On the role of bound states in dark matter freeze-out[1,2]

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What this talk is (isn't) about

Consider pair annihilation in the non-relativistic regime

e.g.
- $\gtrsim 10$ TeV WIMPs interacting with Z^0-exchange
- *co-annihilation partners of WIMPs charged under QCD*
- attractive interactions mediated by Higgs exchange
- massive dark sectors charged under extra U(1)
- quarkonium-like non-Abelian dark sectors
- bottom quarks generated in heavy ion collisions

$\Rightarrow$ Physics motivation: large mass naturally guarantees experimental non-detection, Boltzmann-suppressed number density, $1/M^2$-suppressed cross section, and chemical freeze-out.

$\Rightarrow$ Technical motivation: effective field theory methods (based on $\pi T \ll M$) make the problem theoretically tractable.

Example of a concrete model with the required properties [3]

χ = DM = Majorana fermion ($\sim$ bino)

η = DM' = "mediator" because couples to SM ($\sim$ stop)

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\rm SM} + \frac{1}{2}\bar{\chi}(i\not{\partial} - M)\chi \\ &+ (D_\mu \eta)^\dagger D^\mu \eta - (M + \Delta M)^2 \eta^\dagger \eta - \lambda_2 (\eta^\dagger \eta)^2 \\ &- \lambda_3 \eta^\dagger \eta H^\dagger H - y\, \eta^\dagger \bar{\chi} a_{\rm R} q - y^* \bar{q} a_{\rm L} \chi \eta \; . \end{aligned}$$

H = SM Higgs, q = SM quark (e.g. the top quark)

$0.01 \lesssim \lambda_3, |y|^2 \lesssim 1$ = portal couplings

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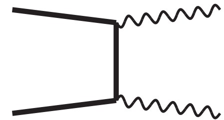
[3] e.g. M. Garny, A. Ibarra and S. Vogl, *Signatures of Majorana dark matter with t-channel mediators*, 1503.01500.

Text-book WIMP is in trouble

Lee-Weinberg equation[4] ($n =$ number density, $H =$ Hubble rate)

$$(\partial_t + 3H)n = -\langle \sigma v_{\text{rel}} \rangle \, (n^2 - n^2_{\text{eq}}) \, .$$



Start from equilibrium at $T \gtrsim M$; linearize around equilibrium:

$$n = n_{\text{eq}} + \delta n \, , \quad n^2 - n^2_{\text{eq}} \approx 2 n_{\text{eq}} \delta n \, .$$

Parametrize cross section:

$$\langle \sigma v_{\text{rel}} \rangle \equiv \frac{\alpha^2}{M^2} \, , \quad M \equiv M_{\text{DM}} \, .$$

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[4] B.W. Lee and S. Weinberg, *Cosmological Lower Bound on Heavy Neutrino Masses*, Phys. Rev. Lett. 39 (1977) 165.

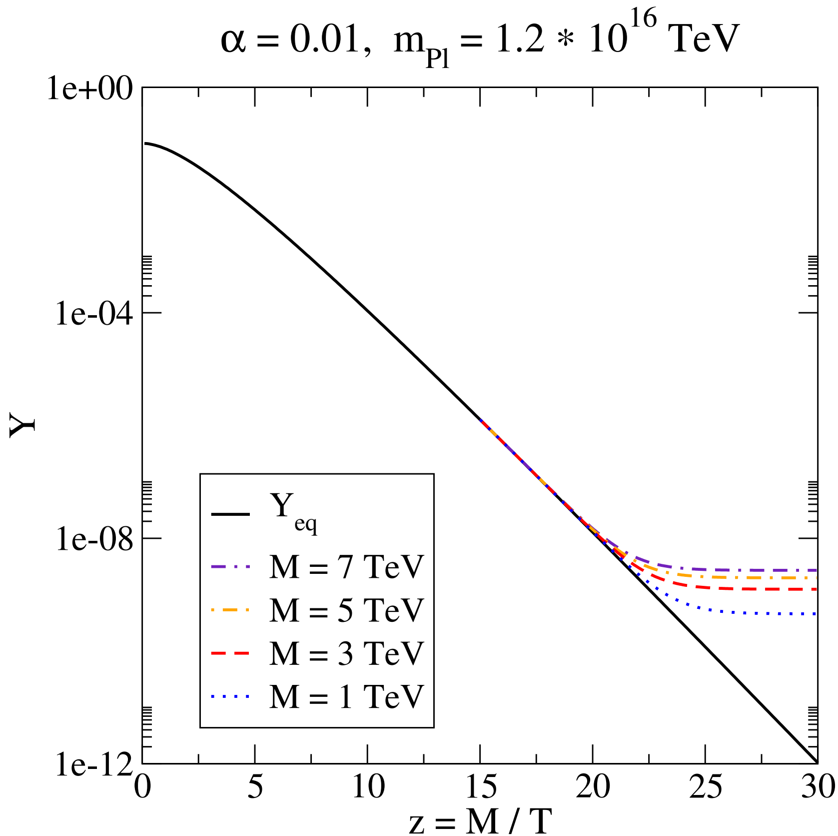
$$\Rightarrow \boxed{(\partial_t + 3H)n \approx -\frac{2\alpha^2 n_{\text{eq}}}{M^2}\,\delta n}\ .$$

