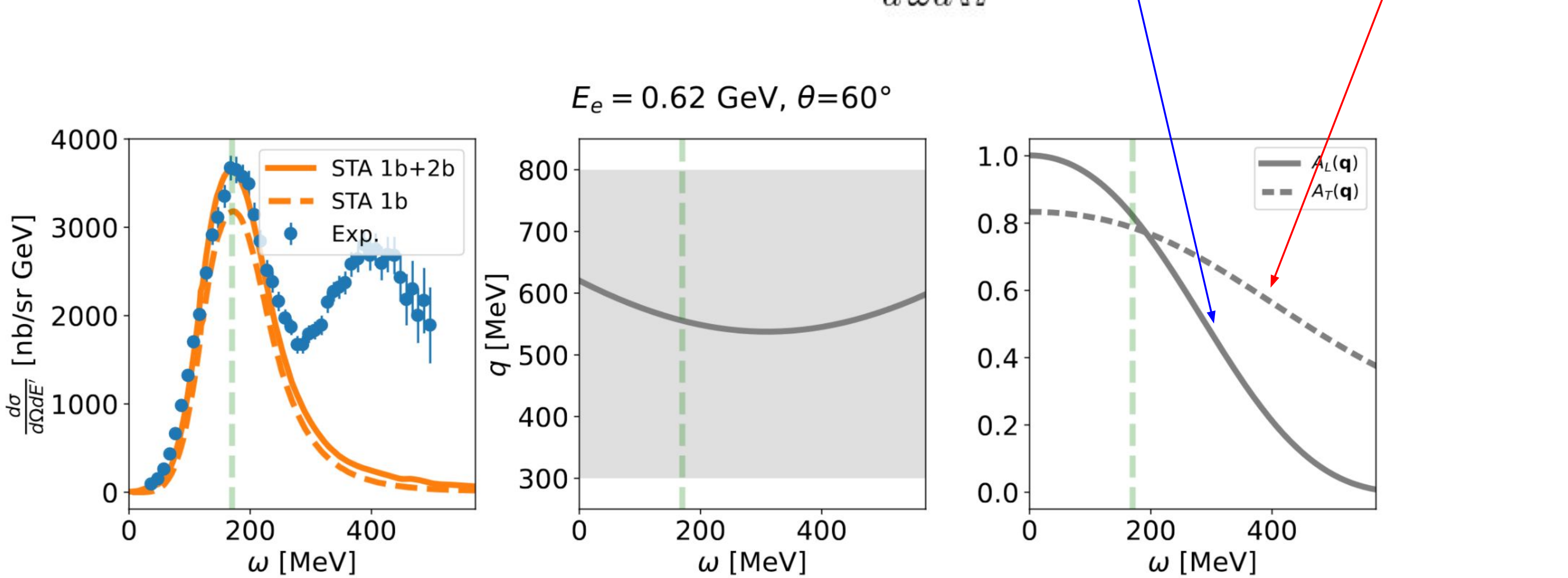


$$\frac{d^2\,\sigma}{d\,\omega d\,\Omega} = \sigma_M\,\left[v_L\,R_L(\mathbf{q},\omega) + v_T\,R_T(\mathbf{q},\omega)\right]$$

Andreoli *et al.* *Phys.Rev.C* 110 (2024) 6, 064004 arXiv:2407.06986

^{12}C cross sections

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$

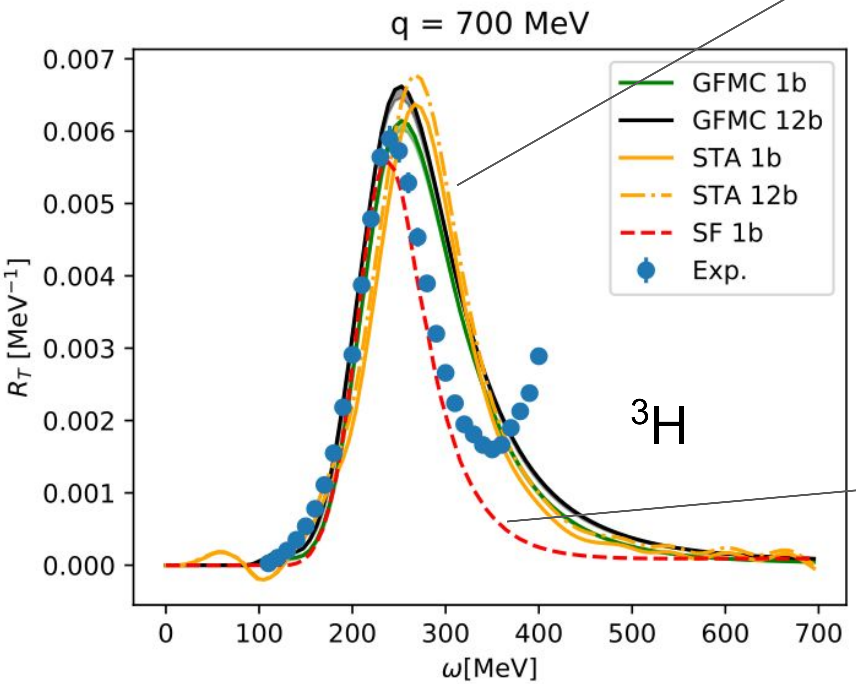


Andreoli *et al.* *Phys.Rev.C* 110 (2024) 6, 064004 arXiv:2407.06986

Data From https://discovery.phys.virginia.edu/research/groups/qes-archive/index.html

Relativistic effects in e-^{3}H scattering

non-relativistic 1b curr

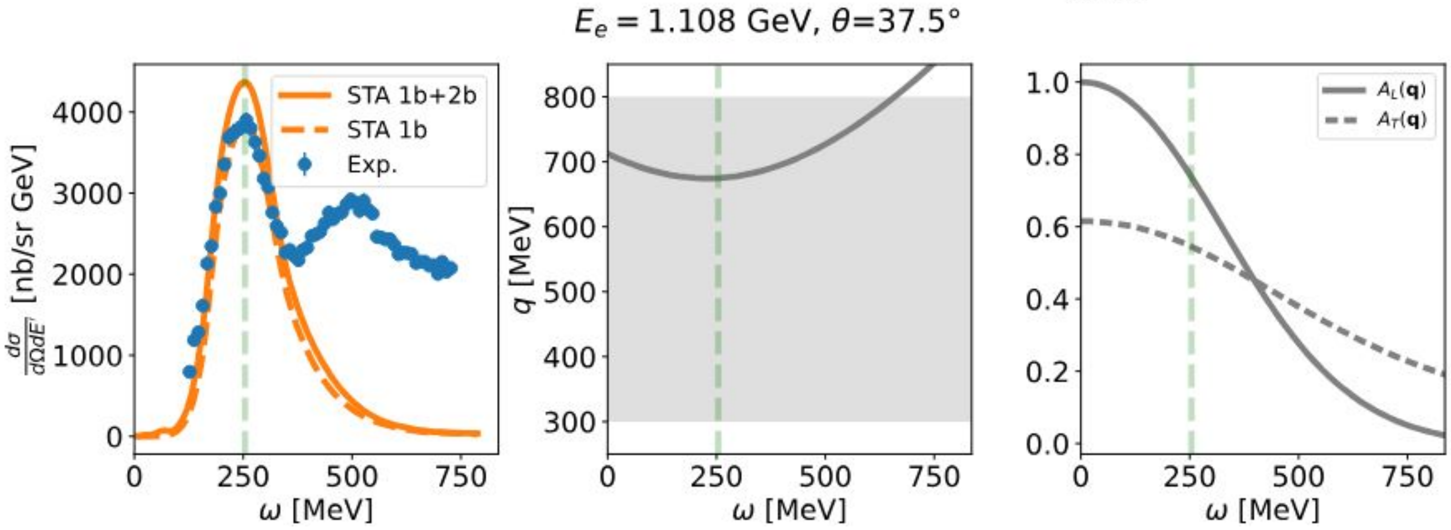


relativistic 1b curr

Andreoli *et al. Phys.Rev.C* 105 (2022) 1, 014002

Relativistic effects in e-^{12}C scattering

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$



Andreoli *et al.* *Phys.Rev.C* 110 (2024) 6, 064004 arXiv:2407.06986

Relativistic corrections

With Ronen Weiss and Lorenzo Andreoli
In preparation

Traditional non relativistic expansion of the covariant single nucleon electromagnetic current assumes initial and final nucleon momentum small.

