The QCD Axion: A Candidate for Dark Matter?

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arXiv:1802.09339

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The Strong CP Problem

Quantum chromodynamics (QCD) decribes the strong interactions remarkably well down to the smallest scales probed so far. Yet it faces a problem. The theory allows for a CP-violating term S_θ in the action,

$$S = S_{\rm QCD} + S_\theta$$

the so-called θ term. In Euclidean space-time S_θ reads

$$S_\theta = i\,\theta\,Q\,, \quad Q = \int d^4x\, q(x) \;\in\; \mathbb{Z}$$

where Q is the topological charge with charge density

$$q(x) = -\frac{1}{64\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,F^a_{\mu\nu}(x)\,F^a_{\rho\sigma}(x)$$

In this formulation θ enters as an arbitrary phase with values $\theta \in [0, 2\pi)$. The problem is that no CP violation has been observed in the strong interactions.

A nonvanishing value of the vacuum angle θ would result in an electric dipole moment d_n of the neutron

Nucleon EM current

$$\langle p', s' | J_\mu | p, s \rangle = \bar{u}(\vec{p}\,', s')\, \mathcal{J}_\mu\, u(\vec{p}, s)$$

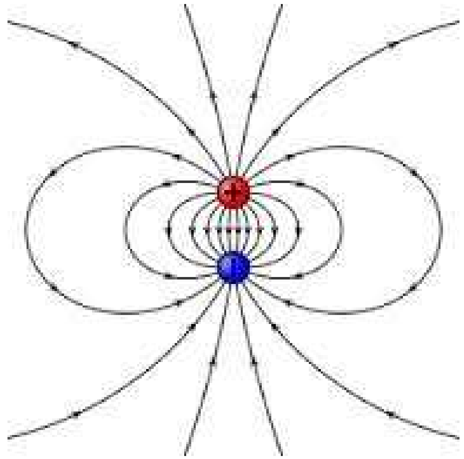
CP violating ↓

$$\mathcal{J}_\mu = \gamma_\mu\, F_1(q^2) + \sigma_{\mu\nu}\, q_\nu\, \frac{F_2(q^2)}{2m_N} + (\gamma q\, q_\mu - \gamma_\mu\, q^2)\, \gamma_5\, F_A(q^2) + \sigma_{\mu\nu}\, q_\nu\, \gamma_5\, \frac{F_3(q^2)}{2m_N}$$

