FONDAZIONE BRUNO KESSLER

ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

QUAdRupole Moment MeasuremenT of mid-Z Transition Elements with Microcalorimeters

QUARTZ

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TECHNION | Helen Diller Quantum Center

Introduction:

Quadrupole moment (*Q*)

Why do we need to seek the best possible determinations of nuclear quadrupole moments?

- Fundamental Nuclear Property:
  Q is an important measurable input for nuclear modeling (Oblate, Prolate)

- Measurements of Electronic Properties in Atomic, Molecular Physics:
  In order to obtain well measured electronic properties – these cannot be determined to accuracy greater than that of the *Q* concerned (Electric field gradient and its symmetry)

- Measurements around Magic Numbers.:
  To probe nuclear deformation, shell evolution, shape coexistence and the emergence of collectivity around this region (measurement across isotopic chains)

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- ❑ **Measurement around Doubly/Singly Magic No.:**
  To probe nuclear deformation, shell evolution, shape coexistence and the emergence of collectivity around this region (measurement across isotopic chains)

Introduction:

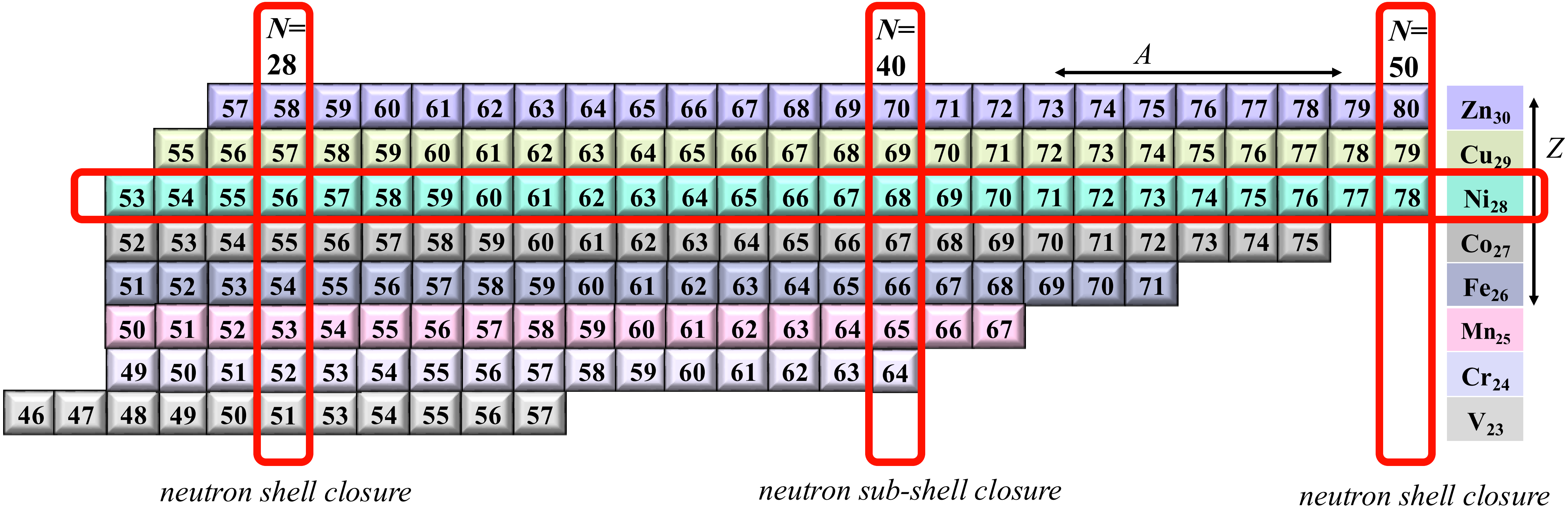
<u>**Quadrupole moment (*Q*)**</u>

A special region of isotopic chain measurement interest is around Ni_{28}

- ❑ Both on the neutron-deficient and neutron-rich sides (across *A*)
- ❑ Adding or Removing a protons near the magic *Z* (across *Z*)

Introduction:

A special region of isotopic chain measurement interest is around Ni_{28}

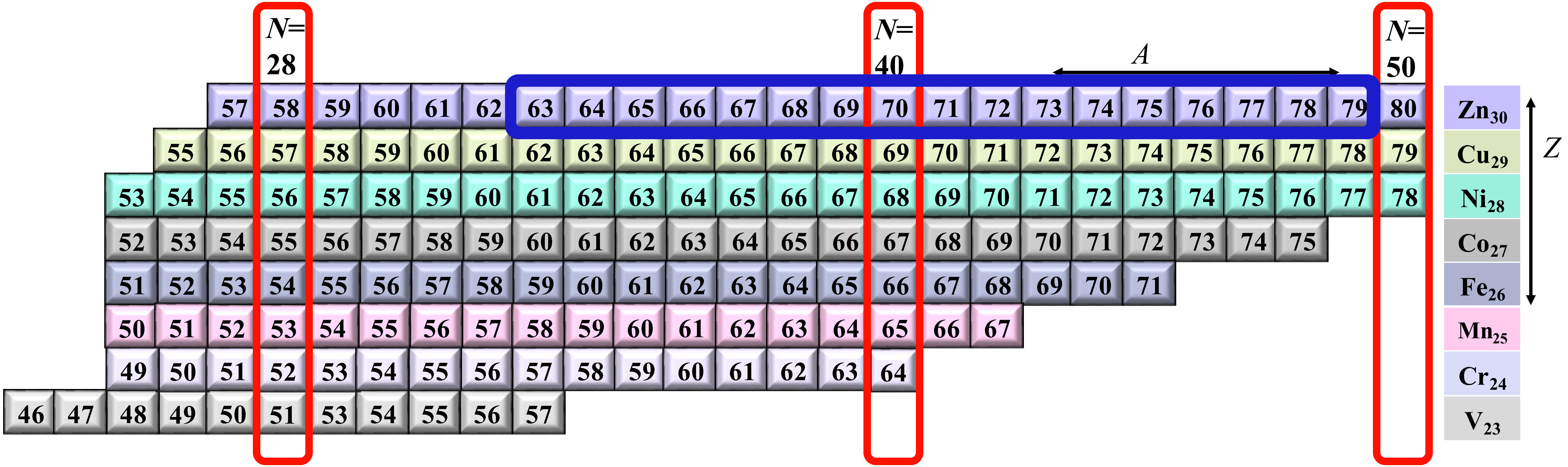


Introduction:

CERN ISOLDE

Measured to determine nuclear moments

1. Wraith, C., et al. *Physics Letters B* 771 (2017): 385-391.
2. Yang, X. F., et al. *Physical review letters* 116.18 (2016): 182502.



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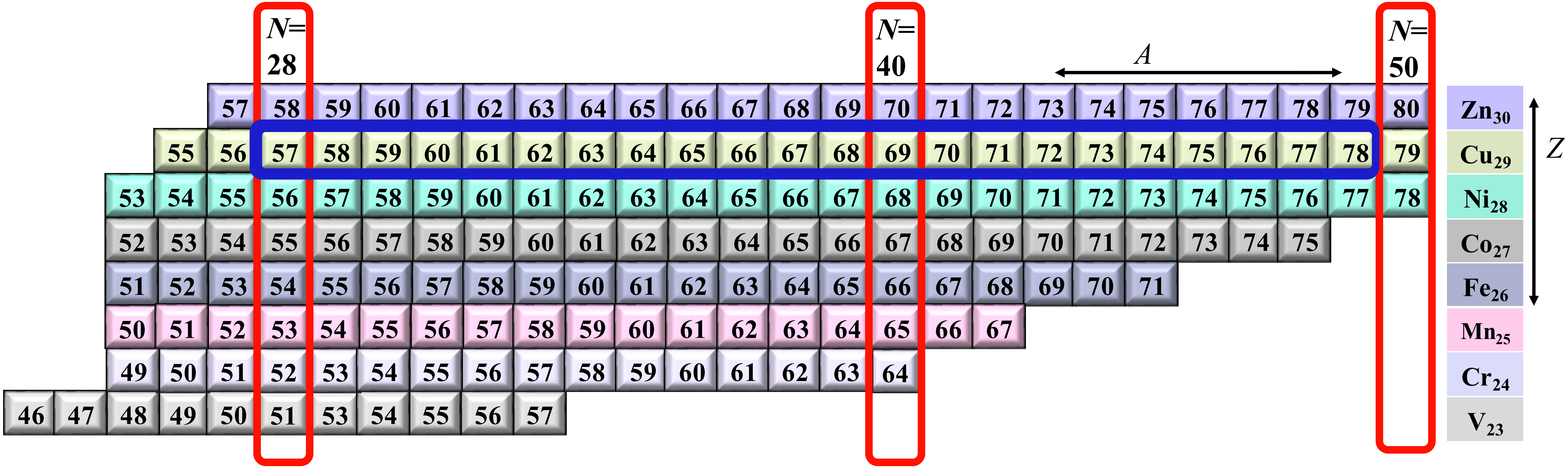
1. Vingerhoets, P., et al. *Physical Review C* 82.6 (2010): 064311.
2. Vingerhoets, P., et al. *Physics Letters B* 703.1 (2011): 34-39.
3. Lozeva, R. L., et al. *Physics Letters B* 694.4-5 (2011): 316-321.
4. Fenta, A. S., et al. *Europhysics Letters* 115.6 (2016): 62002.



5. Bissell, M. L., et al. *Physical Review C* 93.6 (2016): 064318.
6. De Groote, R. P., et al. *Physical Review C* 96.4 (2017): 041302

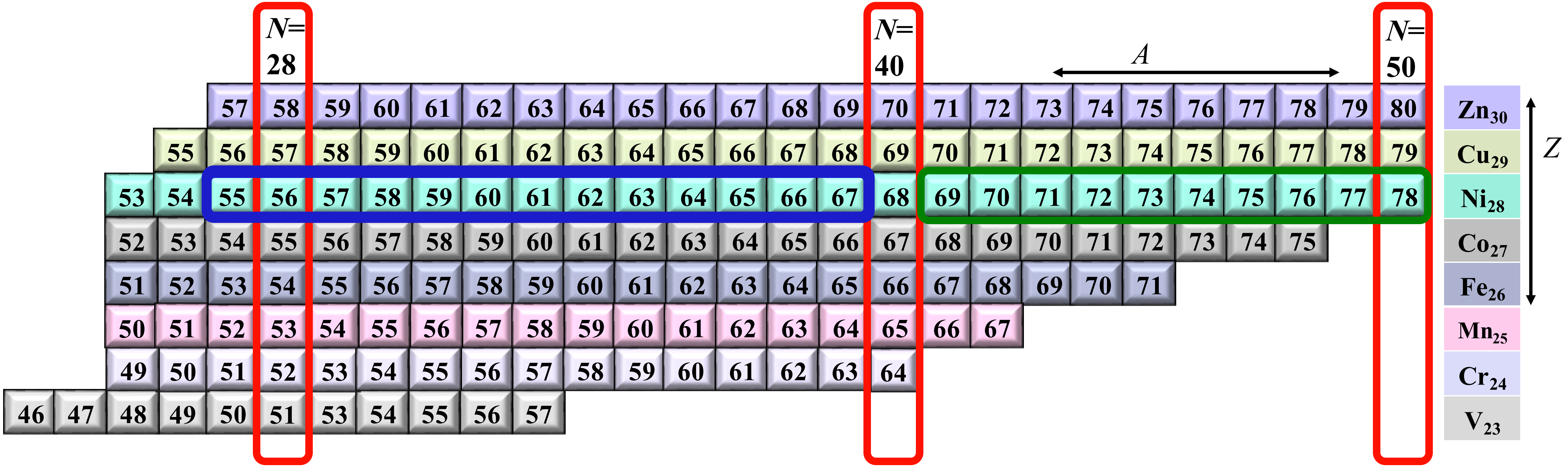
Introduction:

CERN ISOLDE

Measured to determine nuclear moments

Measurement Planned (ISOLDE)

1. Müller, P., et al. *Physics Letters B* 854 (2024): 138737.



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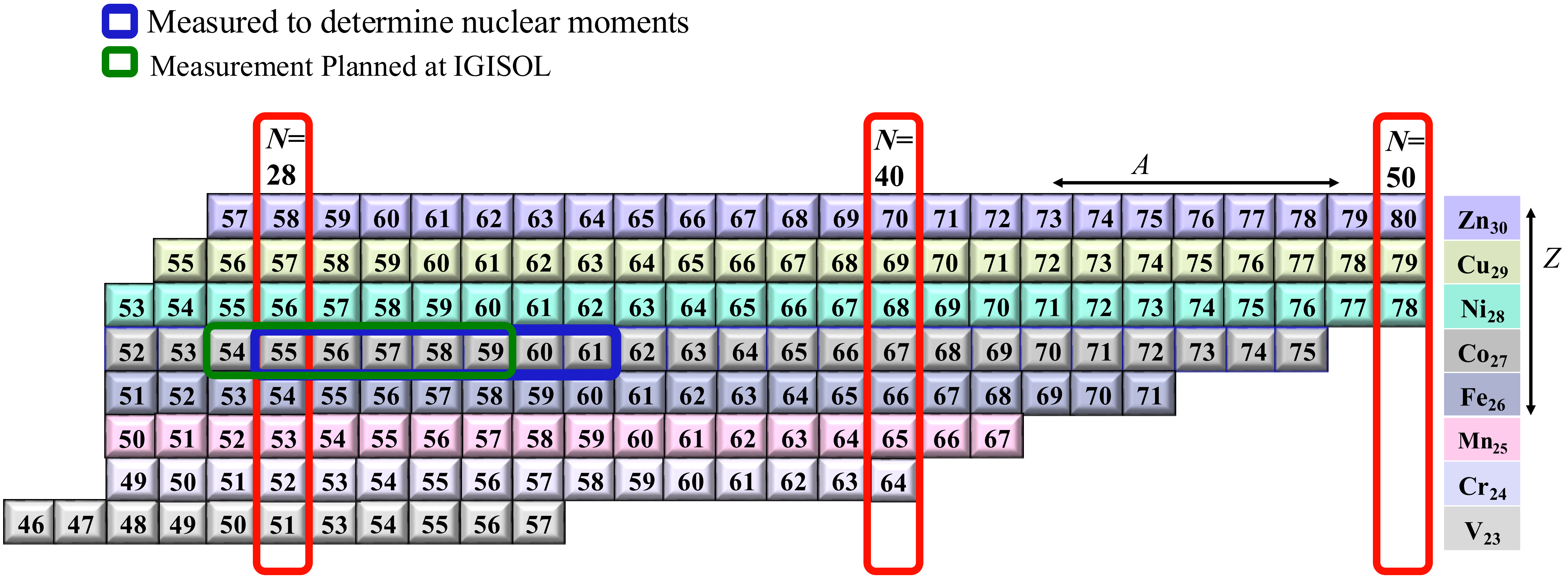