$$j^\mu = e\bar{u}(\boldsymbol{p}'s')\left(e_N\gamma^\mu + \frac{i\kappa_N}{2m_N}\sigma^{\mu\nu}q_\nu\right)u(\boldsymbol{p}s)$$

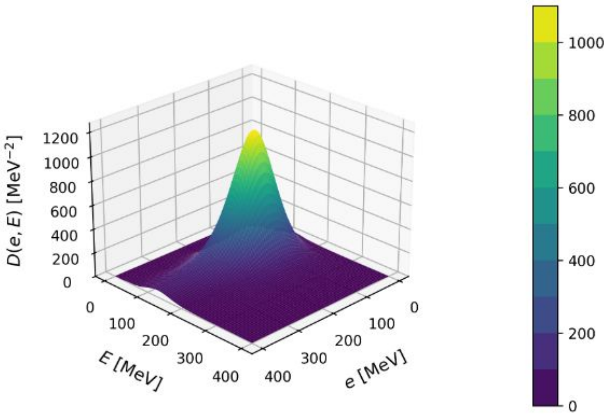
$$\boldsymbol{p}' = \boldsymbol{p} + \boldsymbol{q}$$

New paradigme where the relativistic correction is obtained expanding the covariant one-nucleon current for high values of momentum transfer, and small values of initial nucleon momentum p. This changed:

1. Expression of the one-body operator
2. Energy conserving delta function

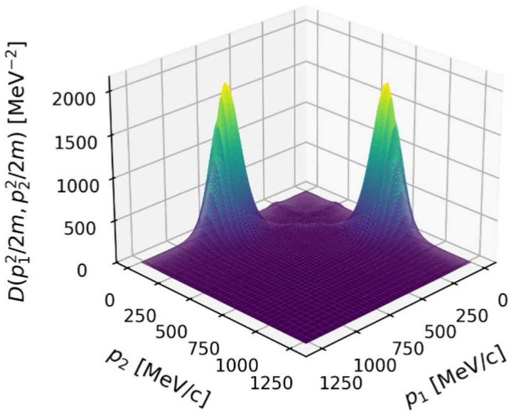
Ronen Weiss
Ed Jaynes Fellow at WashU

Implementation single nucleon current

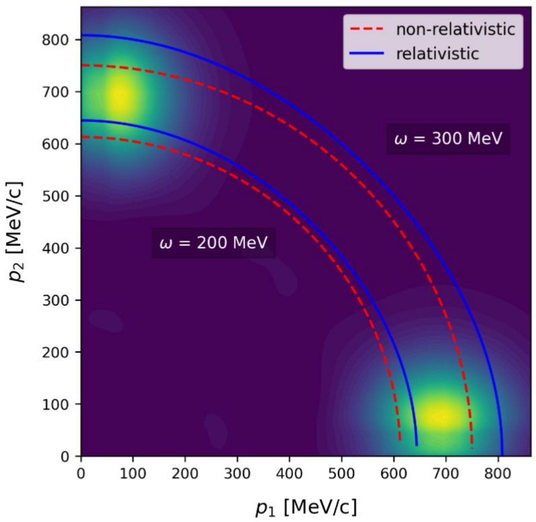


Response density vs relative and c.m. energy of the struck pair

^{4}He Transverse response density at q = 700 MeV/c

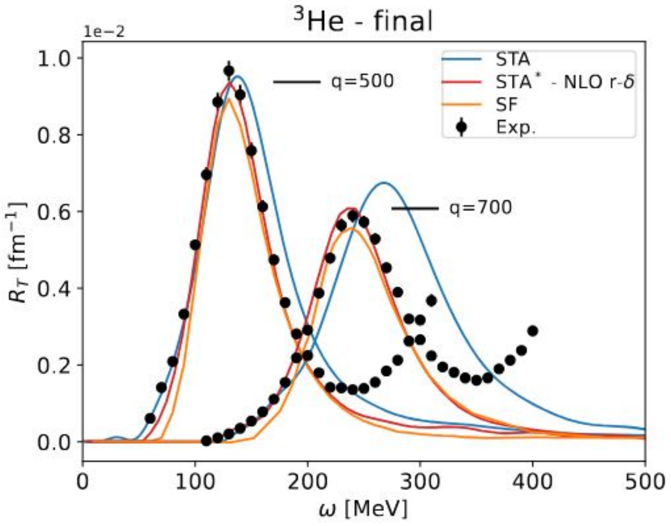


Response density vs momenta of individual nucleons



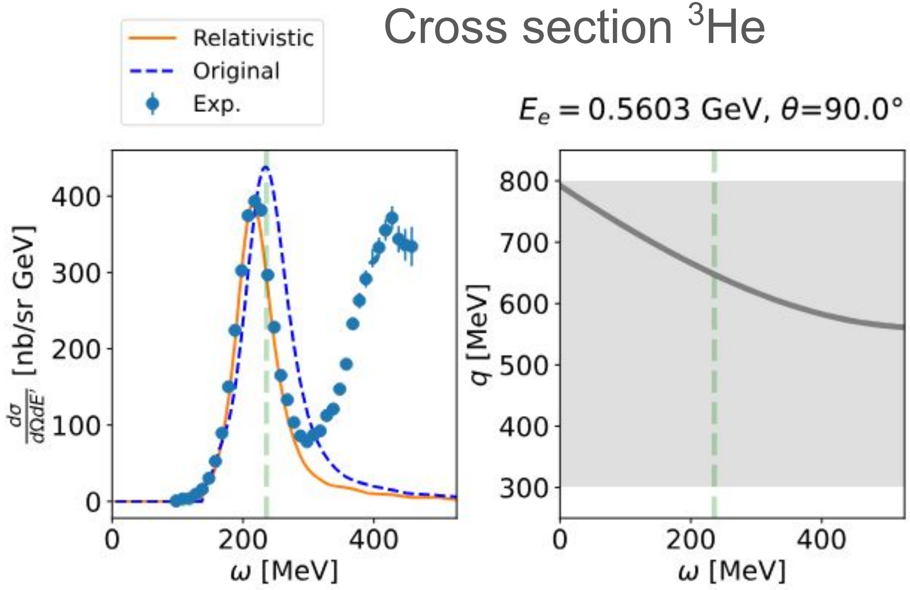
Preliminary

Application to e-^{3}H scattering

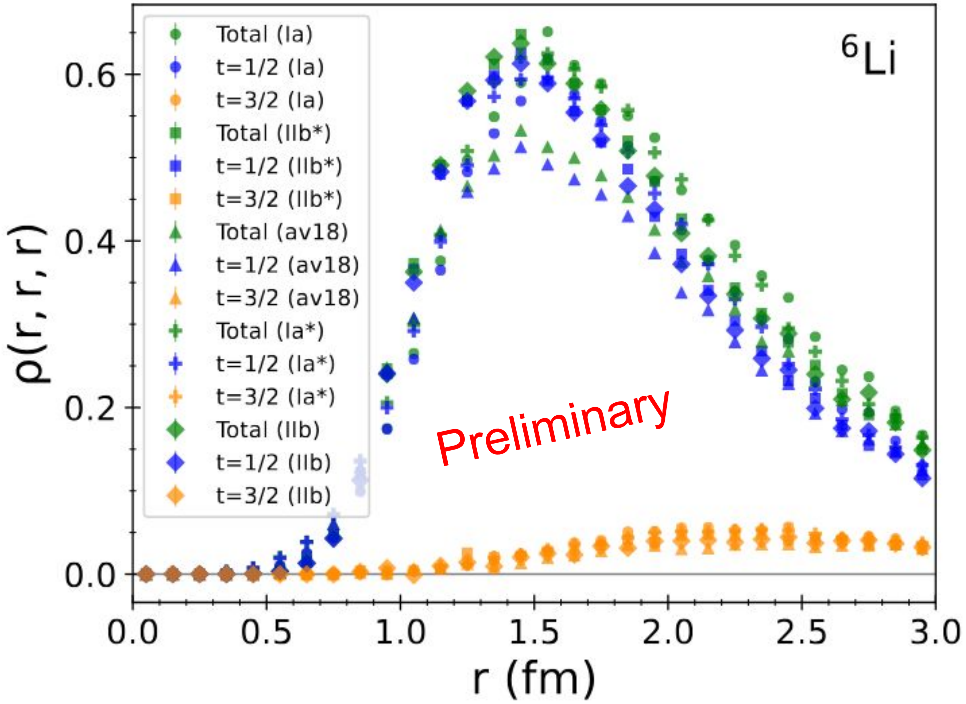


Preliminary

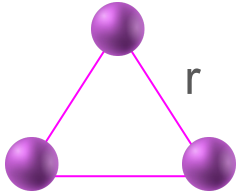
Cross section ^{3}He

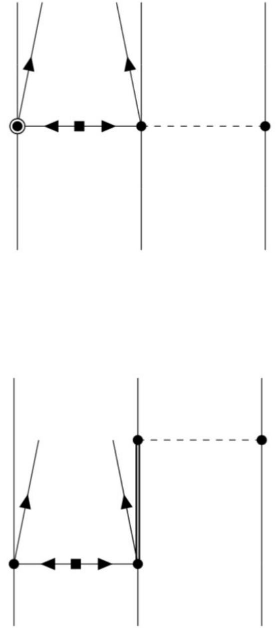


Three-body densities



Graham Chambers-Wall in preparation

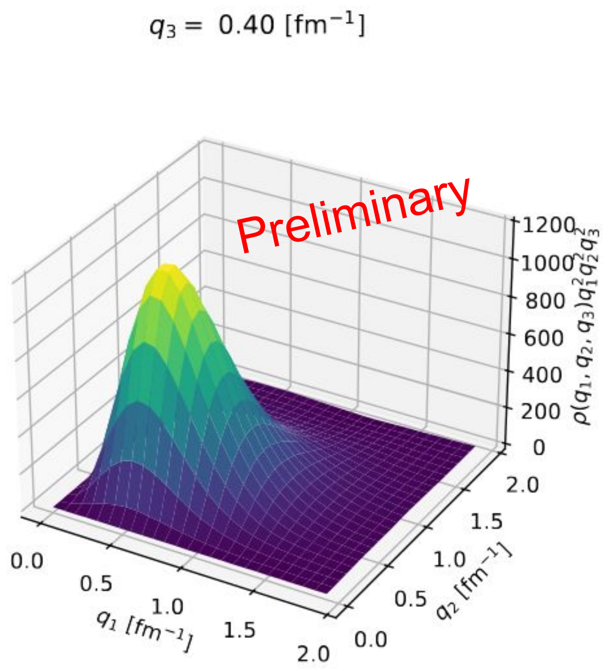


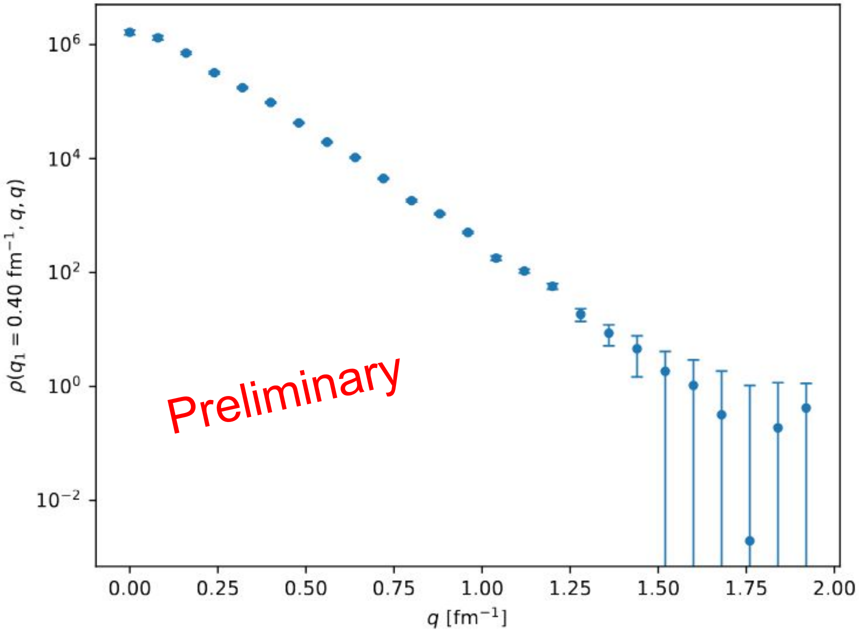


Ronen Weiss, and Stefano Gandolfi *Phys.Rev.C* 108 (2023) 2, L021301

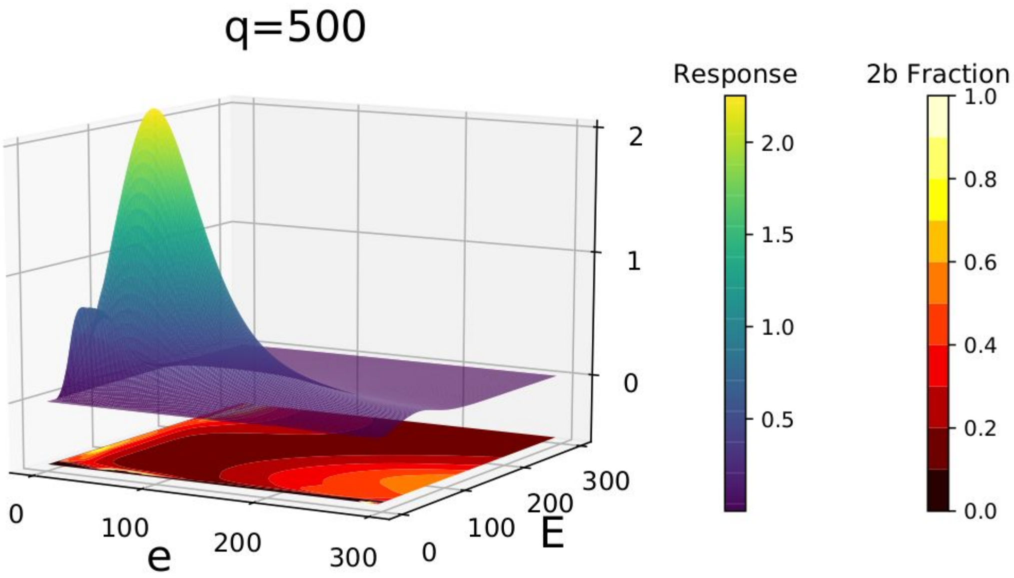
With AFDMC

^{4}He Three-Body Momentum Distribution





Transverse Response Density: two-body physics



STA: regime of validity

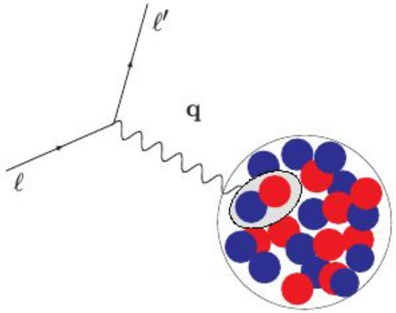
The typical (conservative estimate) energy (time) scale in a nucleus with A correlated nucleons in pairs is

$\varepsilon_{pair} \sim 20$ MeV ($t \sim 1/\varepsilon_{pair}$)

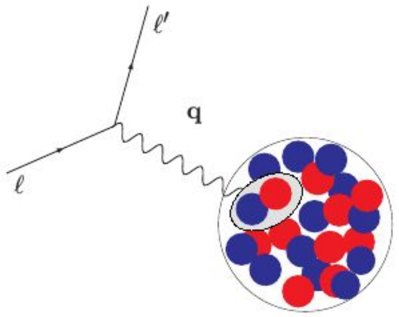
This sets a natural expansion parameter in the QE region characterized by ω_{QE}