The equilibrium number density is a known function of T, M:

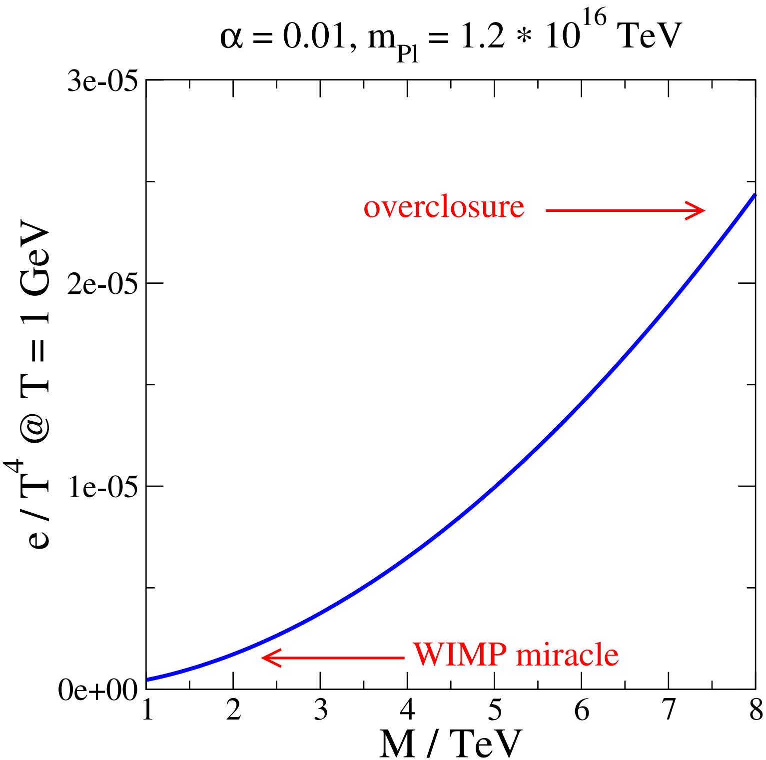
$$n_{\text{eq}} \propto \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{1}{e^{\sqrt{p^2+M^2}/T} \pm 1} \approx \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}\ .$$

The differential equation has a "thermal fixed point" at $n = n_{\text{eq}}$ but cannot keep close to it for $\alpha^2 n_{\text{eq}}/M^2 \ll H$.

Numerical solution shows a "freeze-out" ($Y \equiv n/s$):



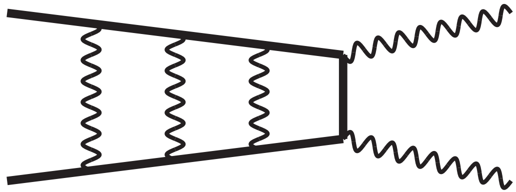
Final energy density ($e \equiv Mn$) compared with radiation $\sim T^4$:



LHC etc push up M, so there is a danger "overclosure".

Could increased $\langle \sigma v_{\text{rel}} \rangle$ help?

