SU(2) gauge theory with N_f=2 fundamental flavour: A minimal template for model building ?

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in collaboration with

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Interdisciplinary approach to QCD-like composite dark matter

ECT*, Trento, 4th October 2018

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Outline

- Introduction
- Benchmark Results :
  - ★Setup
  - ★Spectrum
- New Results :
  - ★Scattering properties and resonances width
- Summary / Outlook

Introduction

Strongly Interacting Dark Matter

- Dark Matter is essential to our current understanding of the Universe evolution

- Properties :
  - ★ Long-lived
  - ★ Electrically neutral
  - ★ Interaction with the Standard Model are suppressed

Why the Lattice ?

✦Lattice simulations provide insights in many strongly coupled theories

✦The lattice can provide information on the dark sector in isolation:

- ★ Low-lying spectrum
- ★ Matrix element relevant for direct detection
- ★ Production cross section ?
- ★ Self interactions ?

✦Price to pay:

- ★The uv completion needs to be fixed.

Motivations to study SU(2) with Nf=2

- ✦Our main original motivation: Composite Higgs / Technicolor framework [Cacciapaglia & Sannino 2014]
- ✦Light asymmetric Dark Matter [Lewis et al.]
- ✦SIMP mechanism [Hochberg et al.]

SU(2)$_C$ with N$_f$=2 fundamental Dirac flavours

- SU(2) gauge theory with N_f = 2 Dirac fermions in the fundamental representation.

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{U}\gamma^\mu D_\mu U + i\overline{D}\gamma^\mu D_\mu D + \frac{m}{2}Q^T(-i\sigma^2)C\,EQ + \frac{m}{2}\left(Q^T(-i\sigma^2)C\,EQ\right)^\dagger$$

- Pseudo-real irrep of SU(2): **global flavour symmetry is upgraded to SU(4)** :

$$Q \equiv \begin{pmatrix} U_L \\ D_L \\ \widetilde{U}_L \\ \widetilde{D}_L \end{pmatrix} \equiv \begin{pmatrix} U_L \\ D_L \\ -i\sigma_2 C\bar{u}_R^T \\ -i\sigma_2 C\bar{d}_R^T \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

- Infinitesimal SU(4) transformation : $Q \longrightarrow \left(1 + i\sum_{n=1}^{15}\alpha^n T^n\right)$

- Generators that leaves the Lagrangian invariant satisfy : $ET^n + T^{nT}E = 0$

- Chiral symmetry breaking pattern : ***SU(4) -> Sp(4)*** ***(SO(6) -> SO(5))***

  (5 Goldtone bosons)

- Wess-Zumino-Witten term allowed (3->2 vertex in EFT)

Hadronic operators

✦Meson operators:

$$\mathcal{O}^{(\Gamma)}_{\overline{U}D} \equiv \overline{U}(x)\Gamma D(x) \,,$$

$$\mathcal{O}^{(\Gamma)}_{\overline{D}U} \equiv \overline{D}(x)\Gamma U(x) \,,$$

$$\mathcal{O}^{(\Gamma)}_{\overline{U}U\pm\overline{D}D} \equiv \frac{1}{\sqrt{2}}\Big(\overline{U}(x)\Gamma U(x) \pm \overline{D}(x)\Gamma D(x)\Big) \,,$$

✦Baryon operators:

$$\mathcal{O}^{(\Gamma)}_{UD} \equiv U^T(x)(-i\sigma^2)C\Gamma D(x)$$

$$\mathcal{O}^{(\Gamma)}_{DU} \equiv D^T(x)(-i\sigma^2)C\Gamma U(x)$$

✦Large euclidean time behaviour :

$$\sum_{\mathbf{x}} \langle O(\mathbf{x},t)O(\mathbf{0},0)\rangle = \sum_n \langle 0|O|n\rangle\langle n|O|0\rangle e^{-m_n t}$$

What do we know ?

- Most notably : spectrum investigations at non vanishing fermions mass
  - ★Goldstone Bosons
  - ★Vector and Axial mesons
  - ★Scalar and Pseudo-scalar mesons
  - ★Pseudo-scalar decay constant

- In this talk :
  - ★Review spectrum predictions
  - ★Goldstone Boson scattering and the vector meson resonance

- Can we bring useful constraints in the context of :
  - ★Dark pion produced by through Dark vector mesons
  - ★SIMP mechanism
  - ★Self-interacting DM ?

Benchmark results

The setup

R. Lewis, C. Pica, F. Sannino, Phys.Rev. D85 (2012) 014504 [arXiv:1109.3513]
A. Hietanen, C. Pica, R. Lewis, F. Sannino, JHEP 1407 (2014) 116 [arXiv:1404.2794]
A. Hietanen, C. Pica, R. Lewis, F. Sannino [arXiv:1308.4130]
R. Arthur, V.D, A. Hietanen, M. Hansen, C. Pica, F. Sannino [arXiv:1602.06559]

- Plaquette action + dynamical Wilson Fermions
- Several volumes $V=L^3 xT$ $\Rightarrow$ extrapolate to infinite volume
- 4 lattice spacings : a $\Rightarrow$ extrapolate to the continuum limit
- Several fermion masses $m_f \longleftrightarrow m_{PS}^2$ $\Rightarrow$ extrapolate to the chiral limit
- Non-perturbative renormalisation
- HiRep code

L. Del Debbio, A. Patella, C. Pica, Phys.Rev. D81 (2010) 094503

Chiral behavior : GB sector

J. Bijnens and J. Lu, JHEP **11** (2009) 116, [arXiv:0910.5424]

- Continuum χPT:

$$\frac{m_{\rm ps}^2}{m_{\rm f}} = 2B\left[1 + \frac{3}{4}x\log\frac{2Bm_{\rm f}}{\mu^2} + b_M x + \mathcal{O}(x^2)\right]$$

$$f = F\left[1 - x\log\frac{2Bm_{\rm f}}{\mu^2} + b_F x + \mathcal{O}(x^2)\right]$$

$$x = \frac{2Bm_{\rm f}}{(4\pi F)^2}$$

- Range of applicability: unknown *a priori*
- In terms of $\tilde{x} = \frac{m_{\rm ps}^2}{(4\pi F)^2}$ the expressions are unchanged at NLO
- Assess discretisation effects:
  - Strategy I:
    - modelling the discretisation effects and doing a global fit:

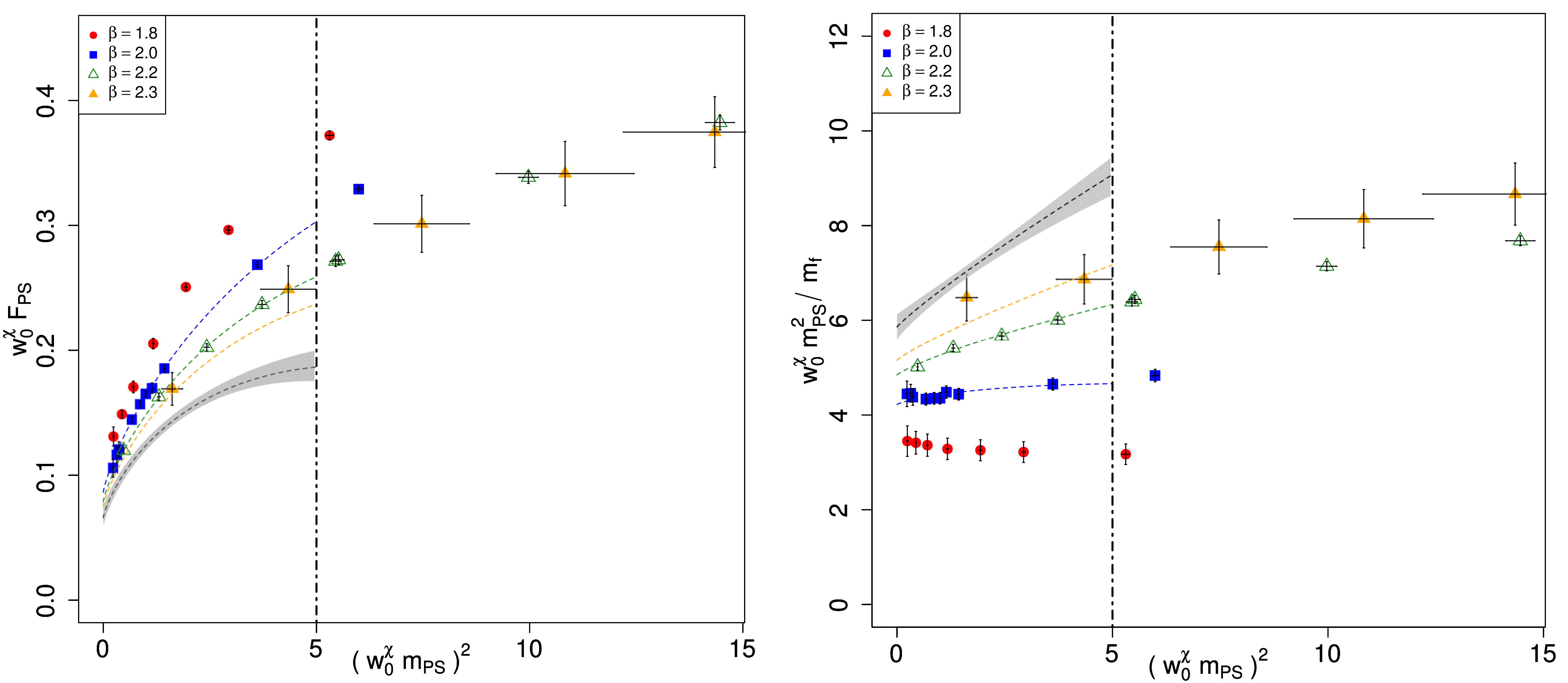
$$\frac{m_{\rm ps}^2}{m_{\rm f}} = 2B\left[1 - a_M\tilde{x}\log\frac{m_{\rm ps}^2}{\mu^2} + b_M\tilde{x} + \delta_M\frac{a}{w_0^\chi} + \gamma_M m_{\rm ps}^2\frac{a}{w_0^\chi}\right]$$

$$F_{\rm ps} = F\left[1 - a_F\tilde{x}\log\frac{m_{\rm ps}^2}{\mu^2} + b_F\tilde{x} + \delta_F\frac{a}{w_0^\chi} + \gamma_F m_{\rm ps}^2\frac{a}{w_0^\chi}\right]$$

