A minimal theory of strongly-coupled dark baryons: spectrum predictions from lattice QFT

Anthony Francis
Renwick James Hudspith
Randy Lewis
Sean Tulin

"Interdisciplinary approaches to QCD-like composite dark matter"

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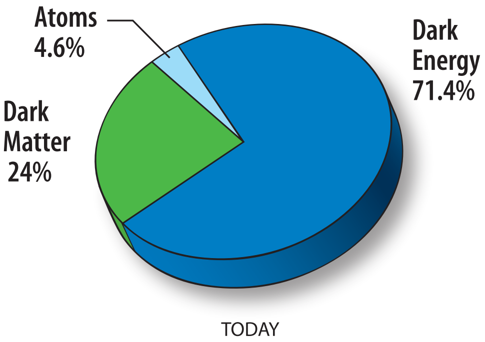
Motivation

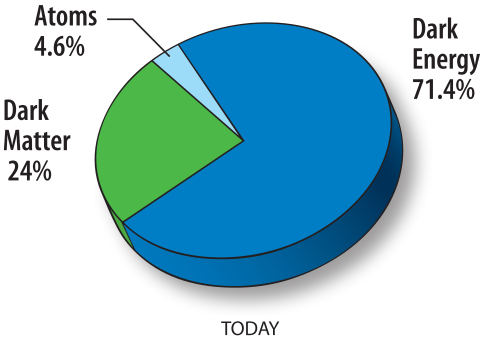
The majority of matter in the universe is made up of dark matter



WMAP #121236

- many DM detection strategies and models suggested
- often motivated by other issues in the SM (e.g. weak scale stability, strong CP)

QCD: mass and stability of luminous matter (atoms) in the universe



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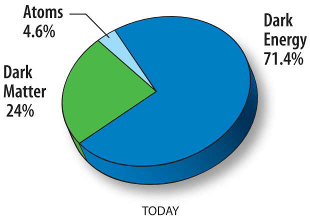
- $\sim$ 99% of the mass of baryonic matter is due to strong interactions
- stability is due to accidental $U(1)_B$ baryon number symmetry

Dark and luminous matter from separate particle physics sectors?

DM as lightest stable state with

- its own (non)-abelian gauge group
- matter representations
- i.e. its own (strong) dynamics



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Consequences:

- interactions within the DM sector generically large
  $\longrightarrow$ self-interactions
- interactions with the SM only through higher-dim. operators
  $\longrightarrow$ possibly feeble

An attractive idea: mass and stability of DM arise through strong dynamics like for LM.

To study strong dynamics in the non-perturbative regime the main tool today is **lattice QFT**.

Early and other realisations along these lines of thought

Review: G.D. Kribs and E.T. Neil '16

- technicolor

  Nussinov '85; Chivukula and Walker '90; Barr, Chivukula and Farhi '90 ...

- mirror baryons

  Chacko, Goh and Harnik '06; Foot '14 ...

- non-abelian dark sectors

  Lewis, Pica and Sannino '12; Hietanen et al. '14; LSD '13, '14; Detmold, McCullough and Pochinsky '14; Appelquist et al. '15 ...

A Minimal model: fewest colors N_c, fewest flavors N_f, smallest non-trivial representation of matter fields

$\longrightarrow$ SU(2) with 1 Dirac fermion in the fundamental representation.

- Notes on $N_c = 2$ theory:
  - shares many features with QCD
  - but, the fundamental rep. is **pseudo-real**
  - enlarged symmetry reflecting trafos from $q \to \bar{q}$

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- $N_c = 2$ with $N_f = 1$ (as we will see later on) :
  - q and $\bar{q}$ form a doublet:

$$\mathcal{Q} \sim \begin{pmatrix} q \\ \bar{q} \end{pmatrix}$$

  - unbroken global $\mathrm{SU(2)_B}$ acting on $\mathcal{Q}$
  - non-abelian generalisation of $\mathrm{U(1)_B}$ baryon number sym.
  - *conservation of baryon number = possible stable DM candidate.*

A minimal model of dark baryons

$$\boxed{\mathcal{L} = -\tfrac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{q}(i\not{D} - m)q + \mathcal{L}_{\text{higher dim}}}$$

SU(2) gauge theory with $N_f = 1$ fund. rep. Dirac fermion

- chiral symmetry for $m = 0$: $\mathrm{U(1)_L} \times \mathrm{U(1)_R}$.
- **But**, SU(2) is pseudo-real
- enlarged U(2) global symmetry.

In general, for $N_c = 2$, we have:

$$\boxed{\mathrm{U(N_f)_L} \times \mathrm{U(N_f)_R} \longrightarrow \mathrm{U(2\,N_f)} = \mathrm{U(1)_A} \times \mathrm{SU(2\,N_f)}}$$

Chiral symmetry breaking reduces $\mathrm{SU(2\,N_f)} \to \mathrm{Sp(2\,N_f)}$ with $(2\,N_f + 1)(N_f - 1)$ Goldstone bosons.

For $N_f = 1$ there are **no** Goldstone bosons.