Much discussed:[5] "Sommerfeld effect"[6]:



$$\langle \sigma v_{\text{rel}} \rangle \longrightarrow \langle \sigma_{\text{tree}}\, v_{\text{rel}}\, S(v_{\text{rel}}) \rangle \ .$$

For attractive Coulomb-like interaction,

$$S(v_{\text{rel}}) \sim \frac{\alpha}{v_{\text{rel}}} \quad \text{for} \quad v_{\text{rel}} \lesssim \alpha \ .$$

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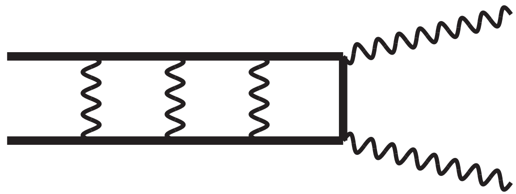
[5] e.g. J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, *Non-perturbative effect on thermal relic abundance of dark matter*, hep-ph/0610249; J.L. Feng, M. Kaplinghat and H.-B. Yu, *Sommerfeld Enhancements for Thermal Relic Dark Matter*, 1005.4678.

[6] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory*, Third Edition, §136; V. Fadin, V. Khoze and T. Sjöstrand, *On the threshold behavior of heavy top production*, Z. Phys. C 48 (1990) 613.

More recent:[7] bound-state contribution



$$M_{\text{bound}} = 2M - \Delta E \;\Rightarrow\; e^{-M_{\text{bound}}/T} > e^{-2M/T} \; .$$

This is quantum mechanics in a statistical background.

(Typically the dark sector contains several species, DM and DM', and perhaps only one of them forms bound states.)

[7] e.g. W. Detmold, M. McCullough and A. Pochinsky, *Dark Nuclei I: Cosmology and Indirect Detection*, 1406.2276; B. von Harling and K. Petraki, *Bound-state formation for thermal relic dark matter and unitarity*, 1407.7874; J. Ellis, F. Luo and K.A. Olive, *Gluino Coannihilation Revisited*, 1503.07142; K. Petraki, M. Postma and M. Wiechers, *Dark-matter bound states from Feynman diagrams*, 1505.00109.

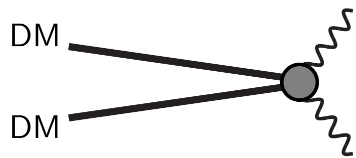
Some quantum statistical physics

Particles in the initial state: most energy is carried by mass.

$$E_{\text{rest}} \sim 2M \; , \quad E_{\text{kin}} \sim \frac{k^2}{2M} \sim T \; .$$

Particles in the final state: all energy is carried by momentum.

$$E_{\text{kin}} \sim 2k \sim 2M \quad \Rightarrow \quad \Delta x \sim \frac{1}{k} \sim \frac{1}{M} \ll \frac{1}{T} \; .$$



Therefore the "hard" annihilation process is *local*.[8]

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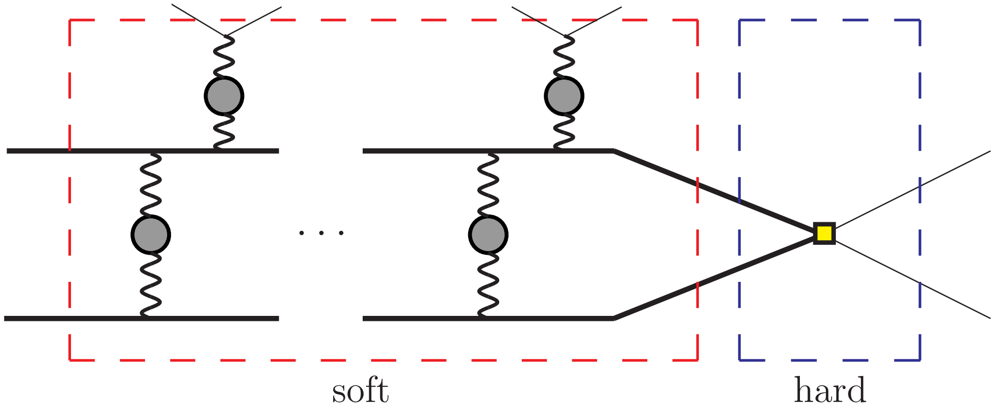
[8] e.g. G.T. Bodwin, E. Braaten and G.P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, hep-ph/9407339; L.S. Brown and R.F. Sawyer, *Nuclear reaction rates in a plasma*, astro-ph/9610256.

But before the annihilation there are “soft” initial-state effects:



“Debye screening”, “Landau damping”, ...

In particular $2 \to 2$ scatterings, absent in vacuum computations of bound-state dissociation, do play an important role.

