SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL FROM QUARKS AND GLUONS TO HADRONS AND NUCLEI
Cluster of Excellence PRiSMA+ Precision Physics, Fundamental Interactions and Structure of Matter
JOHANNES GUTENBERG UNIVERSITÄT MAINZ
Deutsche Forschungsgemeinschaft DFG
STRONG-2020

Nuclear charge radii and CKM unitarity tests

Misha Gorshteyn (Gorchtein)

JGU Mainz

Based on:
2502.17070
2501.15274
2412.05932
2407.09743
2311.00044
2309.16893
2212.02681
2208.03037

With
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Petr Navratil
Mehdi Drissi
John Behr
Arup Chakraborty
Vaibhav Katyal

New perspectives in charge radii of light nuclei, ECT* Tento, July 28-August 1, 2025

Outline

Precision tests of the Standard Model with CKM unitarity

V_{ud} from nuclear β decays and nuclear radii

Nuclear polarization in muonic atoms, light vs heavy

Results, comparisons

Outlook

Standard Model: making sense of beta decays

Discovery of radioactivity 1896 (Becquerel) —> Contact theory 1934 (Fermi)

—> Parity violation 1956-7 (Lee-Yang, Wu)

—> V - A theory 1957 (Sudarshan, Marshak, Gell-Mann, Feynman); S-PS not excluded

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Radiative corrections to muon decay: important evidence for V-A theory
RC to muon decay - UV finite for V-A but divergent for S-PS

Muon lifetime $\tau_\mu = 2196980.3(2.2)ps$ —> Fermi constant $G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

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1-loop RC to spectrum:

$$\Delta P^0 d^3p = \frac{\alpha}{2\pi} P^0 d^3p \left[6 \ln \frac{\Lambda}{M_p} + \text{finite} \right]$$

UV cut-off

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Is weak interaction universal for leptons and hadrons?

1967: Sirlin applied current algebra:
general UV behavior of β decay rate at 1-loop $$\frac{\alpha}{2\pi} P^0 d^3p \; 3[1+2\bar{Q}]\ln(\Lambda/M)$$

$\bar{Q}$: average charge of fields involved: $1+2\bar{Q}_{\mu,\nu_\mu} = 0$ but $1+2\bar{Q}_{n,p} = 2$

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Eventually, massive weak bosons render RC to beta decay UV-finite: $\Lambda = M_{W,Z}$

Precision, Universality and CKM unitarity

In SM the same coupling of W-boson to leptons and quarks, $G_V = G_\mu$

Before RC were included: $G_V \sim 0.98 G_\mu$

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Cabibbo: strength shared between 2 generations

$$|G_V^{\Delta S=0}| = \cos\theta_C G_\mu$$

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Cabibbo unitarity: $\cos^2\theta_C + \sin^2\theta_C = 1$

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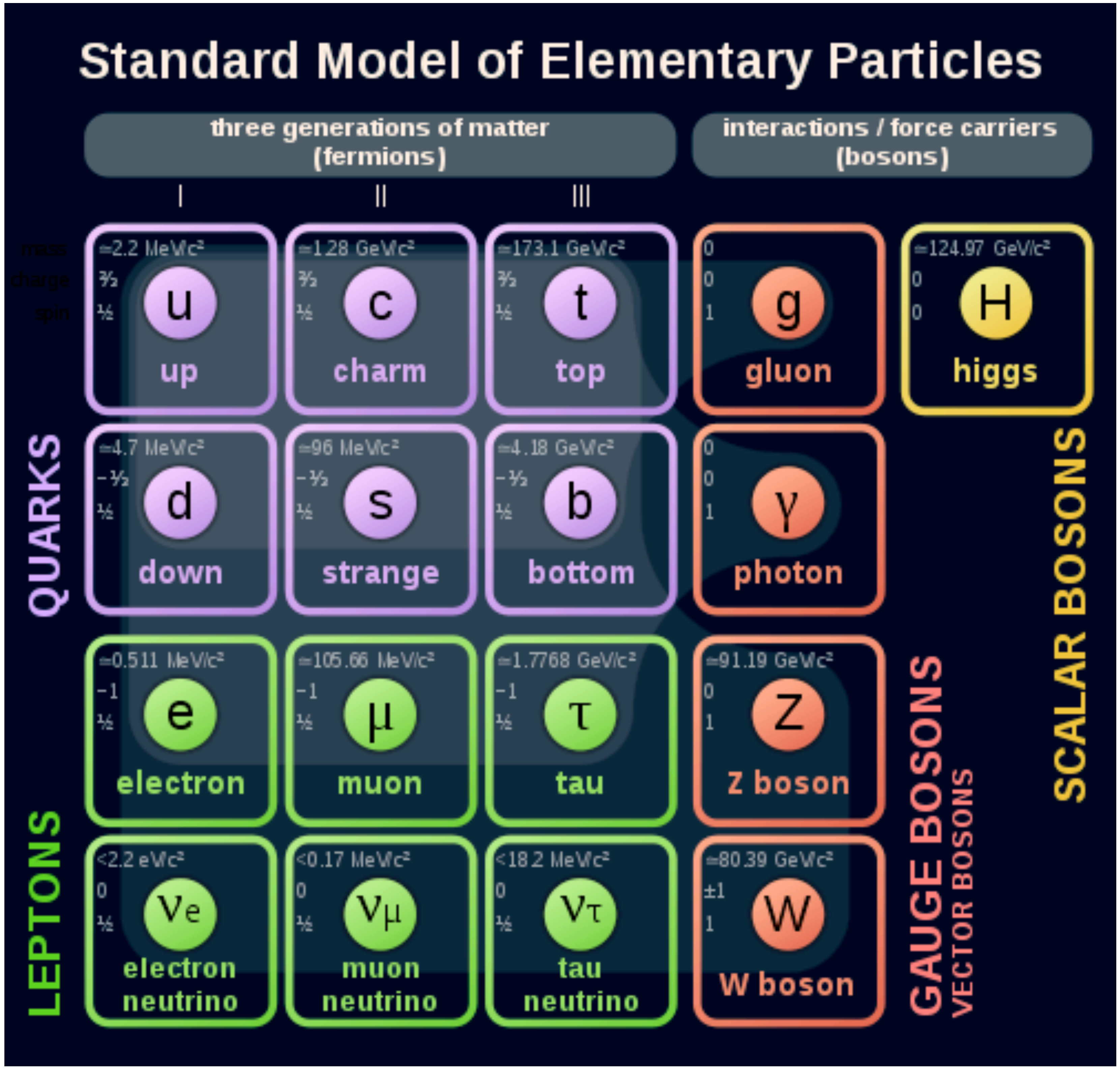
Kobayashi & Maskawa: 3 flavors + CP violation — CKM matrix V

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM unitarity - completeness of the SM: $VV^\dagger = \mathbf{1}$

Top row unitarity constraint: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Detailed understanding of β decays largely shaped the Standard Model



Status of Cabibbo unitarity anno 2024

$$\underset{\sim 0.95}{|V_{ud}|^2} + \underset{\sim 0.05}{|V_{us}|^2} + \underset{\sim 10^{-5}}{|\cancel{V_{ub}}|^2} = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

V_{ud} and V_{us} determinations
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$\dfrac{K \to \mu\nu}{\pi \to \mu\nu}$: $\quad |V_{us}/V_{ud}| = 0.2311(5)$

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PDG $[S = 2.5]$: $\quad |V_{us}| = 0.2243(8)$

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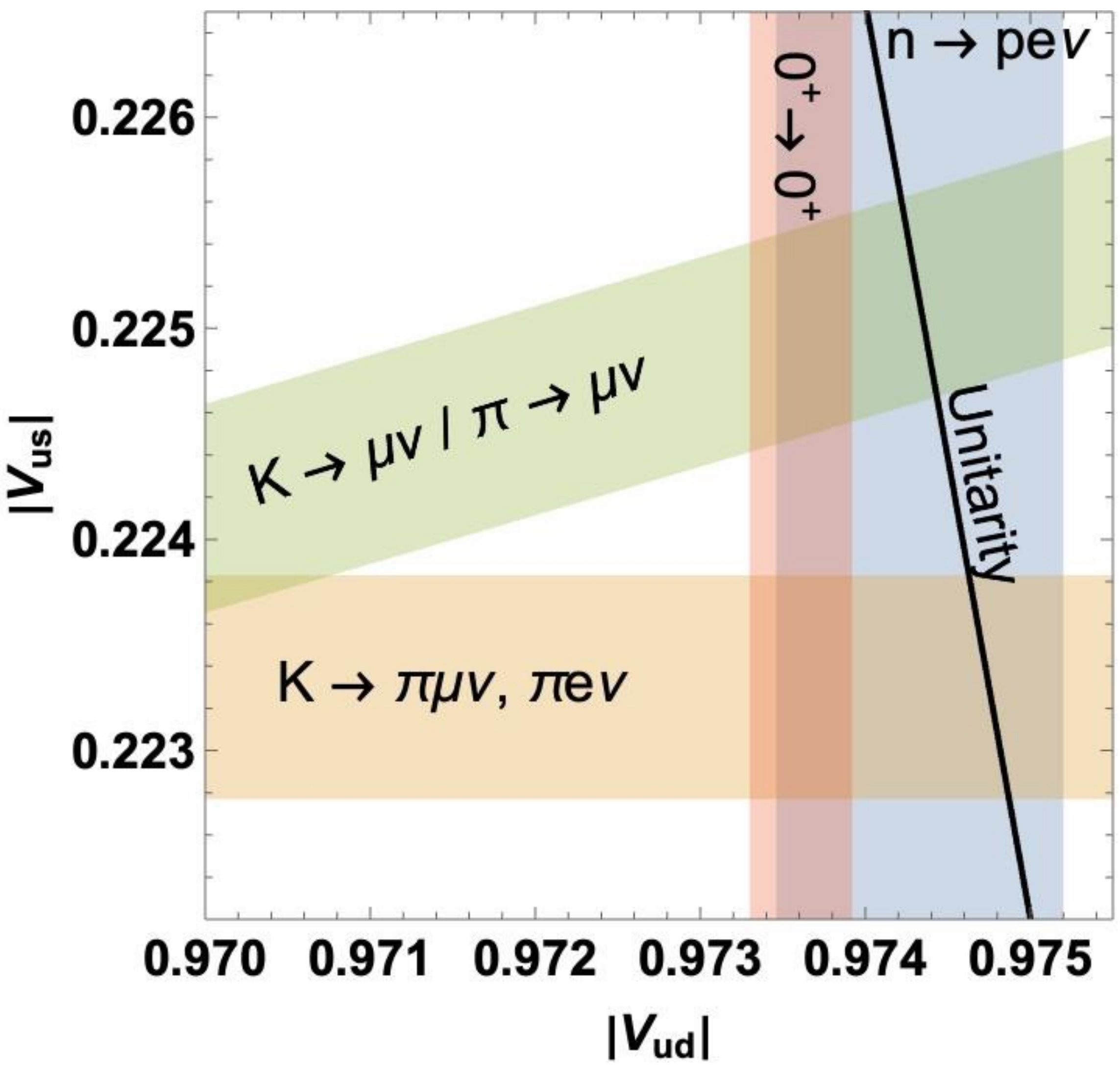

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CAA summary - 3 anomalies!

3 observables: $|V_{us}|^{K\ell 3}$, $|V_{us}/V_{ud}|^{K\mu 2}$, V_{ud}
2 quantities to determine: V_{us}, V_{ud}
▶ **3 ways to test unitarity**

$$\Delta^{(1)}_{\rm CKM} = |V_{ud}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1$$ **= –0.00176(56) –3.1σ**

$$\Delta^{(2)}_{\rm CKM} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K_{\mu 2}}\right)^2\right] - 1$$ **= –0.00098(58) –1.7σ**

$$\Delta^{(3)}_{\rm CKM} = |V_{us}^{K_{\ell 3}}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K_{\mu 2}}}\right)^2 + 1\right] - 1$$ **= –0.0164(63) –2.6σ**

Can it be a signal of BSM?