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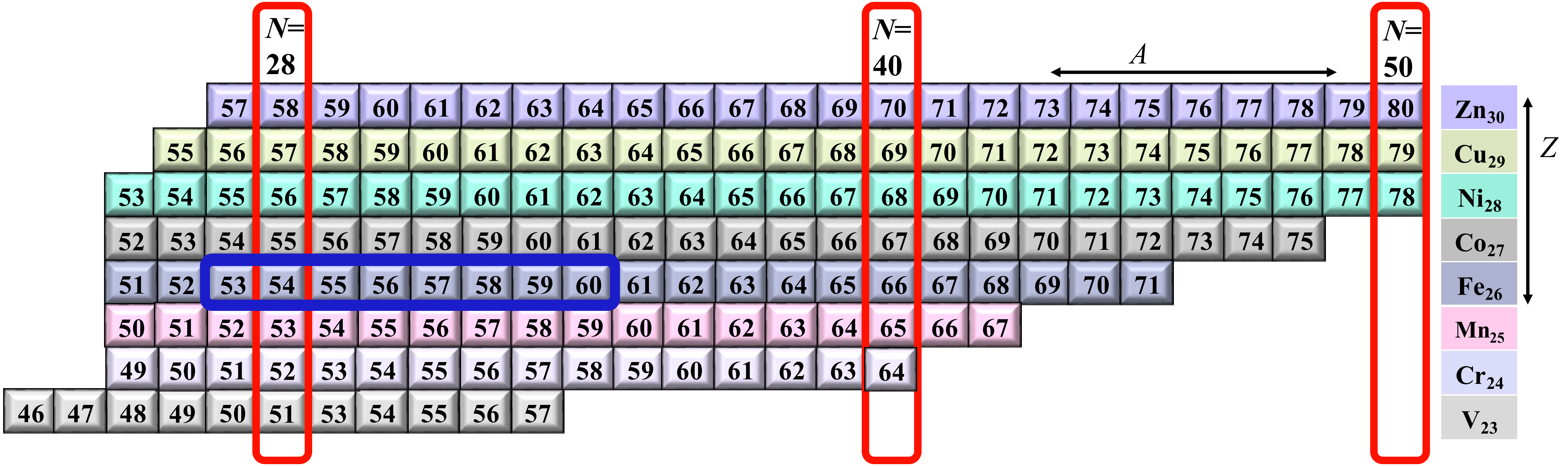
Measured to determine nuclear moments



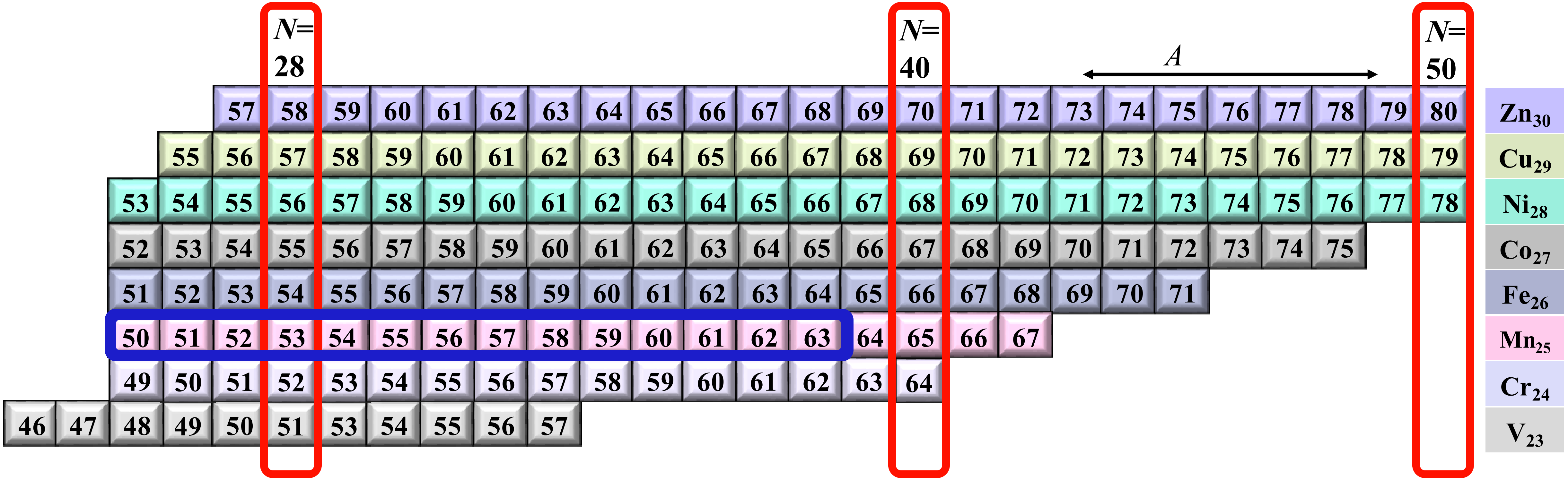
Introduction:

CERN ISOLDE

Measured to determine nuclear moments

1. Babcock, Carla, et al. *Physics Letters B* 760 (2016): 387-392.
2. Charlwood, F. C., et al. *Physics Letters B* 690.4 (2010): 346-351.

IGISOL



Introduction:

- Measured to determine nuclear moments
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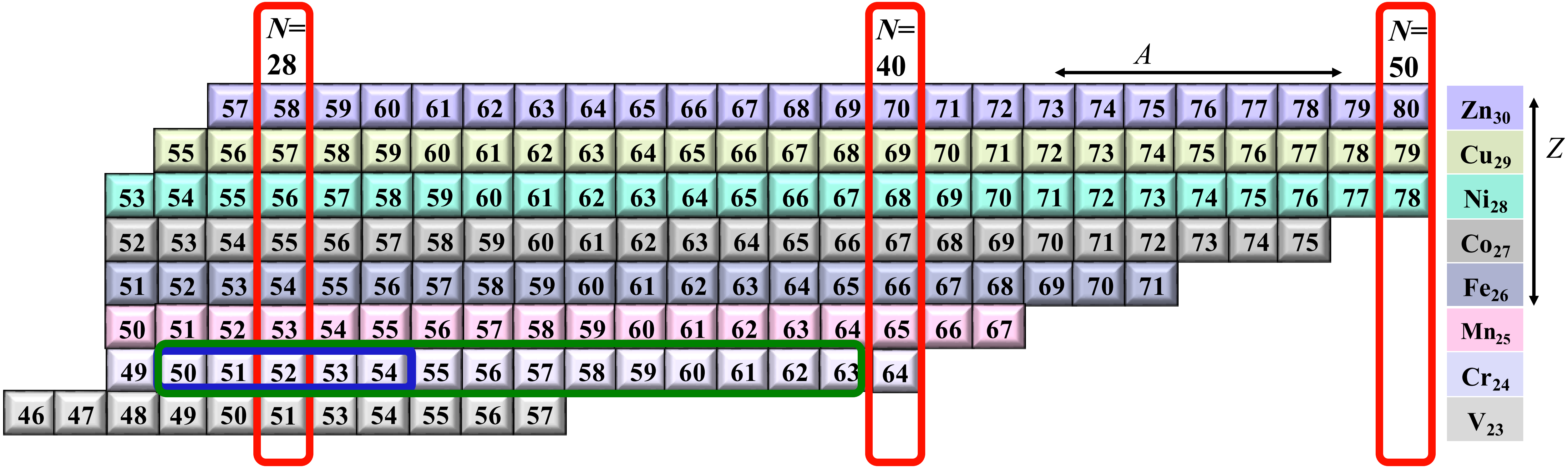
1. Towsley, C. W., D. Cline, and R. N. Horoshko. *Nuclear Physics A* 250.2 (1975): 381-404.



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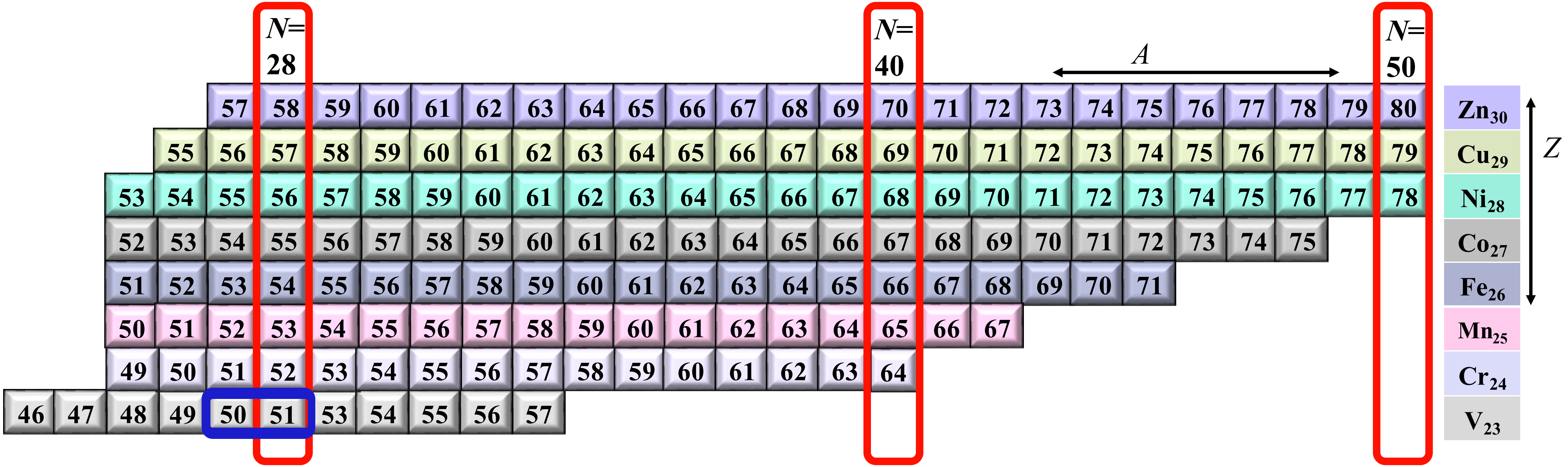
Measured to determine nuclear moments

1. Childs, William J. *Physical Review* 156.1 (1967): 71.



Introduction:

<u>How is quadrupole moment determined for these isotope chains</u>?

Experiments measure

Hyperfine coupling ⇒ **Spectroscopic Quadrupole Moments**

B (Quadrupole coupling constant) $\propto eQ \times V_{zz}$

Theory

V_{zz}: *principal component of the Electric Field Gradient (EFG) tensor*

<u>Depends on the electron cloud</u>

Motivation:

B (Quadrupole coupling constant) $\propto eQ \times V_{zz}$;

Measured in experiment

Theory

For two isotopes,

Q_1 & Q_2; Quadrupole moments

B_1 & B_2; measured coupling constants

$$\frac{B_1}{B_2} = \frac{Q_1}{Q_2} \quad \Rightarrow \quad Q_2 = \frac{Q_1}{B_1} \times B_2$$

Motivation:

$$\frac{B_1}{B_2} = \frac{Q_1}{Q_2} \quad \Rightarrow \quad Q_2 = \frac{Q_1}{B_1} \times B_2$$

A reference isotope, calibrate the whole isotope chain (i);

$$Q_i = \frac{Q_{ref}}{B_{ref}} \times B_i^{measured}$$

$$Unc.Q_i = Q_i\sqrt{\left(\frac{Unc.Q_{ref}}{Q_{ref}}\right)^2 + \left(\frac{Unc.B_{ref}}{B_{ref}}\right)^2 + \left(\frac{Unc.B_i}{B_i}\right)^2}$$

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A reference isotope, calibrate the whole isotope chain (i);

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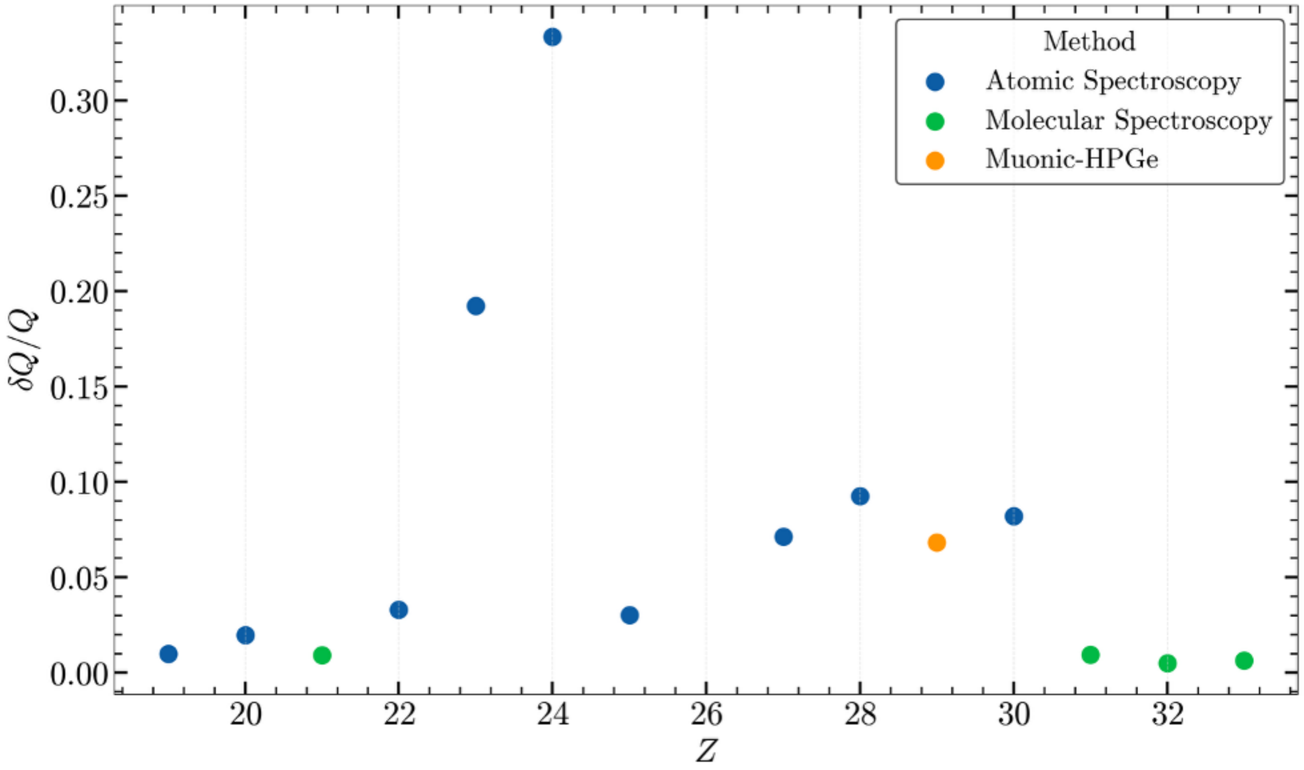
Accuray is limited by accuracy in Q_{ref} Given that B_i is measured precisely

$$Unc.Q_i = Q_i\sqrt{\left(\frac{Unc.Q_{ref}}{Q_{ref}}\right)^2 + \left(\frac{Unc.B_{ref}}{B_{ref}}\right)^2 + \left(\frac{Unc.B_i}{B_i}\right)^2}$$