A "linear response" analysis shows that this "inclusive" rate equals the thermal expectation value of the annihilation operator:

$$\langle \sigma v_{\text{rel}} \rangle \quad \sim \quad \frac{\alpha^2}{M^2 n_{\text{eq}}^2} \langle \mathcal{O}^\dagger(0) \mathcal{O}(0) \rangle_T$$

$$\equiv \quad \frac{\alpha^2}{M^2 n_{\text{eq}}^2} \frac{1}{Z} \sum_{m,n} e^{-E_m/T} \langle m | \phi^\dagger \phi^\dagger \overbrace{|n\rangle\langle n|}^{\Rightarrow \mathbb{1}} \phi\phi | m \rangle \; .$$

Here $|m\rangle$ are eigenstates containing a DM-DM pair, and $\phi\phi$ annihilates the DM-DM pair.

How to estimate $\langle \mathcal{O}^\dagger(0)\mathcal{O}(0)\rangle_T = \langle \phi^\dagger\phi^\dagger\phi\,\phi\rangle_T$ in practice?

(i) as it is, with lattice NRQCD (see later);

(ii) within perturbation theory, it is preferable to derive a "spectral representation" for this local expectation value.

The 2-body problem can be reduced to a 1-body problem:

$$E_m \quad =: \quad E' + \underbrace{\left[2M + \frac{k^2}{4M}\right]}_{\text{center-of-mass energy}} \quad .$$

Converting $\sum_m$ into integrals over E' and k and carrying out the integral over k we are left with

$$\langle \mathcal{O}^\dagger(0)\mathcal{O}(0)\rangle_T = e^{-2M/T}\left(\frac{MT}{\pi}\right)^{3/2}\int_{-\Lambda}^{\infty}\frac{\mathrm{d}E'}{\pi}e^{-E'/T}\rho(E') \ .$$

The "spectral function" $\rho(E')$ represents the solution of a Schrödinger equation for a Green's function.

$$\begin{aligned} \left[H_T - i\,\Gamma_T(r) - E'\right] G(E';\mathbf{r},\mathbf{r}') &= \delta^{(3)}(\mathbf{r}-\mathbf{r}')\ , \\ \lim_{\mathbf{r},\mathbf{r}'\to\mathbf{0}} \mathrm{Im}\, G(E';\mathbf{r},\mathbf{r}') &= \rho(E')\ . \end{aligned}$$

Here the Hamiltonian has a standard from

$$H_T = -\frac{\nabla_{\mathbf{r}}^2}{M} + V_T(r)\ , \quad r = |\mathbf{r}|\ ,$$

whereas $-i\Gamma_T(r)$ accounts for real scatterings with the plasma.

In practice we are doing rather simple computations.

$$V_T(r) = -\underbrace{\frac{g_s^2 C_F}{4\pi}\Big[m_D}_{\text{“Salpeter correction”}} + \underbrace{\frac{\exp(-m_D r)}{r}}_{\text{“Debye screening”}}\Big],$$

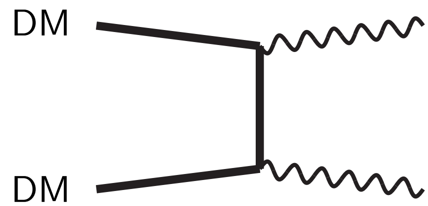
$$\Gamma_T(r) = \underbrace{\frac{g_s^2 C_F T}{2\pi}\int_0^\infty \frac{dz\, z}{(z^2+1)^2}\left[1 - \frac{\sin(z m_D r)}{z m_D r}\right]}_{\text{“Landau damping”}}.$$

Relation to indirect non-detection

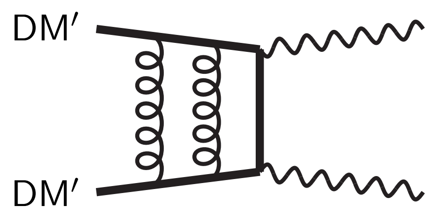
Why is cosmology different from the present day?

Long ago: $t \sim 10^{-12}$ s, $T \sim 100$ GeV.

