Precision Physics with Few-Electron Ions: Testing the Standard Model and Beyond

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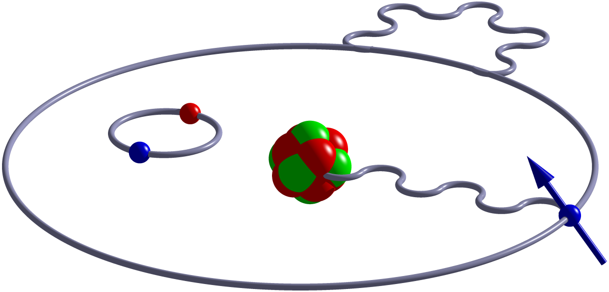
ECT* Workshop on
"New perspectives in the charge radii determination for light nuclei"
July 29, 2025

Highly charged ions

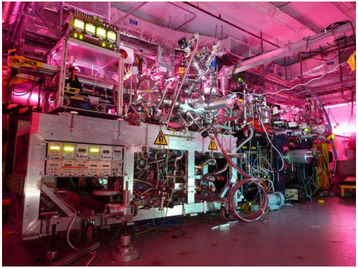


Size: $r = \frac{a_0}{Z}$, with a_0 being the Bohr radius ($a_0 = 5.26 \times 10^{-11}$ m)
$\rightarrow$ smaller radii due to the Coulomb attraction of the nucleus

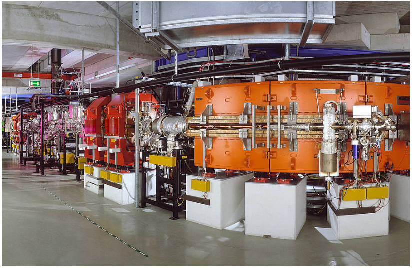
$\rightarrow$ somewhere between electronic H (a_0) and muonic H ($a_0/207$)
Energy scale: $E = -\frac{1}{2} m_\mathrm{e} c^2 (Z\alpha)^2 \approx m_e c^2$: relativistic effects
($\alpha \approx \frac{1}{137}$: fine-structure constant)

"high-energy atomic physics"

Generation of HCI in **laboratory experiments**:



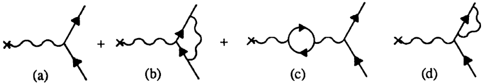
Electron beam ion trap (EBIT) at the Max Planck Institute for Nuclear Physics, Heidelberg



Heavy ion storage ring ESR of the GSI Helmholtz Centre for Heavy Ion Research in Darmstadt, Germany

The g factor of the free electron

Interaction energy of an electron with an external **magnetic field**

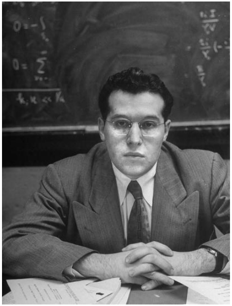


At the one-loop level, it is only corrected by the vertex diagram

$$\Delta E = -\langle \vec{\mu} \rangle \cdot \vec{B} \,,$$

with the magnetic moment μ, the Bohr magneton $\mu_B = \frac{e\hbar}{2mc}$

$$\langle \vec{\mu} \rangle = g \mu_B \langle \vec{S} \rangle \,.$$



Thus the ***g*-factor of the free electron up to the one-loop order** is

$$g_{\text{free}} = 2 + \frac{\alpha}{\pi} \approx 2(1 + 0.00116141)$$

The α/π term is the Schwinger term (Schwinger, 1947)

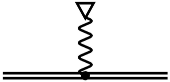
$$2 \quad + \quad \frac{\alpha}{\pi}$$

Dirac and Feynman at the Relativity conference, Jabłonna Palace, 1962

The bound-electron g factor

Bound-electron g factor theory = $\underbrace{\text{free } g \text{ factor theory}}_{\text{perturbative } B \text{ field}} + \underbrace{\text{Lamb shift theory}}_{\text{strong Coulomb field}}$

For a Coulomb potential, the Dirac g-factor for the 1s state (G. Breit, 1928):



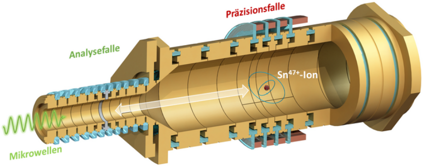
$$g_{\mathrm{D}} = \frac{2}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) = 2 - \frac{2}{3}(Z\alpha)^2 - \frac{1}{6}(Z\alpha)^4 + \ldots$$

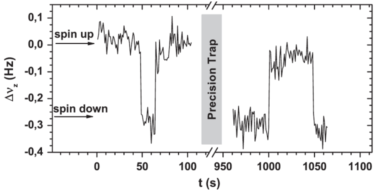
Double line: Coulomb-Dirac (wave function or) propagator:

with an arbitrary number of interactions with the nuclear potential

Penning trap measurement of the g factor

See talk of Fabian Heisse yesterday!