Motivation:

Uncertainty in the reference Q_{ref} from Stone's Table



https://www-nds.iaea.org/nuclearmoments/

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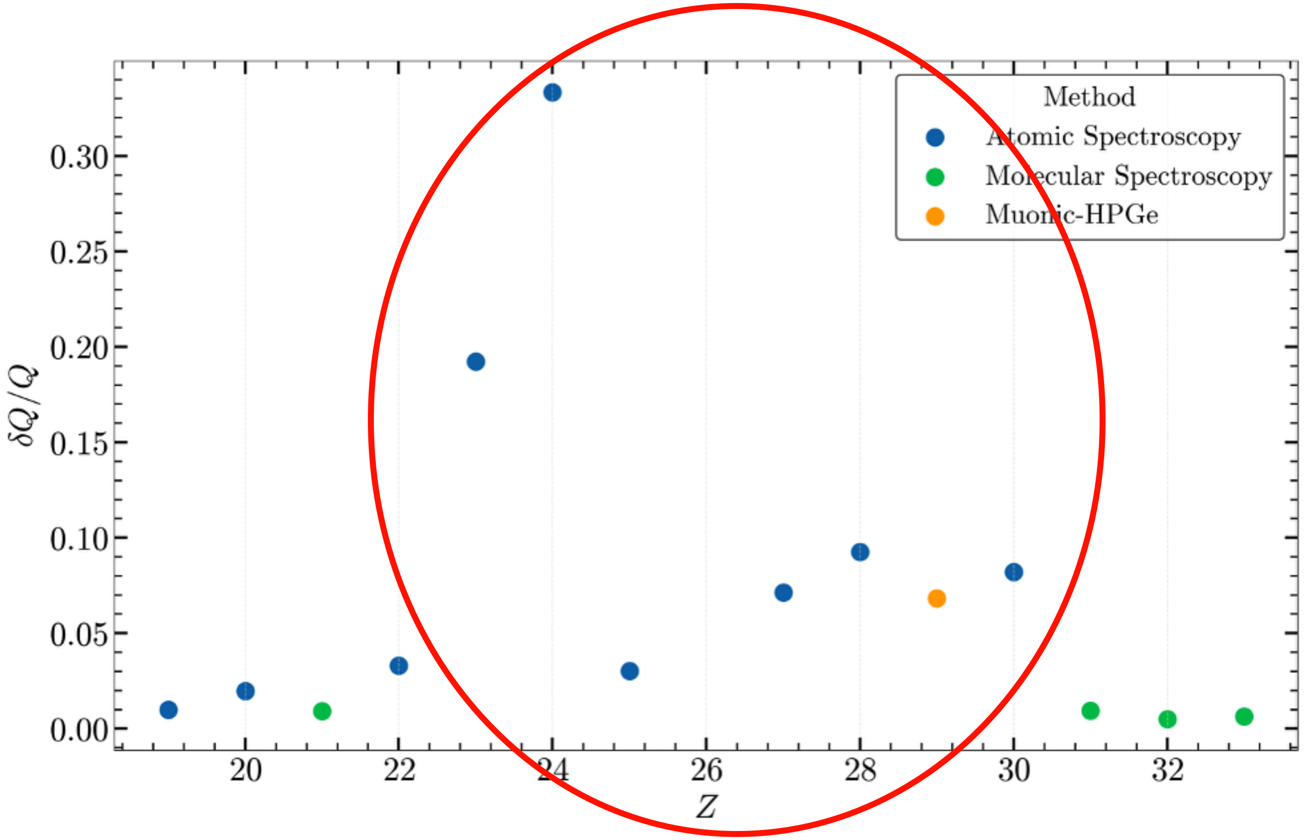
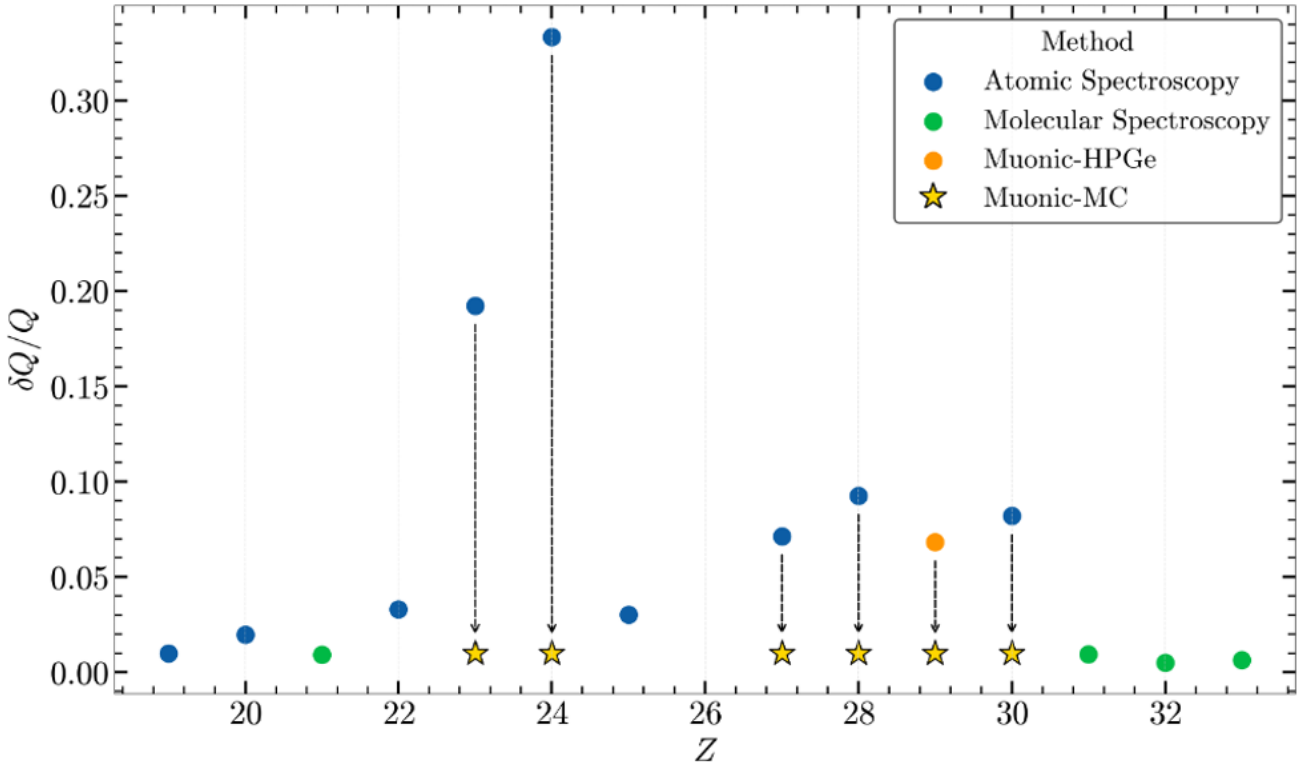


Upcoming and Previous measurements are limited by these uncertainties

https://www-nds.iaea.org/nuclearmoments/

QUARTZ AIM:

To improve the precision in the reference Q_{ref} of mid-Z transition elements



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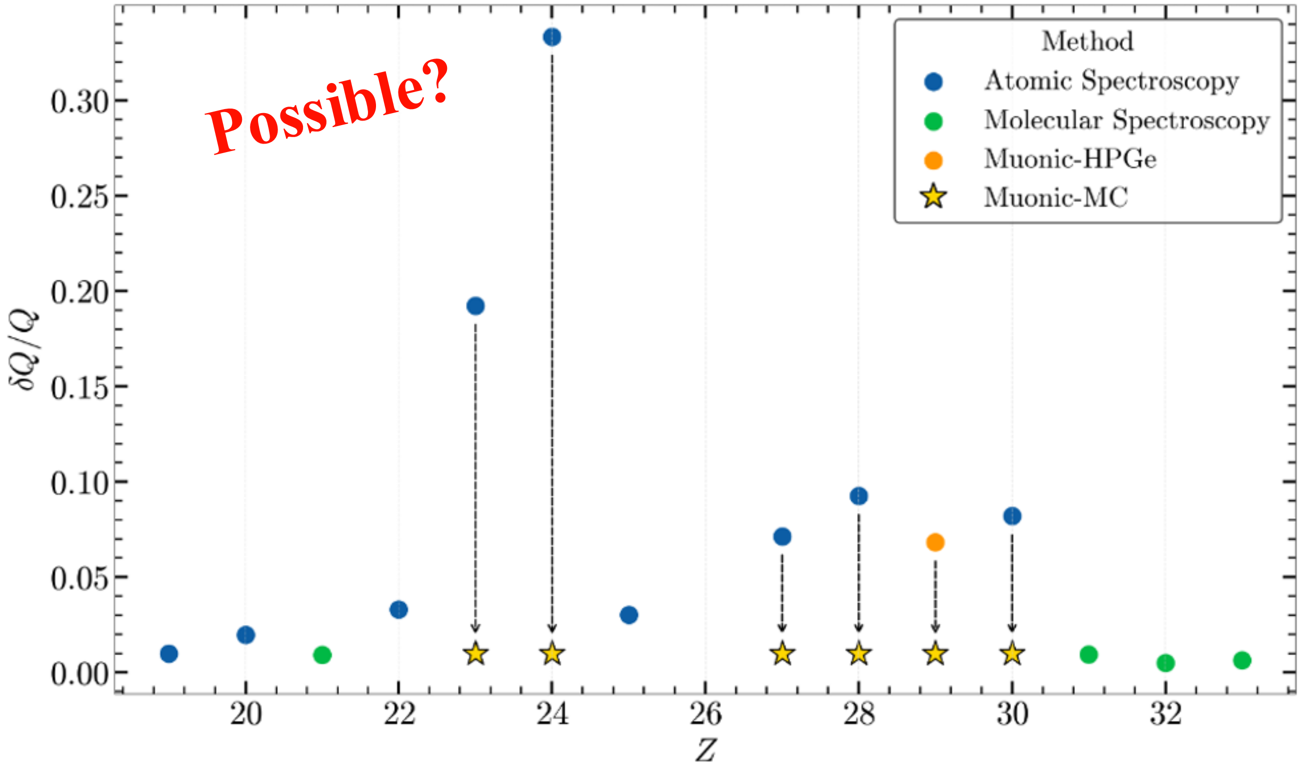

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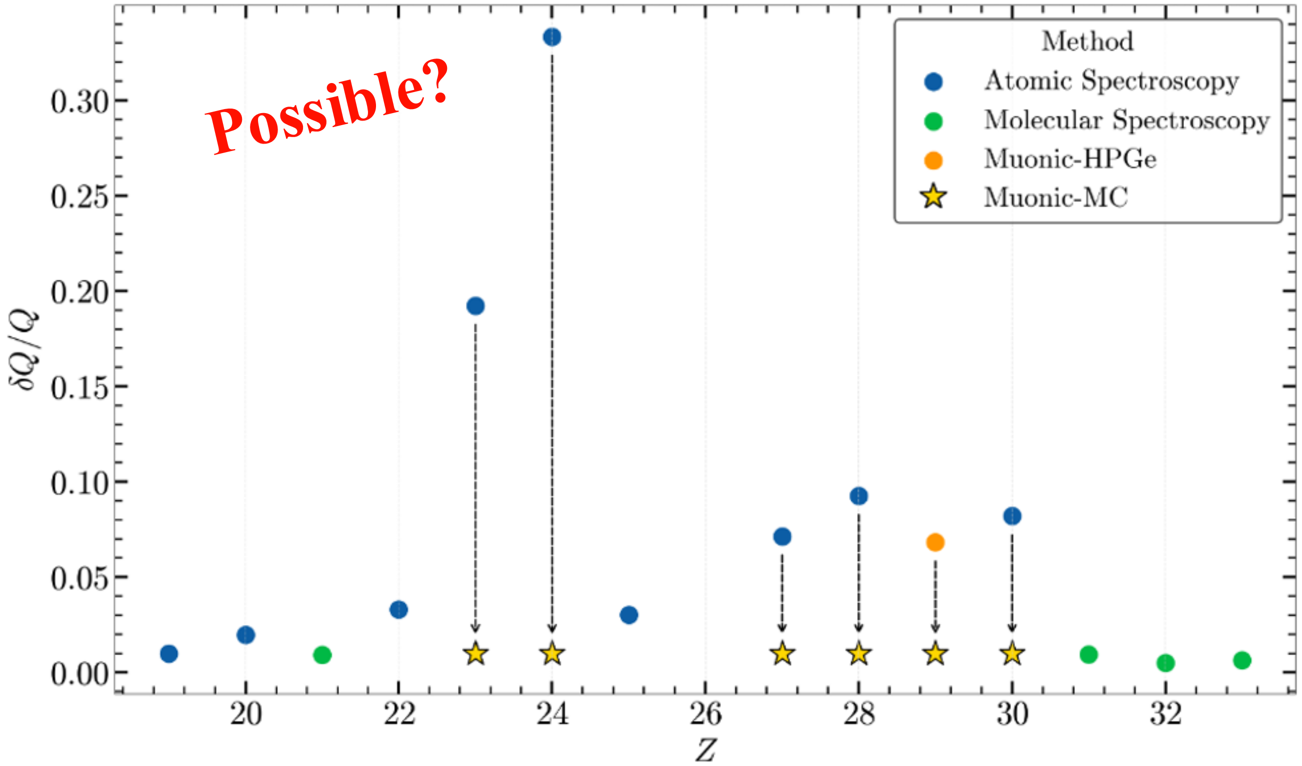
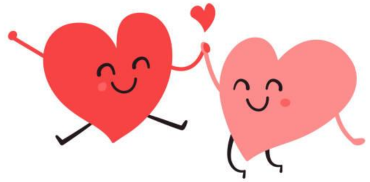