CAA in presence of RH currents

Cirigliano et al.
PLB 838 (2023)

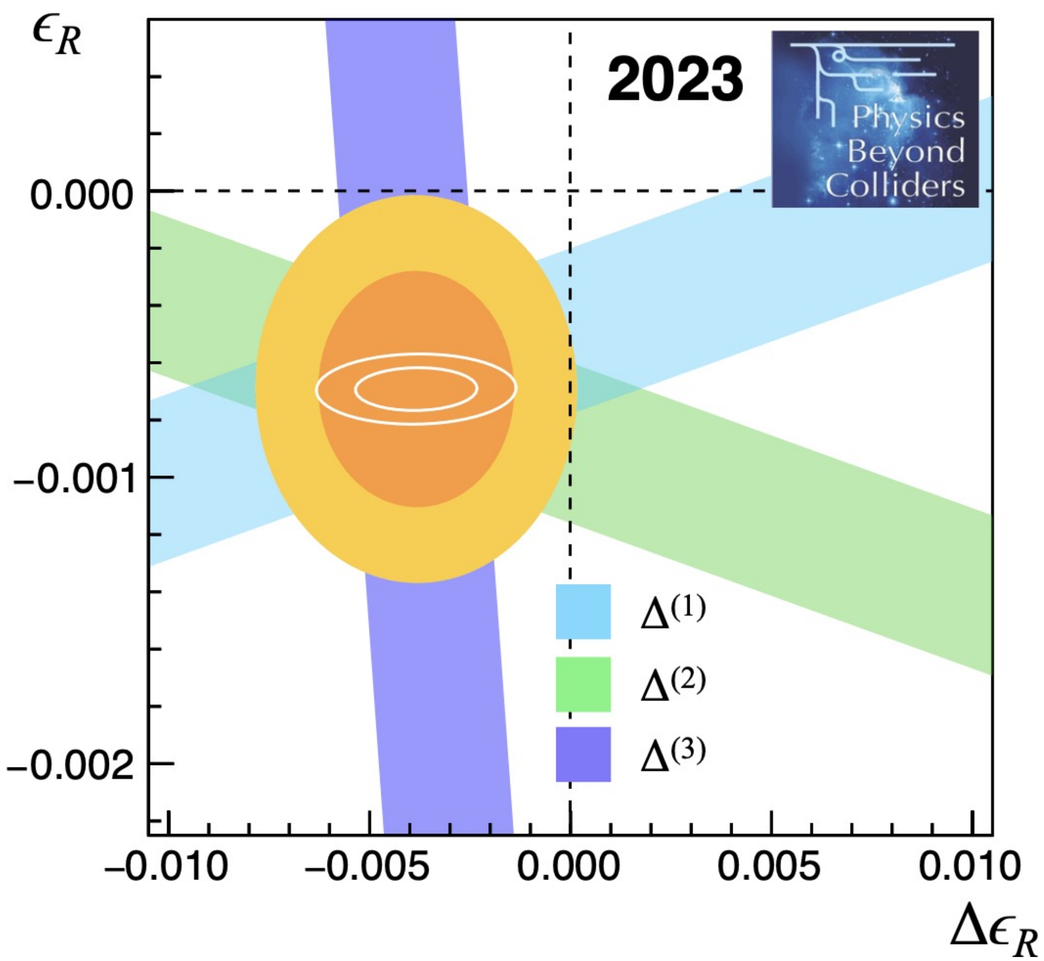
- In SM, W couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}$-$K_{\mu 2}$ difference
- Define ϵ_R = admixture of RH currents in non-strange sector
  $\epsilon_R + \Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(2)}_{\rm CKM} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(3)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)$$

$$r \equiv \left(\frac{1+\Delta^{(2)}_{\rm CKM}}{1+\Delta^{(3)}_{\rm CKM}}\right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

From current fit:

$\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)
$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)
$\epsilon_R = \Delta\epsilon_R = 0$ excluded at 3.1σ



Review the "σ" that defines the significance of the Cabibbo angle anomaly!

Are all SM contributions under control?

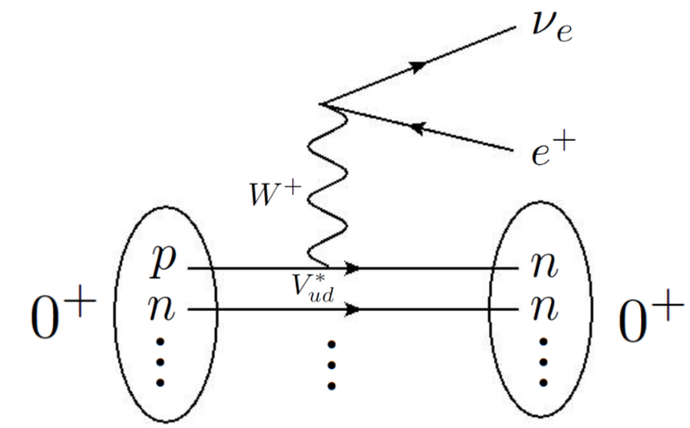
Nuclear radii and V_{ud}:

An impressive interplay of

Theory + Experiment

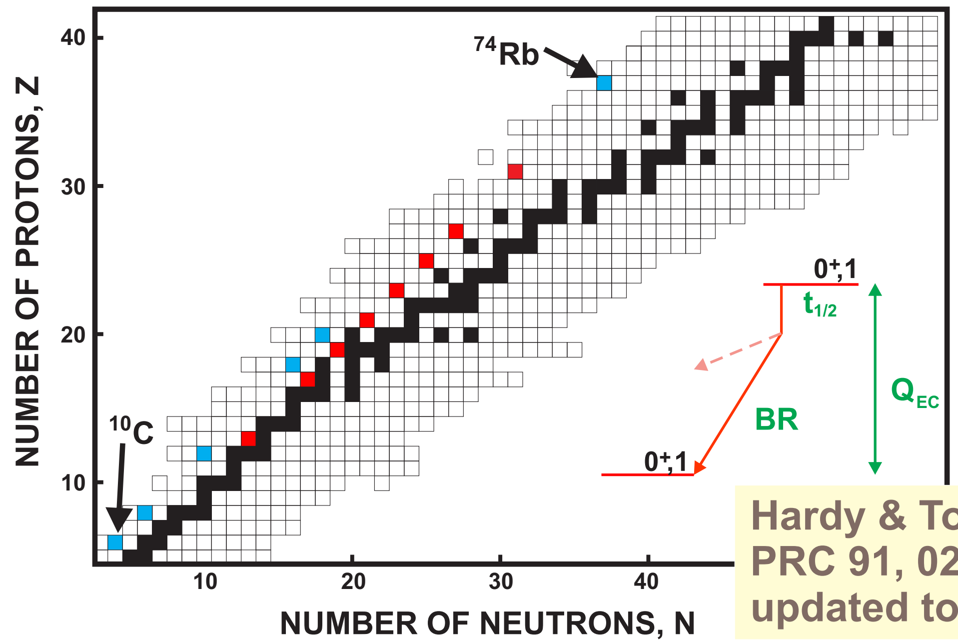
V_{ud} from superallowed decays

Superallowed 0^+-0^+ nuclear decays:
- only conserved vector current
- many decays
- all rates equal modulo phase space

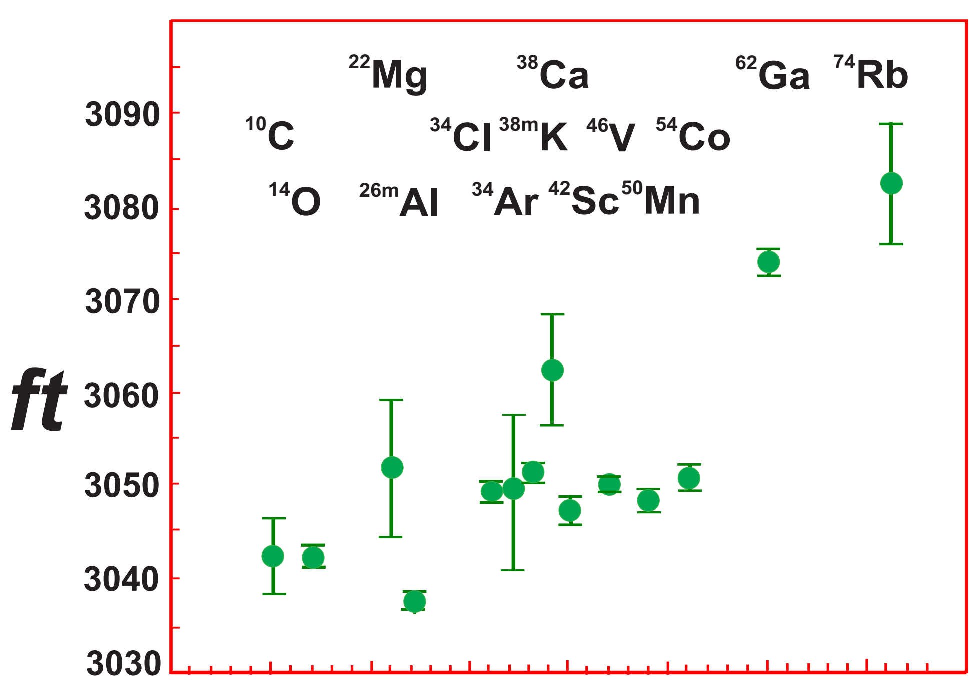


Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- **8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**



Hardy & Towner PRC 91, 025501 (2015) updated to 2017



ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken

a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
 (proton and neutron distribution not the same)

V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \text{RC} + \text{ISB}) = \mathscr{F}\text{t}(1 + \Delta_R^V) = \text{ft}(1 + \delta_R')(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

~ Measured (→ ft) QED (→ δ_R') Isospin-breaking (→ δ_C) Nuclear structure (→ δ_{NS}) Universal RC (→ Δ_R^V)

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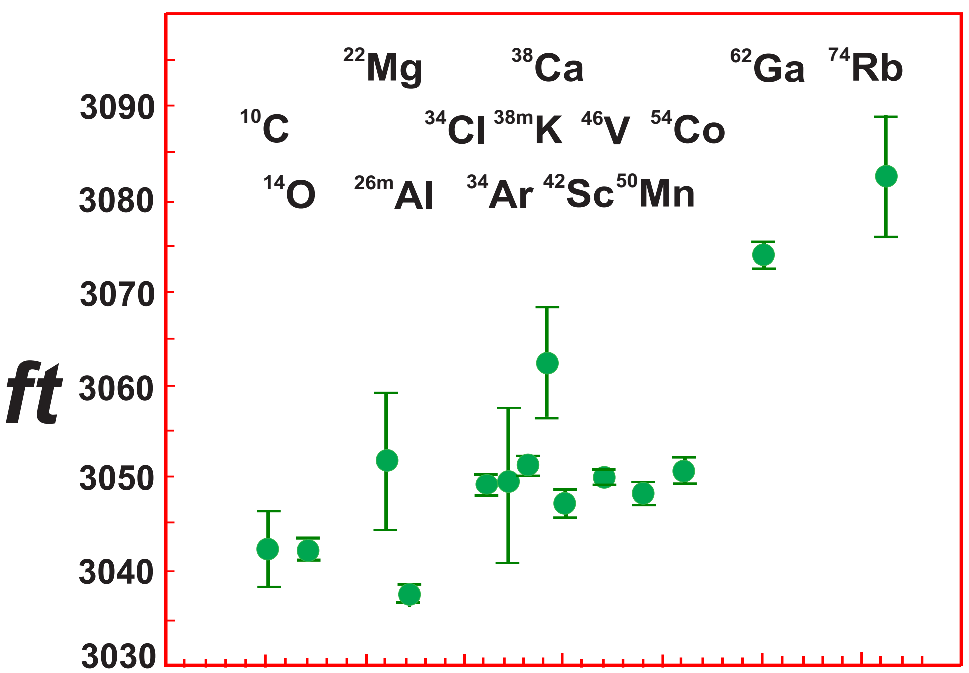

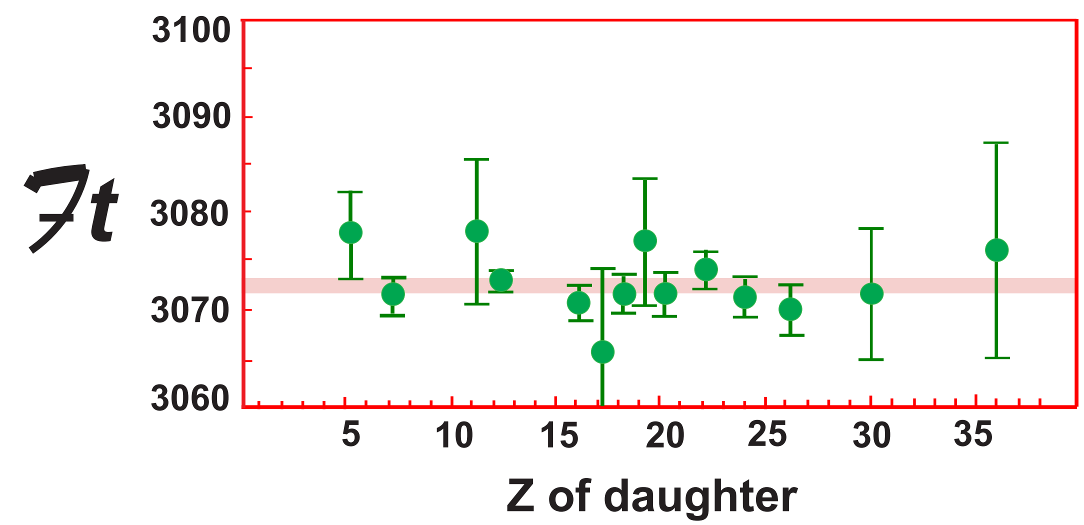

Average of 14 decays Hardy, Towner 1972 - 2020

Pre-2018: $\overline{\mathscr{F}t} = 3072.1 \pm 0.7\, s$

PDG 2022: $\overline{\mathscr{F}t} = 3072 \pm 2\, s$

$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.9737\,(1)_{exp,\,nucl}(3)_{NS}(1)_{RC}[3]_{total}$$

QED + FNS corrections to β-spectrum

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \, |\vec{p}_e| E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

Unperturbed beta spectrum

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Fermi function: e^+ in Coulomb field of daughter

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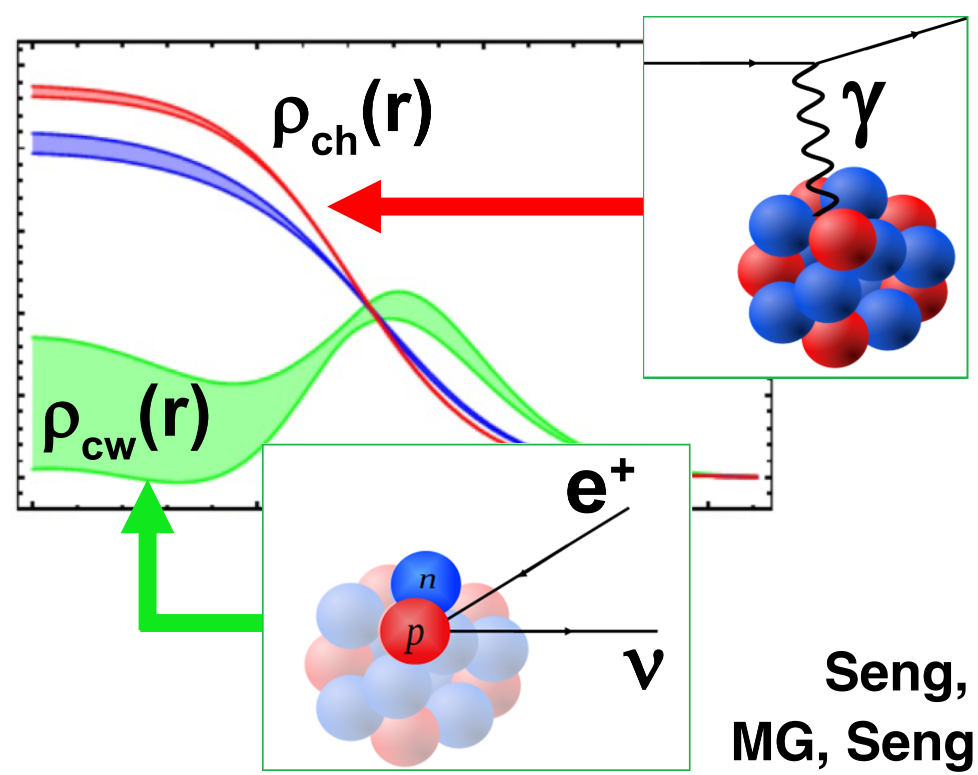
Recent development:
isospin symmetry + known charge distributions

