Interdisciplinary approach to QCD-like composite dark matter

ECT* Villazzano 1 October 2018

# Axions and topology in QCD

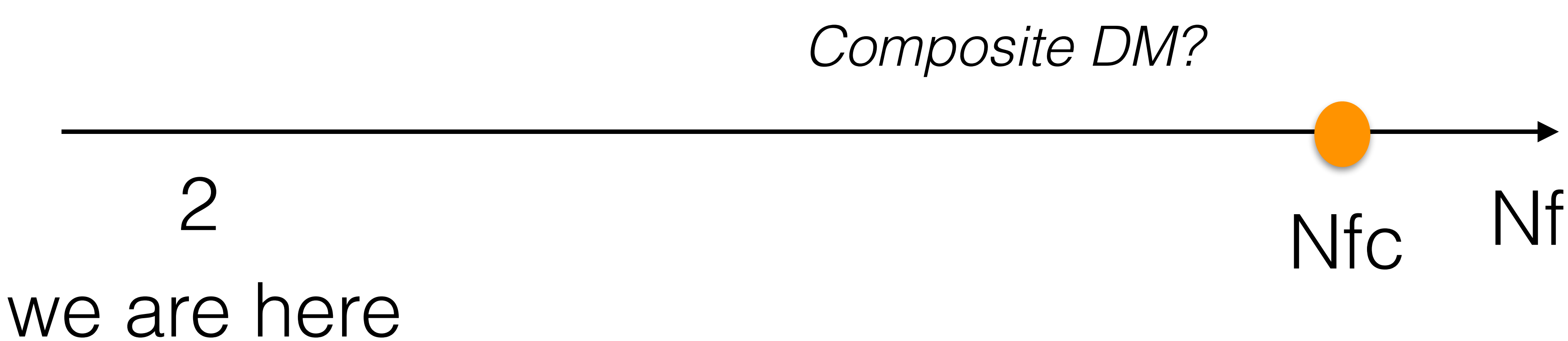
*Maria Paola Lombardo*

INFN Firenze

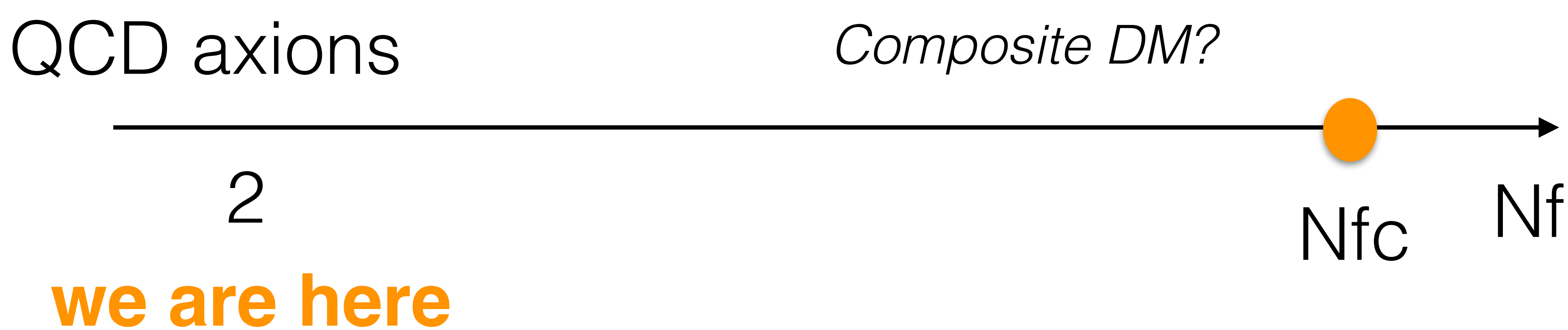
Istituto Nazionale di Fisica Nucleare

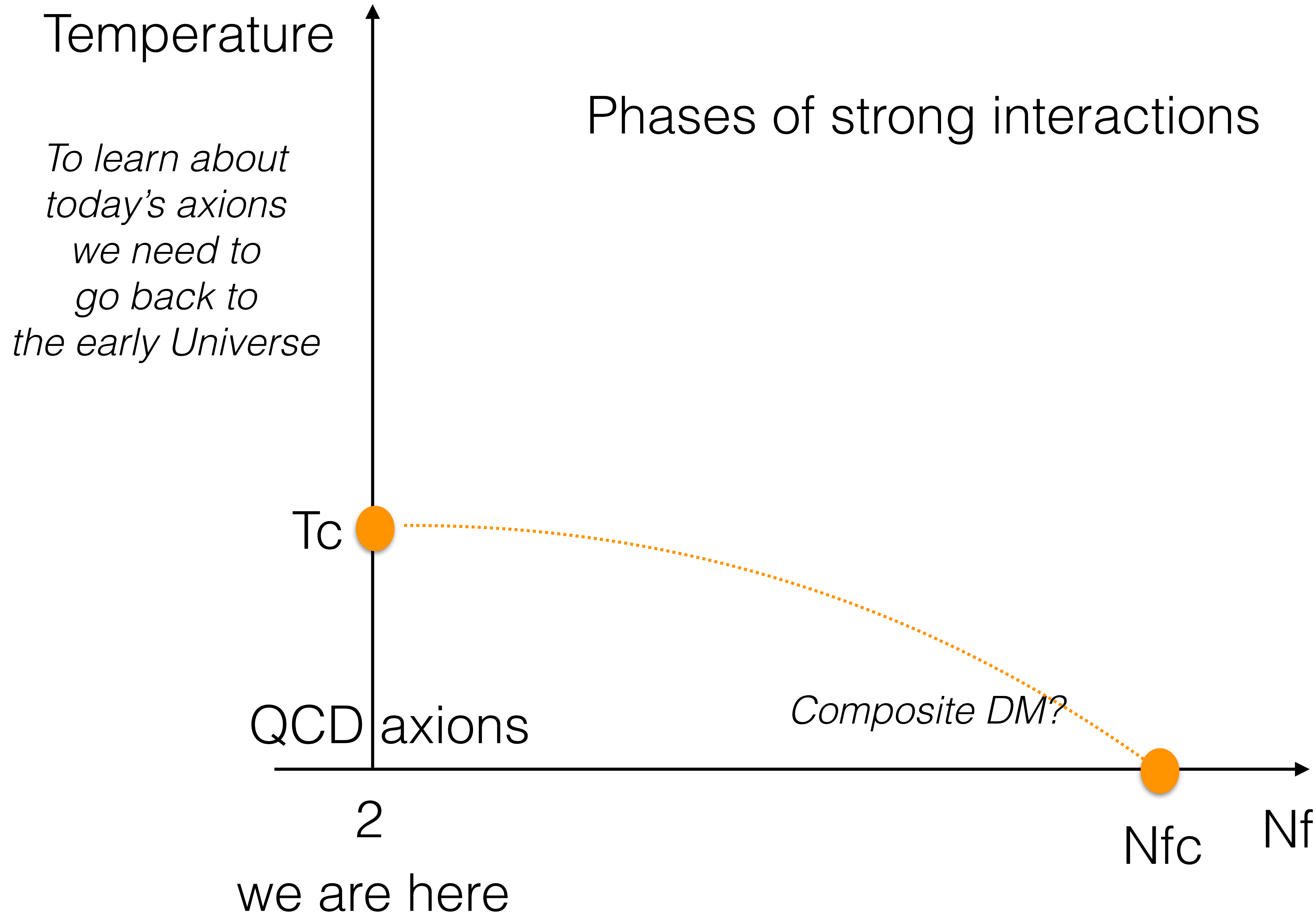
EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

# Phases of strong interactions



## Phases of strong interactions




Temperature
Phases of strong interactions
To learn about today's axions we need to go back to the early Universe
Tc
QCD axions
Composite DM?
2
we are here
Nfc
Nf

# Different entangled topics

QCD phenomenology, hadron spectrum

Axions

History of the Universe -
particle cosmology

Phases and Topology of QCD

Calculational tool: Lattice simulations

Different entangled topics

QCD phenomenology, hadron spectrum

**Axions**

History of the Universe - particle cosmology

Phases and **Topology of QCD**

Calculational tool: Lattice simulations

# Plan

Axions
Topology in QCD

Results:

Topological Susceptibility
Bounds on the QCD axion's mass
Beyond Susceptibility - towards the axion's potential

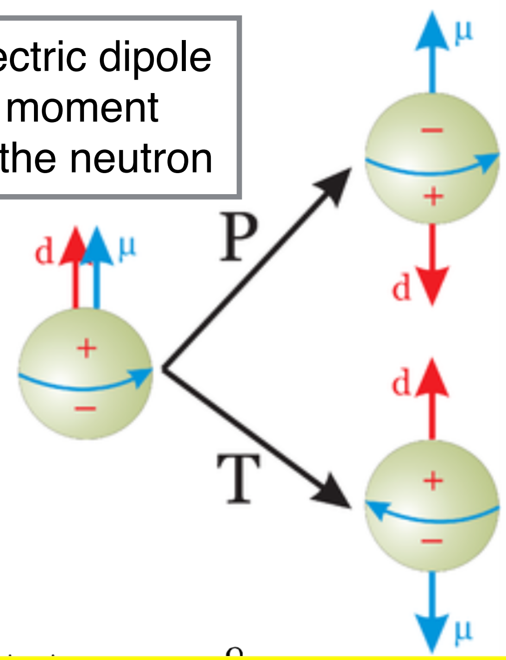
# Axions ‘must’ be there (?)

## $\theta$ term, strong CP problem and topology

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

electric dipole moment of the neutron




$$|d_n| < 2.9 \times 10^{-26} \; e\,\mathrm{cm}$$

$$d_n(\theta) \sim e\,\theta \frac{m_u m_d}{(m_u + m_d)m_n^2} \Rightarrow |\theta| < 10^{-9}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T\sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)]$$

$$\left.\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T) = (< Q^2 > - < Q >^2)/V$$

# Axions 'must' be there: solution to the strong CP problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

Ammitted but $\theta < 10^{-9}$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

Postulate axions, coupled to Q:

$$\mathcal{L}_{\text{axions}} = \frac{1}{2}(\partial_\mu a)^2 + \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T\sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)]$$

Axion potential

$$m_a^2(T)f_a^2 = \left.\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T),$$

$$f_A \gtrsim 4 \times 10^8 \ \text{GeV}$$

weakly coupled

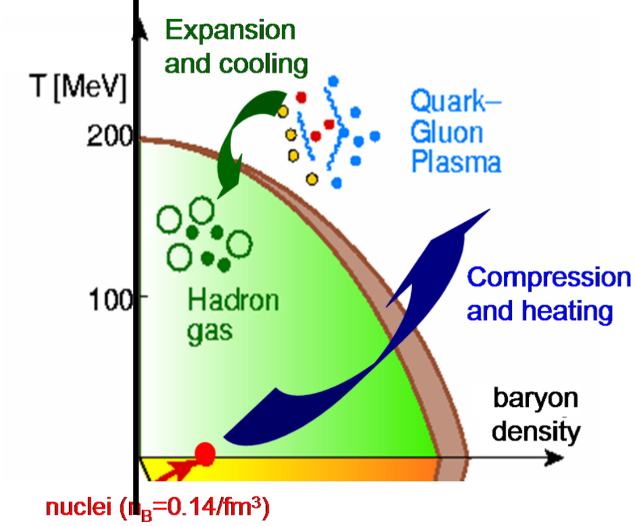
## Temperatures:

150 MeV < T < 500 MeV

..and beyond

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

Quark Gluon Plasma:
Topology



Time from Big Bang

Temperature

Hubble parameter

$$H(T) \simeq T^2/M_P$$

Axions's freezout

$$3H(T) = m_a(T)$$

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

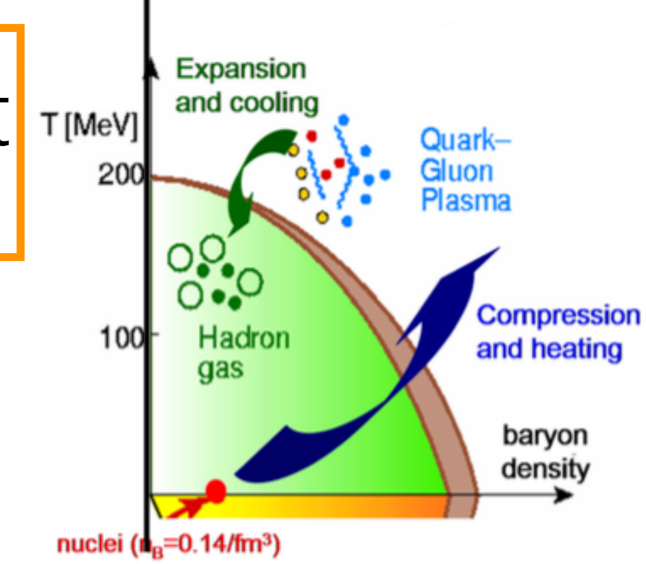
**Quark Gluon Plasma: Topology**

**Axions' mass and density today**

After freezout $\frac{n_a}{s}$ constant

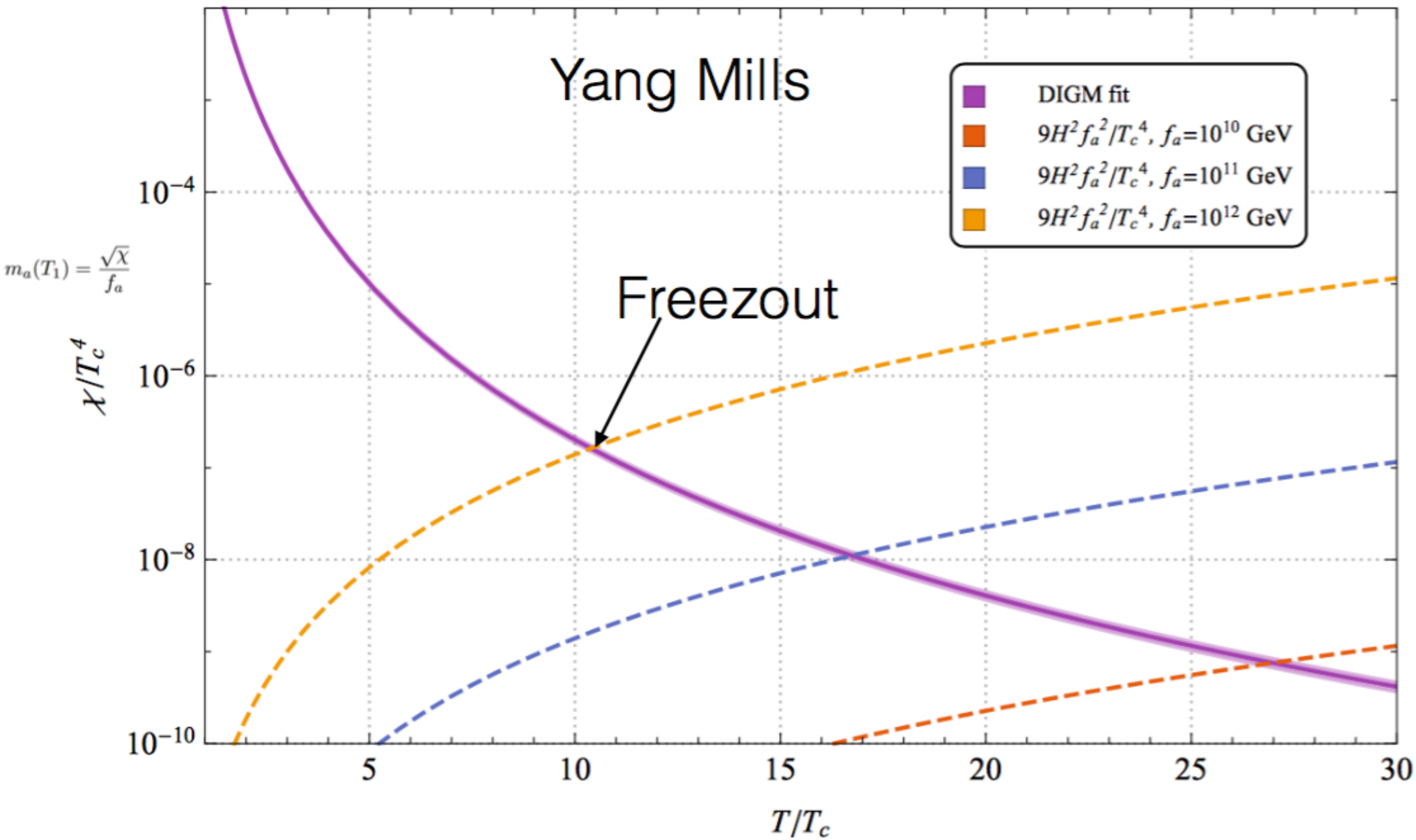
$$\rho_{a,0} = \frac{n_a}{s} m_a s_0$$

Wantz, Shellard 2010



Axion freezout : $3H(T) = m_a(T) = \sqrt{\chi(T)}/f_a$

First numerical study: Berkowitz Buchoff Rinaldi 2015




Axion density at freezout controls axion density today

Cold Dark Matter candidates might have been created after the inflation

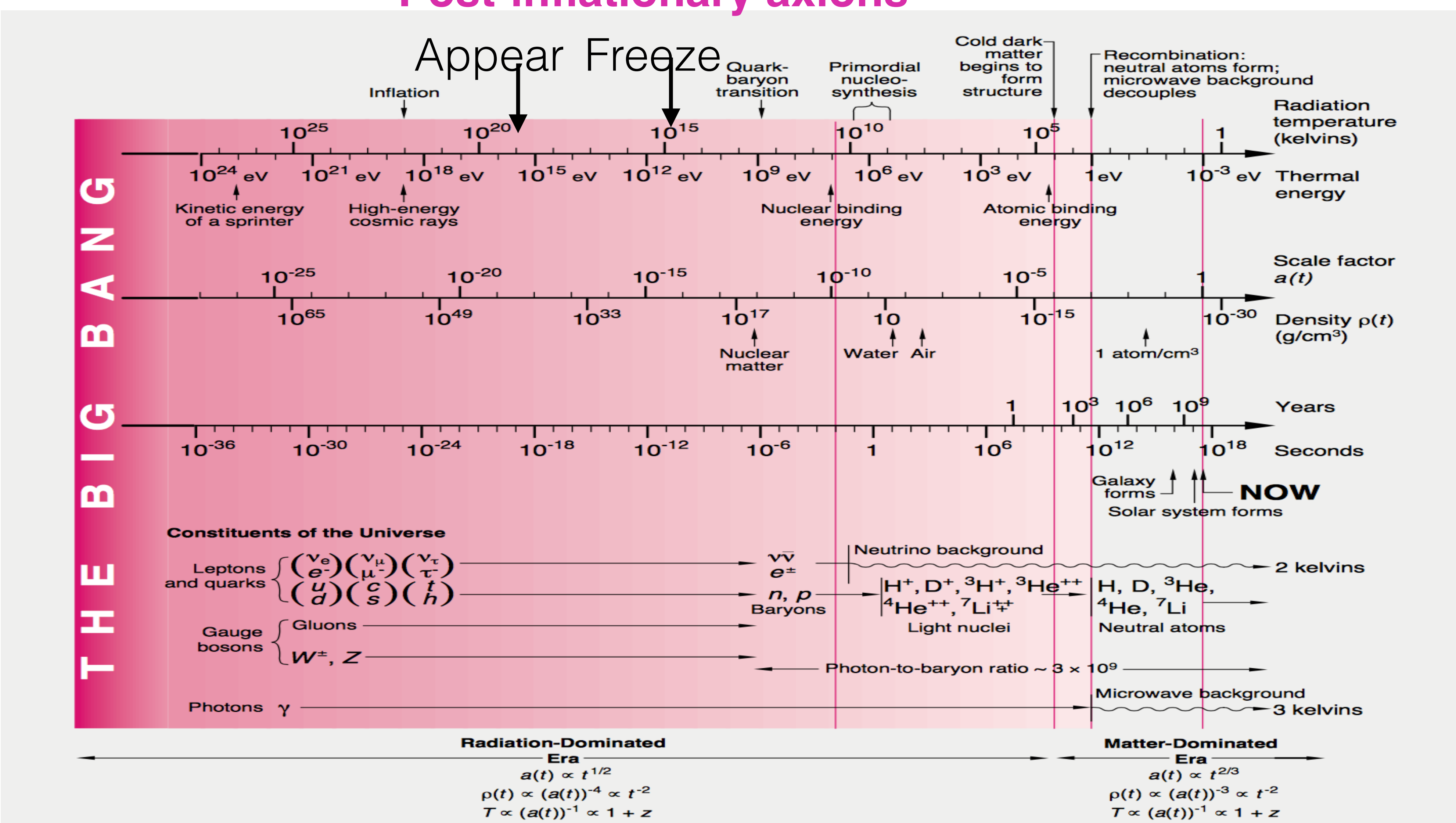
Several CDM candidates are highly speculative - but one, **the axion**, is

**Theoretically well motivated in QCD**
**Amenable to quantitative estimates once QCD topological properties are known:**

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

**Post-inflationary axions**

Appear Freeze



# QCD topology and phenomenology

Hadron cosmology:

Origin of mass

**Almost all hadrons can be described taking into account chiral symmetry breaking and confining potential**

Quarks Hadrons Nuclei ***time***

QCD transition

Nucleosynthesys

Chiral symm. breaking
Confinement:

Chiral perturbation theory +
Potential models
=
Hadron spectrum

Hadron cosmology:

Origin of mass

**Almost all hadrons can be described taking into account chiral symmetry breaking and confining potential**

**With an important exception**

Quarks — Hadrons — Nuclei — ***time***

QCD transition

Nucleosynthesys

Chiral symm. breaking
Confinement:

Chiral perturbation theory +
Potential models
=
Hadron spectrum

Pseudoscalar light spectrum:
eight pseudoGoldstones

$$SU(3)_L X SU(3)_R \to SU(3)_V$$

$\chi PT$ predicts

$$m_\pi^2 \propto (m_u + m_d)\Lambda_{QCD}$$
$$m_K^2 \propto (m_s + m_{u,d})\Lambda_{QCD}$$
$$m_\eta^2 \propto \frac{1}{3}(m_u + m_d + 4m_s)\Lambda_{QCD} \ ,$$

| Particle name | Particle symbol | Antiparticle symbol | Quark content | Rest mass (MeV/c²) |
|---|---|---|---|---|
| Pion[6] | $\pi^+$ | $\pi^-$ | $u\bar{d}$ | 139.570 18 ±0.000 35 |
| Pion[7] | $\pi^0$ | Self | $\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a] | 134.9766 ±0.0006 |
| Eta meson[8] | $\eta$ | Self | $\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ [a] | 547.862 ±0.018 |
| Eta prime meson[9] | $\eta'$(958) | Self | $\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ [a] | 957.78 ±0.06 |
| Kaon[12] | $K^+$ | $K^-$ | $u\bar{s}$ | 493.677 ±0.016 |
| Kaon[13] | $K^0$ | $\bar{K}^0$ | $d\bar{s}$ | 497.614 ±0.024 |

$U(1)_A$

should be broken
as well
producing a 9th
Goldstone BUT:

**Exception!**

$\eta'$ is too heavy

# Topology, $\eta'$ and the $U_A(1)$ problem:

The $U_A(1)$ symmetry $q \to e^{i\alpha\gamma_5} q$

would be broken by the (spontaneously generated) $\bar{q}q$ :

the candidate Goldstone is the $\eta'$

too heavy!! (900 MeV)

| Particle name | Particle symbol | Antiparticle symbol | Quark content | Rest mass (MeV/$c^2$) |
|---|---|---|---|---|
| Pion[6] | $\pi^+$ | $\pi^-$ | $u\bar{d}$ | 139.570 18 ±0.000 35 |
| Pion[7] | $\pi^0$ | Self | $\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a] | 134.9766 ±0.0006 |
| Eta meson[8] | $\eta$ | Self | $\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ [a] | 547.862 ±0.018 |
| Eta prime meson[9] | $\eta'$(958) | Self | $\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ [a] | 957.78 ±0.06 |
| Kaon[12] | $K^+$ | $K^-$ | $u\bar{s}$ | 493.677 ±0.016 |
| Kaon[13] | $K^0$ | $\bar{K}^0$ | $d\bar{s}$ | 497.614 ±0.024 |

BUT:
the divergence of the current $j_5^\mu = \bar{q}\gamma_5\gamma_\mu q,$

contains a mass independent term ↓

$$\partial_\mu j_5^\mu = m\bar{q}\gamma_5 q + \frac{1}{32\pi^2} F\tilde{F}.$$

IF $\frac{1}{32\pi^2}\int d^4x F\tilde{F} \neq 0$

The $U_A(1)$ symmetry is **explicitly** broken

Topology, $\eta'$ and the $U_A(1)$ problem:

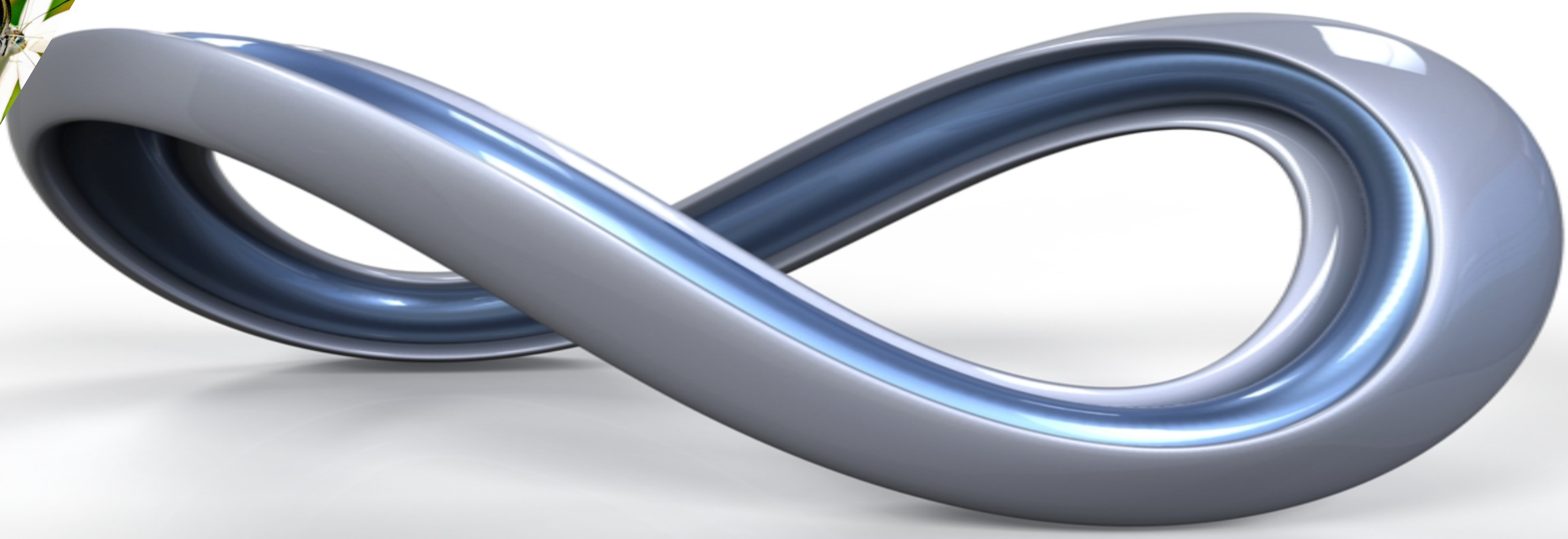
It can be proven that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F} = Q$$ **Gluonic definition**

and

$$Q = n_+ - n_-$$ **Fermionic definition**

$F\tilde{F}$



# Topology, $\eta'$ and the $U_A(1)$ problem:

It can be proven that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F} = Q$$

**Gluonic definition**

and

$$Q = n_+ - n_-$$

**Fermionic definition**

The $\eta'$ mass may now be computed from the decay of the correlation

$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x)\tilde{F}(x) F(y)\tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2}\chi_t^{\mathrm{qu}}$$

# Topology, $\eta'$ and the $U_A(1)$ problem:

It can be proven that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F} = Q$$ **Gluonic definition**

and

$$Q = n_+ - n_-$$ **Fermionic definition**

The $\eta'$ mass may now be computed from the decay of the correlation

Q(x) Q(y)

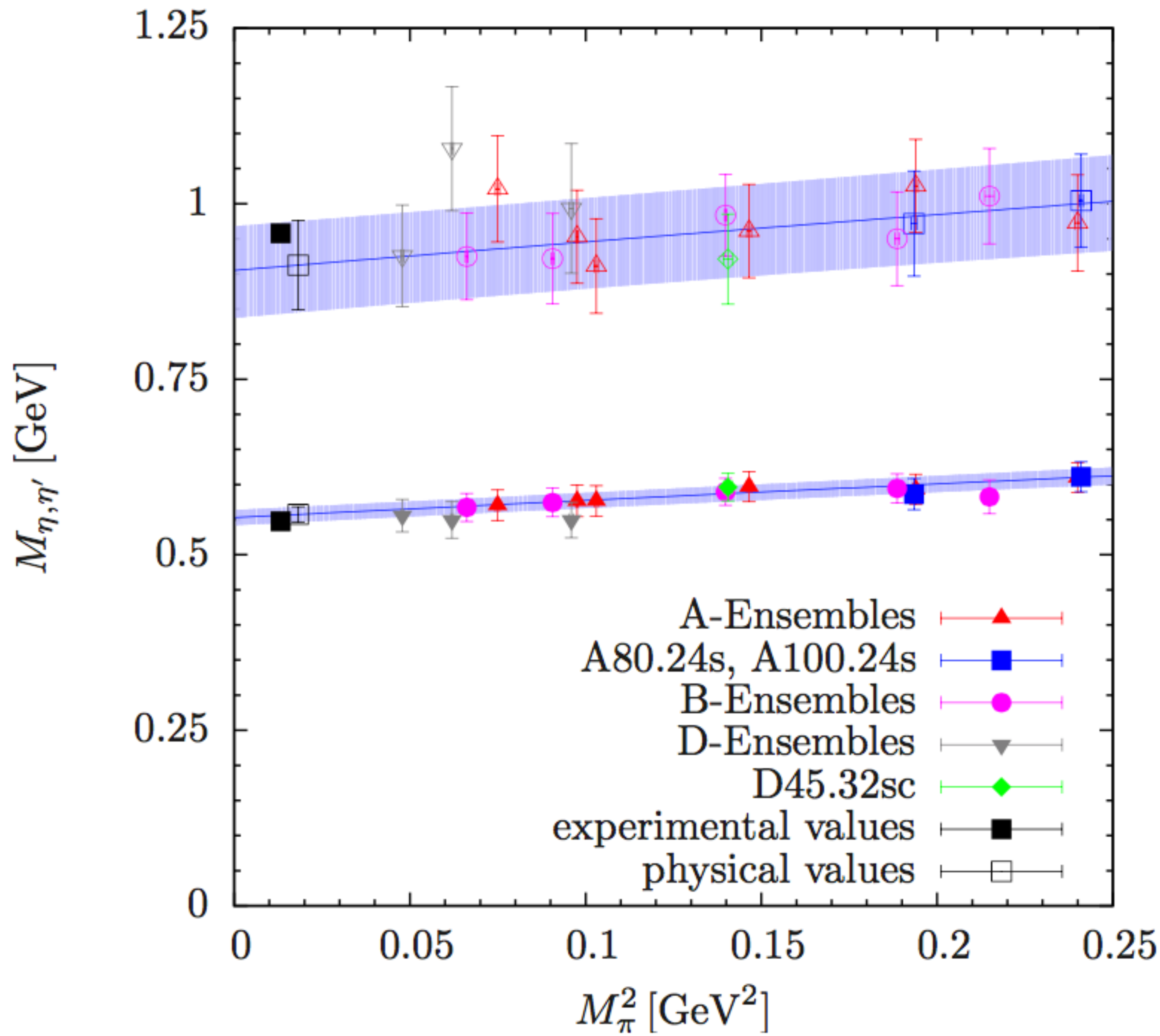
$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x)\tilde{F}(x) F(y)\tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2}\chi_t^{\mathrm{qu}}$$

Successful at T=0

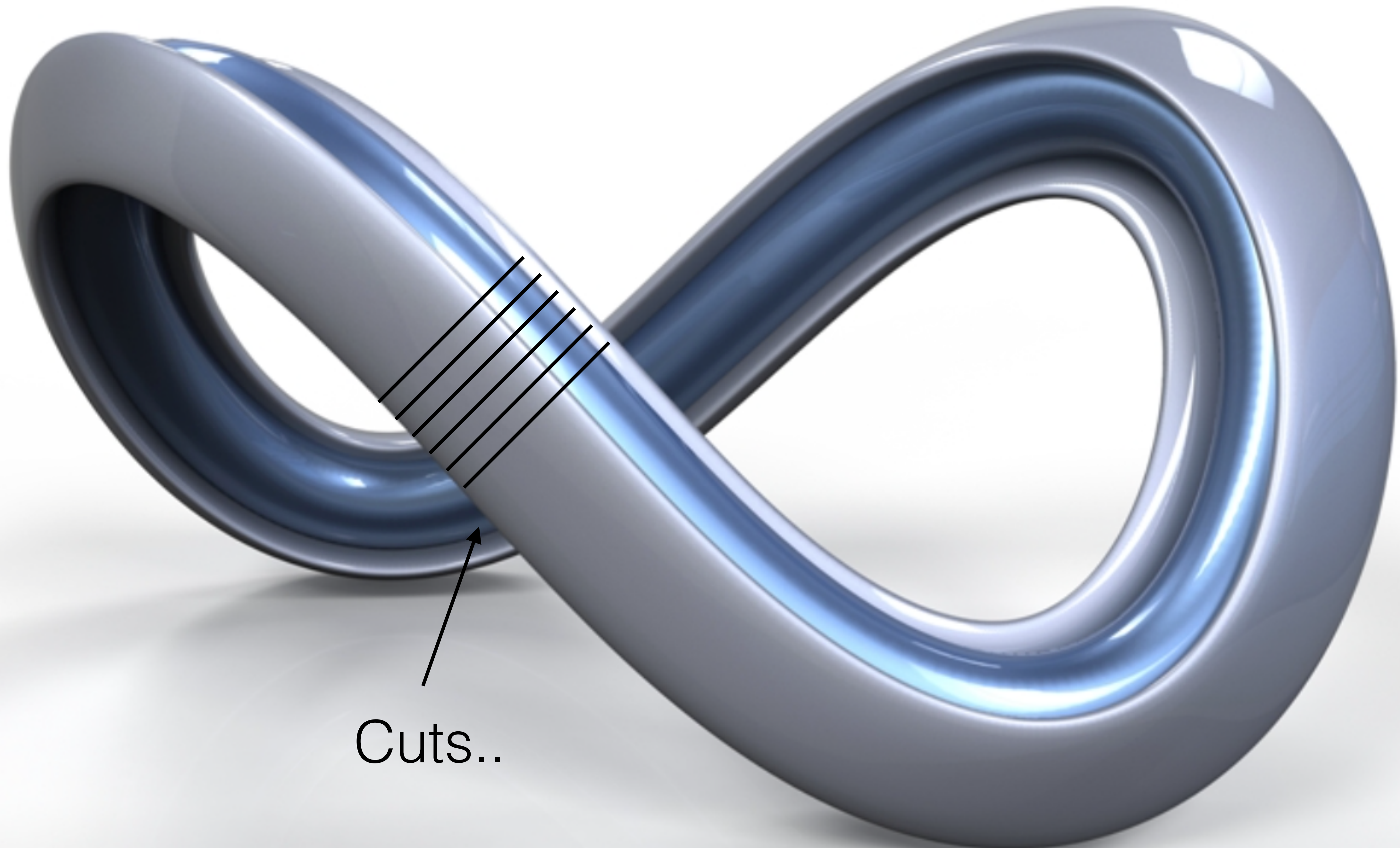
# Topology, $\eta'$ and the $U_A(1)$ ~~problem~~... solution




ETMC 2017

# Results

Topology on a lattice




…It is difficult to identify different topological sectors

..and large temperatures require huge statistics..