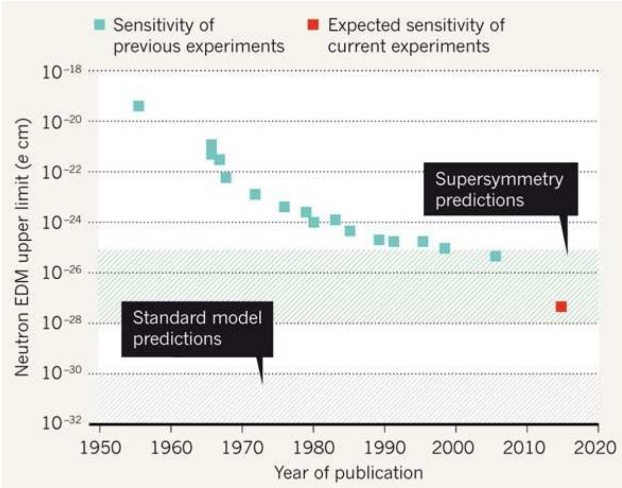
anapole dipole

Dipole moment

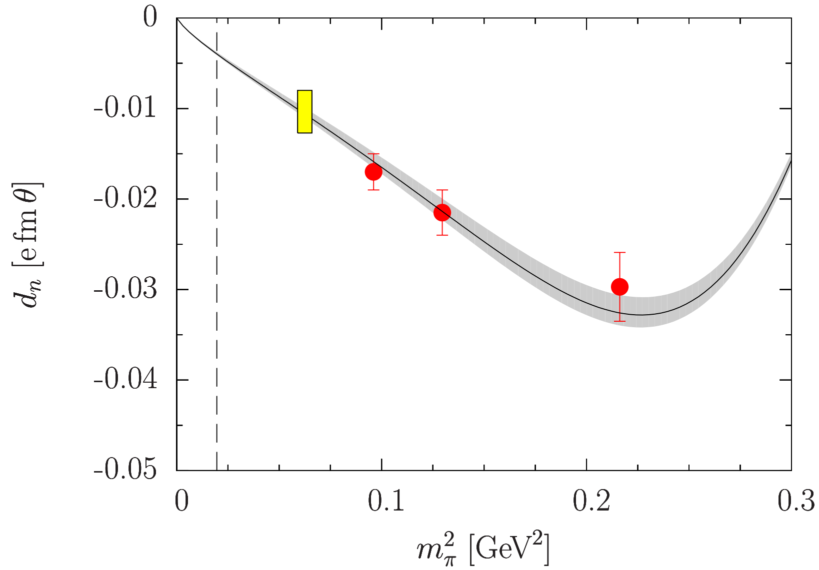
$$d_n = \frac{e\, F_3(0)}{2m_N} \;\propto\; e_q\, \ell$$



Experiment



Lattice



arXiv:1502.02295

Current experimental limits on $|d_n|$, paired with lattice calculations, lead to the upper bound $|\theta| \lesssim 7.4 \times 10^{-11}$. This anomalously small number is referred to as the strong CP problem, which is one of the most intriguing problems in particle physics.

The Case for Axions

In the Peccei-Quinn theory the CP violating action S_θ is augmented by the axion interaction

$$S_\theta \to S_\theta + S_{\mathrm{Axion}} = \int d^4x \left[\frac{1}{2}\left(\partial_\mu \phi_a(x)\right)^2 + i \underbrace{\left(\theta + \frac{\phi_a(x)}{f_a}\right) q(x)}_{\text{dimension five !}}\right]$$

Under a $U_{\mathrm{PQ}}(1)$ transformation the axion field translates to

$$\phi_a(x) \to \phi_a(x) + \delta f_a$$

which leaves the classical action invariant and is called shift symmetry. Transforming $\phi_a(x)$ to $\phi_a(x) - \theta f_a$ cancels the CP violating term in the action. This leaves us with the action

$$S = S_{\mathrm{QCD}} + S_{\mathrm{Axion}}\,, \quad S_{\mathrm{Axion}} = \int d^4x \left[\frac{1}{2}\left(\partial_\mu \phi_a(x)\right)^2 + i\,\frac{\phi_a(x)}{f_a}\,q(x)\right]$$

It is expected that QCD induces an effective potential for ϕ_a, $U_{\mathrm{eff}}(\phi_a)$, whose minimum is at $\phi_a = 0$, thus restoring CP symmetry.

A necessary condition for the Peccei-Quinn theory to solve the strong CP problem is that QCD allows basically all values of ϕ_a/f_a to exist

This appears not to be the case. Writing

$$\bar{\phi}_a = \frac{1}{V} \int d^4x\, \phi_a(x)$$

the range of allowed ϕ_a values can be estimated from the effective theory with Gaussian distributed topological charge Q, described by the partition function

$$Z = \frac{1}{\sqrt{2\pi\langle Q^2\rangle}} \int dQ\, d\bar{\phi}_a\, \exp\left\{ -Q^2/2\langle Q^2\rangle - i\,(\bar{\phi}_a/f_a)\,Q - (m_a^2/2)\,\bar{\phi}_a^2\, V \right\}$$

including a hypothetical mass term. This predicts

$$\langle \bar{\phi}_a^2 \rangle \propto \frac{1}{(\chi_t/f_a^2 + m_a^2)\, V}\,, \quad \chi_t = \langle Q^2\rangle/V$$

stating that ϕ_a is expected to assume small values only, which decrease with the inverse power of the volume

~~shift symmetry~~

Axion Phenomenology: The Common Lore

If the shift symmetry, $\phi_a \to \phi_a + \delta f_a$, were exact, the axion would be exactly massless, $m_a = 0$, and any value of $\bar{\phi}_a$ would be equally acceptable from the energetic point of view