Only the outer protons can decay:
all neutron states in the core occupied

Photon probes the entire nuclear charge



Seng, 2212.02681
MG, Seng 2311.16755

Impact of precise nuclear radii on Ft and V_{ud}

Recent measurement at ISOLDE **Plattner et al, arXiv: 2310.15291**

IS in Al 27-26m $3s^2 3p\ ^2P_{3/2}$ —> $3s^2 4s\ ^2S_{1/2}$ transition

$$\delta\nu^{27,26m} = F\delta\langle r^2\rangle^{27,26m} + M\frac{m_{26m} - m_{27}}{m_{27}(m_{26m} + m_e)}$$

$$\delta\nu^{27,26m} = 377.5(3.4)\ \mathrm{MHz}$$

↓

Wilfried's talk

$$R_c(^{26m}\mathrm{Al}) = 3.130(15)\ \mathrm{fm}$$

Previously guessed $R_c(^{26m}Al) = 3.040(20)\,\mathrm{fm}$

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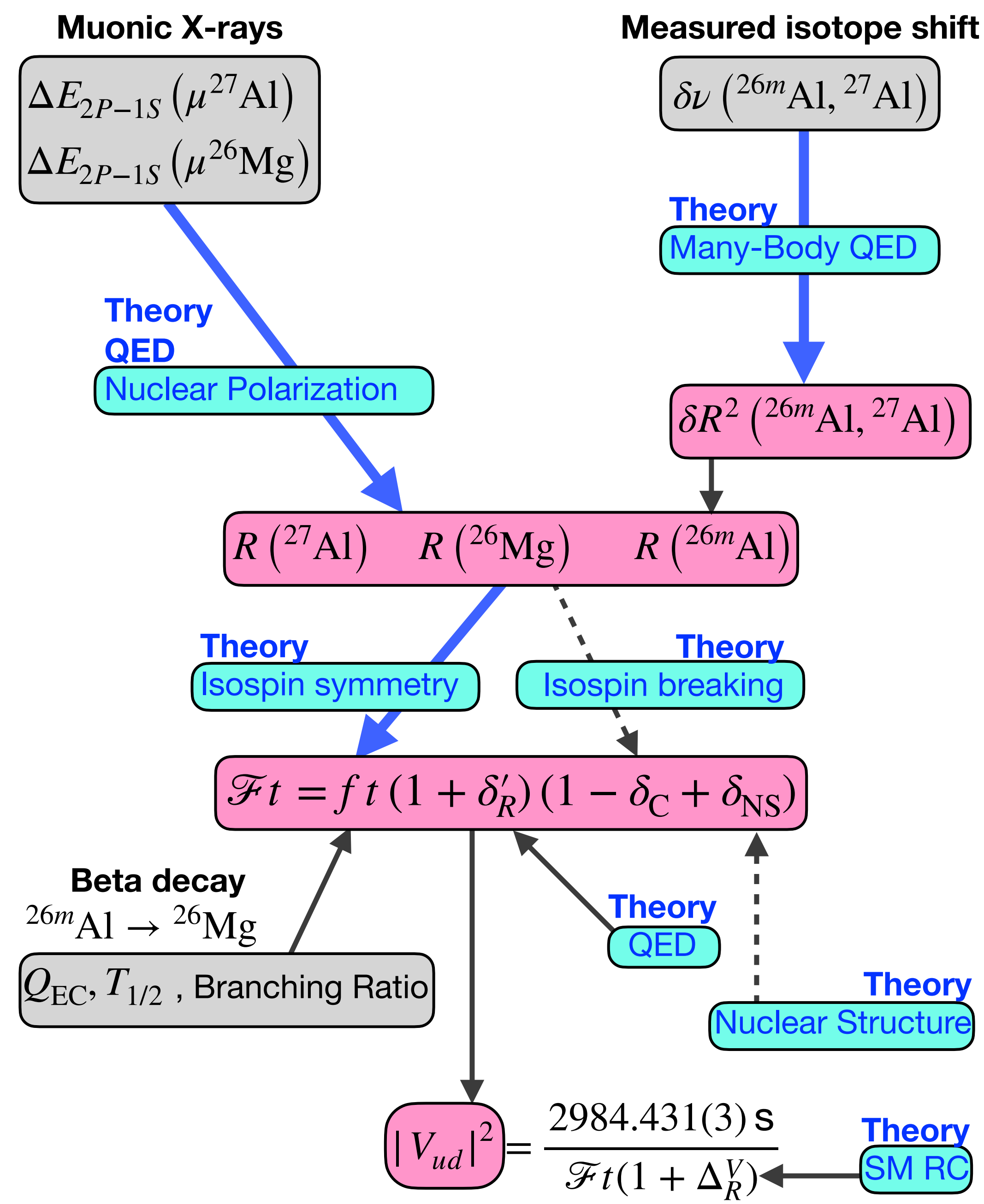
Many theory ingredients to translate atomic measurements into Vud



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We re-examined ~ALL ingredients **MG et al, arXiv: 2502.17070**

Careful reevaluation of f-value (QED)
isotope shift factors F, M (Many-body QED)
charge radii of Al-27, Mg-26 (Nuclear theory)

Major impact on Ft value uncovered

$$\mathscr{F}t[^{26m}\text{Al} \to {}^{26}\text{Mg}] = 3072.4(1.1)_{\text{stat}}\,\text{s} \to 3070.0(1.2)_{\text{stat}}\,\text{s}$$

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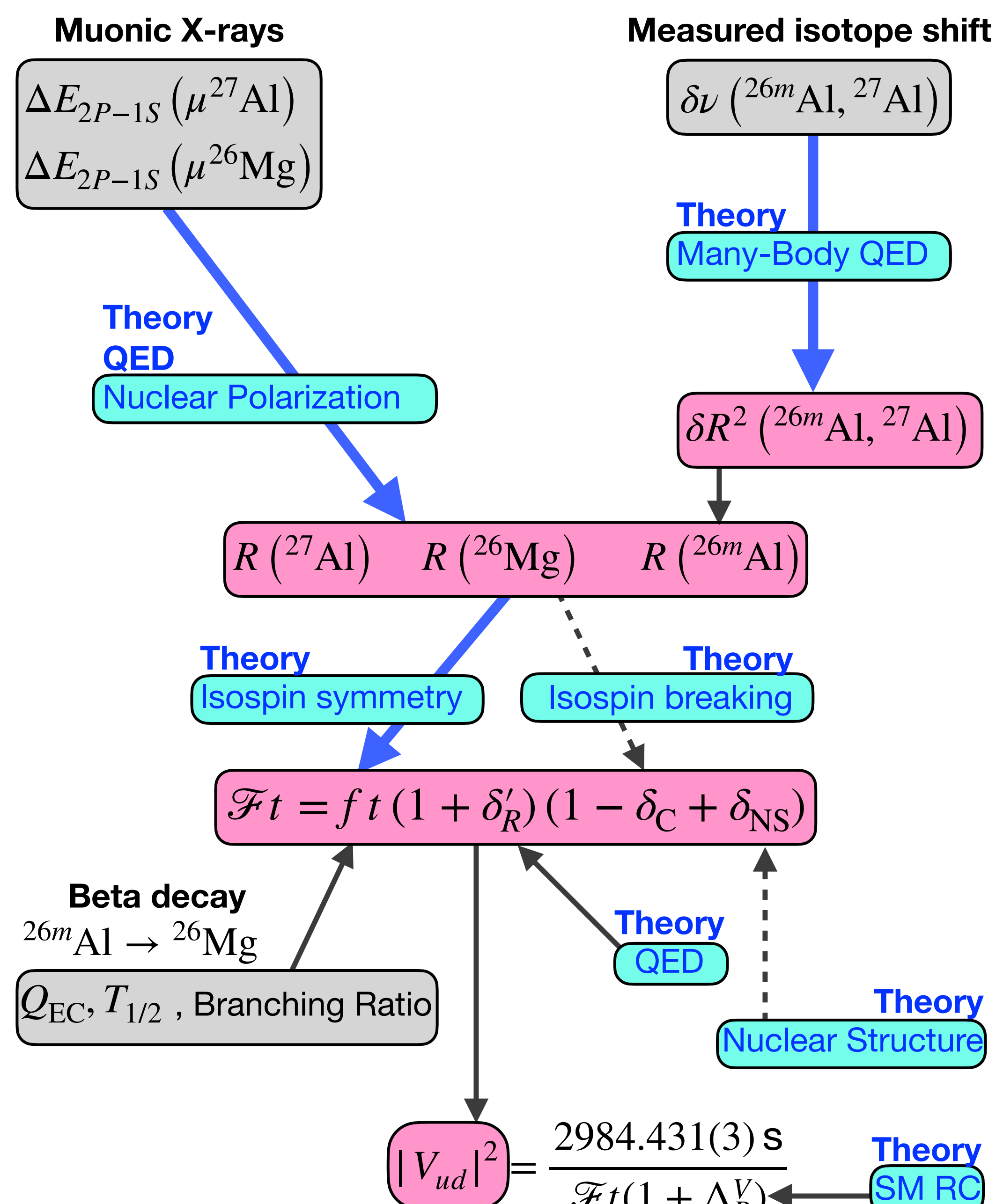

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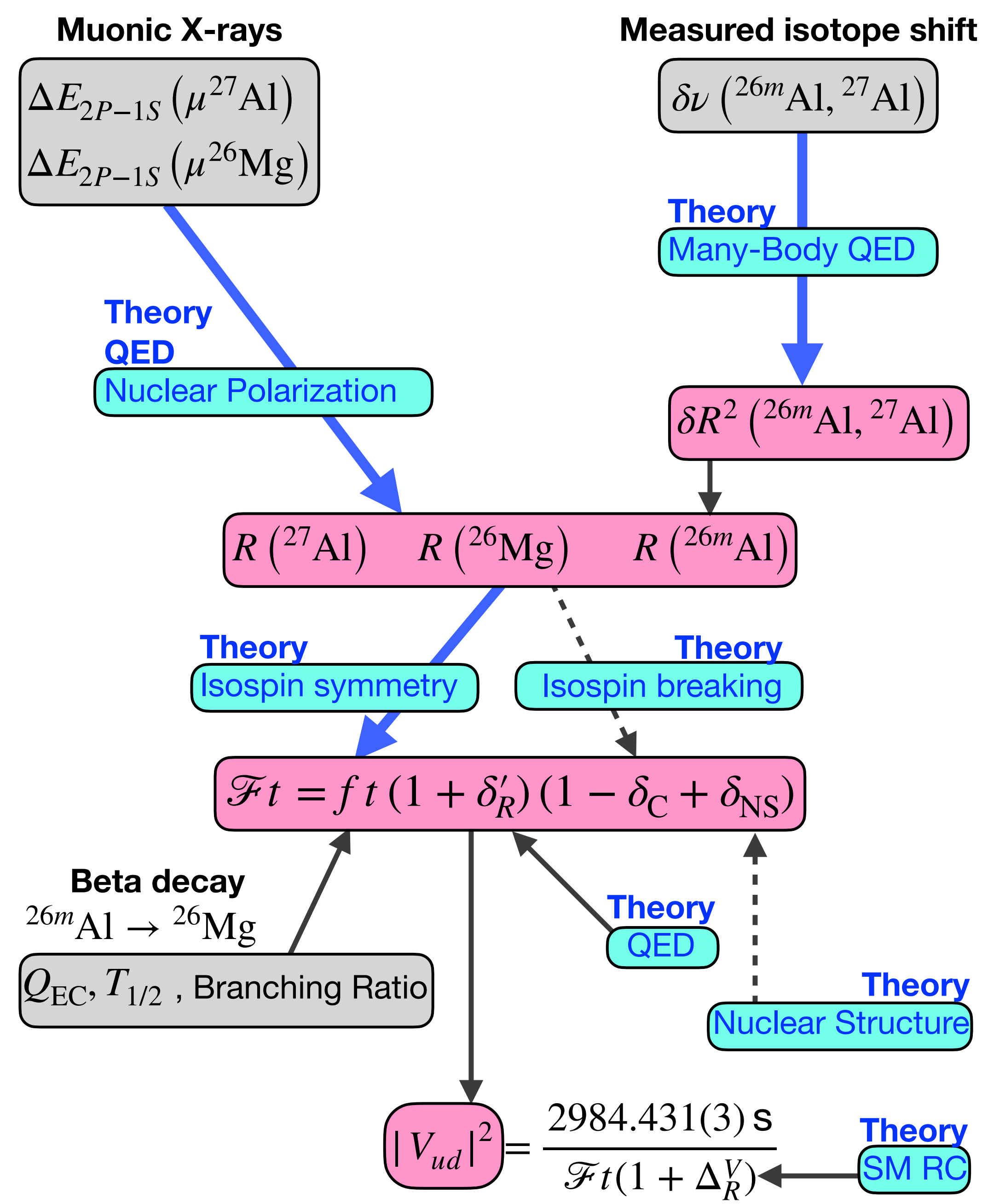