    - Define four data subsets to control fit stability
  - Strategy II:
    - set $\delta_{M,F}$ & $\gamma_{M,F}$ to zero
    - fit each lattice spacings independently
    - study fit parameters as a function of the lattice spacing

Goldstone bosons : mass and decay constant

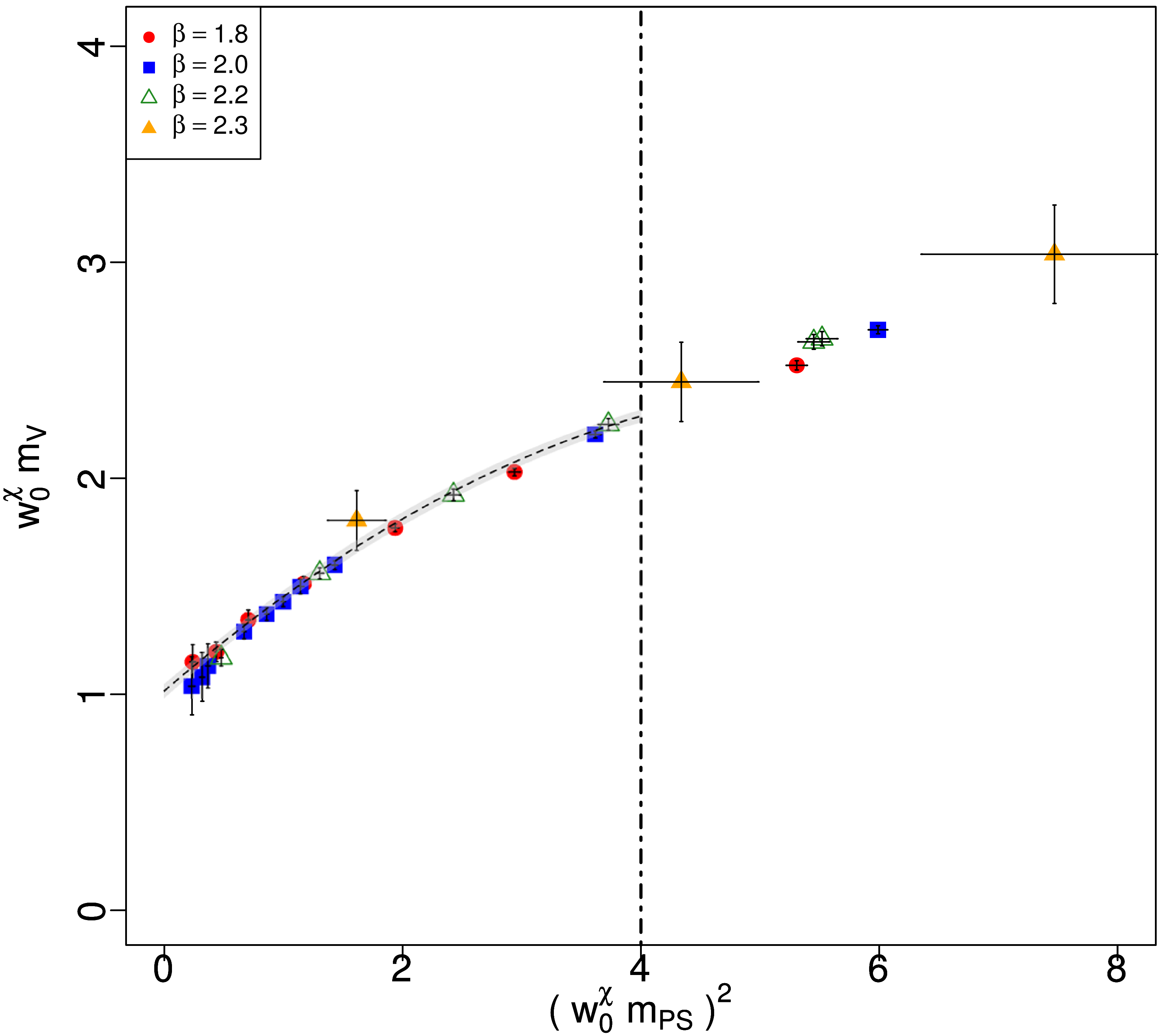
R. Arthur, VD, A. Hietanen, C. Pica, F. Sannino [arXiv:1602.06559]



- Large cut-off effects

Vector meson

R. Arthur, VD, A. Hietanen, C. Pica, F Sannino [arXiv:1602.06559]

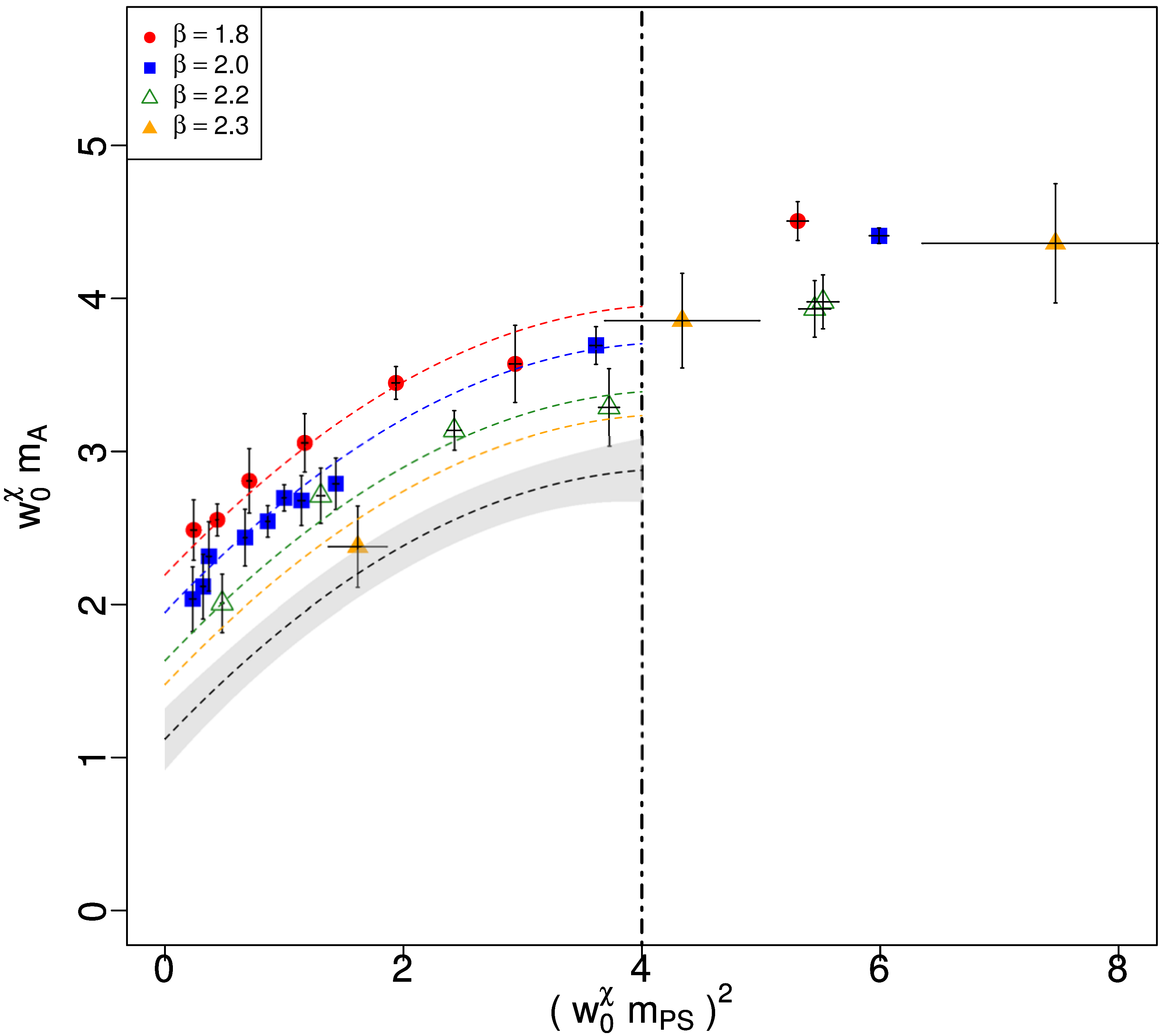


- Vector meson stable in the regime of our simulations ($m_V < 2\, m_{PS}$)
- All 4 lattice spacings are consistent
- m_V=13.1(2.2) F_{PS}

$$w_0^\chi m_X = w_0^\chi m_X^\chi + A(w_0^\chi m_{\rm ps})^2 + B(w_0^\chi m_{\rm ps})^4 + C\frac{a}{w_0}$$

Axial meson

R. Arthur, VD, A. Hietanen, C. Pica, F Sannino [arXiv:1602.06559]



- Axial meson stable in the regime of our simulations ($m_A < 2\, m_{PS}$)
- Significant cut-off effects
- m_A=14.5(3.6) F_{PS}

General remarks

- Scalar sector might be the way to discover new physics
- In the real world :
  - $m_\sigma/f_{PS} \sim 5$ and width is large
  - The ratio depends a lot on the quark mass
- Here :
  - 0++ is not light because of symmetry reasons
  - N_c dependence is not know
- Technical issues :
  - Rigorous treatment of resonances on the lattice is expensive and difficult (see later)
  - Disconnected contributions are noisy

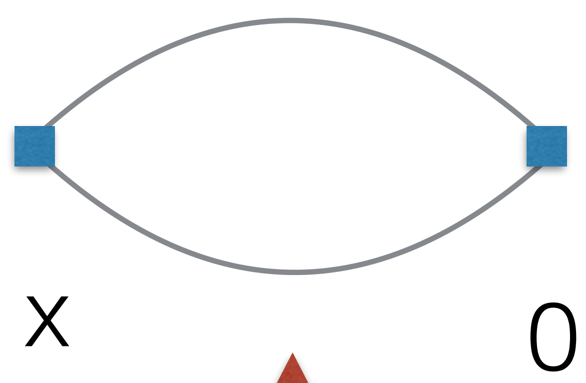
Very challenging !