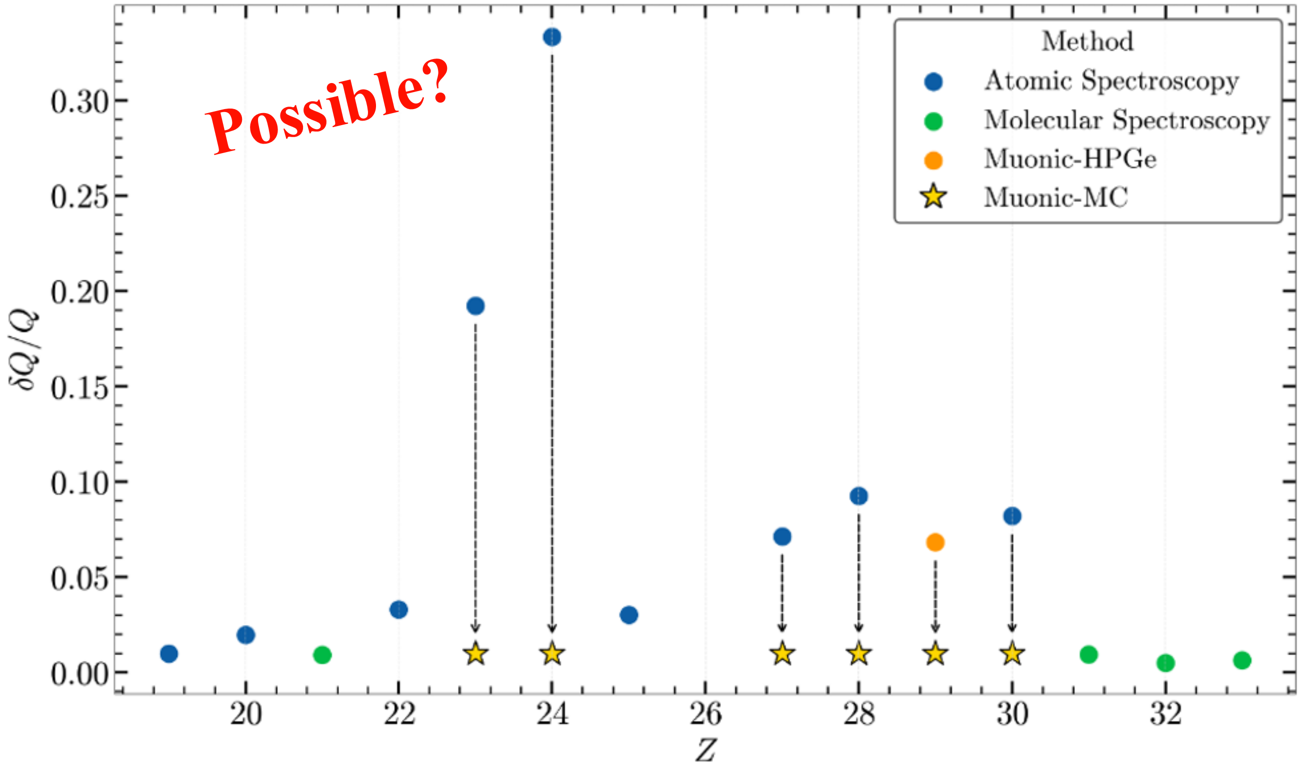


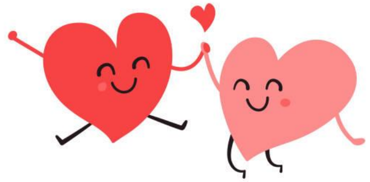
Muonic atoms
+
Microcalorimeters

QUARTZ AIM:

To improve the precision in the reference Q_{ref} of mid-Z transition elements

A technique being exploited by QUARTET and HEATES





Muonic atoms
+
Microcalorimeters

Concept:

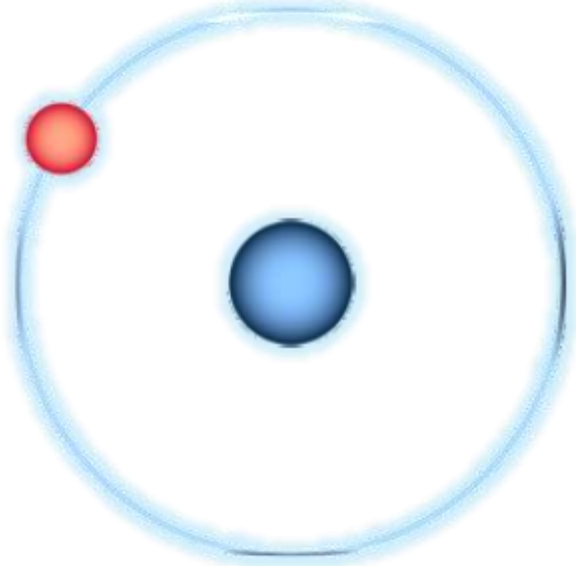
In muonic atoms:

Enhanced sensitivity to nuclear effects

Hyperfine interaction;

Spectroscopic Electric Quadrupole coupling $\propto \left(\frac{m_u}{m_e}\right)^3$

(Exists only when nuclear spin (I) ≥ 1 & $J \geq 1$)



Muonic ≈ Hydrogen-like atoms => EFG is strong and can be calculated with high precision

Concept:

Range of transition energy in the selected elements:

$2p \rightarrow 1s$	$2s \rightarrow 2p$	$3d \rightarrow 2p$
$\approx$ MeV	$\approx$ tens of keV	$\approx$ hundreds of keV

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$2p \rightarrow 1s$	$2s \rightarrow 2p$	$3d \rightarrow 2p$
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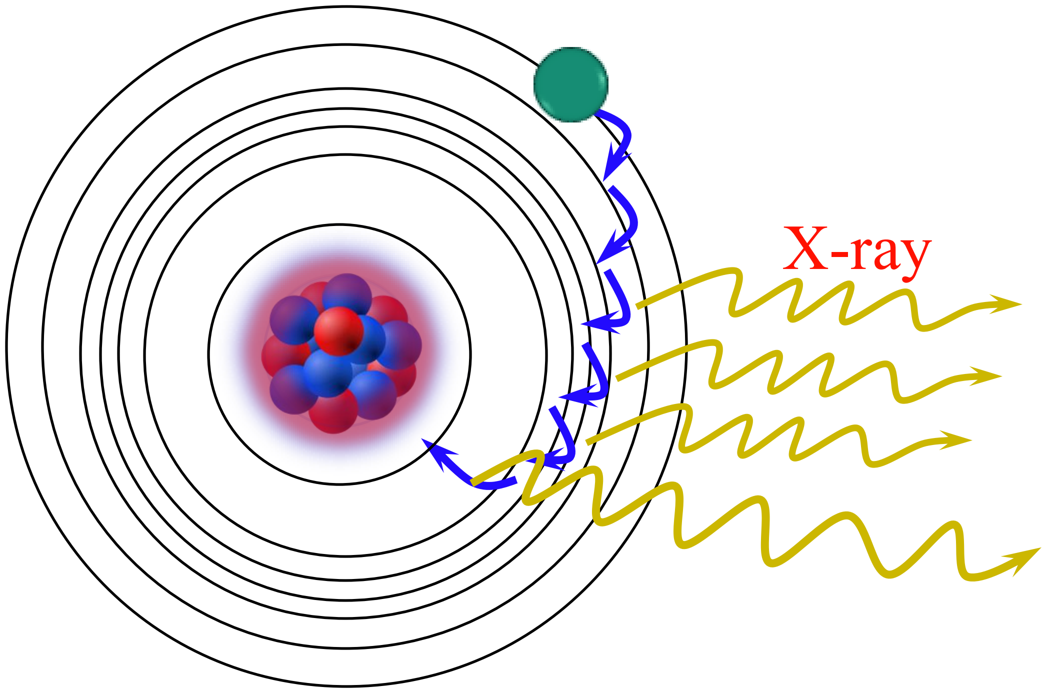
Enhanced nuclear coupling effects and transitions suitable to benefit from MC

Suitable energy range for our experimental setup (detector)

Concept:



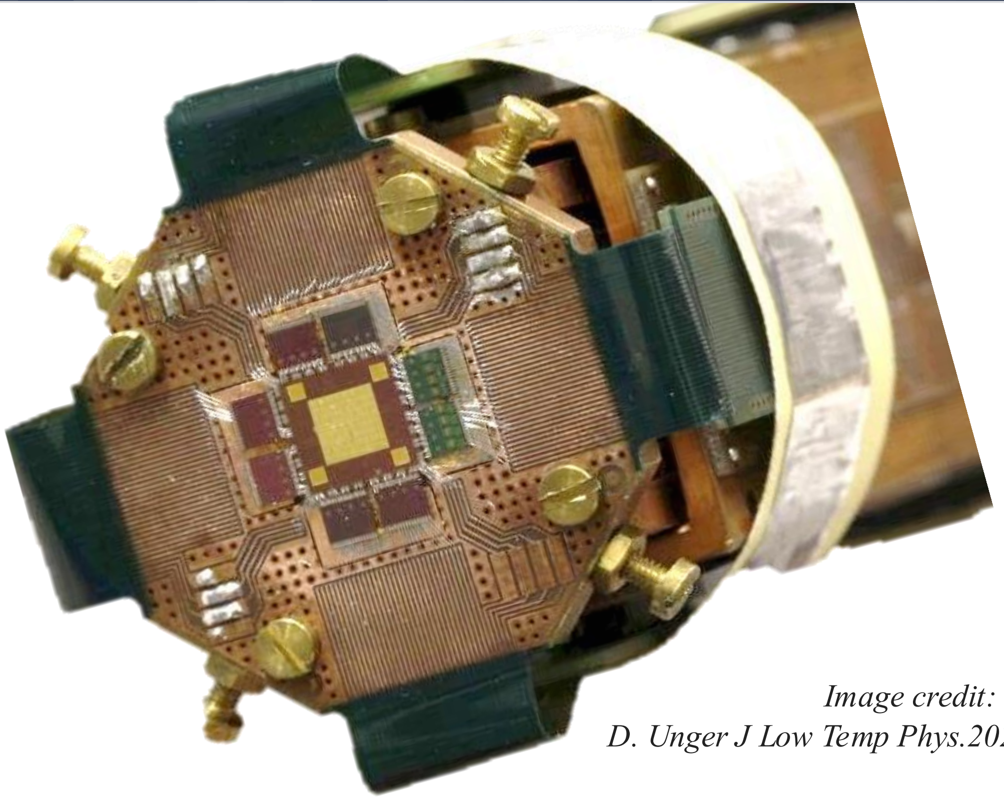


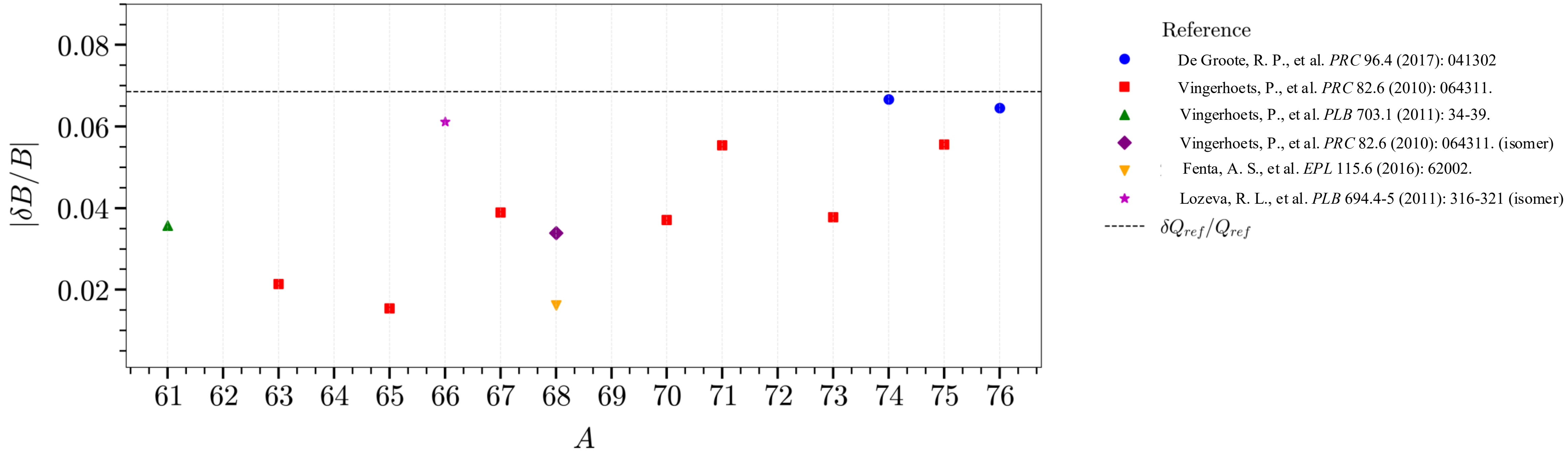
Image credit:
*D. Unger J Low Temp Phys.2024;***216**, 344-351

Muonic Atom ($2s \rightarrow 2p$) + MC: To determine precise Q_{ref} is possible

Potential Impact:

Example of Copper isotope and isomers measurements;

cases where ***B*** is measurement with more accurate compared to $\boldsymbol{Q_{ref}}$



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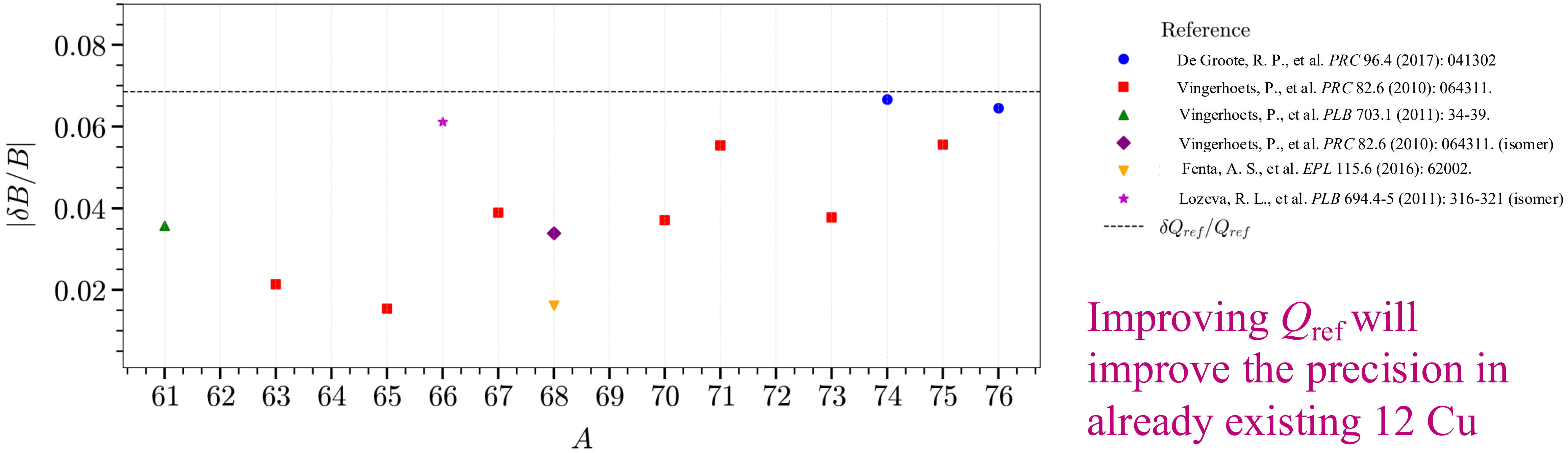


Improving Q_{ref} will improve the precision in already existing 12 Cu isotopes/isomers Q values

Theory Inputs:

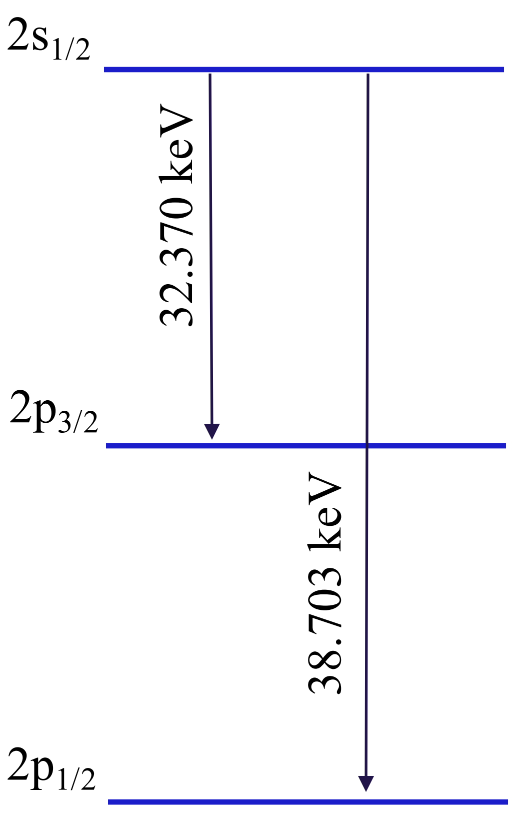
$$\delta E_{FS} = \delta E_{Dirac} + \delta E_{QED} + \delta \mathrm{E}_{TPE} + \delta E_{\mathrm{recoil}}$$

$$E_{HFS} = E_{M1} + E_{E2}$$

- QED (Vacuum Polarization) Correction
- Hyperfine Mixing
- Nuclear Polarization Correction (Dynamic Hyperfine Mixing)

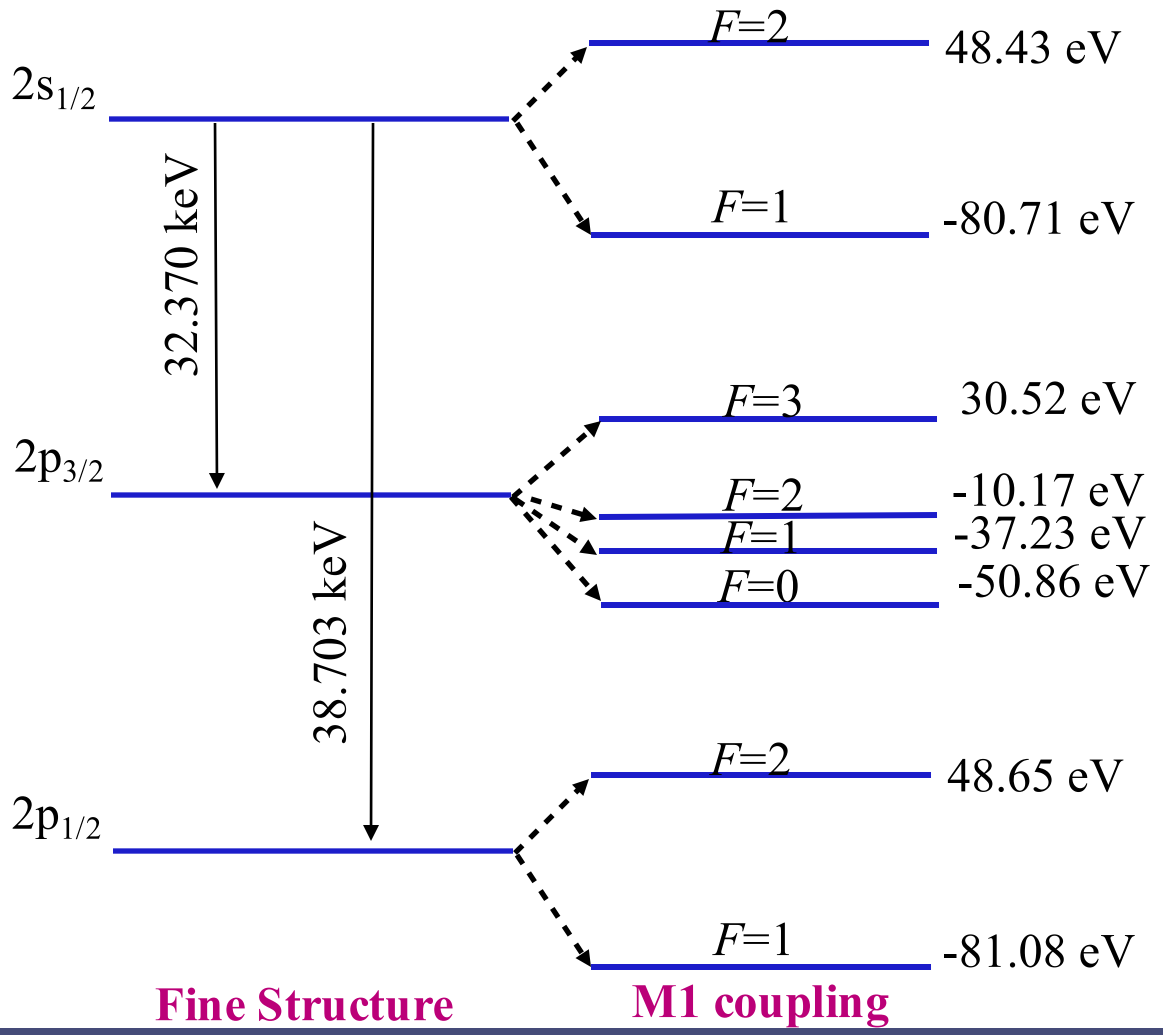
$M1$: *Magnetic dipole coupling*
$E2$: *Electric quadrupole coupling*

Case Study ^{63}Cu:



Fine Structure

Case Study ^{63}Cu:



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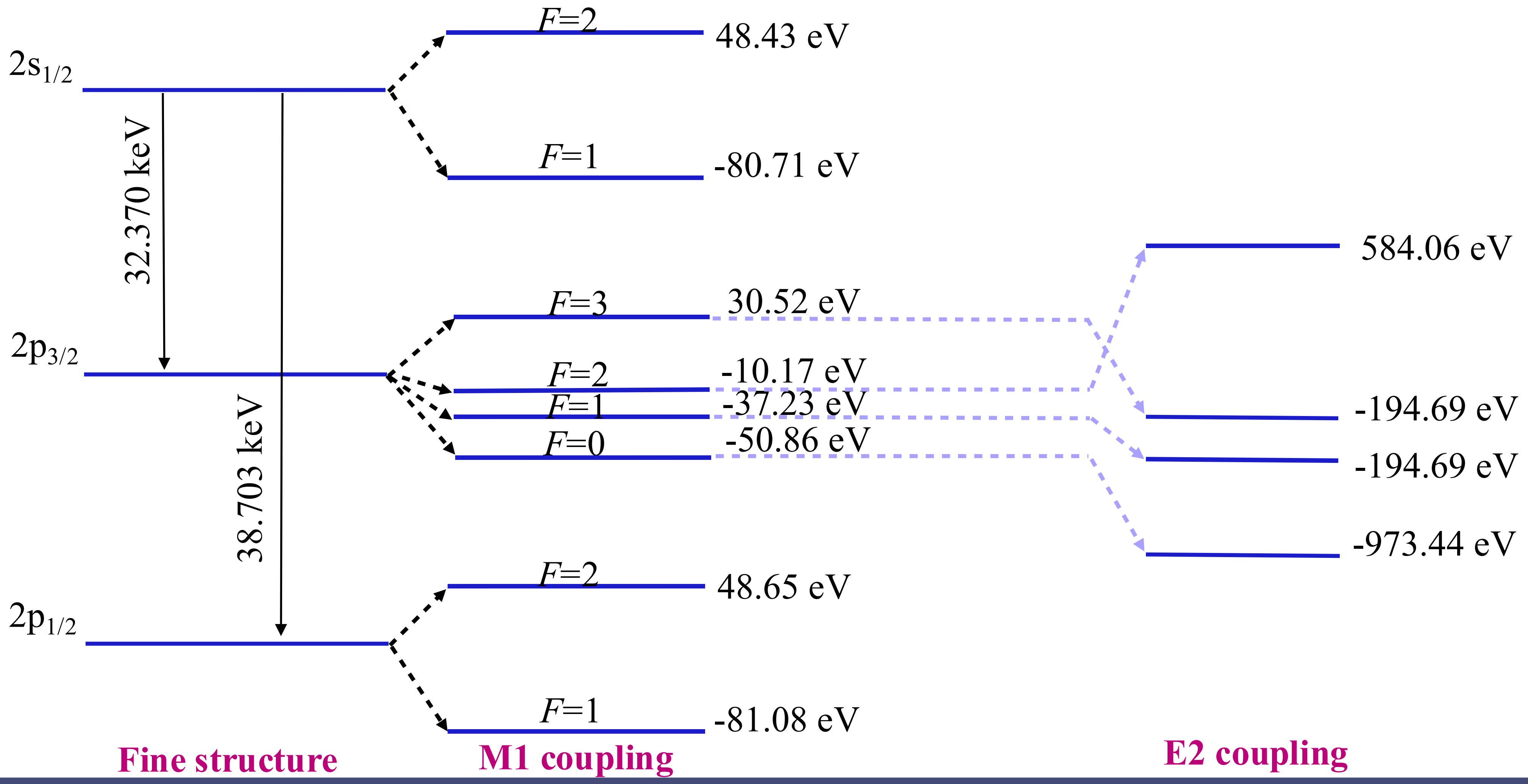

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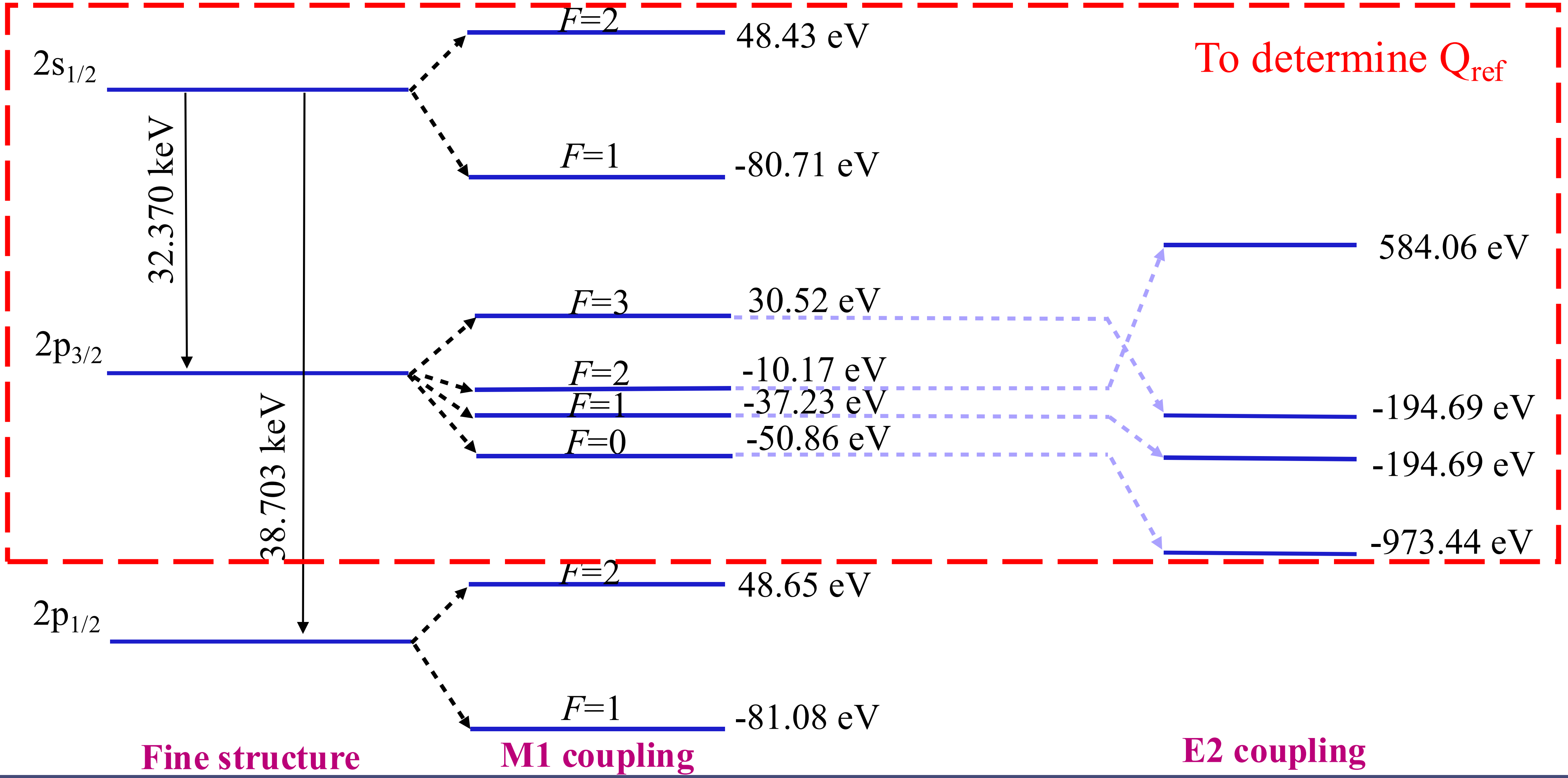