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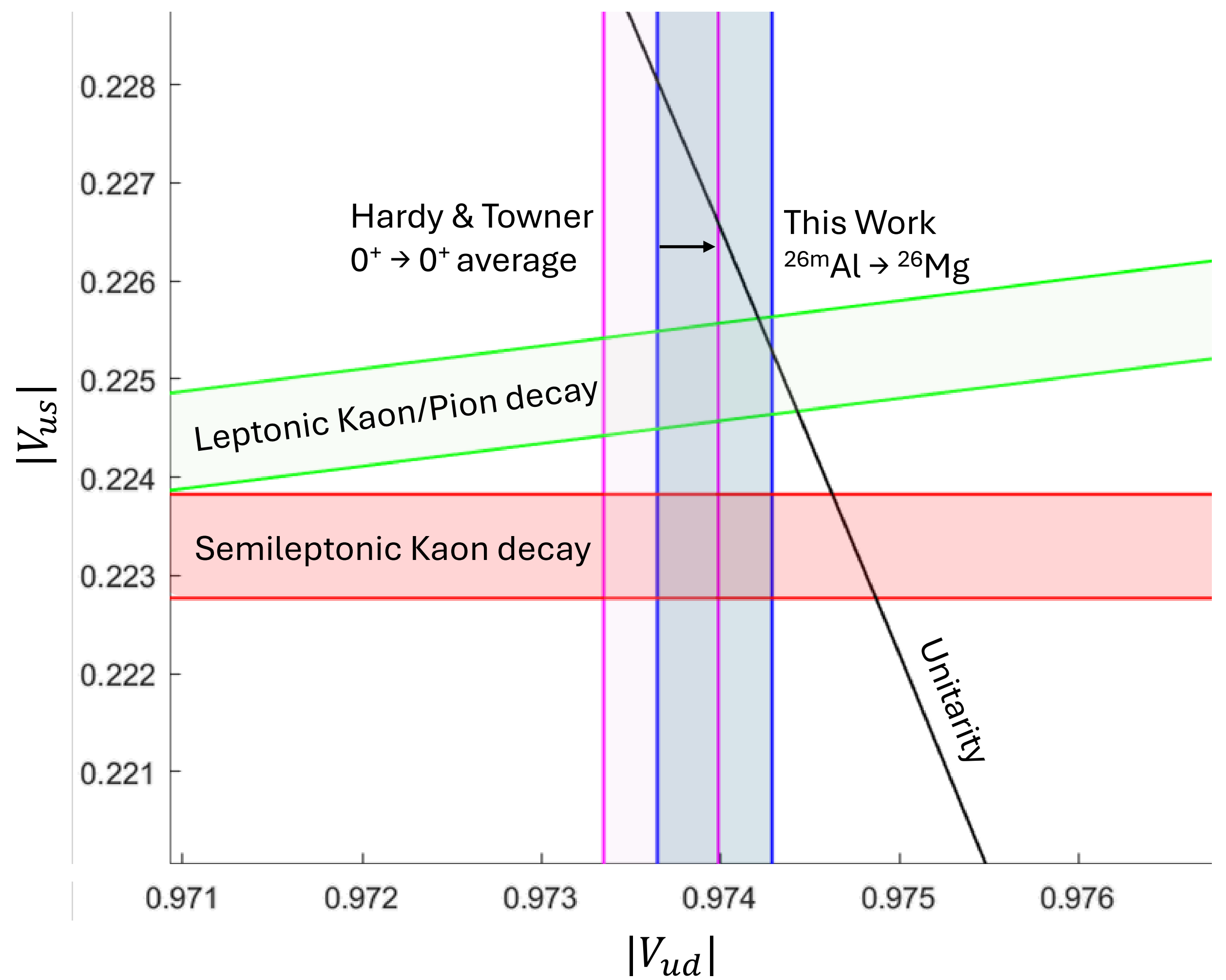
$$\mathcal{F}t[^{26m}\mathrm{Al} \to {}^{26}\mathrm{Mg}] = 3072.4(1.1)_{\mathrm{stat}}\,\mathrm{s} \to 3070.0(1.2)_{\mathrm{stat}}\,\mathrm{s}$$

Al-26m—> Mg-26 is the most precisely measured transition —> impacts the V_{ud} determination!

One radius makes a difference in BSM search!

Anno 2025: Cabibbo anomaly disappears — 2.5σ to 1.3σ (?)

$$|V_{ud}|^2 + |V_{us}|^2 = 0.9985(7) \longrightarrow |V_{ud}|^2 + |V_{us}|^2 = 0.9991(7)$$



MG et al, arXiv: 2502.17070

But: only f was revisited; need to check δ_{NS} and δ_C

Test of isospin symmetry in $T = 1, O^{+}$isotriplet

Isospin symmetry was assumed— but we know that it is slightly broken!
Why isospin limit is good enough for QED corrections to spectrum?

Shape factor and finite size effects are ~small corrections to Fermi function
1-2% ISB effect on top of a RC may be assumed negligible (but needs to be tested)

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ISB dominated by Coulomb repulsion between protons

Nuclear Hamiltonian: $H = H_0 + V_{\rm ISB} \approx H_0 + V_C$

Miller, Schwenk 0805.0603; 0910.2790; Auerbach 0811.4742; 2101.06199; Seng, MG 2208.03037; 2304.03800; 2212.02681

Coulomb potential for uniformly charged sphere

$$V_C \approx -\frac{Ze^2}{4\pi R_C^3}\sum_{i=1}^{A}\left(\frac{1}{2}r_i^2 - \frac{3}{2}R_C^2\right)\left(\frac{1}{2} - \hat{T}_z(i)\right)$$

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Same operator generates nuclear radii

$$R_{p/n,\phi} = \sqrt{\frac{1}{X}\langle\phi|\sum_{i=1}^{A} r_i^2\left(\frac{1}{2} \mp \hat{T}_z(i)\right)|\phi\rangle}$$

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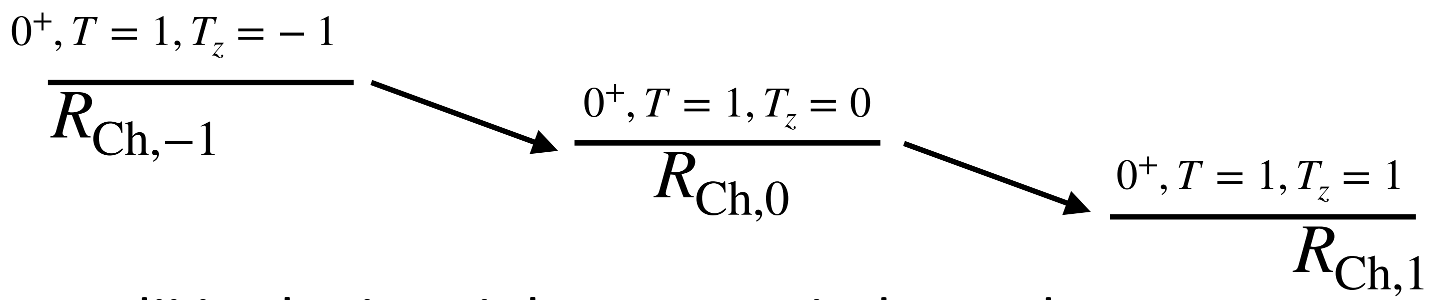

In isospin limit nuclear radii in the isotriplet are not independent

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2 \ = 0 \quad \text{if isospin symmetry exact}$$

Test requires that all 3 nuclear radii in the isotriplet are known;
Currently only the case for A=38 system

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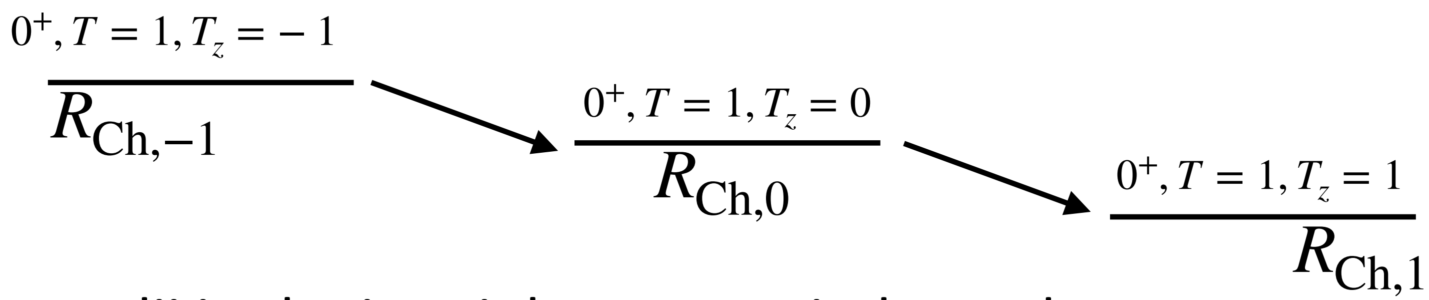

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26	$^{26}_{14}$Si	$^{26m}_{13}$Al: 3.130(15)^f	$^{26}_{12}$Mg: 3.0337(18)^a	4.11(15)
30	$^{30}_{16}$S	$^{30}_{15}$P(ex)	$^{30}_{14}$Si: 3.1336(40)^a	N/A
34	$^{34}_{18}$Ar: 3.3654(40)^a	$^{34}_{17}$Cl	$^{34}_{16}$S: 3.2847(21)^a	3.954(68)
38	$^{38}_{20}$Ca: 3.467(1)^c	$^{38m}_{19}$K: 3.437(4)^d	$^{38}_{18}$Ar: 3.4028(19)^a	3.999(35)
42	$^{42}_{22}$Ti	$^{42}_{21}$Sc: 3.5702(238)^a	$^{42}_{20}$Ca: 3.5081(21)^a	4.64(39)
46	^{46}Cr	^{46}V	^{46}Ti: 3.6070(22)^a	N/A

$$\frac{1}{2}\left(20 \times 3.467(1)^2 + 18 \times 3.4028(19)^2\right) - 19 \times 3.437(4)^2 = -0.00(12)(14)(52)$$

Improvement of K-38m radius necessary! (Plans at TRIUMF on IS K-38m, K-37?)

Data-driven approach to ISB

ISB dominated by Coulomb repulsion between protons

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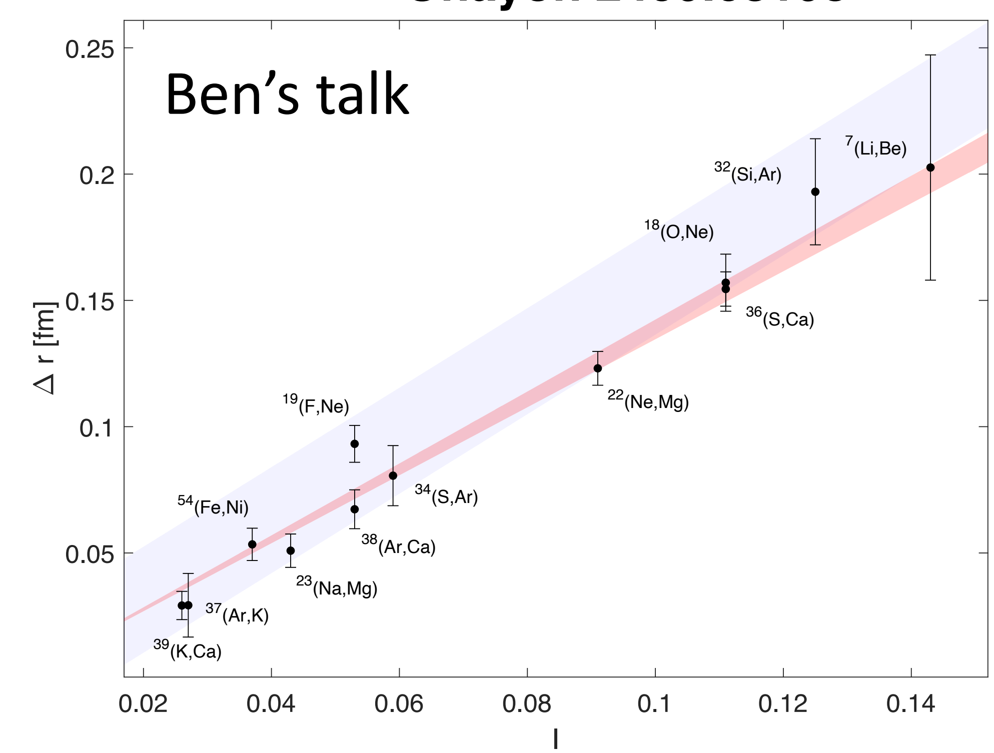
Superallowed isotriplets contain mirrors
Use info about radii of other mirror nuclei