$\varepsilon_{pair} / \omega_{QE}$

The STA neglects terms of order $\mathcal{O}((\varepsilon_{pair} / \omega_{QE})^2)$



Short-Time-Approximation



Short-Time-Approximation:

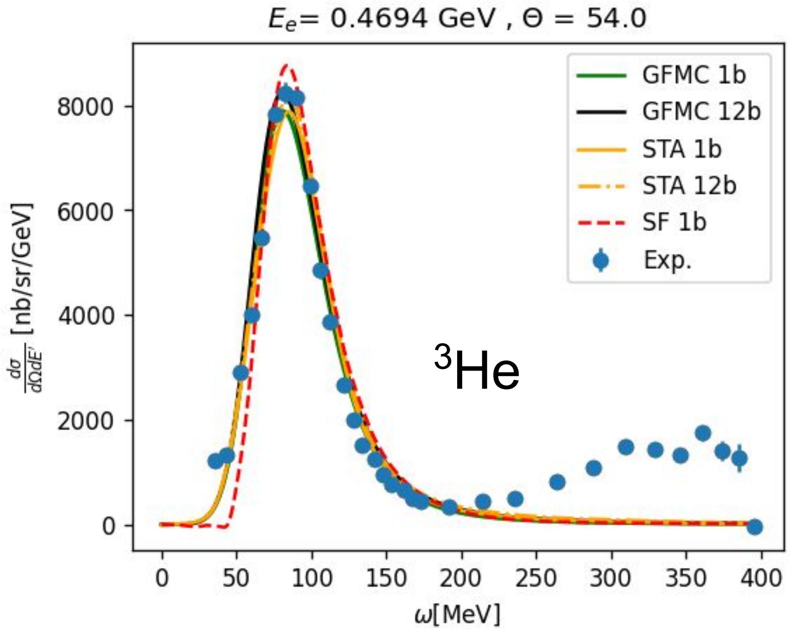
- Based on Factorization
- Retains two-body physics
- Correctly accounts for interference

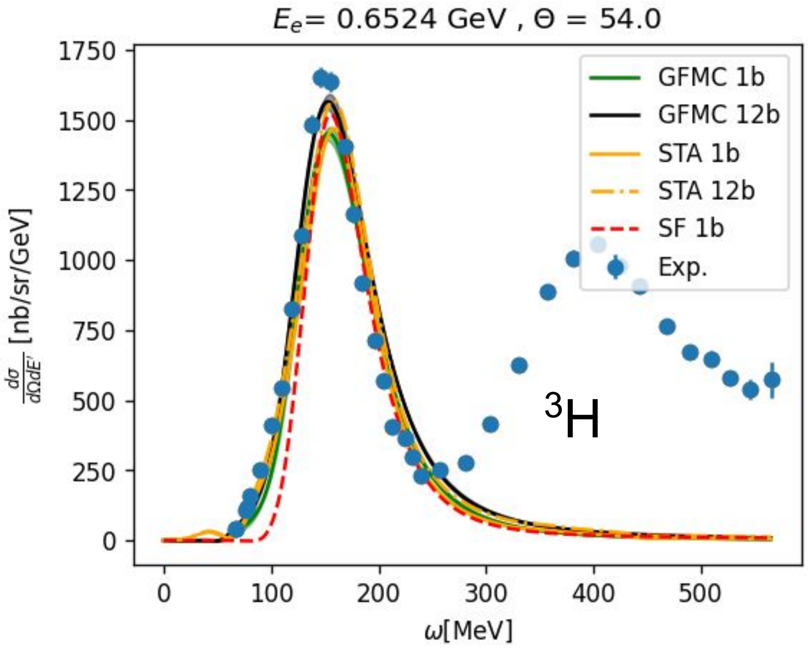
$$R(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_0)t} \langle 0|O^\dagger e^{-iHt} O|0\rangle$$

$$O_i^\dagger e^{-iHt} O_i + O_i^\dagger e^{-iHt} O_j + \boxed{O_i^\dagger e^{-iHt} O_{ij}} + O_{ij}^\dagger e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i<j} v_{ij}$$

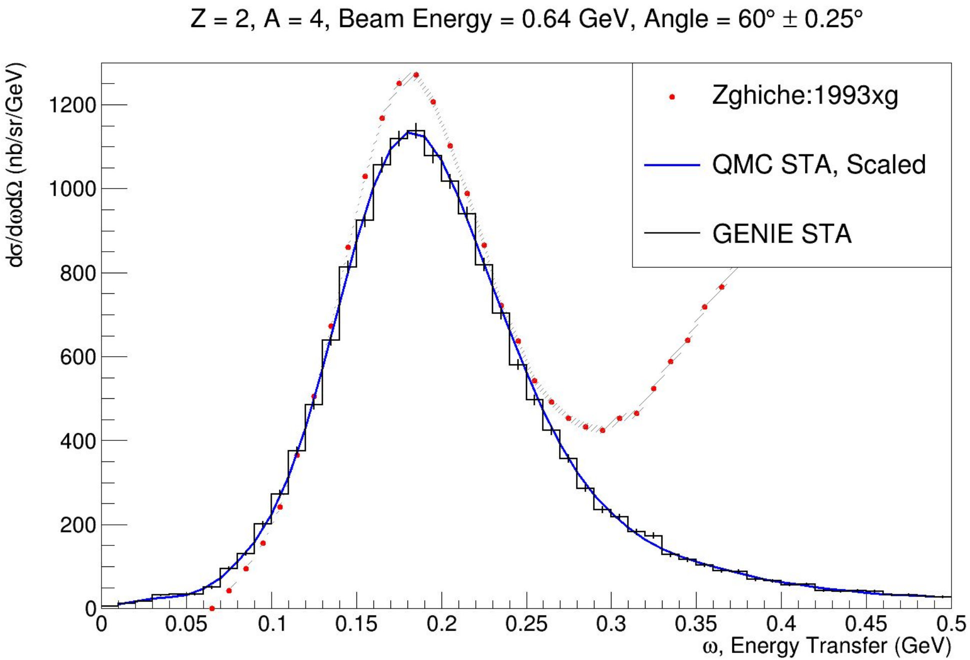
GFMC SF STA: Benchmark & error estimate in A=3





Andreoli *et al. Phys.Rev.C* 105 (2022) 1, 014002

GENIE validation using e-scattering

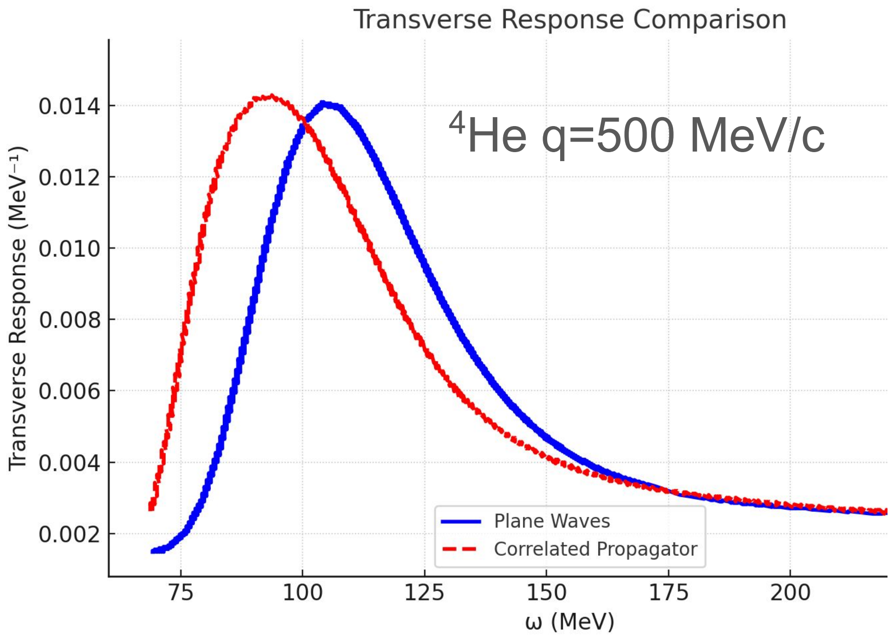


Barrow, Gardiner, SP et al. PRD 103 (2021) 5, 052001

- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE (a Monte Carlo neutrino event generator)
- Here, we use electromagnetic processes (for which data are available) to validate the generator