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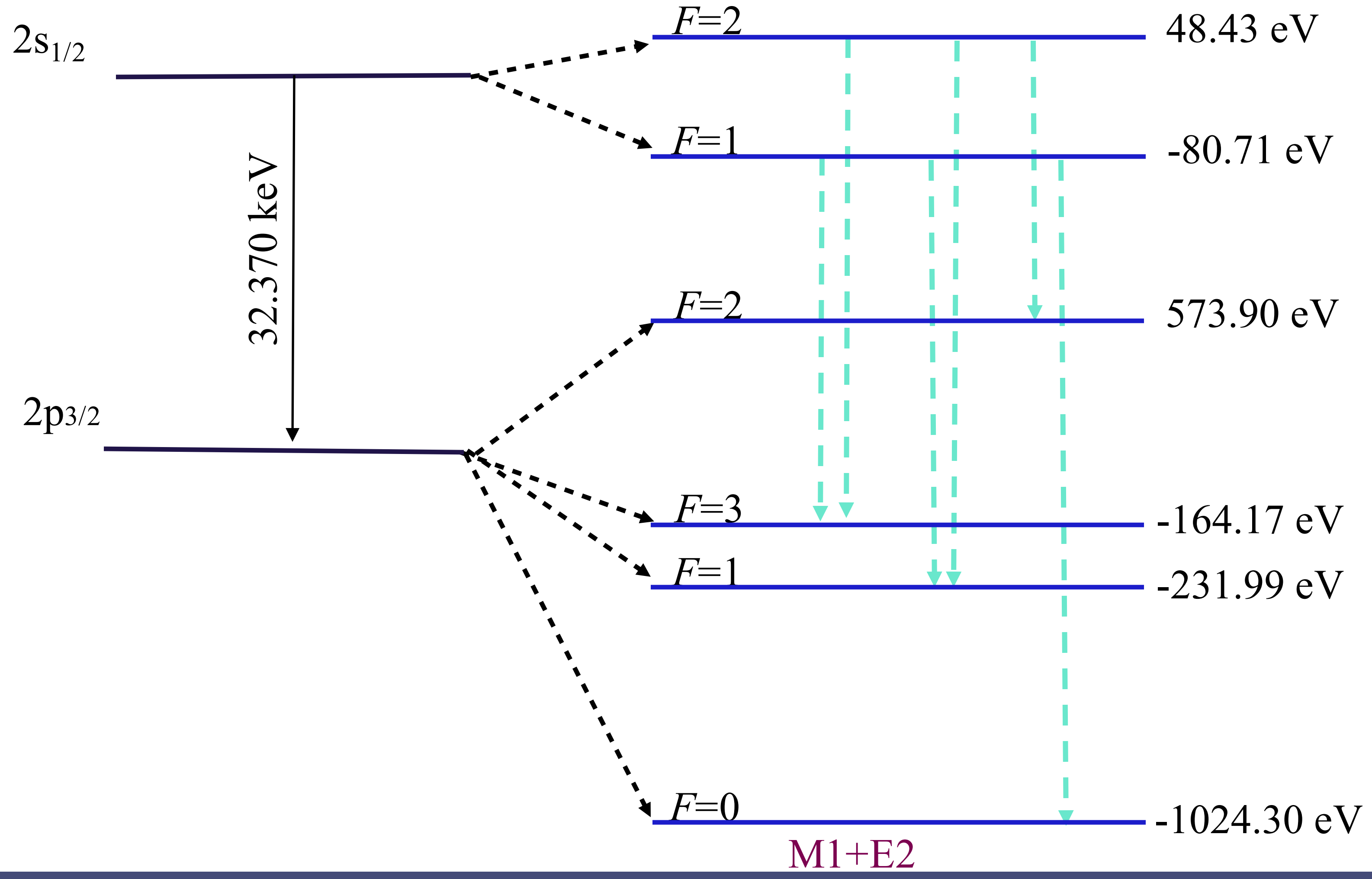
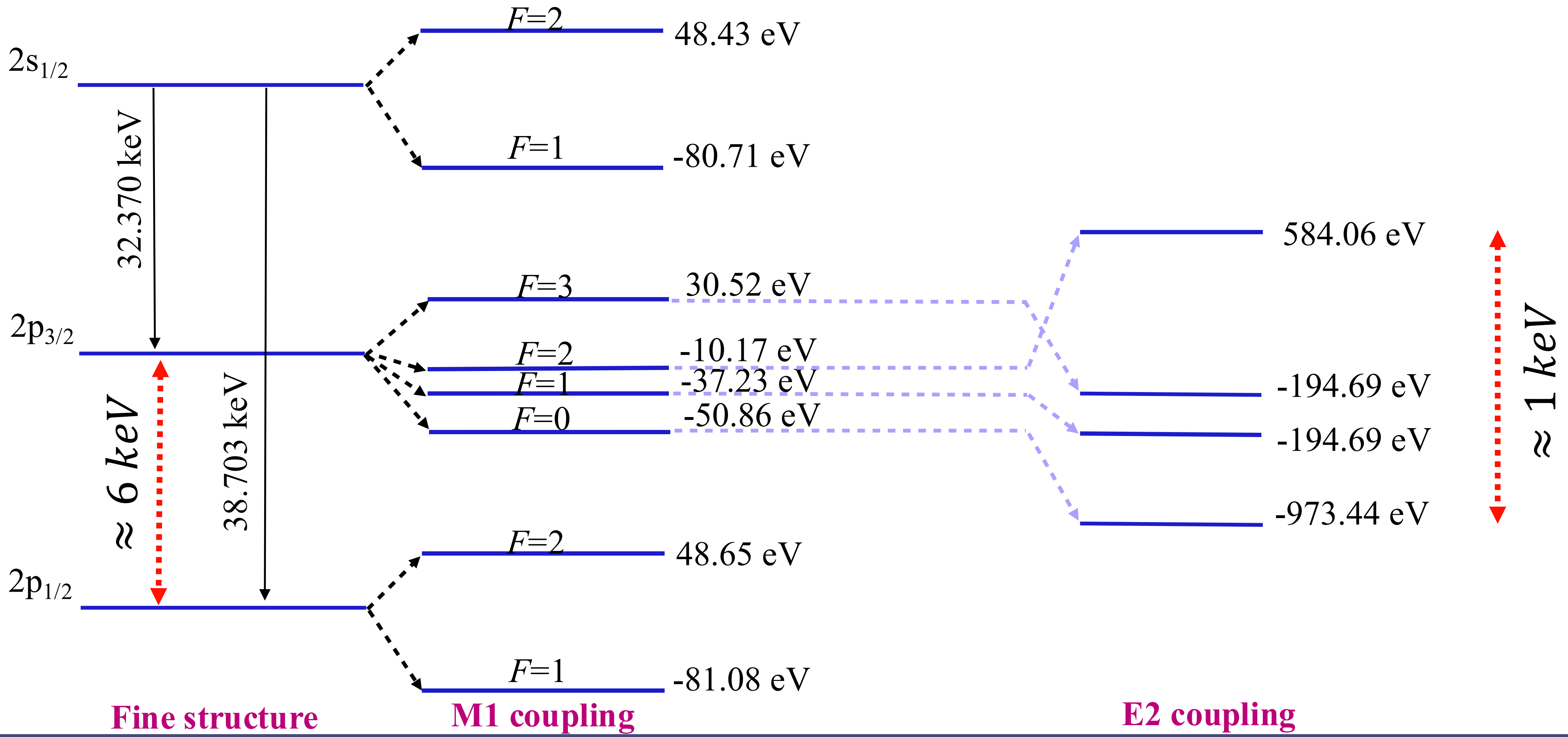


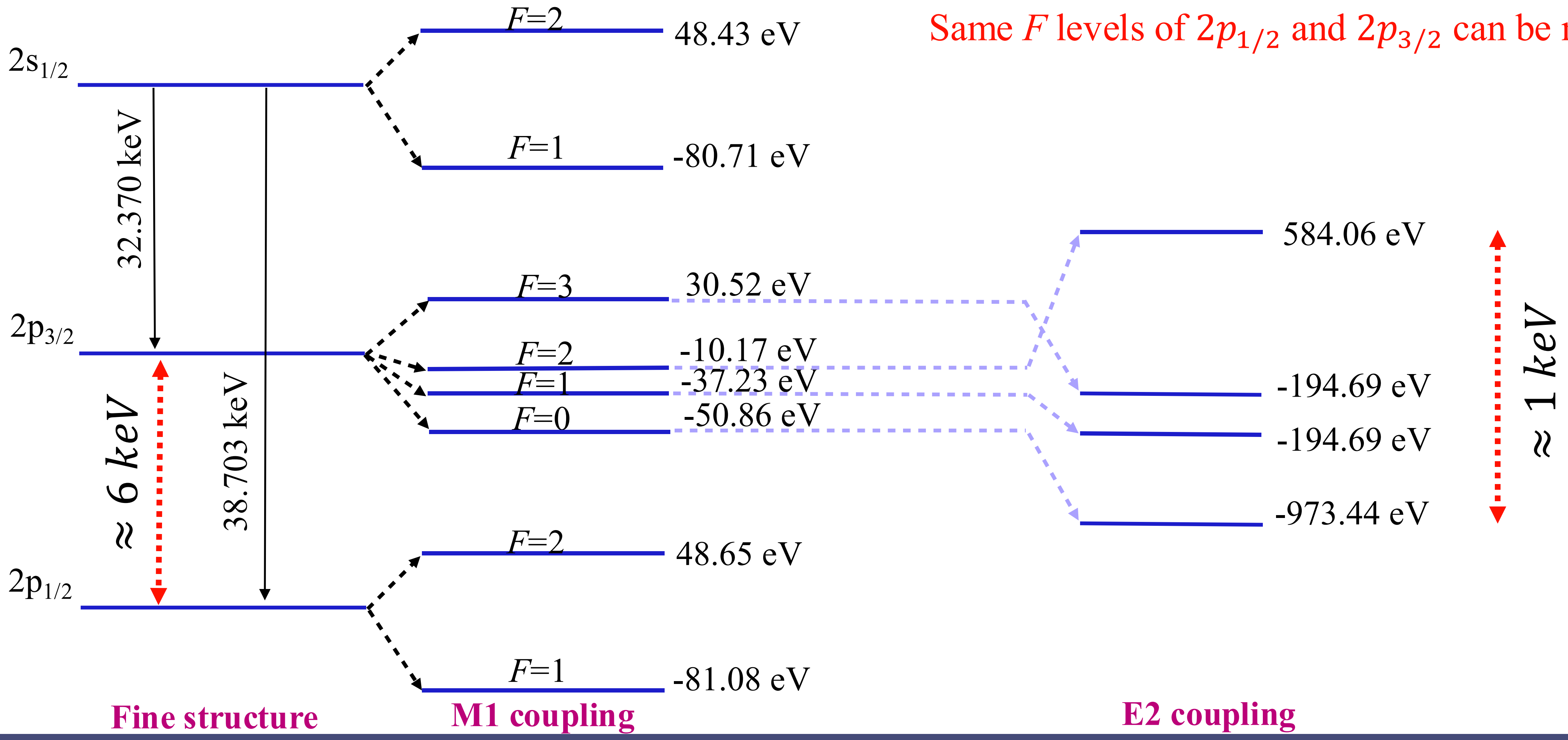
6 hyperfine transitions

Hyperfine Mixing ^{63}Cu:



Hyperfine Mixing ^{63}Cu:

Same F levels of $2p_{1/2}$ and $2p_{3/2}$ can be mixed



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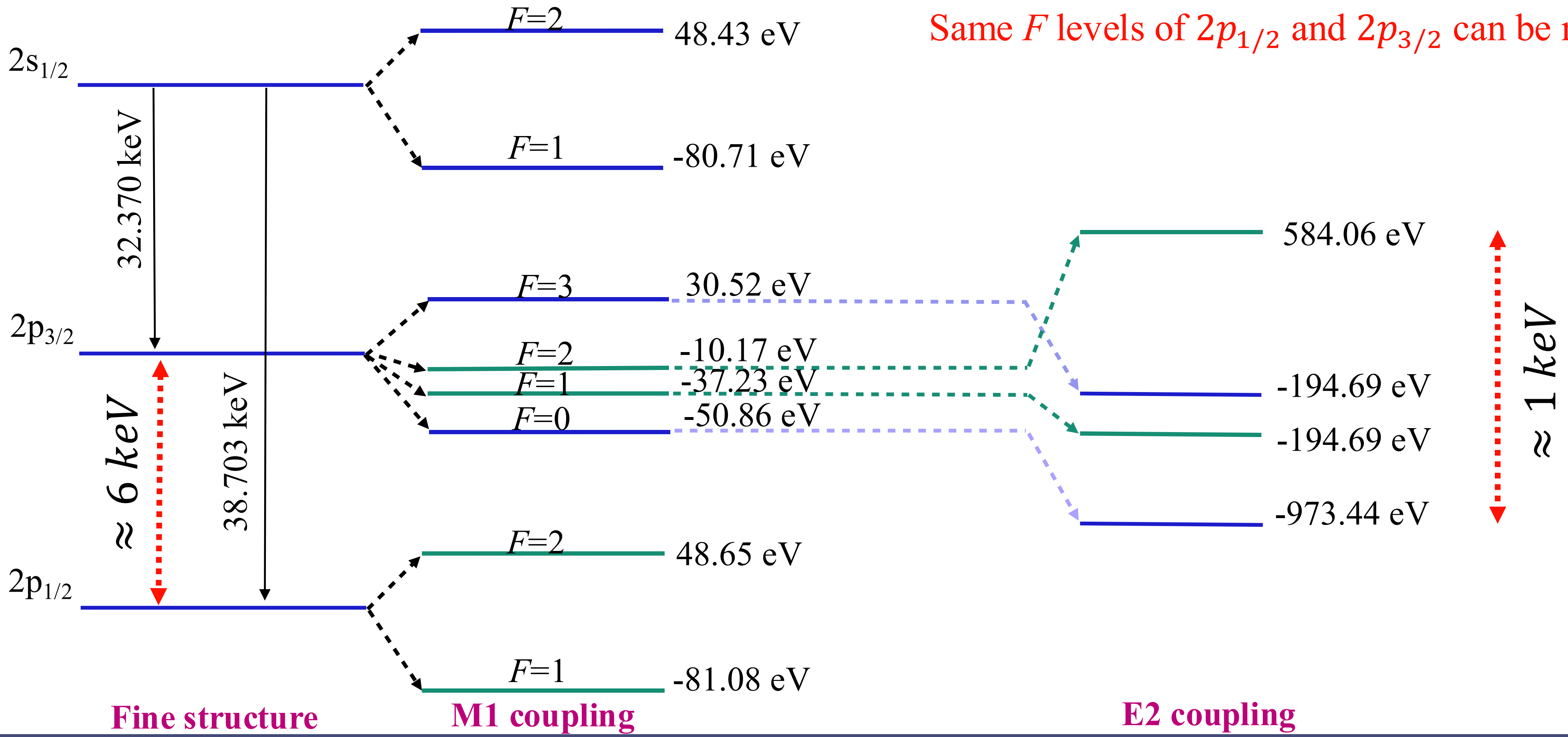


Hyperfine Mixing ^{63}Cu:

$H_{HFS}^{(1)}$: Magnetic Dipole Interaction
$H_{HFS}^{(2)}$: Electric Quadrupole Interaction

Diagonal: $< J\ P \mid H_{HFS}^{(1)} \mid J\ P >, \; < J\ P \mid H_{HFS}^{(2)} \mid J\ P >$

Off-diagonal: $< J'\ P \mid H_{HFS}^{(1)} \mid J\ P >, \; < J'\ P \mid H_{HFS}^{(2)} \mid J\ P >$

$$\begin{pmatrix} < J|H_{HFS}^{(1)}|J > + < J|H_{HFS}^{(2)}|J > + E_{J\,FS} & < J|H_{HFS}^{(1)}|J' > + < J|H_{HFS}^{(2)}|J' > \\ < J'|H_{HFS}^{(1)}|J > + < J'|H_{HFS}^{(2)}|J > & < J'|H_{HFS}^{(1)}\left|J' > + < J'|H_{HFS}^{(2)}\right|J' > + E_{J'FS} \end{pmatrix}$$

Re-diagonalization of the Hamiltonian

$\Downarrow$

Eigenvalues and eigenvectors as new energies and mixing coefficients

Case Study ^{63}Cu:

Theoretical information about the hyperfine line

Positions ✓

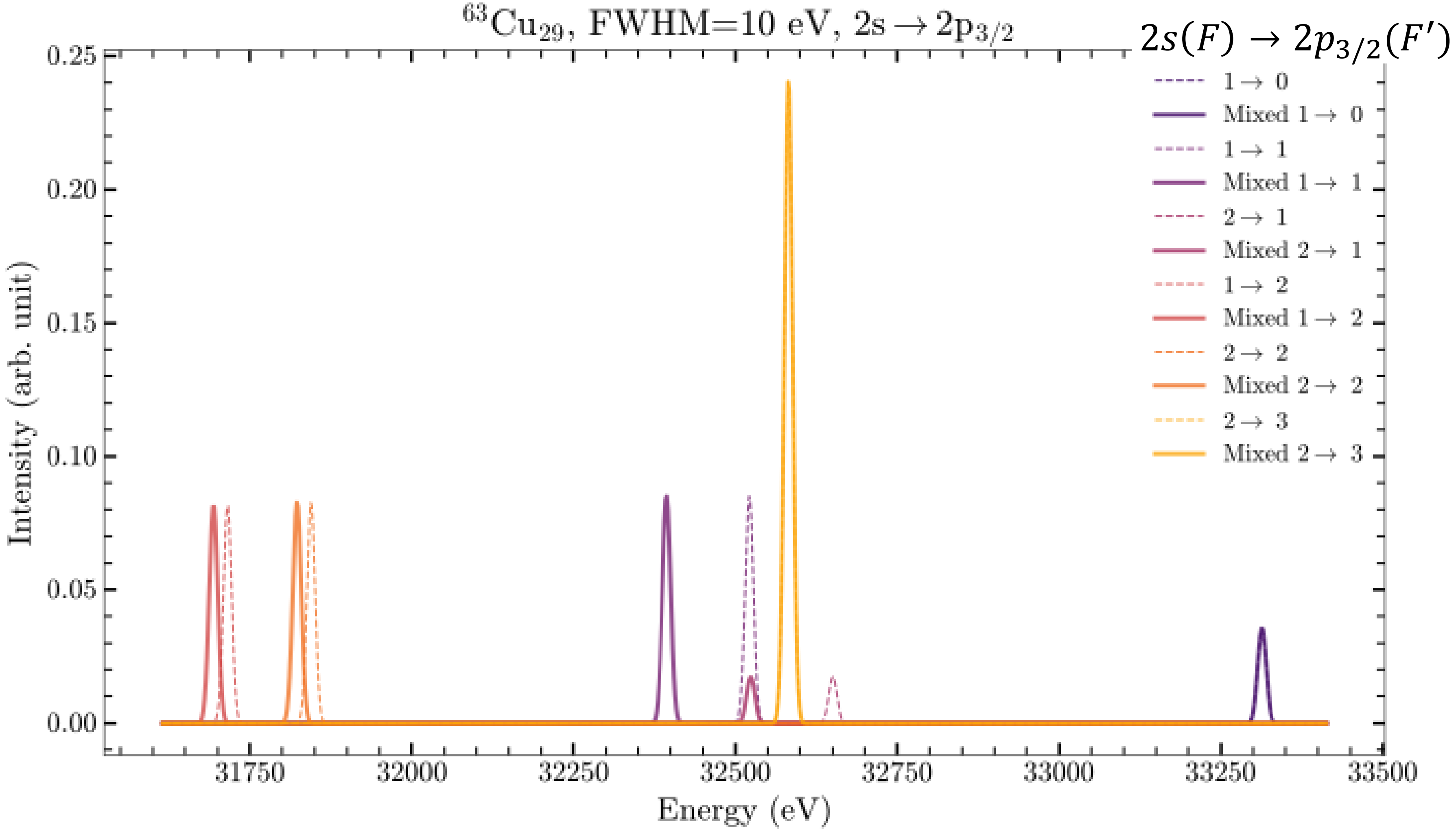
Natural width ✓

Intensities ✓

Detector: $\mathrm{FWHM}(eV)^{*} = 9.9 + 2 \times 10^{-4}\ \mathrm{E}^2$ (E in keV)

* Sikorsky, Tomas, et al. *Physical Review Letters 125.14 (2020): 142503.*

Theoretically simulated spectra of ^{63}Cu:



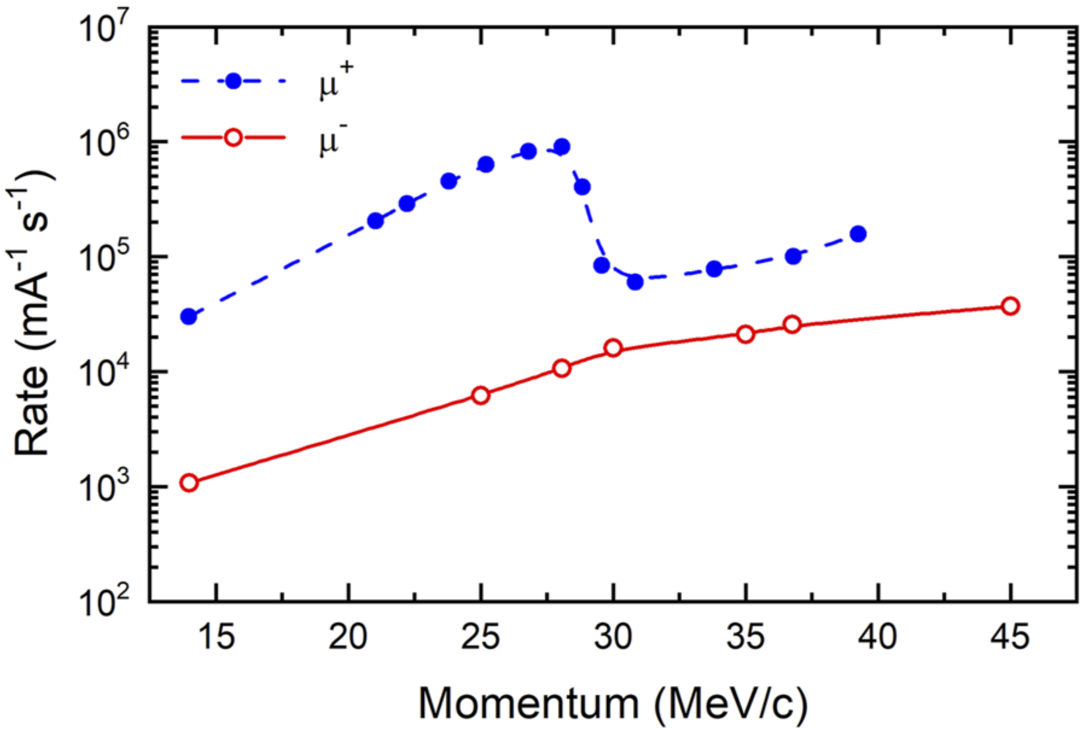
Checking feasibility of ^{63}Cu Q Measurement:

To simulate the signal count rate

- PiE1 Beamline: Negative Muon Rate vs. Momentum
- Implantation Depth vs. Muon Momentum
- X-Ray Transmission through Target
- Solid Angle: Geometrical acceptance considerations
- Detector's Quantum Efficiency
- Background
- Branching Fraction $2s \rightarrow 2p_{3/2}$
- *2s* Population

Checking feasibility of ^{63}Cu Q Measurement:

PiE1 Beamline: Negative Muon Rate vs. Momentum



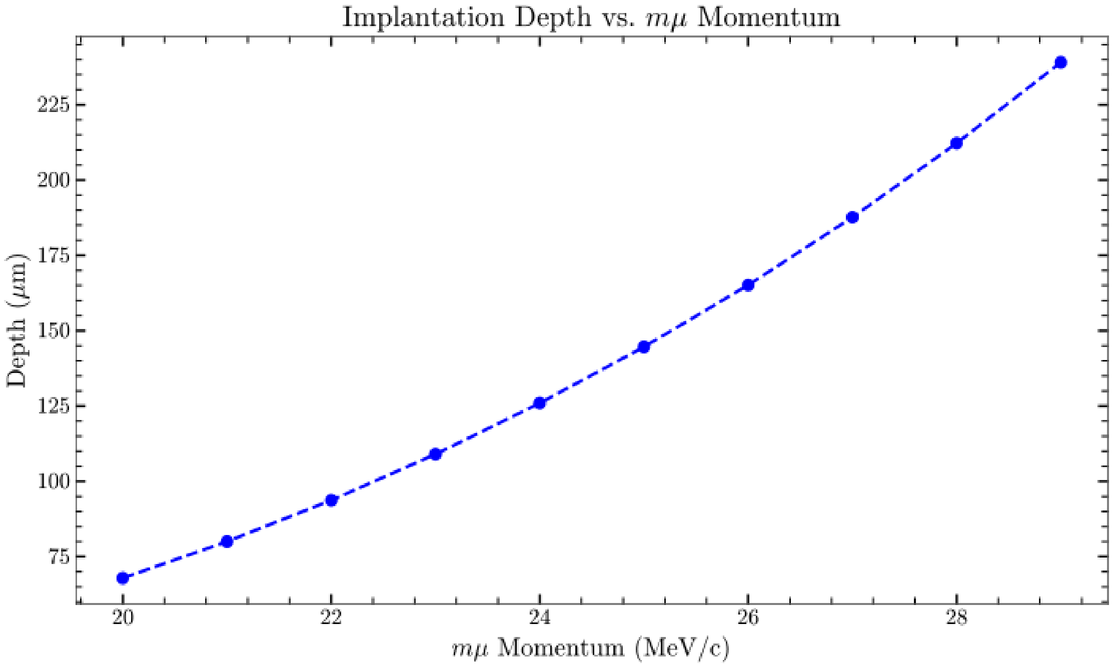
Current $\approx 1\ mA^{-1}$

L. Gerchow et.al. Rev. Sci. Instrum. 94, 045106 (2023)

Checking feasibility of ^{63}Cu Q Measurement:

Geant4 Simulations

Implantation Depth vs. Muon Momentum



Solid Angle

≈ 1E-04

2s Population

≈ 0.03 (Vogel Cascade Code)