(Unlike $N_f = 2$, which has 5 Goldstone bosons, including the baryons.)

Enlarged U(2) global symmetry

To see this more clearly, one may introduce

$$\mathcal{Q} = \begin{pmatrix} q_L \\ -i\sigma^2 C \bar{q}_R^T \end{pmatrix}, \qquad E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(C is charge conjugation) and write the fermionic part of $\mathcal{L}$ as:

$$\mathcal{L}_{\text{fermion}} = \bar{\mathcal{Q}} i \not{D} \mathcal{Q} - \frac{m}{2} \left(\mathcal{Q}^T i\sigma^2 C E \mathcal{Q} + \bar{\mathcal{Q}} i\sigma^2 C E \bar{\mathcal{Q}}^T \right)$$

- kinetic term: invariant under U(2) trafo on $\mathcal{Q}$.
- mass term: decompose $\mathrm{U}(2) = \mathrm{U}(1)_\mathrm{A} \times \mathrm{SU}(2)_\mathrm{B}$.
  - $\mathrm{U}(1)_\mathrm{A} \Leftrightarrow$ rotation of $\mathcal{Q}$ by overall phase.
  - $\mathrm{SU}(2)_\mathrm{B} \Leftrightarrow$ baryonic isospin w/ $\mathrm{U}(1)_\mathrm{B}$ as subgroup.
- $\mathrm{U}(1)_\mathrm{A}$ broken for $m \neq 0$, but $\mathrm{SU}(2)_\mathrm{B}$ remains intact as E is invariant.

$\mathsf{SU(2)_B} \Leftrightarrow$ *baryonic isospin implies:*

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix} \sim \begin{pmatrix} qq \\ \frac{1}{\sqrt{2}}(q\bar{q} + \bar{q}q) \\ \bar{q}\bar{q} \end{pmatrix}$$

$\Rightarrow$ The lightest qq state is part of a spin-1 iso-triplet.

$\mathsf{U(1)_A}$ *implies:* Including the axial anomaly, no chiral symmetries at $m_q = 0$. The would-be Goldstone boson η gets a mass from the anomaly even in the "chiral" limit.

Hadronic states of the model

Here we consider the hadronic states $J^P = 0^\pm$ and $1^\pm$ constructed from the meson interpolating operators:

scalar (0^+)	$\mathcal{O}_S = \bar{q}q = \frac{1}{2}\left(\mathcal{Q}^T i\sigma^2 CE\mathcal{Q} + \bar{\mathcal{Q}} i\sigma^2 CE\bar{\mathcal{Q}}^T\right)$
pseudoscalar (0^-)	$\mathcal{O}_P = \bar{q}\gamma_5 q = -\frac{1}{2}\left(\mathcal{Q}^T i\sigma^2 CE\mathcal{Q} - \bar{\mathcal{Q}} i\sigma^2 CE\bar{\mathcal{Q}}^T\right)$
vector (1^-)	$\mathcal{O}_V^\mu = \bar{q}\gamma^\mu q = \bar{\mathcal{Q}}\gamma^\mu\tau^3\mathcal{Q}$
axial vector (1^+)	$\mathcal{O}_A^\mu = \bar{q}\gamma^\mu\gamma^5 q = \bar{\mathcal{Q}}\gamma^\mu\mathcal{Q}\,.$

In analogy to QCD we denote: η ($\mathcal{O}_P$), ρ ($\mathcal{O}_V^\mu$), a_1 ($\mathcal{O}_A^\mu$).

Except for the vector channel the operators are iso-singlet.

The vector is part of an iso-triplet, including meson and diquark operators:

$$\mathcal{O}_V^{a\mu} = \bar{\mathcal{Q}}\gamma^\mu \tau^a \mathcal{Q}$$

$\Longrightarrow$ The lightest baryon of the theory has $J^P = 1^-$.

The tensor current is ($\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$)

$$\mathcal{O}_T^{\mu\nu} = \bar{q}\sigma^{\mu\nu}q = \mathcal{Q}^T E\tau^3 C\sigma^{\mu\nu}(i\sigma^2)\mathcal{Q} - \bar{\mathcal{Q}}E\tau^3 C\sigma^{\mu\nu}(i\sigma^2)\bar{\mathcal{Q}}^T$$

$\Longrightarrow$ It transforms under $\mathrm{SU(2)_B}$ like $\mathcal{O}_V^\mu$.