## QCD topology, long standing focus of strong interaction:

-learning about:
fundamental symmetries, $\eta'$mass, strongCP problem —> axions

-hampered by technical difficulties

## Recent developments:

-first results for dynamical fermions at high temperature:

Trunin *et al.* **J.Phys.Conf.Ser. 668 (2016) no.1, 012123**
Bonati *et al.* **JHEP 1603 (2016) 155**
Borsany *et al.* **Nature 539 (2016) no.7627, 69**
Petreczky *et al.* **Phys.Lett. B762 (2016) 498**
Burger *et al.* **Nucl.Phys. A967 (2017) 880**
Taniguchi *et al.* **Phys.Rev. D95 (2017) no.5, 054502**
Burger *et al.* arXiv 180506001

TmFT

+work in progress

# The Hot Twisted Mass project

**Chiral observables and topology in hot QCD with two families of quarks**
A. Trunin, F. Burger, E. M. Ilgenfritz, M. P. Lombardo
arXiv:1805.06001

**Topology (and axion's properties) from lattice QCD with a dynamical charm**
A. Trunin, F. Burger, E. M. Ilgenfritz, M. P. Lombardo and M. Müller-Preussker.
Nucl.Phys. **A967** (2017) 880-883

**Topological susceptibility from $N_f = 2 + 1 + 1$ lattice QCD at nonzero temperature**
A. Trunin, F. Burger, E. M. Ilgenfritz, M. P. Lombardo and M. Müller-Preussker.
J. Phys. Conf. Ser. **668**, no. 1, 012123 (2016)

**Towards the quark–gluon plasma Equation of State with dynamical strange and charm quarks**
F. Burger, E. M. Ilgenfritz, M. P. Lombardo, M. Müller-Preussker and A. Trunin.
J. Phys. Conf. Ser. **668**, no. 1, 012092 (2016)

**Equation of state of quark-gluon matter from lattice QCD with two flavors of twisted mass Wilso**
F. Burger *et al.* [tmfT Collaboration].
Phys. Rev. D **91**, no. 7, 074504 (2015)

**Towards thermodynamics with $N_f = 2 + 1 + 1$ twisted mass quarks**
F. Burger, G. Hotzel, M. Müller-Preussker, E. M. Ilgenfritz and M. P. Lombardo.
PoS Lattice **2013**, 153 (2013)

**Thermal QCD transition with two flavors of twisted mass fermions**
F. Burger *et al.* [tmfT Collaboration].
Phys. Rev. D **87**, no. 7, 074508 (2013)

**Phase structure of thermal lattice QCD with $N_f = 2$ twisted mass Wilson fermions**
E.-M. Ilgenfritz, K. Jansen, M. P. Lombardo, M. Müller-Preussker, M. Petschlies, O. Philipsen and L. Zeidlewicz.
Phys. Rev. D **80**, 094502 (2009)

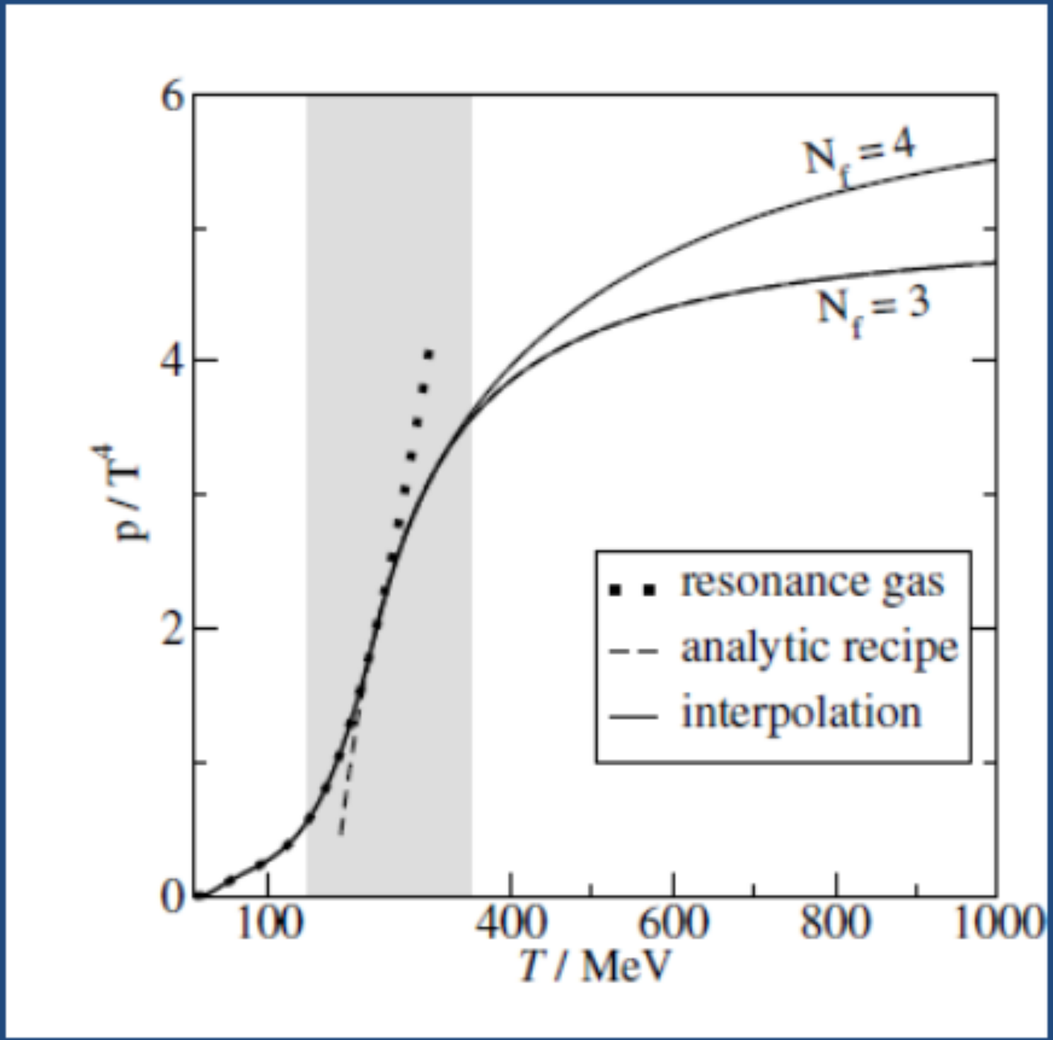
# Why Nf = 2 +1 +1 ?

Tc

≈200MeV

| 340 –380 MeV<br>RHIC AuAu<br>200 GeV | 420-480 MeV<br>LHC<br>2.76 TeV | 500- 600MeV<br>LHC hot spots<br>2.76 TeV | → | 1 GeV<br>LHC<br>7 TeV |
|---|---|---|---|---|

## Quark Gluon Plasma @ Colliders




Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

**Laine Schroeder 2006**

# Nf = 2+1+1 Wilson fermions with a twisted mass term

Frezzotti Rossi 2003

'twisted' mass terms in flavor space:

$i\mu\tau_3\gamma_5$ for two degenerate light flavors

$i\mu_\sigma\tau_1\gamma_5 + \tau_3\mu_\delta$ for two heavy flavors

are added to the standard Wilson Lagrangian

Consequences:

-simplified renormalization properties

-automatic O(a) improvement

-control on unphysical zero modes

Successful phenomenology at T=0

ETMC collaboration 2003—

# Nf = 2 +1+1 Setup

Fixed varying scale

For each lattice spacing we explore a range of temperatures 150MeV — 500 MeV by varying Nt

We repeat this for three different lattice spacings following ETMC T=0 simulations.

Four pion masses

| Number of flavours | $m_{\pi^\pm}$ |
|---|---|
| $N_f = 2+1+1$ | 210 |
| | 260 |
| | 370 |
| | 470 |
| $N_f = 2$ | 360 |
| | 430 |

Advantages: we rely on the setup of ETMC T=0 simulations. Scale is set once for all.

Disadvantages: mismatch of temperatures - need interpolation before taking the continuum limit

| $T = 0$ (ETMC) nomenclature | $\beta$ | $a$ [fm] [6] | $N_\sigma^3$ | $N_\tau$ | $T$ [MeV] | # confs. |
|---|---|---|---|---|---|---|
| A60.24 | 1.90 | 0.0936(38) | $24^3$ | 5 | 422(17) | 585 |
| | | | | 6 | 351(14) | 1370 |
| | | | | 7 | 301(12) | 341 |
| | | | | 8 | 263(11) | 970 |
| | | | | 9 | 234(10) | 577 |
| | | | | 10 | 211(9) | 525 |
| | | | | 11 | 192(8) | 227 |
| | | | | 12 | 176(7) | 1052 |
| | | | $32^3$ | 13 | 162(7) | 294 |
| | | | | 14 | 151(6) | 1988 |
| B55.32 | 1.95 | 0.0823(37) | $32^3$ | 5 | 479(22) | 595 |
| | | | | 6 | 400(18) | 345 |
| | | | | 7 | 342(15) | 327 |
| | | | | 8 | 300(13) | 233 |
| | | | | 9 | 266(12) | 453 |
| | | | | 10 | 240(11) | 295 |
| | | | | 11 | 218(10) | 667 |
| | | | | 12 | 200(9) | 1102 |
| | | | | 13 | 184(8) | 308 |
| | | | | 14 | 171(8) | 1304 |
| | | | | 15 | 160(7) | 456 |
| | | | | 16 | 150(7) | 823 |
| D45.32 | 2.10 | 0.0646(26) | $32^3$ | 6 | 509(20) | 403 |
| | | | | 7 | 436(18) | 412 |
| | | | | 8 | 382(15) | 416 |
| | | | | 10 | 305(12) | 420 |
| | | | | 12 | 255(10) | 380 |
| | | | | 14 | 218(9) | 793 |
| | | | | 16 | 191(8) | 626 |
| | | | $40^3$ | 18 | 170(7) | 599 |
| | | | $48^3$ | 20 | 153(6) | 582 |

# Topological and chiral susceptibility

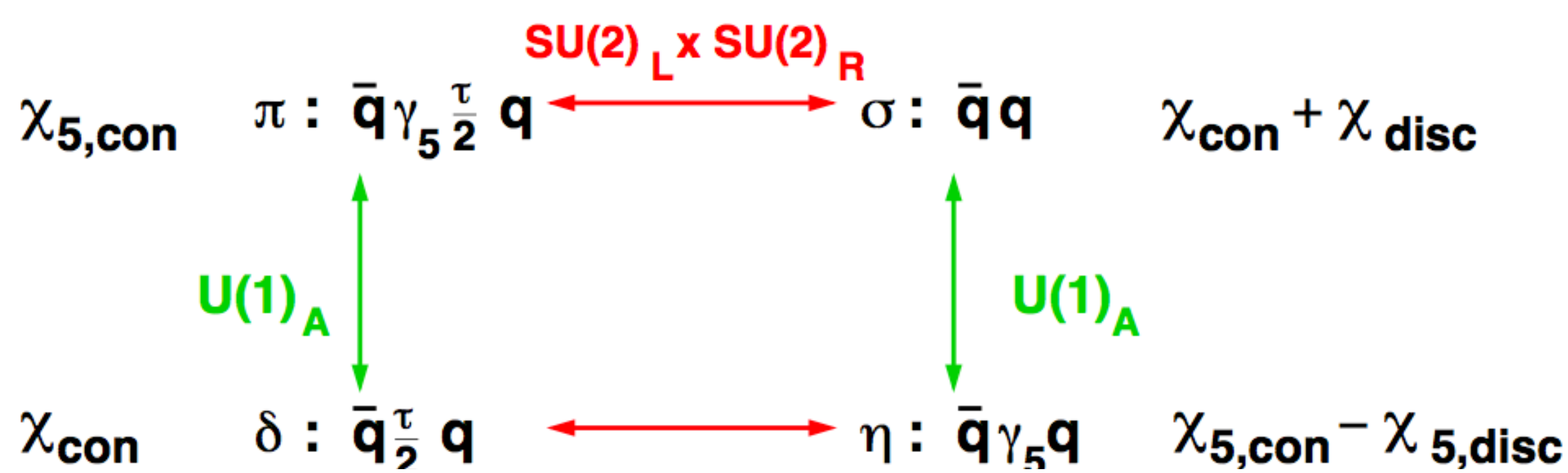
Kogut, Lagae, Sinclair 1999

HotQCD, 2012

$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{5,disc}$$

From:

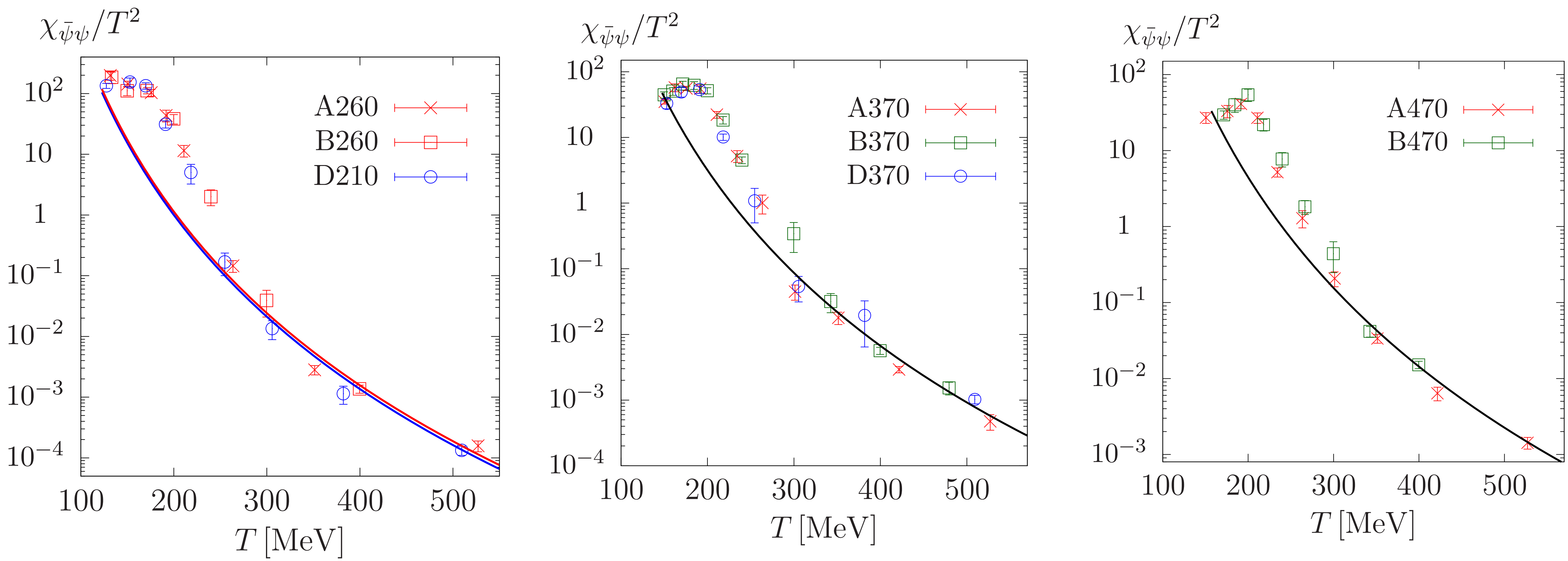
$$m \int d^4x \bar{\psi}\gamma_5\psi = Q_{top}$$




$$\chi_\pi - \chi_\delta = \chi_{\rm disc} = \chi_{5,{\rm disc}} \,, \qquad \text{for} \quad T \geq T_c \,, \; m_l \to 0$$