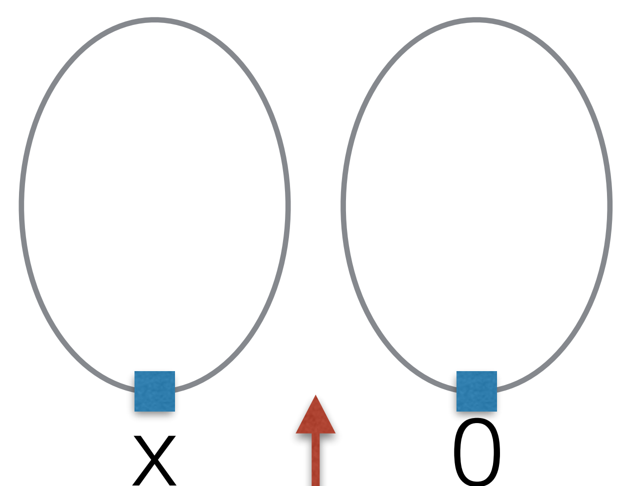
Masses (II)

Forgetting about the fact that the σ is unstable

- Simplest fermionic interpolating field : $J = \bar{u}u + \bar{d}d$ (iso-singlet, σ)
- Correlator : $C_{2\mathrm{pts}}(t) = \sum_{\vec{x}} \langle J(t,\vec{x})\bar{J}(0)\rangle$
- After integration over the fermions :

$$C_{2\mathrm{pts}}(t) = \sum_{\vec{x}} \mathrm{tr}\{\mathrm{S}(\mathrm{x},0)\mathrm{S}(0,\mathrm{x})\} + \mathrm{N_f} \sum_{\tilde{\mathrm{x}}} \mathrm{tr}\{\mathrm{S}(\mathrm{x},\mathrm{x})\}\mathrm{tr}\{\mathrm{S}(0,0)\}$$





=connected contribution

=disconnected contribution

« infinitely many gluons » exchanged

- connected contribution : two-point function $J = \bar{u}u - \bar{d}d$ (iso-vector, a_0)

Masses (II)

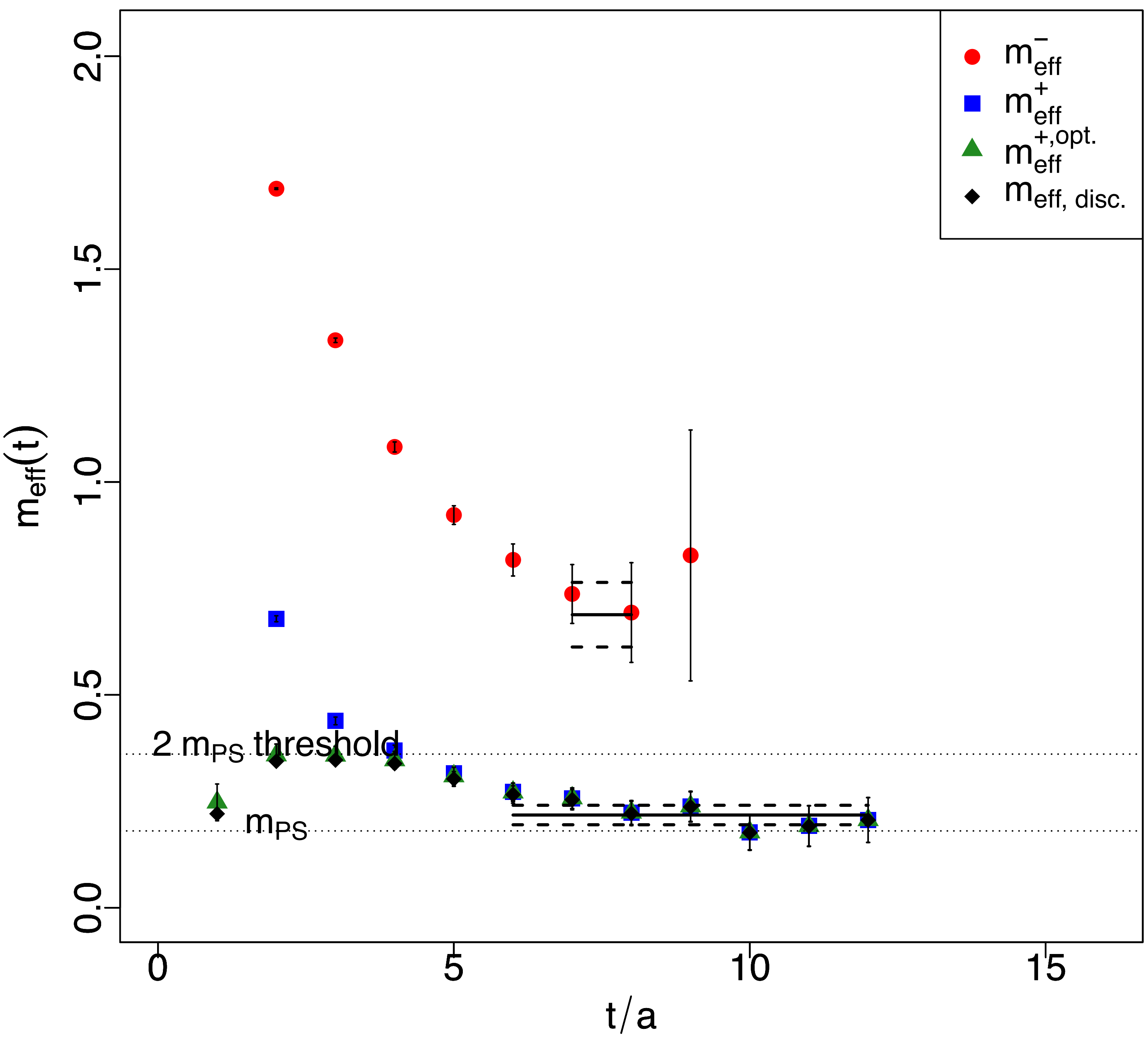
- Disconnected contribution estimated using stochastic estimators (64 volume sources/conf.)
- Same calculation can be used to compute η'
- Interpolating field $J = \bar{u}\gamma_5 u + \bar{d}\gamma_5 d$ (iso-scalar)

- *not* a GB because of the $U(1)_A$ anomaly

results on 3 new states of the theory : σ, a_0, η'