Further notes:

- Spontaneous symmetry breaking: chiral condensate $\chi_c = \langle\bar{q}q\rangle \neq 0$
- No violation of baryonic isospin, since $\bar{q}q$ is iso-singlet
- χ_c breaks global $\mathrm{U_A(1)}$, but $\mathrm{U_A(1)}$ is anomalous
  $\Longrightarrow$ η acquires additional mass at $m_q = 0$

SM coupling and DM stability

The dark sector and SM can couple through higher dimensional operators,

$$\boxed{\mathcal{O}_{S,P}|H|^2 \quad \text{(leading dim.-5:)}}$$

VEV $\langle H \rangle = v/\sqrt{2}$ gives an additional contribution to m_q
$\longrightarrow$ possible **CP-violating** phase (Dirac mass $\leftrightarrow$ VEV contribution)

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$$\mathcal{L} \supset -m\,\bar{q}_R q_L - m^*\,\bar{q}_L q_R - \frac{1}{M}\bar{q}q|H|^2$$

After chiral rotation (to retrieve positive, real mass):

$$\mathcal{L} \supset -m_q\,\bar{q}q - \frac{1}{M}\left(\cos\phi\,\mathcal{O}_S + \sin\phi\,\mathcal{O}_P\right)\left(vh + \tfrac{1}{2}h^2\right)$$

CP violation becomes a coupling between $\mathcal{O}_{S,P}$ and the Higgs boson h.

(Consequences later on, but: P-coupling might cosmologically destabilize the η.)

What about dark matter stability? Is the ρ a viable DM candidate?

For DM to be stabilized by accidental symmetry, symmetry must be preserved including dim.-5.

- dim.-5 may induce decay more rapidly than age of the universe (even for suppression scale $M \sim M_{\text{Planck}}$).
- dim.-6 operators are ok, if M is large enough (just like the proton).

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In this model:

- $\mathcal{O}_{S,P}|H|^2$ does not allow ρ-decay ($\mathrm{SU(2)_B}$).
- Leading $\mathrm{SU(2)_B}$ violation includes $\mathcal{O}_V^{a\mu}$ at dim.-6 by Lorentz sym.

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$\Longrightarrow$ ρ is **safe**, all other (iso-singlet) states are not, e.g. η.

Note, coupling to SM through a singlet scalar does not change this.

Also coupling through a Z' gauge boson only destabilizes the (meson) ρ^0 while the (diquark) $\rho^{\pm}$ remain stable DM candidates.

Calculating the spectrum

To begin the calculation of the one-flavor SU(2) theory spectrum, we choose the lattice action:

$$S_{\text{Wilson}} = \underbrace{\frac{\beta}{2}\sum_{x,\mu,\nu}\left(1-\frac{1}{2}\text{ReTr}\,\hat{U}_{\mu\nu}(x)\right)}_{S_{\text{gauge}}} + \underbrace{\sum_{x,y}\bar{\psi}_x M(x,y)\psi_y}_{S_{\text{fermion}}}\,.$$

with (Wilson) fermion matrix:

$$M(x,y) = (4+m_0)\delta_{x,y} - \frac{1}{2}\sum_{\mu=1}^{4}\left((1-\gamma_\mu)U_{x,\mu}\delta_{x+\mu,y} + (1+\gamma_\mu)U^\dagger_{x,\mu}\delta_{x-\mu,y}\right)$$

Coupling $\beta(a) = 4/g^2$ depends on a and sets the ultraviolet cut-off.

m_q is additively renormalized and must be found from numerical calculation.

Three step procedure:

1. Generate a set of gauge field configurations using S_{Wilson}
2. Calculate solutions to the Dirac equation S^{-1} on each
3. Contract the solutions using Wick's theorem to form hadrons given a set of creation/annihilation operators $\mathcal{O}_{\mathrm{had}}$

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The resulting correlation function may be written as:

$$\begin{aligned} C_{\mathcal{O}_1\mathcal{O}_2}(t) &= \frac{1}{L^3}\sum_x \langle \mathcal{O}_1(x,t)\mathcal{O}_2^\dagger(0,0)\rangle, \\ &= \sum_n \frac{\langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle}{2m_n}\left(e^{-m_n t} \pm e^{-m_n(T-t)}\right). \end{aligned}$$

Note, $m_{\text{had}} = m_{\text{ground}} = m_0$ can be extracted by fitting an exponential Ansatz to the long-distance region.

Calculating $N_f = 1$ is a special technical challenge

- requires use of RHMC (slow compared to other methods)
- low-lying Dirac eigenmodes destabilize the algorithm

$\Longrightarrow$ Cannot go as low in m_q as with $N_f \geq 2$.

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- suppressed signal for the annihilation diagrams

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Breaking new ground:

- scales, masses, (NP) renormalisation unknown
- intuition not necessarily useful

$\Longrightarrow$ Initial determination and estimation required

Our lattice setup:

- $\beta = 2.2$ with lattice sized $12^3 \times 32$ (and $12^3 \times 48$).
- scans in bare mass m_0 for $m_0 \in [-0.880 : -0.105]$

Goals:

- Tune the bare quark mass, in particular find $m_q = 0$
- Determine a reasonable scale, e.g. dark $a\Lambda_{\overline{\mathrm{MS}}}$
- Calculate m_{had}, f_{had} ($\mathrm{had} = \pi, \eta, \rho, a_1, \mathrm{scalar}$) and $\partial_{m_q} m_\rho$

Numerical setup:

- HPC calculations using the `HiRep`-software package
  Del Debbio, Patella and Pica, PRD81 ('10)
- RHMC for the generation of dynamical $\mathrm{SU}(2)$ one flavor gauge configurations, $\mathcal{O}(10^5)$ MDU
- time-diluted Z_2-stochastic wall sources with 64 hits on $\mathcal{O}(10^4)$ independent configurations for spectroscopy

(The disconnected piece was too noisy for the a_1 and *scalar* channels and dropped here.)