$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{disc}$$

# Chiral susceptibility

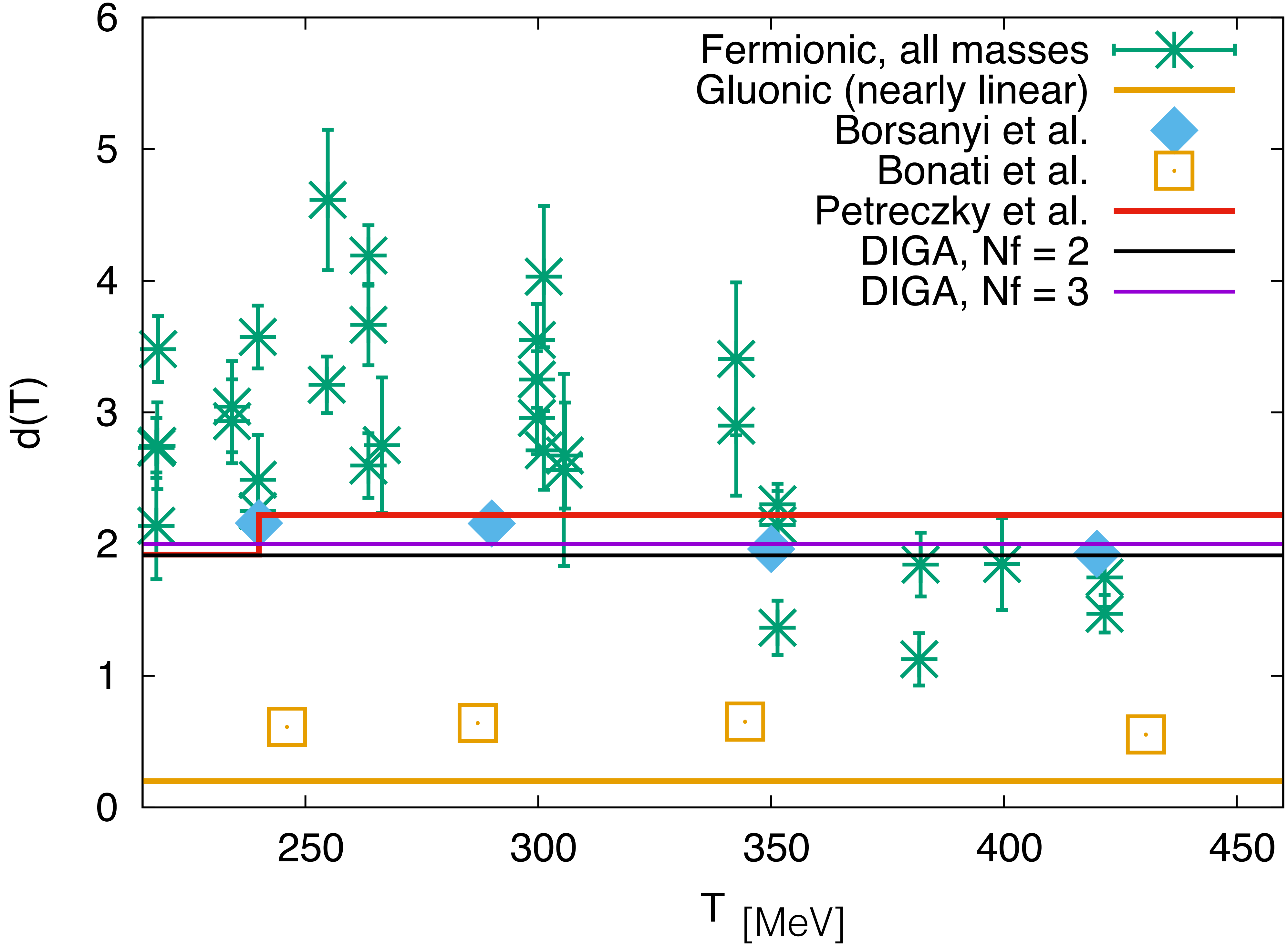



Within errors, no discernable spacing dependence

Effective exponent d(T):

Comparisons with other results :