Effective mass : scalar channel

R. Arthur, VD, A. Hietanen, C. Pica, F. Sannino [arXiv:1607.06654]

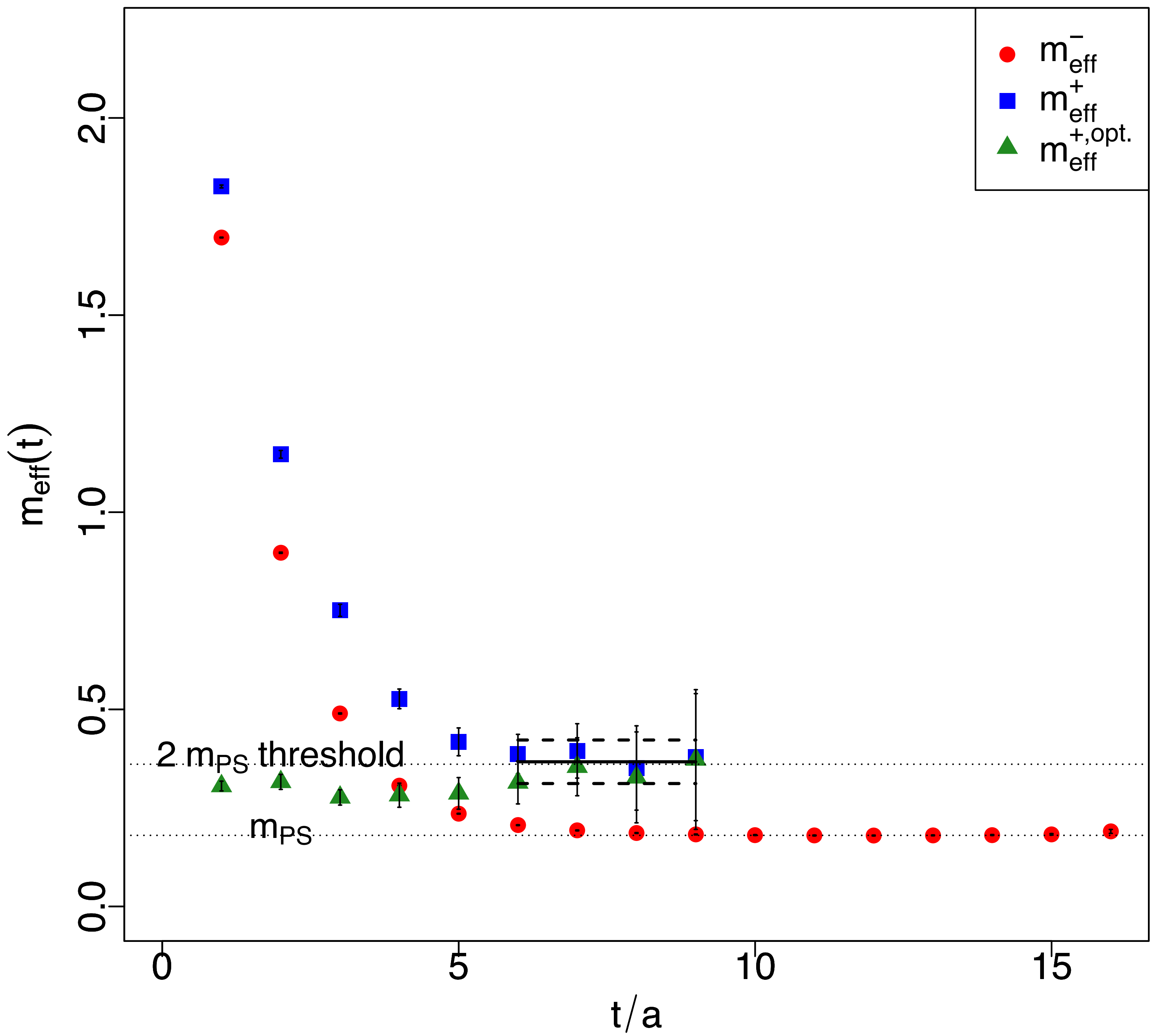


- Most chiral run: β=2.0, m=-0.958, V=32^4
- σ is stable in our setup
- $m_\sigma \sim m_{PS}$!

No need to consider the resonance analysis for the σ

Effective mass : pseudoscalar channel

R. Arthur, VD, A. Hietanen, C. Pica, F. Sannino [arXiv:1607.06654]

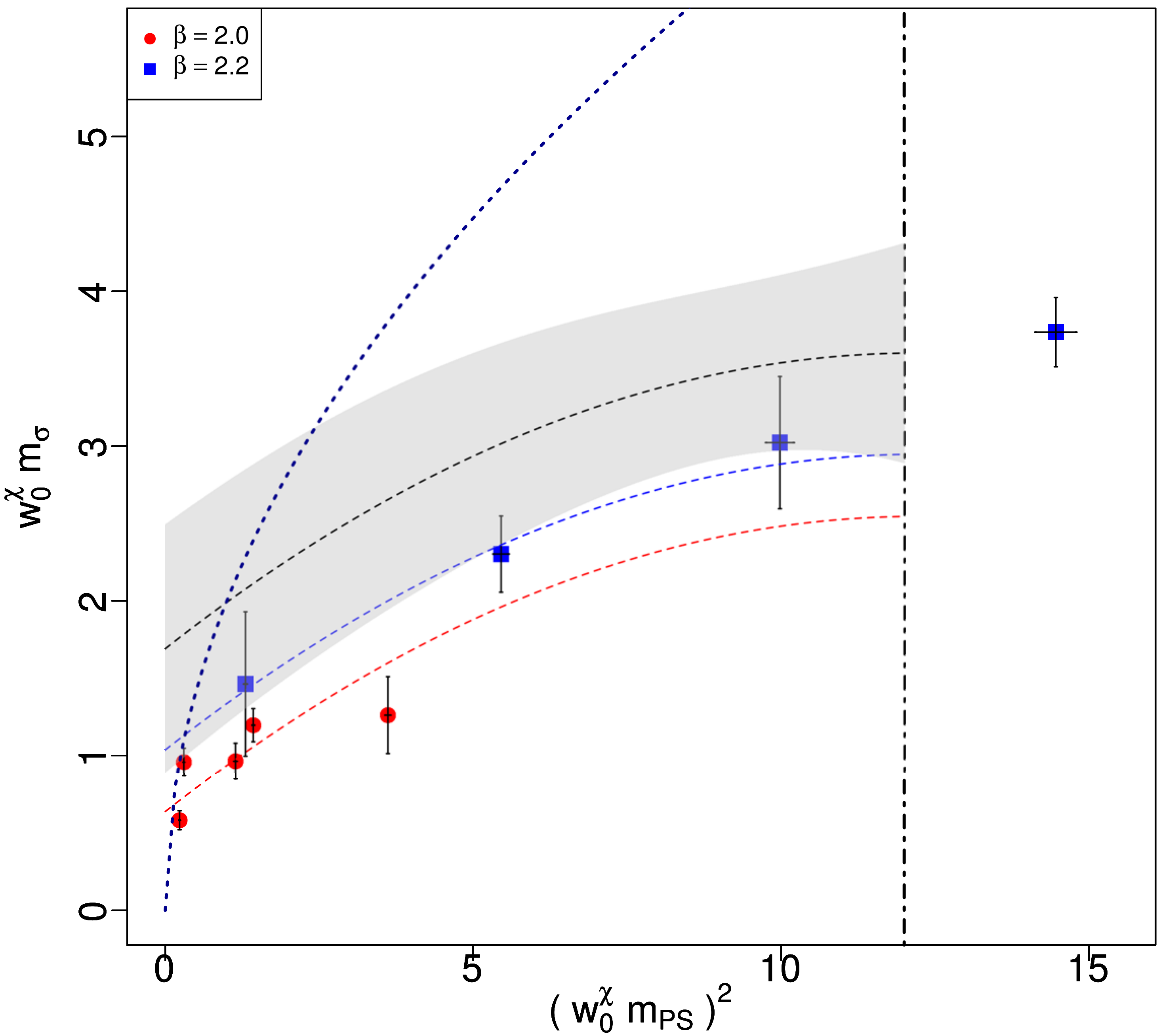


- Most chiral run: β=2.0, m=-0.958, V=32^4
- η' signal

Contribution from the disconnected diagrams is sizeable only for our lightest masses

σ resonance : 0(0+)

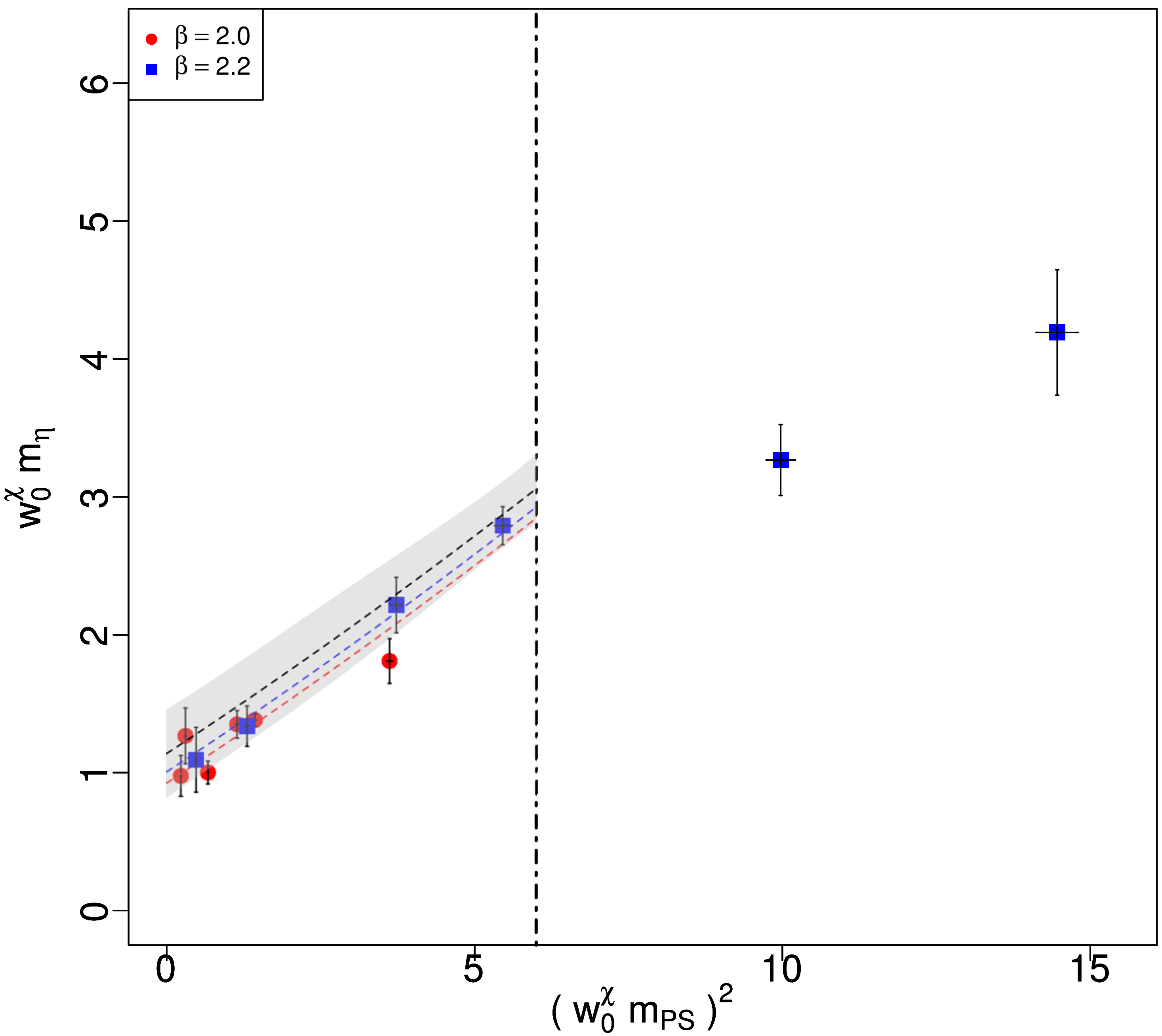
- σ stable in our simulation
- Polynomial global fit including 2 lattice spacing
- m_σ = 21.7(10.8) F_{PS}