$$\Delta_I = r_{N,Z}(I) - r_{Z,N}(I) = 1.382(34) \times I \text{ fm}$$

$$I = (N - Z)/A$$

Agrees with ab-initio nuclear theory (Novario, 2111.12775) but more precise

Ohayon 2409.08193



Test of isospin symmetry using mirror fit

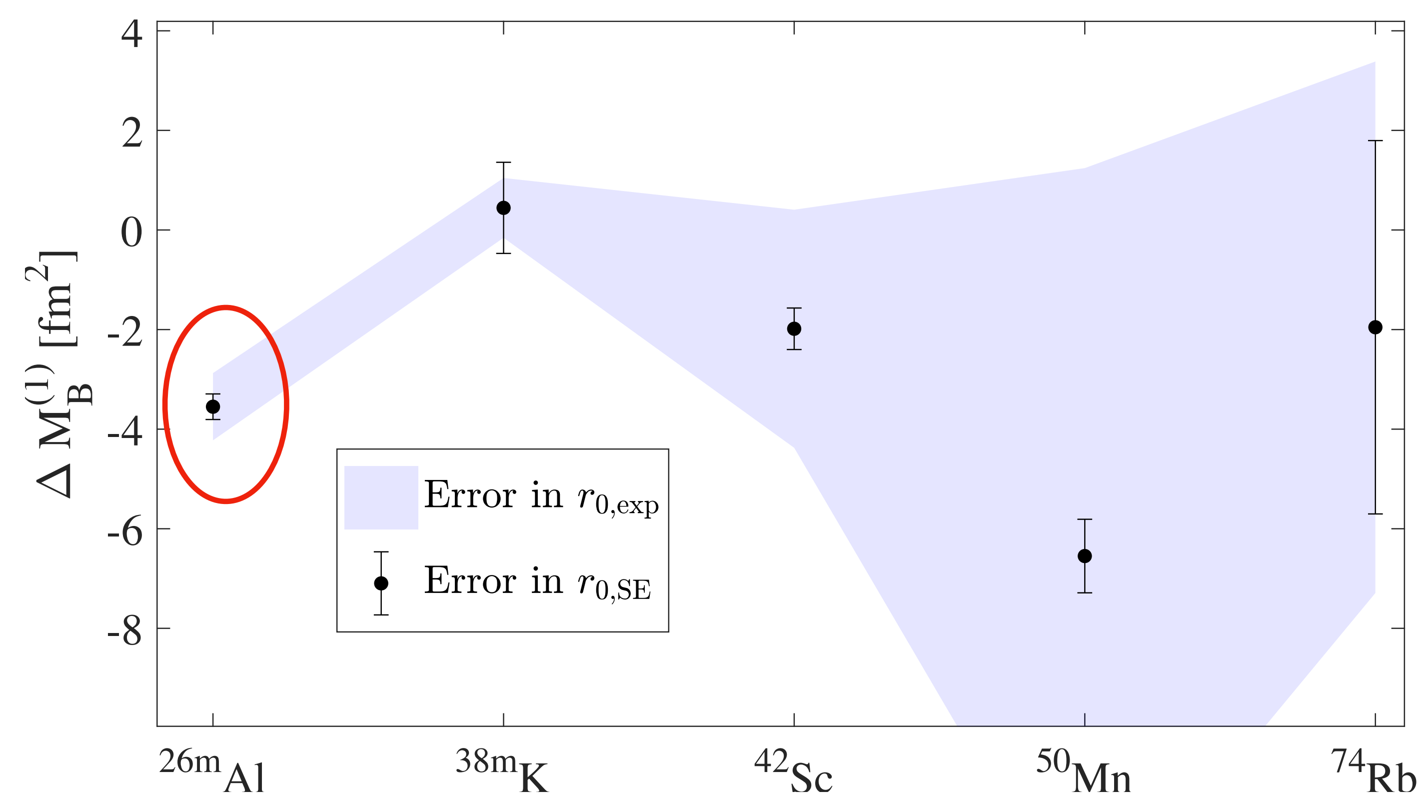
Fill in missing entries using mirror fit

$$r_{-1}^2 - r_{+1}^2 = \Delta_I(2r_{+1} + \Delta_I)$$

$$r_{0,\mathrm{SE}}^2 = r_{+1}^2 + \frac{Z_{-1}}{2Z_0}\Delta_I(2r_{+1} + \Delta_I)$$

	r_{-1} fm		$r_{0,\mathrm{SE}}$ fm	$r_{0,\mathrm{exp}}$ fm		r_{+1} fm	$\Delta M_B^{(1)}$ fm^2	r_{CW}^2 fm^2	Ref. [38]
$^{10}_{6}$C	2.638(36)	$^{10}_{5}$B*	2.531(38)		$^{10}_{4}$Be	2.361(36)		9.72(25)	N/A
$^{14}_{8}$O	2.706(11)	$^{14}_{7}$N*	2.623(10)		$^{14}_{6}$C	2.508(09)		10.41(12)	N/A
$^{18}_{10}$Ne	2.934(09)	$^{18}_{9}$F*	2.863(07)		$^{18}_{8}$O	2.777(07)		12.08(12)	13.4(5)
$^{22}_{12}$Mg	3.071(05)	$^{22}_{11}$Na*	3.017(05)		$^{22}_{10}$Ne	2.948(04)		13.24(12)	12.9(7)
$^{26}_{14}$Si	3.137(04)	$^{26m}_{13}$Al	3.088(04)	3.132(08)	$^{26}_{12}$Mg	3.030(03)	−3.5(0.7)	13.77(12)	N/A
$^{30}_{16}$S	3.224(07)	$^{30}_{15}$P*	3.181(06)		$^{30}_{14}$Si	3.132(06)		14.50(13)	N/A
$^{34}_{18}$Ar	3.365(11)	$^{34}_{17}$Cl	3.328(04)		$^{34}_{16}$S	3.284(04)		15.66(13)	15.6(5)
$^{38}_{20}$Ca	3.469(04)	$^{38m}_{19}$K	3.440(07)	3.437(05)	$^{38}_{18}$Ar	3.402(06)	0.6(1.1)	16.58(13)	16.0(3)
$^{42}_{22}$Ti	3.576(05)	$^{42}_{21}$Sc	3.545(04)	3.558(16)	$^{42}_{20}$Ca	3.510(04)	−2.0(2.4)	17.46(13)	21.5(3.6)
$^{46}_{24}$Cr	3.670(05)	$^{46}_{23}$V	3.642(05)		$^{46}_{22}$Ti	3.610(04)		18.29(14)	N/A
$^{50}_{26}$Fe	3.719(04)	$^{50}_{25}$Mn	3.693(04)	3.728(41)	$^{50}_{24}$Cr	3.664(04)	−6.6(7.8)	18.73(14)	23.2(3.8)
$^{54}_{28}$Ni	3.741(05)	$^{54}_{27}$Co	3.715(04)		$^{54}_{26}$Fe	3.688(04)		18.93(14)	18.3(9)
$^{58}_{30}$Zn	3.820(03)	$^{58}_{29}$Cu*	3.797(03)		$^{58}_{28}$Ni	3.773(03)		19.66(14)	N/A
$^{62}_{32}$Ge	3.927(06)	$^{62}_{31}$Ga	3.906(06)		$^{62}_{30}$Zn	3.883(06)		20.65(15)	N/A
$^{74}_{38}$Sr	4.205(12)	$^{74}_{37}$Rb	4.187(12)	4.194(17)	$^{74}_{36}$Kr	4.168(12)	−1.9(6.5)	23.32(19)	19.5(5.5)

At present can test 5 isotriplets
A=26 shows significant ISB (??)
Others consistent with 0 within errors



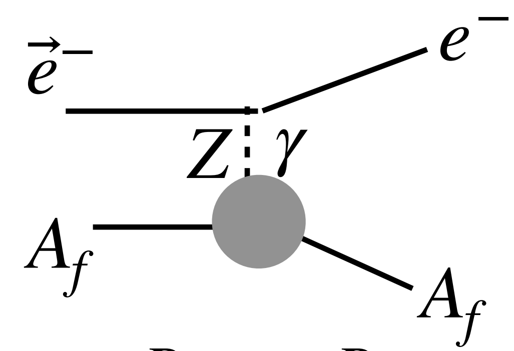
Closing in on ISB: neutron radii from PVES

Another ISB combination involves neutron radius vs proton radius of the mirror companion

$$\Delta M_A^{(1)} = \frac{N_1}{2}\left[R_{n,1}^2 - R_{p,-1}^2\right]$$

Neutron radius of stable daughter:
from parity-violating e-scattering

$$A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\frac{Q_W}{Z}\frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$



$R_{\rm NW} \approx R_n$

Z-boson couples to neutrons, photon - to protons;
PV asymmetry at low Q^2 sensitive to the difference $\langle r_{n,1}^2\rangle - \langle r_{p,1}^2\rangle$ - neutron skin

Cadeddu et al, 2407.09743: feasibility study for PVES on ^{12}C at Mainz
3-5% PV asymmetry at backward angles —> **0.3-0.5% R_n extraction possible**
PVES on stable superallowed daughters (Mg-26, Fe-54, …) + mirror fit — test of ISB!

3-fold cross check: $R_{p,-1}$ from $R_{n,1}$ (i) mirror fit (ii) and IS measurement (iii)

Reference radii from muonic atoms: Nuclear Polarization

Gorchtein 2501.15274

See also talks by
Saori, Natalia, Sonia, Mehdi, Vadim, Chen Ji, …

Nuclear Charge Radii from μ atoms

Lepton feels pointlike Coulomb potential far outside the nucleus

Finite size effects modify this potential in the vicinity of the nucleus

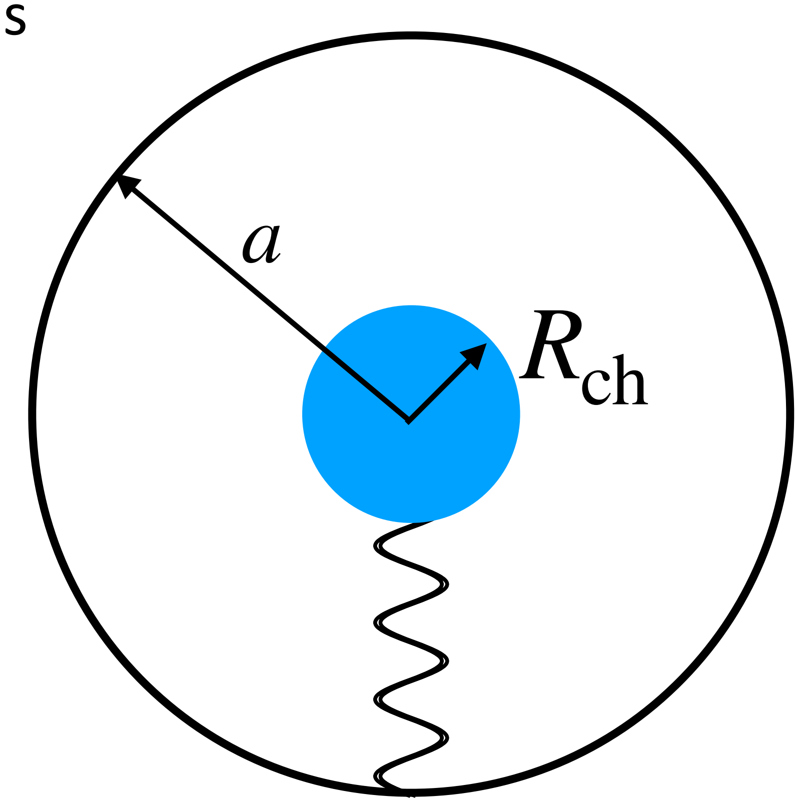
Interplay between atomic and nuclear radii

$$a_{1S}^{eA} = (Z\alpha m_{er})^{-1} \approx 500\,000\,\mathrm{fm}\,Z^{-1}$$

$$\Downarrow$$

$$a_{1S}^{\mu A} = (Z\alpha m_{\mu r})^{-1} \approx 250\,\mathrm{fm}\,Z^{-1}$$

$$R_{\mathrm{ch}} \approx 1.1\,\mathrm{fm} \times A^{1/3}$$



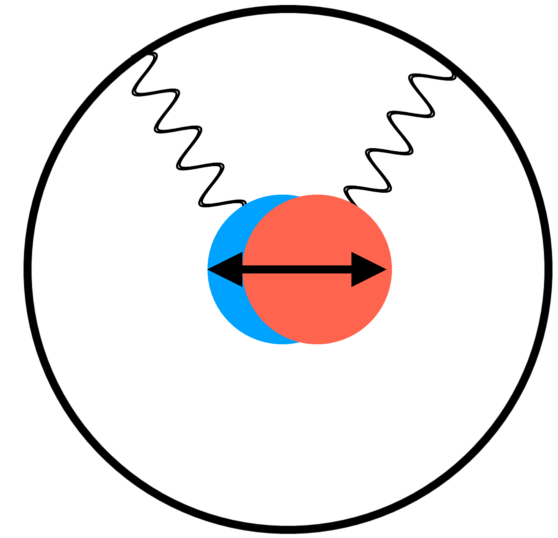
From Z ~ 50 $R_{\mathrm{ch}} \approx a_{1S}^{\mu}$ — very sensitive to nuclear radii

$$\Delta E_{1S} \propto Z\alpha m_r (R_{\mathrm{ch}}/a_{1S}^{\mu})^2$$

For precision: include higher-order corrections (QED + nuclear structure)

QED: numerical solutions of Dirac/Schroedinger radial equations, or analytical $Z\alpha$-expansion

In presence of nuclear polarization



Muon may induce polarization of the nucleus

Structure constant α_{E1} —> electric dipole polarizability

Charges inside nucleus are displaced against each other

α_{E1} has dimension of volume

$$\Delta E_{1S} \propto -Z\alpha m_r \, \alpha_{E1}/(a^{\mu}_{1S})^3$$

Empirical scaling (giant dipole resonance) $\alpha_{E1} \approx 0.00225 A^{5/3}\,\mathrm{fm}^3$

Effectively shifts the extracted radius by

$$\frac{\delta R_{\rm ch}}{R_{\rm ch}} \propto \frac{\alpha_{E1}}{2R^2_{\rm ch}\, a^{\mu}_{1S}} \propto \frac{Z\alpha m_r\, 0.00225\,\mathrm{fm}^3 \times A^{5/3}}{2\times(1.1\,\mathrm{fm}\times A^{1/3})^2} \sim 3.6 ZA \times 10^{-6}$$

Typical precision $\delta R/R \sim 10^{-4}$ —> precision requirement on NP $\quad 10^4 \frac{\delta R_{\rm ch}}{R_{\rm ch}} \sim 7 \frac{Z}{10}\frac{A}{20}$

Accuracy of calculated NP reflects directly in the precision of nuclear radii (not via this formula)

Nuclear polarization - basics

2nd order PT $\Delta E_p = \sum_{N \neq 0} \langle 0' \,|\, \Delta H_c \,|\, N \rangle \left[\sum_n \frac{|n\rangle\langle n|}{\epsilon_0 - \epsilon_n - \omega_N} \right] \langle N \,|\, \Delta H_c \,|\, 0' \rangle$