The basic assumption is that the shift symmetry gets spontaneously broken by the vacuum energy U_{eff}

$$Z(\bar{\phi}_a) = \frac{1}{\sqrt{2\pi\langle Q^2\rangle}} \int dQ \, \exp\left\{-Q^2/2\langle Q^2\rangle - i\,(\bar{\phi}_a/f_a)\,Q\right\} = \exp\left\{-V\,U_{\text{eff}}(\bar{\phi}_a)\right\}$$

which leads to

$$U_{\text{eff}}(\bar{\phi}_a) = \chi_t\left[1 - \cos(\bar{\phi}_a/f_a)\right] \approx (\chi_t/2f_a^2)\,\bar{\phi}_a^2$$

generating a mass for the axion

$$m_a^2 = \frac{\partial^2}{\partial\bar{\phi}_a^2} U_{\text{eff}}(\bar{\phi}_a)\Big|_{\bar{\phi}_a=0} = \frac{\chi_t}{f_a^2}$$

No local interaction with QCD !

Accordingly, the QCD axion is interpreted as the Goldstone boson of the broken $U_{\text{PQ}}(1)$ symmetry

Axion mass

$T = 0$

$$m_a^2 = \frac{\chi_t}{f_a^2} \approx \frac{1.2 \cdot 10^{-3}\,\text{GeV}^4}{f_a^2}$$

$T_c > T > 0$

$$\frac{m_a^2(T)}{m_a^2} = \frac{\chi_t(T)}{\chi_t} = 1 - \frac{3T^2}{2f_\pi^2} J_1\left(\frac{m_\pi^2}{T^2}\right)$$

Gasser & Leutwyler

$T \gg T_c$

$$\frac{m_a^2(T)}{m_a^2} = \frac{\chi_t(T)}{\chi_t} \approx 1.8\left(\frac{T_c}{T}\right)^3$$

$T \lesssim 1\,\text{GeV}$ ↑ Axions begin to oscillate

$f_a \approx 1 \cdot 10^{12}\,\text{GeV}$ ↑ Axions only source of dark matter

Bonati et al.

[Borsanyi et al.]

In the following we will treat the axion field as a dynamical degree of freedom with the purpose to solve the strong CP problem, whether it arises from the spontaneously broken $U_{\mathrm{PQ}}(1)$ Peccei-Quinn symmetry or from a more fundamental theory, and focus on QCD interactions

$$\text{Cut-off} \approx \frac{\pi}{a} = 8\ \text{GeV}$$

The QCD Axion on the Lattice

In Euclidean quantum field theory the axion mass m_a is given by the large-time decay of the correlation function

$$\int d^3\vec{x}\ \langle \phi_a(\vec{x},t)\, \pi(0)\rangle \simeq A\, \mathrm{e}^{-m_a t} \qquad \pi\ :\ \text{any pseudoscalar source}$$

with the equation of motion Peskin & Schroeder

$$\frac{\partial^2}{\partial t^2} \int d^3\vec{x}\ \langle \phi_a(\vec{x},t)\, \pi(0)\rangle = \frac{i}{f_a} \int d^3\vec{x}\ \langle q(\vec{x},t)\, \pi(0)\rangle\ , \quad t > 0$$

Taking $\pi = q$ and employing the axial anomaly, this leads to

$$m_a = -\lim_{t\to\infty} \frac{1}{t} \log \int d^3\vec{x}\, \langle q(\vec{x},t)\, q(0)\rangle = -\lim_{t\to\infty} \frac{1}{t} \log \int d^3\vec{x}\, \langle P(\vec{x},t)\, P(0)\rangle$$

with $P = (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \cdots)$. Thus, the axion will strongly mix with the pseudoscalar meson sector