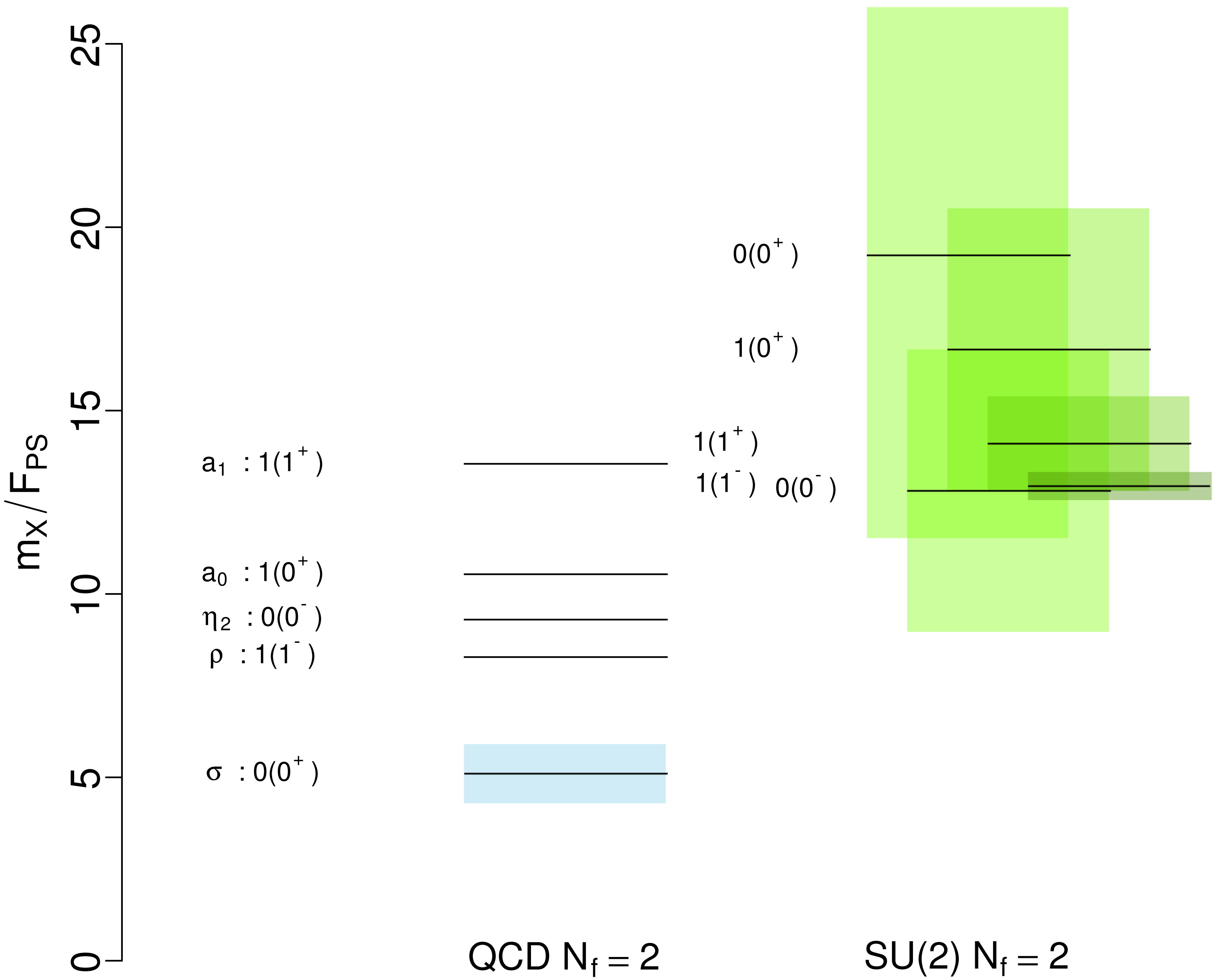
$$w_0^\chi m_X = w_0^\chi m_X^\chi + A(w_0^\chi m_{\rm ps})^2 + B(w_0^\chi m_{\rm ps})^4 + C\frac{a}{w_0}$$

η’ : 0(0-)

- 2 lattice spacings
- $m_\eta > m_{ps}$ only at our lightest fermion masses.
- no cutoff effects are seen.
- m_η=14.6(4.7) F_{PS}



Summary and comparison with N_C=3



- Shifted upward compared to QCD ?
- Neglects the decay of resonances

New Results: improved setup

Scattering

- In the TC framework the Goldstone boson equivalence theorem : at large energies, external vector bosons states equivalent to Goldstone Boson states

- e.g : $\rho \to \pi^+\pi^-$ corresponds to vector resonance in W^+W^- scattering

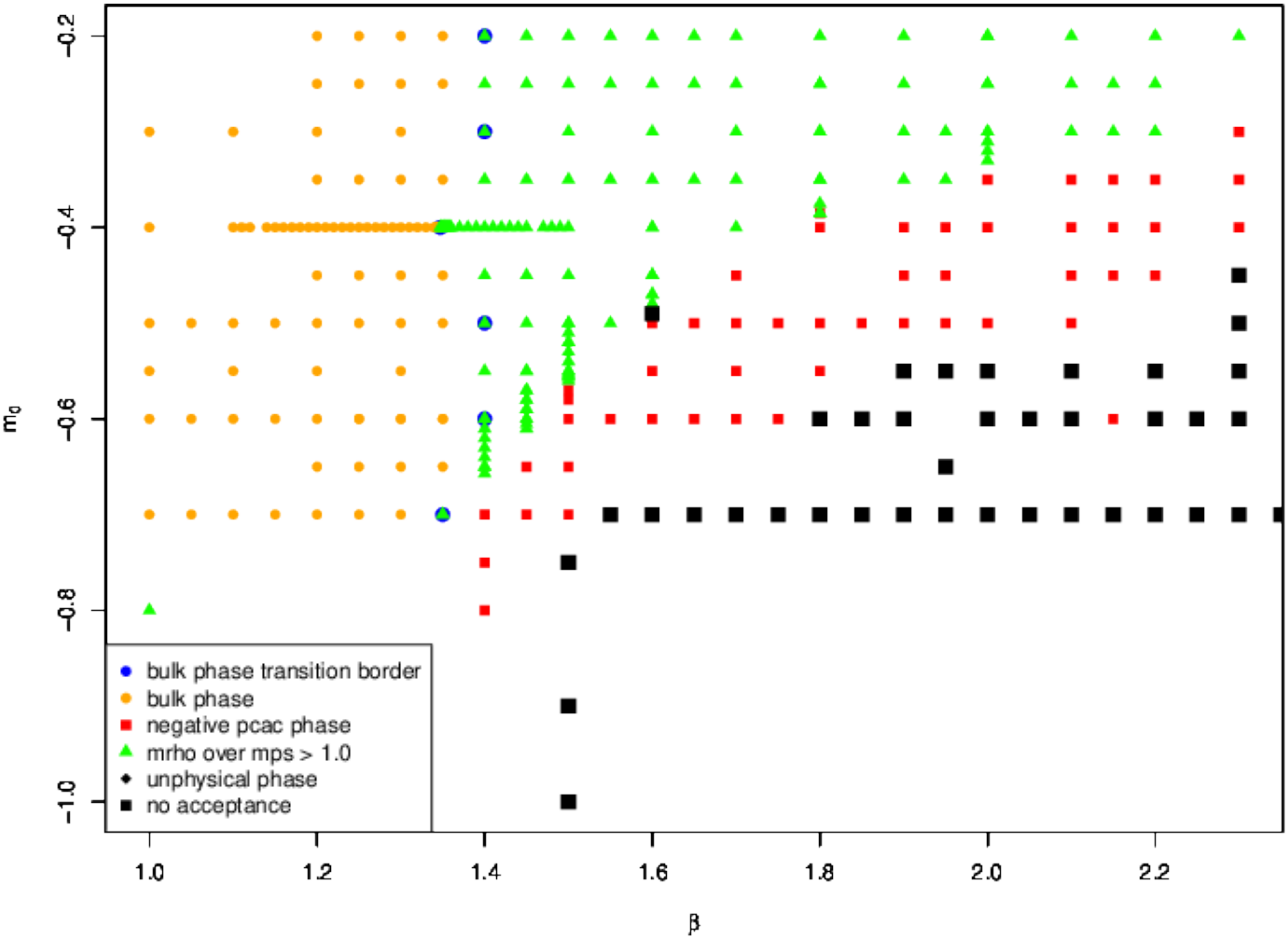
- Effective Lagrangian $\mathcal{L}_{eff} = g_{\rho\pi\pi}\rho^{\mu}_{[ij]}\partial_\mu\pi_i\pi_j$