DM annihilation:



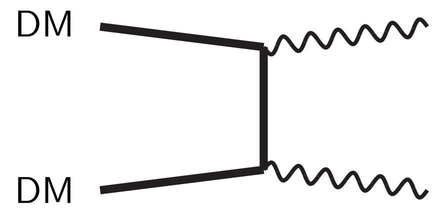
DM' annihilation:



DM $\leftrightarrow$ DM' is in thermal equilibrium $\Rightarrow$ annihilation can proceed through the heavier DM' channel if this is more efficient.

Today: $t \sim 10^{17}$ s, $T \ll$ eV.

DM annihilation is active in galactic centers, but with small $\langle \sigma v_{\text{rel}} \rangle$ (e.g. p-wave).

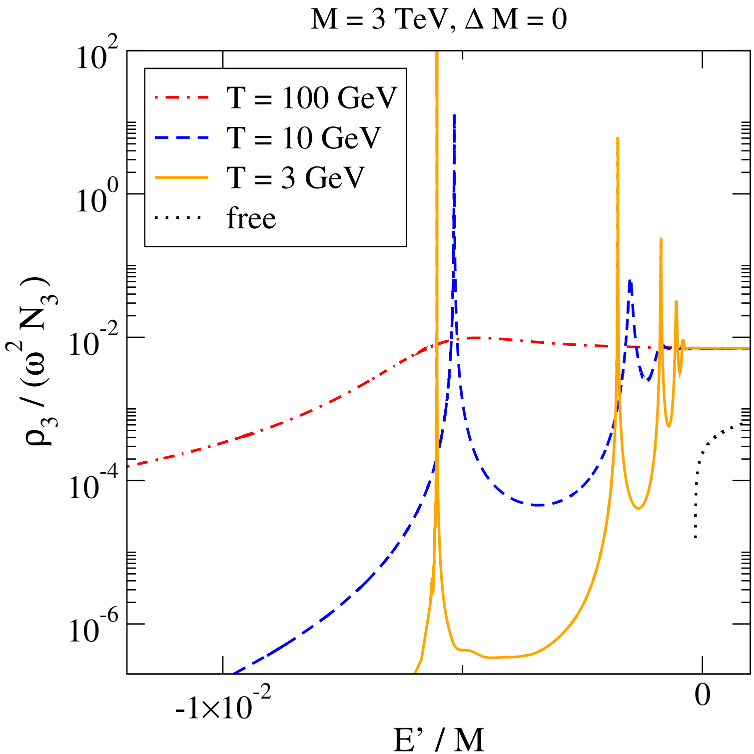


DM' decayed long ago, and plays no practical role in cosmology (however it can be searched for at the LHC).

Denote $\Delta M \equiv M_{\text{DM'}} - M$

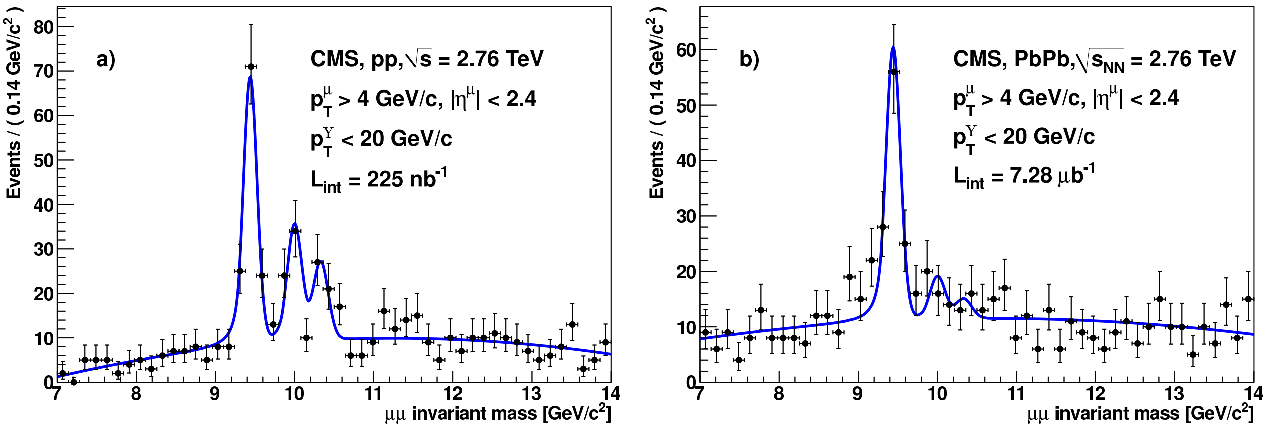
$\Rightarrow \ \epsilon_{\text{min}} \leq \Delta M / M \ll 1$ leads to interesting effects.