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$

Correlated pairs vs uncorrelated pairs

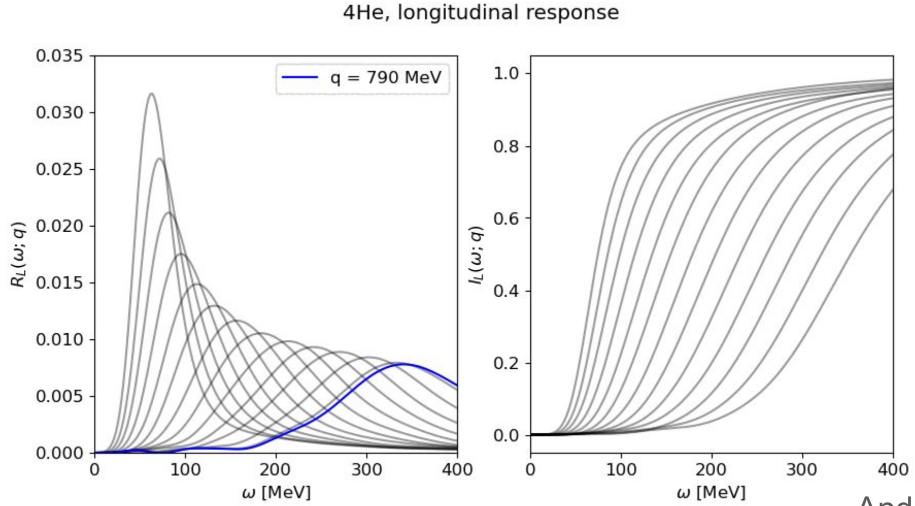


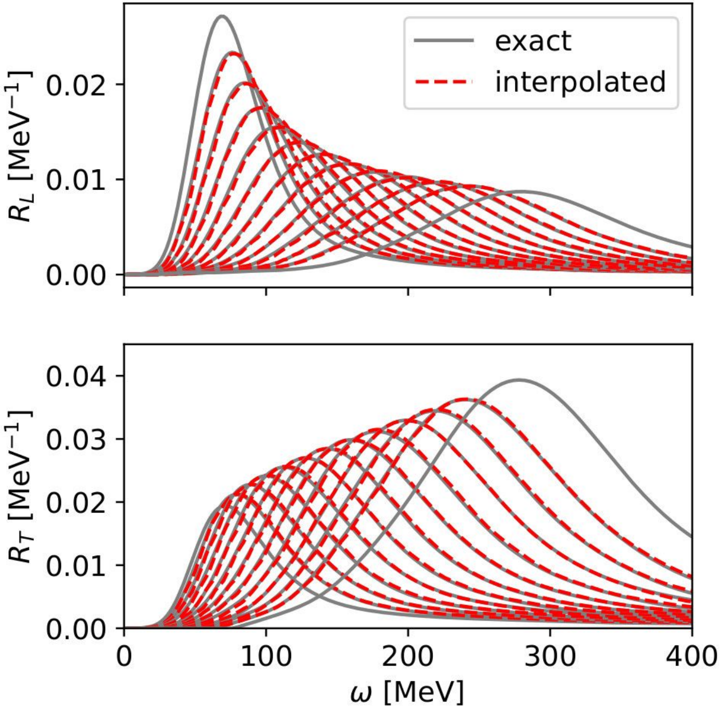
Scattering from uncorrelated vs correlated nucleon pairs

^{12}C cross sections: interpolation scheme

We have coarse grid in q for ^{12}C. We use an interpolation scheme tested on He4.

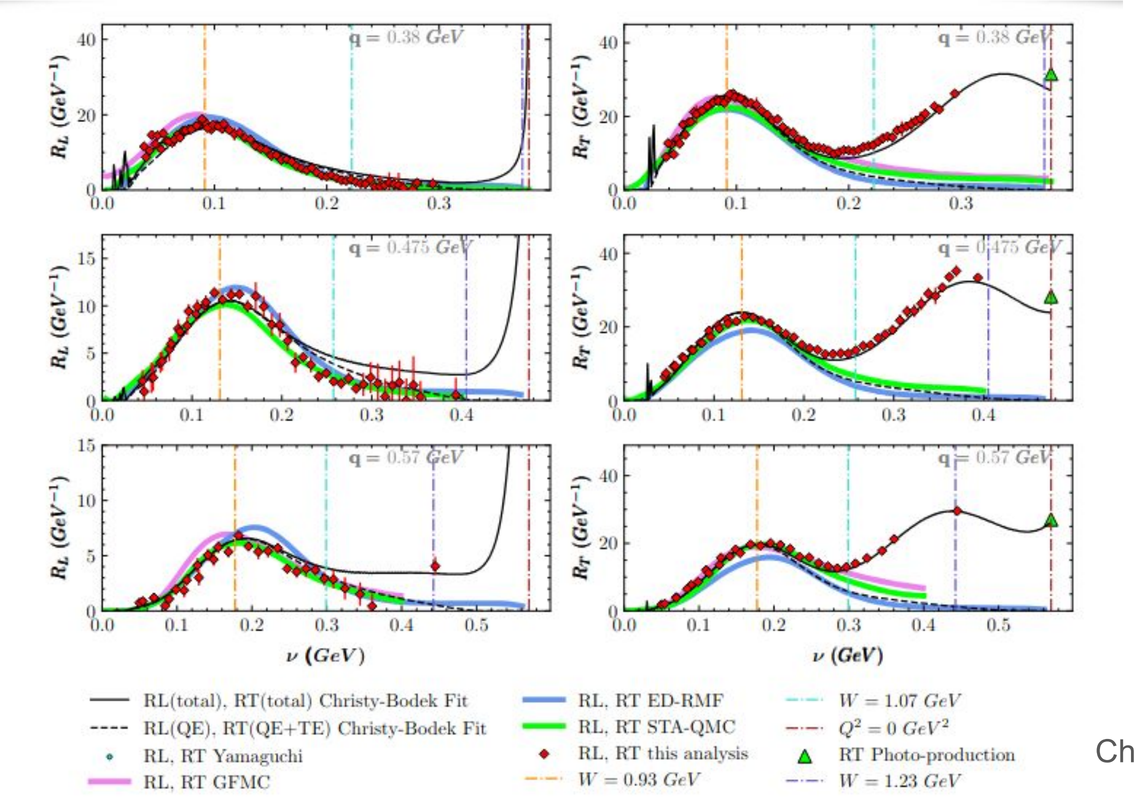
$$I_{L/T}(\omega;\mathbf{q}) = \frac{\int_0^\omega R_{L/T}(\omega';\mathbf{q})d\omega'}{\int_0^\infty R_{L/T}(\omega';\mathbf{q})d\omega'}$$