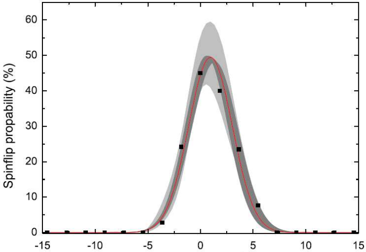
Larmor frequency:

$$\nu_{\rm L} = g\mu_B B \frac{1}{2\pi} = g\frac{e}{4\pi m_e}B\,,$$

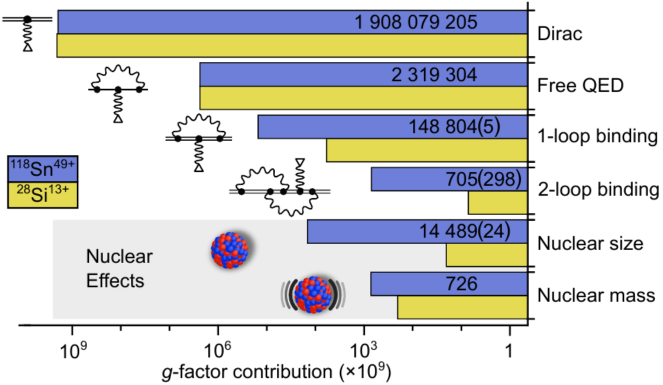
Cyclotron frequency:

$$\nu_{\rm c} = \frac{qB}{2\pi m_{\rm ion}}\,,$$

$$\rightarrow \quad g_{\rm exp} = 2\frac{\nu_{\rm L}}{\nu_{\rm c}}\frac{m_e}{m_{\rm ion}}\frac{q}{e}$$



g factor of hydrogenlike Sn^{49+}



$$g_{\rm exp} = 1.910\,562\,058\,962(73)_{\rm stat}(42)_{\rm sys}(910)^{1}_{\rm ext}\,;$$

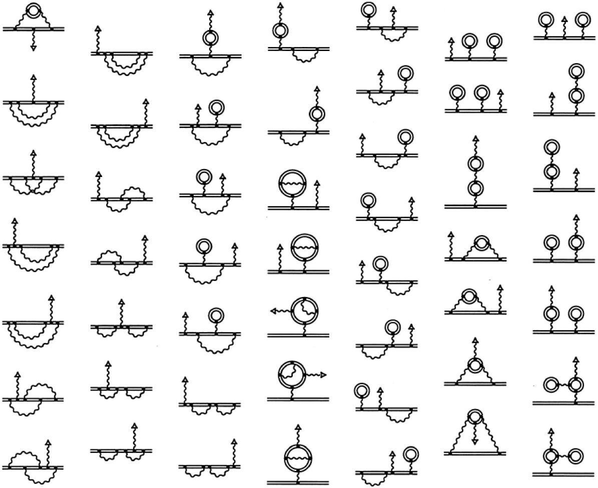
$$g_{\rm theo} = 1.910\,561\,82(30)^{1}$$

$$g_{\rm theo} = 1.910\,561\,98(4)^{2} \quad \text{improved two-loop QED theory}$$

[1] J. Morgner, B. Tu, C. M. König *et al.*, Nature **622**, 53 (2023);

[2] B. Sikora, V. A. Yerokhin, C. H. Keitel, Z. Harman, Phys. Rev. Lett. **134**, 123001 (2025)

Two-loop corrections in a nonperturbative Coulomb field

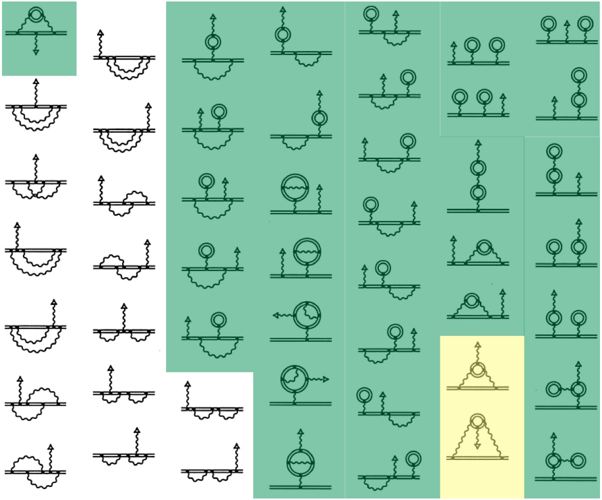


50 total diagrams

(29 inequivalent diagrams)