$$\chi_{top}^{1/4} = aT^{-d(T)}$$

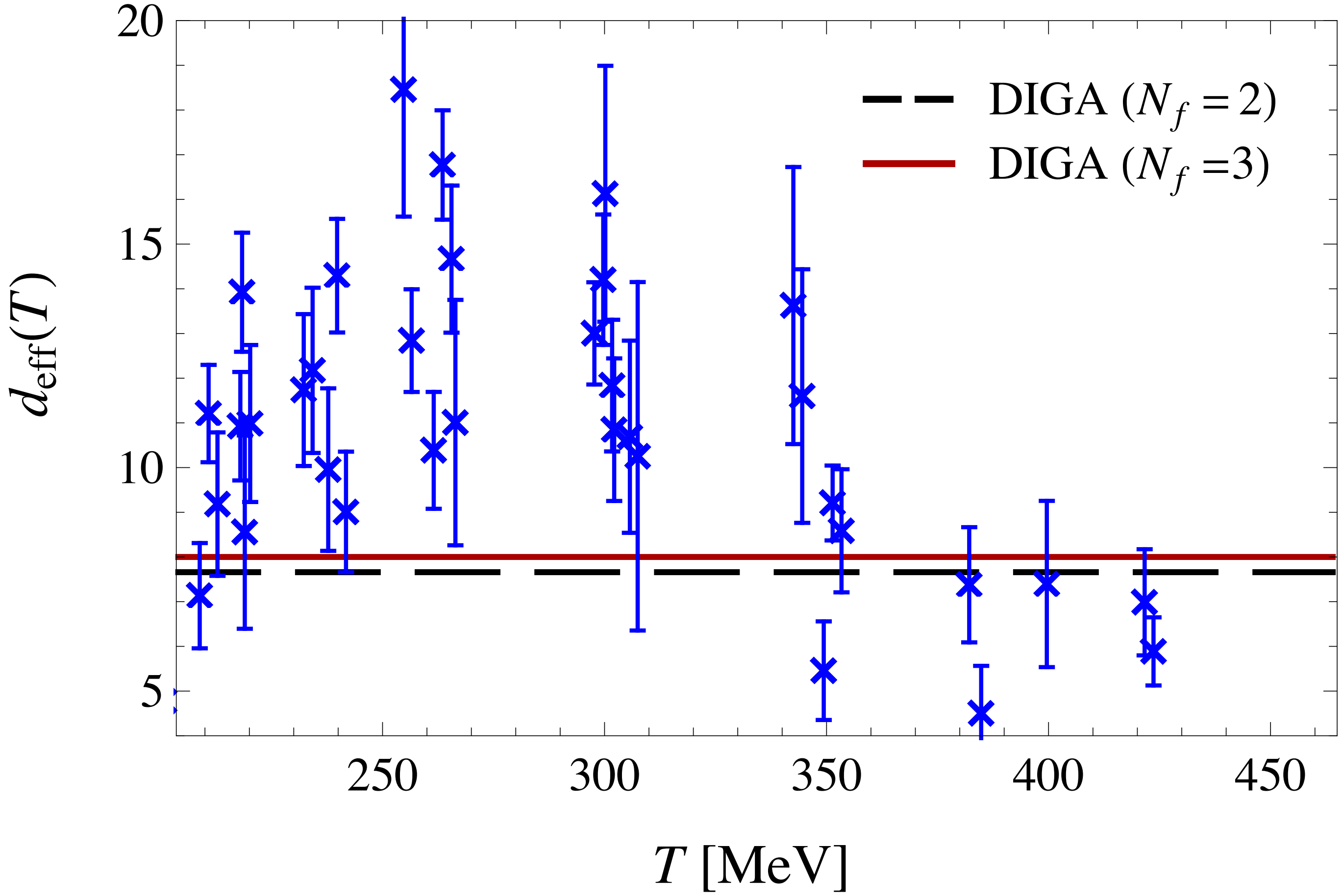


# Power-law decay?

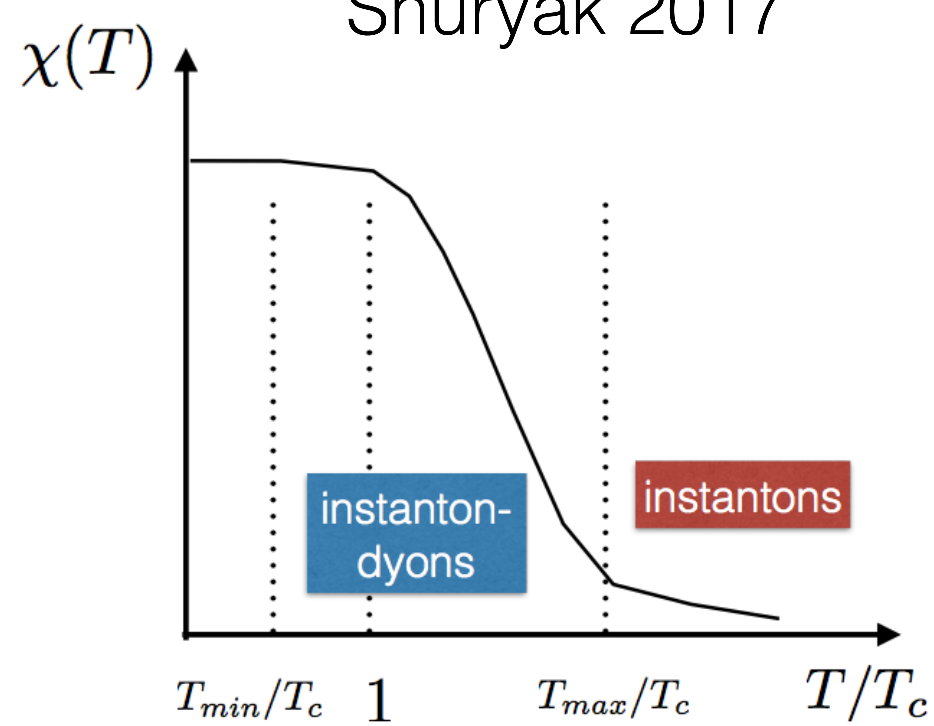
$$\chi^{0.25}(T) = aT^{-d(T)}$$

For instanton gas $d(T) \equiv const \simeq (7 + \frac{N_f}{3})$

$$d(T) = -T \frac{d}{dT} \ln \chi^{0.25}(T)$$

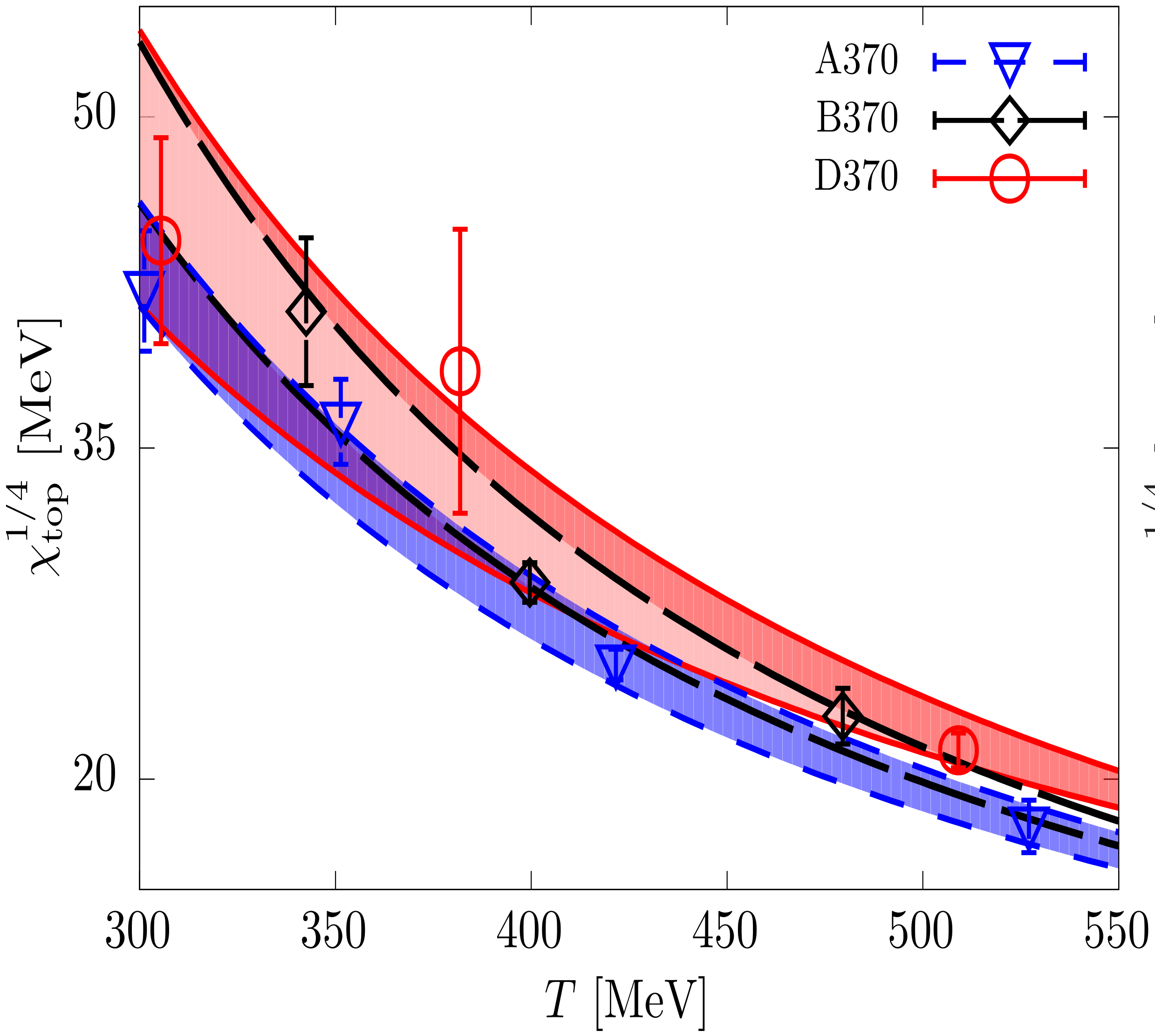


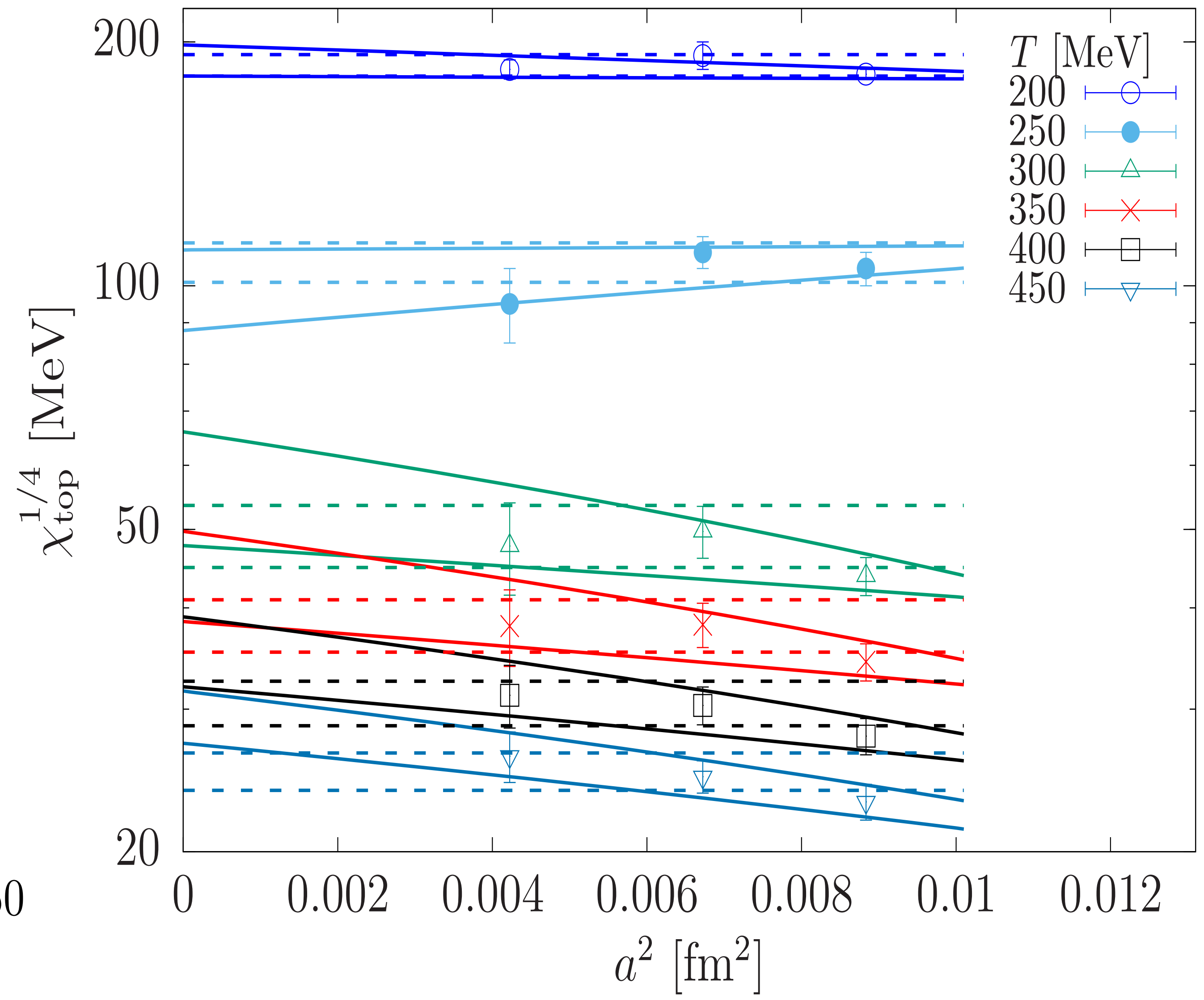

Possibly consistent with instant -dyon? Shuryak 2017




Faster decrease before DIGA sets in

# Continuum limit *(details)*



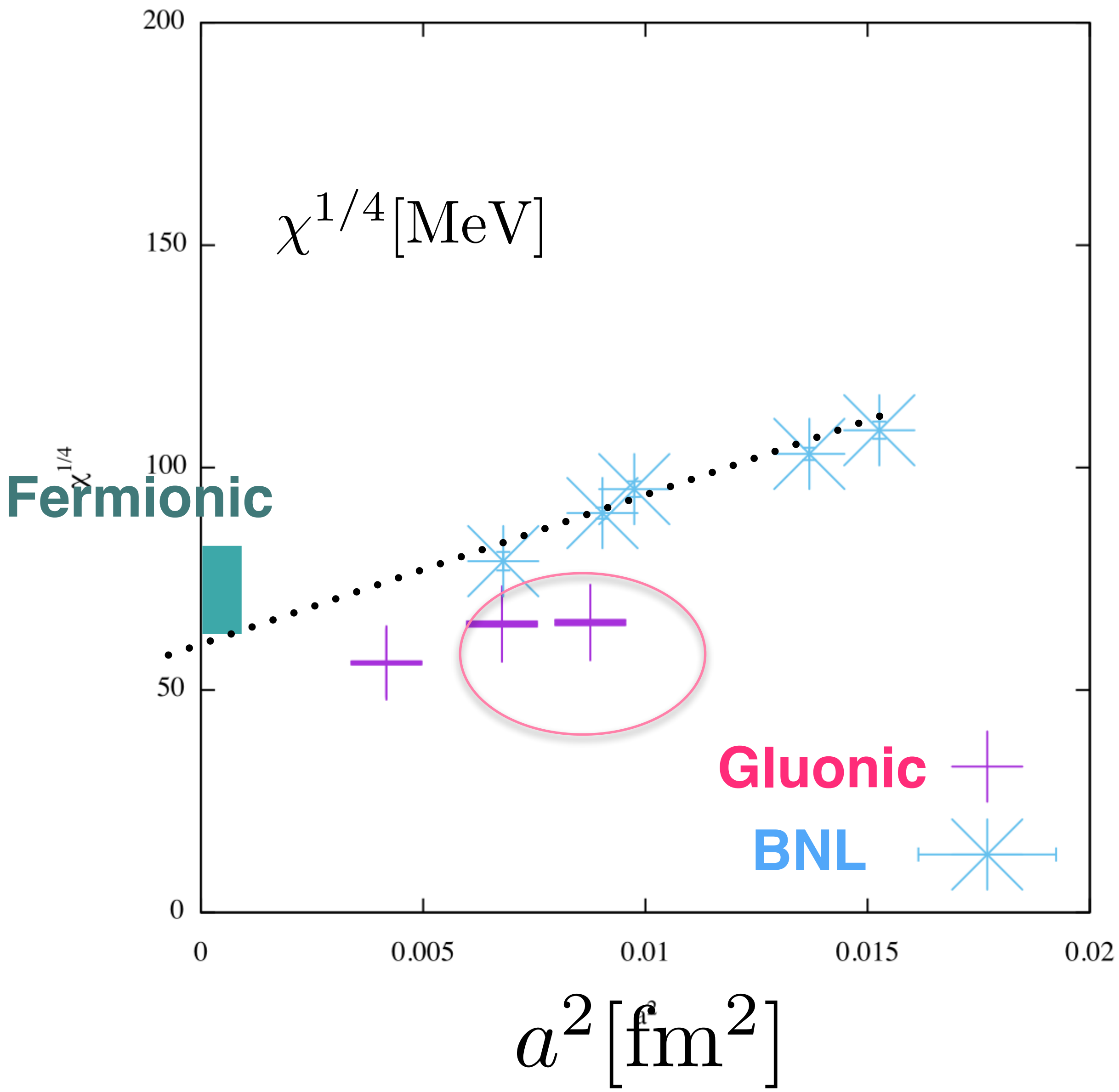

*the B ensemble is indeed representative of continuum*

# Continuum limit *(details)*

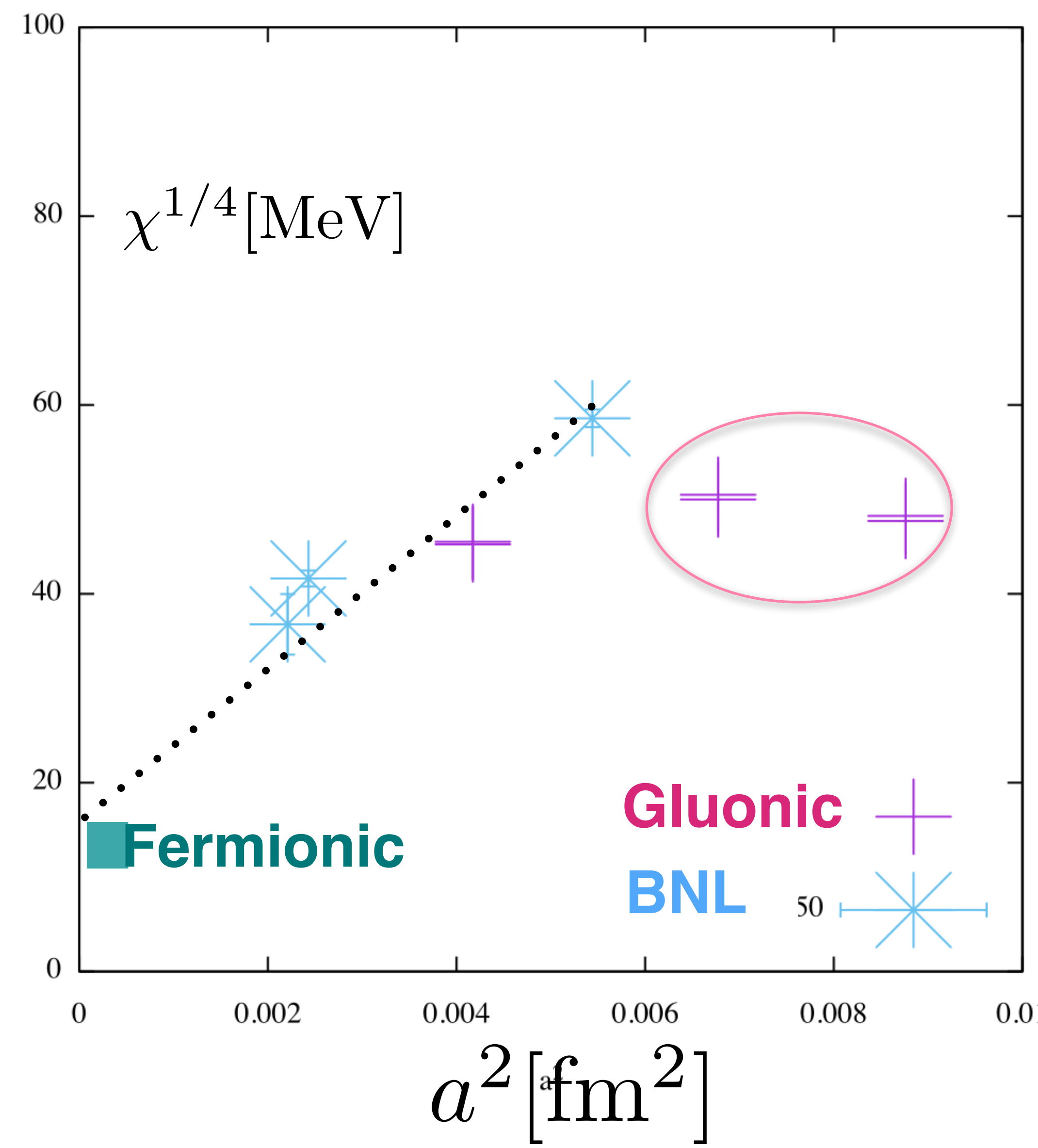
## Comparison with BNL results

numerical data courtesy *S. Sharma*

199 < T < 210 MeV




333 < T < 350 MeV

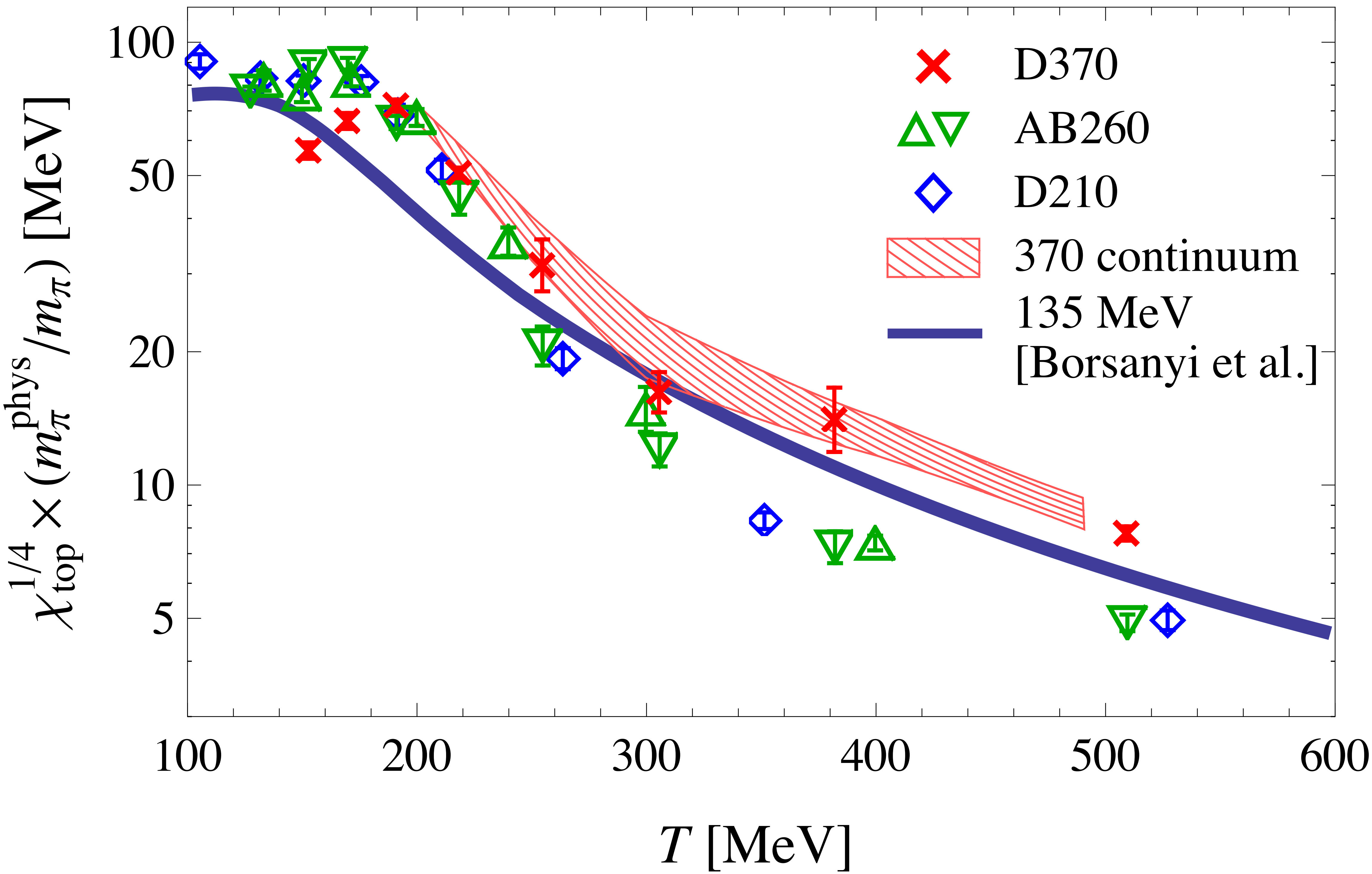



*dotted lines to guide the eye*

Results for physical pion mass

Rescaled according to

$$\chi_{\rm top} = m_l^2 \chi^{\rm disc}_{\bar{\psi}\psi} = \sum_{n=0} a_n m_\pi^{4(n+1)}.$$