Action

At finite lattice spacing a the field-theoretic topological charge Q is ill defined. The topological interaction term can be rotated into the quark mass matrix using the axial anomaly, which preserves the shift symmetry. Neglecting operators of dimension six and higher:

$$S_{\text{Axion}} = a^4 \sum_x \Big[\frac{1}{2}\big(\partial_\mu \phi_a(x)\big)^2 - i\,\frac{\phi_a(x)}{3 f_a}\,\hat{m}\big(\bar{u}(x)\gamma_5 u(x) + \bar{d}(x)\gamma_5 d(x) + \bar{s}(x)\gamma_5 s(x)\big)\Big]$$

with $\hat{m}^{-1} = \left(m_u^{-1} + m_d^{-1} + m_s^{-1}\right)/3$. S_{Axion} is complex, but lends itself to numerical simulations for imaginary values of the axion decay constant $f_a^* = i f_a$. As $\langle \bar{\phi}_a^2 \rangle \propto 1/V$, the result can be analytically continued to real numbers of f_a for sufficiently large volumes. This leaves us with

$$S_{\text{Axion}} = a^4 \sum_x \Big[\frac{1}{2}\big(\partial_\mu \phi_a(x)\big)^2 + \frac{\hat{m}}{3}\,\frac{\phi_a(x)}{f_a^*}\,\big(\bar{u}(x)\gamma_5 u(x) + \bar{d}(x)\gamma_5 d(x) + \bar{s}(x)\gamma_5 s(x)\big)\Big]$$

PQWW/DFSZ action

By a redefinition of the quark fields $\psi_f(x)$

$$\psi_f(x) \to \exp\left\{-i\gamma_5 \frac{\phi_a(x)}{f_a}\frac{c_f}{2}\right\} \psi_f(x)\,, \quad c_f = \frac{\hat{m}}{3m_f}$$

the topological interaction term

$$\frac{\phi_a(x)}{f_a}\, q(x)$$

in S_{Axion} is eliminated and moved into the quark mass matrix

$$m_u\, \bar{u}(x) \exp\left\{-i\gamma_5\, c_u \frac{\phi_a(x)}{f_a}\right\} u(x) + m_d\, \bar{d}(x) \exp\left\{-i\gamma_5\, c_d \frac{\phi_a(x)}{f_a}\right\} d(x) \\ + m_s\, \bar{s}(x) \exp\left\{-i\gamma_5\, c_s \frac{\phi_a(x)}{f_a}\right\} s(x)$$

S. Weinberg, 'The Quantum Theory of Fields', Vol. 2

Simulation parameters

We use nonperturbatively $O(a)$ improved Wilson fermions. As a first step, we focus on the SU(3) flavor symmetric point

$$m_u = m_d = m_s \qquad m_q = 1/2\kappa - 1/2\kappa_c\,,\ \kappa_c = 0.12110$$

with

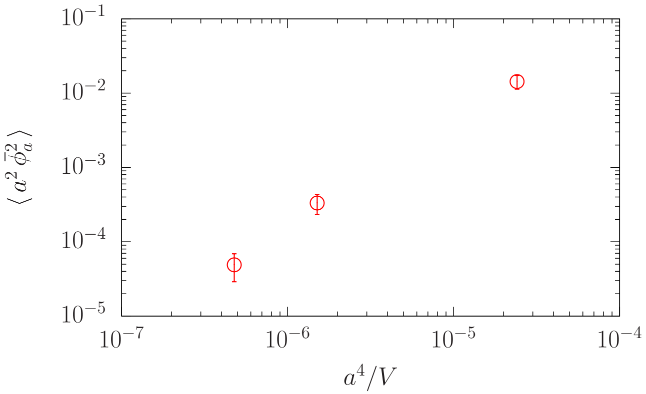
$$m_\pi^2 = m_K^2 = m_\eta^2 = \left(m_\pi^{2\,\mathrm{phys}} + 2m_K^{2\,\mathrm{phys}}\right)/3 \approx [420\,\mathrm{MeV}]^2$$