Wilson Clover fermions + Symanzik improved gauge

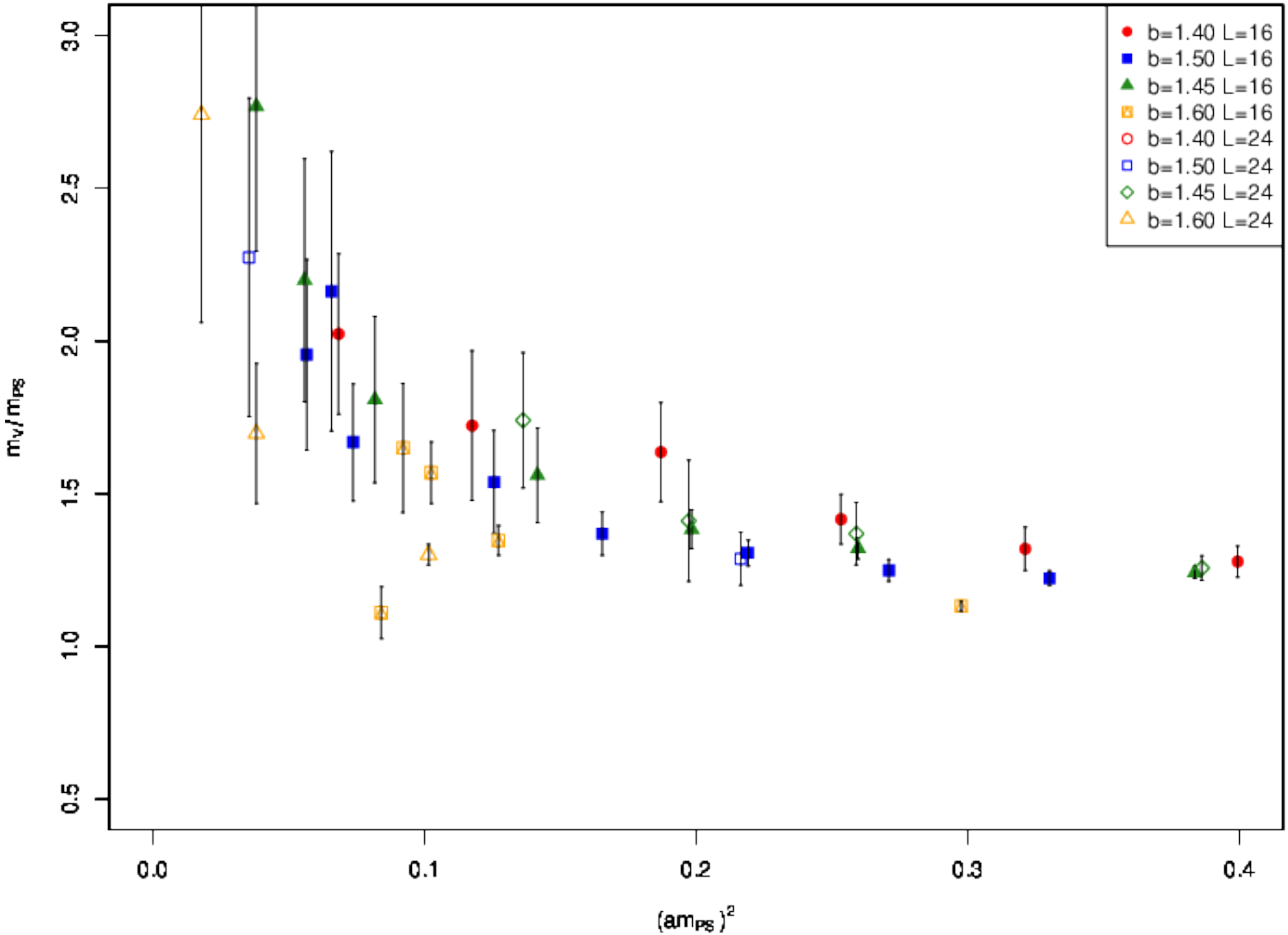
b	m0	L	T	Blk_size	stat	ampac	dampac	amps	damps	afps	dafps	amV	damV
1.25	-0.6000	16	32	100	6500	0.2851	0.0004	1.303	0.002	0.233	0.0024	1.468	0.007
1.30	-0.6000	16	32	100	6500	0.2321	0.0005	1.154	0.002	0.203	0.0025	1.324	0.009
1.35	-0.6000	16	32	100	6500	0.1701	0.0006	0.956	0.003	0.168	0.0025	1.129	0.013
1.35	-0.7000	16	32	10	1666	0.0357	0.0009	0.423	0.006	0.105	0.0035	0.655	0.081
1.40	-0.6000	16	32	10	1500	0.0984	0.0006	0.686	0.003	0.124	0.0022	0.862	0.020
1.40	-0.6100	16	32	10	1392	0.0846	0.0008	0.632	0.005	0.118	0.0032	0.808	0.031
1.40	-0.6200	16	32	10	1218	0.0696	0.0009	0.567	0.006	0.109	0.0034	0.748	0.039
1.40	-0.6300	16	32	10	1621	0.0552	0.0007	0.503	0.005	0.102	0.0024	0.713	0.040
1.40	-0.6400	16	32	10	1795	0.0413	0.0007	0.433	0.005	0.093	0.0025	0.708	0.070
1.40	-0.6500	16	32	10	1585	0.0250	0.0010	0.343	0.010	0.081	0.0040	0.591	0.082
1.40	-0.6570	16	32	10	2779	0.0137	0.0008	0.262	0.010	0.067	0.0036	0.529	0.066
1.45	-0.5500	16	32	10	2000	0.0898	0.0005	0.619	0.003	0.105	0.0016	0.769	0.012
1.45	-0.5700	16	32	20	4156	0.0631	0.0005	0.510	0.003	0.093	0.0013	0.673	0.017
1.45	-0.5800	16	32	10	1791	0.0493	0.0007	0.445	0.005	0.085	0.0019	0.616	0.028
1.45	-0.5900	16	32	10	1602	0.0346	0.0009	0.376	0.007	0.077	0.0027	0.587	0.058
1.45	-0.6000	16	32	10	1766	0.0197	0.0010	0.286	0.010	0.064	0.0033	0.517	0.076
1.45	-0.6050	16	32	10	2371	0.0110	0.0011	0.236	0.013	0.051	0.0049	0.520	0.090
1.45	-0.6100	16	32	20	3424			0.195	0.015			0.541	0.083
1.50	-0.5000	16	32	10	1500	0.0869	0.0006	0.575	0.004	0.091	0.0019	0.703	0.013
1.50	-0.5100	16	32	10	1500	0.0731	0.0007	0.521	0.005	0.084	0.0019	0.650	0.017
1.50	-0.5200	16	32	10	1513	0.0600	0.0007	0.468	0.005	0.078	0.0018	0.611	0.019
1.50	-0.5300	16	32	10	1500	0.0459	0.0008	0.407	0.006	0.072	0.0019	0.557	0.027
1.50	-0.5400	16	32	10	1325	0.0330	0.0010	0.354	0.009	0.062	0.0026	0.545	0.058
1.50	-0.5500	16	32	10	1750	0.0190	0.0010	0.272	0.013	0.050	0.0028	0.453	0.048
1.50	-0.5525	16	32	10	2214	0.0141	0.0010	0.256	0.012	0.041	0.0028	0.555	0.115
1.50	-0.5550	16	32	20	4117	0.0120	0.0011	0.238	0.017	0.038	0.0035	0.465	0.066
1.60	-0.4000	16	32	10	1500	0.0993	0.0005	0.546	0.004	0.071	0.0012	0.617	0.008
1.60	-0.4500	16	32	10	1810	0.0407	0.0007	0.357	0.007	0.048	0.0013	0.480	0.014
1.60	-0.4700	16	32	10	2432	0.0162	0.0011	0.320	0.015	0.022	0.0015	0.502	0.023
1.60	-0.4800	16	32	10	2106	0.0018	0.0016	0.304	0.027	0.002	0.0020	0.501	0.046
1.60	-0.4900	16	32	10	2866	-0.0163	0.0006	0.290	0.012	-0.033	0.0020	0.322	0.021
1.80	-0.2500	16	32	5	500	0.1393	0.0004	0.566	0.004	0.053	0.0009	0.602	0.005
1.80	-0.3000	16	32	5	500	0.0893	0.0004	0.501	0.006	0.045	0.0010	0.548	0.007
1.80	-0.3500	16	32	10	2500	0.0333	0.0003	0.451	0.006	0.019	0.0003	0.464	0.008
1.80	-0.3850	16	32	10	1917	-0.0062	0.0005	0.429	0.007	-0.004	0.0003	0.461	0.007
2.00	-0.2000	16	32	5	500	0.1266	0.0002	0.533	0.004	0.048	0.0009	0.578	0.004
2.00	-0.2500	16	32	5	500	0.0757	0.0003	0.331	0.008	0.038	0.0011	0.365	0.008
2.00	-0.3000	16	32	5	500	0.0250	0.0006	0.234	0.013	0.034	0.0014	0.314	0.020
1.45	-0.5500	24	48	10	1828	0.0899	0.0003	0.621	0.002	0.106	0.0016	0.781	0.025
1.45	-0.5700	24	48	10	1684	0.0628	0.0004	0.509	0.002	0.093	0.0018	0.697	0.052
1.45	-0.5800	24	48	10	1485	0.0487	0.0004	0.444	0.003	0.085	0.0023	0.627	0.088
1.45	-0.5900	24	48	10	1020	0.0340	0.0009	0.369	0.007	0.078	0.0044	0.643	0.081
1.50	-0.5200	24	48	10	1225	0.0603	0.0005	0.465	0.004	0.079	0.0023	0.599	0.040
1.50	-0.5560	24	48	10	1673	0.0100	0.0007	0.188	0.009	0.048	0.0029	0.428	0.096
1.60	-0.4500	24	48	10	2693	0.0404	0.0002	0.319	0.002	0.051	0.0008	0.414	0.010
1.60	-0.4700	24	48	10	1991	0.0151	0.0005	0.195	0.007	0.036	0.0012	0.331	0.043
1.60	-0.4800	24	48	10	1758			0.134	0.012			0.367	0.085

- Reduce cutoff effects
- Or simulate at coarser lattice spacing with larger physical volumes…

Phase diagram: bare parameters



Decaying vector meson !