*Using the B ensemble as representative of continuum*

$$\chi_{\rm top} \simeq A\, T^{-d}$$

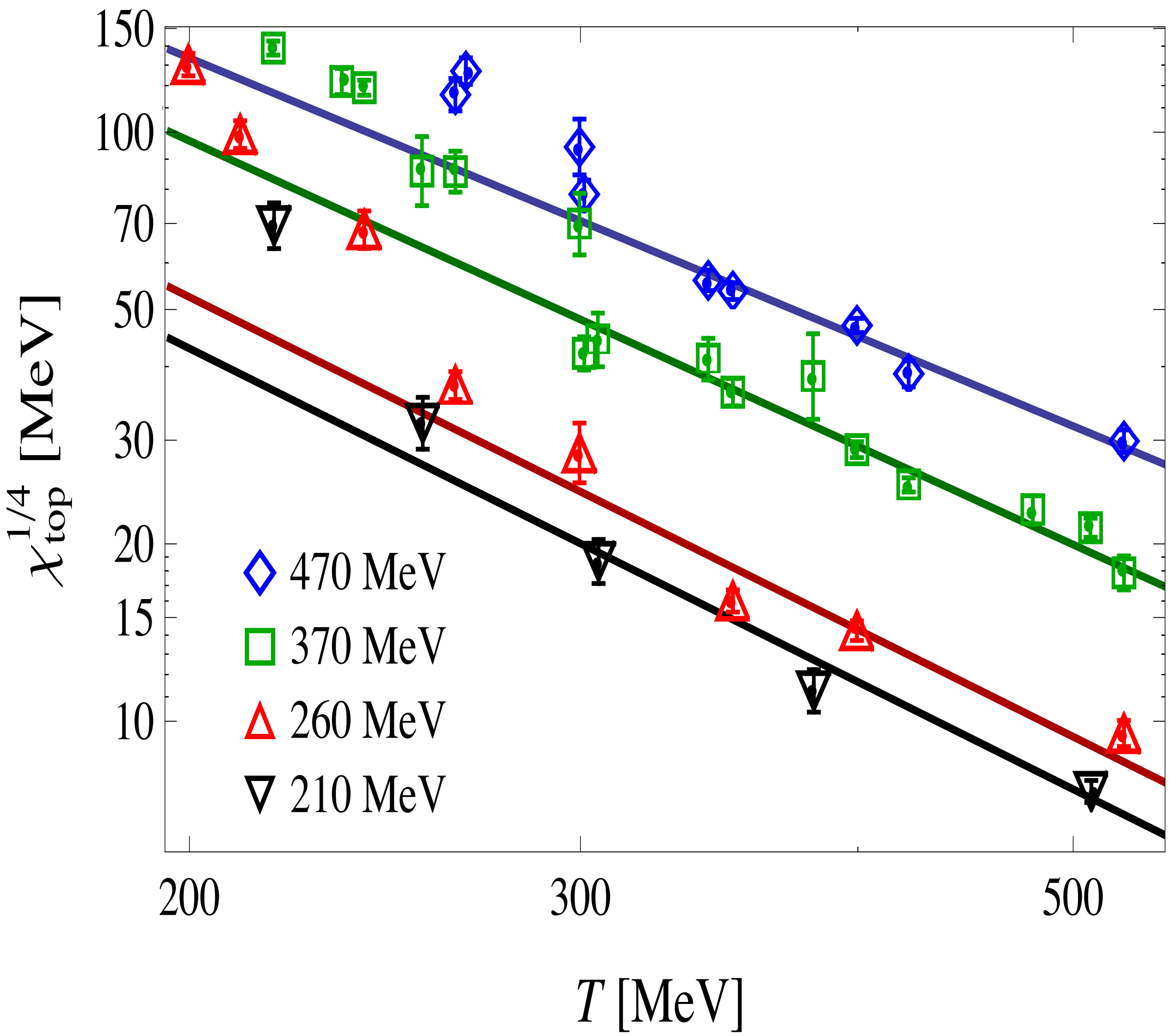
$$d = (6.26, 6.88, 7.52, 7.48)$$

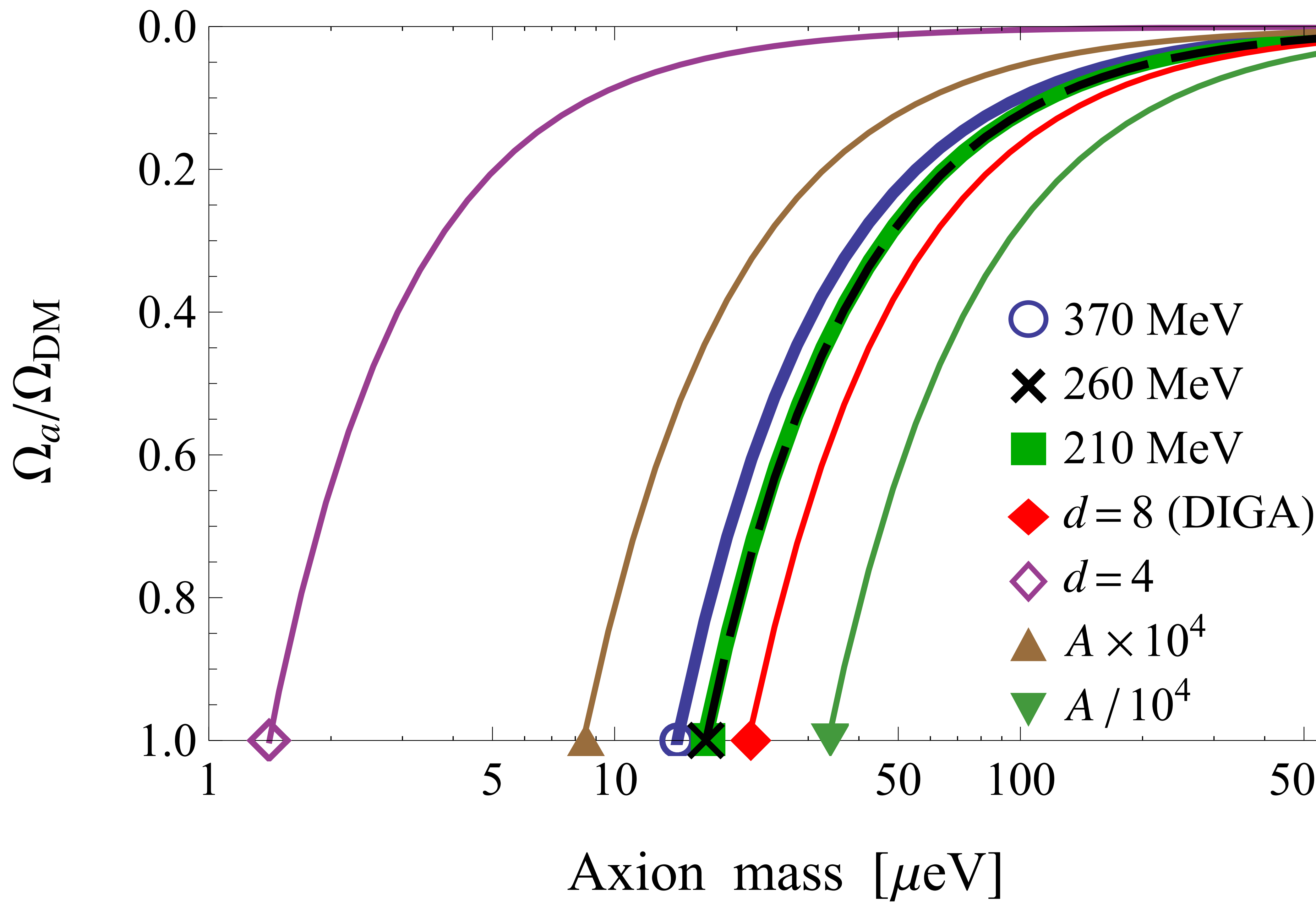
$$m_\pi = (470, 370, 260, 210)\ \text{MeV}$$

$$\rho_{a,0} = \frac{n_a}{s} m_a s_0$$

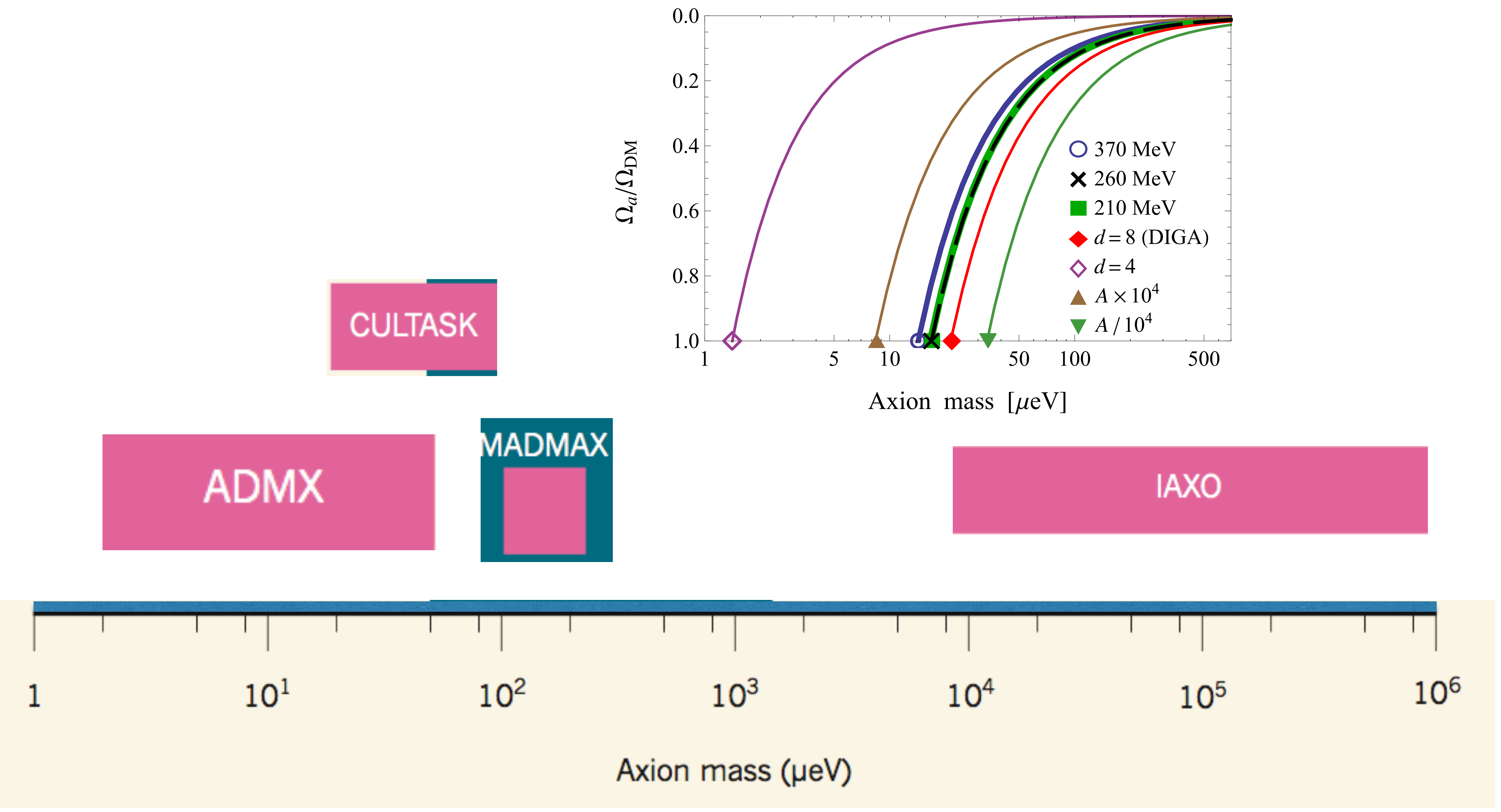
$$m_a = \sqrt{\chi(T)}/f_a$$

$$\rho_a(m_a) \propto m_a^{-\frac{3.053+d/2}{2.027+d/2}}$$





0.0
0.2
0.4
0.6
0.8
1.0
$\Omega_a/\Omega_{\rm DM}$
1
5
10
50
100
500
Axion mass [$\mu$eV]
370 MeV
260 MeV
210 MeV
$d = 8$ (DIGA)
$d = 4$
$A \times 10^4$
$A / 10^4$



$\Omega_a/\Omega_{DM}$
0.0
0.2
0.4
0.6
0.8
1.0
1
5
10
50
100
500
Axion mass [$\mu$eV]
370 MeV
260 MeV
210 MeV
$d = 8$ (DIGA)
$d = 4$
$A \times 10^4$
$A / 10^4$
CULTASK
ADMX
MADMAX
IAXO
1
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
$10^6$
Axion mass (μeV)

## ..towards the axion's potential

$$Z_{QCD}(\theta,T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T \sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta,T)]$$

Axion potential

$$m_a^2(T) f_a^2 = \left.\frac{\partial^2 F(\theta,T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T), \quad f_A \gtrsim 4\times 10^8 \text{ GeV}$$

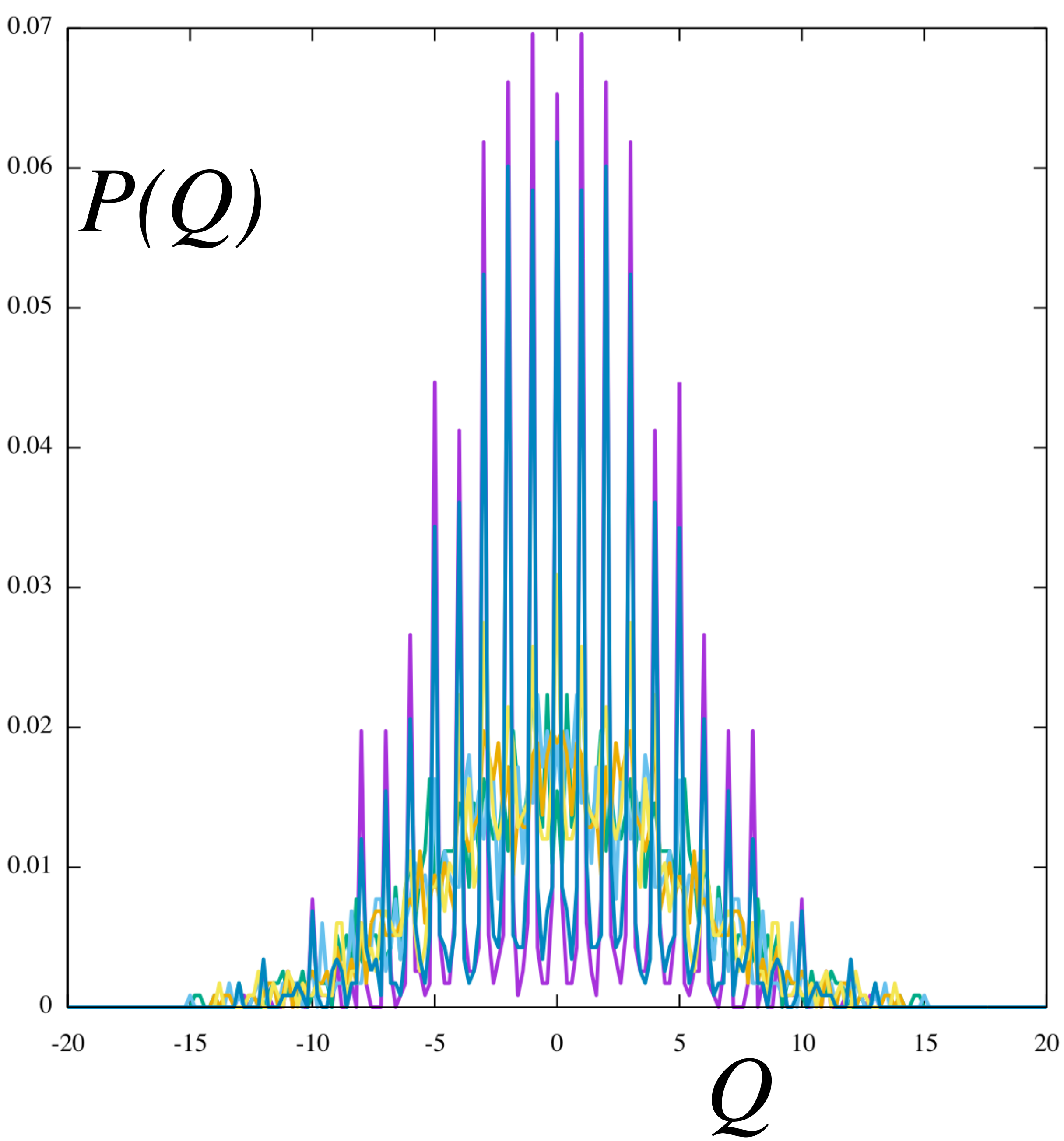
weakly coupled

# Distribution of the topological charge *P(Q)*

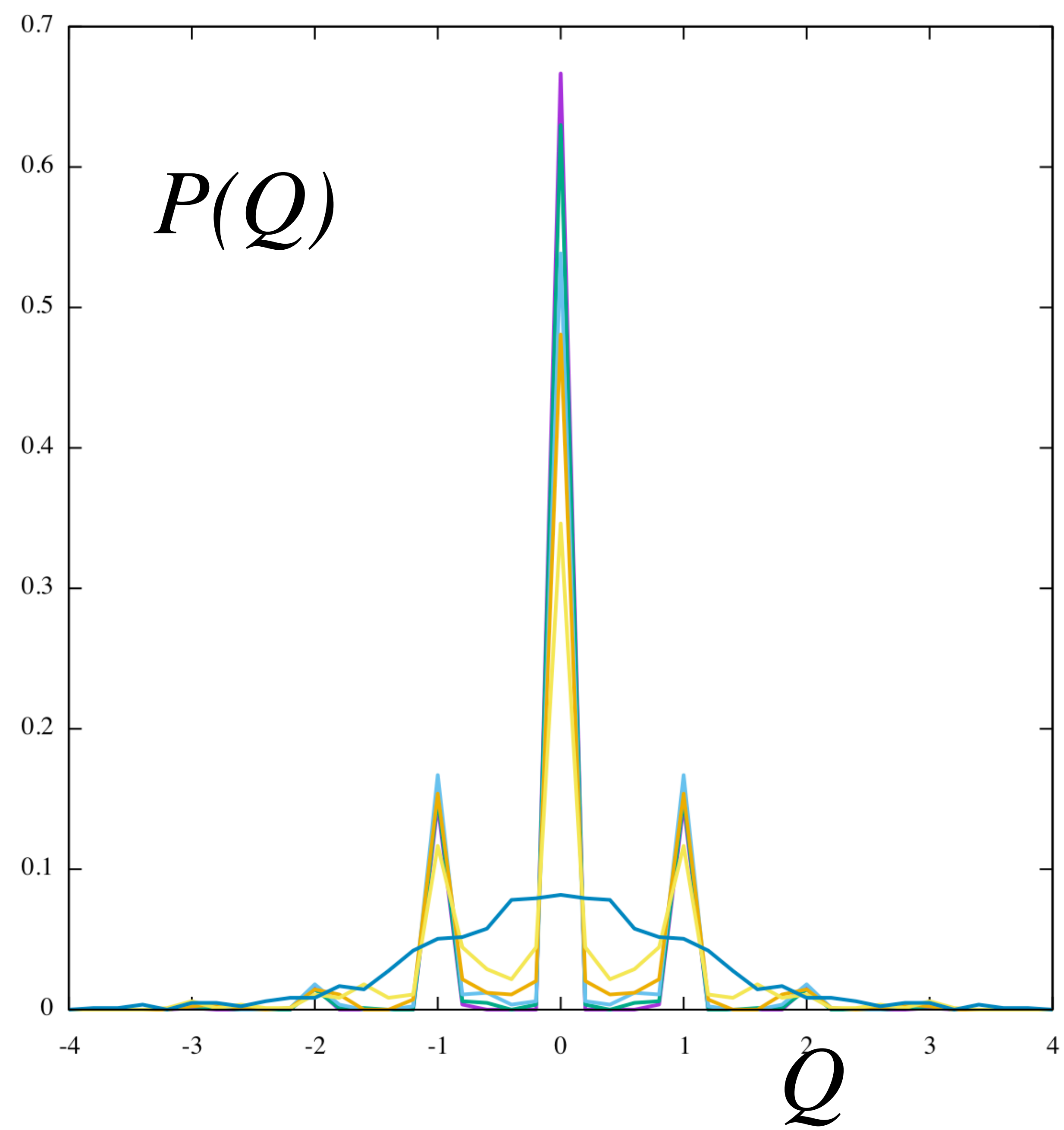
## cluster around integers as cooling proceeds