The DM' bound-state spectrum is T-dependent.



A nice relation to heavy ion collision experiments

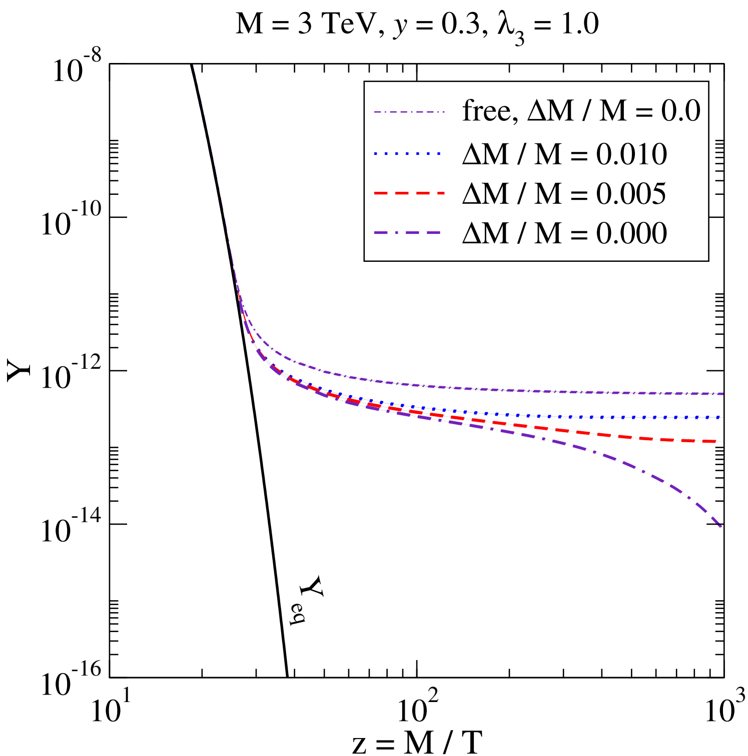
S. Chatrchyan et al. [CMS Collaboration], *Suppression of excited Υ states in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV*, Phys. Rev. Lett. 107 (2011) 052302 [1105.4894].



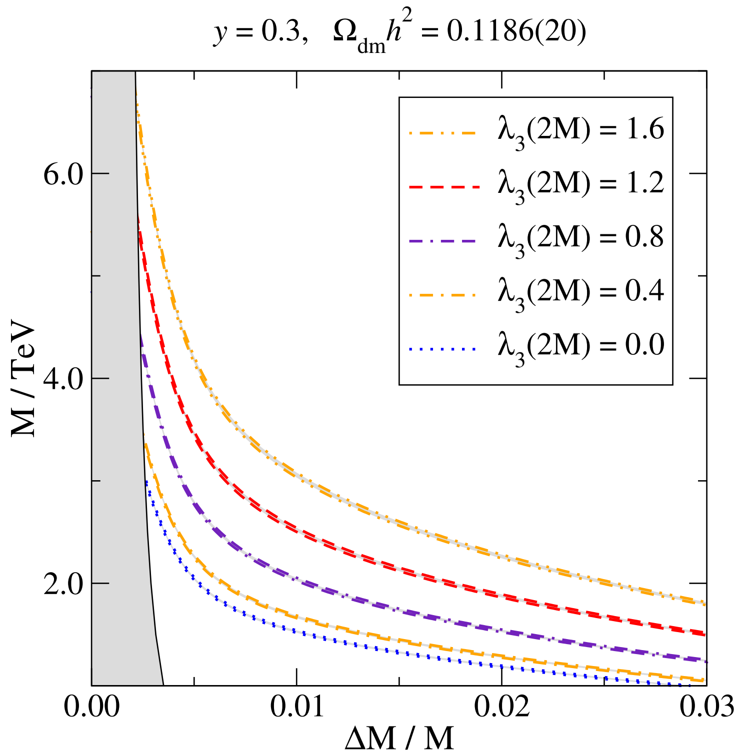
This follows a general pattern predicted theoretically.[9]

[9] e.g. F. Karsch, D. Kharzeev and H. Satz, *Sequential charmonium dissociation*, Phys. Lett. B 637 (2006) 75 [hep-ph/0512239].