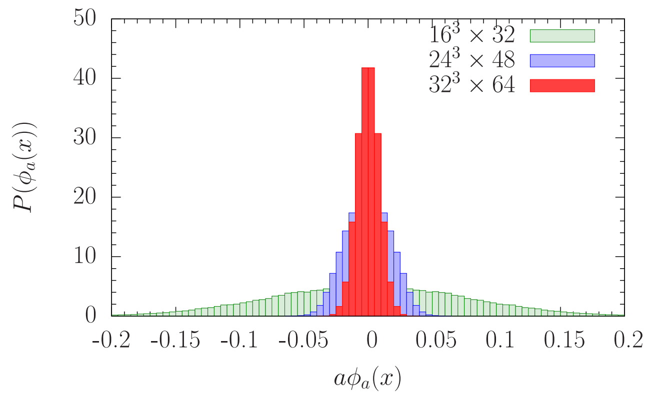
Lattices

#	$a^{-4}V$	κ	$1/af_a^*$
1	$12^3 \times 24$	0.12090	0.01825
2	$12^3 \times 24$	0.12090	0.1825
3	$12^3 \times 24$	0.12090	1.825
4	$24^3 \times 48$	0.12090	0.01825
5	$24^3 \times 48$	0.12090	0.1825
6	$24^3 \times 48$	0.12090	1.825
7	$32^3 \times 64$	0.12090	0.1825

$a = 0.074(2)$ fermi

Remnant (shift) symmetry?

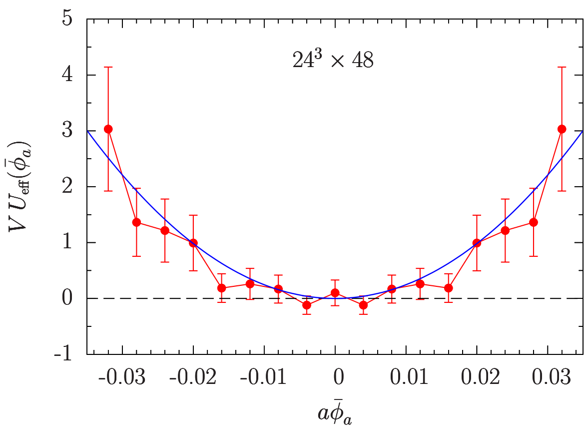




To solve the strong CP problem, the axion field ϕ_a would have to cover the full range $0 \leq |\phi_a/f_a| \leq \pi$

Effective potential

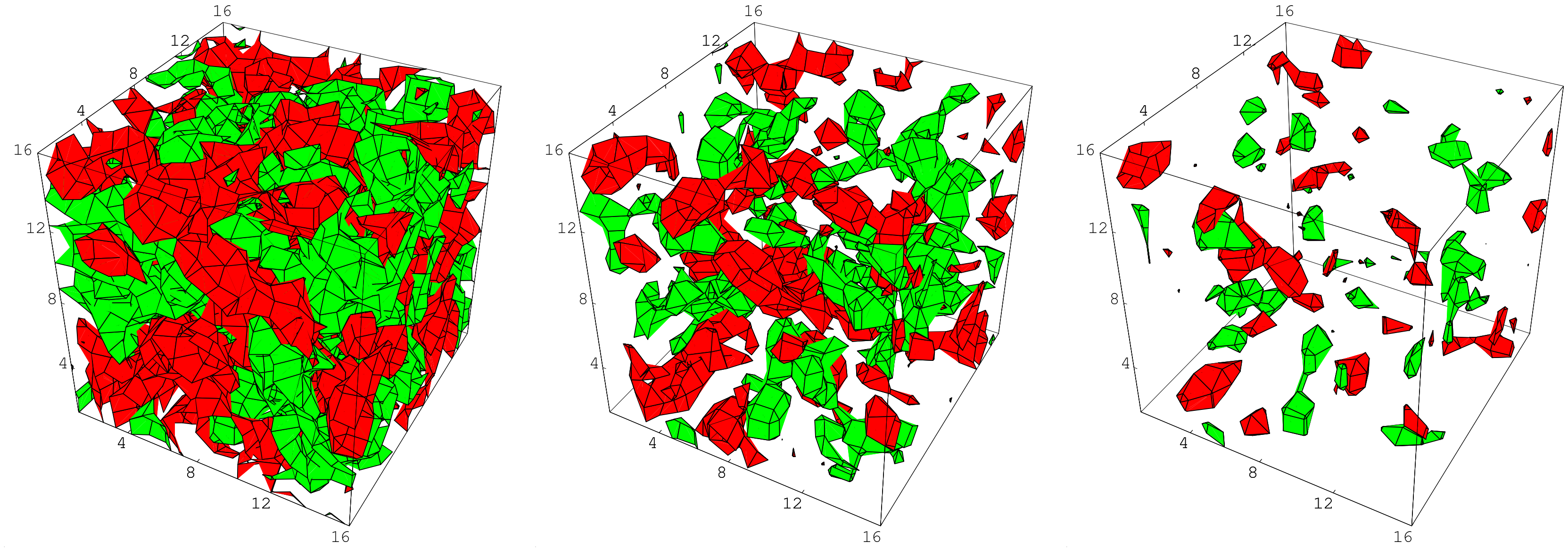
Local interaction with QCD now included !