(results for a = 0.06 fm)

T=153 MeV




T=253 MeV



# Gradient flow

Lüscher, Lüscher Weisz

Evolve the link variables in a fictitious flow time:

$$\dot{V}_{x,\mu}(t) = -g_0^2 \left[\partial_{x,\mu} S_{\mathrm{Wilson}}(V(t))\right] V_{x,\mu}(t),$$

Monitor $\langle E \rangle = \frac{1}{2N_\tau N_\sigma^3} \sum_{x,\mu,\nu} \mathrm{Tr}[F_{\mu\nu}(x) F^{\mu\nu}(x)]$ as a function of $t$

Stop flowing when $t^2 \langle E \rangle \big|_{t=t_0} = 0.3$

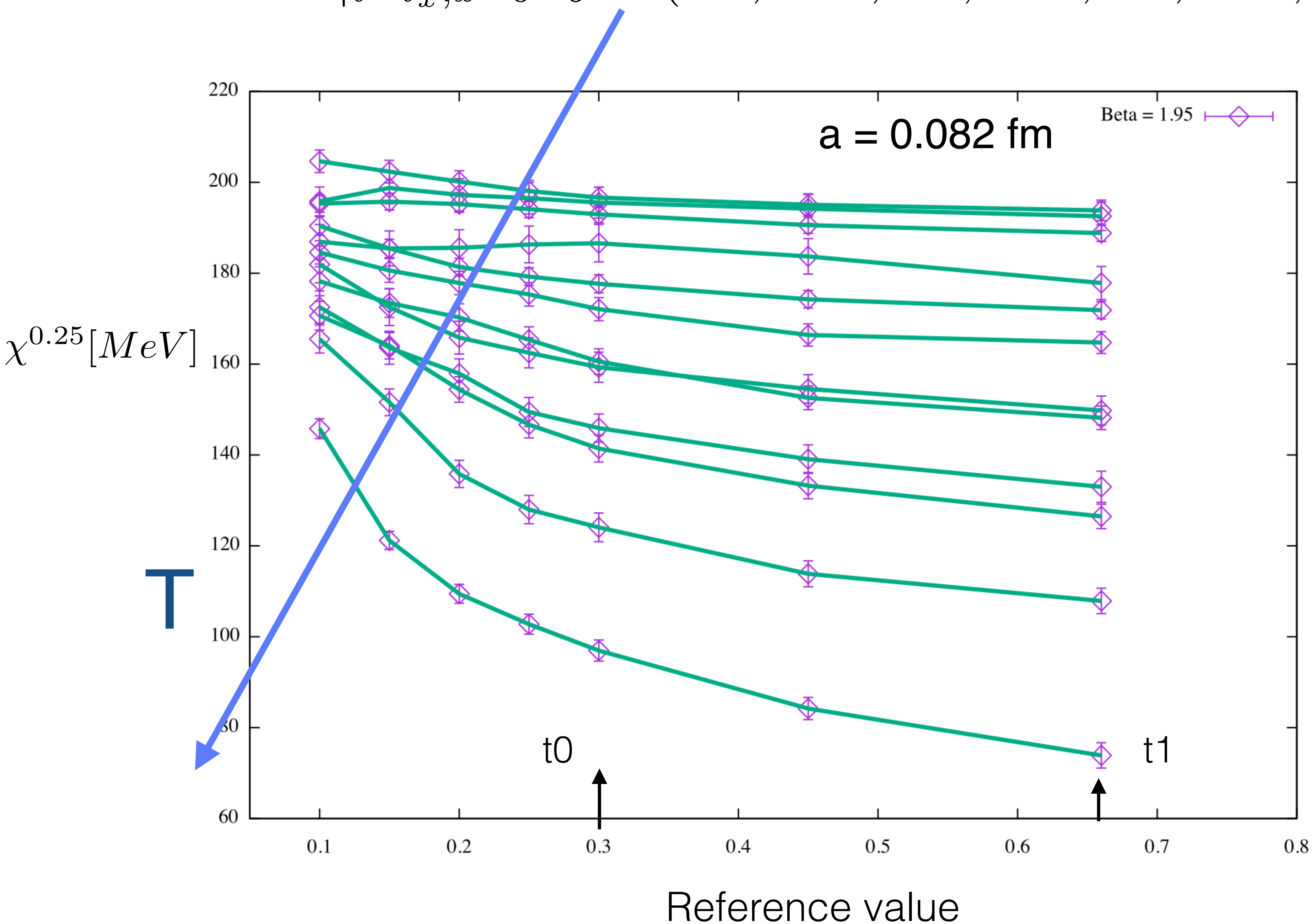
Observables $< O(t) >$ renormalized at $\mu = 1/\sqrt{8t}$

**Continuum limit of $< O(t) >$ is independent on the chosen reference value**

Caveat: note comments by Kanaya et al.

# Flowing towards the plateau

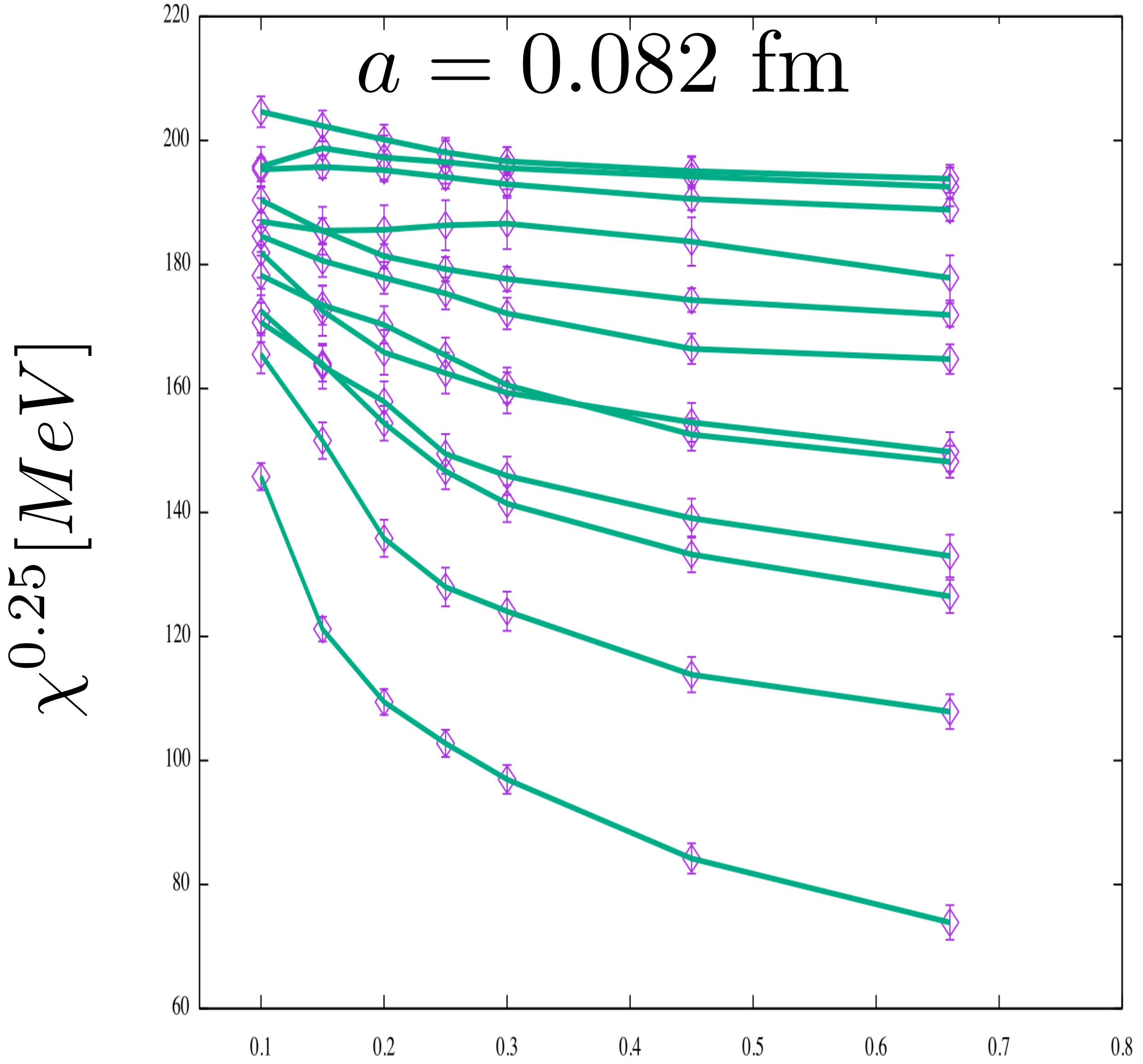
$$t^2 < E > |_{t=t_x, x=0-6} = (0.3, 0.66, 0.1, 0.15, 0.2, 0.25, 0.45)$$



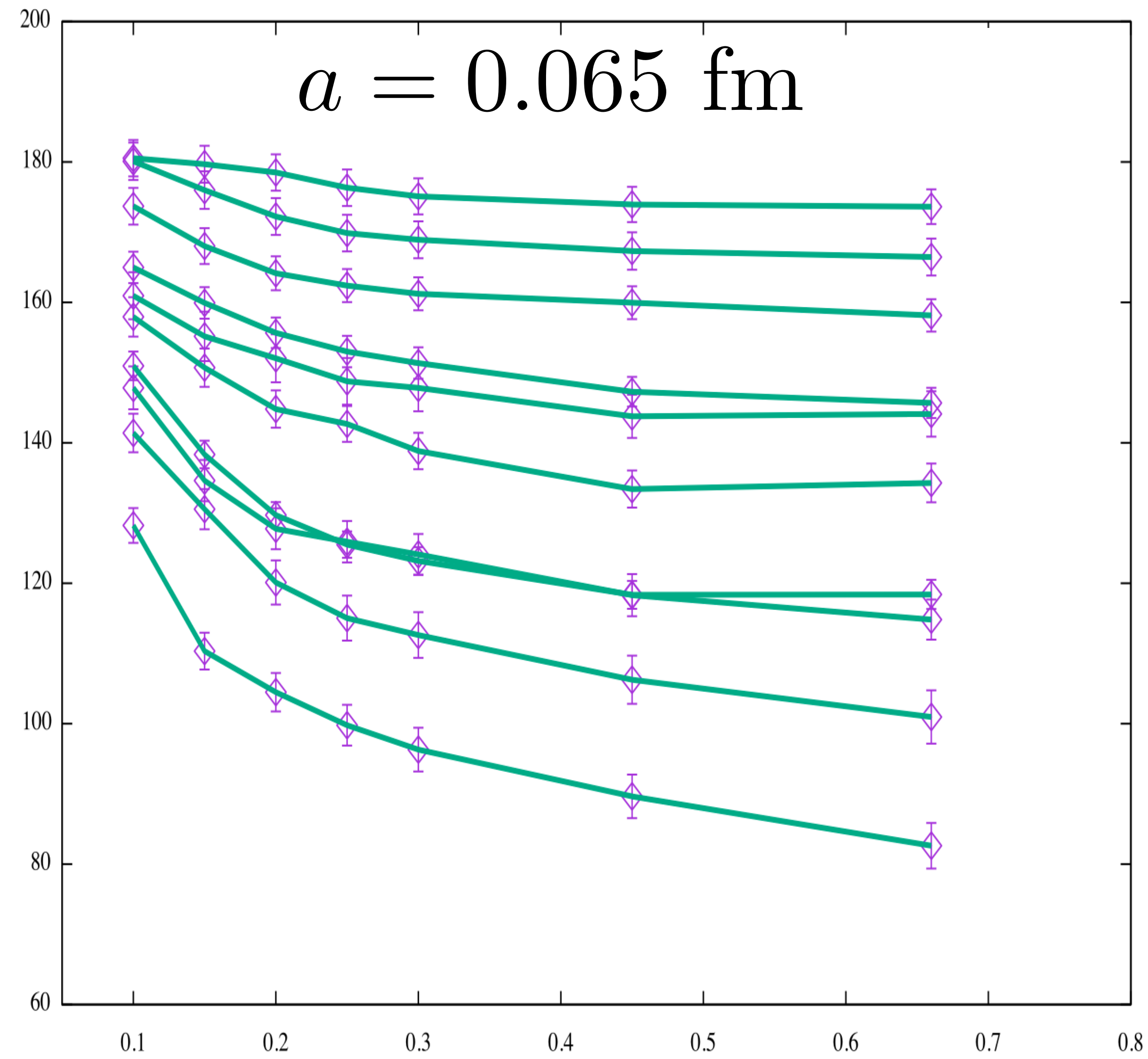

We focus on t(0.3) = t0 and t(0.66)= t1

# On finer lattices, plateau is almost reached:

Gradient method coincides with cooling



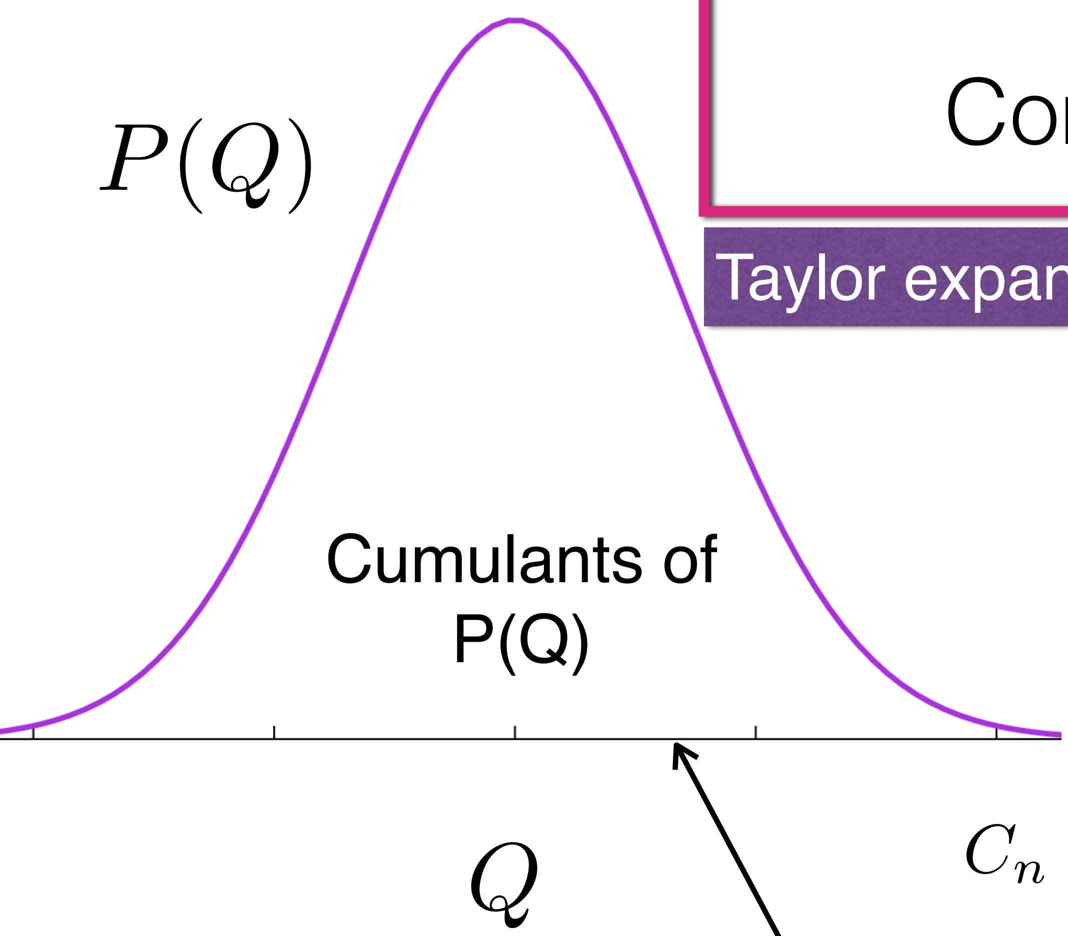

Reference value




Reference value

# Computing $F(\theta)$

**Taylor expansion, and cumulants of the topological charge distribution**




$$e^{-F(\theta)} = \langle e^{i\theta Q} \rangle$$

$$P_\nu = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta\nu} e^{-F(\theta)} \qquad Q = \nu$$

$$C_n = (-1)^{n+1} \frac{1}{V} \frac{d^{2n}}{d\theta^{2n}} F(\theta)\Big|_{\theta=0} \equiv < Q^{2n} >_{conn}$$