Finding the massless point $m_q = 0$

In $N_f \geq 2$ theories there is usually a pseudo-Goldstone boson,
$\longrightarrow$ use non-singlett nature and AWI for accurate determination of m_q.

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But, in $N_f = 1$ this particle is absent from the physical spectrum.

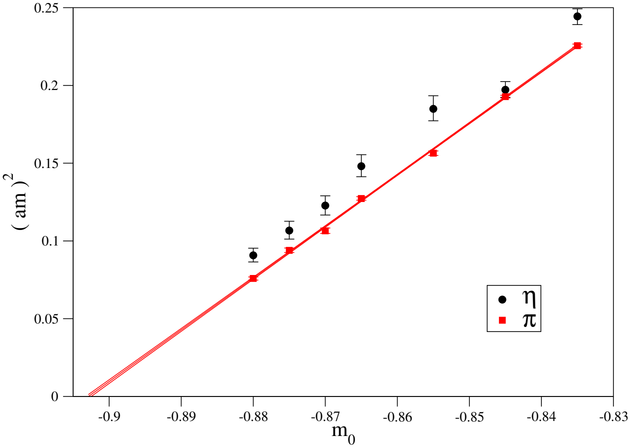
Here, we calculate this unphysical particle, dubbed π, to define the critical quark mass m_c via its vanishing $m_\pi = 0$.
$\longrightarrow$ We define:

$$m_q = m_0 - m_c \ , \quad m_c = m_0|_{m_\pi = 0}$$

We perform a 1-exp. fit for m_π in $m_0 \in [-0.880 : -0.845]$ and extrapolate linearly in $m_0 \to m_c$.

We find:

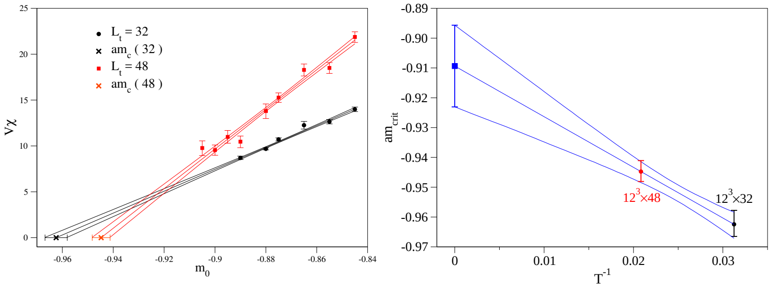
$$\boxed{m_c = -0.9029(4)}$$



We find:

$$m_c = -0.9029(4)$$

Note, the difference $\frac{m_\eta^2 - m_\pi^2}{\Lambda_{\overline{MS}}^2} \approx 0.25$ remains finite even at $m_q = 0$



Alternatively, m_c can be defined via the topological susceptibility:

$$\chi = \frac{\chi_c}{\sum_f^{N_f} m_f^{-1}} \quad \underset{N_f=1}{\longrightarrow} \quad \chi = \chi_c m, \quad m_c = m_0\big|_{\chi=0}$$

Using the `slab`-method to determine χ and extrapolating $T = 32$ and 48, we find:

$$m_c = -0.909(14)\ [\chi] \quad \text{vs.} \quad m_c = -0.9029(4)\ [m_\pi]$$

$\Longrightarrow$ Both values are in agreement, we use the one determined via m_π.

Determining a scale

Dimensionful quantities given in units a, e.g. am_{had} or af_{had}.
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No physical scale is known for our dark sector model. We use: **dark confinement scale** $\Lambda_{\overline{\mathbf{MS}}}$ set via the dark string tension $\sqrt{\sigma}$.

For one-flavor SU(2) we find in perturbation theory:

$$\Lambda_{\overline{\text{MS}}} = 0.7712\sqrt{\sigma}$$

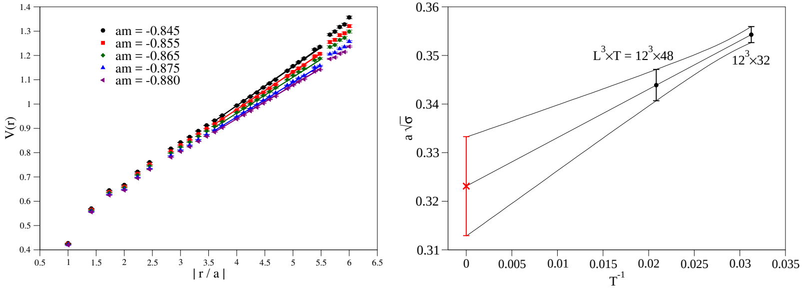
$\sqrt{\sigma}$ is the slope of the linear part of the potential between a static quark-antiquark pair, calculable on the lattice via Wilson loops:

$$W(r,\tau) \underset{\tau \gg 0}{=} Ae^{-aV(r)\tau}.$$

by fitting a Cornell-type potential $V(r) = \frac{A}{r} + B + \sigma r$.