(Slide: courtesy of V. Debierre)

Two-loop corrections in a nonperturbative Coulomb field



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Diagrams with 0&1 self-energy loops → Treated in [V.A. Yerokhin, ZH, Phys. Rev. A **88**, 042502 (2013)]

(with free VP ($e^- e^+$) loops)

[A. Czarnecki, R. Szafron, Phys. Rev. A 94, 060501(R) (2016)]

(Slide: courtesy of V. Debierre)

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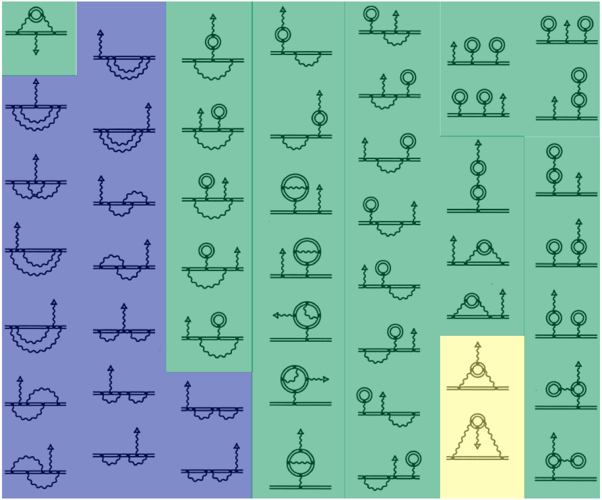

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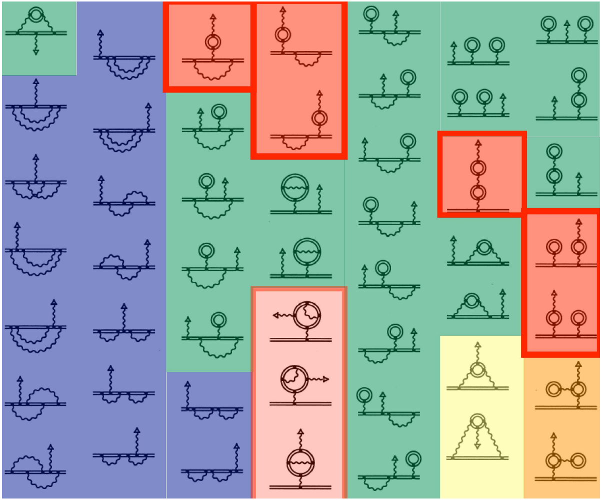
[A. Czarnecki, R. Szafron, Phys. Rev. A **94**, 060501(R) (2016)]

Diagrams with 2 self-energy loops
[B. Sikora, V. A. Yerokhin, C. H. Keitel, Z. Harman, Phys. Rev. Lett. **134**, 123001 (2025); B. Sikora, V. A. Yerokhin, N. S. Oreshkina, H. Cakir, C. H. Keitel, Z. H., Phys. Rev. Research **2**, 012002(R) (2020)]

(Slide: courtesy of V. Debierre)

Two-loop corrections in a nonperturbative Coulomb field



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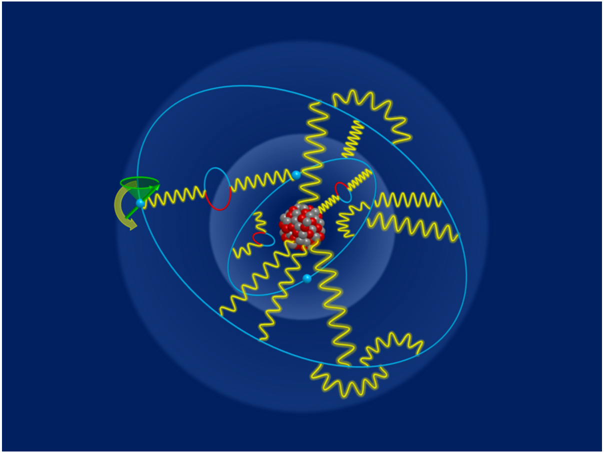
[A. Czarnecki, R. Szafron, Phys. Rev. A **94**, 060501(R) (2016)]