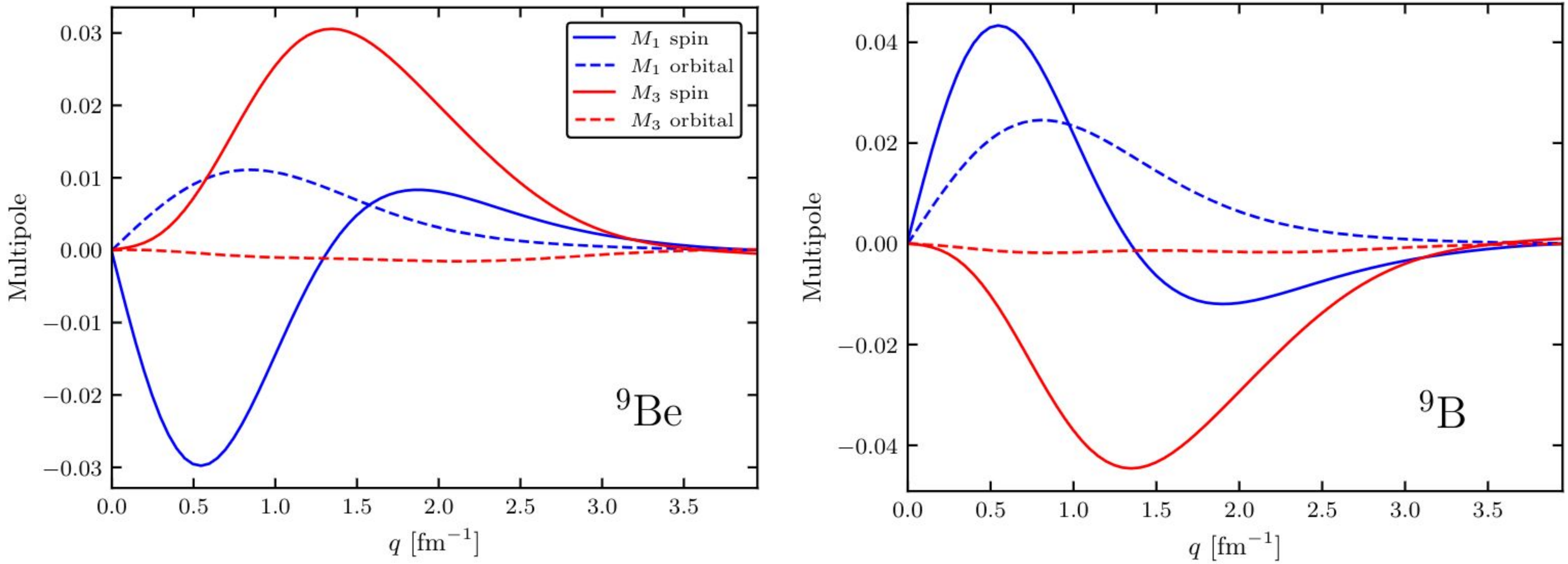
Andreoli *et al.* *Phys.Rev.C* 110 (2024) 6, 064004 arXiv:2407.06986

^{12}C comparison with the data



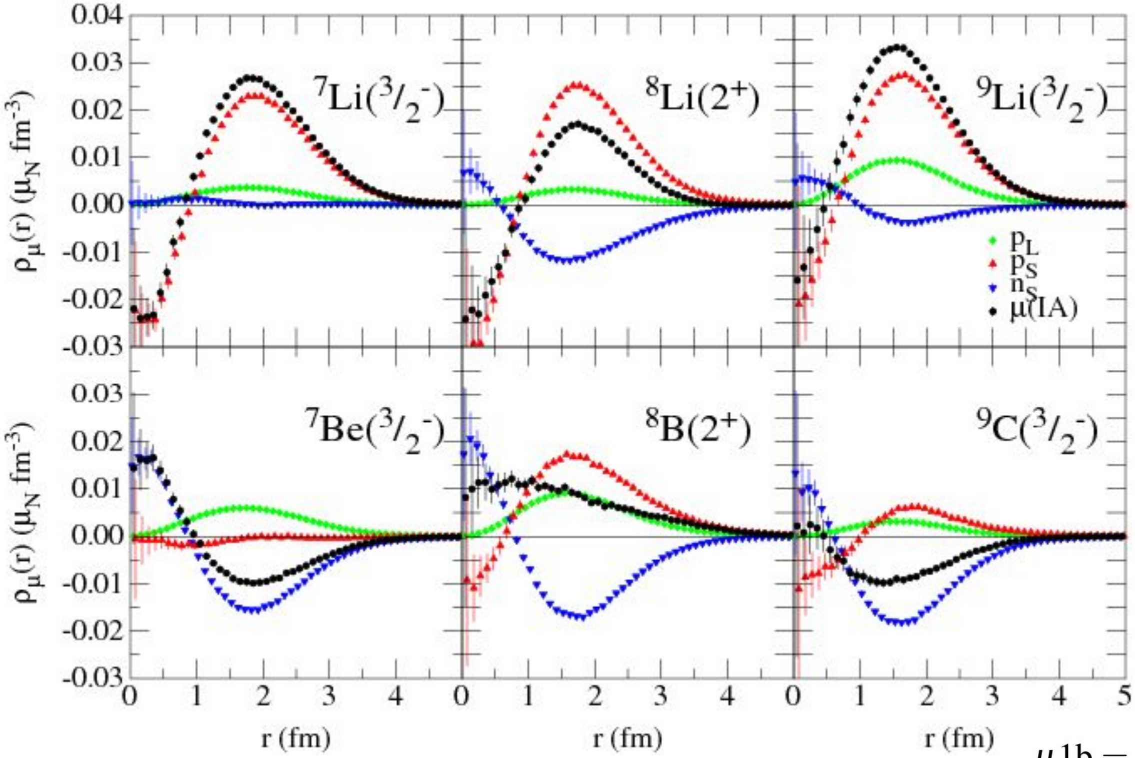
Christy, Bodek et al (2025)

One-body interference in M1 vs M3



M1 vs M3; spin magnetization (solid line) vs orbital (dashed line)

One-body magnetic density

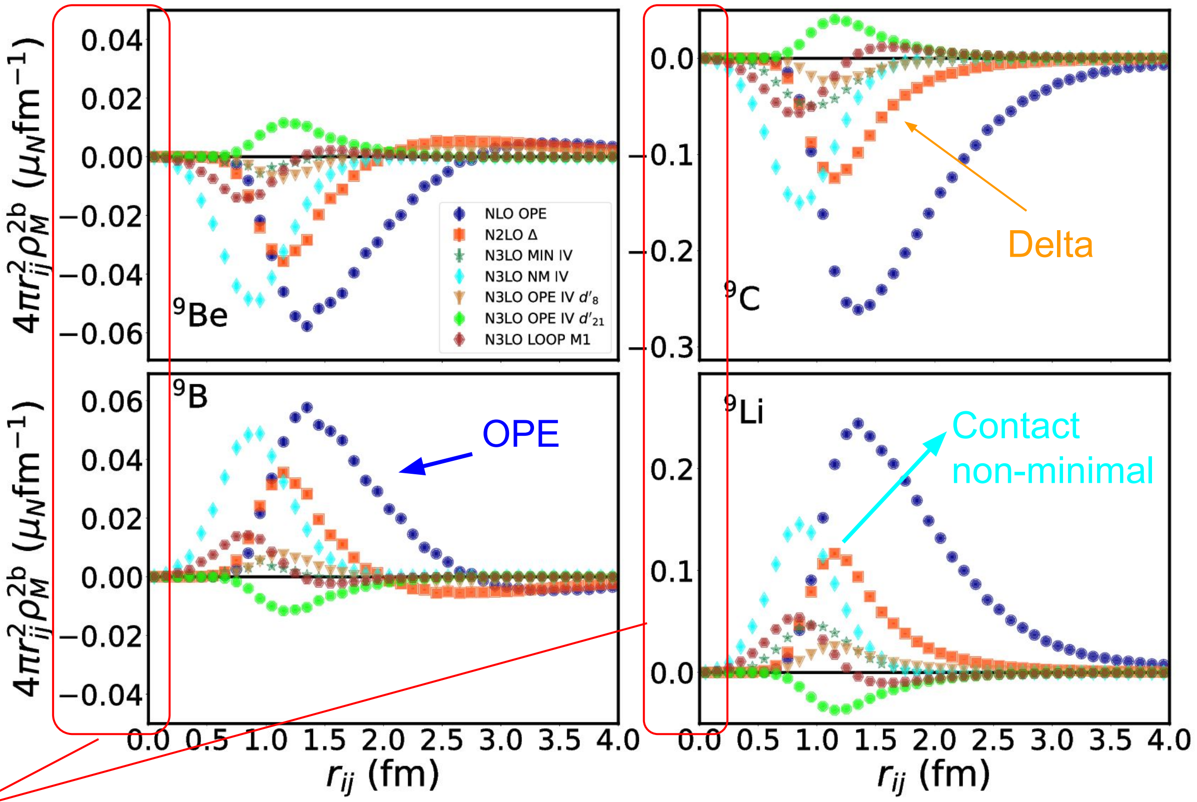


$$\mu^{1b} \propto \int \rho_M^{1b}(r)dr$$

r single particle coordinate from the c.m.