We find:

$$\boxed{a\sqrt{\sigma} = 0.323(10) \quad \Longrightarrow \quad a\Lambda_{\overline{\text{MS}}} = 0.249(8)}$$



Here, we perform a combined fit to the Wilson loops on all our ensembles between $m_0 \in [-0.880 : -0.845]$ (including both $T = 32$ and $T = 48$ data)

$$V(r) = B(1 + c_1 m_q) + a^2\sigma(1 + c_2 m_q)r$$

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Hadron masses and decay constants

$$C_{\mathcal{O}_1\mathcal{O}_2}(t) = \sum_n \frac{\langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle}{2m_n}\left(e^{-m_n t} \pm e^{-m_n(T-t)}\right)$$

- hadron masses m_n via exp. fit, here: $m_{\text{ground}} = m_0$
- decay rates from the matrix elements $\langle 0|\mathcal{O}_1|n\rangle$

In particular, we determine:

$$Z_A\langle 0|\mathcal{O}_A^t(0)|\eta\rangle = f_\eta m_\eta, \quad Z_V\langle 0|\mathcal{O}_V^{ai}(0)|\rho^a\rangle = f_\rho m_\rho \hat{e}_i,$$
$$Z_P\langle 0|\mathcal{O}_P(0)|\eta\rangle = f_P\frac{m_\eta^2}{m_q}.$$

Masses $m_{\eta,\rho,\ldots}$ and respective decay rates via combined multi-state (n=3) fits. The fit parameters are linked to the decay constants via:

$$f_\eta = Z_A A_{AP}\sqrt{\frac{2}{m_\eta^1}}, \quad f_\rho = Z_V A_{VV}\sqrt{\frac{2}{m_\rho^1}}, \quad f_P = Z_P A_{PP}\sqrt{\frac{2m_q}{(m_\eta^1)^2}}$$

Performing n=1 state fits gave compatible, yet less precise results.

$$C_{\mathcal{O}_1\mathcal{O}_2}(t) = \sum_n \frac{\langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle}{2m_n}\left(e^{-m_n t} \pm e^{-m_n(T-t)}\right)$$

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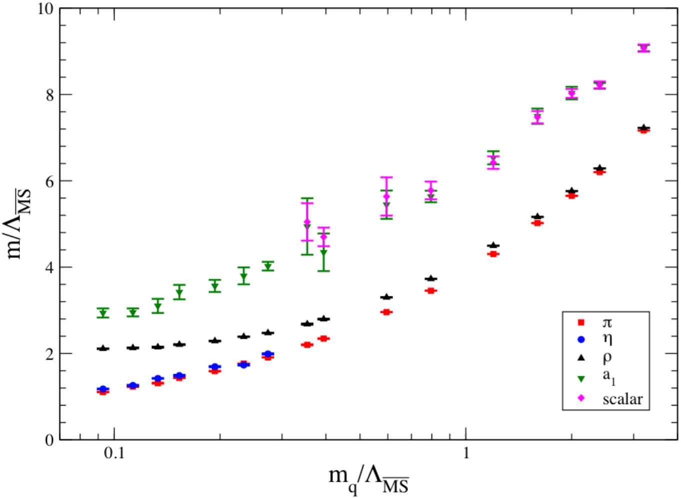
The renormalisation constants $Z_{A/V/P}$ are known at 1-loop in PT:

$$Z_{A/V/P} = 1 - \frac{g_0^2}{16\pi^2}\frac{3}{4}C_{A/V/P}$$

with coefficients $C_A = 15.7$, $C_V = 20.62$, and $C_P = -6.71$.

Martinelli and Zhang, '83; Del Debbio et al.,'08; Hietanen et al., '14

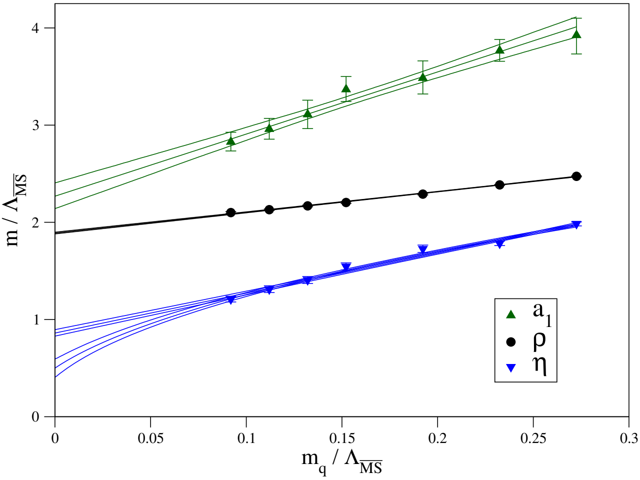
Mass spectrum - results



- QCD-like hierarchy: $m_P < m_V < m_A < m_S$
- HQ region: hyperfine splittings shrink

Note, possible large cut-off effects at $am_q \gtrsim 1$

Mass spectrum - "chiral" extrapolations



- Extrapolations linear and quadratic in m_q. Better χ^2/d.o.f. for quadratic in m_η.