Also in progress: Diagrams that vanished in the free VP loop approach & calculate lowest nonvanishing contribution
[V. Debierre, B. Sikora, H. Cakir, N. S. Oreshkina, V. A. Yerokhin, C.H. Keitel, Z. H., Phys. Rev. A **103**, L030802 (2021)]

g factor of lithiumlike Sn^{47+}



$$g_{\text{theo}} = 1.980\,354\,796(12)$$
$$g_{\text{exp}} = 1.980\,354\,799\,750(84)_{\text{stat}}(54)_{\text{sys}}(944)_{\text{ext}}$$

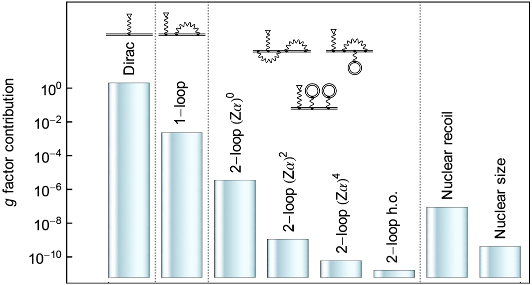
J. Morgner, V. A. Yerokhin, C. König *et al.*, Science **388**, 6750, 945 (2025)

High-precision determination of the electron mass

The mass of the electron can be expressed by the mass and charge of the $^{12}C^{5+}$ ion, the experimentally measured cyclotron and Larmor frequencies, and the theoretical g-factor as

$$m_e = \frac{g}{2}\frac{e}{Q}\frac{\nu_{\rm cycl}}{\nu_{\rm L}} m_{\rm ion}$$

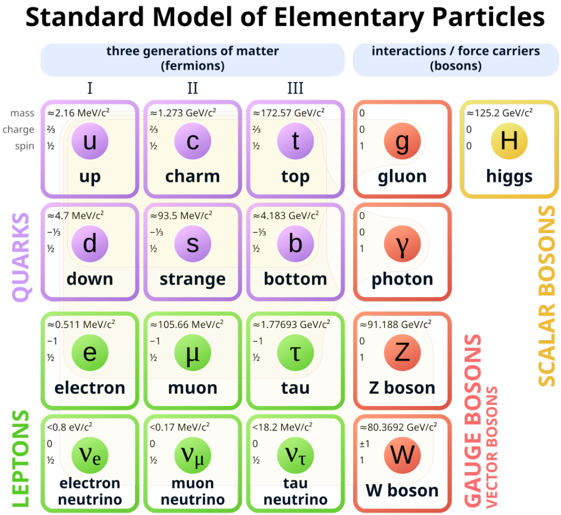
- $e/Q = 1/6$;
- $m_{\rm ion}$ is known very well ($m_{^{12}\rm C\ atom} \equiv 12$ u);
- $\nu_{\rm cycl}/\nu_{\rm L}$ is measured very precisely;
- the g-factor is taken from theory



The resulting value $m_e = 0.000\,548\,579\,909\,069\,4(128)_{\rm stat}(86)_{\rm sys}(13)_{\rm theo}$ u surpasses the earlier CODATA value by more than an order of magnitude and largely defines the new CODATA value

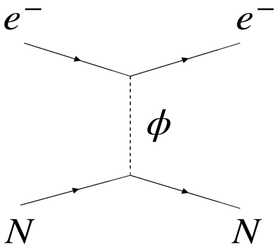
- S. Sturm, F. Köhler, J. Zatorski, A. Wagner, Z. H., G. Werth, W. Quint, C. H. Keitel, K. Blaum, Nature **506**, 467 (2014)
- F. Köhler, S. Sturm, A. Kracke, G. Werth, W. Quint, and K. Blaum, J. Phys. B **48**, 144032 (2015)

Beyond QED



However in Nature there should be something more, maybe new scalars or vectors?