If $\Delta M/M$ is too small, late times become problematic.

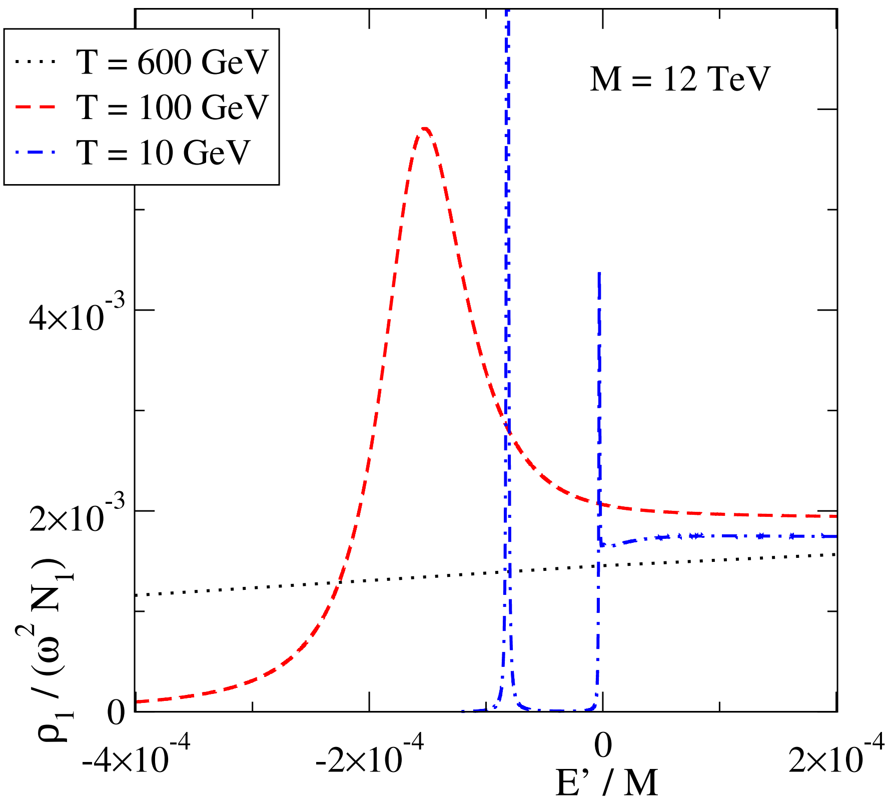


With small but not too small $\Delta M/M$, large M is possible.

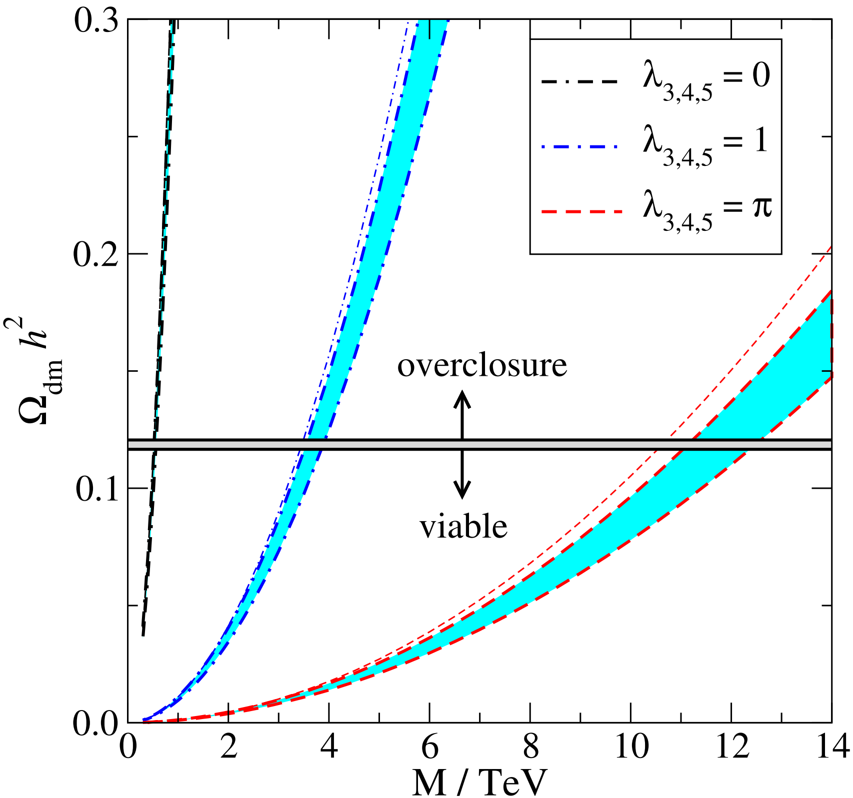


Extensions

The same can happen with weak interactions (2HDM)