$$V\,U_{\rm eff}(\bar{\phi}_a) = -\log P(\bar{\phi}_a) + c$$

$$m_a = O(\Lambda_{\rm QCD})$$

Independent of f_a^2



Isosurfaces of positive (red) and negative (green) topological charge density of a single time slice for $|q(x)|/|q_{\rm max}| > 0,\ 0.2$ and 0.3

arXiv:0912.2281

The quantum axion field $\phi_a(x)$ follows the fluctuations of the topological charge density $q(x)$ of QCD

For configurations with total charge $Q = 0$ even !

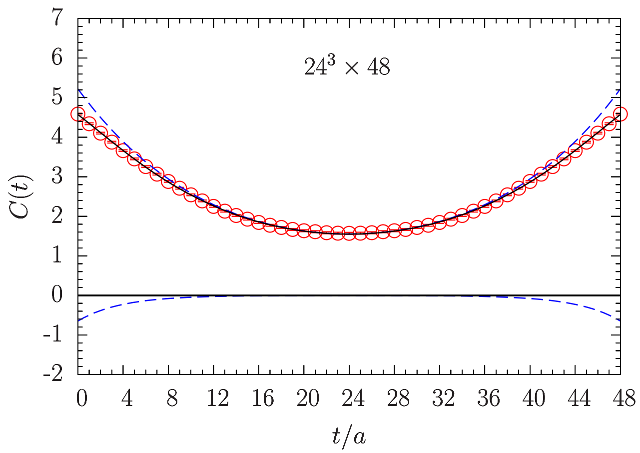
The QCD Axion Mass

The axion mass m_a is obtained from the correlation function $C(t) = a^2 \sum_{\vec{x}} \langle \phi_a(\vec{x}, t)\, \phi_a(0) \rangle$, which we parameterize as

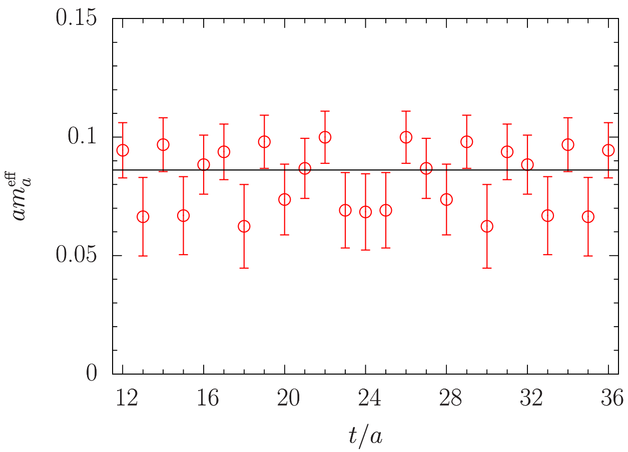
$$C(t) = A \cosh\big(-m_a \tau\big) + B \cosh\big(-m_{\eta'} \tau\big)\,, \quad \tau = t - T/2$$