- A proposed fifth fundamental force
  Massive spinless boson ϕ (mass range unknown)
  Couples electrons to nucleons
- Relevance to high-energy physics
  - They are light dark matter candidates
  - Known to be addressable by low-energy experiments since their proposal:
    P. W. Graham, D. E. Kaplan, S. Rajendran, Phys. Rev. Lett. **115**, 221801 (2011)
  - Similarly, pseudoscalars: axions

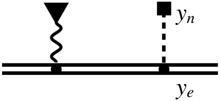


- Yukawa potential seen by electrons

$$V_\phi(\mathbf{r}) = -\hbar c\, \alpha_{\rm NP} A \frac{e^{-\frac{m_\phi c}{\hbar}|\mathbf{r}|}}{|\mathbf{r}|}$$

  - m_ϕ: mass of the boson
  - $\alpha_{\rm NP} = \frac{y_e y_n}{4\pi}$ coupling constant: coupling **electron-neutron** or **electron-proton**, $\frac{y_e y_p}{4\pi}$

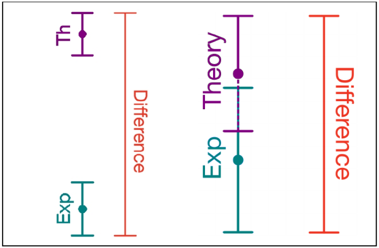
Correction to g factor:



$$\Delta g_{1s} = -\frac{4}{3}\alpha_{\rm NP}\frac{(Z\alpha)}{\gamma} A \left(1 + \frac{m_\phi}{2Z\alpha m_e}\right)^{-2\gamma} \times \left[3 - 2\frac{(Z\alpha)^2}{1+\gamma} - \frac{2\gamma}{1+\frac{m_\phi}{2Z\alpha m_e}}\right]$$

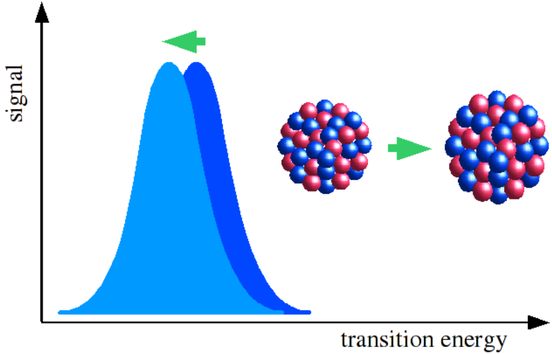
V. Debierre, Z.H., C. H. Keitel, Phys. Lett. B **807**, 135527 (2020)

If a disagreement between experiment and standard model theory is found, that MIGHT BE due to this term:

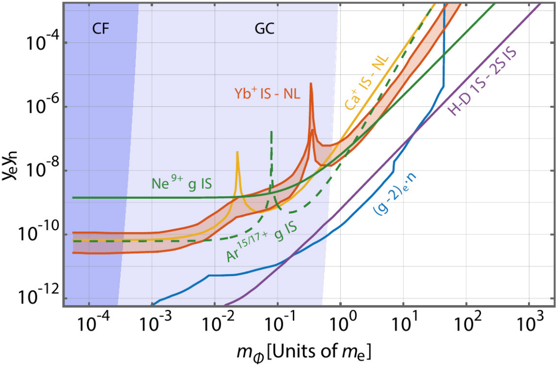


Recent **ALPHATRAP experiment** on the isotope shift (IS) of H-like Ne,
$\Delta g = g({}^{20}_{10}\mathrm{Ne}^{9+}) - g({}^{22}_{10}\mathrm{Ne}^{9+})$:

- Differences of g factors of two similar ions can be measured very **accurately**, since several systematic effects cancel;
- Different isotopes: different number of **neutrons**, sensitivity to such new physics
- Same number of protons: **QED** (“old physics”) largely cancels



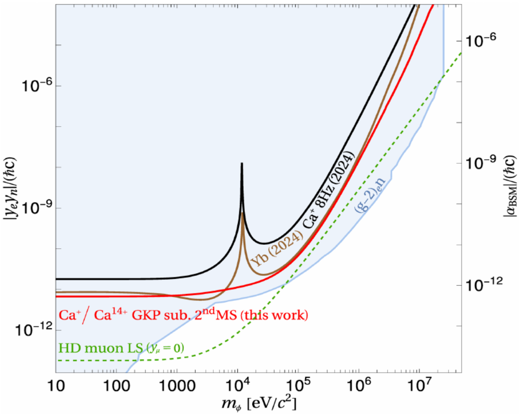
Bounds on the coupling strength extracted from the experimental Δg:



- IS-NL: isotope shift nonlinearity experiments
  Ca^{+}: J. Berengut *et al.*, Phys. Rev. Lett. **120**, 091801 (2018);
  Yb^{+}: I. Counts *et al.*, Phys. Rev. Lett. **125**, 123002 (2020)
- H-D: hydrogen-deuterium isotope shift, laser spectroscopy
  C. Delaunay *et al.*, Phys. Rev. D **96**, 115002 (2017)
- $Ar^{15/17+}$: projected possible bound from g factor isotope shift of H-like and Li-like Ar