$$\mu 1\mathrm{b} = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

Two-body magnetic densities

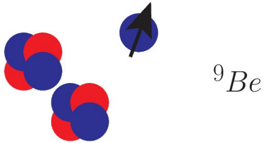


Note the scale

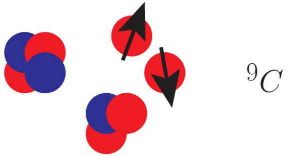
Chambers-Wall, King, Gnech et al. PRC 2024 2407.04744

$$\mu^{2b} = \int dr_{ij} 4\pi r_{ij}^2 \rho_M^{2b}(r_{ij})$$

Cluster effects suppress the two-body contribution for A=9,T=1/2

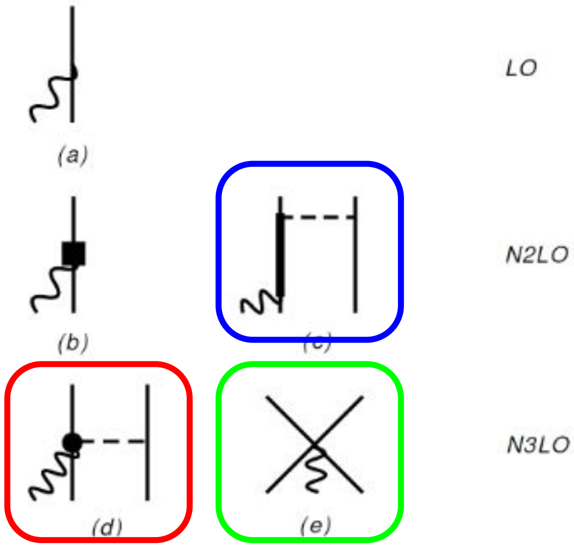


$[441] = [\alpha, \alpha, n(p)]$



$[432] = [\alpha, {}^3\mathrm{He}({}^3\mathrm{H}), pp(nn)]$

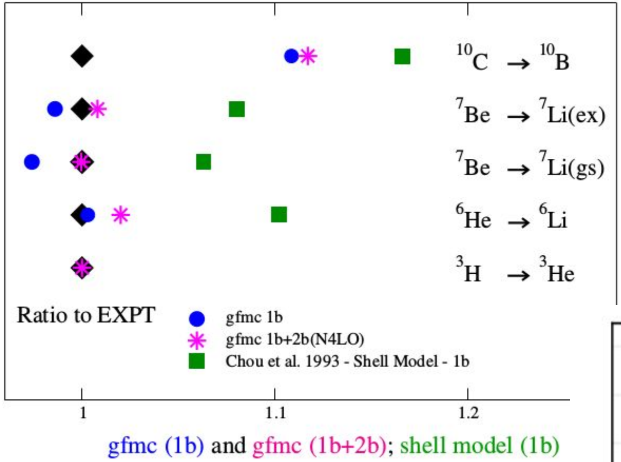
Axial currents with Δ at tree-level



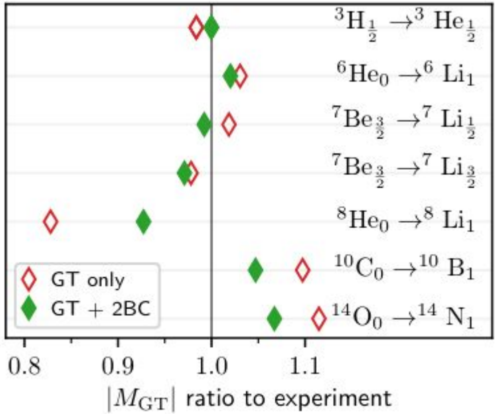
Two body currents of one pion range (red and blue) with c_3 c_4 from Krebs *et al.* Eur.Phys.J.(2007)A32

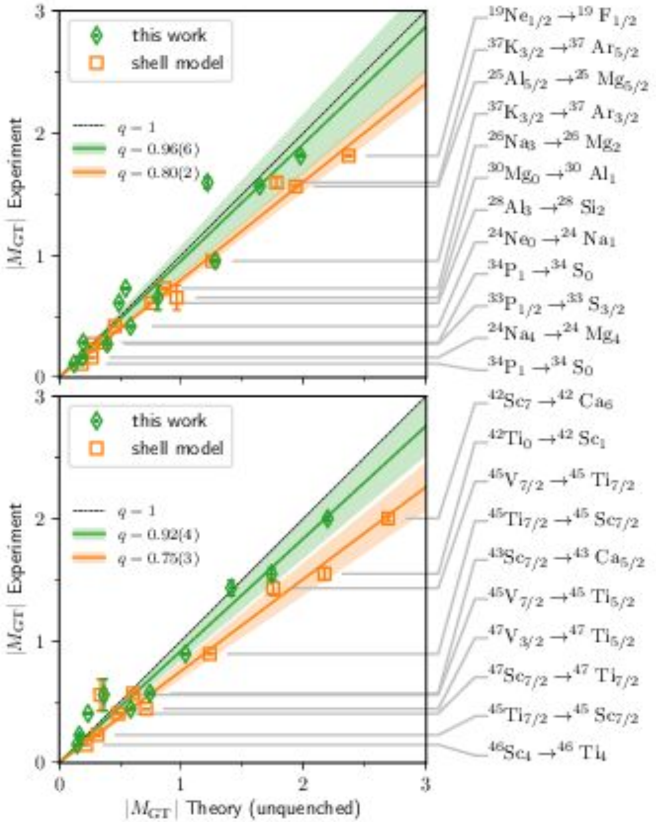
Contact current involves the LEC $\boldsymbol{c_D}$