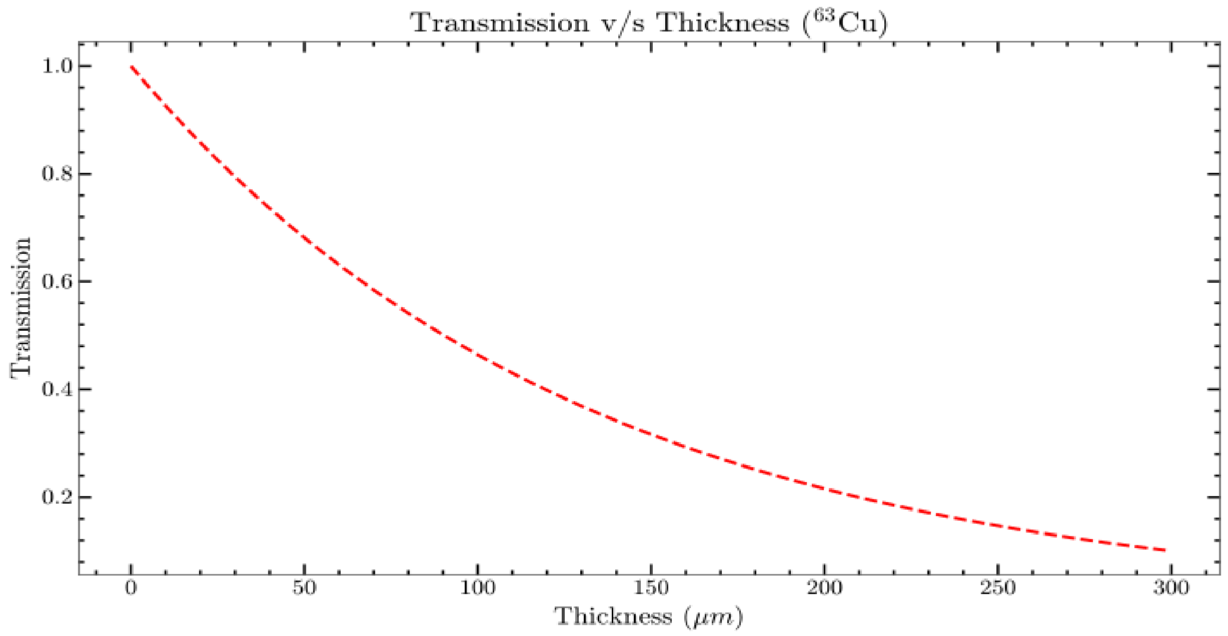
Branching Fraction
$2s \rightarrow 2p_{3/2}$

≈ 0.26 of *2s* population

(Knyazeva, V. A., *et al. Phys. Rev. A* 106.1 (2022): 012809)

Checking feasibility of ^{63}Cu Q Measurement:

X-Ray Transmission through Target



Detector's Quantum Efficiency

≈ 0.9 (Quartet Progress report)

Checking feasibility of ^{63}Cu Q Measurement:

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PiE1 Beamline: Negative Muon Rate vs. Momentum

Implantation Depth vs. Muon Momentum

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Solid Angle: Geometrical acceptance considerations

Detector's Quantum Efficiency

Background

2s Population

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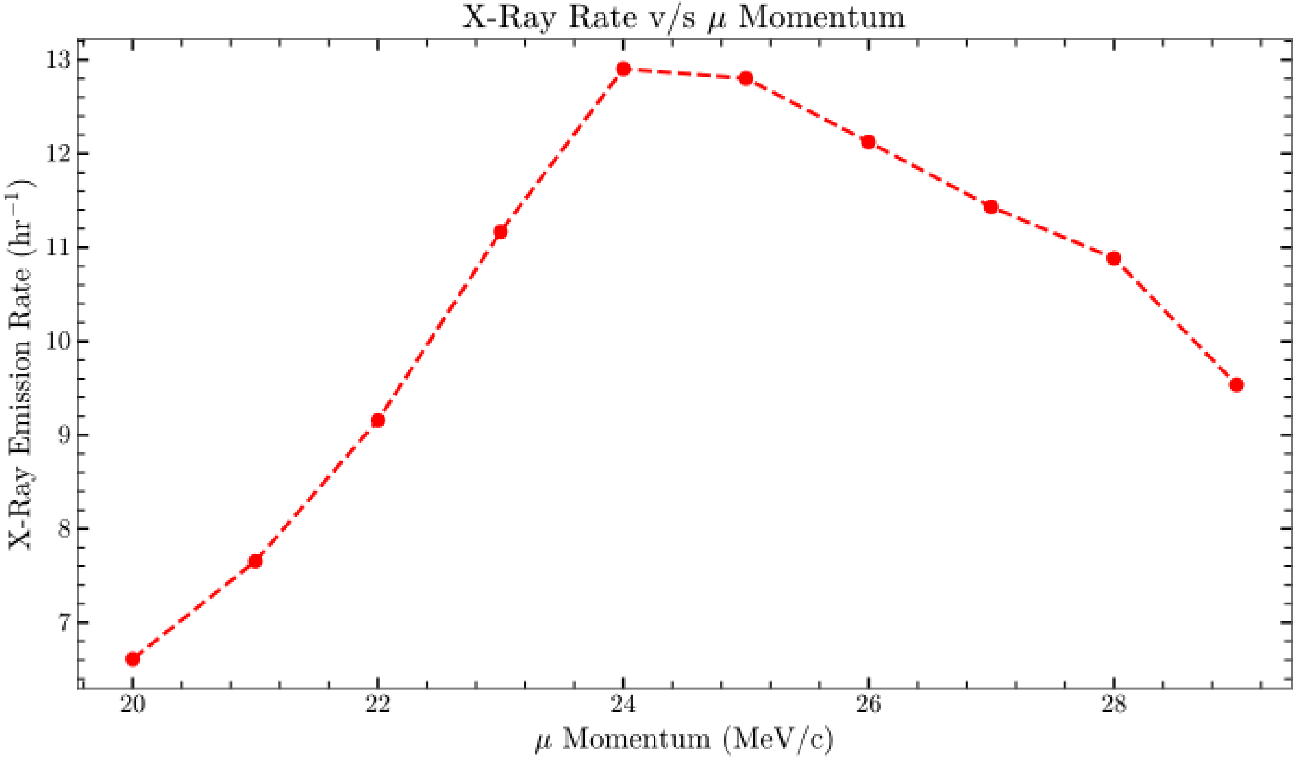
Checking feasibility of ^{63}Cu Q Measurement:

$$\boldsymbol{Photon\ emission\ rate} = Muon\ rates\ (s^{-1}mA^{-1})\ \times Current(mA) \times transmission\ \times QE\ \times 2s\ fraction\ \times BF_{2s\to 2p_{3/2}} \times Solid\ angle$$

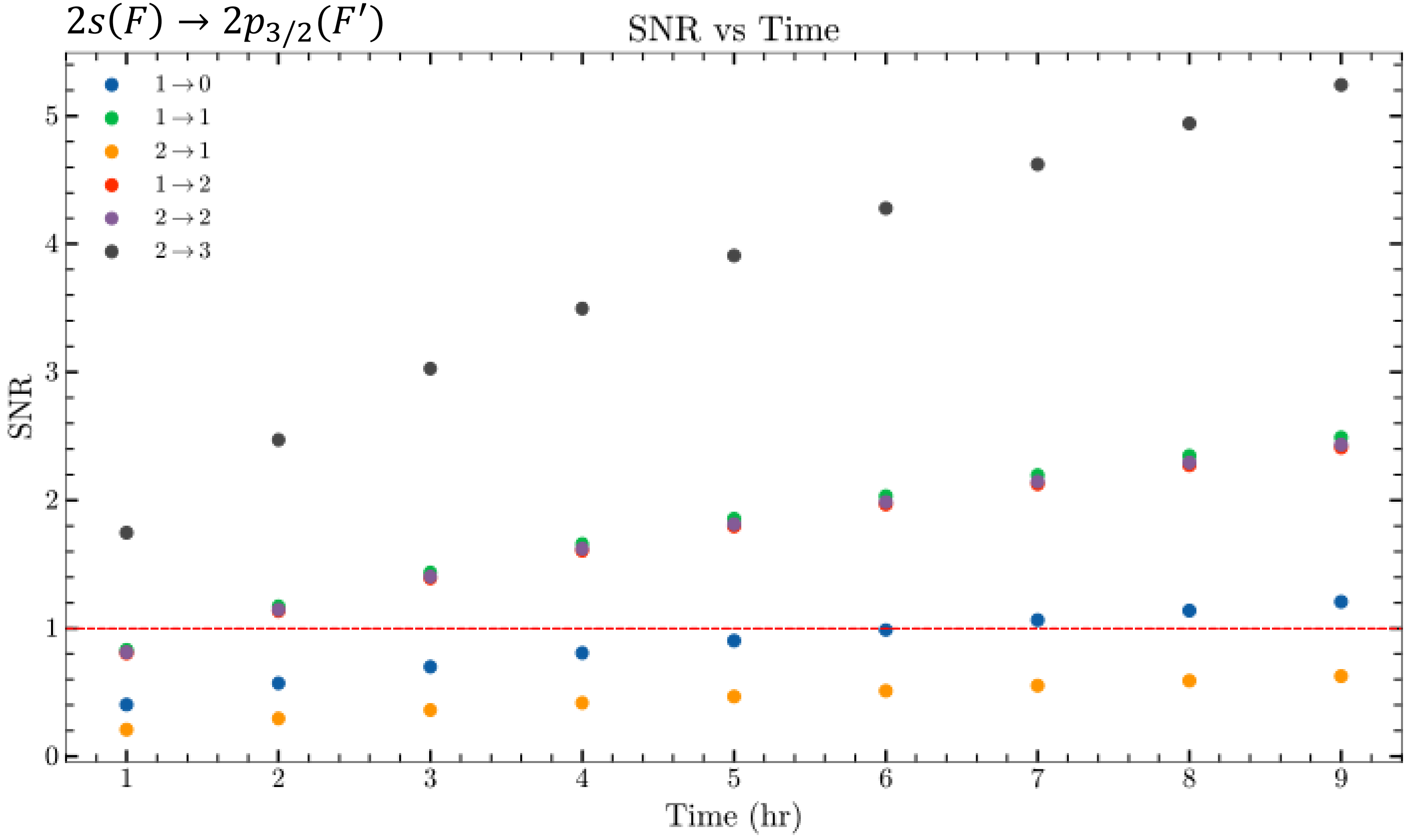


Remember:
Distribution among the 6 HFS line

Checking feasibility of ^{63}Cu Q Measurement:

Estimated time of the measurement:

$2s(F) \rightarrow 2p_{3/2}(F')$



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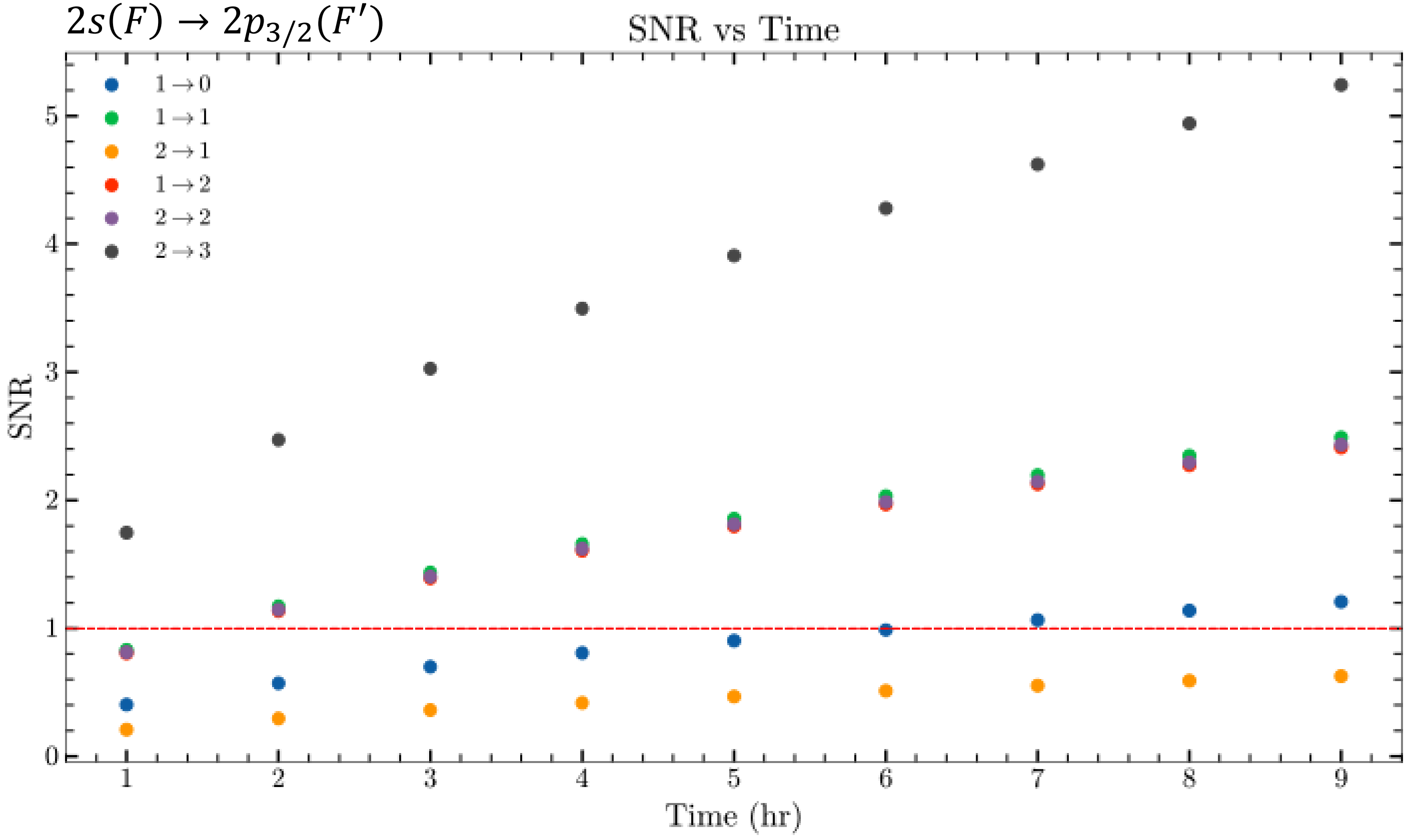

Possible Measurements

Thank You,

Comments and Questions?