Ericson, Hüfner 1972
Friar 1977

Perturbation: transition induced by Coulomb interaction $\Delta H_c(\vec{r}) = -\alpha \int \frac{d^3 \vec{r}_N}{|\vec{r} - \vec{r}_N|} \hat{\rho}(\vec{r}_N)$

First approximation:

nucleus much smaller than atom

nuclear energy splittings much larger than atomic energy

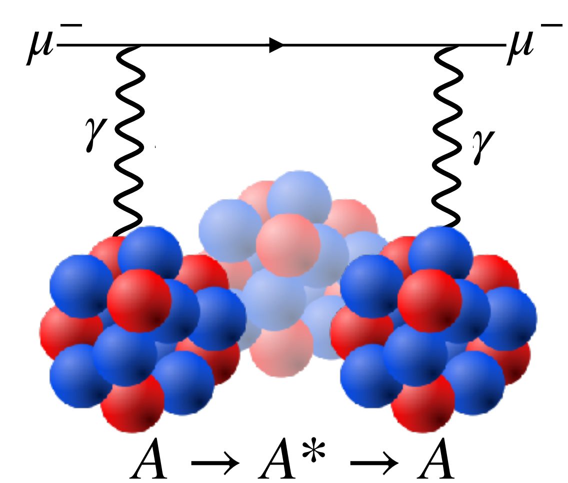
$$\left[\sum_n \frac{|n\rangle\langle n|}{\epsilon_0 - \epsilon_n - \omega_N} \right]$$

Npol-induced potential - δ-function at origin; relativistic treatment of nuclear system

Start with leading-order result:

$$\Delta E_{n\ell} = \frac{8\alpha^2 m}{i\pi} \phi_{n\ell}(0)\,|^2 \int d^4q \frac{(q^2 - \nu^2)T_2 - (q^2 + 2\nu^2)T_1}{q^4(q^4 - 4m^2\nu^2)}$$

Bernabeu-Jarlskog 1974; Rosenfelder 1983



Nuclear polarization - basics

Im parts of forward Compton amplitudes

~ photoabsorption data

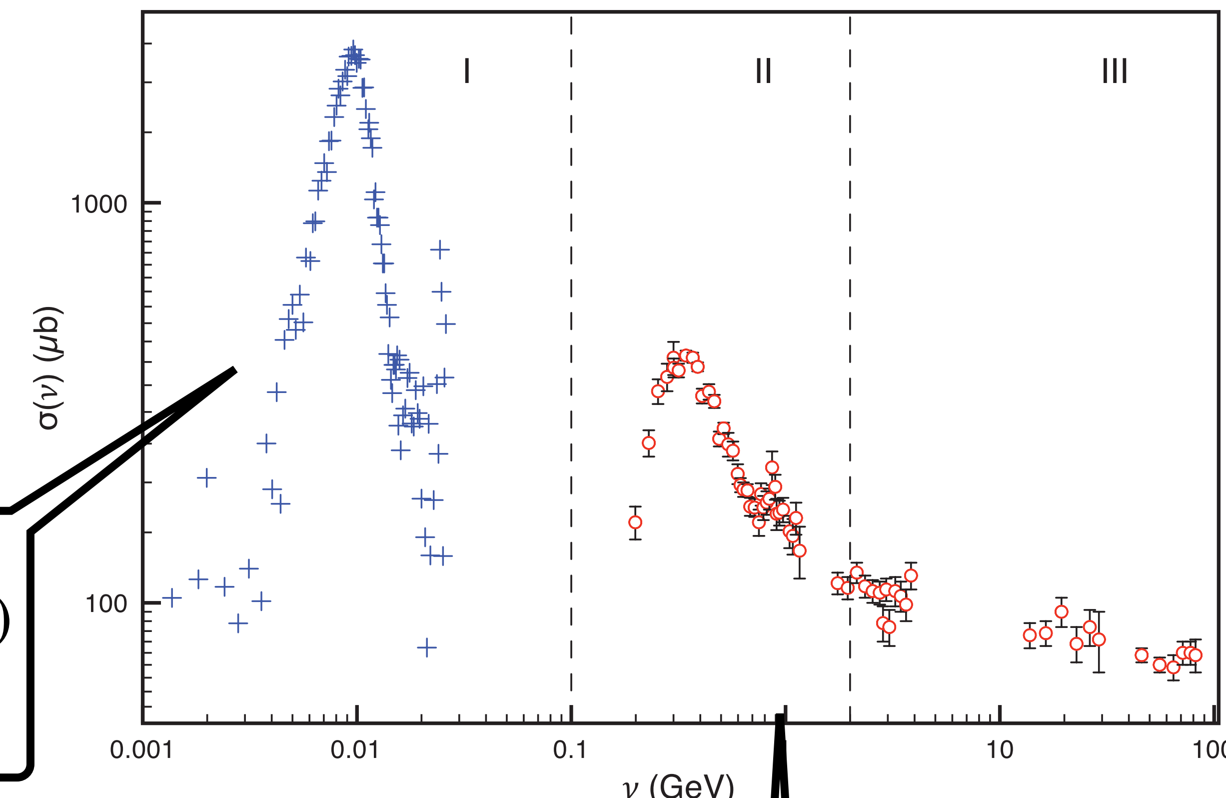
$$\Delta E_{n\ell} = \frac{8\alpha^2 m}{i\pi} \phi_{n\ell}(0)|^2 \int d^4q \frac{(q^2-\nu^2)T_2 - (q^2+2\nu^2)T_1}{q^4(q^4-4m^2\nu^2)}$$

$$\operatorname{Im} T_1(\nu, q^2) = \frac{1}{4M} F_1(\nu, q^2) \qquad \operatorname{Im} T_2(\nu, q^2) = \frac{1}{4\nu} F_2(\nu, q^2)$$

Real photoabsorption data:

Nuclear range $\nu < 140\,\mathrm{MeV}$

Hadronic range $\nu \geq 140\,\mathrm{MeV}$



Nonrelativistic
Migdal sum rule

$$\alpha_{E1} = \frac{1}{2\pi^2} \int_{\mathrm{thr}}^{\nu_{mas}} \frac{d\nu}{\nu^2} \sigma_\gamma(\nu)$$

Baldin sum rule (relativistic)

$$\alpha_E + \beta_M = \frac{1}{2\pi^2} \int_{\mathrm{thr}}^{\infty} \frac{d\nu}{\nu^2} \sigma_\gamma(\nu)$$

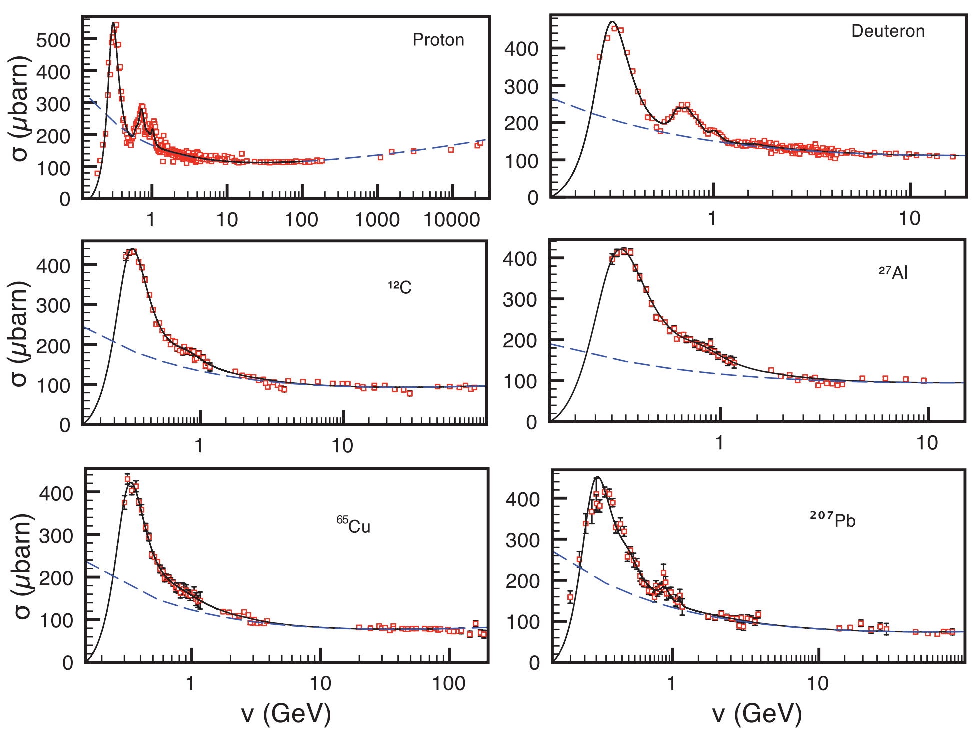
Total photoabsorption data in hadronic range: scales as ~A

Nuclear polarizability scales as $A^{5/3}$

A-scaling of total photoabsorption in hadronic range

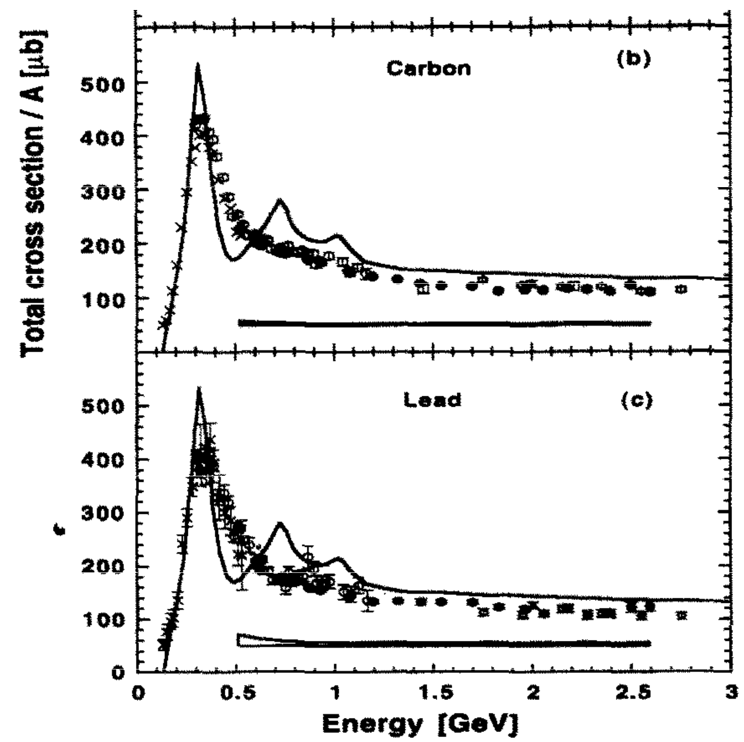
Fit to nuclear photoabsorption — CS per nucleon

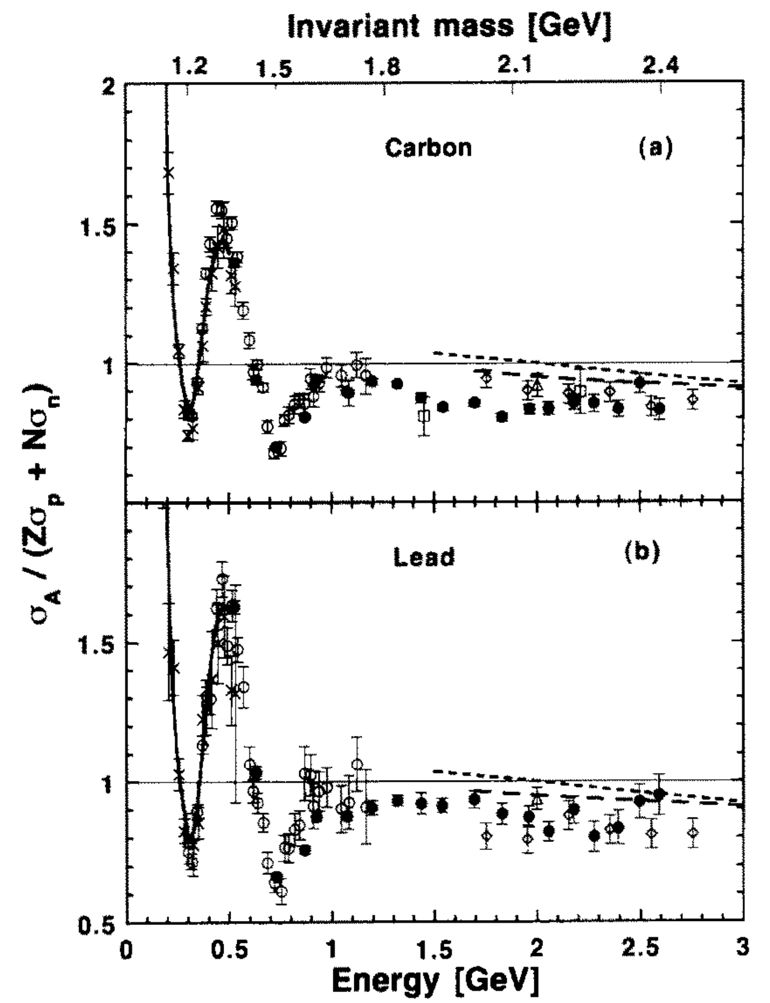
MG et al, 1110.5982



Total hadronic photoabsorption on carbon and lead in the shadowing threshold region

M. Mirazita[a], H. Avakian[a], N. Bianchi[a,*], A. Deppman[a], E. De Sanctis[a], V. Gyurjyan[a], V. Muccifora[a], E. Polli[a], P. Rossi[a], R. Burgwinkel[b], J. Hannappel[b], F. Klein[b], D. Menze[b], W. Schwille[b], F. Wehnes[b]





Oscillating around A_{eff} = A in resonance region;