T. Sailer, V. Debierre, Z. H., F. Heiße, C. König, J. Morgner, B. Tu, A. V. Volotka, C. H. Keitel, K. Blaum, S. Sturm, Nature **606**, 479 (2022)

Very recent results from King plot nonlinearity

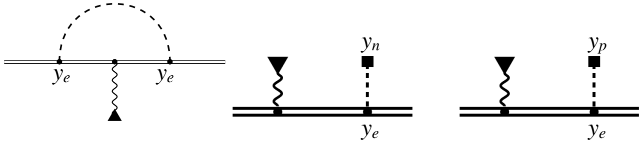


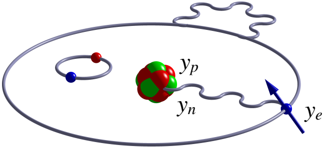
- C^{14+} and C^+ ions: A. Wilzewski *et al.*, Phys. Rev. Lett. **134**, 233002 (2025)
  Experiments: group of Piet O. Schmidt
- Yb^+ ions: M. Door, C. Yeh, M. Heinz *et al.*, Phys. Rev. Lett. **134**, 063002 (2025)
  Experiments: group of T. E. Mehlstäubler

How to constrain the **protonic** coupling?

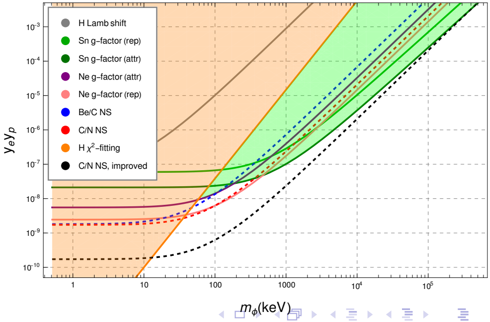
A highly charged ion contains all fermions: at least one electron, protons and neutrons. Total shift of g factor:

$$\Delta g = \underbrace{\Delta g_{ee}}_{\sim y_e^2} + \underbrace{\Delta g_{en}}_{\sim y_e y_n} + \underbrace{\Delta g_{ep}}_{\sim y_e y_p}$$



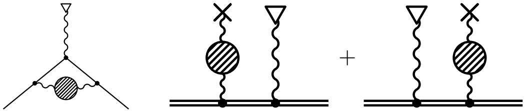


Data for the g factor of H-like Ne^{9+}: Heisse *et al.*, Phys. Rev. Lett. **131**, 253002 (2023) ⇒ H data from: **Robert Potvliege**, see his talk on Friday!



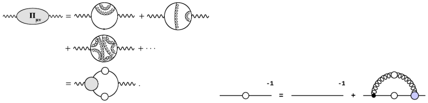
Hadronic vacuum polarization correction to the g factor

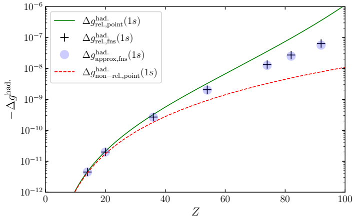
Searching for BSM physics; but: SM $\neq$ QED!



Our calculations so far: based on hadronic polarization functions extracted from collider experiments (similarly as for muon $g-2$) [S. Breidenbach, E. Dizer, H. Cakir, Z. Harman, Phys. Rev. A **106**, 042805 (2022); E. Dizer, Z. Harman, Phys. Rev. A **108**, 042808]

In progress: ab initio QCD calculation by the group of **Jan Pawlowski**, Heidelberg university by iteratively solving the Dyson-Schwinger equations



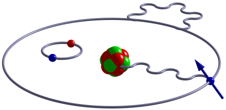


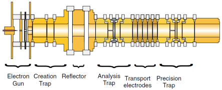
[In analogy to: T. Goecke *et al.*, Phys. Lett. B **704**, 211 (2011)]

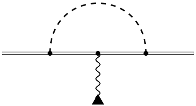
Above $Z=14$, hadronic VP in the bound-e^- g factor is larger than in the free-e^- g factor! (But you will need better radii...)