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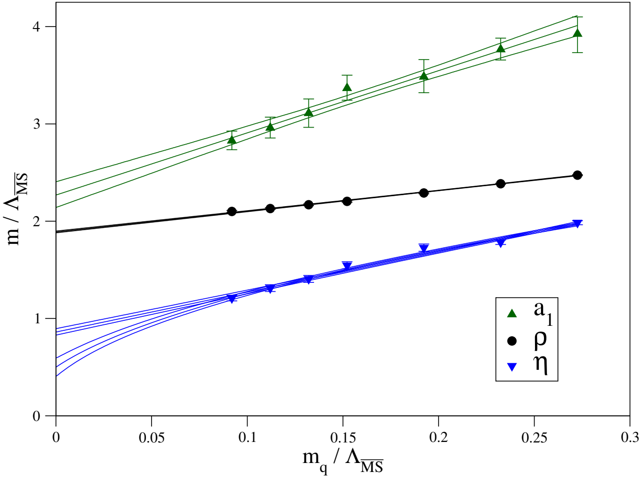
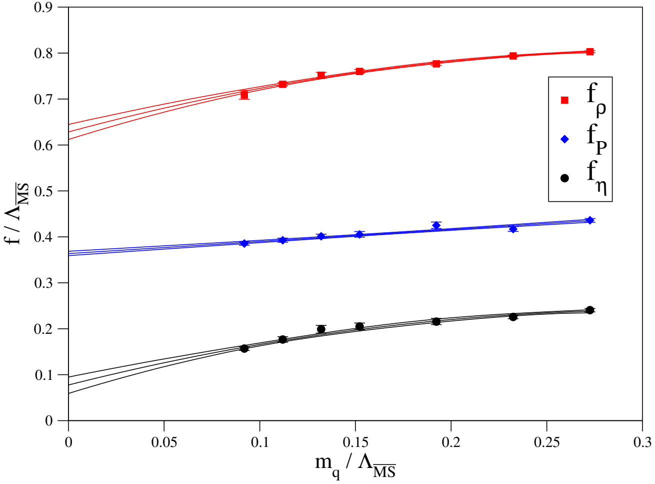


At $m_q = 0$ we find:

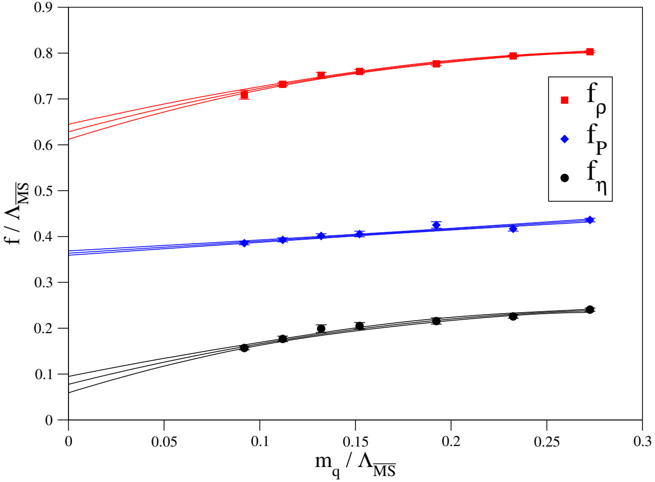
$$\boxed{m_\eta/\Lambda_{\overline{MS}} = 0.50(9)/0.86(3),\ \ m_\rho/\Lambda_{\overline{MS}} = 1.889(9),\ \ m_{a_1}/\Lambda_{\overline{MS}} = 2.27(13)}$$

Decay constants - "chiral" extrapolations



- Extrapolations linear and quadratic in m_q. Better description for quadratic in f_η and f_ρ.

Decay constants - "chiral" extrapolations



At $m_q = 0$ we find:

$$\boxed{f_\eta/\Lambda_{\overline{MS}} = 0.078(18), \quad f_\rho/\Lambda_{\overline{MS}} = 0.628(16), \quad f_P/\Lambda_{\overline{MS}} = 0.364(5)}$$

Dark sector phenomenology

Direct detection and Higgs decay

- DM candidate is an iso-triplet vector ρ_μ^a
- At lowest order $\mathcal{O}_{S,P}$ can couple to the Higgs
- CP phase ϕ is arbitrary $\longrightarrow$ treat scales $M_S = M/\cos\phi$ and $M_P = M/\sin\phi$ separately

Low-energy effective DM Lagrangian (including the scalar operator):

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{4}\rho^a_{\mu\nu}\rho^{a\mu\nu} + \frac{1}{2}m_\rho^2\,\rho^a_\mu\rho^{a\mu} - \frac{1}{2}\lambda_S\,\rho^a_\mu\rho^{a\mu}\left(|H|^2 - \tfrac{1}{2}v^2\right)$$

with $\rho^a_{\mu\nu}$ is the field strength tensor and $\lambda_S = \langle\rho|\bar{q}q|\rho\rangle/M_S$ is a coupling term (which we can determine from our data).