$\pi\pi$ scattering

- Consider the process : $$\pi^a \pi^b \longrightarrow \pi^c \pi^d$$

- In (2 flavour) QCD pion's belong to ”3“, and the two pion operators can be classified according to :

$$3 \times 3 = 1 + 3 + 5$$

- In our case GB belong to “5” dimensional irrep of SP(4) :

$$5 \times 5 = 1 + 10 + 14$$

- There are still 3 channels

Phase shift & cross section

- Differential cross-section in terms of scattering amplitude:

$$\frac{d\sigma}{d\Omega} = |A(p,\Omega)|^2$$

- Partial wave decomposition:

$$A(p,\Omega) = \sum_{l,m} 4\pi A_l(p) Y^*_{lm}(\theta_p,\phi_p) Y_{lm}(\theta,\phi)$$

- Phase shift definition: $A_l(p) = \dfrac{e^{2i\delta_l(p)} - 1}{2ip} = \dfrac{e^{i\delta_l(p)}\sin\delta_l(p)}{p}$

- Relation with effective coupling and width

$$\tan\delta_1 = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{p^3}{E_{CM}(m_\rho^2 - E_{CM}^2)}, \; p = \sqrt{E_{CM}^2/4 - m_\pi^2}$$

$$\Gamma_\rho = \Gamma_R(s)\Big|_{s=m_\rho^2} = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{p_\rho^3}{m_\rho^2}\,, \quad p_\rho = \sqrt{m_\rho^2/4 - m_\pi^2}\,.$$

Lattice calculation in a nutshell

[Luscher, Rummukainen-Gottlieb,…]

- Non interacting pions with momenta **p** and **0**:

$$E_1^0 = \sqrt{m_{\rm PS}^2 + \mathbf{p}^2} + m_{\rm PS}$$

$$E_2^0 = \sqrt{m_{\rm V}^2 + \mathbf{p}^2}$$

- In a finite box the energy levels are shifted by the interaction. The energy shift is related to the two-pion scattering phase shift δ in infinite volume.

- Total momentum: $P = \dfrac{2\pi}{L}\mathbf{e_z}$

$$\tan\delta_1(E_{CM}) = \frac{\gamma\pi^{3/2}q}{\mathcal{Z}_{00}^{\mathbf{d}}(1;q^2) + (2q^{-2}/\sqrt{5})\mathcal{Z}_{20}^{\mathbf{d}}(1;q^2)} \qquad E_{CM} = 2\sqrt{m^2 + \mathbf{p}^{*2}} \qquad \mathbf{p}^{*2} \equiv \left(q\frac{2\pi}{L}\right)^2$$

Lattice calculation in a nutshell

- Lattice approach, compute:

$$C(t) = \begin{pmatrix} \langle 0|\ (\pi\pi)^\dagger(t)\ (\pi\pi)(t_S)\ |0\rangle & \langle 0|\ (\pi\pi)^\dagger(t)\ \rho_3(t_S)\ |0\rangle \\ \langle 0|\ \rho_3^\dagger(t)\ (\pi\pi)(t_S)\ |0\rangle & \langle 0|\ \rho_3^\dagger(t)\ \rho_3(t_S)\ |0\rangle \end{pmatrix}$$

with $\quad (\pi\pi)(t) = \frac{1}{\sqrt{2}}\Big(\pi^-(\mathbf{p},t)\pi^+(\mathbf{0},t) - \pi^+(\mathbf{p},t)\pi^-(\mathbf{0},t)\Big)$

- Large time behaviour:

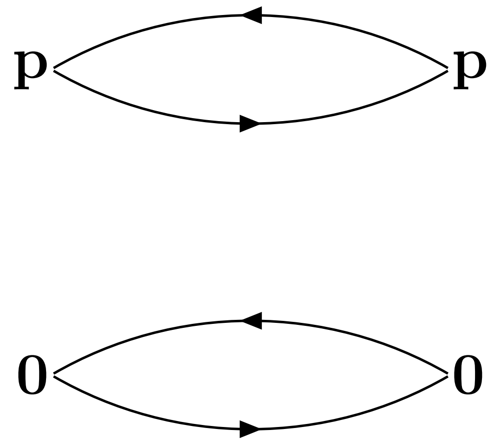
$$C_{ij}(t) \equiv \langle 0 \mid O_i^\dagger(t) O_j(0) \mid 0\rangle = \sum_{n,m} \langle 0 \mid O_i^\dagger \mid n\rangle (e^{-E_n t}\delta_{mn}) \langle m \mid O_j \mid 0\rangle$$

- Solve the generalised eigenvalue problem:

$$C_{ij}^{-1}(t_0) C_{jk}(t) = V_{in}^{-1} diag \left(e^{-E_n(t-t_0)}\right)_{nm} V_{mj}$$

Wick contractions

$C_{11}(t) =$



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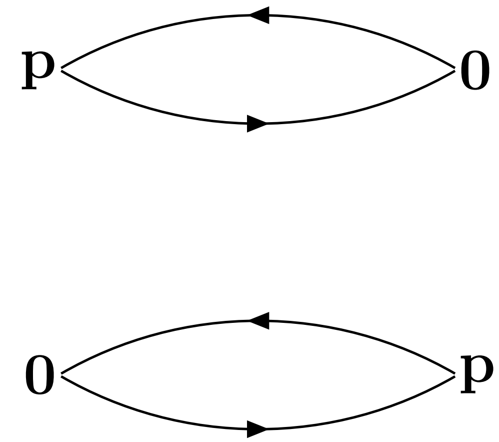

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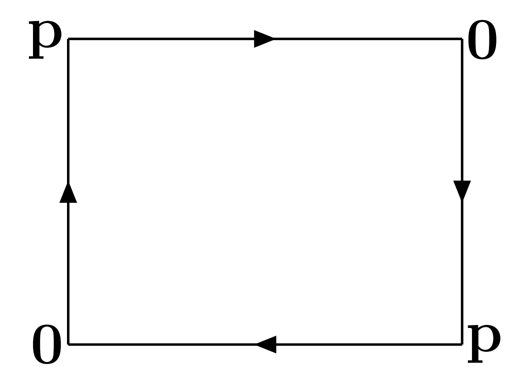

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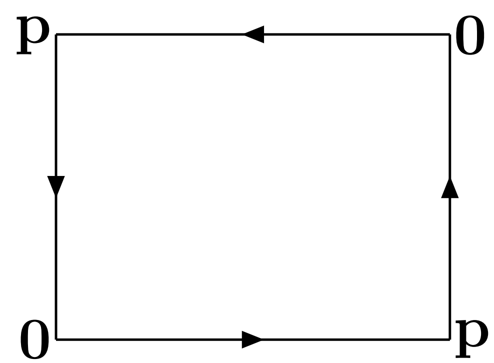

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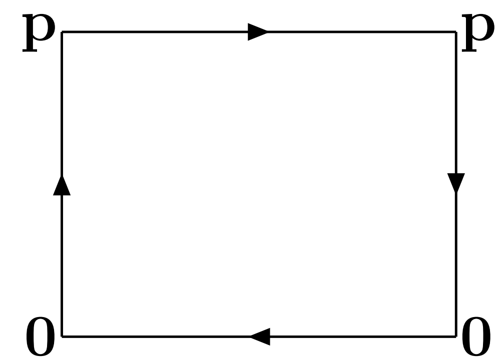

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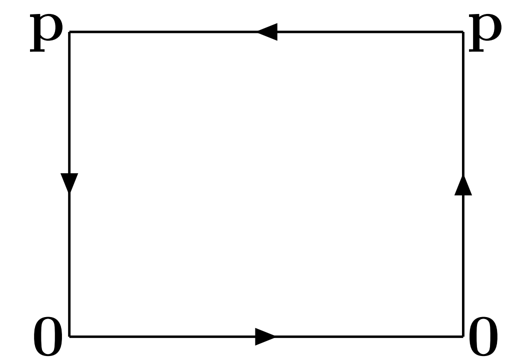
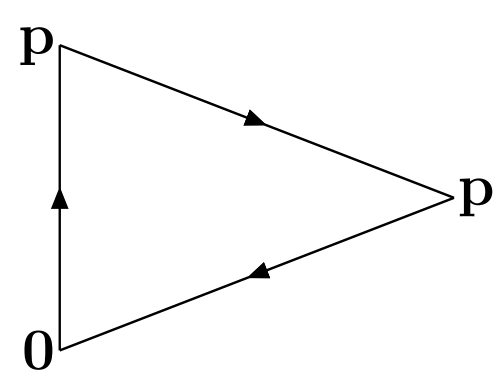


$C_{12}(t) = -C_{21}^{*}(t) =$



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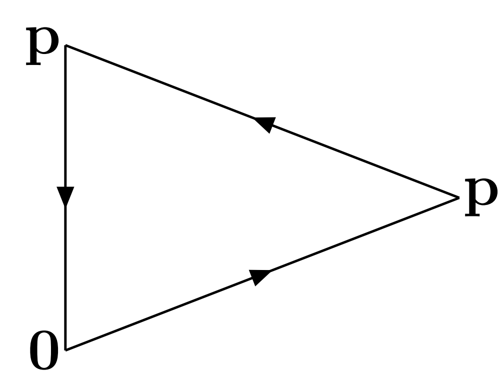
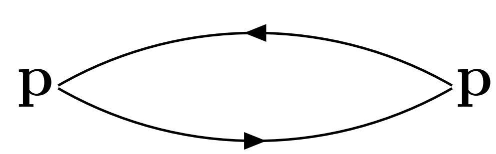