T is the temporal extent of the periodic lattice

Correlation function



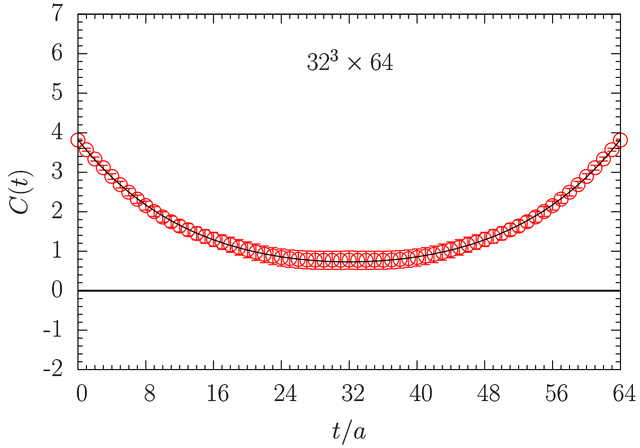
Effective mass



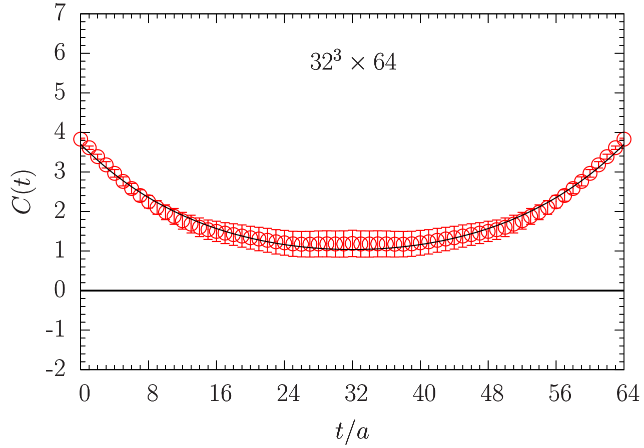
Small, but visible coupling to η' meson

$$am_a^{\rm eff} = \operatorname{arcosh}\Big[\frac{C(t-a)+C(t+a)}{2C(t)}\Big]$$

All



$Q = 0$



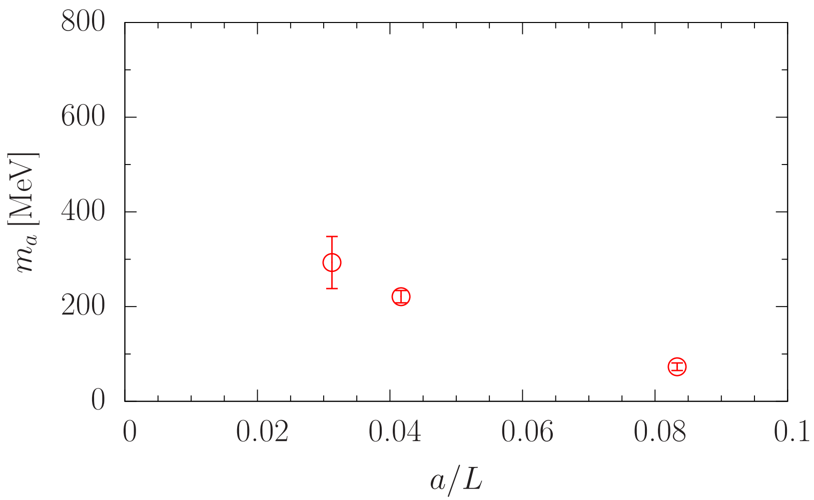
$$\cancel{m_a^2 = \frac{\chi_t}{f_a^2}}$$

The theory undergoes spontaneous symmetry breaking as a result of quantum fluctuations, known as Coleman-Weinberg mechanism