Summary

- Accurate **test of QED** in strong fields with the g factor of highly charged ions – very recent calculation of two-loop self-energy in nonperturbative nuclear fields
- Testing the QED of many-body systems
- Determination of **fundamental constants**, e.g. m_e, α
- Competitive tests of hypothetical **new physics** possible through g-factor measurements
- High flexibility: fermion-resolved determination of coupling constants: coupling to **electrons, protons, neutrons**
- **Hadronic vacuum polarization** visible in foreseeable future







Collaboration

- **Theory, MPIK:**
  B. Sikora, V. A. Yerokhin, M. Moretti, H. Cakir, V. Debierre, N. S. Oreshkina, C. H. Keitel

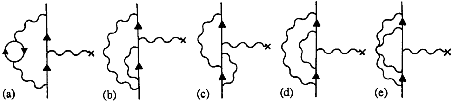
- **Theory, St. Petersburg, Russia:**
  I. I. Tupitsyn

- **Theory, Warsaw and Poznan, Poland:**
  K. Pachucki, M. Puchalski

- **Theory, MPQ/LMU München/Pulkovo:**
  S. Karshenboim
- **Penning trap experiments, MPIK/Mainz University/GSI:**
  S. Sturm, F. Heisse, F. Köhler-Langes, A. Kracke (Wagner), G. Werth, W. Quint, K. Blaum, *et al.*

Bedankt voor uw andacht!
感谢聆听，欢迎提问！
Dziękuję za uwagę!
Grazie per l'attenzione!
İlginiz için teşekkürler!
ध्यान देने के लिए आपका धन्यवाद।
Köszönöm a figyelmet!
Merci à tous pour votre attention !
Muchas gracias por su atención!
Mulţumesc pentru atenţie!
Obrigado pela atenção!
Спасибо за внимание!
Thank you for your attention!
Vielen Dank für Ihre Aufmerksamkeit!

Two-loop diagrams:



A. Peterman, Helv. Phys. Act **30**, 407 (1957);
C. M. Sommerfield, Ann. Phys. **5**, 26 (1958)

Three$^+$-loop diagrams:

S. Laporta, E. Remiddi, Phys. Lett. B **379**, 283 (1996) [3 loops, analytical]
T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, Phys. Rev. Lett. **109**, 111807 (2012) [numerical]
S. Laporta, Phys. Lett. B **772**, 232 (2017) [4 loops, semi-analytical, 1100 digits given]

Current best experimental value:

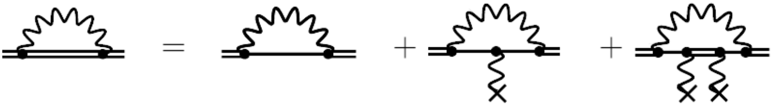
$$g_{\rm exp} = 2.002\,319\,304\,361(6)$$

(rel. accuracy=$3 \cdot 10^{-12}$)
Accurate value for the **fine-structure constant**: $g_{\rm exp}$ and corresponding multi-loop free-electron QED calculations

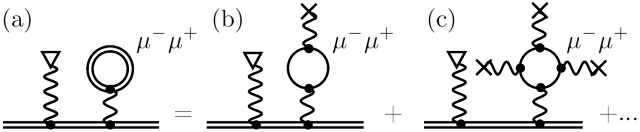
D. Hanneke, S. Fogwell, and G. Gabrielse, Phys. Rev. Lett. **100**, 120801 (2008)

QED theory of the bound-electron g factor

Bound-state QED in the Furry picture:
relate the diagrams to (known) free QED

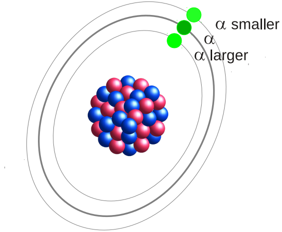


self-energy function	vertex function	Coulomb-Dirac propagator
$\Sigma(p)$	$\Gamma_\mu(p,p')$	$G^{2+}(\mathbf{x},\mathbf{y},E)$
momentum space	momentum space	coordinate space



free-loop approximation	virtual
polarization function $\Pi(q^2)$	light-by-light
$\rightarrow$ Uehling potential $V_{\mathrm{U}}(r)$	scattering

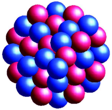
Towards the determination of the fine-structure constant from the bound-electron g-factor

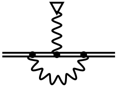
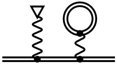
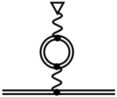