Beta decay



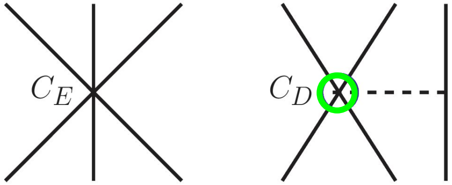
SP *et al*. PRC97(2018)022501



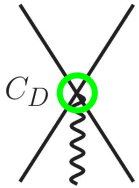


P. Gysbers *Nature Phys.* 15 (2019)

Three-body Force and the Axial Contact Current



Three-body force



Axial two-body contact current

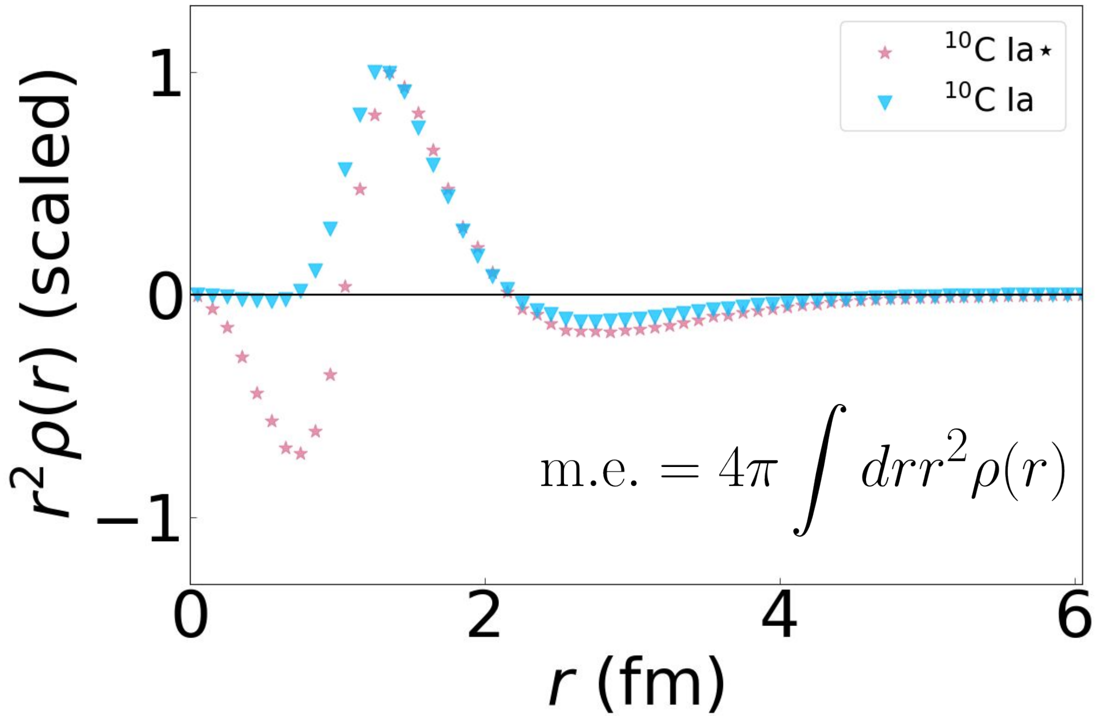
LECs c_D and c_E are fitted to:

- **trinucleon B.E.** and ***nd*** **doublet scattering length** in **NV2+3-Ia**
- **trinucleon B.E.** and **Gamow-Teller matrix element of tritium** **NV2+3-Ia***

Baroni *et al.* PRC98(2018)044003

Energies A=8-10 slightly better with non-starred models

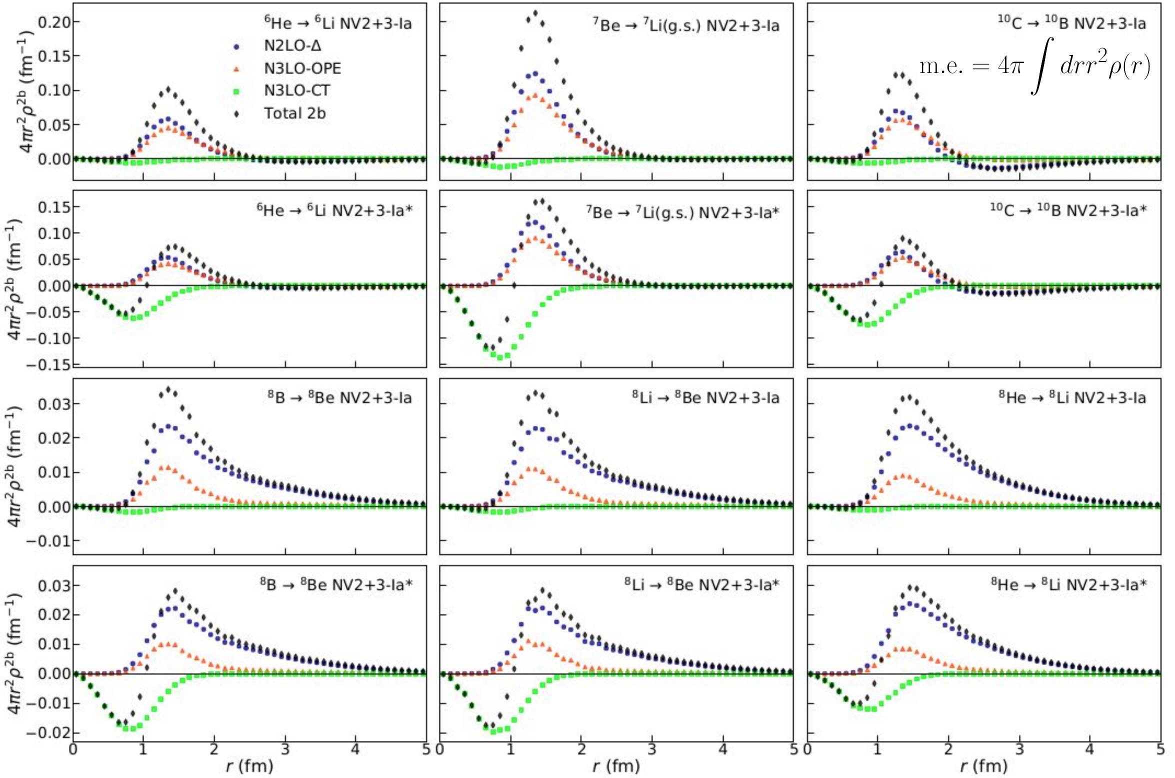
Scaled two-body transition densities



Different fitting procedures lead to different short range behaviours.

Garrett King *et al.* PRC102(2020)025501

Axial Two-body Transition Density



Garrett King *et al.* PRC102(2020)025501

NV2+3-Ia ; **NV2+3-Ia***

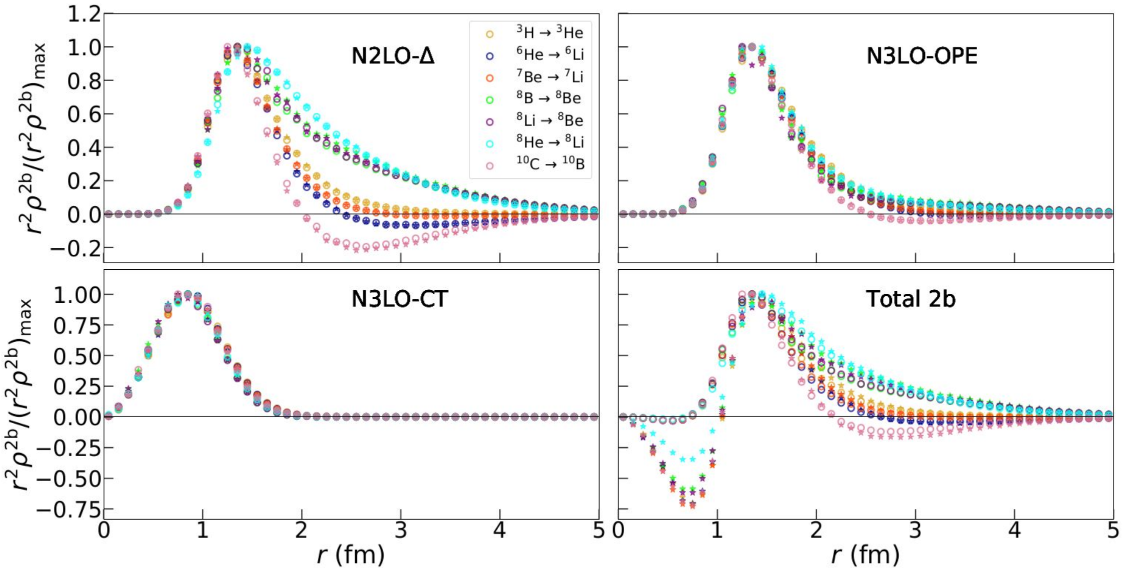
enhanced contribution from contact current in the starred model gives rise to nodes in the two-body transition density

Two-body axial currents

long-range at N2LO and N3LO

contact current at N3LO

Scaling & Universality of Short-Range Dynamics



Garrett King *et al.* PRC102(2020)025501

NV2+3-Ia empty circles; NV2+3-Ia* stars
Different colors refer to different transitions

Quantum Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: H = T + v_{ij} + V_{ijk}

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using the trial wave function:

$$|\Psi_V\rangle = \left[\mathcal{S}\prod_{i<j}(1 + U_{ij} + \sum_{k\neq i,j} U_{ijk})\right]\left[\prod_{i<j} f_c(r_{ij})\right]|\Phi_A(JMTT_3)\rangle$$

Further improve the trial wave function by eliminating spurious contaminations via a Green's Function Monte Carlo propagation in imaginary time

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n\psi_n$$

$$\Psi(\tau \to \infty) = a_0\psi_0$$

Carlson, Wiringa, Pieper *et al.*