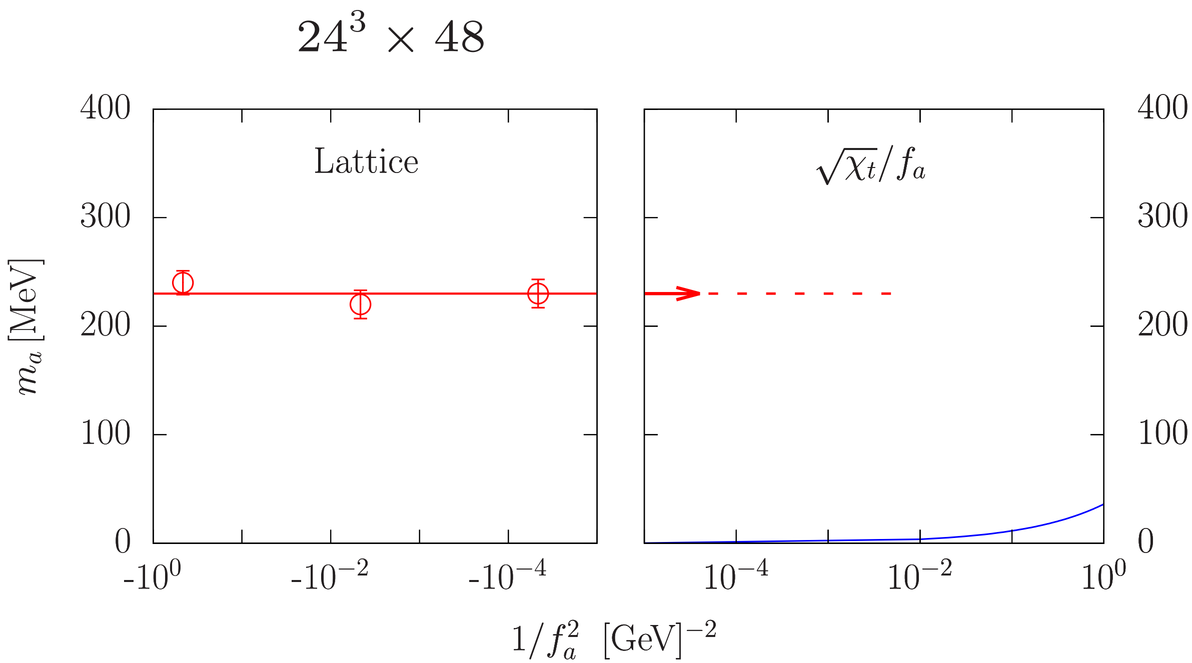
Present results

#	$a^{-4}V$	$1/f_a^*$ [GeV^{-1}]	$\chi_t^{1/4}$ [MeV]	m_a [MeV]	$m_{\eta'}$ [MeV]
1	$12^3 \times 24$	0.0068	119 ± 4	62 ± 2	
2	$12^3 \times 24$	0.068	121 ± 6	73 ± 8	
3	$12^3 \times 24$	0.68	108 ± 8	66 ± 4	
4	$24^3 \times 48$	0.0068	153 ± 11	230 ± 13	700 ± 110
5	$24^3 \times 48$	0.068	148 ± 9	221 ± 13	660^{+50}_{-350}
6	$24^3 \times 48$	0.68	151 ± 8	238 ± 11	670 ± 120
7	$32^3 \times 64$	0.068	152 ± 9	293 ± 55	

$V \rightarrow \infty$: m_a shows strong upward tendency



Continuation to real values of f_a $\quad (1/f_f^2 > 0)$



How is it possible that $m_a \approx$ constant and $m_a = O(\Lambda_{\text{QCD}})$?

- Topological charge density $q(x)$ largely independent of $1/f_a$
- $\phi_a(x)$ follows fluctuations of $q(x)$ — Coleman-Weinberg
- Kinetic term $\big(\partial_\mu \phi_a(x)\big)^2$ in S_{Axion} does not wipe out fluctuations of $\phi_a(x)$

Conclusions

- The axion is a hypothetical particle postulated by the Peccei-Quinn theory to resolve the strong CP problem in QCD. If axions exist and have low mass, they are a candidate for dark matter as well

- So far our knowledge of the properties of axions rested on semi-classical arguments and effective theory. In this work we have subjected the theory to a quantum mechanical test on the lattice for the first time

- Our results on the axion mass, $m_a = O(\Lambda_{\rm QCD})$, are found to be in conflict with current axion phenomenology and experiment. They suggest that the mass is largely generated by quantum fluctuations through a Coleman-Weinberg type mechanism, rather than by the vacuum energy

- A further striking result is that QCD allows only small values of $\phi_a(x)/f_a$ to exist, so that the θ term, $(\phi_a(x)/f_a + \theta)\, q(x)$, will not be able to relax to zero, which thwarts the Peccei-Quinn solution of the strong CP problem

- This questions the validity and use of the Peccei Quinn theory, and the existence of a very light axion, which would qualify as a dark matter candidate