Effects are then on the 10% level



$\langle \mathcal{O}^\dagger(0)\mathcal{O}(0)\rangle_T$ can be measured with lattice NRQCD

$\Rightarrow$ denote by $\bar{S}_i$ enhancement factor over pQCD in channel i, G^θ = propagator, α, γ = colour indices, i, j = spin indices

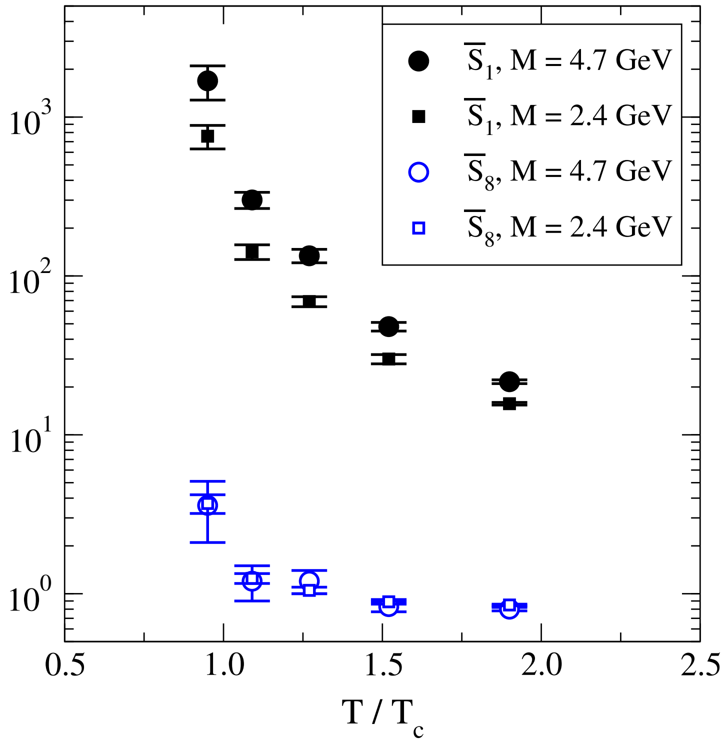
$$P_1 \equiv \frac{1}{2N_c} \mathrm{Re} \left\langle G^\theta_{\alpha\alpha;ii}(\beta, \mathbf{0}; 0, \mathbf{0}) \right\rangle ,$$

$$P_2 \equiv \frac{1}{2N_c} \left\langle G^\theta_{\alpha\gamma;ij}(\beta, \mathbf{0}; 0, \mathbf{0}) G^{\theta\dagger}_{\gamma\alpha;ji}(\beta, \mathbf{0}; 0, \mathbf{0}) \right\rangle ,$$

$$P_3 \equiv \frac{1}{2N_c^2} \left\langle G^\theta_{\alpha\alpha;ij}(\beta, \mathbf{0}; 0, \mathbf{0}) G^{\theta\dagger}_{\gamma\gamma;ji}(\beta, \mathbf{0}; 0, \mathbf{0}) \right\rangle$$

$$\Rightarrow \quad \bar{S}_1 = \frac{P_2}{P_1^2} , \quad \bar{S}_8 = \frac{N_c^2 P_3 - P_2}{(N_c^2 - 1) P_1^2} .$$

Large effect confirmed in the attractive "singlet" channel



Summary

• Apart from model uncertainties, generic dark matter studies contain theoretical uncertainties.

• Both quantum-mechanical effects (bound states, multiple interactions) and statistical physics phenomena (Debye screening, $2 \to 2$ scatterings on plasma particles) may play a role.

• For instance, a strongly interacting DM' may increase $\langle \sigma v_{\text{rel}} \rangle$. The below-threshold ("bound-state") contribution is typically at least as large as the Sommerfeld effect.

• Model-specific studies are needed for definite conclusions.