Taylor coefficients of

$$F(\theta) = V \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{(2n)!} C_n$$

$$P_\nu = \frac{e^{-\frac{\nu^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \left[ 1 + \frac{1}{4!} \frac{\tau}{\sigma^2} \mathrm{He}_4(\nu/\sigma) \right]$$

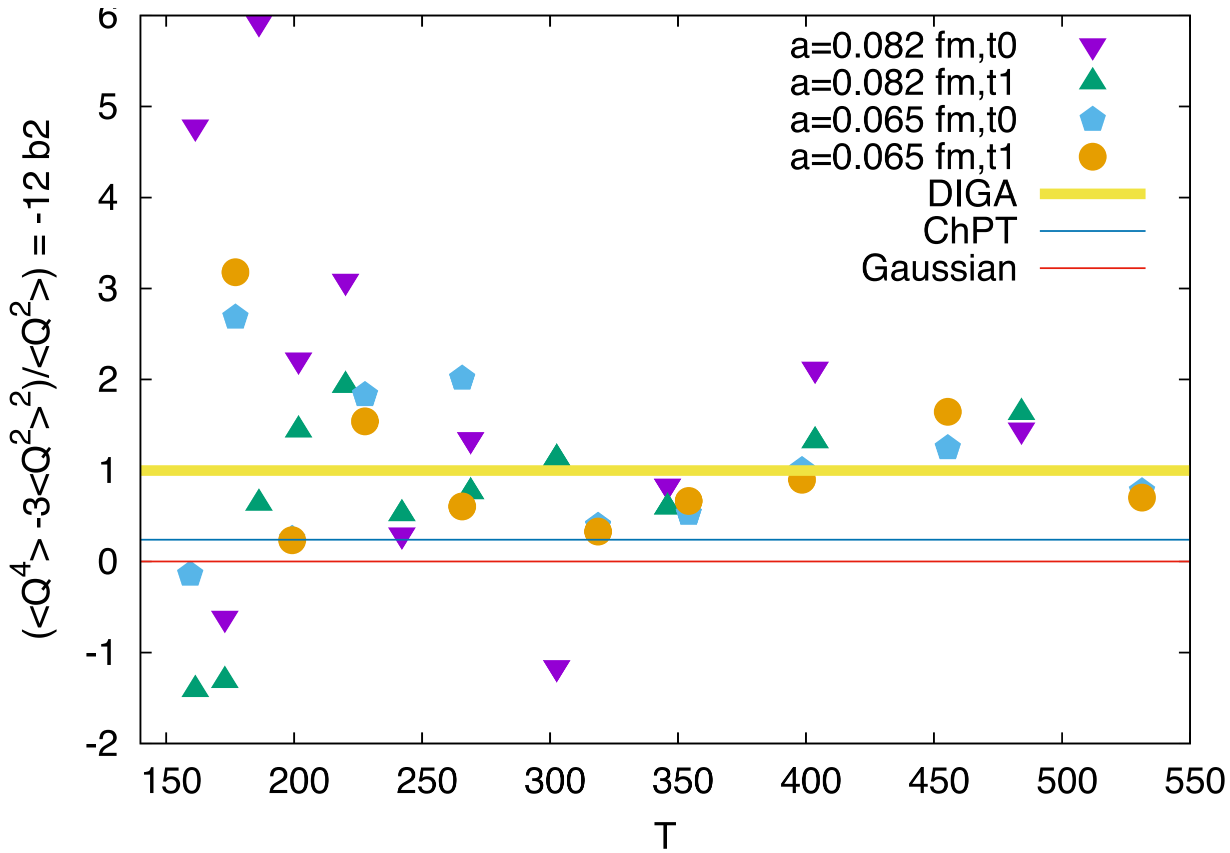
$\sigma^2 = VC_1$ and $\tau = C_2/C_1$ $\quad$ *P(Q)* is Gaussian for $V \to \infty$

$F(\theta)$ is 'hidden' in *P(Q)*'s cumulants

# Instanton potential - cumulants' ratio b2

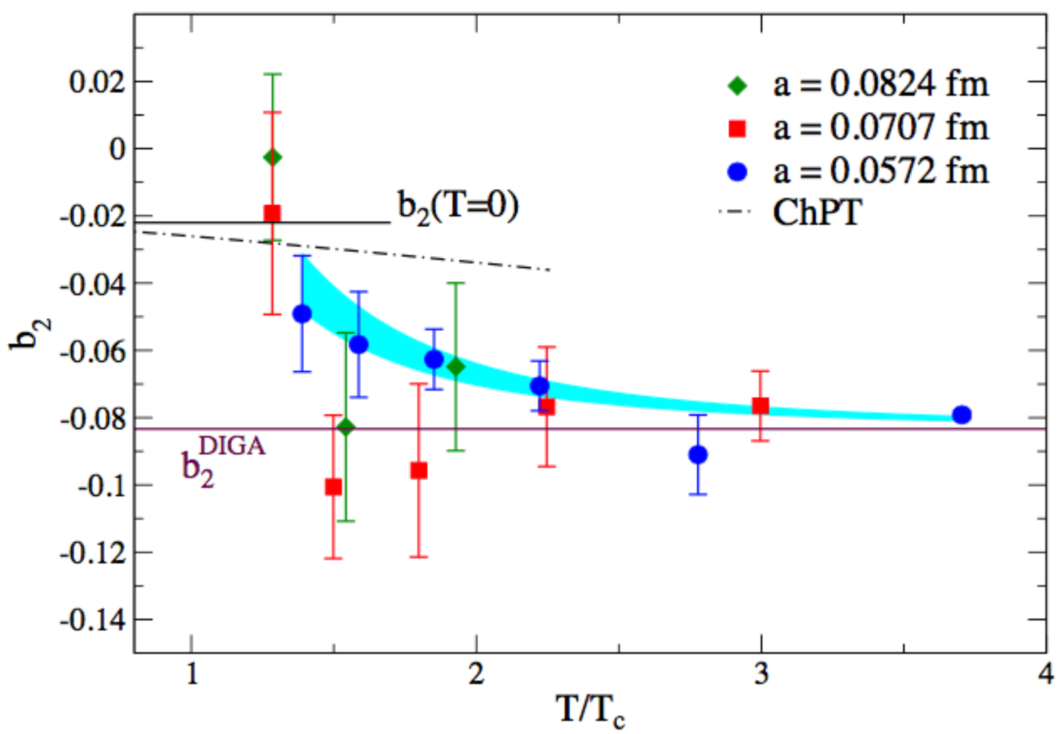
DIGA predicts

$$F(\theta, T) - F(0, T) = \chi(T)(1 - cos(\theta)) \longrightarrow b_2 = -1/12$$




b2 = -1/12

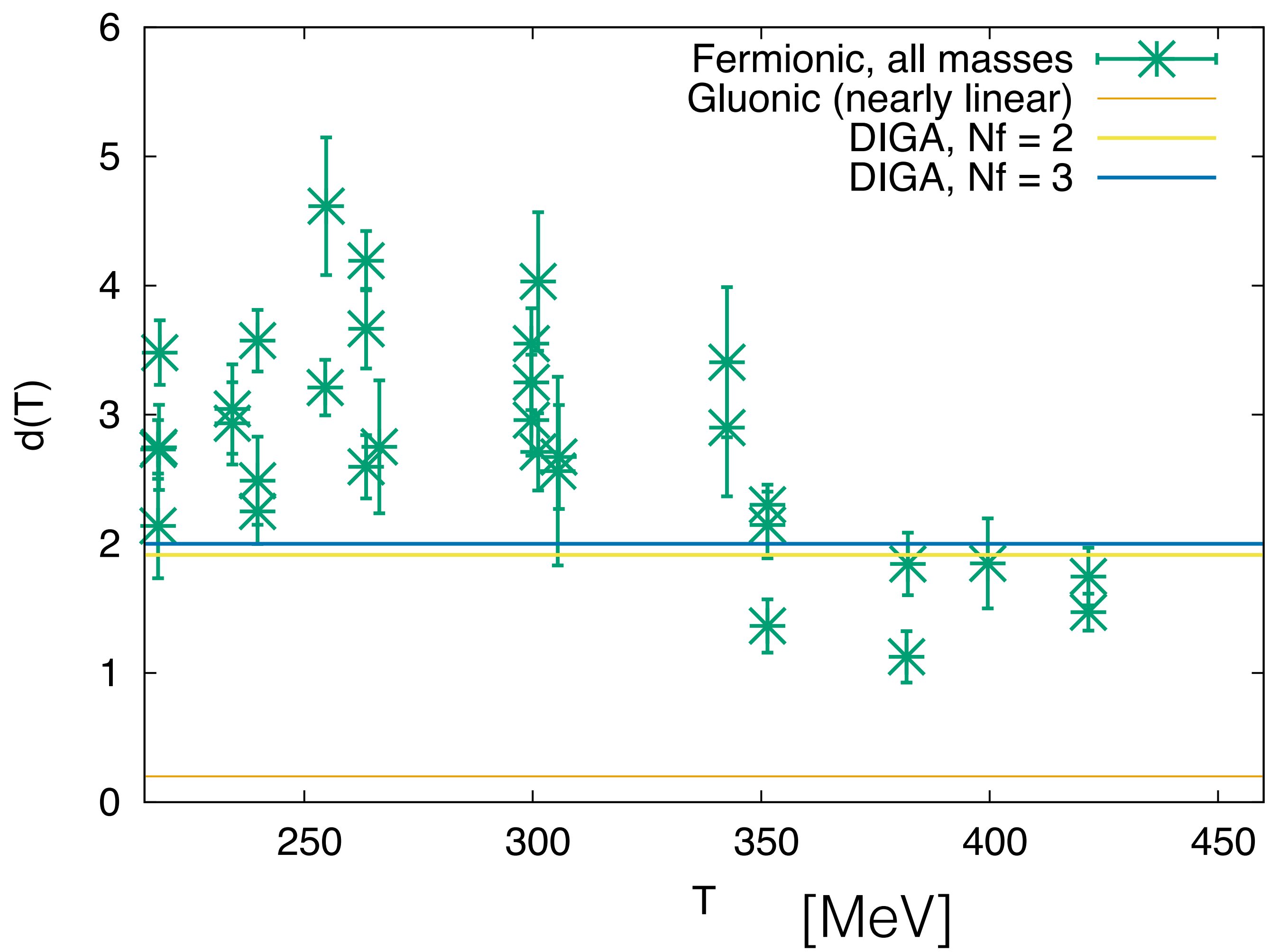
DIGA limit for T > 350 MeV




Consistent with Bonati et al.

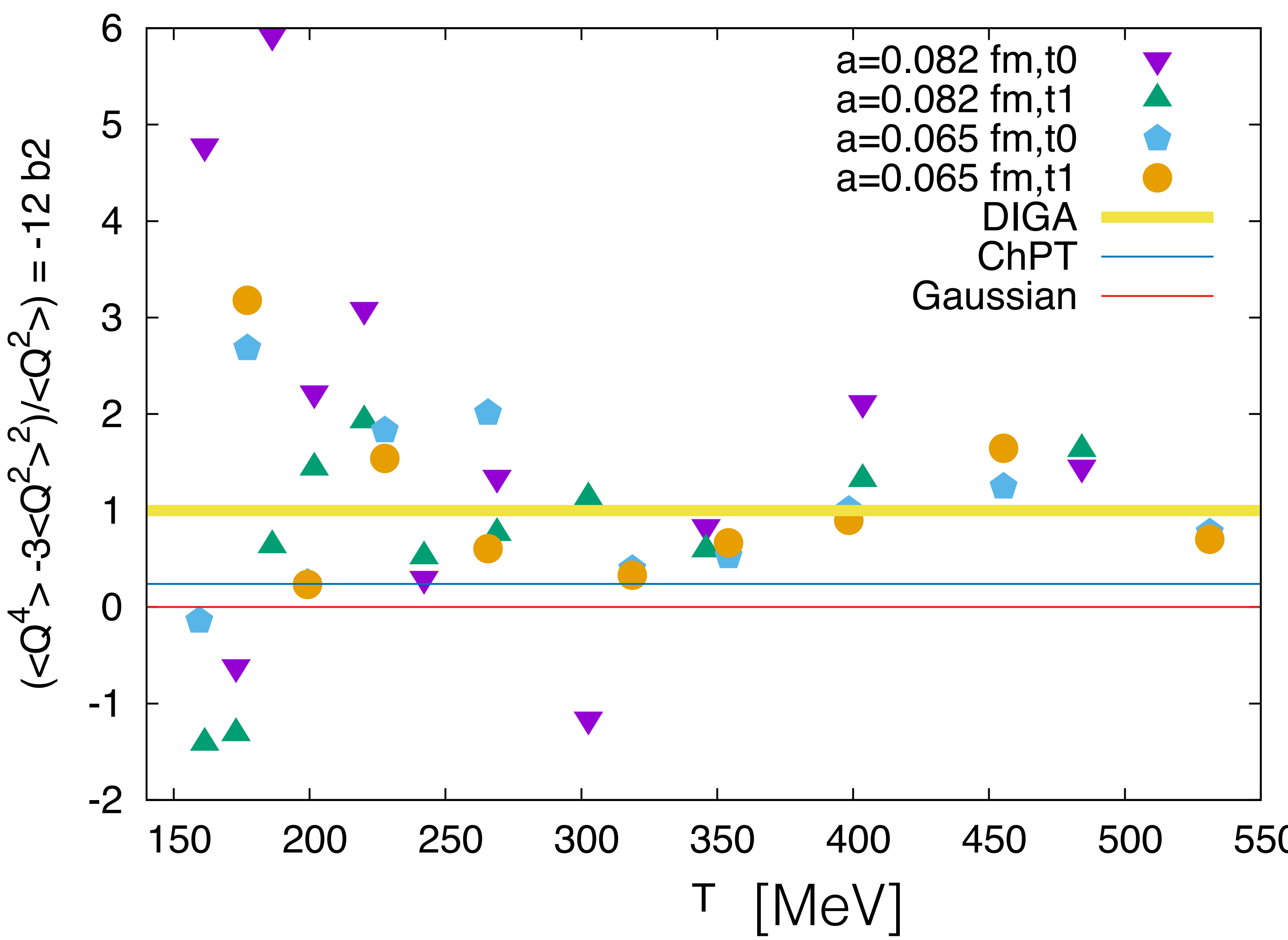
Effective exponent :

$$\chi_{top}^{1/4} = aT^{-d(T)}$$




Same DIGA onset seen in b2 ≈ 350 MeV

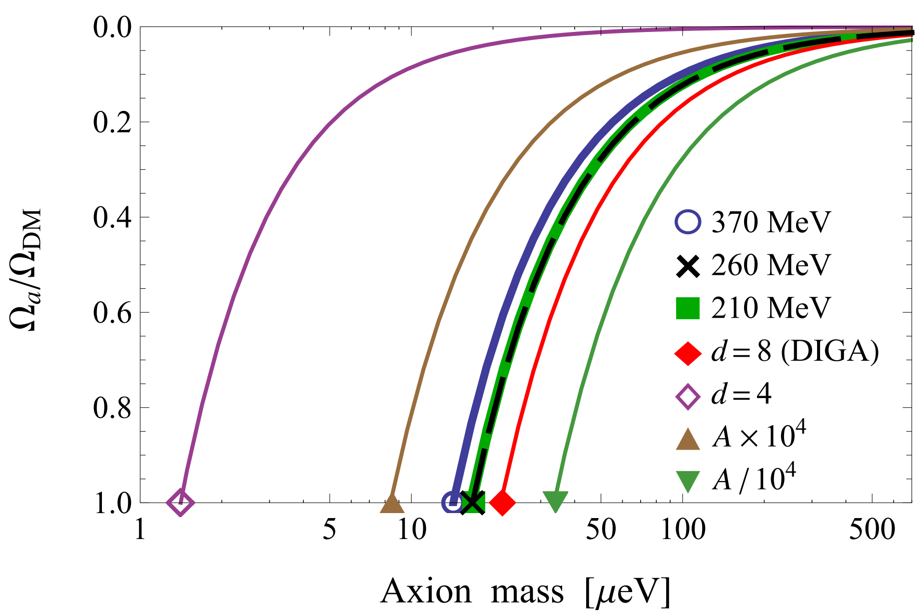
Results for $F(\theta)$ coherent with d(T)



# Parting remarks

*Topology in hot QCD is a rich field*
which is only recently **becoming truly quantitative:**
**there is room for improvement!**

*Topology constraints the **QCD axion mass***




*Questions:*
requirements from phenomenology?
axion potential?