Note, this looks like hidden vector DM coupled via the Higgs portal. There, the Higgs interaction often sets the DM relic abundance, which limits λ_S. Here, this is not the case. The strong dynamics within the dark sector sets the relic density. Further details later on.

The spin-independent DM-nucleon cross section is

$$\sigma_{\rho N} = \frac{\lambda_S^2 m_N^4 f_N^2}{4\pi m_h^4 (m_\rho + m_N)^2}$$

(m_h: Higgs mass, $f_N \approx 0.3$: Higgs-nucleon coupling, $\lambda_S = \langle\rho|\bar{q}q|\rho\rangle / M_S$)

Using the Feynman-Hellman theorem:

$$\langle\rho|\bar{q}q|\rho\rangle = \frac{\partial m_\rho^2}{\partial m_q} = 2m_\rho \underbrace{\frac{\partial m_\rho}{\partial m_q}}_{f_S}$$

f_S can be calculated from our data by

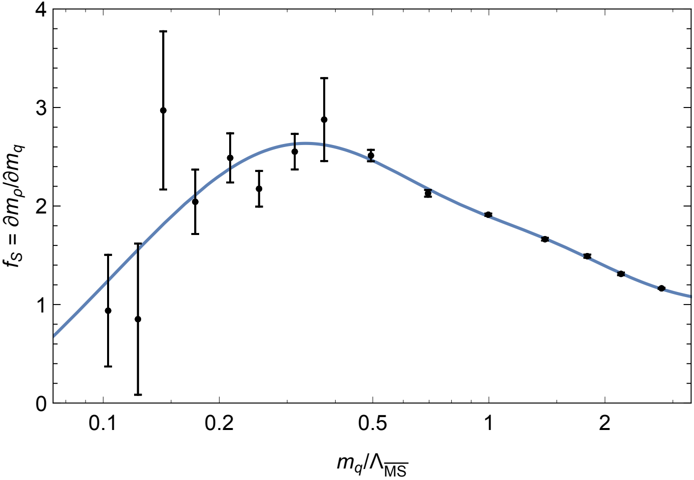
- Fitting our data for m_ρ and taking the derivative
- Computing the derivative via finite differences

We find:

$$\boxed{f_S \approx 1 - 3}$$

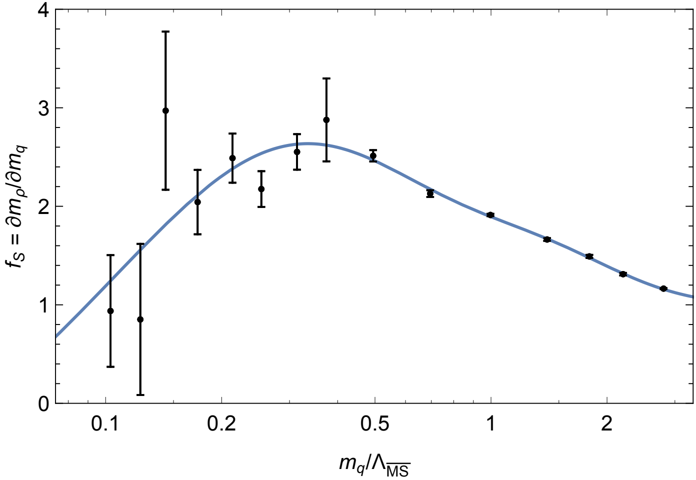
(We expect $f_S \approx 2$ for large m_q as $m_\rho \approx 2m_q$.)

Scalar form factor



- Beware of possibly lattice artefacts for $m_q \gg$
- Beware of systematics from $m_q \to 0$ extrapolation for $m_q \ll$

Scalar form factor



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(m_h: Higgs mass, $f_N \approx 0.3$: Higgs-nucleon coupling, $\lambda_S = 2m_\rho f_S / M_S$, $\boxed{f_S \approx 1 - 3}$)

Direct detection limits most constraining for weak-scale DM mass.

- XENON1T: upper bound for spin-independent cross section.
- $\sigma_{\rho N} = 4.1 \times 10^{-47}$ cm^2 for 30 GeV DM mass
- $\boxed{M_S > 28 \text{ TeV}}$

Higgs@LHC constraings for low mass DM.