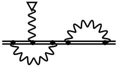
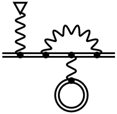
- In atoms/ions: Binding energies, wave functions and thus all properties depend on α – or, actually, $Z\alpha$
- Accurately determine the value α from atomic properties
  e.g. from the bound-electron g-factor – can be measured to very high accuracy
- Leading (Dirac) g-factor:
  $g_\mathrm{D} = {}^2/_3 \left(1 + 2\sqrt{1-(Z\alpha)^2}\right)$

Determining α from the g-factor

Principle of determining α:

$g_{\text{exp}} \overset{!}{=} g_{\text{theo}} = {}^{2}\!/_{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) +$  $+$

 $+$  $+$  $+$  $+$

 $+$  $+ \ldots \Rightarrow$ solve for α

- **Different physics** to determine α than in the case of the *free-electron* g-factor: dominant dependence not from a radiative correction (α/π), but from the binding $(Z\alpha)$
- **Enhanced sensitivity** as compared to the *free-electron* g-factor

- **Problem:** nuclear parameters (e.g. $\langle r^2 \rangle$) are not known accurately
- **Solution:** weighted difference of H- and Li-like ions (same Z):

$$\delta_\Xi g = g(2s) - \Xi g(1s) \,,$$

with the weight Ξ theoretically chosen to suppress nuclear size effects

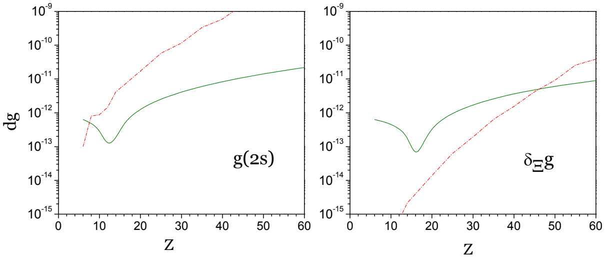
- Simplest approximation: $\Xi = \frac{1}{8} = 0.125$ – because: $|\psi_{ns}(r=0)|^2 \propto \frac{1}{n^3}$
- Accurate formula (incl. relativity, QED and $e^- - e^-$ interaction:

$$\Xi = 2^{-2\gamma-1} \left[1 + \frac{3}{16} (Z\alpha)^2 \right] \left(1 - \frac{2851}{1000} \frac{1}{Z} + \frac{107}{100} \frac{1}{Z^2} \right) ,$$

where $\gamma = \sqrt{1-(Z\alpha)^2}$

error due to $\delta\langle r^2\rangle$ + distr. $\rightarrow$

error due to present $\delta\alpha$ $\rightarrow$



$\Rightarrow$ α can be significantly improved

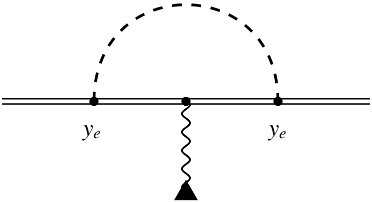
Earlier idea: weighted difference of **heavy** H- and B-like ions

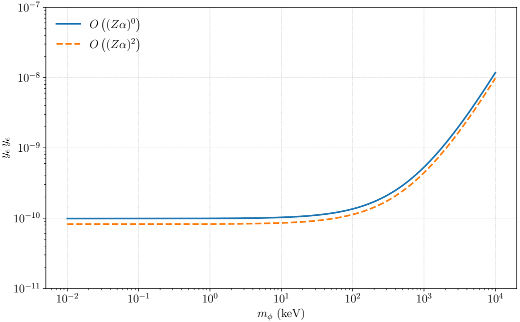
V. M. Shabaev, D. A. Glazov, N. S. Oreshkina *et al.,* Phys. Rev. Lett. **96**, 253002 (2006)

- V. A. Yerokhin, E. Berseneva, Z. H., I. I. Tupitsyn, C. H. Keitel, Phys. Rev. Lett. **116**, 100801 (2016); Phys. Rev. A **94**, 022502 (2016)
- V. A. Yerokhin, C. H. Keitel, Z. H., J. Phys. B **46**, 245002 (2013)

However, if the new boson couples electrons and nucleons, it also couples the electron to itself: **self-interaction** correction of order y_e^2

- The correction due to this vertex correction can be also calculated (M. Moretti, Z. Harman, C. H. Keitel, to be published)
- Best bound so far: from the free-electron g factor – highest experimental and theoretical accuracy





Experiment from: Fan *et al.*, Phys. Rev. Lett. **130**, 071801;
Theory from e.g.: Volkov Phys. Rev. D **100**, 096004 (2019); CODATA