Shadowing (A_{eff} < A) at high energies

For μ-atoms: $\bar{\nu} = \sigma_{-1}/\sigma_{-2} \sim 500\,\mathrm{MeV}$

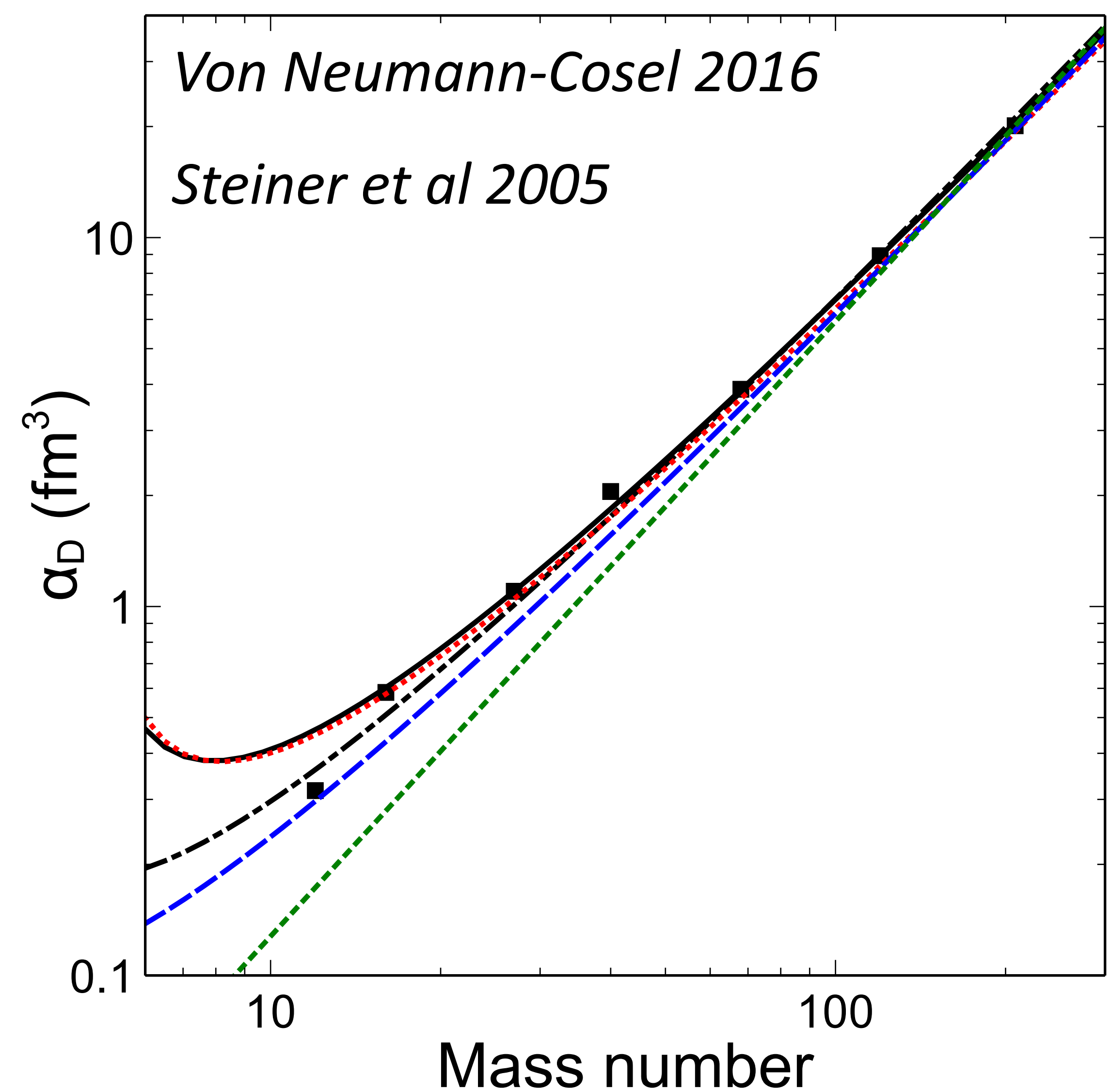
A-scaling of total photoabsorption in nuclear range

Dipole polarizability: external input (nuclear theory or data)

Fit from oxygen to lead

$$\alpha_{E1} = \frac{0.0518\,\mathrm{MeV\ fm}^3 A^2}{S_v(A^{1/3} - \kappa)}$$

$$S_v = 27.3(8)\ \mathrm{MeV}\ \text{and}\ \kappa = 1.69(6)$$



Some lighter nuclei: data (Ahrens et al, 1974)

	$\hat{E}$ (MeV)	$\sum_{-2}$ (mb/MeV)	±(%)
Li	100	0.196	1.1
	140	0.197	1.1
	210	0.198	1.1
Be	100	0.192	2.5
	140	0.194	2.5
	210	0.195	2.5
C	100	0.313	1.7
	140	0.316	1.7

Nuclear polarization - leading order

Loop integral is evaluated in two different ways for nuclear and hadronic parts
—> *nuclear* polarization (NP) and *nucleon* polarization (nP)

Hadronic: exact relativistic expression; direct use of real and virtual photoabsorption data

Evaluated on H, He isotopes — use the A-scaling of cross section to extrapolate to arbitrary A

$$\left[\Delta E_{2S}^{\rm hadr}\right]_{\mu D} = -28(2)\,\mu{\rm eV} \longrightarrow \left[\Delta E_{nS}^{\rm nP}\right]_{\mu A} = -28(2)\,\mu{\rm eV}\,\frac{|\phi_{nS}^{\mu A}(0)|^2}{|\phi_{2S}^{\mu D}(0)|^2}\,\frac{A}{2}$$

Carlson, Vanderhaeghen 2011; Carlson, MG, Vanderhaeghen 2013, 2016; …

Nuclear shadowing ($A_{\rm eff}$ < A) concentrated at high energies, ~does not affect Npol

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Nuclear: keep dominant longitudinal response

$$\Delta E_{nS}^{NP} = -8\alpha^2|\phi_{nS}(0)|^2\int_0^\infty \frac{d\mathbf{q}}{\mathbf{q}^2}\int_0^\infty \frac{d\nu S_L(\nu,\mathbf{q})}{\nu+\mathbf{q}^2/2m}$$

$$S_L(\nu,\mathbf{q}) = \mathbf{q}^2\,\frac{\sigma_\gamma(\nu)}{4\pi^2\alpha\nu}F^2(\mathbf{q})$$

$$\alpha_{E1} = \frac{1}{2\pi^2}\int_{\rm thr}^{\nu_{mas}}\frac{d\nu}{\nu^2}\sigma_\gamma(\nu)$$

Leading-order nuclear polarization

$$\Delta E_{nS}^{\rm NP} = -2\pi\alpha|\phi_{nS}(0)|^2\alpha_{E1}\sqrt{2m\bar\nu}\,e^{\beta^2(\bar\nu)}{\rm Erfc}(\beta(\bar\nu))$$

E.g., Rosenfelder 1983

Nuclear polarization - beyond leading approximation

Approximation scheme:

define small parameters

$$\epsilon_1 = Z\alpha m_r R_{\rm ch} = R_{\rm ch}/a^{\mu}_{1S}$$

$$\epsilon_2 = (Z\alpha)^2 \frac{m_r}{2\nu_N} = \left| \frac{E_{1S}}{E^{\rm Nucl.\,Exc.}_{\mu}} \right|$$

Nuclear polarization - beyond leading approximation

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$$\epsilon_2 = (Z\alpha)^2 \frac{m_r}{2\nu_N} = \left| \frac{E_{1S}}{E_{\mu}^{\text{Nucl. Exc.}}} \right|$$

Corrections in ϵ_1: variation of atomic WF over nucleus volume

$$F_R = \int_0^{\infty} r^2 dr e^{-2Z\alpha m_r r} \rho_{\text{Nuc}}(r)$$

Nuclear polarization - beyond leading approximation

Approximation scheme:

define small parameters

$$\epsilon_1 = Z\alpha m_r R_{\rm ch} = R_{\rm ch}/a^{\mu}_{1S}$$

$$\epsilon_2 = (Z\alpha)^2 \frac{m_r}{2\nu_N} = \left| \frac{E_{1S}}{E_\mu^{\rm Nucl.\,Exc.}} \right|$$

Corrections in ϵ_1: variation of atomic WF over nucleus volume

$$F_R = \int_0^\infty r^2 dr e^{-2Z\alpha m_r r} \rho_{\rm Nuc}(r)$$

Corrections in ϵ_2: keep Coulomb energy in the Green's function

$$\left[\sum_n \frac{|n\rangle\langle n|}{\epsilon_0 - \epsilon_n - \omega_N} \right]$$

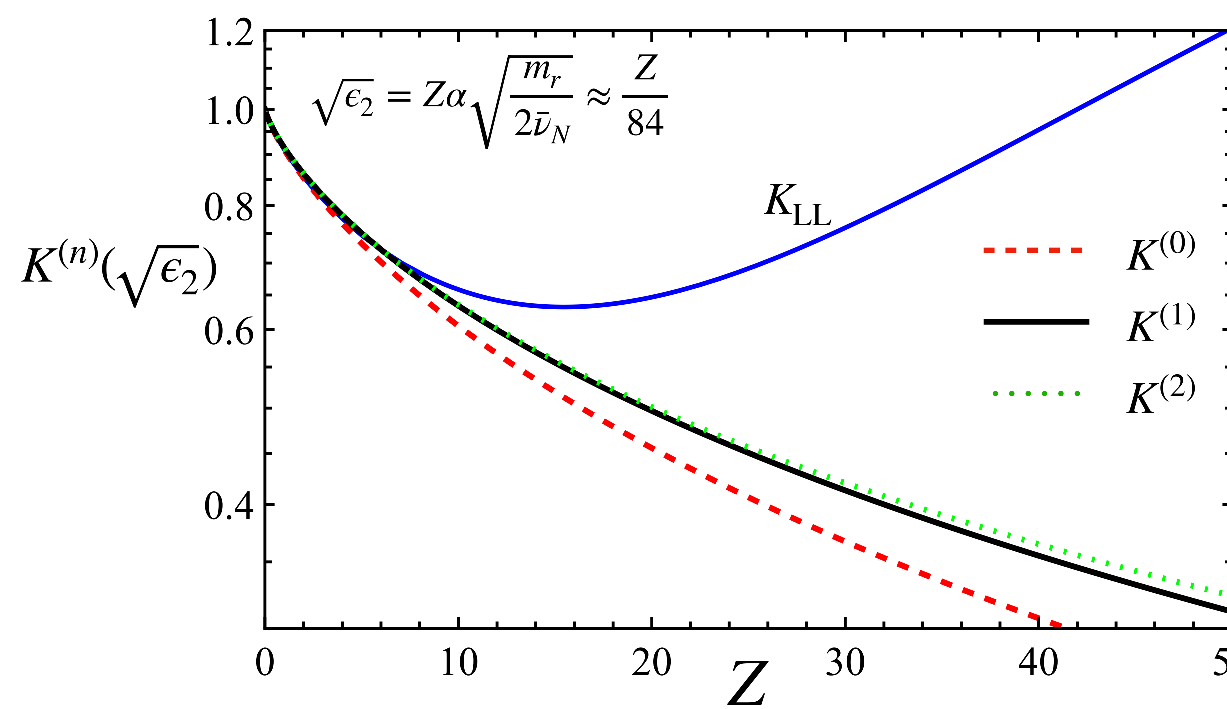
Obtained via radial integral with Coulomb GF and atomic WF

$$K = -\sqrt{\frac{\nu_N}{2m_r}} \int_0^\infty dr \int_0^\infty dr' \phi_{nS}(r) \frac{g_1(-\nu_N, r, r')}{rr'} \phi_{nS}(r')$$

*New closed-form expressions for CD correction

Until now taken in leading-log approximation

from Friar 1977



Nuclear polarization - beyond leading approximation

Final expression: leading-order + corrections

$$\Delta E_{nS}^{\mathrm{TOT}} = \Delta E_{nS}^{\mathrm{NP}}\, F_R(\epsilon_1)\, K^{(1)}(\sqrt{\epsilon_2}) + \Delta E_{nS}^{\mathrm{nP}}\, F_R(\epsilon_1) K^{(1)}(\sqrt{\epsilon_2^n});$$

$$\epsilon_2^n = (Z\alpha)^2 m_r / 2\nu_n$$

$$\nu_n \approx 500 \ \mathrm{MeV}$$

All ingredients have simple parametrization in terms of few input parameters

Easy to use and reproduce! Evaluate and compare to entries in Fricke, Heilig (used to extract radii)

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Rinker, Speth 1978:

$$\Delta E_a = \frac{\alpha^2 B^2 k^2 Z}{2M} \langle r^{2k-2} \rangle \left[\frac{Z}{A} \langle E_N^{(b)} - E_N^{(a)} \rangle_{\tau=0}^{-2} + \frac{N}{A} \langle E_N^{(b)} - E_N^{(a)} \rangle_{\tau=1}^{-2} \right]$$

Energy-weighted (TRK) sum rule to normalize

Polarizability ~ inverse energy sum rule —> enhanced sensitivity to low-lying states (PDR)

Long-range part of the induced dipole potential $\sim \alpha_{E1}/r^4$ taken between atomic WF

Already noted in Ericson, Hüfner 1972

Results, Uncertainties, Comparisons

Predictions for Npol for $3 \leq Z \leq 41$ — not the final answer (which is 42)

Uncertainties:

Polarizability 10%; F_R (Gauss vs hard sphere), Coulomb distortion (higher orders in ϵ_2)

If a "better" dipole polarizability at hand — simply rescale the NP contribution

Z−Element	A	α_{E1} (fm^3)	$-\Delta E_{1S}^{NP}$	$-\Delta E_{1S}^{nP}$	Total NP	Entry in [7]	$\sigma_{\rm exp}$
4−Be	9	0.192(19)[a]	0.44(4)(0)(0)	0.063(6)(0)(0)	0.50(4)	1.0(3)	10
5−B	10	0.230(23)[a*]	0.99(10)(0)(1)	0.13(1)(0)(0)	1.12(10)	1.0(3)	7
6−C	12	0.313(31)[a]	2.1(2)(0)(0)	0.27(3)(0)(0)	2.4(2)	2.5(7)	0.5
7−N	14	0.405(40)[a*]	3.8(4)(0)(1)	0.48(5)(0)(0)	4.3(4)	3.0(9)	5
8−O	16	0.580(58)[a]	7.8(0.8)(0.1)(0.1)	0.79(8)(1)(1)	8.6(8)	5.0(1.5)	4
9−F	19	0.700(70)	11.9(1.2)(0.1)(0.2)	1.28(13)(1)(1)	13.2(1.2)	9.0(2.7)	2
10−Ne	20	0.741(74)	15.7(1.6)(0.2)(0.3)	1.78(18)(2)(1)	17.5(1.6)	19(6)	5
	21	0.783(78)	17.0(1.7)(0.2)(0.4)	1.88(19)(2)(1)	19(2)	18(5)	4
	22	0.823(82)	18.0(1.8)(0.2)(0.4)	1.98(20)(2)(1)	20(2)	18(5)	4
11−Na	23	0.870(87)	23.3(2.3)(0.3)(0.6)	2.64(26)(4)(1)	26(3)	25(8)	2
12−Mg	24	0.915(91)	30.0(3.0)(0.5)(0.8)	3.46(35)(6)(2)	33(3)	38(11)	2
	25	0.961(96)	31.3(3.1)(0.5)(0.8)	3.61(36)(6)(2)	35(3)	31(9)	3
	26	1.01(10)	32.3(3.2)(0.5)(0.9)	3.75(38)(6)(2)	36(3)	33(10)	3
13−Al	27	1.10(11)[a]	42.2(4.2)(0.8)(1.2)	4.80(48)(9)(3)	48(5)	40(12)	2

Close agreement with F&H for light elements

Should not be taken for granted: approaches are different

Nucleon polarization surprisingly large ~10% — has been neglected until now!