- Higgs decays into (long-lived) dark states that escape detection
- we have the Higgs invisible width:

$$\Gamma(h \to \mathrm{inv}) = \frac{m_h v^2}{4\pi M^2}$$

(Assuming $m_q, \Lambda_{\overline{\mathrm{MS}}} \ll m_h/2$ and all dark states escape. Very different to hidden vector models, where $\Gamma(h \to \mathrm{inv}) \propto m_{\mathrm{DM}}^{-4}$)

- Constraint on Higgs invisible branching fraction below 23%
- $\boxed{M > 40\ \mathrm{TeV}}$

Impact of the dark η

- η couples via $\mathcal{O}_P|H|^2 \Longrightarrow$ meta-stable dark state
- strongly constrained from CMB, if they decay to visible SM particles

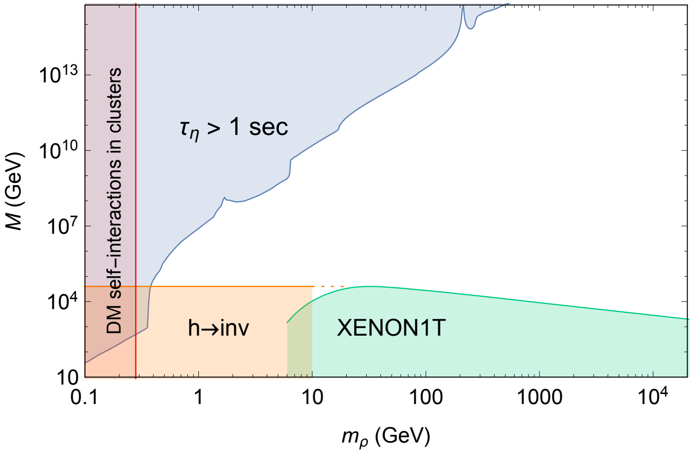
The total η width is (We restrict $\tau_\eta < 1$ s.)

$$\Gamma_\eta = \tau_\eta^{-1} = \sin^2\theta_{h\eta}\Gamma_h(m_\eta) + \Gamma(\eta \to hh)$$

The mixing angle $\theta_{h\eta}$ is: $\tan 2\theta_{h\eta} = \frac{2v\langle 0|\mathcal{O}_P|\eta\rangle}{M_P(m_h^2 - m_\eta^2)}$, $\langle 0|\mathcal{O}_P|\eta\rangle$ from our data

Invisible Higgs decay and $\tau_\eta < 1$ s yields the lower limits

$$\boxed{m_\eta > 228 \text{ MeV} \quad \text{and} \quad m_\rho > 320 \text{ MeV}}$$



- chose: $m_q/\Lambda_{\overline{\rm MS}} = 0.1$, $\phi = \pi/4$, $m_\eta \approx 0.57 m_\rho$, $f_P \approx 0.39$, $f_S \approx 1$
- other choices do not greatly shift the shaded regions

Self-interactions

- dark scattering through strong dynamics
- These self-interactions can leave an observable imprint on DM halos
- need: $\sigma_{\text{elastic}}/m$

We assume $\sigma_{\text{elastic}} \sim 4\pi\Lambda_{\overline{\text{MS}}}^{-2}$ ($\Lambda_{\overline{\text{MS}}}$ sets size of ρ) and use $m_\rho > 2\Lambda_{\overline{\text{MS}}}$ to give the lower bound:

$$\boxed{\sigma_{\text{elastic}}/m \gtrsim 16\pi/m_\rho^3}$$

Relaxed massive clusters provide the strongest constraint

- $\sigma_{\text{elastic}}/m \approx 0.1\text{cm}^2/g$
- we take $\sigma_{\text{elastic}}/m < 0.5\text{cm}^2/g$
- $\boxed{m_\rho > 280\ \text{MeV}}$

$\rho\rho \to \eta\eta$ annihilation and relic density

- In the early universe the dark sector forms a thermal dark QGP, similar to QCD.
- DM relic density may be frozen out before or after T_c^{dark}

In the "after" case, i.e. $m_q \lesssim \Lambda_{\overline{\text{MS}}}$, our model features:

- $\rho\rho \to \eta\eta$ annihilation and subsequent SM decay
- can set the relic density ...
- ... if kinematically allowed, $m_\rho > m_\eta$.
- We find this holds for all of our values of m_q. (Unlike QCD where $m_\rho < m_{\eta'}$)

Conclusions

One-flavor SU(2) theory as (minimal) composite dark matter model.

Features:

- dark sector hadrons are mostly singlet
- DM candidate is part of iso-triplet $\rho^{\pm,0}$

Lattice study:

- determined the scale $\Lambda_{\overline{\mathsf{MS}}}$ via σ_{dark}
- singlet/triplet hadron masses and decay constants
- chiral and heavy m_q regimes
- scalar form factor via Feynman-Hellman

Neglected systematics/to-do list $\Longrightarrow$ *Future updates*

- lattice spacing effects
- finite volume effects
- dark nuclear physics / dark scattering

Conclusions

One-flavor SU(2) theory as (minimal) composite dark matter model.

DM stability:

- accidental $\mathrm{SU}(2)_B$ baryon number is preserved including dim.-5
- DM candidate is as stable as the proton

CP violation and η decay:

- w/ dim.-5 op. there is a CP phase that mixes the η with the Higgs
- rapid decay of η in the early universe, before nucleosynthesis

Annihilation channel:

- efficient annihilation to set the DM relic density, $\rho\rho \to \eta\eta$
- kinematically allowed since we observe $m_\rho > m_\eta$ for any quark mass

Thank you for your attention!