$C_{22}(t) =$



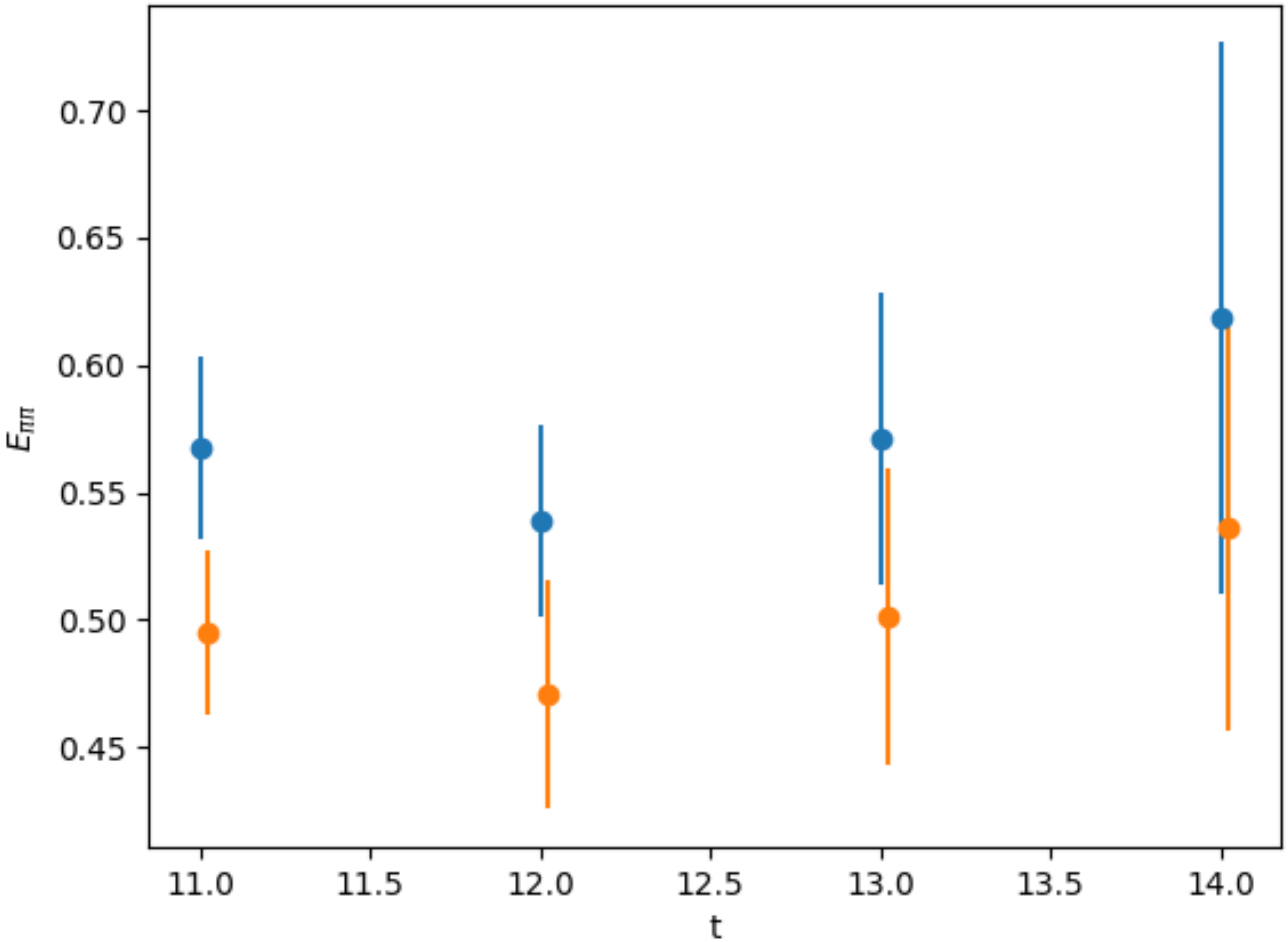
Benchmark ensemble

β	1.45
m_0	-0.6050
c_{sw}	1.0
am_π	0.2114(8)
am_ρ^{naive}	0.444(9)
am_{pcac}^0	0.01110(7)
af_π	0.0564(3)
# trajectories	1600
# analysed	140

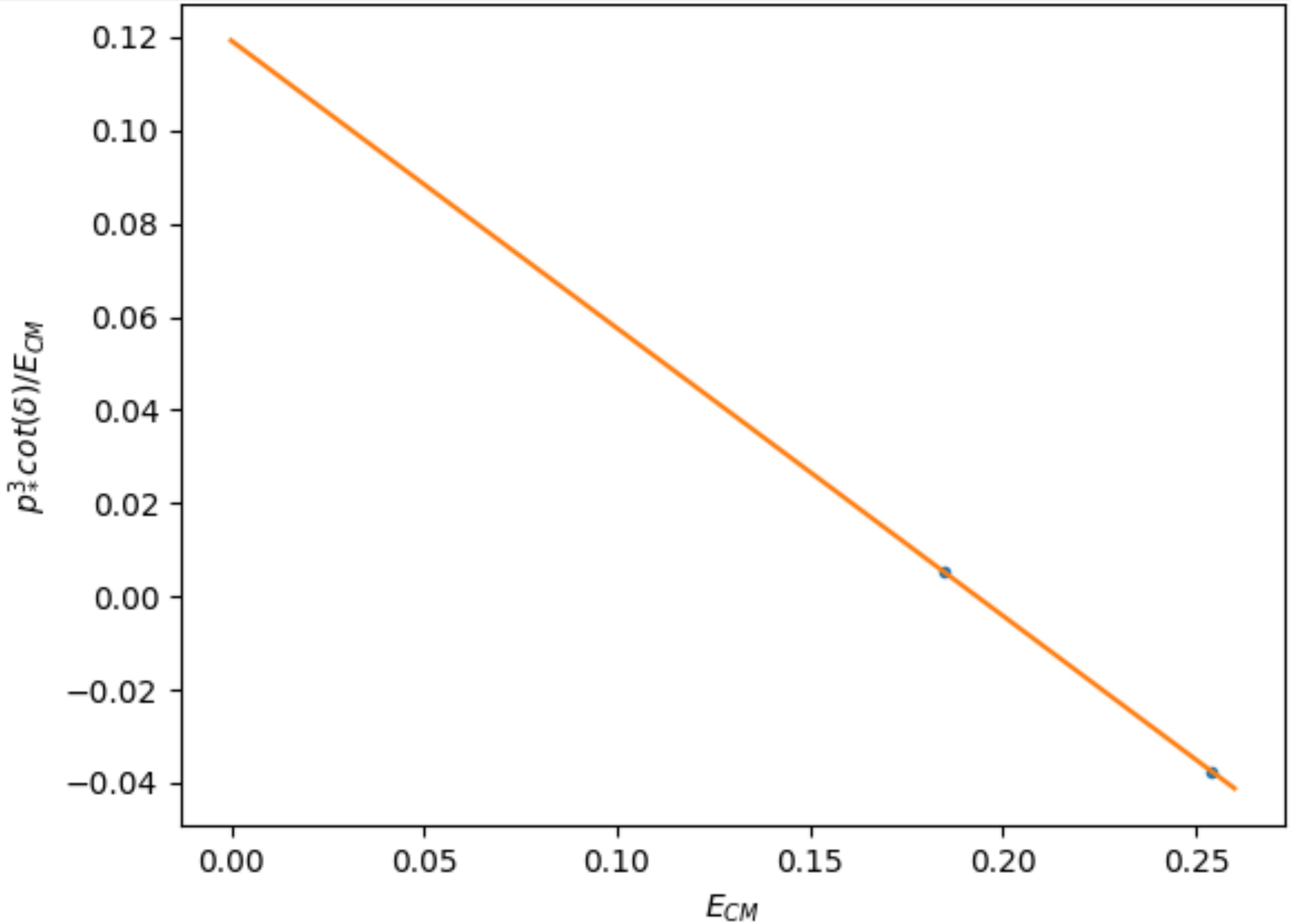
- For the time being, we use $p = \frac{2\pi}{L}\mathbf{e_z}$
- We plan two use one or two more moving frames

Energy levels

Eigenvalue $\lambda_i(t) = \exp\left(-E_i(t - t_0)\right),\ t_0 = 10.$
Effective mass $E_i(t) = \ln \lambda_i(t)/\lambda_i(t+1)$



Preliminary: phase shift



Fitting central values only (for now) PRELIMINARY:

$$g_{\rho\pi\pi} = 5.527$$
$$M_\rho = 0.440$$

Summary & Conclusions

- Lattice as a laboratory to explore non-perturbative dynamics of gauge theories

- Prediction of the spectrum for a range of underlying fermion mass
- Decay channels are often closed for kinematic reasons (scalar, vector, axial are stable in most of our simulations)
- Benchmark predictions in the chiral limit for spin-1 resonances:
  - ★ $m_V/F_{\mathrm{PS}} = 13.1(2.2)$; $m_A/F_{\mathrm{PS}} = 14.5(3.6)$
- Preliminary results in the spin-0 sector :
  - ★ $m_\sigma/F_{\mathrm{PS}} = 21.7(10.8)$
  - ★ $m_\eta/F_{\mathrm{PS}} = 14.6(4.7)$
- Ongoing/Outlook :
  - ★ Update results on the spectrum
  - ★ Preliminary results on the vector meson resonance
  - ★ Generalisation to other resonances?