Z−Element	A	α_{E1} (fm^3)	$-\Delta E^{NP}_{1S}$	$-\Delta E^{nP}_{1S}$	Total NP	Entry in [7]	$\sigma_{\rm exp}$
17−Cl	35	1.47(15)	98.5(9.9)(2.9)(3.4)	11.9(1.2)(0.3)(0.1)	110(11)	-	-
	37	1.58(16)	106(11)(3)(4)	12.6(1.3)(0.4)(0.1)	119(12)	-	-
18−Ar	36	1.53(15)	116(12)(4)(4)	14(1.4)(0.4)(0.1)	130(12)	118(36)	24
	38	1.64(16)	124(12)(4)(5)	15(1.5)(0.5)(0.1)	139(14)	107(32)	24
	40	1.75(18)	132(13)(4)(5)	16(1.6)(0.5)(0.1)	148(15)	126(38)	25
19−K	39	1.70(17)	141(14)(5)(5)	18(1.8)(0.6)(0.2)	159(16)	119(36)	32
	41	1.81(18)	150(15)(5)(6)	18(1.8)(0.6)(0.2)	168(17)	132(40)	28
20−Ca	40	1.75(18)	160(16)(6)(6)	20(2.0)(0.7)(0.2)	181(18)	142(40)	25
	42	1.87(19)	170(17)(6)(7)	21(2.1)(0.8)(0.2)	191(19)	166(50)	29
	43	1.93(19)	176(18)(7)(7)	21(2.1)(0.8)(0.2)	198(20)	145(43)	27
	44	2.00(20)	180(18)(7)(7)	22(2.2)(0.8)(0.2)	203(21)	175(52)	26
	46	2.12(21)	193(19)(7)(8)	23(2.3)(0.8)(0.2)	216(22)	156(47)	107
	48	2.25(22)	206(21)(8)(8)	24(2.4)(0.9)(0.2)	230(24)	153(46)	26

Nucleon polarization from Ca on exceeds exp. precision!

$Z-$Element	A	α_{E1} (fm^3)	$-\Delta E_{1S}^{NP}$	$-\Delta E_{1S}^{nP}$	Total NP	Entry in [7]	σ_{exp}
26−Fe	54	2.65(26)	371(37)(21)(19)	48(5)(3)(1)	419(47)	362(109)	48
	56	2.79(28)	384(38)(22)(20)	49(5)(3)(1)	433(49)	403(121)	44
	57	2.86(29)	391(39)(22)(20)	50(5)(3)(1)	441(50)	390(117)	56
	58	2.93(29)	397(40)(23)(20)	50(5)(3)(1)	447(50)	400(120)	54
27−Co	59	3.00(30)	433(43)(26)(23)	56(6)(4)(2)	489(56)	438(131)	50
28−Ni	58	2.93(29)	459(46)(29)(25)	59(6)(4)(1)	518(60)	437(131)	46
	60	3.07(31)	467(47)(30)(25)	61(6)(4)(1)	528(61)	461(138)	45
	61	3.14(31)	476(48)(30)(26)	62(6)(4)(1)	538(63)	426(138)	54
	62	3.22(32)	484(48)(31)(26)	62(6)(4)(1)	546(64)	458(138)	45
	64	3.36(34)	502(50)(33)(27)	64(6)(4)(1)	566(66)	438(138)	49
29−Cu	63	3.29(33)	506(51)(35)(29)	68(7)(5)(1)	574(68)	538(161)	47
	65	3.44(34)	530(53)(36)(30)	70(7)(5)(1)	600(71)	489(147)	49
41−Nb	93	5.78(58)	1264(126)(156)(92)	177(18)(20)(3)	1441(223)	1127(338)	16

Agreement worse for larger Z, nP contribution important!

If disagree with other calculations, also extracted radii disagree

What do we learn from comparing two theory calculations?

If disagree — which one's right? If agree — what if both wrong?

Nuclear polarization - how good is good enough?

Compare to more advanced calculation by Natalia et al:

	NP	Natalia		MG
^{35}Cl ΔE_{1S}:	104(24) eV		vs.	99(11) eV
^{37}Cl ΔE_{1S}:	100(23) eV		vs.	106(12) eV

Take the simple model to the extreme: ^{208}Pb

Should not work great: $\varepsilon_1 = 1.76$ and $\varepsilon_2 = 1.33$

	NP	Natalia et al 2504.19977		MG
^{208}Pb ΔE_{1S}:		5.7(6) keV	vs.	4.9(7) keV

nP (nucleon polarization) ~10%, unexpectedly large

Simple E1 polarizability approach gives the bulk of NP
Fine details (magnetic, higher multipoles) important for IS — but are below uncertainty!

Light vs. Heavy

Anything heavier than lithium: $$[\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m_r + V(r)] \, |\psi_{n\kappa m}\rangle = E_{n\kappa} |\psi_{n\kappa m}\rangle$$

numerical solution of Dirac eq. with “realistic” charge distribution (e.g. 2pF, 3pF, …)

Nuclear polarization: ~effective potential between muon Dirac (or Schrödinger) WF

But **uncorrelated** with bound-state QED calculation

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2. Npol: part of two-photon exchange elastic + inelastic — Pachucki et al., 2212.13782
3. Computed systematically via η-expansion (slow convergence!)

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$$\delta^{\rm NR}_{\rm pol} = \sum_{N\neq N_0}\int d^3R\; d^3R'\rho^p_N(\boldsymbol{R})W(\boldsymbol{R},\boldsymbol{R}',\omega_N)\rho^p_N(\boldsymbol{R}')$$

Hernandez et al, 1909.05717

$$W(\boldsymbol{R},\boldsymbol{R}',\omega_N) = -\,Z^2|\phi_\mu(0)|^2\int\frac{d^3q}{(2\pi)^3}\left(\frac{4\pi\alpha}{q^2}\right)^2\left(1-e^{i\boldsymbol{q}\cdot\boldsymbol{R}}\right)\frac{1}{\frac{q^2}{2m_r}+\omega_N}\left(1-e^{-i\boldsymbol{q}\cdot\boldsymbol{R}'}\right)$$

$$= -\,\frac{\pi}{m_r^2}(Z\alpha)^2|\phi_\mu(0)|^2\left(\frac{2m_r}{\omega_N}\right)^{3/2}\frac{1}{\eta}\left(e^{-\eta}-1+\eta-\frac{1}{2}\eta^2\right)$$

$$\eta = \sqrt{2m_r\omega}\,|\vec{R}-\vec{R}'|$$

Point-Coulomb extracted at step 1, has to be subtracted — cancellations inherent to the method

Example of cancellation: $\mu\ ^{6,7}\mathrm{Li}^{2+}$

δ^A_{Zem} — elastic piece

δ^A_{pol} — inelastic piece

Alarming 95% cancellation!

Maybe OK because 100% correlated

Intrinsic approximations may destroy

this correlation — dangerous!

Compare with my simple model

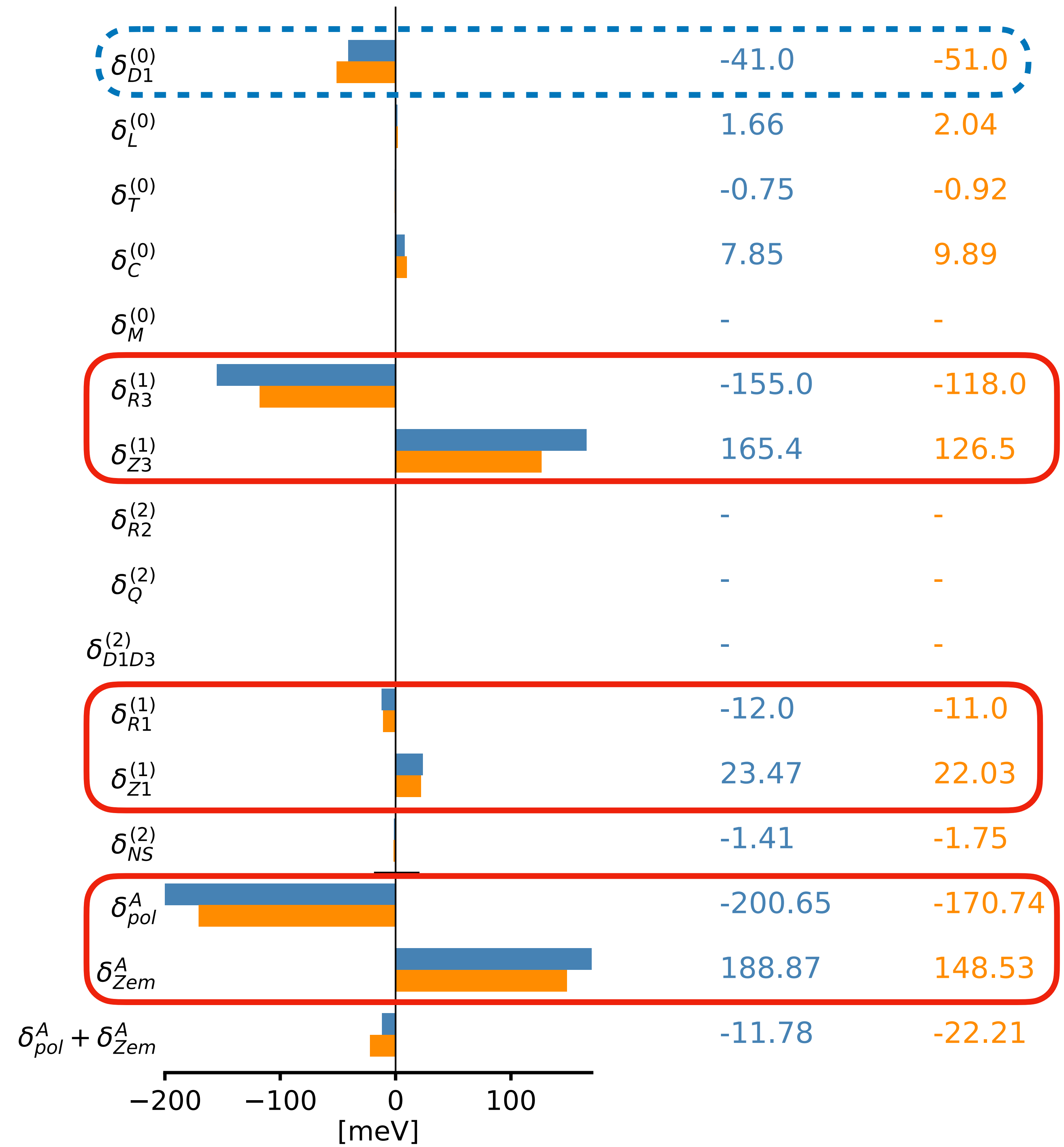
	$\mu\ ^{6}\mathrm{Li}^{2+}$	$\mu\ ^{7}\mathrm{Li}^{2+}$
$\Delta E^{NP}_{2P-2S} =$	23(2) meV	24(2) meV

Factor 2 off for Li-6, OK for Li-7

Relativistic effects expected to matter

for lightest systems

Li Muli et al, 1910.14370



Conclusions & Outlook

Presumable quote by Wolfgang Pauli:

Nothing is worse than a wrong theory describing data

But:

How about agreement between two wrong calculations?

- Nuclear charge radii: crucial input to SM tests and BSM searches at low energies
- Cabibbo (CKM) unitarity and V_{ud}: nuclear corrections current bottleneck - use R_{ch} as input
- Nuclear polarization crucial to extraction of R_{ch} from atomic transitions
- Are uncertainties of NP firmly under control?
- NP is related to dispersion corrections in e-scattering and to NS correction in β-decay
- Look for a uniform treatment of all of these
- Ab-initio methods are hot right now: (potentially) very accurate and systematically improvable — are not easy to understand and are very expensive computationally; viable recipe for nuclear radii tables? — no single ab-initio method covers full nuclear chart
- Generally, μ atoms difficult: nuclear and atomic scales are not well separated; full-blown ab-initio nuclear calculation per se is not enough to guarantee precision
- Methods used in light and heavy μ-atoms are different - should be reconciled